

Connection between Gravitational Waves and Gamma-ray Bursts

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Outline

Triggered GW searches

- Motivations

- Sensitivity improvements

Gamma-ray bursts

- Astrophysics

- GW emission

LIGO/Virgo

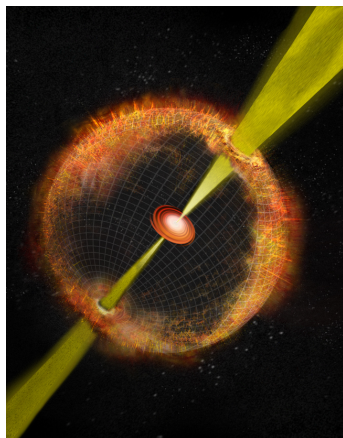
- Methods

- Results

- Prospects

GRB+GW as astrophysical probes

Summary



Credit: Bill Saxton, NRAO/AUI/NSF

Motivations for triggered GW searches

● Astrophysics

▶ We know about:

- Gamma-ray bursts
- Soft gamma repeaters
- Supernovae
- Pulsars

▶ GWs give information on core mass dynamics

● Detector noise

- ▶ Instrumental/environmental noise is limiting
- ▶ Any information helps removing some noise

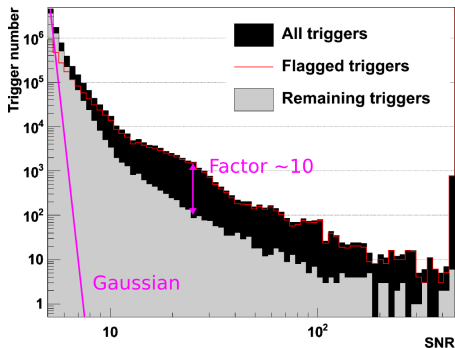
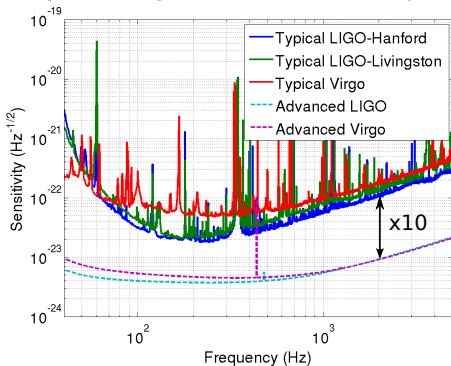
Example of sensitivity improvement unrelated to GRBs

- Young pulsars (neutron stars)
 - ▶ Crab (SN 1054)
 - ▶ Vela (SN $\sim 10^4$ yr ago)
 spin frequency is precisely observed in radio
- The rotation period is decreasing
→ loss of rotational energy
- LIGO 2005-2007: less than 2% of Crab energy loss is due to GW emission (Abbott et al., 2010)
- Virgo 2009-2010: less than 40% of Vela energy loss is due to GW emission (Abadie et al., 2011b)
- Without any radio observation the limits on energy loss higher by $\sim 10^2 - 10^3$ (Abadie et al., 2011a)

⇒ EM observation enhance GW searches sensitivity



Improving GW sensitivity

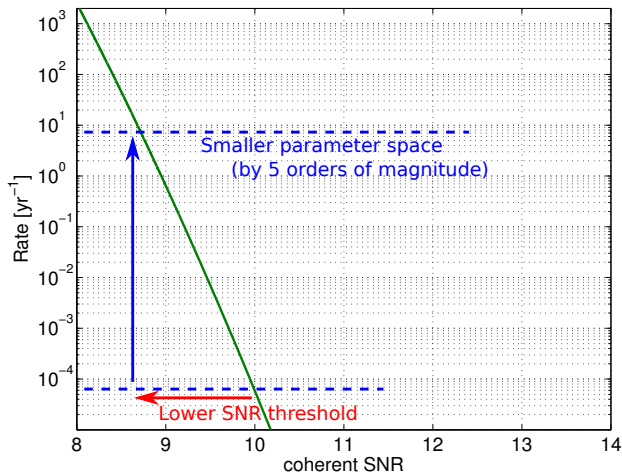


smallest observable GW amplitude $\propto \sqrt{S(f)} \times \text{SNR}_{\text{threshold}}$

- Astrophysical triggers, GW models, etc ... changes search parameter space

⇒ $\text{SNR}_{\text{threshold}}$ depends on the search hypothesis

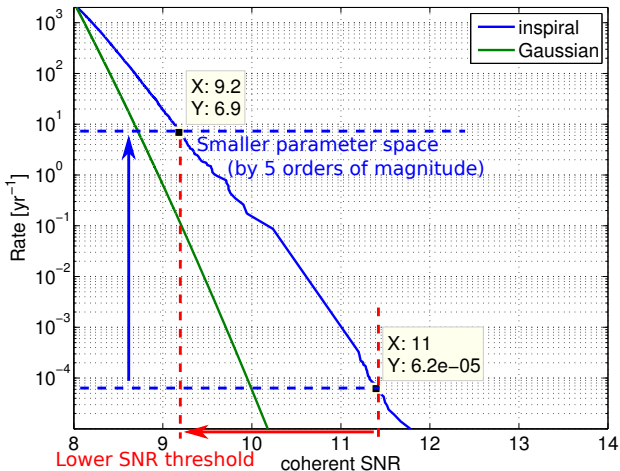
Triggered search in Gaussian noise



Localization in time: $\frac{\text{few minutes}}{\text{few months}} \sim 10^{-5}$

⇒ Improves sensitivity by 15%, 50% in volume

Well cleaned real noise (inspiral + χ^2 test)

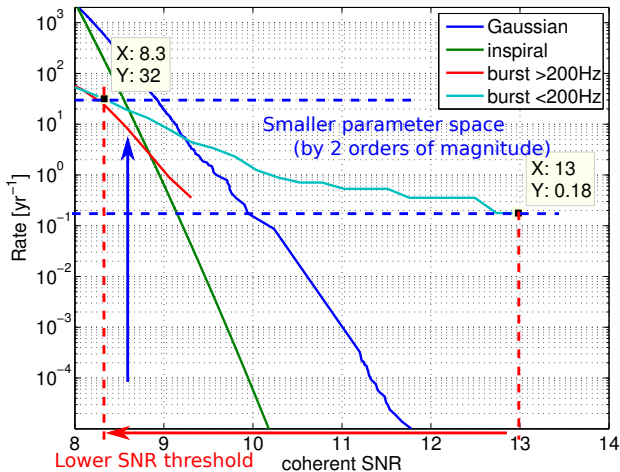


Localization in time:

