Neutron Stars: Structure

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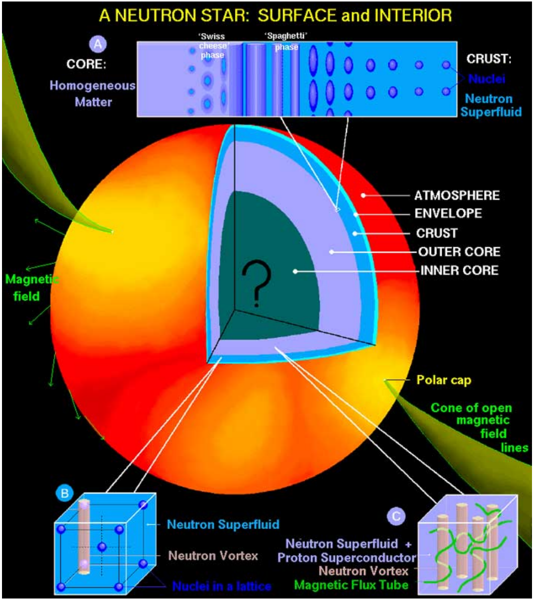
Neutron star structure: why it is important!

- Neutron stars relevant to GW emission in *two* ways:
- **GW emission from NS-X binary (X=NS or BH) (Toni's lecture)**:
  - For late stage in inspiral, finite-size effects potentially detectable
  - Structure information encoded in 'Love numbers'
  - EoS also affects outcome of merger
- **GW emission from individual star (Leonardo's lecture)**
  - Produced by mountain or excitation of normal modes of oscillation
  - Star may or may not be in a binary
  - Emission extremely sensitive to stellar structure
- Observed in radio, optical, X-ray, Gamma-ray
  $\Rightarrow$ **ideal objects for multi-messenger astronomy**

Origin and statistics

- Neutron stars produced in core collapse
- Progenitor star must lie in mass range $\sim 8 \rightarrow \sim 20 M_{\odot}$
- On *evolutionary grounds*, expect *mimumum mass* $\sim 1.2 M_{\odot}$
- Birth rate $\sim$ 1 per century $\Rightarrow \sim 10^9$ in Galaxy
- $\Rightarrow$ closest $\sim$ 10's of parsecs from Earth

Anatomy of a neutron star



Picture credit: Dany Page

Some key numbers

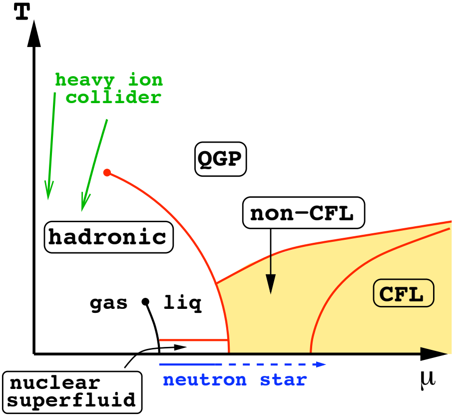
- Mass $M \sim 1.4 M_\odot \sim 2.8 \times 10^{33}$ g, radius $R \sim 10^6$ cm
- Density $\rho \sim 10^{15}$ g cm^{-3}; compare with $\rho_{\rm nuc} \approx 2.7 \times 10^{14}$ g cm^{-3}, $\Rightarrow \rho/\rho_{\rm nuc} \gtrsim 3$
- Fastest observed rotation rates $\nu \sim 700$ Hz $\Rightarrow \frac{v_{\rm equatorial}}{c} \sim 0.15$
- Compactness $M/R \sim 0.2$
- Temperature $T_{\rm core} \sim 10^8$ K for younger stars $\Rightarrow T/T_{\rm Fermi} \sim 10^{-4}$
- Magnetic field strength $B \sim 10^9$—10^{15} G; 10^{12} G for 'typical' star $\Rightarrow$
$$\frac{B^2/(8\pi)}{\rho} \sim 10^{-13}$$

The Equation of State

- Basic ingredient in calculating structure is the *Equation of State* (EoS)
- Realistic NS model will contain multiple species: neutrons, protons, electrons, muons, hyperons ...
- For calculating gross features, EoS often simplified to relation $P = P(\rho, T)$.
- ‘Low temperature’ allows approximation $P = P(\rho)$, a ‘barotropic’ EoS.
- Modelling of the EoS is a large and vigorous industry

The Big Picture: The QCD phase diagram

Neutron stars *may* harbour exotic states of matter (and may not even be ‘neutron stars’!):



