All-Sky CW searches: The Frequency Hough Transform

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As already explained this morning..

"All-sky"searches are blind searches, done without making any strong assumption on the NS (neutron star) parameters, with hierarchical approaches (cut computational load at the cost of a sensitivity loss);

They are computationally bounded (grid/cloud/Einstein@Home)

We need to explore a portion of the source parameter space as large as possible:

All-sky

Frequency from ~10 Hz up to (1.5-2) kHz

Spin-down age as smallest as possible (e. g. < 10^3 – 10^4 years)

This cannot be done with fully coherent methods, computationally unfeasible because the number of points is huge (~10^{31}).

- **Different hierarchical** approaches have been developed which try to satisfy two requirements:
    - drastically reduce the computing power needed;
    - not lose too much in sensitivity
- The key idea is that of dividing data in a number of 'short' segments and combine them incoherently.
- In the incoherent step a rough exploration of the parameter space is done and candidates are selected.
- Candidates are then followed with a more refined search.

$$h_{0,\min} \approx \frac{10}{N^{1/4}} \sqrt{\frac{S_n}{T_{FFT}}}$$

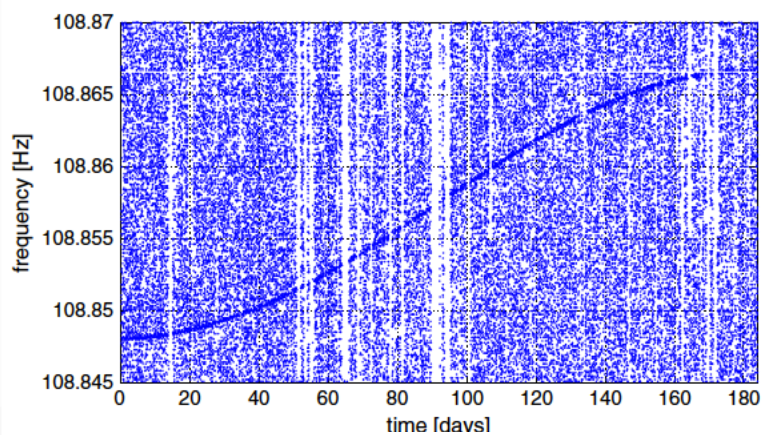
N: number of segments

T_{FFT}: length of short pieces

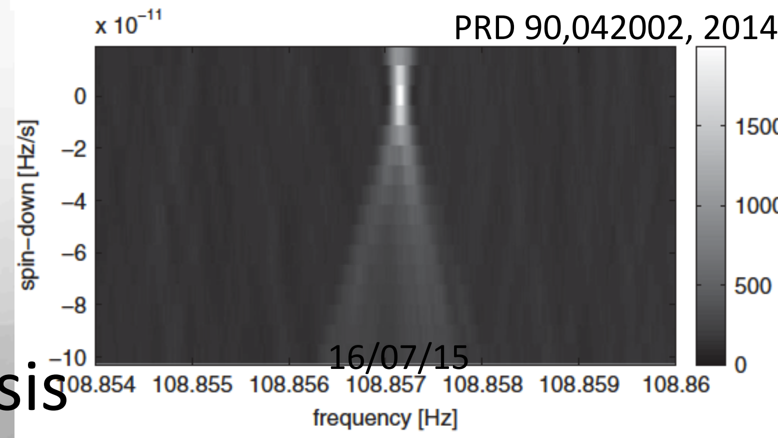
For the incoherent step, we use the **Hough transform**, a pattern recognition method originally developed in the 60' s to analyze tracks in bubble chambers.

- *It realizes a mapping between the detector time-frequency plane and the source parameter space.*

- The "Frequency Hough" (FH) takes Doppler-corrected 'peaks' in the detector time/frequency plane
- Each peak is transformed into a 'strip' in the source frequency/spin-down plane
- Significant points in the Hough map are selected for further analysis



PRD 90,042002, 2014



The Frequency Hough (FH) procedure is described in PRD,90,042002 (2014).
Used for VSR2/VSR4 low frequency analysis PRD,93,042007 (2016)
And used in the S6 MDC (arXiv:1606.00660v2. Sub to PRD)

We construct a set of FFT data bases (the SFDB “short FFT data base”)

- T_{FFT}=8192 s for [10-128] Hz
- T_{FFT}=4096 s for [128-512] Hz
- T_{FFT}=2048 s for [512-1024] Hz
- T_{FFT}=1024 s for [1024-2048] Hz

to optimize the sensitivity as a function of the frequency

The input to the Hough are the peakmaps

Shortly: From the SFDB the peakmap is built with a threshold (of 1.58) on the square root of the equalized spectra.

To build a peak-map for a given dataset we need first to have built the Short FFT Database (SFDB). The SFDB contains a set of N windowed and interlaced FFTs covering the frequency range between 0 and 2048 Hz, with a length such that the power of CW signal present into the data would be confined in a single frequency bin (e.g. duration of 1048 s). For each FFT the SFDB contains also an auto-regressive (AR) estimation of the average spectrum S_i^{AR} .
The AR spectrum estimator has the following good properties:

1. It is not much affected by peaks present in the frequency domain;
2. It is able to follow noise level variations (both slow or more rapid).

References:
CQG 22, S1197 (2005) : mainly section 5
CQG 22, S1255 (2005): section 2

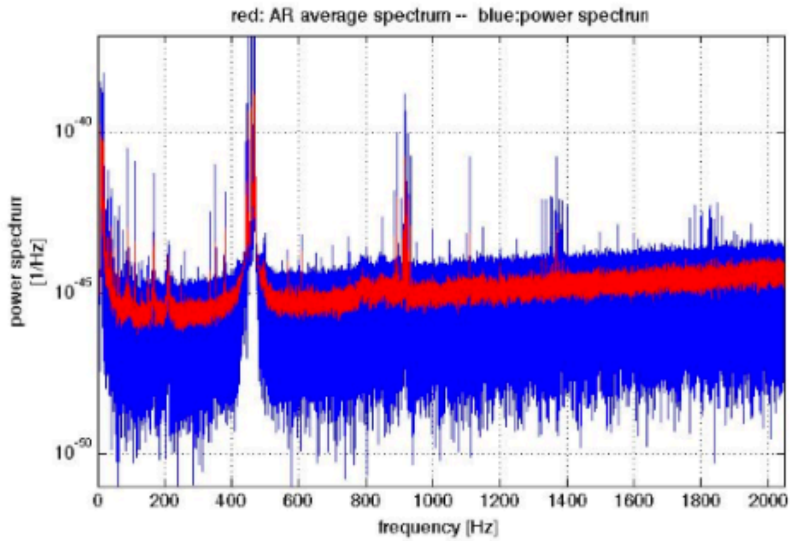


Figure 1: A typical spectrum from VSR4 data (blue) and the AR average spectrum (red)

The construction of the AutoRegressive spectrum

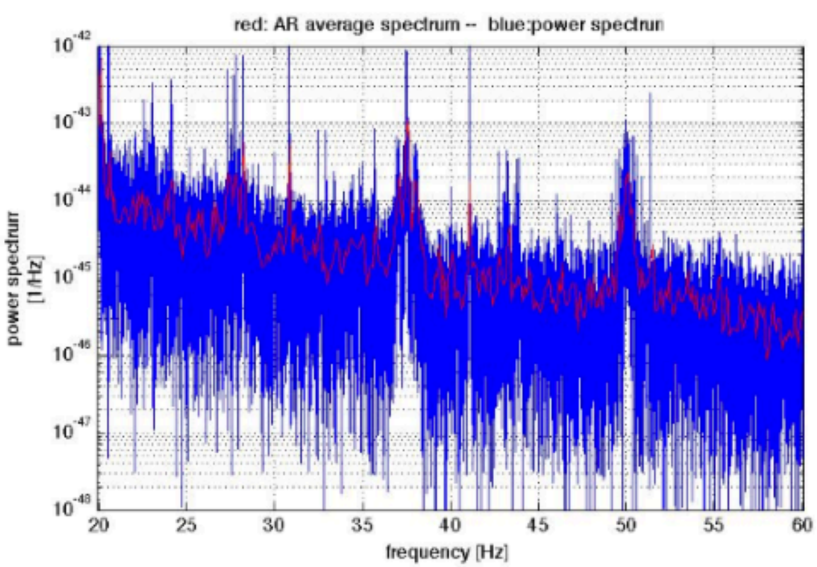


Figure 2: zoom of Figure 1 at low frequency

For each FFT we compute the AR spectrum $S_i=|FFT|^2$ and compute the ratio

$$R_i(j)=\frac{S_i}{S_i^{AR}}, \forall i=1,..N$$

where the index j runs over the frequency bins of a given AR spectrum. This numbers typically fluctuate around 1 and are significantly different from 1 when a spectral peak is present.

The $R_i(j)$ are compared to a threshold θ and we select all i and all j that satisfy the criterion

$R_i(j) > \theta$ AND $R_i(j)$ is a local maxima

Each selected pair (i,j) defines a peak, completely identified by a frequency value and the beginning time of the corresponding FFT. The amplitude of the peak is not taken into account.

The choice of selecting local maxima (instead e.g. of "all what is above the threshold") has two advantages and one theoretical disadvantage:

- less sensitivity to spectral disturbances (i.e. better robustness);
- lower computing cost;
- small theoretical sensitivity loss.

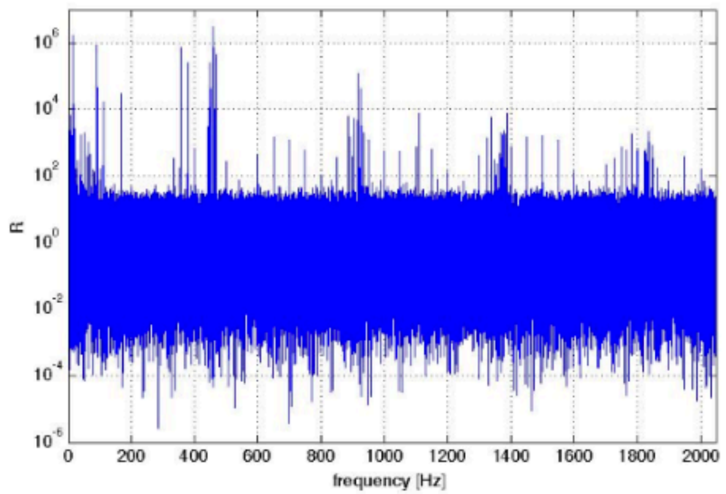


Figure 4: ratio R for a given FFT (from VSR4 data).

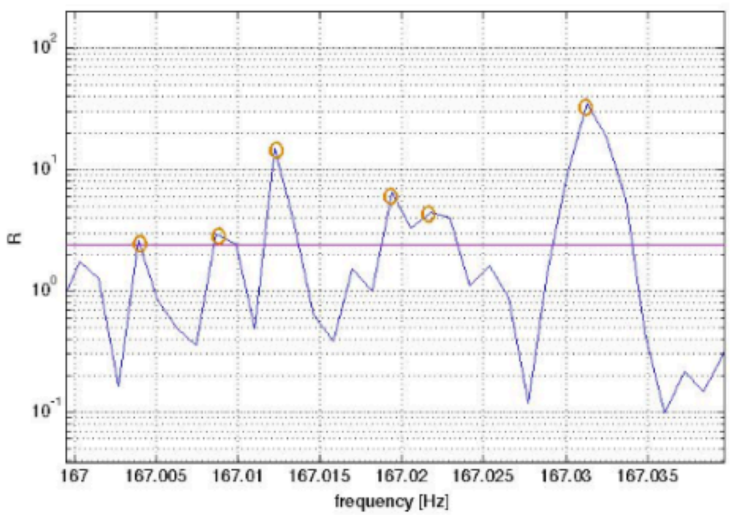
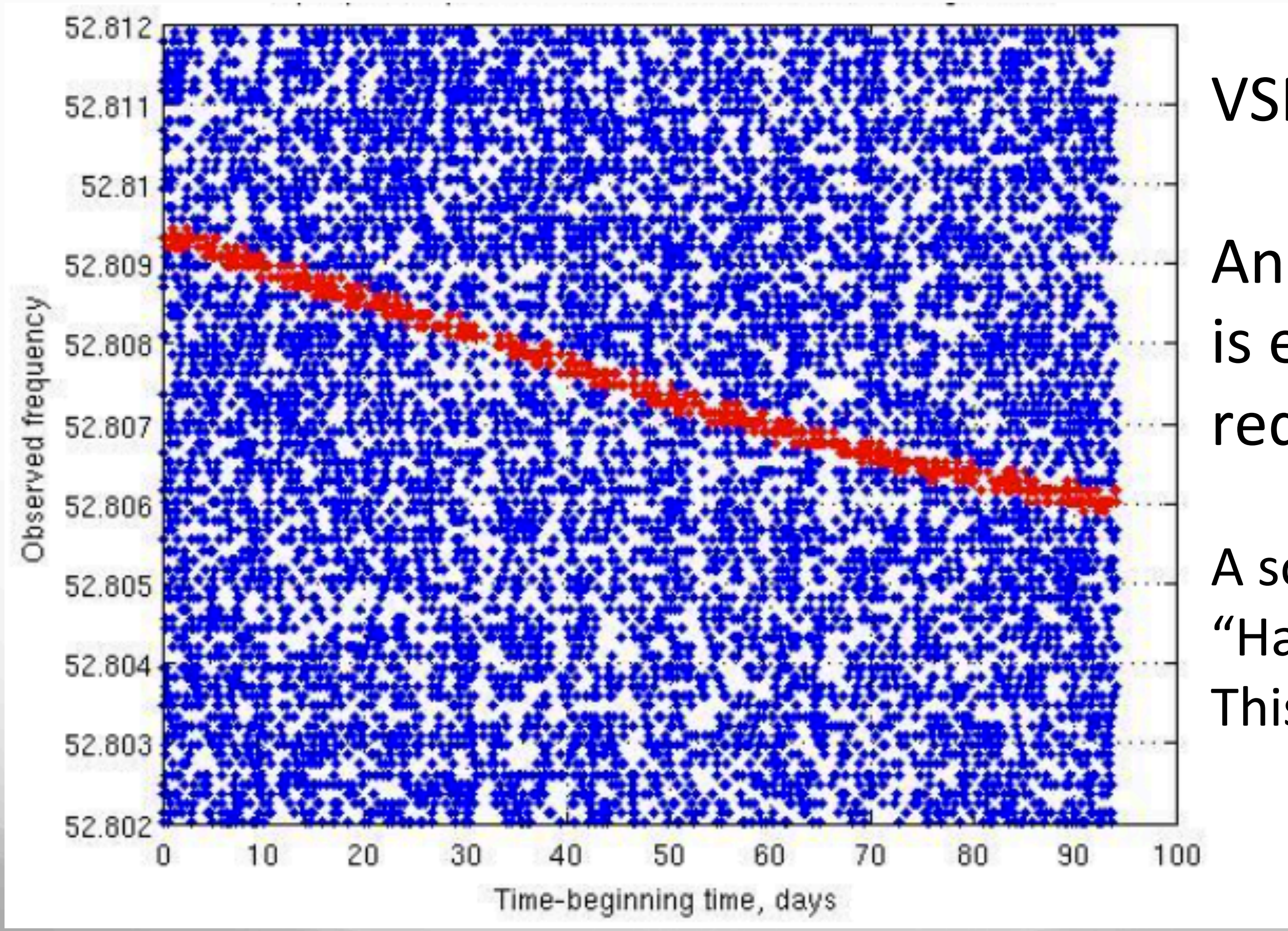


Figure 5: zoom of Fig.4. The orange circles identify the selected peaks, i.e. local maxima above the threshold given by the horizontal line.