$\frac{\text{few minutes}}{\text{few months}} \sim 10^{-5}$

⇒ Improves sensitivity by 20%, 70% in volume

Real data, no GW model



Localization in time:

$$\frac{1 \text{ day}}{\text{few months}} \sim 10^{-2}$$

\Rightarrow Improves sensitivity by 60%, factor 4 in volume

Astrophysical trigger

⇒ reduction in search parameter space

⇒ gain in sensitivity

- especially for non-Gaussian data

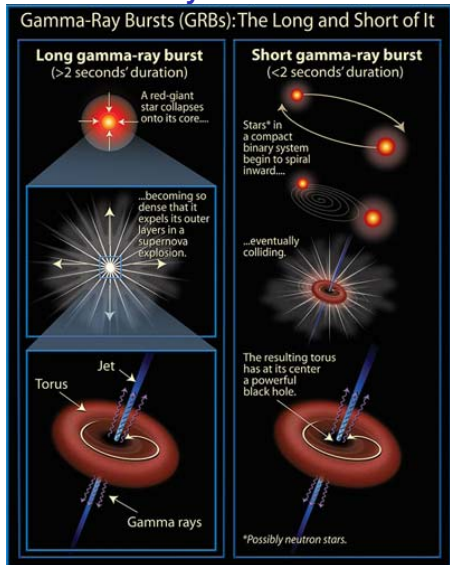
Practical example on GRB case

Astrophysical inputs & analysis strategy

Goal: Find GW associated with GRBs

- What to look for?
 - ▶ GW signal waveform
 - ▶ GW signal amplitude
 - ▶ GW signal polarization
- Where to look for?
 - ▶ GRB sky localization
 - ▶ Timing between GRB trigger and GW trigger
 - ⇒ Understand both EM and GW emission
- Is it worthwhile to search?
 - ▶ GRB progenitors distance distribution
 - ▶ Is it better than blind (all-sky, all-time) search?

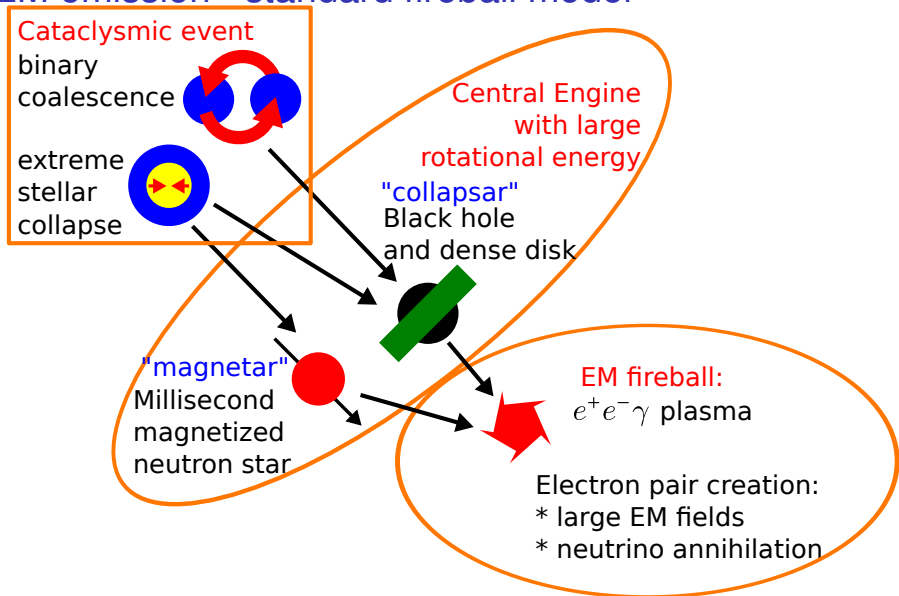
Gamma-ray burst models



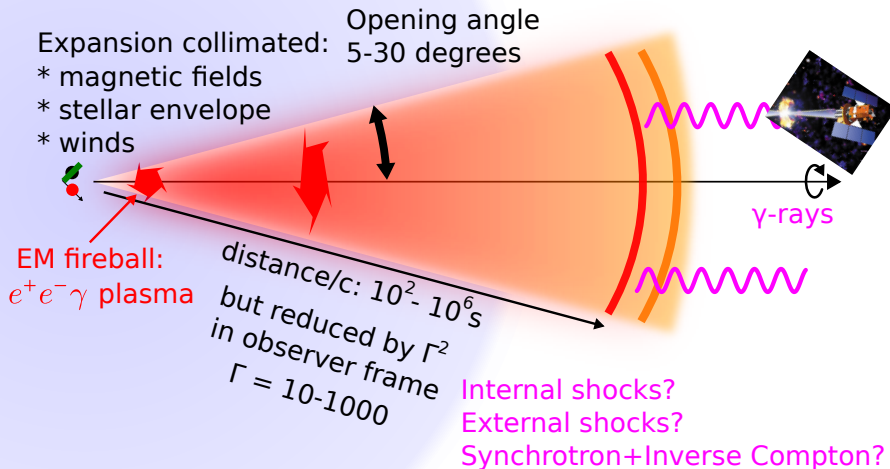
credit: Ute Kraus

- Long GRBs
- ⇒ Massive rapidly spinning star collapse and explosion
- Short GRBs
- ⇒ Coalescence of a neutron star and a compact object
 - ▶ small fraction is actually neutron star quakes ($\lesssim 15\%$)
- GWs see the core of the mass distribution dynamics
- Measured gamma emission:
 $\sim 10^{51} \text{ erg} = 10^{-3} M_{\odot} c^2$

EM emission - standard fireball model

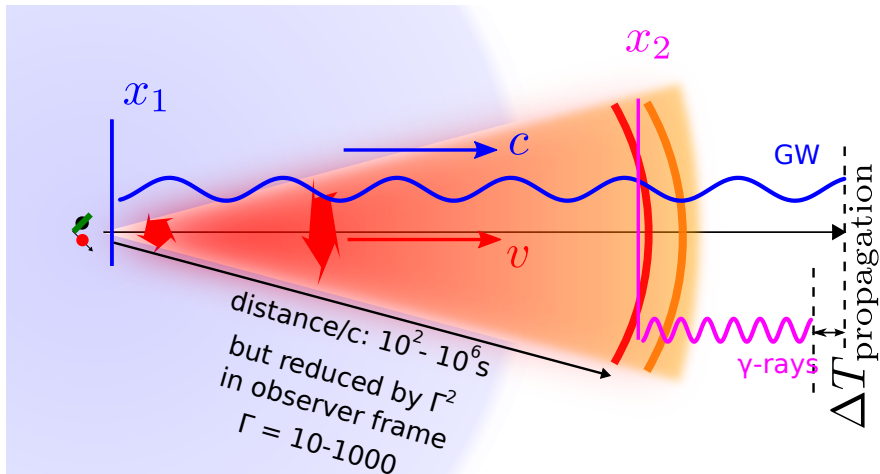


EM emission - standard fireball model



Relativistic contraction

$$\Gamma = \frac{1}{\sqrt{1 - (v/c)^2}}, \quad \Delta T_{\text{propagation}} = \frac{x_2 - x_1}{v} - \frac{x_2 - x_1}{c} \underset{\Gamma \gg 1}{\approx} \frac{x_2 - x_1}{2c\Gamma^2}$$