Picture credit: Alford et al

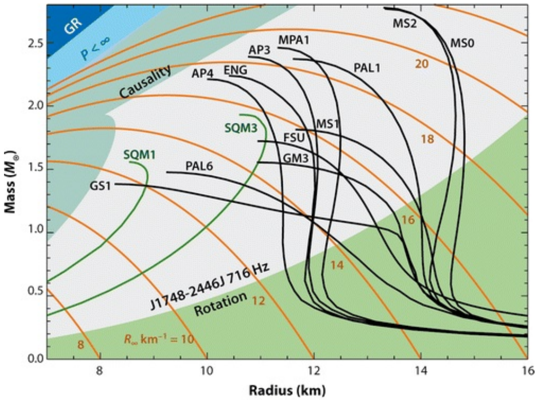
Calculating the equilibrium structure

- In Newtonian theory, find equilibrium structure by solving equations of force balance for a self-gravitating sphere:

$$
\begin{aligned}
0 &= -\frac{1}{\rho}\nabla_a P - \nabla_a \Phi, & (1)\\
\nabla^2 \Phi &= 4\pi G \rho & (2)\\
P &= P(\rho) & (3)
\end{aligned}
$$

- Combination of these equations into one ODE gives *Lane-Emden equation*.
- Relativistic analogue is the *Tolman-Oppenheimer-Volkov* (TOV) equations.
- For given EoS, get a one-parameter family of solutions . . .

The Mass-Radius Diagram



Lattimer JM. 2012.
Annu. Rev. Nucl. Part. Sci. 62:485–515

The maximum mass

- General relativity sets a maximum mass for any given EoS.
- Highest known accurate mass is that of PSR J1614-2230, with $M = 1.97 \pm 0.04 M_\odot$ (Demorest et al 2010)
- Some evidence for $M = 2.4 M_\odot$ for the 'Black Widow' pulsar, but systematics difficult to quantify.

Bounds on density, compactness, . . .

- Can identify other 'excluded regions' of the M-R plane.
- Example: **Compactness:**
  - Must have $M < 2R$ or else system is a black hole!
- Example: **causality of the EoS:**
  - Sound speed must be sub-luminal:

$$\frac{dP}{d\rho} < c^2$$

- Example: **Rapid rotation:**
  - Fastest spinning pulsar is J1748 $- 2446ad$, at 716 Hz
  - This must be less than the rotational break-up velocity: $\Omega^2_{\text{critical}} R < GM/R^2$.

The crust

- Crust consists of a crystalline lattice, divided into layers.
- Atomic number Z and mass number A increase with depth, with deeper nuclei being more neutron-rich.

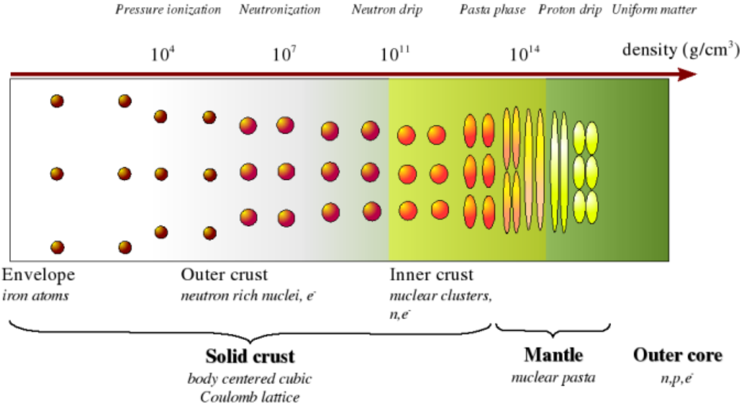


Figure: Chamel & Haensel (2008)

The crust cont...

- Crust has a non-zero shear modulus $\mu \sim 10^{29}$ dyn cm^{-3}; implies elasticity weak compared to gravity:

$$\frac{\mu}{GM\rho/R} \sim 10^{-6}$$

  but can still be vital for astrophysics (starquakes & mountains; Leonardo's lecture).

- At densities greater than *neutron drip* $\rho_{\rm ND} \sim 4 \times 10^{11}$ g cm^{-3}, neutrons not completely confined to nuclei; they form a neutron superfluid, *coexisting* with crust. Believed to play crucial role in pulsar glitches.
- Deep in crust, topology of nuclei changes, giving *pasta phases*.

The magnetosphere

- Magnetic field extends outside of star.
- Rotating magnetised sphere induces E-fields $\Rightarrow$ particles ripped from surface of star, and accelerated along field lines.
- Within *light cylinder*, defined by $c = \Omega\varpi$, field lines closed; outside, open.
- Electromagntic radiation produced by particle acceleration along these field lines.

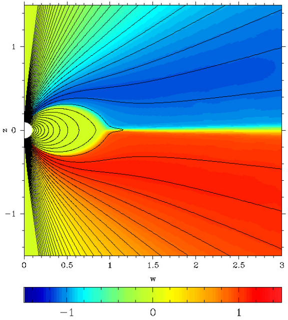


Figure: Komissarov (2006)

Summary

- Assuming neutron star matter is hadronic, gross features reasonably well understood.
- Balance of gravity verses pressure gives overall size and sets many key features.
- ‘Finer details’ important in explaining observations, e.g. superfluidity, crustal elasticity, and magnetic fields.
- These will form focus of my second lecture.