An example of the construction of the peakmap

An example of input peakmap



VSR4 data

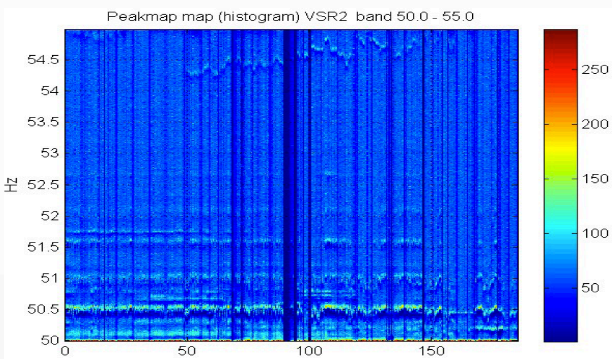
An injected signal is evidenced in red

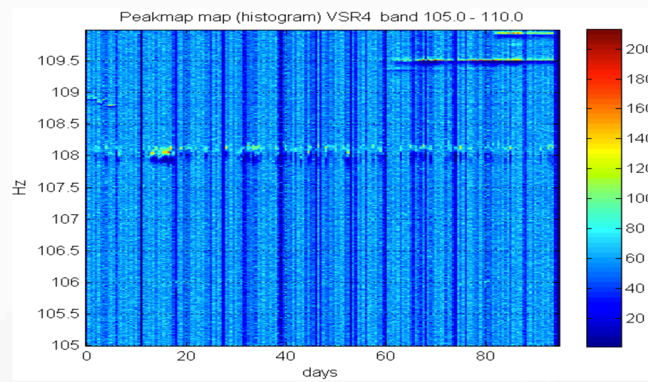
A so-called "Hardware injection" This is pulsar_5

Peakmaps: cleaning

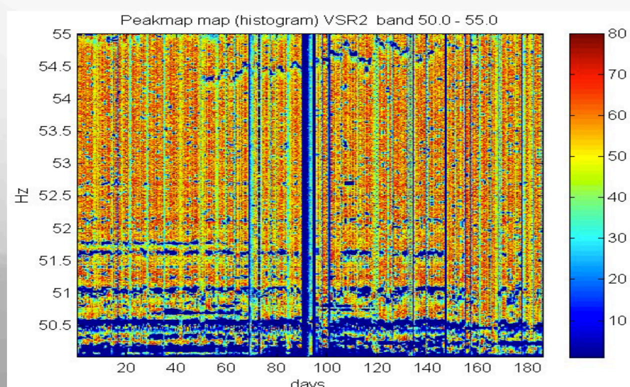
- Two cleaning steps on the peakmaps:
  - removal of “wide/wondering lines”;
  - removal of persistent lines with constant frequency

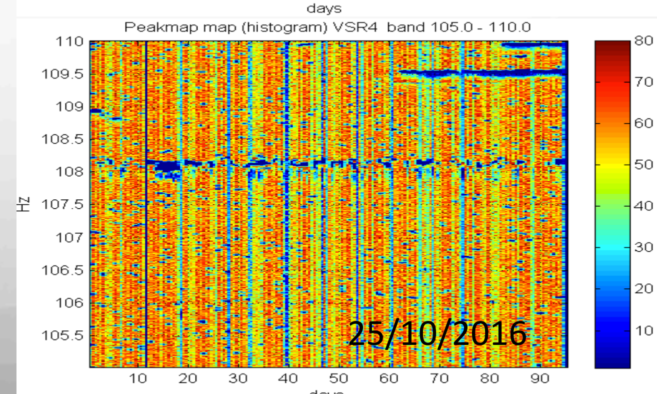
Peakmap “gross histogram” before veto

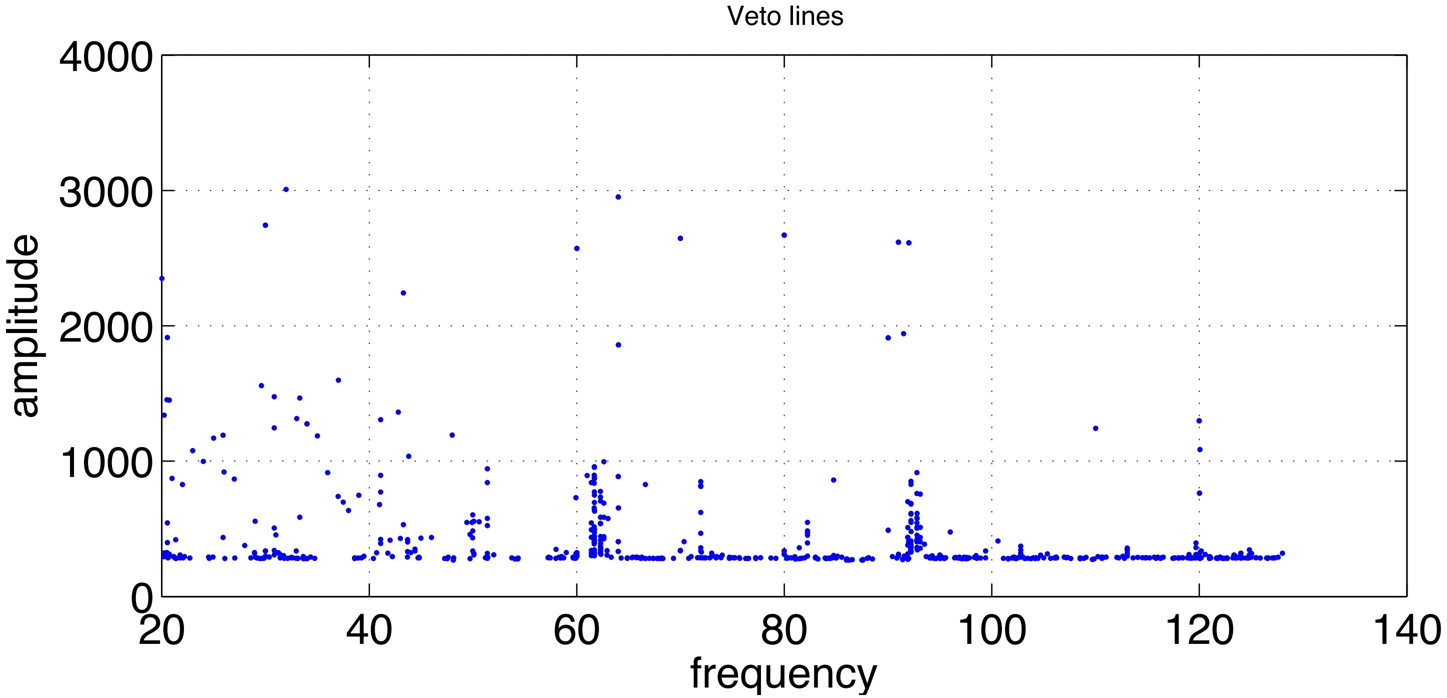




After veto



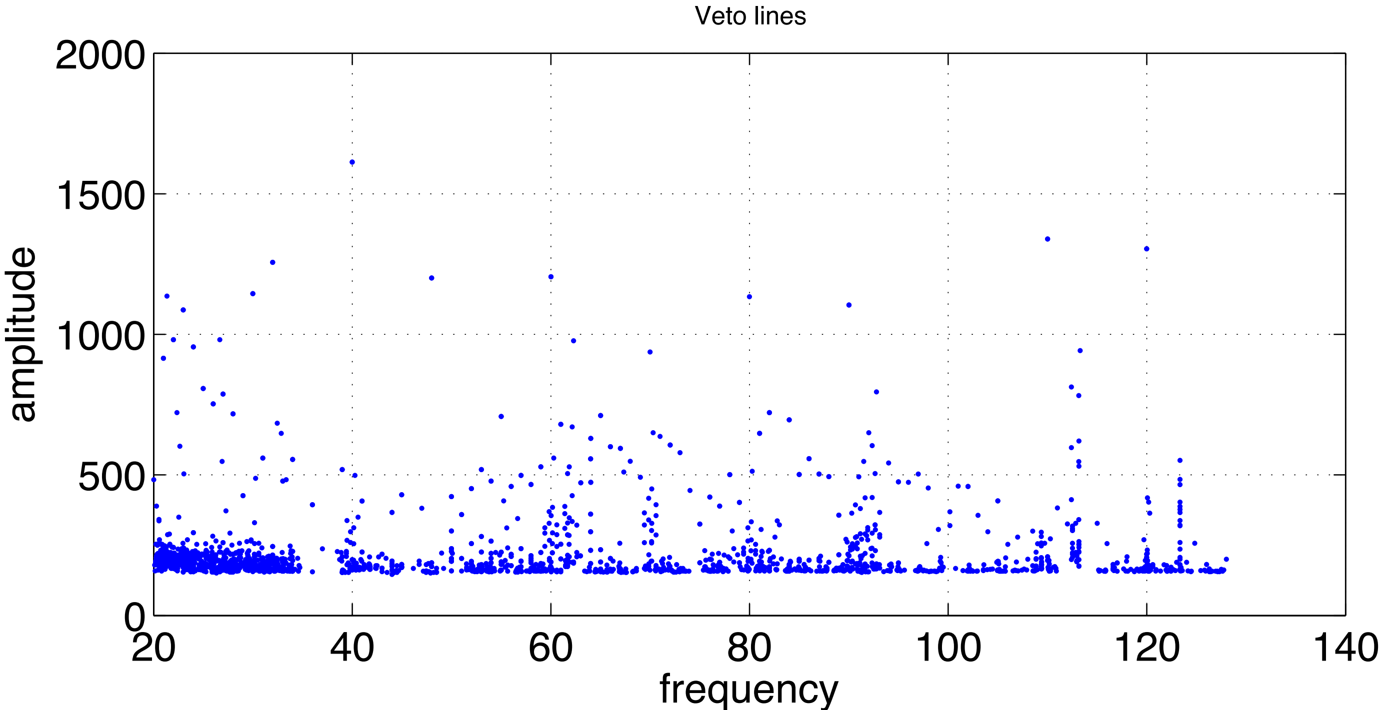




VSR2
710 lines

Lines vetoed

Amplitude is the number of FFTs where the line was present



VSR4
1947 lines

The FH: a transformation from the (time - observed frequency) plane to the (frequency - spin down) plane

Hough plane: frequency, corrected for the Doppler from a given source location , and first spin-down parameter (d)

For a given source location, and for source frequency f_0 at the time t, the observed frequency is:

$$f = f_0 + d * t$$

that is, a straight line in the plane f_0, d

The f_0 - d plane

$$d = -\frac{f_0}{t} + \frac{f}{t}$$

For each peak in the peakmap, defined by *(f,t)* we have a straight line in the ***(f0,****d)* plane

The slope is 1/t → **Choice of the time origin**

Peaks have a width Δf →

Every peak goes into a stripe among two parallel straight lines:

$$-\frac{f_0}{t} + \frac{f - \Delta f/2}{t} \le d \le -\frac{f_0}{t} + \frac{f + \Delta f/2}{t}$$

Hough map construction

- For every position in the sky → construct the Doppler shifted peakmap, from the original peakmap;
  - for every point in the shifted peakmap (f , t):
  - for every d → find f_0
  - construct the differential map (next slide)
  - sum over the bins (along the frequency direction, from left to right) → integral map

see 2008 Class. Quantum Grav. 25 184015

Construction of the map (*direct differential method*)

- For each d value, the map is incremented by 1 in the point

$$f_0 = f - \frac{\Delta f}{2} - d \cdot t$$

- and decremented by 1 in the point

$$f_0 = f + \frac{\Delta f}{2} - d \cdot t$$

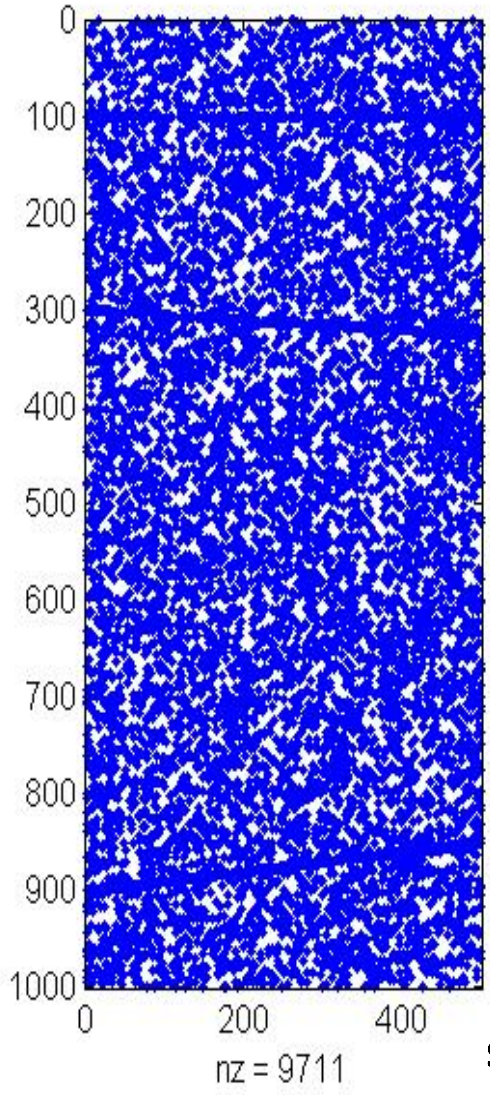
An important property of the method

- The increasing of resolution in the estimation of the frequency of the source, respect to the frequency resolution Δf ***does not have a relevant computational cost***:

- It affects only the *SIZE* of the computed map.In fact it has a negligible cost when evaluating the integral histogram, as the number of additions is proportional to the number of bins. The cost of additioning is dominated by the number of candidates in the peakmap