Gravitational source quadrupolar approximation

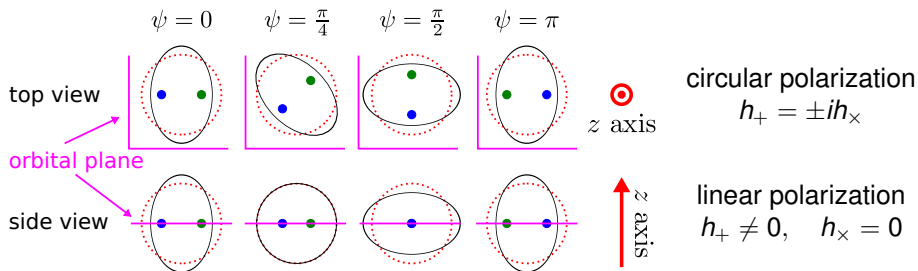
Approximation: far field + slow moving source

- Mass distribution quadrupolar moment

$$I_{ij} = \int (x_i x_j - \frac{1}{3} \delta_{ij} \delta_{km} x^k x^m) \rho(x) d^3 x.$$

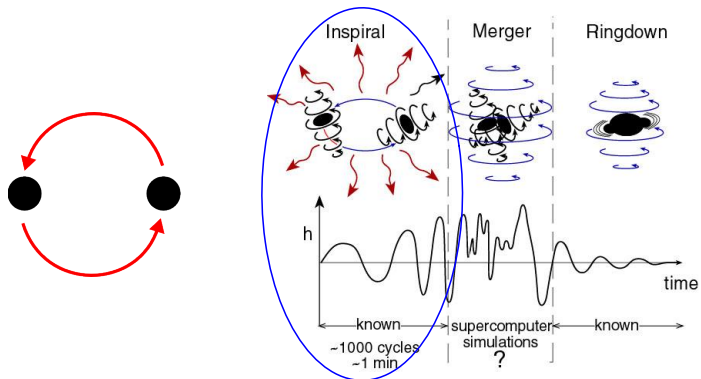
- Source of gravitational waves

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \longrightarrow h_{jk}^{TT} = \frac{2G}{c^4} \frac{1}{r} \underbrace{P_{jkmn}}_{\text{projection}} \ddot{i}^{mn}(t - \frac{r}{c}),$$



GW emission - coalescence scenario

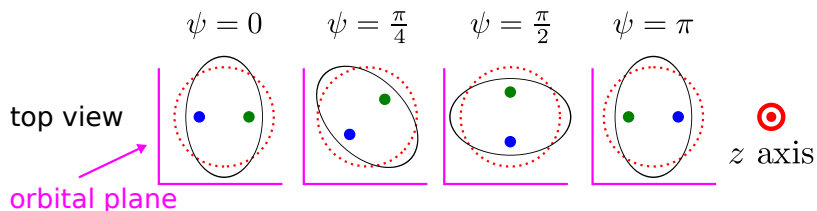
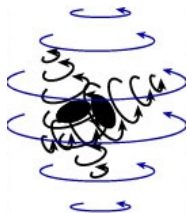
- Binary system of two compact objects (NSNS or NSBH)



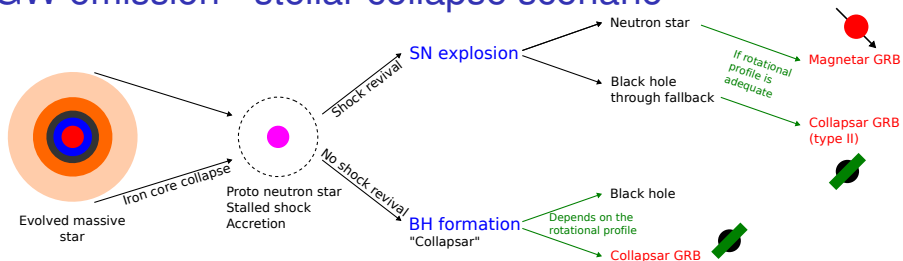
- Lose energy by GW radiation
- GW emission enters sensitive band ($\gtrsim 50$ Hz) < 50 s before coalescence
- GW at merger, ringdown \rightarrow high frequency ($M_{\text{BH}} \lesssim 20 M_{\odot}$) \rightarrow low SNR

GW emission - coalescence scenario

- GRB central engine formed in $\lesssim 1$ s
- ⇒ merger $[-1, 0]$ s prior to jet launch
- 1-2 second to produce γ -rays
- ⇒ Inspiral ends $\simeq [-3, 0]$ s prior to GRB
- GRB observed \rightarrow rotation axis points at observer
- ⇒ **GW well known** and **circularly polarized**
up to inclination of $60^\circ \rightarrow$ loose constraint
(jet opening angle $\lesssim 30^\circ$)



GW emission - stellar collapse scenario



- Magnetar central engine / Proto neutron star

- ▶ bar mode instability in the star
- ▶ neutron star core fragmentation

- Black hole and accretion disk

- ▶ Disk fragmentation
- ▶ Disk precession

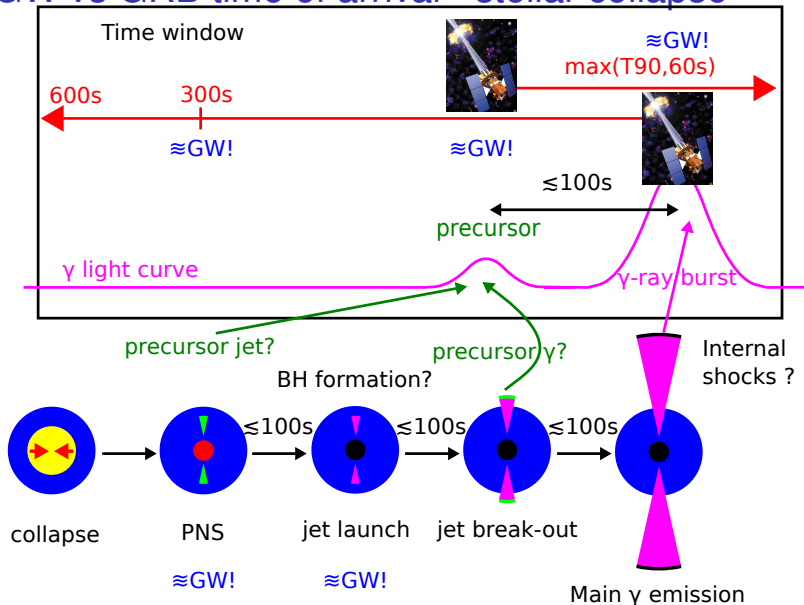
⇒ **circular polarization** along rotation axis