A first introductory example

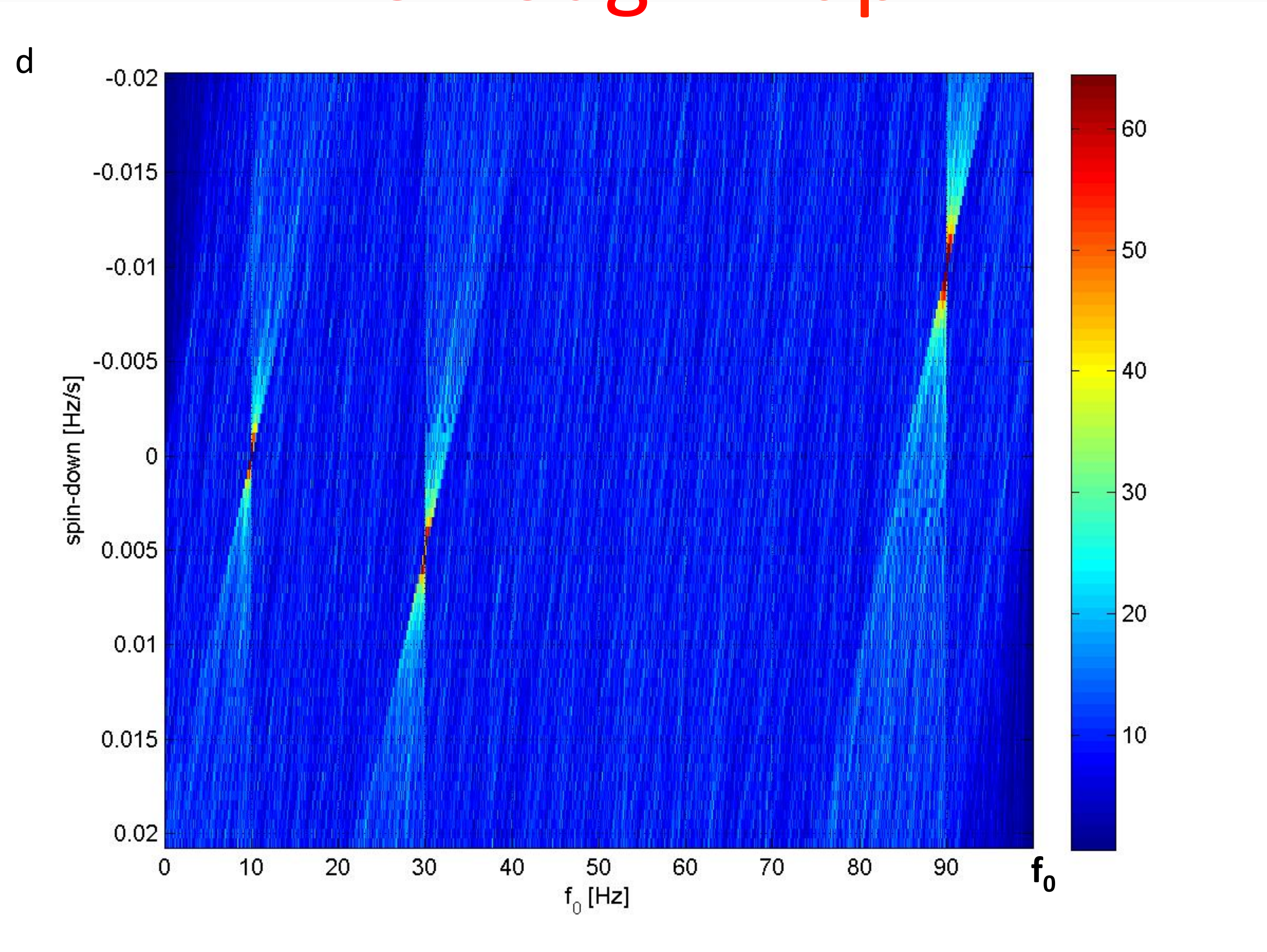
freq*10



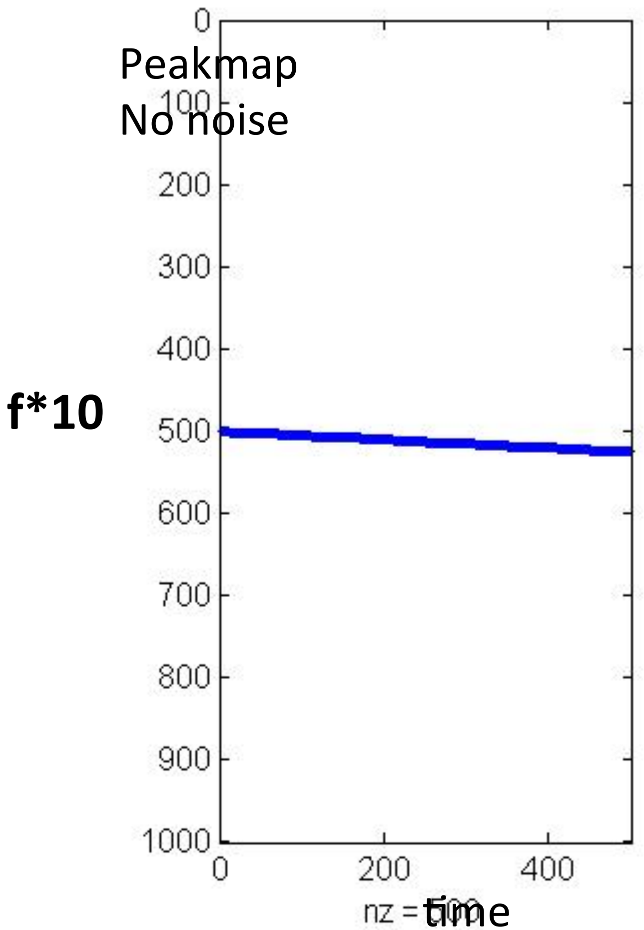
seconds

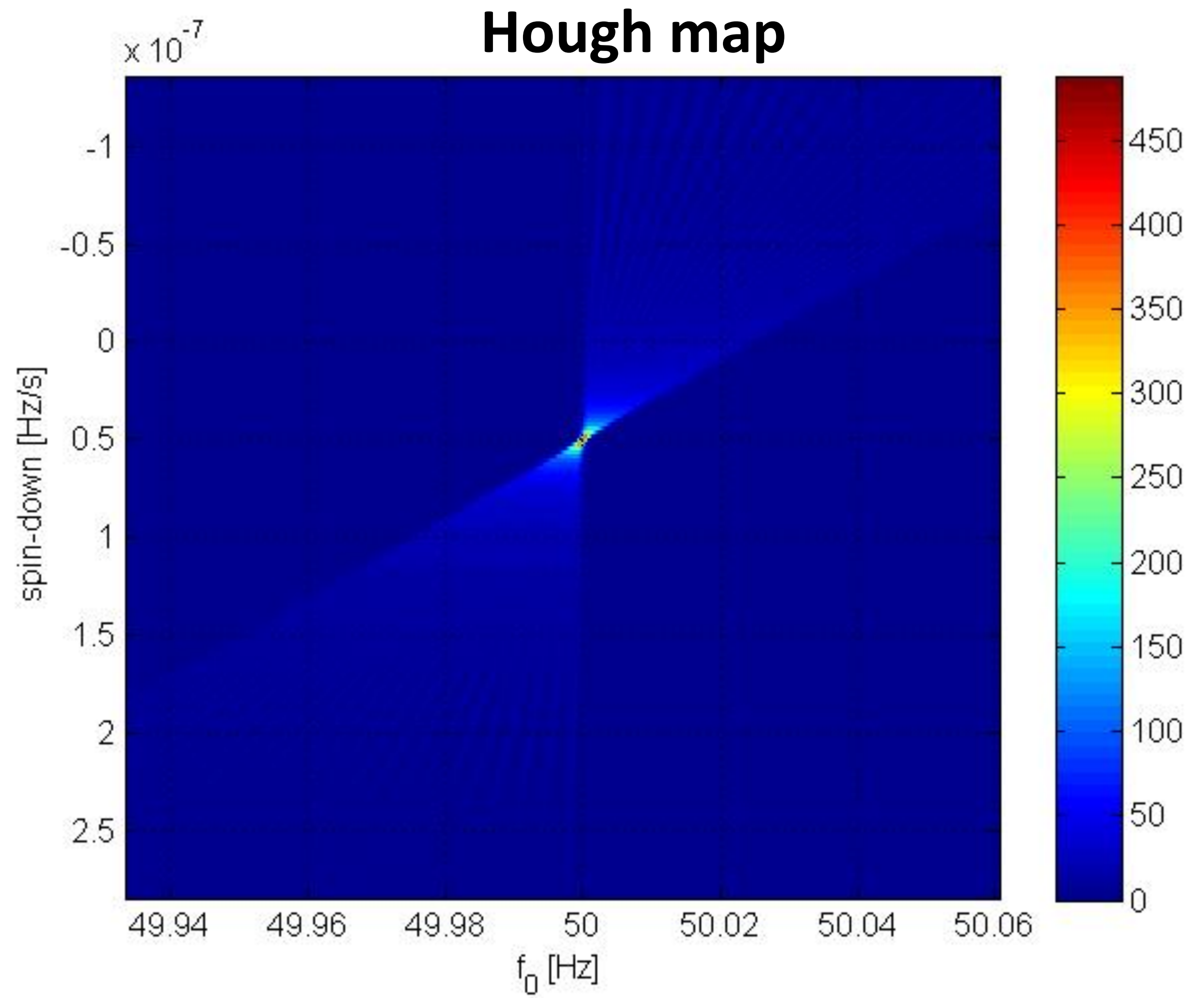
3 lines with different spin-down, frequency and SNR

The Hough map



Study the efficiency of the method





Efficiency versus frequency resolution factor

Resolution factor		Average normalized to the expected maximum
1		0.78
2		0.85
5		0.92
10		**0.93**
20		0.93
50		0.94

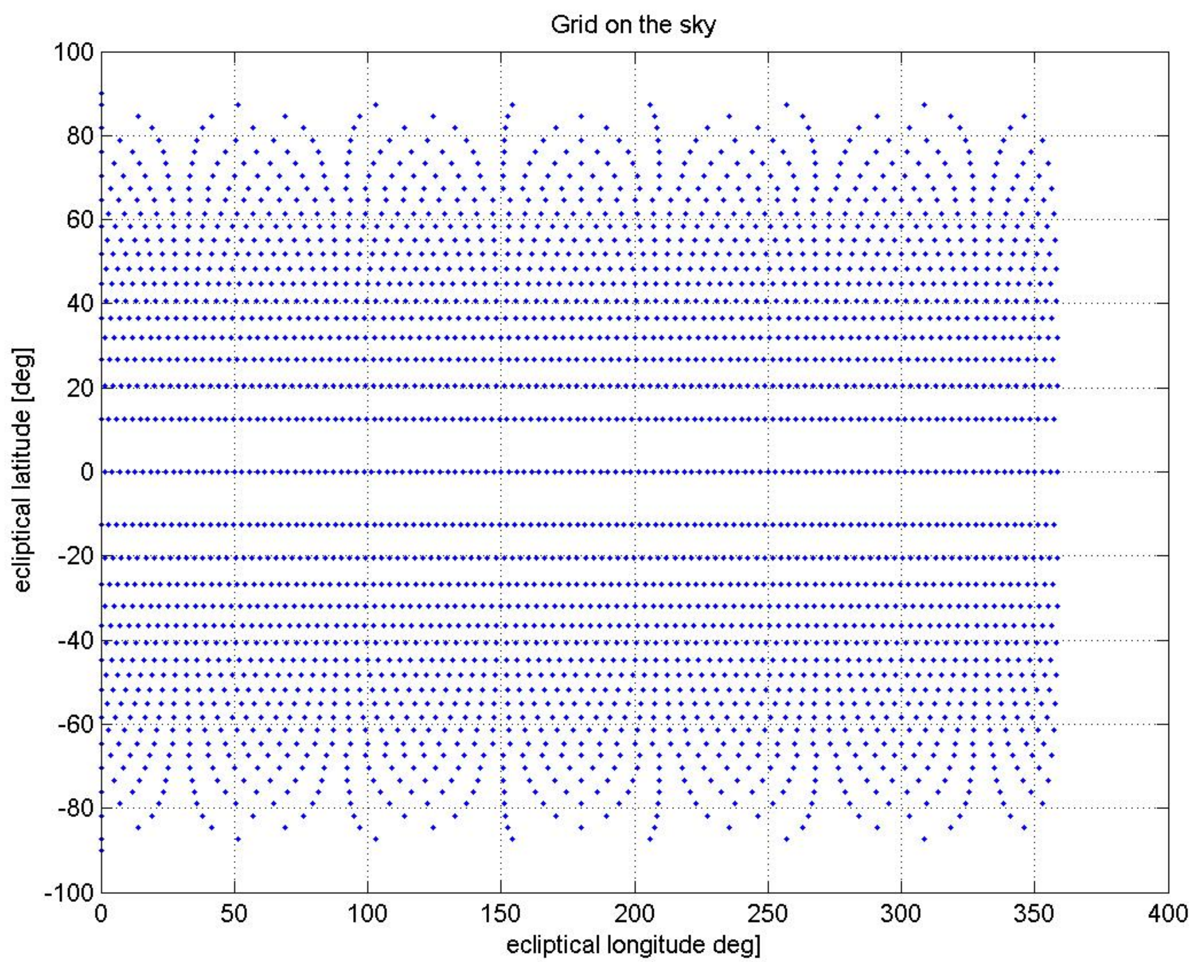
Efficiency versus spin down resolution (fixed res=10)

K ($\Delta d = k^*\Delta d_0$)		Average normalized to the expected maximum
1		0.93
0.5		0.96
0.2		0.97
0.1		0.98

Δd_0 is the natural spin down resolution, given by df/T_{obs}

The optimal grid on the sky

(here: N_D=20, N_{sky}=2902 points)



$$N_D = \frac{2f_{max}}{10000}$$

Bins in the Doppler band

The Sky grid

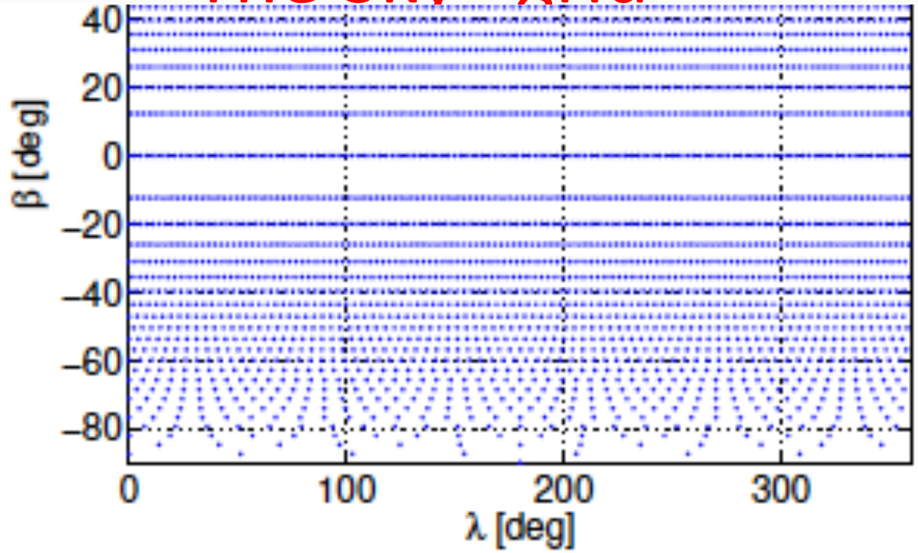
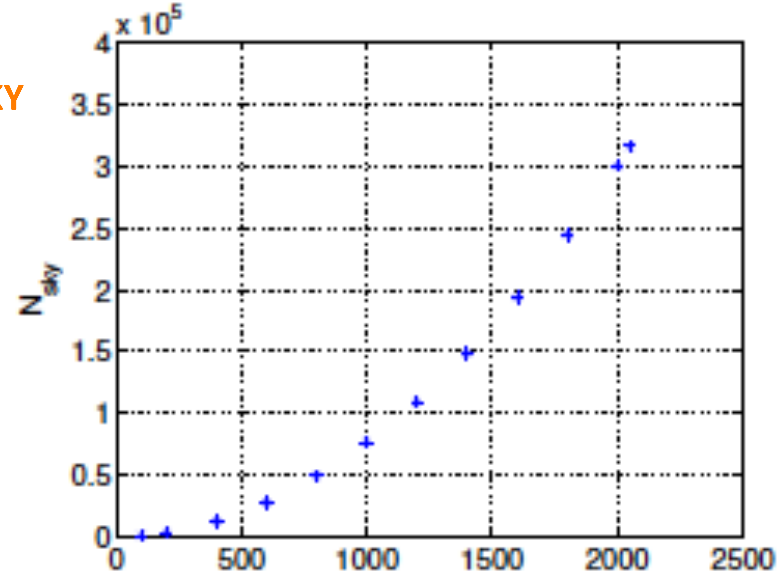
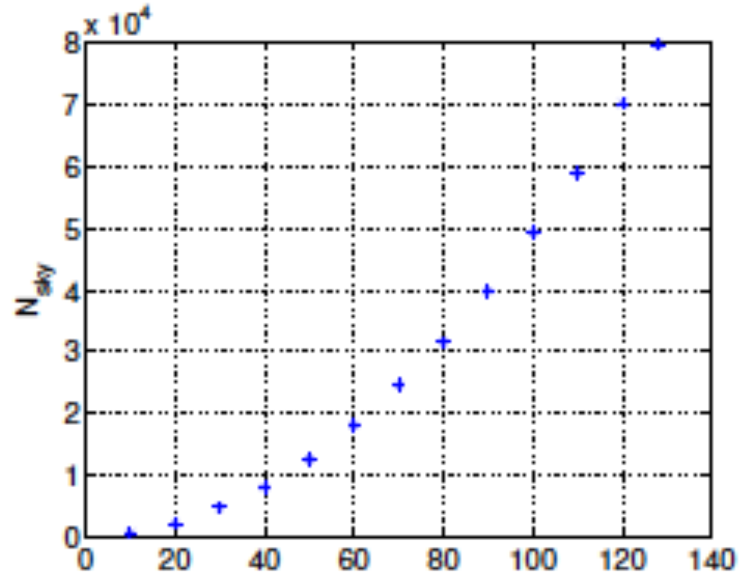


FIG. 7: Sky grid, for T_{FFT}=1024 s and frequency 200 Hz. $K_{sky} = 1$. x-axis: Ecliptical longitude, degrees; y-axis: Ecliptical latitude, degrees.