⇒ Emitted GW energy $\lesssim 10^{-2} M_{\odot} c^2$

- Other emission mechanism but no prospects for extra-galactic reach

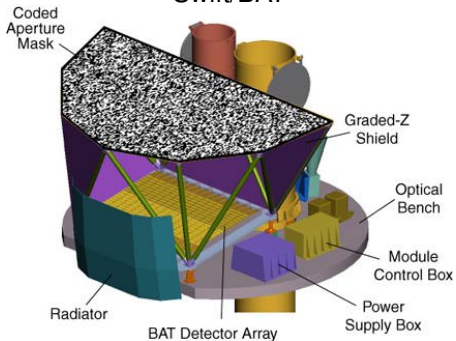
- ▶ Out of frequency band (Neutrino, normal modes, ...)
- ▶ Too small amplitude (Core bounce, SASI, ...)

GW vs GRB time of arrival - stellar collapse



GRB sky localization - two technologies

Swift/BAT



FoV $\sim 10\%$ of the sky
errors $\lesssim 0.3^\circ$

Fermi/GBM



$\sim 70\%$ of the sky
errors $\lesssim 5^\circ$

Astrophysical inputs summary

● Short GRBs

- ▶ GWs: inspiral waveform
- ▶ Inspiral ends a few seconds before start of GRB

⇒ parameter space reduced by
 $\sim 30 \times 6 \text{ s}/1 \text{ year} \simeq 10^{-5}$
 ⇒ at least factor 1.2 sensitivity gain

● Long GRBs

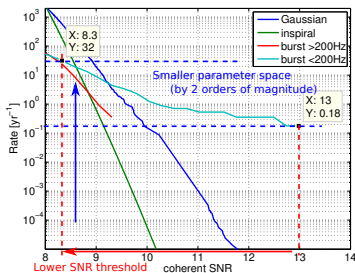
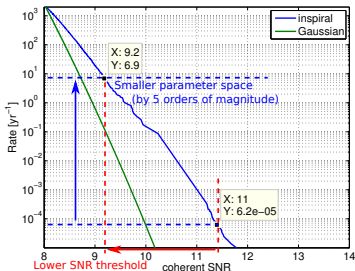
- ▶ GWs: circularly polarized
- ▶ Up to a few minutes before GRB trigger

⇒ parameter space reduced by
 $\sim 400 \times 660 \text{ s}/1 \text{ year} \simeq 10^{-2}$
 ⇒ at least factor 1.6 sensitivity gain

- ▶ GW amplitude highly uncertain

● All GRBs

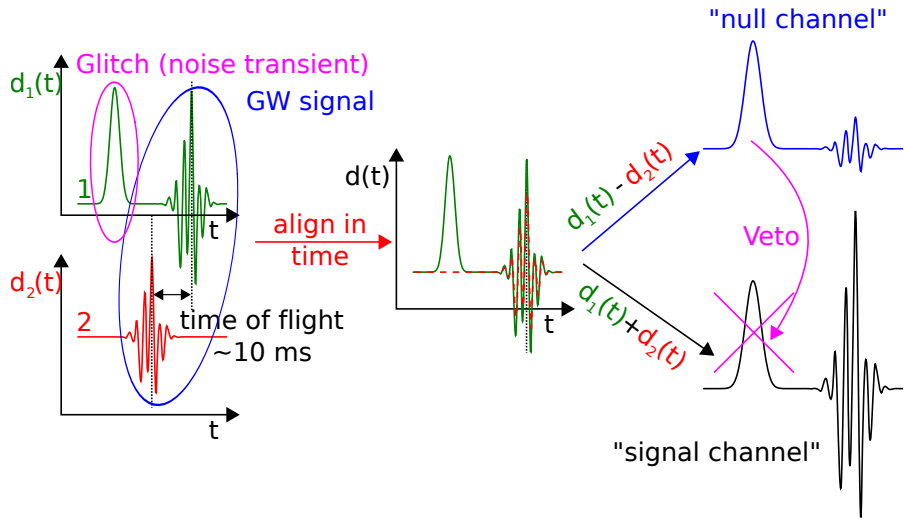
- ▶ Located on the sky with
 - $\sim 0.3^\circ$ precision ($\sim 25\%$)
 - $\sim 5^\circ$ precision ($\sim 75\%$)



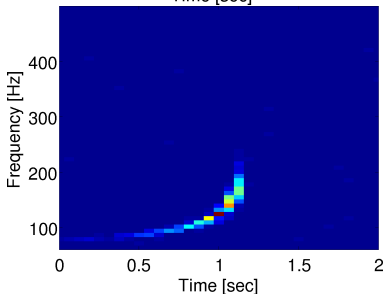
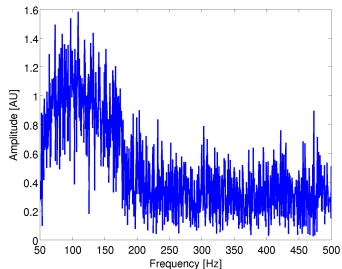
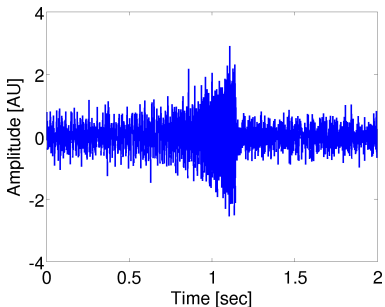
Two complementary searches

- Broad in scope – covers most possibilities
 - ▶ “burst” searching method – any signal shapes
 - ▶ Limited to 60 – 500 Hz band, $\lesssim 1$ s duration
 - ▶ Assumes circular polarization
 - ▶ **Loose** time coincidence between γ -rays and GW
 $T_\gamma - T_{\text{GW}} \in [-600, \max(T_{90}, 60)]$ s
- Focused on short GRBs – binary coalescence
 - ▶ Inspiral waveform templates, NS-NS and NS-BH
 - ▶ **Tight** time coincidence between γ -rays and GW
 $T_\gamma - T_{\text{GW}} \in [-5, 1]$ s
 - ▶ More sensitive to inspiral signals by factor ~ 2
- GW data combined coherently in both searches
- (Abadie et al., 2012b)