N_{SKY}





In the end:Efficiency of the search

Without the frequency resolution gain

Sky	*frequency and spin-down*	*Total*
0.860	0.880	**0.757** (*squared*=0.573)

FH

Sky	*frequency and spin-down*	*Total*
0.900	0.965	**0.869** (*squared*=0.754)

The Adaptive implementation:

```
function [hdf0]=hfdf_adaptivehough2010(antenna,sour,verb,hmap,peaks,ttdays,factdop)
%   HFDF_HOUGH  creates a linear peakmap
%
%   hmap        hough map structure
%    .fr    [minf df enh nf] min fr, original step, enhancement
%               factor, number of fr
%    .d       [mind dd nd] min d, step, number of d
%   peaks(3,n)    peaks of the peakmap as [t,fr,pmean]
%   factdop(nt)   Doppler correction factor (if present)
```

....

```
binh_df0(id,a) = binh_df0(id,a)+1;
binh_df0(id,b) = binh_df0(id,b)-1;
```

Non adaptive

```
binh_df0(id,a) = binh_df0(id,a)+radpat./mm;
binh_df0(id,b) = binh_df0(id,b)-radpat./mm;
```

Adaptive

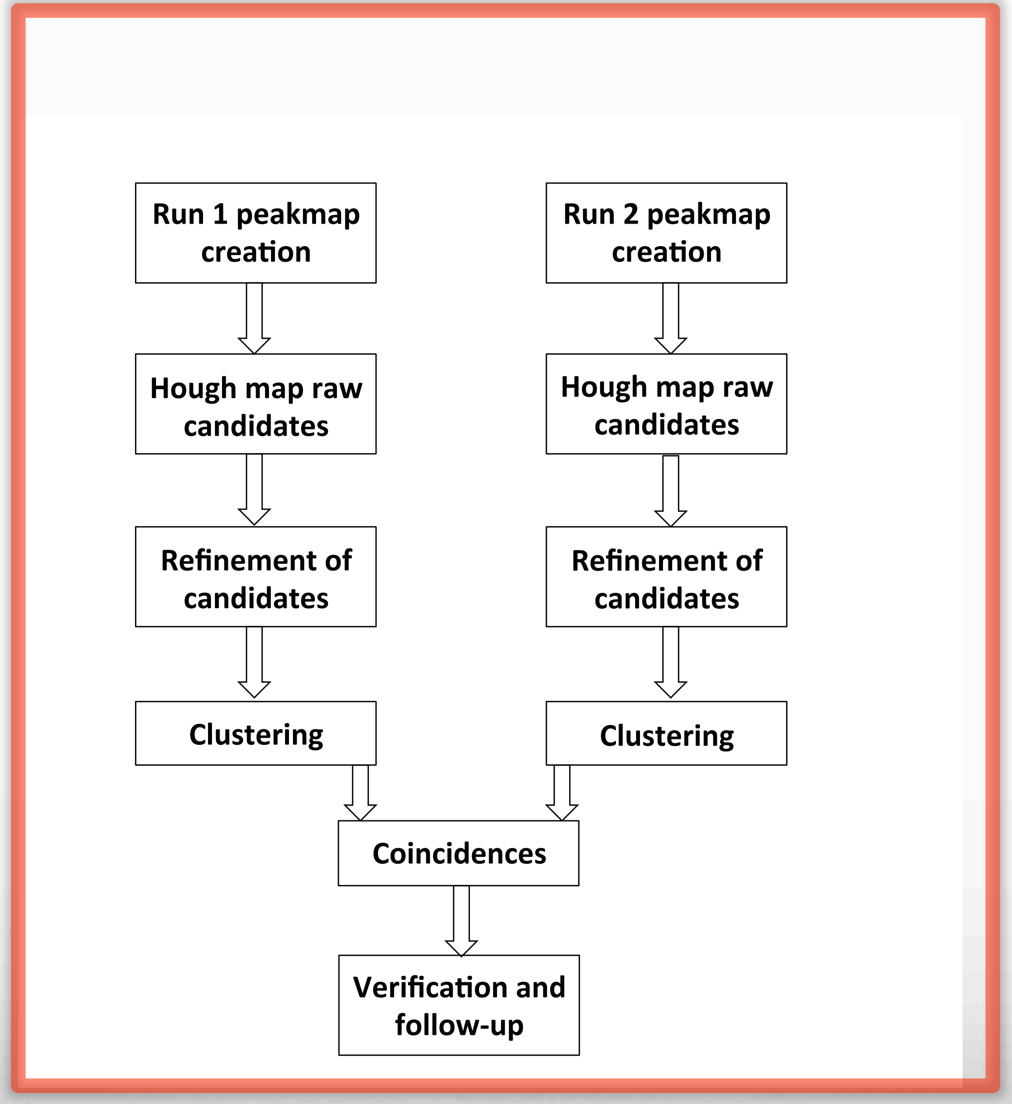
- radpat is the amplitude radiation pattern for that direction and detector (when used it is for circular polarization)
- and mm is the noise around the considered

frequency, estimated from the AR procedure used to construct the peakmaps.

We are now using the median of these mm values in sub-bands around the considered frequency.

Note that the distribution is different in the two cases,
the expected distribution for the adaptive implementation is Gaussian, with less tails compared to the binomial distribution of the non-adaptive implementation.

Scheme of the pipeline

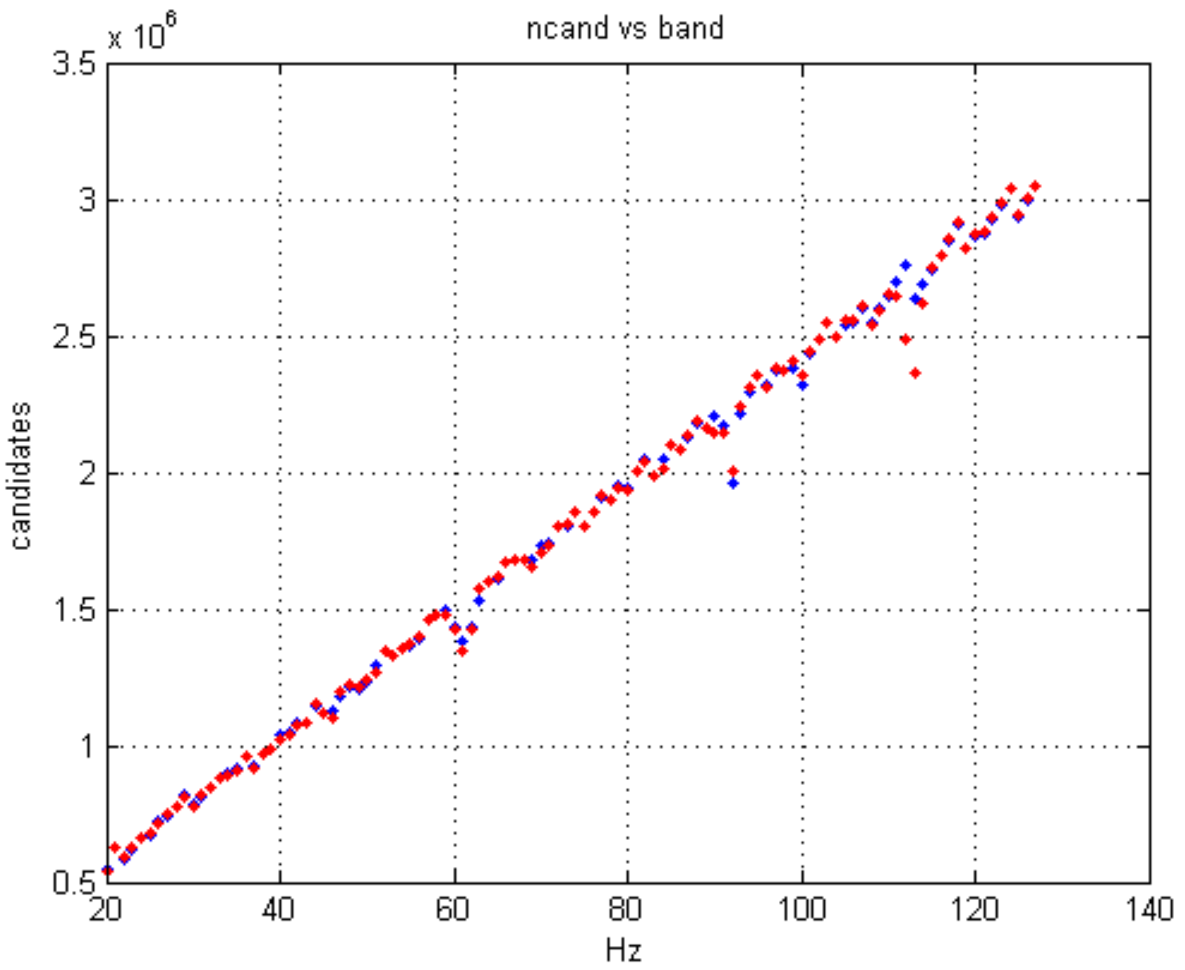


The procedure used to select candidates

- The practical problem due to the presence of artifacts (it is impossible to remove them all, clearly) might be to have in some cases the candidates concentrated in sub-bands of the 1 Hz band, which means to blind the search in some other sub-bands. We use a procedure which overcomes this problem.

- We fix the number N_{1Hz} of candidates to be selected in each **1 Hz band** and, in each band, these candidates are then uniformly selected over the Sky grid : **$N = N_{1Hz} / N_{SKY}$**

Example: Candidates for VSR2 (red) VSR4(blue)



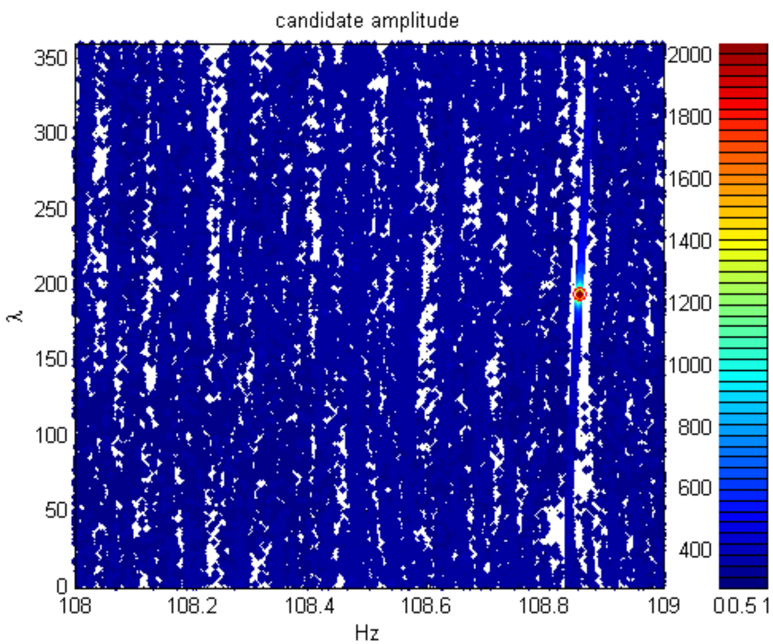
N_VSR2=194457048

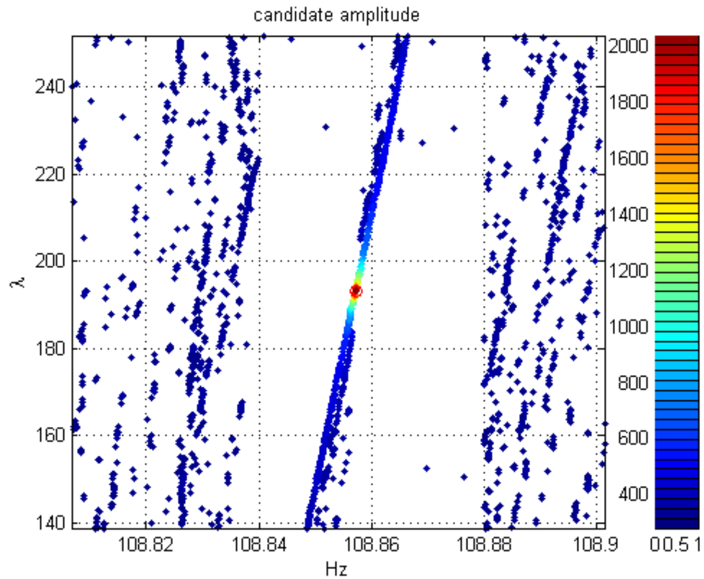
N_VSR4=193855645

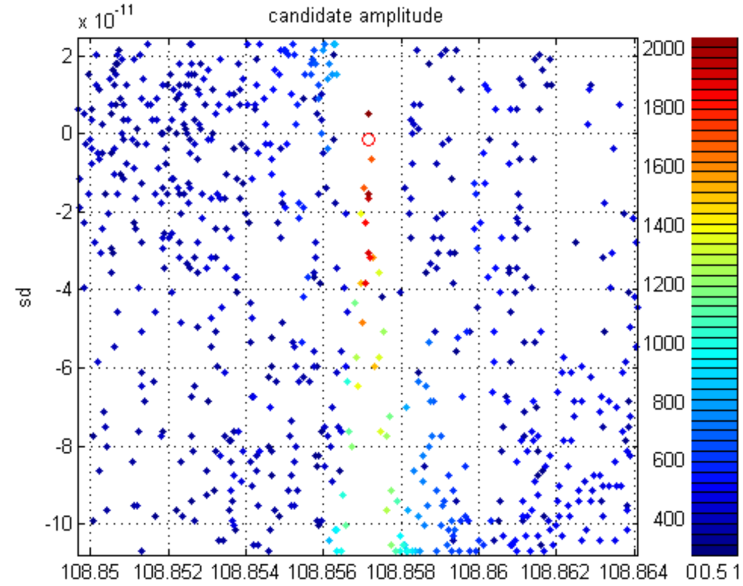
With the choice done we expected 200 10^6 candidates/run

Example: the HI called pulsar 3

Candidates [108-109] Hz
Here the sub-bands have been obtained
by dividing the 1 Hz band into 23 pieces







Computed with hfdf_hough

For all beta and lambda

Example: parameters used for the low-frequency All-Sky search

VSR2

- Beginning mjd 55020
- Final mjd 55205
- Epoch 55112
- NFFT=3611 (not 3896)
- Spin down step (natural, that is , df/Tobs)

  $7.63\ 10^{-12}$ Hz/s
- Spin down range (15 steps)
  [$-9.91\ 10^{-11}$; $1.52\ 10^{-11}$] Hz/s

VSR4

- Beginning mjd 55715
- Final mjd 55810
- Epoch 55762
- NFFT=1893 (not 1978)
- Spin down step (natural, that is , df/Tobs)

  $1.50\ 10^{-11}$ Hz/s
- Spin down range (8 steps *)
  [$-1.05\ 10^{-10}$; $1.50\ 10^{-11}$] Hz/s

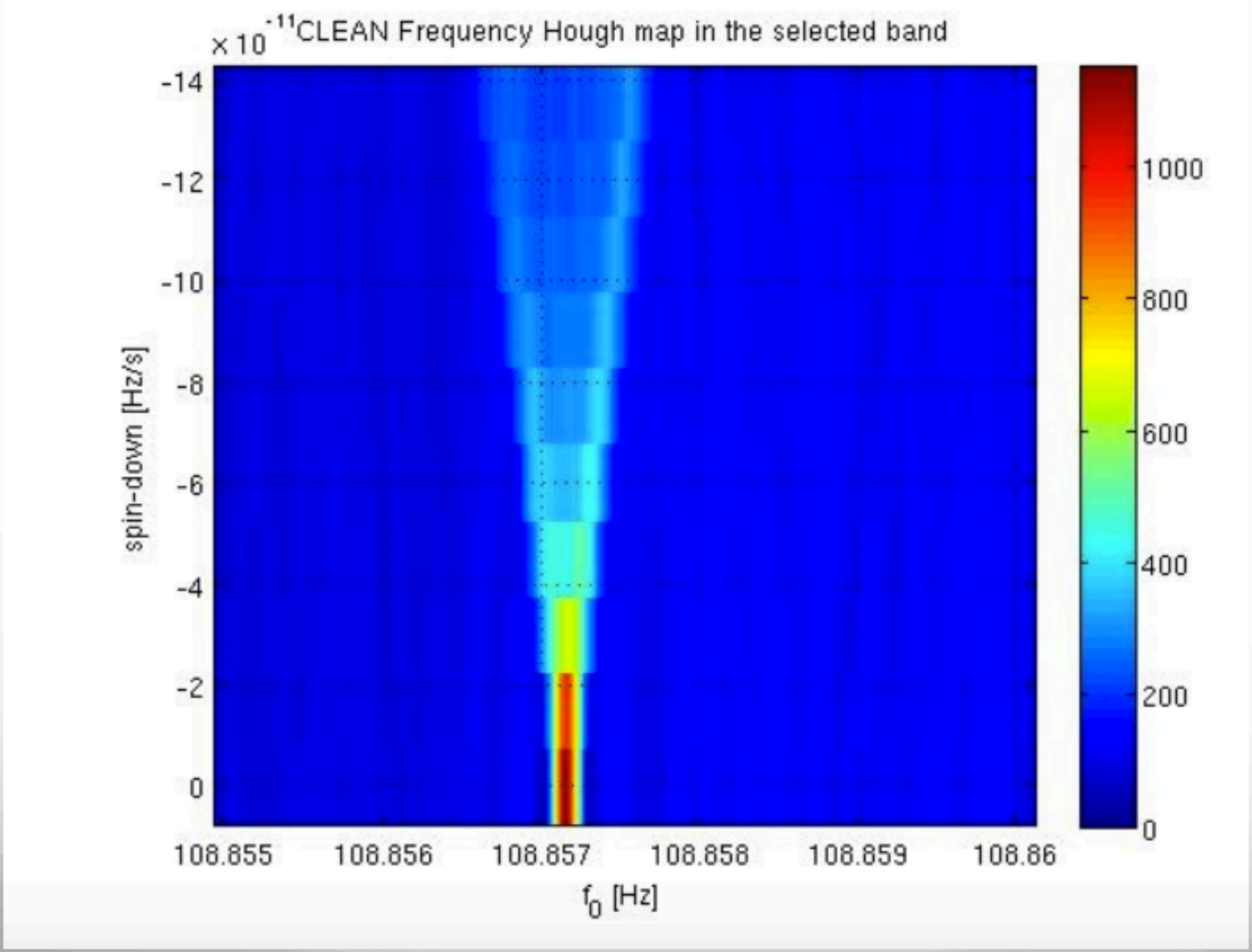
*: to cover the same spin down range in the two runs.

Robust statistics

- All the choices (t-f veto, known lines veto, weights veto) have been done using the median instead of the mean, which is more robust versus the presence of tails in the distribution.
- But not only: the dispersion parameter has also been computed using the median:

```
function m=robstat(x,p)
m(1)=median(x);
m(2)=median(abs(x-m(1)))/0.6745; % norm to 1
sigma for normal data
```

Hough map on pulsar_3



But…

This is part of your work today (and/or tomorrow)

You will be using (Virgo) VSR4 data
Detailed Instructions available in the Moodle
@ CW-Hands on Lecture

Study of Hardware injections

- In the band [0-128] Hz we have 3 hardware in injections (see the lectures material);
- If SNAG is properly installed and added to the Matlab path, you will see them by typing:
  - pulsar_5
  - pulsar_8
  - pulsar_10 (too faint to be used here)

You can choose **pulsar_5** or **pulsar_8** and proceed with the following analysis:

HFDF_JOB_vGRAW

hfdf_correct=HFDF_JOB_vGRAV(fmin,verb,SearchGrid,adapt)

%**fmin** =0 for HI, negative for software injections. A value to run on whatever you want.   Default is -1 (soft. inj)

%**verb**: from 0 positive (if higher more plots are done or things written). Default is 1

%**SearchGrid:** 0 or 1 (0: only exact source position) 1: reduced grid around the true position. Default is 0

%**adapt:** 0 or 2 (0= no adaptivity). Default is 2.

HFDF_JOB_vGRAW

- Run the code by typing: **HFDF_JOB_vGRAW(0,2)**
- Which means: run it on one HI, produce “many” figures. Run the analysis only for the exact source position (which is clearly much faster than running on a grid). And run the adaptive Hough.
- The goal of this exercise is to see peakmaps before and after the right Doppler correction, using quite strong signals (good to be able to visualize the result, without having to run further analysis to select the proper candidate, as is typically needed) . See the Hough map, again for a strong signal. And to familiarize a bit with the “candidate” structure, which contains the parameters of the candidates selected at this stage of the analysis

HFDF_JOB_vGRAW

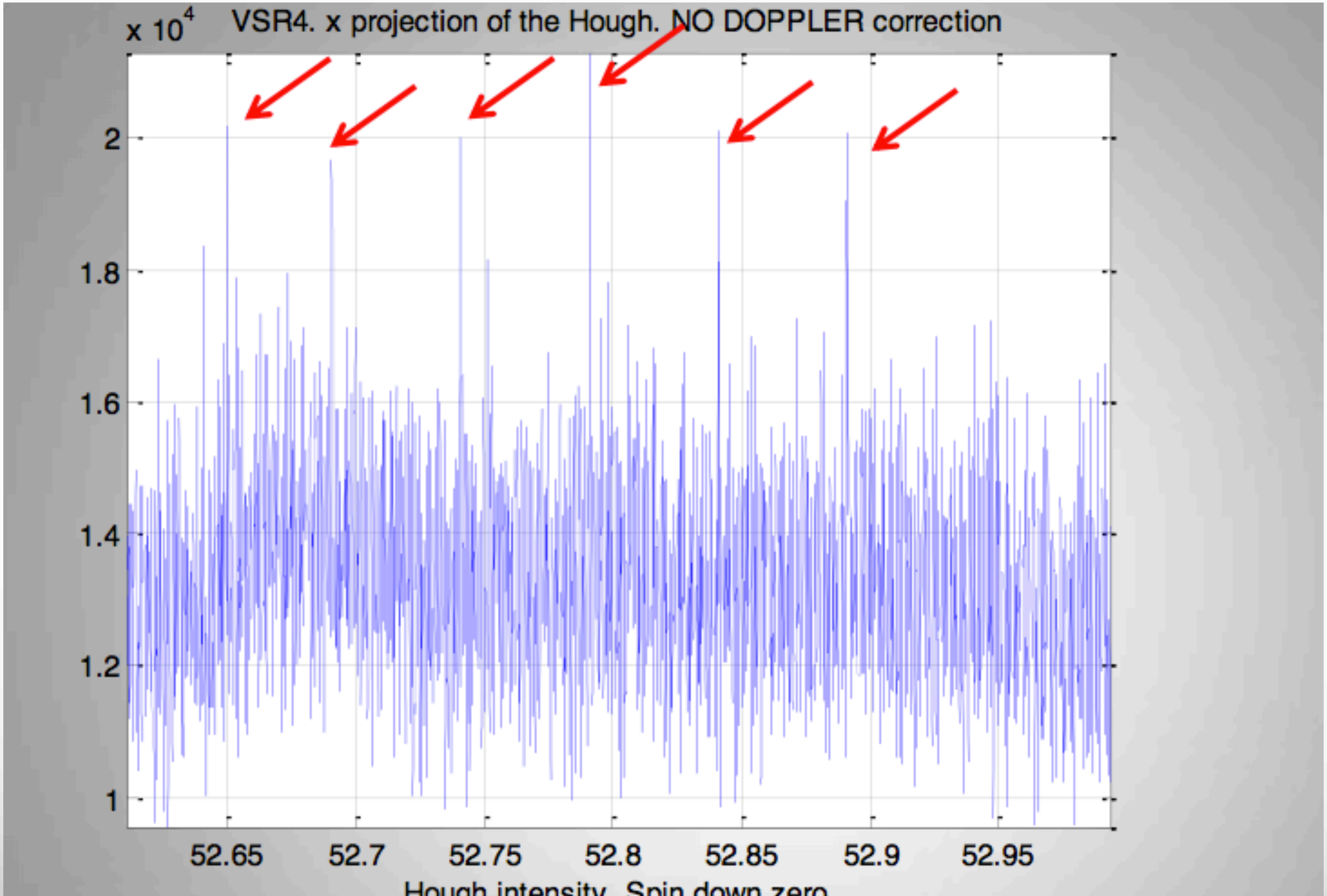
- When prompted type: pulsar_3 (or pulsar_5)
- 5 figures will be produced. Looking at Fig.5 (zoom it more !) you will see the injected signal (Doppler corrected,so it is a straight line now). And also –if you have used pulsar_3- discover “missing dots” due to a vetoed big disturbance present in the data.

Pulsar_5 is well visible also with a proper zoom of fig. 2 (the original peakmap, not Doppler corrected)

- Hough map are clear in both cases.
- On the screen you see some parameters of the candidate found: frequency, spin down, distance from the injection, Hough amplitude, CR

The directory Results is produced and contains a .mat file with all the candidates selected. We will load one of these files later on. “EC” in the name stands for “Exact Coordinates”

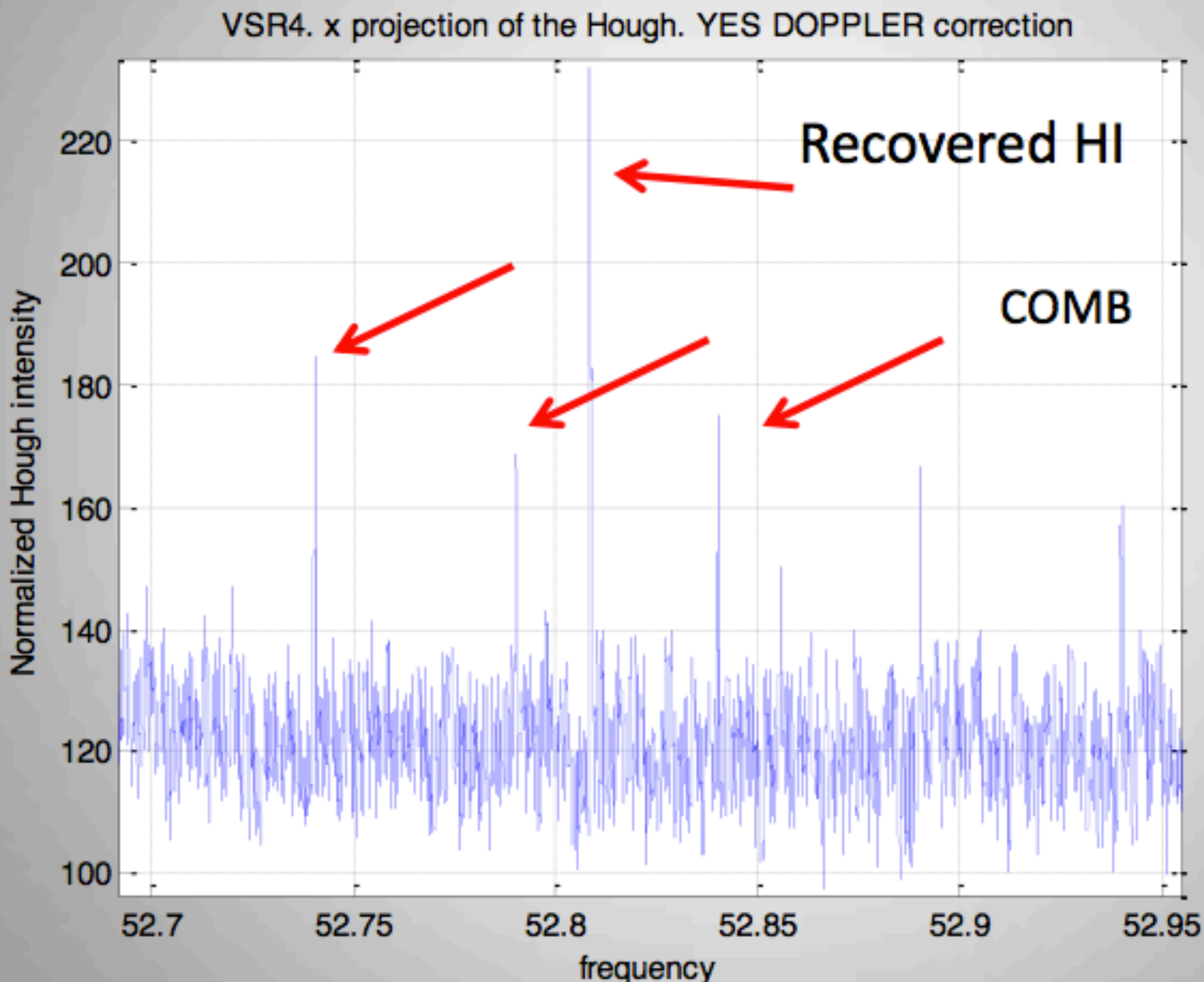
The effect of the Doppler correction and Hough transform on disturbances:



Before..

Comb @ 50mHz at 0.41 Hz (52.741 52.791 52.841 ...)

With the Doppler correction the relative amplitude of these disturbances changes:



After

What if we use the wrong parameters ?

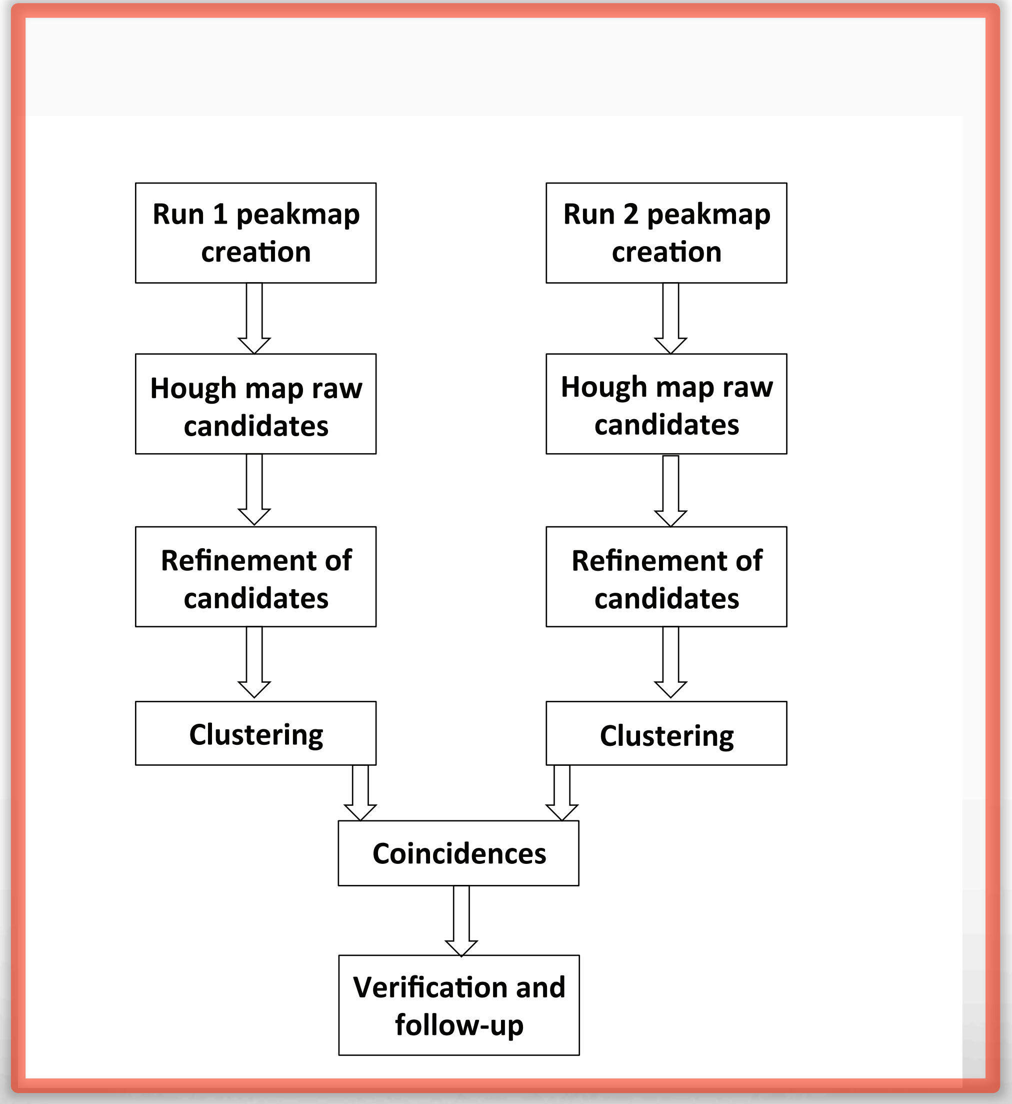
- Go to line 257-258:
  - SLong=lambda;
  - SLat=beta;

-→Modify SLong, by choosing a random value between [0-360] deg and SLat, [-90-90].

Rerun the code, as before.