Coherent GW analysis



Excess wrt Gaussian noise → Time frequency maps

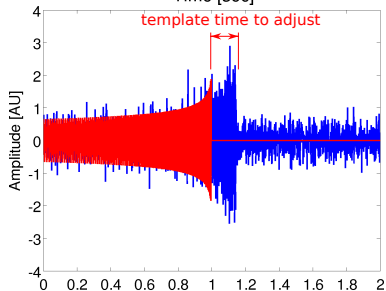
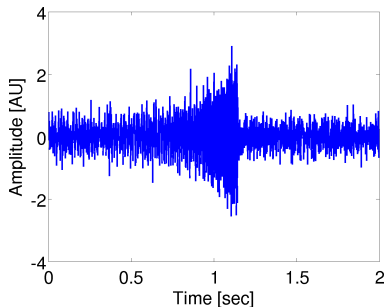


● Burst search

- ▶ Concentrate signal energy in a small number of pixels
- ▶ Sum energy over clusters of “loud” pixels

⇒ Ranking statistic

Excess wrt Gaussian noise → match with templates



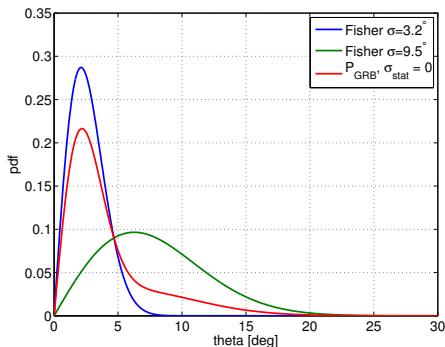
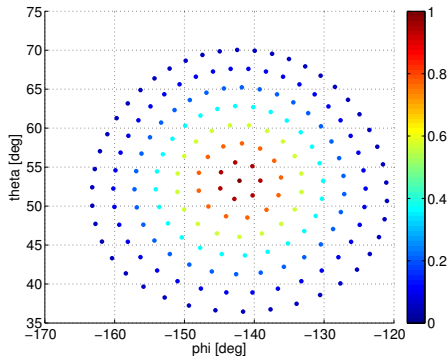
● Coalescence search

- ▶ Adjust template time, parameters (masses, ...)
- ▶ Sum coherently energy using waveform template
- ▶ Check that residual is consistent with Gaussian noise (χ^2)

⇒ Ranking statistic

GRB sky localization

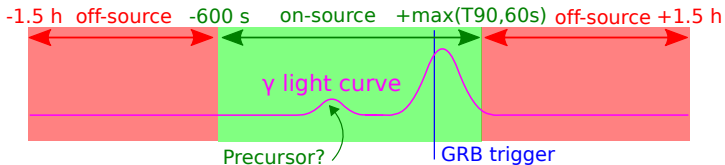
- Swift: errors $\sim 0.3^\circ \Rightarrow$ negligible for GW searches
- Fermi/GBM: large errors \Rightarrow apply coherent analysis to grid covering error box



$$P_F(\theta; \kappa) = \frac{\kappa \sin \theta}{e^\kappa - e^{-\kappa}} e^{\kappa \cos \theta}, \quad \kappa \simeq \frac{1}{(0.66\sigma)^2}$$

- systematic $\sigma_{\text{core}} = 3.2^\circ$, $f_{\text{core}} = 0.7$, $\sigma_{\text{tail}} = 9.5^\circ$

GRB triggered GW burst search



- Known **position** and **time**

- ▶ Reduced time → reduced background
- ▶ Position → simplify coherent analysis
 - time delays between detectors constrained by sky location box
 - ~ 20% sensitivity improvement (Waż et al., 2012)

⇒ Burst search sensitivity improved by a factor ~ 2 (instead of 1.6)

⇒ Inspiral search sensitivity improved by a factor ~ 1.4 (instead of 1.2)

- **On-source data**

- ▶ Search for potential GW events

- **Off-source data**, time slides

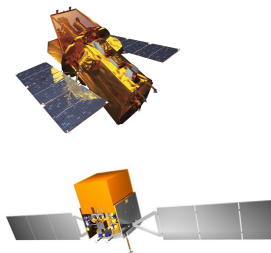
- ▶ Measurement of event background distribution

- Repeated independently for each GRB

Data sample

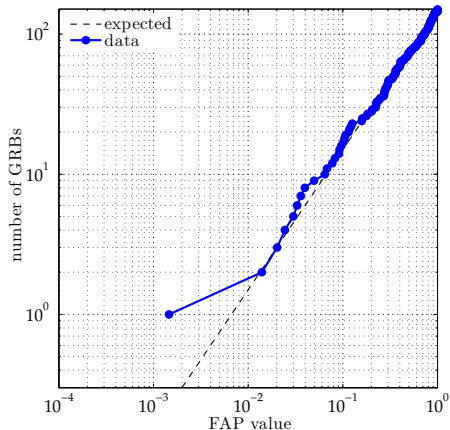


- July 2009 – October 2010
- Network of three GW detectors
 - ▶ LIGO Hanford
 - ▶ LIGO Livingston
 - ▶ Virgo, Italy
- 404 GRBs observed by γ -ray satellites
 Gamma-ray burst Coordinates Network
 - ▶ Swift
 - ▶ Fermi
 - ▶ ...
- 154 GRBs with good data from at least two GW detectors
- includes 26 short GRBs



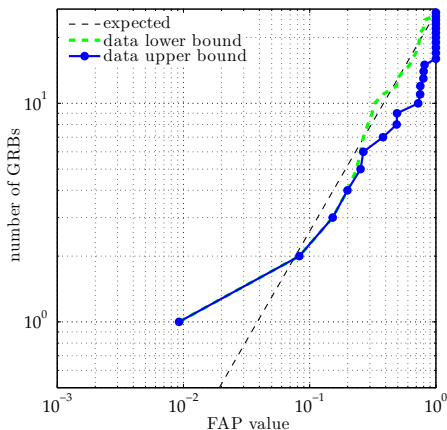
Event distribution consistent with background

GW bursts search



25% background probability

Binary coalescence search



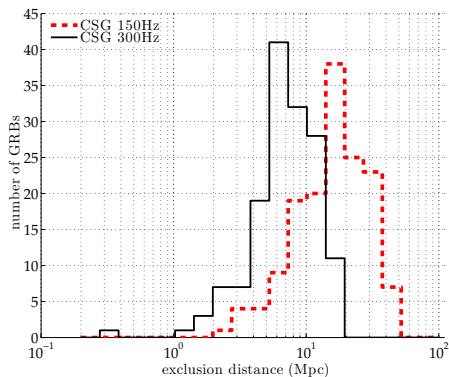
8% background probability

(Abadie et al., 2012b)

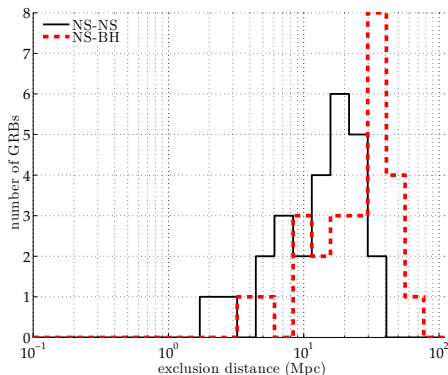
GW burst non detection consequences

GRB progenitor distance exclusion

Unmodeled GW bursts
with $E_{\text{GW}} = 10^{-2} M_{\odot} c^2$



Binary system coalescence



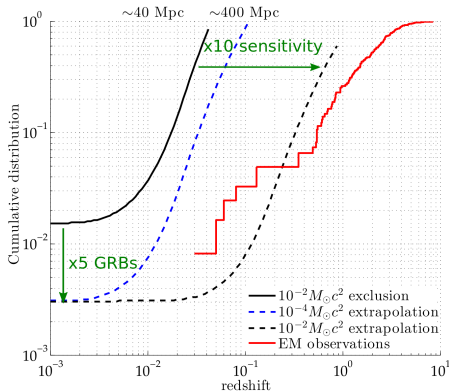
	burst 150Hz	burst 300Hz	NS-NS	NS-BH
median (Mpc)	17	7	7	28

(Abadie et al., 2012b)

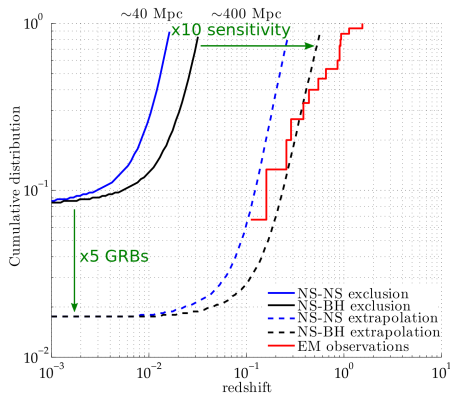
Expectations & Prospects

● 2009-2010 results

Unmodeled GW bursts



Binary coalescence



● Prospects for advanced detectors (Abadie et al., 2012b)

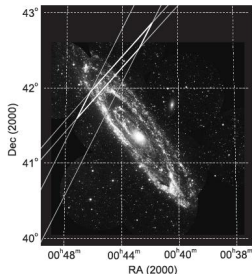
- ▶ $\times 10$ sensitivity, $\times 5$ number of GRBs
- ▶ long GRBs, possible if optimistic GW emission
- ▶ short GRBs, quite possible, especially if significant NS-BH fraction

GRB070201 / GRB051103

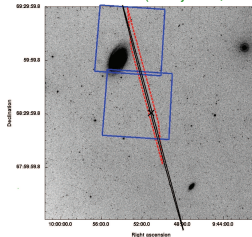
Significant previous non detections

- Short GRBs,
 - ▶ GRB070201 sky location overlap with M31, (Andromeda **770 kpc**)
 - ▶ GRB051103 sky location overlap with M81 (**~ 3.6 Mpc**)
- no GW found
 - ⇒ **Binary coalescence in M31 excluded** at >99% confidence level (Abbott et al., 2008)
 - ⇒ **Binary coalescence in M81 excluded** at 98% confidence level (Abadie et al., 2012a)
- **Compatible with**
 - ▶ Neutron star quake in M31/M81 (Soft gamma-repeater)
 - ▶ Coalescence in galaxy behind M31/M81

GRB070201 error box (Mazets et al., 2008)



GRB051103 error box (Hurley et al., 2010)



What might we learn from GW-GRB observation?

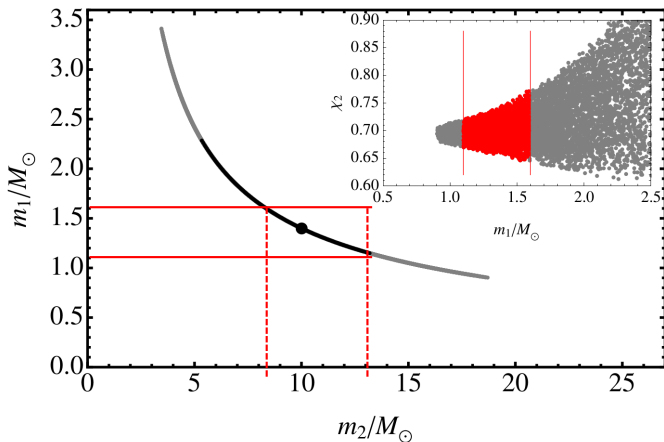
Models for short/long GRBs remain uncertain

- long GRBs
 - ▶ localization in star forming regions
 - ▶ associations with supernova
 - ▶ **but also some long GRBs with strong limits on supernova**
($< 10^{-3}$ typical luminosity)
- short GRBs
 - ▶ localization in galaxies with old stellar population
 - ▶ lack of supernova
 - ▶ **observational confirmation weaker than for long GRBs**

Potential lessons from GW-GRB detection

- Confirm the binary coalescence model for short GRBs
- Measure typical GRB jet opening angle
- Measure BH spin
- Precise measurement of GW speed, $\Delta v/c \sim 10^{-16}$
- Measure of Hubble's constant, distance \leftrightarrow redshift relation

Measuring BH spin



(Hannam et al., 2013)

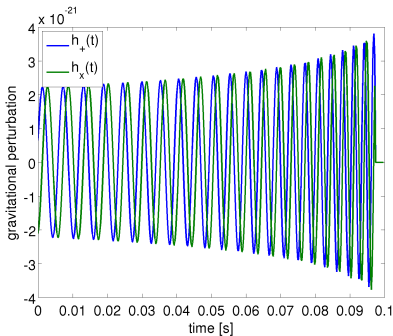
- Binary coalescence: large degeneracies between parameters
- GRB: one of the bodies is a NS, $m \sim 1.4 M_\odot$
- more precise measurement of other parameters

Measuring Hubble's constant with GWs

All potential GWs sources $z \lesssim 0.1$: $H_0 = c \frac{z}{D_L}$

$$\begin{bmatrix} h_+(t) \\ h_\times(t) \end{bmatrix} = \underbrace{\frac{A(t; (1+z)\mathcal{M})}{D_L}}_{\text{enveloppe}} \underbrace{\begin{bmatrix} (1 + \cos^2 \iota) \cos(\Psi(t)) \\ 2 \cos \iota \sin(\Psi(t)) \end{bmatrix}}_{\text{polarized oscillations}}$$