Try to see or evaluate the difference in the peakmap after the (now wrong) Doppler correction and in the final Hough amplitude and CR result

Scheme of the pipeline



Candidate clustering and coincidences

- For computational efficiency reasons, candidates of the two runs are *clustered* before making coincidences.

- A cluster is a collection of candidates such that each of them has a distance d in the parameter space less than d_{clust}=2 from at least one other candidate of the same collection.

$$d = \|\vec{c}_1 - \vec{c}_2\| = \sqrt{k_\lambda^2 + k_\beta^2 + k_f^2 + k_{\dot{f}}^2}$$

$$k_\lambda = \frac{|\lambda_2 - \lambda_1|}{\delta\lambda}$$

$$\delta\lambda = \frac{d\lambda_1 + d\lambda_2}{2}$$

And similarly for the other parameters

○ Coincident candidates

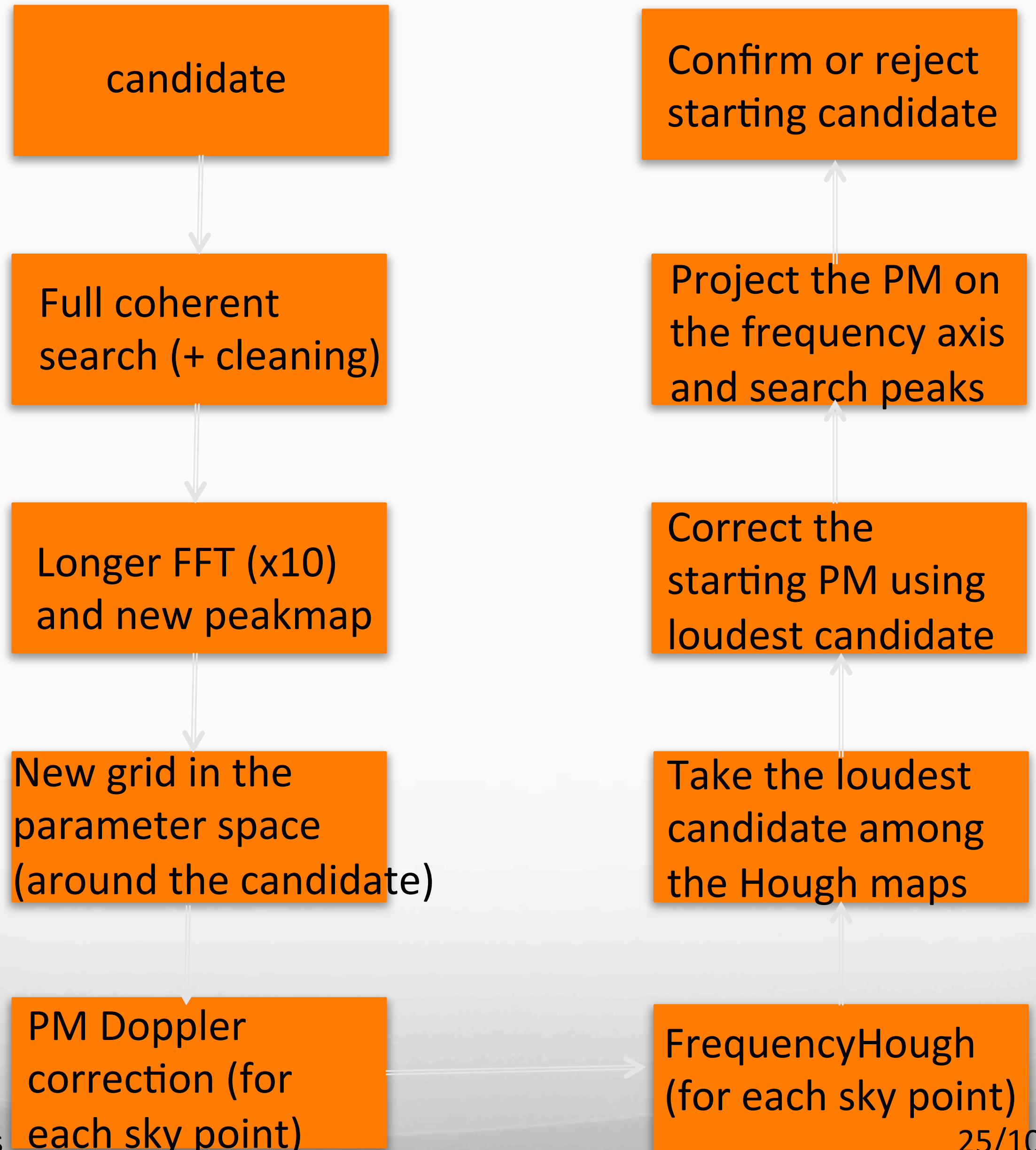
are selected in the following way:

- determine coincident cluster pairs (i.e. those having at least a pair of candidates with distance less than d_{coin}=3).

- On the basis of global cluster parameters, pairs of clusters which cannot be in coincidence are not taken into account (thus reducing the computational effort).

- For each pair of coincident clusters take as coincident candidates those with the **smallest distance** (i.e. we select one pair of coincident candidates for each pair of coincident clusters).

Follow-up



We will not discuss this today.

It’ s important that you know that →

Conclusion

- Once we have the coincidences we use a ranking procedure to select only the most significant ones
- We check is disturbances were surviving up to this level and in case exclude them
- On the survived candidates, we run the follow up procedure, which means –shortly-that we run a more sensitive analysis around the found parameters
- So far, we have in the end excluded all the candidates and set up Upper Limits, as the only result on the analysis

…..hopefully we will soon or later arrive to a point where we will not be able to exclude all the candidates…and additional intestigations will be needed…In the end: ask astronomers to point at that position ?

Other proposed exercises:

- Create software injected signals, by choosing some parameters and their number
  - Run the Hough code, by using only the exact position of the source. Check if these signals have been recovered as Candidates and what is the distance between the candidate recovered and the injection

- Create only 1 injected signals and run on this the Hough code over a Sky Grid around the correct position. Compare the results

- ***We will now read together instructions posted in the Moodle***…

Backup slides

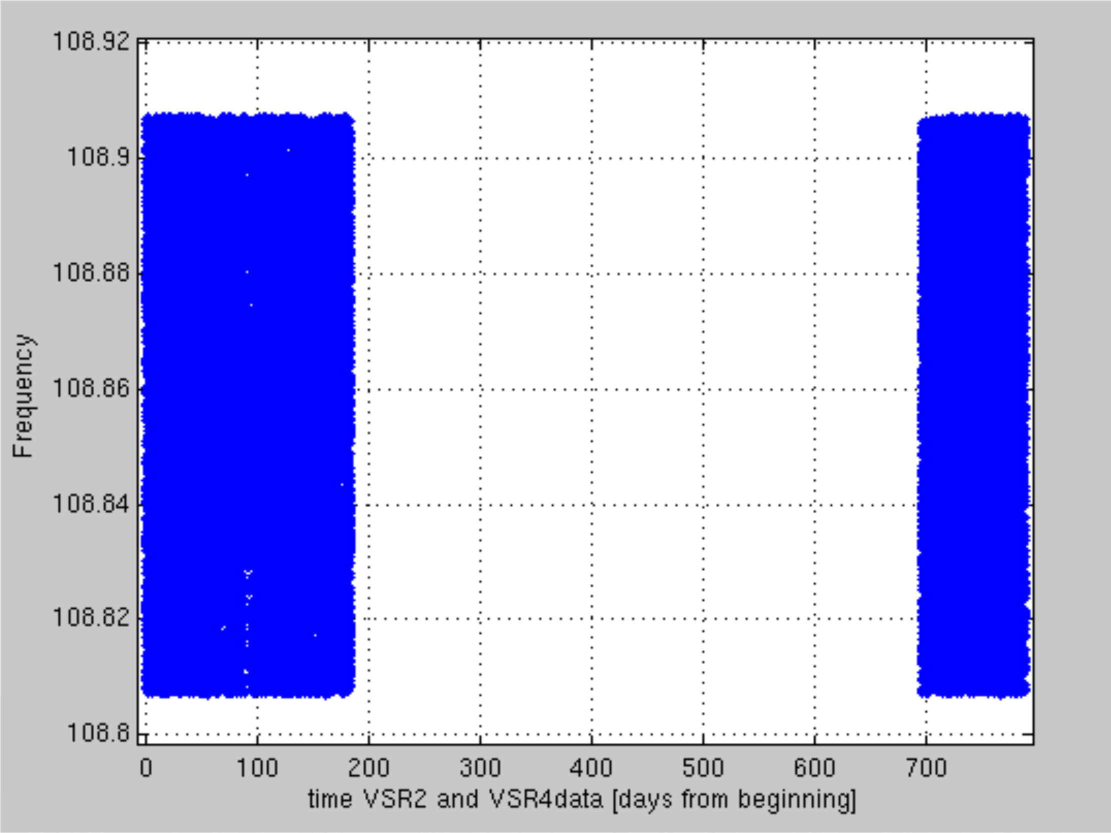
2. **From the corrected data compute new, longer, FFTs and a new peakmap**. Enhancement factor: 10 → T_{FFT}=81920 seconds

Done separately for VSR2 and VSR4.
We consider ±0.05 Hz around candidate frequency.

We select peaks with a threshold of 2.34 (while we used 1.58 in the first step).

The resulting probability of selecting a noise peak is less than 4E-3, i.e. ~20 times smaller than with the initial threshold.

3. **A single peakmap is built** from the two separate peakmaps (one from VSR2 and one from VSR4).



This peakmap is the input of a new FrequencyHough step.

4. **A new grid is built** in the parameter space (around the candidate)

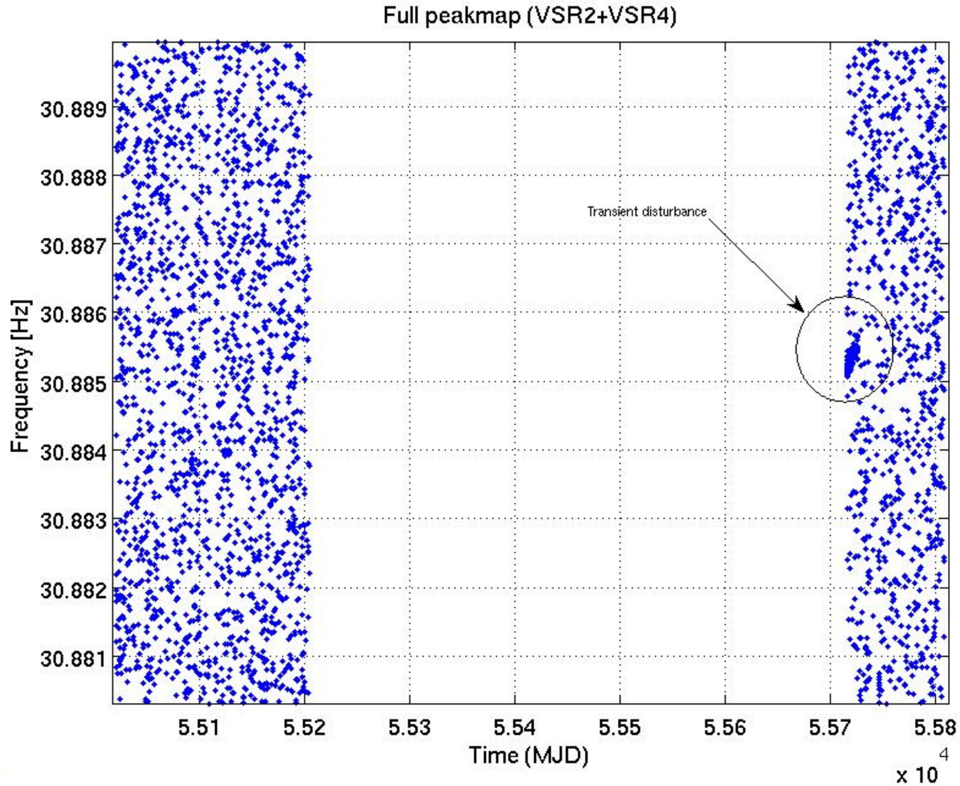
Frequency resolution: Df = 1/(10*Tfft) = 1.22* 10^{-6} Hz
The covered frequency band is 0.1Hz around candidate frequency.

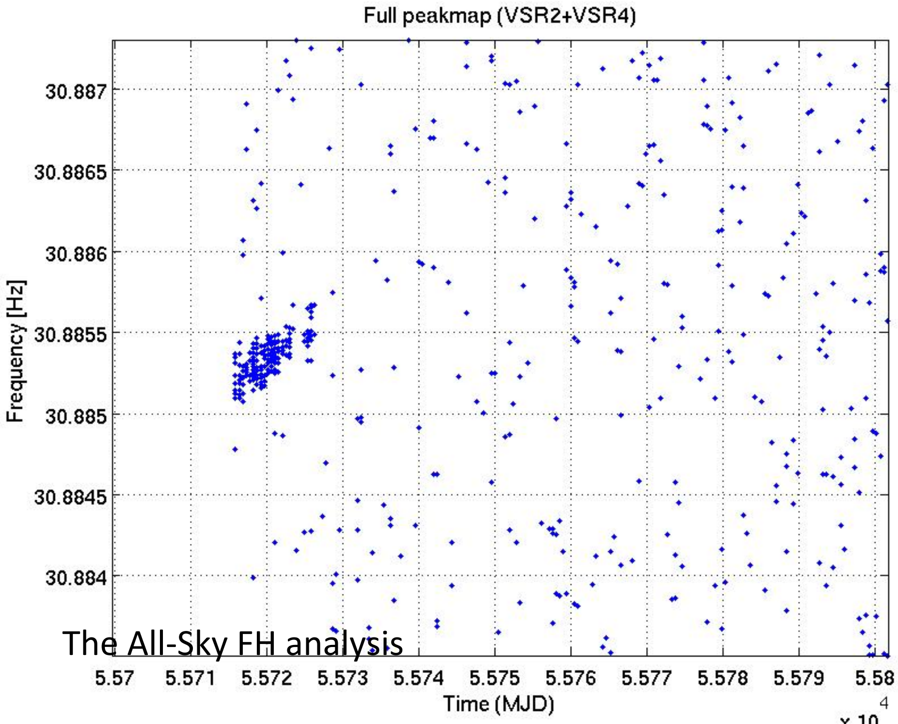
Spin-down resolution: Dfdot=1.5 10^{-12} Hz/s.
It covers ±1 coarse bin around candidate spin-down

Sky: 41*41=1681 patches around candidate position.
They cover ±0.75 times the coarse patch.