- $A(t; (1+z)\mathcal{M})$ - GW shape sets absolute amplitude of the waveform
- D_L - luminosity distance
- ι - binary inclination angle - degenerate with luminosity distance (polarization is hard to measure)
- z - redshift - degenerate with the mass of the binary



Measuring Hubble's constant with GWs

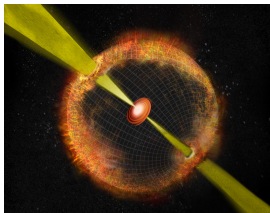
$$\begin{bmatrix} h_+(t) \\ h_\times(t) \end{bmatrix} = \frac{A(t; (1+z)\mathcal{M})}{D_L} \begin{bmatrix} (1 + \cos^2 \iota) \cos(\Psi(t)) \\ 2 \cos \iota \sin(\Psi(t)) \end{bmatrix}$$

Several approaches

- Combine GW and GRB observation (Nissanke et al., 2010)
 - ▶ **redshift** given by EM observations
 - ▶ GW shape yields absolute amplitude
 - Measure D_L from GW amplitude
 - ▶ γ -ray observation means binary close to face-on
 - helps breaking the D_L vs inclination degeneracy
- Use GW information alone (Taylor et al., 2012)
 - ▶ Assume \mathcal{M} known - binary neutron star system
 - Measure **redshift** from GW shape
 - ▶ GW shape yields absolute amplitude
 - Measure D_L from GW amplitude
 - ▶ Dozens of events per year
 - helps breaking the D_L vs inclination degeneracy
- **In both cases $\sim 10\%$ precision on H_0**
- **Measurement independent of cosmic ladder**

Summary

- Astrophysical triggers:
 - ▶ more sensitive GW search
 - ▶ more interesting interpretations
- ⇒ Require study EM & GW phenomenology
- Both short and long GRB may produce GWs
- Non detections already provide information (GRB070201, GRB051103)
- Good prospects for advanced LIGO/Virgo
- With more detections come interesting measures:
BH spin, jets, Hubble's constant



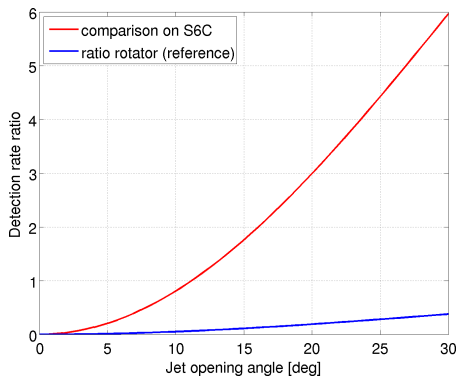
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Relevance of triggered search vs all-sky search

Is a GRB triggered search a good of detecting GWs from binary coalescences / star collapse?

- Triggered search misses progenitors beaming away from Earth
- Triggered search is more sensitive

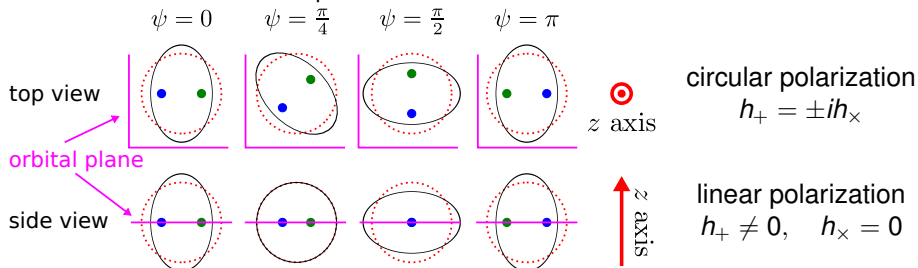


Astrophysical framework

- GRB progenitors (CBC, hypernova,...) are **standard GW sirens** \Rightarrow fixed E_{GW}
- Uniform distribution in space
- Rotator GW emission pattern (binary, bar mode, ...)

$$\begin{pmatrix} h_+ \\ h_\times \end{pmatrix} \propto \begin{pmatrix} 1 + \cos^2 \iota \\ 2 \cos \iota \end{pmatrix}$$

- ▶ face on \rightarrow circular polarization
- ▶ edge on \rightarrow linear polarization
- ▶ inclination $\iota \rightarrow$ elliptical



EM & GW beaming

$$\begin{pmatrix} h_+ \\ h_\times \end{pmatrix} \propto \begin{pmatrix} 1 + \cos^2 \iota \\ 2 \cos \iota \end{pmatrix}$$

- GW power flux dependence on ι , **slight GW beaming**

$$F(\iota) = \frac{(2 \cos \iota)^2 + (1 + \cos^2 \iota)^2}{8}, \quad F(0) = 1, \quad F(\pi/2) = 1/8$$

- γ -ray emission in a cone around rotation axis, **top hat emission**
 - ▶ two sided jet $\rightarrow \iota \in [0, \pi/2]$
 - ▶ jet of opening angle θ_j (thought to be in $5 - 30^\circ$ range)
 - ▶ $\iota < \theta_j \rightarrow$ GRB detected on Earth
 - ▶ $\iota > \theta_j \rightarrow$ progenitor dark in γ -rays (missed by extrtrag search)

Theoretical comparison

Issue

- All-sky searches for GW from all progenitors
 - Extrinsic searches only for progenitors with $\iota < \theta_j$
- ⇒ Gain in sensitivity ↔ loss in GW source density rate

Analysis toy model

- Forget about ITF antenna patterns
- Analysis detection based on a sharp threshold on h_{rss}
 - ▶ At given inclination ι analysis efficiency drops from 1 to 0 at horizon distance $r(\iota)$
 - ▶ Simple dependence on inclination: $r(\iota) = \sqrt{F(\iota)}r(0)$
- Hopefully $r_{\text{extrinsic}}(\iota) > r_{\text{all-sky}}(\iota)$ for $\iota < \theta_j$
- Effective search volume, marginalize over inclination
 - ▶ For all-sky

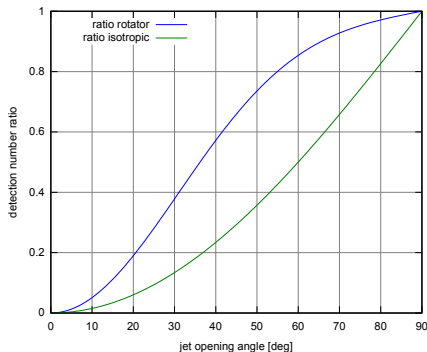
$$V_{\text{all-sky}} = \int_{\iota=0}^{\pi/2} \frac{4\pi}{3} r_{\text{all-sky}}^3(\iota) \sin(\iota) d\iota$$