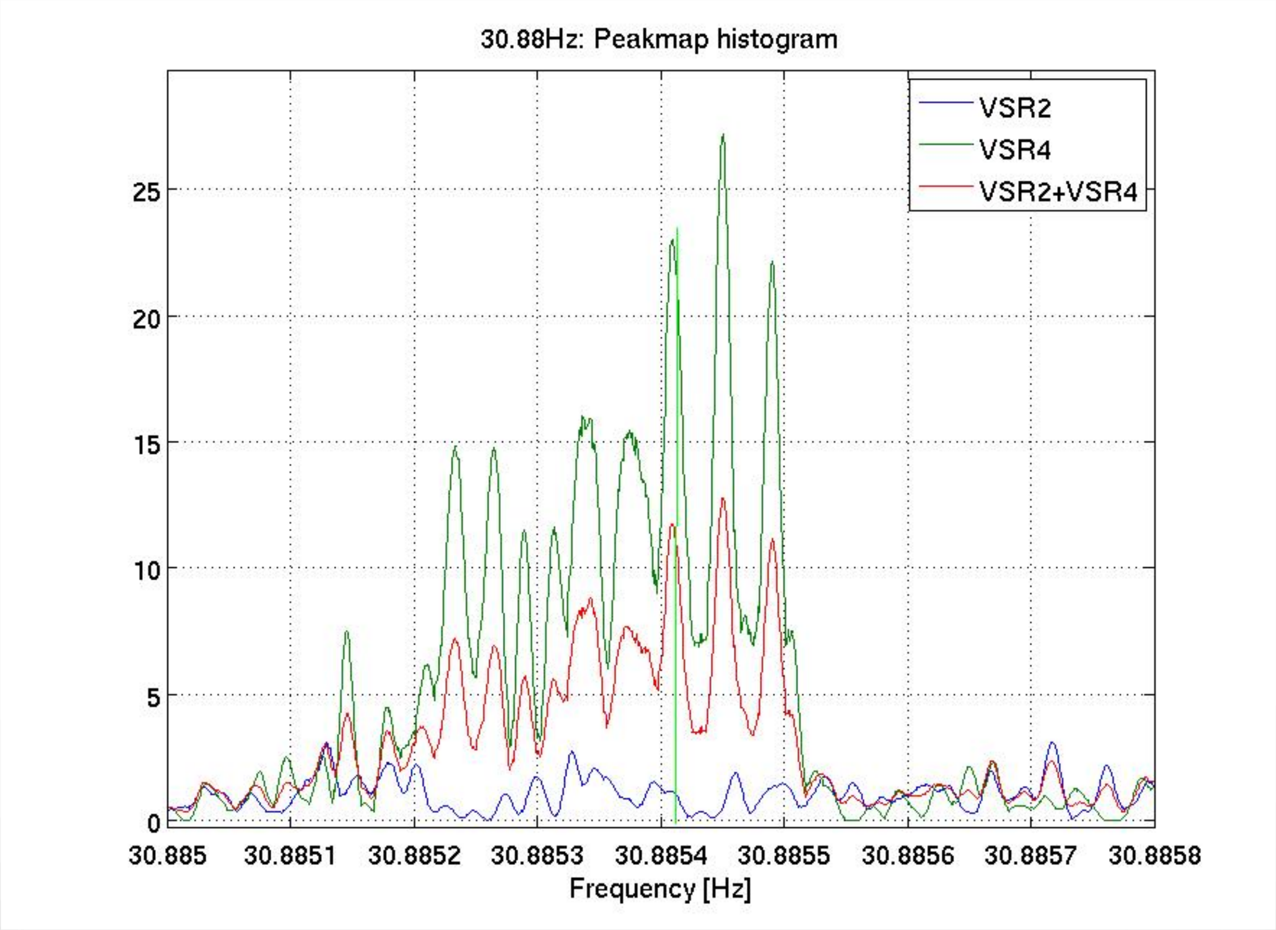
Looking at the peakmap:

There is an excess of points at the right frequencies over about 10 days at the beginning of VSR4.



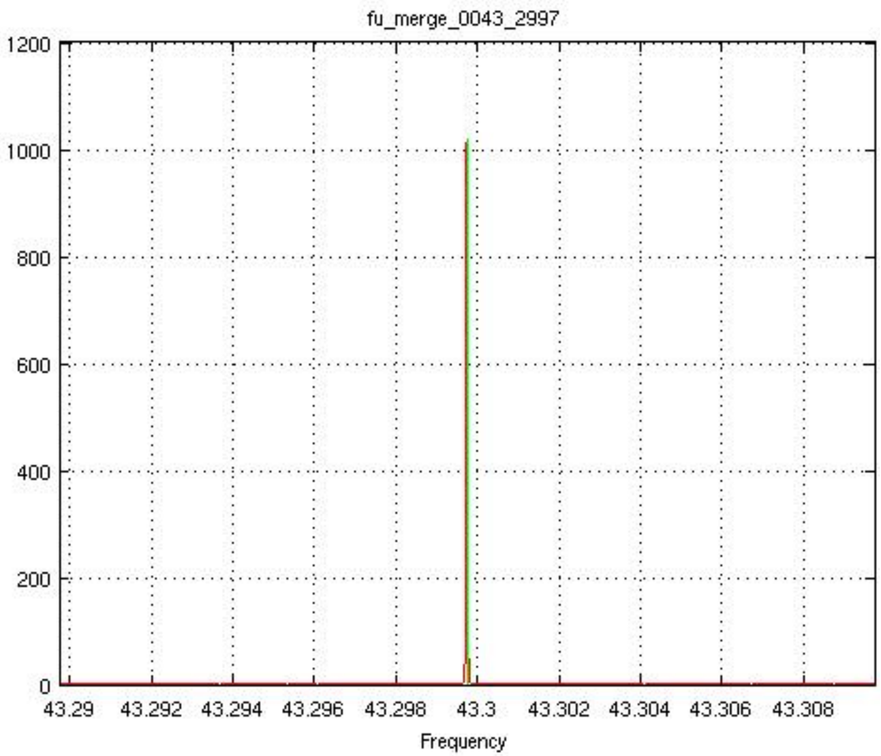


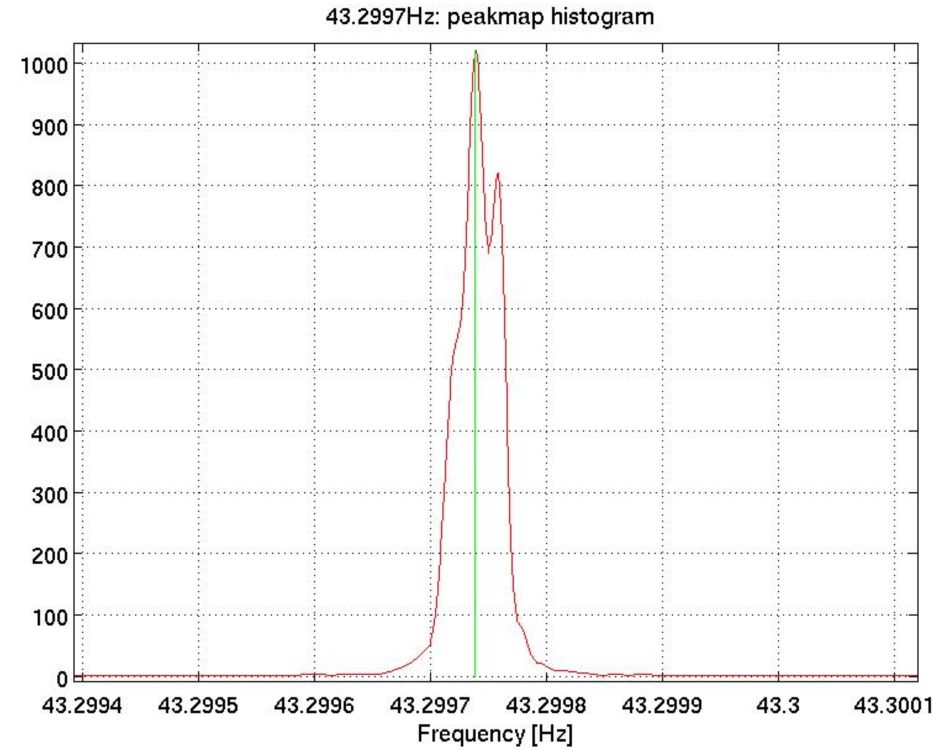
Indeed, by looking at the two runs separately we have a significant candidate only in VSR4:



We conclude this candidate is due to noise.

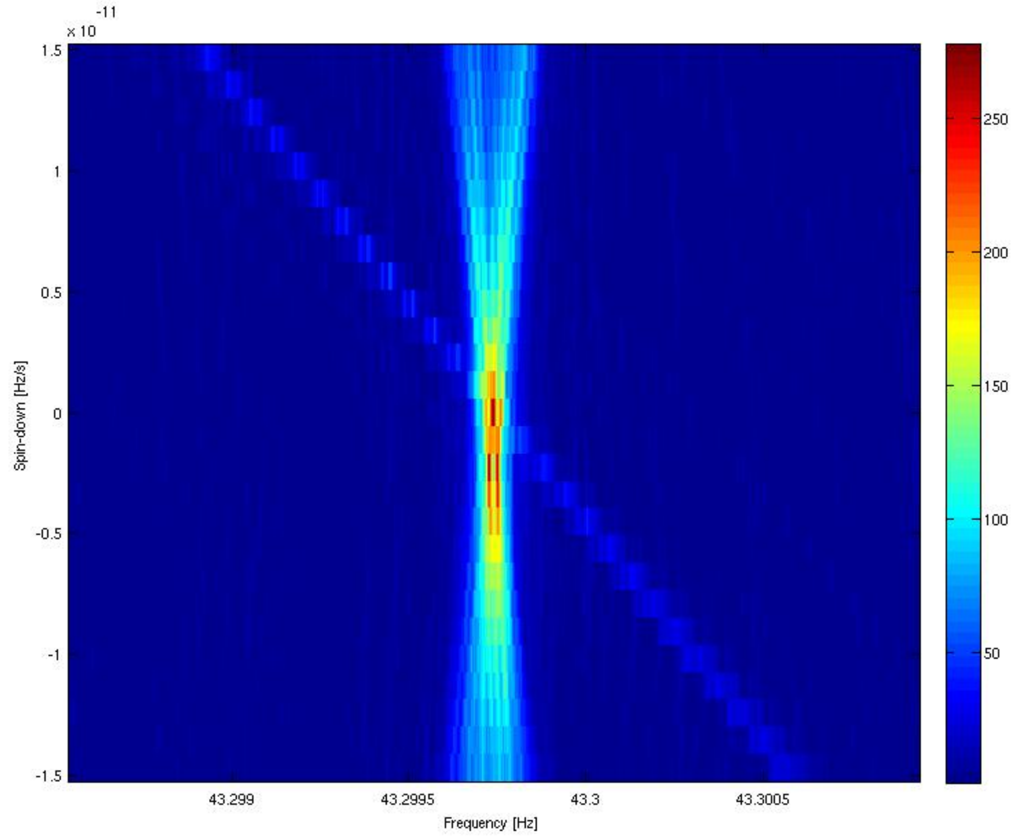
Candidate 43.30Hz





p	f	fdot	l	b
<5.5E-3	43.29974	-5.04E-12	198.30	89.42

The candidate is at the ecliptical pole, where constant frequency “signals” tend to collect.



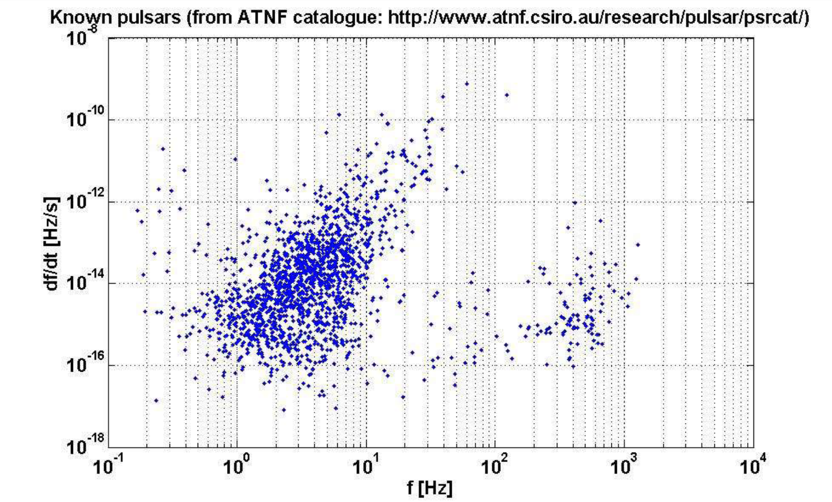
The signal appears to be very strong in VSR2 and very weak in VSR4.

This is also clear from the peakmaps, see next slide.

✧ We classify as *continuous* (CW) those GW signals with duration much longer than the typical observation time of detectors.

✧ CW are typically emitted by sources with a mass quadrupole moment varying in time in a quasi-periodical way.

✧ For Earth-bound detectors the most interesting sources of CW involve *distorted* spinning neutron stars (NS), isolated or in binary systems.



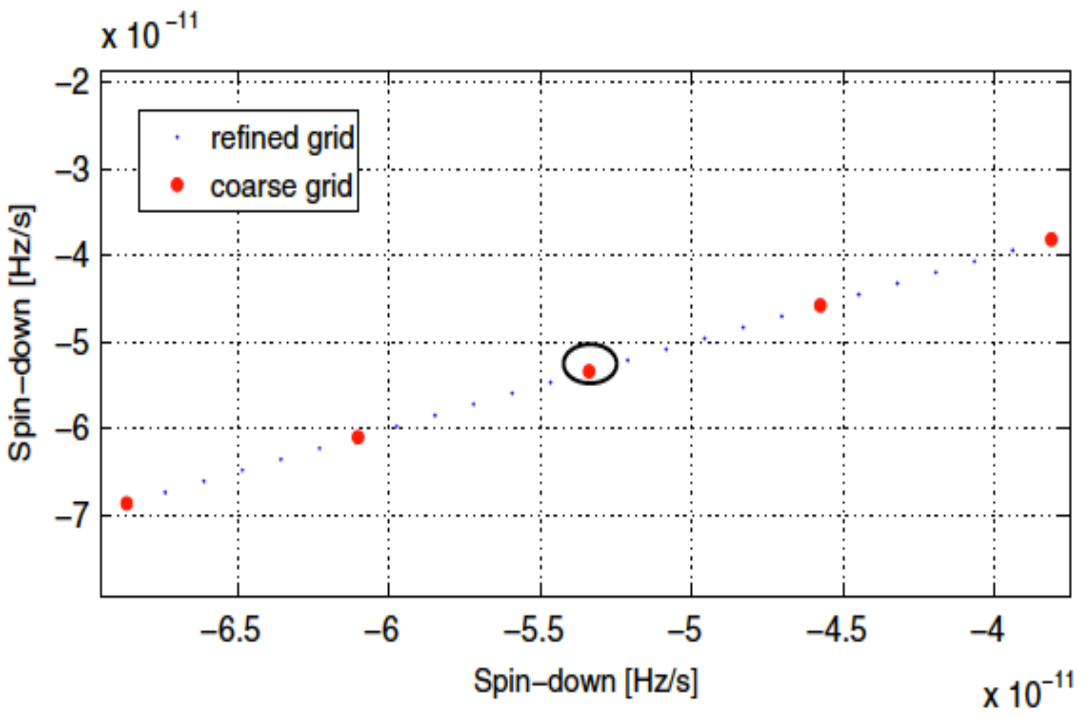
We know that potential sources of CW exist: 2500+ NS are observed (mostly pulsars) and $O(10^8 - 10^9)$ are expected to exist in the Galaxy

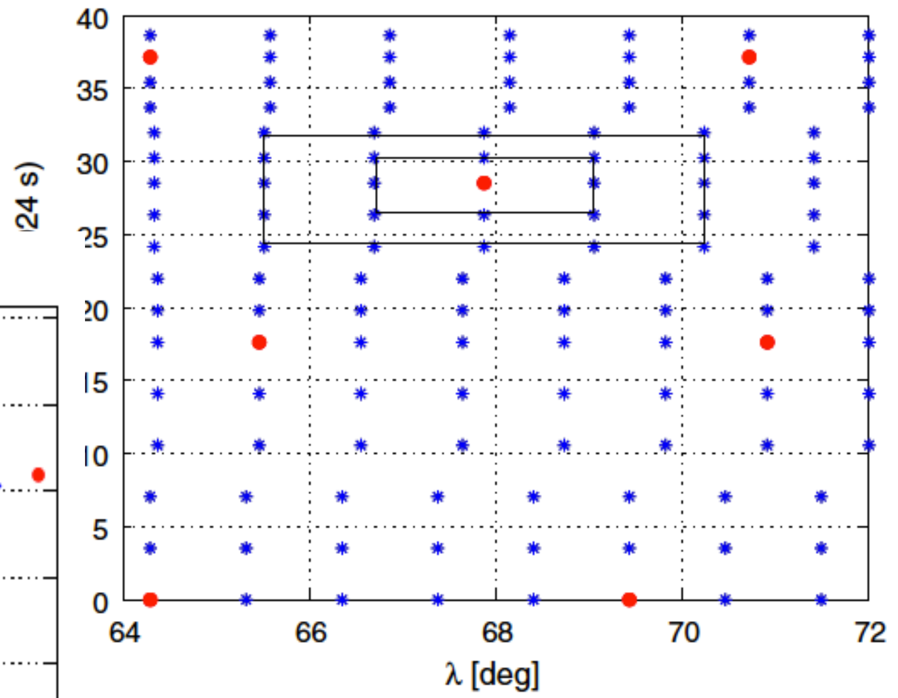
Incoherent vs coherent

$$h_{SNR=1} = h_{SNR=1}^{(OD)} \cdot \sqrt[4]{\frac{T_{obs}}{T_{coh}}} \propto T_{coh}^{-\frac{1}{4}}$$