- ▶ For extrinsic

$$V_{\text{extrinsic}} = \int_{\iota=0}^{\theta_j} \frac{4\pi}{3} r_{\text{extrinsic}}^3(\iota) \sin(\iota) d\iota$$

Detection rate ratio

- Detection rate ratio $R(\theta_j)$ for equal horizons: $r_{\text{all-sky}} = r_{\text{extrig}}$



$$R(\theta_j) = \frac{V_{\text{extrig}}(\theta_j)}{V_{\text{all-sky}}}$$

$$r(\iota) = \sqrt{F(\iota)} r(0)$$

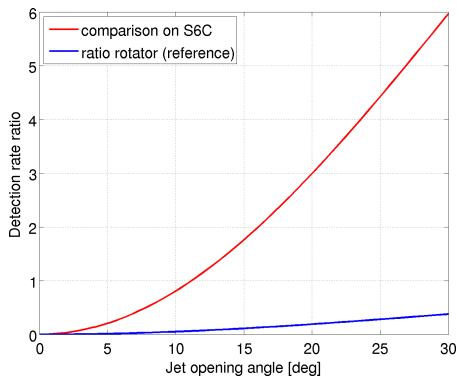
$$F_{\text{rotator}}(\iota) = \frac{(2 \cos \iota)^2 + (1 + \cos^2 \iota)^2}{8}$$

$$F_{\text{isotropic}} = 1$$

- For other sensitivity ratio, multiply curve by $(r_{\text{extrig}}/r_{\text{all-sky}})^3$
- ⇒ GW beaming helps the extrig approach by a factor ~ 3 (in the small opening angle limit)

Relevance of triggered search vs all-sky search

- Triggered search misses progenitors beaming away from Earth
- Triggered search is more sensitive



- ⇒ interesting even for small jet opening angles
 - Reference: fraction found by all-sky search with γ -ray counterpart
- ⇒ Two approaches see (mostly) independent events

Signal to noise ratio – SNR

- Perfectly known signal $s(t) \leftrightarrow \tilde{s}(f)$

$$\text{SNR}_{\text{optimal}}^2 = 2 \int_{-\infty}^{\infty} \frac{|\tilde{s}(f)|^2}{A(|f|)^2} df$$

- Whitened signal/data

$$\tilde{d}^w(f) = \tilde{d}(f) \times \frac{\sqrt{2}}{A(|f|)}$$

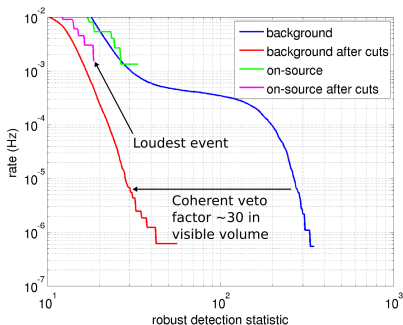
- Matched filtering: scalar product between template and data

$$\text{SNR} = \frac{\int_{-\infty}^{\infty} \tilde{s}^w(f)^* \tilde{d}^w(f) df}{\sqrt{\int_{-\infty}^{\infty} |\tilde{s}^w(f)|^2 df}}$$

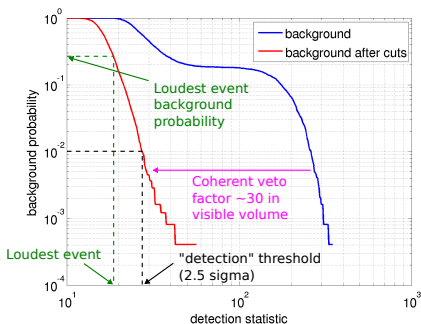
- $d(f) = n(f)$ – SNR normally distributed
 - $d(f) = s(f) + n(f)$ – distribution mean is shifted by $\text{SNR}_{\text{optimal}}$
- ⇒ $\text{SNR}_{\text{optimal}}$ – detectability in perfect conditions

Background estimation

Event rate above threshold



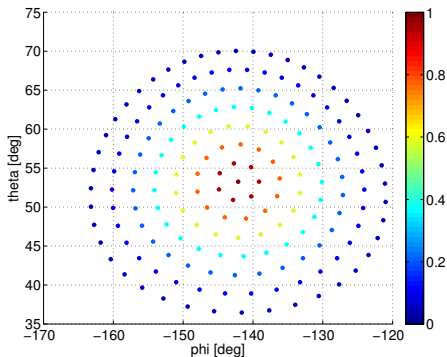
background probability in window



Background probability \simeq background event rate \times time window length

Loudest event in on-source window \Rightarrow Effective clustering over the window

Sky position error box



$$L(\mathbf{d} | \odot, \sigma_h) = \frac{|\mathbf{e}^\odot \cdot \mathbf{d}|^2}{1 + 1/(\sigma_h |\mathbf{f}^\odot|)^2} - \log(1 + \sigma_h^2 |\mathbf{f}^\odot|^2),$$

$$L(\mathbf{d} | \text{circular}) = 2 \log \sum_{\sigma_h \in \mathcal{A}} \frac{\max \{ \exp [\frac{1}{2} L(\mathbf{d} | \odot, \sigma_h)], \exp [\frac{1}{2} L(\mathbf{d} | \ominus, \sigma_h)] \}}{|\mathcal{A}|}.$$

Sensitivity estimation - signal models

- Compact object coalescence (inspiral)

- ▶ BH-NS: $m_{\text{BH}} = 10 \pm 6 M_{\odot}$
 $m_{\text{NS}} = 1.4 \pm 0.4 M_{\odot}$
- ▶ NS-NS: $m_{\text{NS}} = 1.4 \pm 0.2 M_{\odot}$

- Extreme stellar collapse

- ▶ Ad-hoc model to sample parameter space – sine-Gaussian

$$\begin{bmatrix} h_+(t + t_0) \\ h_{\times}(t + t_0) \end{bmatrix} = A_0 \begin{bmatrix} \underbrace{\cos(2\pi f_0 t)}_{\text{rotation}} (1 + \underbrace{\cos^2 \iota}_{\text{inclination}}) \\ \underbrace{\sin(2\pi f_0 t)}_{\text{rotation}} \underbrace{2 \cos \iota}_{\text{inclination}} \end{bmatrix} \underbrace{\exp \left[-\frac{(2\pi f_0 t)^2}{2Q^2} \right]}_{\text{envelope}}$$

- ▶ $f_0 = 2f_{\text{rot}} = 100, 150, 300 \text{ Hz}$, $Q = 9$
- ▶ $E_{\text{GW}} = 10^{-2} M_{\odot} c^2$, distance $\rightarrow A_0$

- Nuisance parameters \rightarrow jitter injections

- ▶ Sky localization error
- ▶ Calibration errors
- ▶ System inclination ι