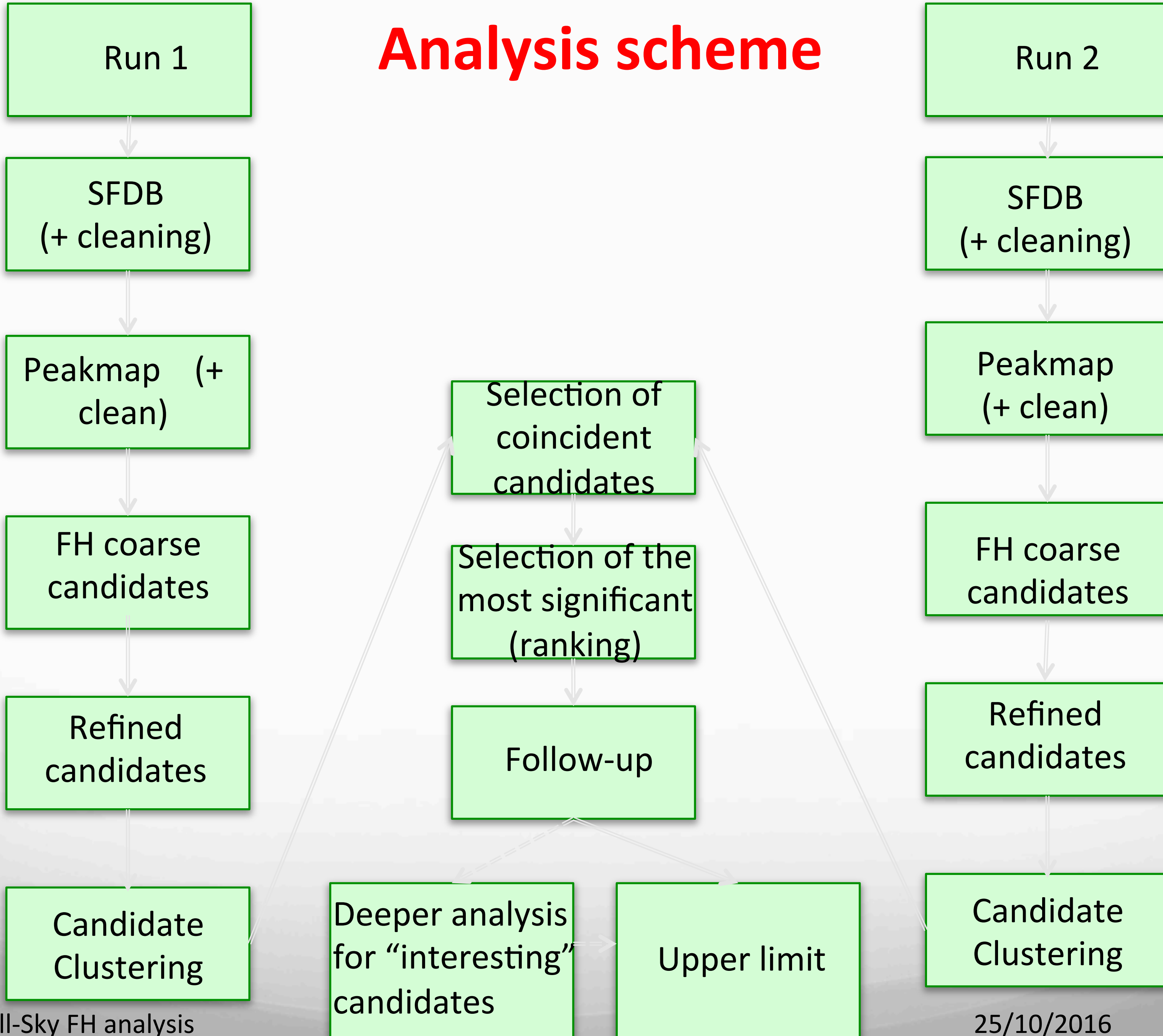
Optimal detection (OD) is for fully coherent searches

Examples of refined grids
Red= coarse
Blue=refined





Analysis scheme



Optimal grid on the sky. How many points ?

$$N_D = \frac{2 f_{\max}}{10000}$$

Bins in the Doppler band

$$N_{sky} = \frac{2\pi^2 N_D{}^2 K^2}{\pi}$$

Optimal grid

$$N_{sky} = 2\pi^2 N_D{}^2 K^2$$

Rectangular grid

Grid on the sky. How many points ?

- The grid can be optimal respect to the frequency variation with the variation of ecliptical longitude λ and latitude β

$$\Delta\lambda = \frac{1}{N_D \cos\beta}$$

$$\Delta\beta = \frac{1}{N_D \sin\beta}$$

$$f(t) \simeq f_0\left(1 + \frac{\vec{v}\cdot\hat{n}}{c}\right)$$

$$\approx f_0\left(1 + \frac{\Omega_{\rm orb} R_{\rm orb} \cos\beta \sin(\Omega_{\rm orb} t)}{c}\right), \quad (35)$$

where $R_{\rm orb}$ is the radius of the Earth orbit. The observed frequency variation during Δt is given by

$$\frac{df}{dt}\Delta t \approx f_0 \frac{\Omega^2_{\rm orb} R_{\rm orb} \cos\beta \cos(\Omega_{\rm orb} t)}{c} \Delta t. \quad (36)$$

The maximum value of this variation is

$$\Delta f_{\rm max} = f_0 \frac{\Omega_{\rm orb} R_{\rm orb} \gamma \cos\beta}{c}. \quad (37)$$

If we fix $\Delta f_{\rm max} = \delta f$ we find the angular resolution along the longitude which is, in radians,

$$\delta\lambda \equiv \gamma = \frac{c}{f_0 \Omega_{\rm orb} R_{\rm orb} T_{\rm FFT} \cos\beta} = 1/(N_D \cos\beta), \quad (38)$$

where N_D is

$$N_D = \frac{f_0 \Omega_{\rm orb} R_{\rm orb} T_{\rm FFT}}{c}. \quad (39)$$

And a similar reasoning
For the latitude

The peakmaps

If P is the probability to have one peak above the threshold theta in the normalized amplitude spectra in the presence of a signal of SNR0= lambda and P(theta;0) is the same in the absence of signals we define

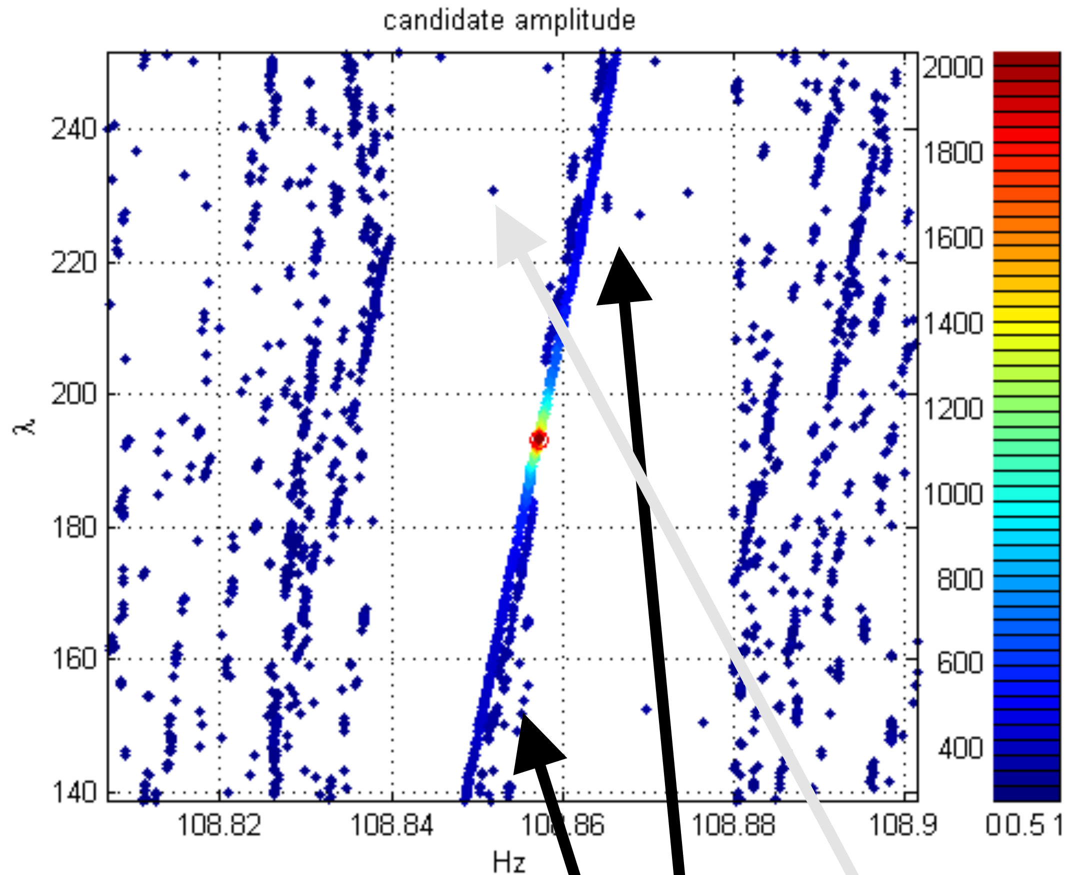
$$\Phi(\theta,\lambda)=\frac{P(\theta,\lambda)-P(\theta,0)}{\sqrt{P(\theta,0)(1-P(\theta,0))}}$$

Threshold: 2.5

Study the efficiency of the method

Parameters:

- $\Delta f = 10^{-3}$ Hz (in the peakmap)
- Source intrinsic frequency f = 50 Hz
- spin down value :DTRUE = $5 * 10^{-8}$ Hz/s
- spin down resolution : $\Delta d = \Delta f / tobs = 2 * 10^{-9}$ Hz/s
- spin down range: dmin= -150* Δd; dmax=150* Δd
- Δf0 = Δf / res Resolution of the Hough plane frequency



Candididates for all bs

Colours: Hough amplitude

- The band [108-109] Hz is divided into 23 small sub-bands (23 is the number of candidates/patch at this frequency).

- Thus we select one or two candidates in each 1/23 Hz (0.043 Hz)sub-bands
- The HI is a huge signal, which implies that the second level candidates are mostly due to the HI and thus not selected, which explains the empty region around the HI
- <u>But there are a few second order candidates selected.</u> The one indicated by the

green arrow is not due to the HI

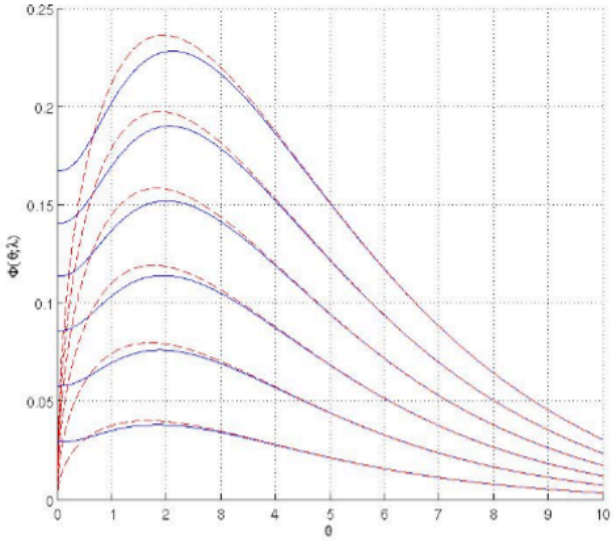


Figure 6: Phi function as a function of the threshold for peaks (blue, continuous) and for "alla bove the threshold" (red, dashed) for different signal amplitudes.

The ideal theoretical choice for the threshold would be to take the value(s) corresponding to the maximum of these curves. These values are listed in columns 2 of the following table, while in column 3 the corresponding probability of selecting a peak are given.

λ	θ	P(θ;0)	θ'	P'(θ';0)
0.10	1.84	0.1349	1.66	0.1901
0.20	1.90	0.1283	1.72	0.1791
0.30	1.94	0.1240	1.78	0.1686
0.40	2.00	0.1178	1.84	0.1588
0.50	2.06	0.1119	1.90	0.1496
0.60	2.12	0.1062	1.96	0.1409

Table 1: Ideal thresholds and peak selection probability for the "local maxima" criterion (2nd and 3rd column) and for the "all above threshold" criterion (4th and 5th columns).

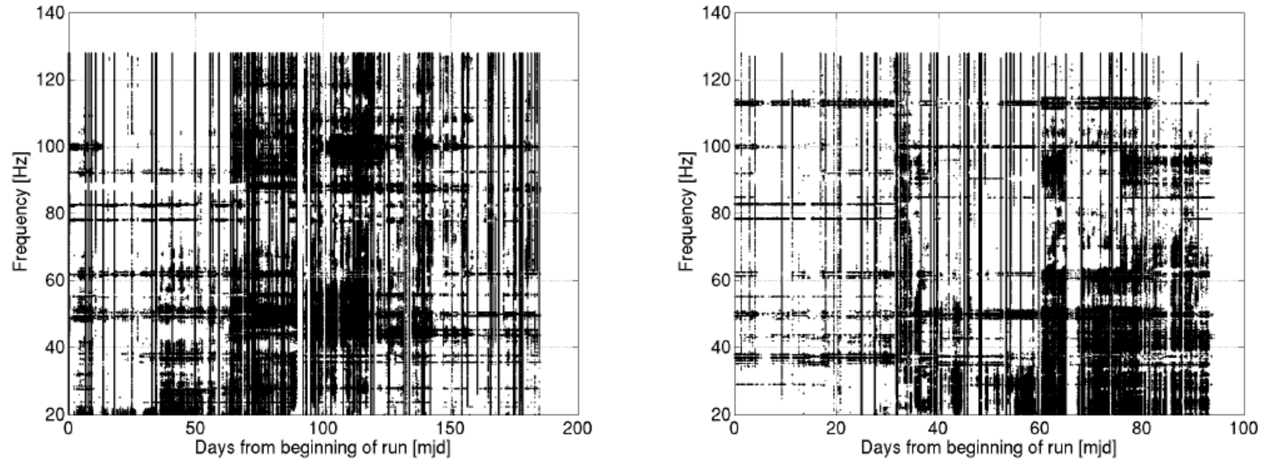


FIG. 4: Time-frequency plot of the peaks removed by the “gross histogram” cleaning procedure for VSR2 (left) and VSR4 (right).

Time/Frequency cleaning

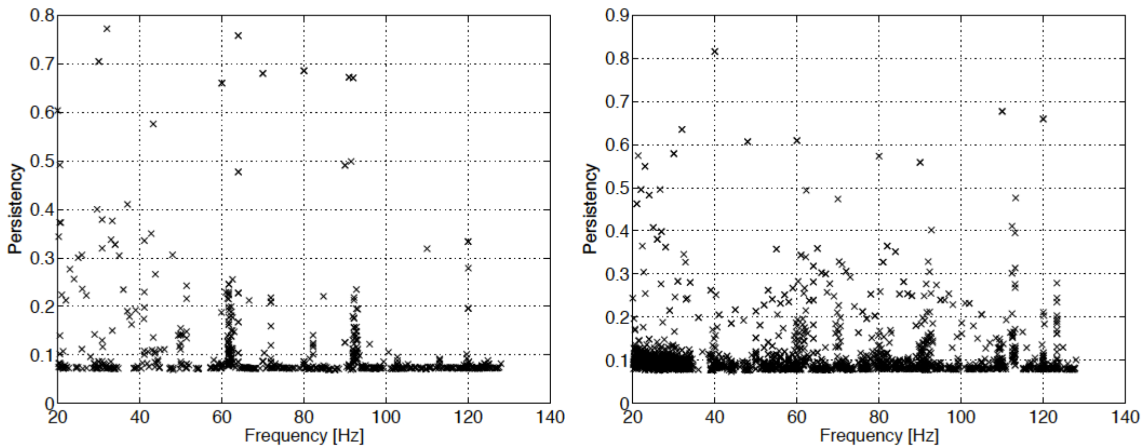


FIG. 6: Lines of constant frequency vetoed on the basis of the persistence, shown in the y-axis, for VSR2 (left) and VSR4 (right). We have removed 710 lines for VSR2 and 1947 lines for VSR4.)

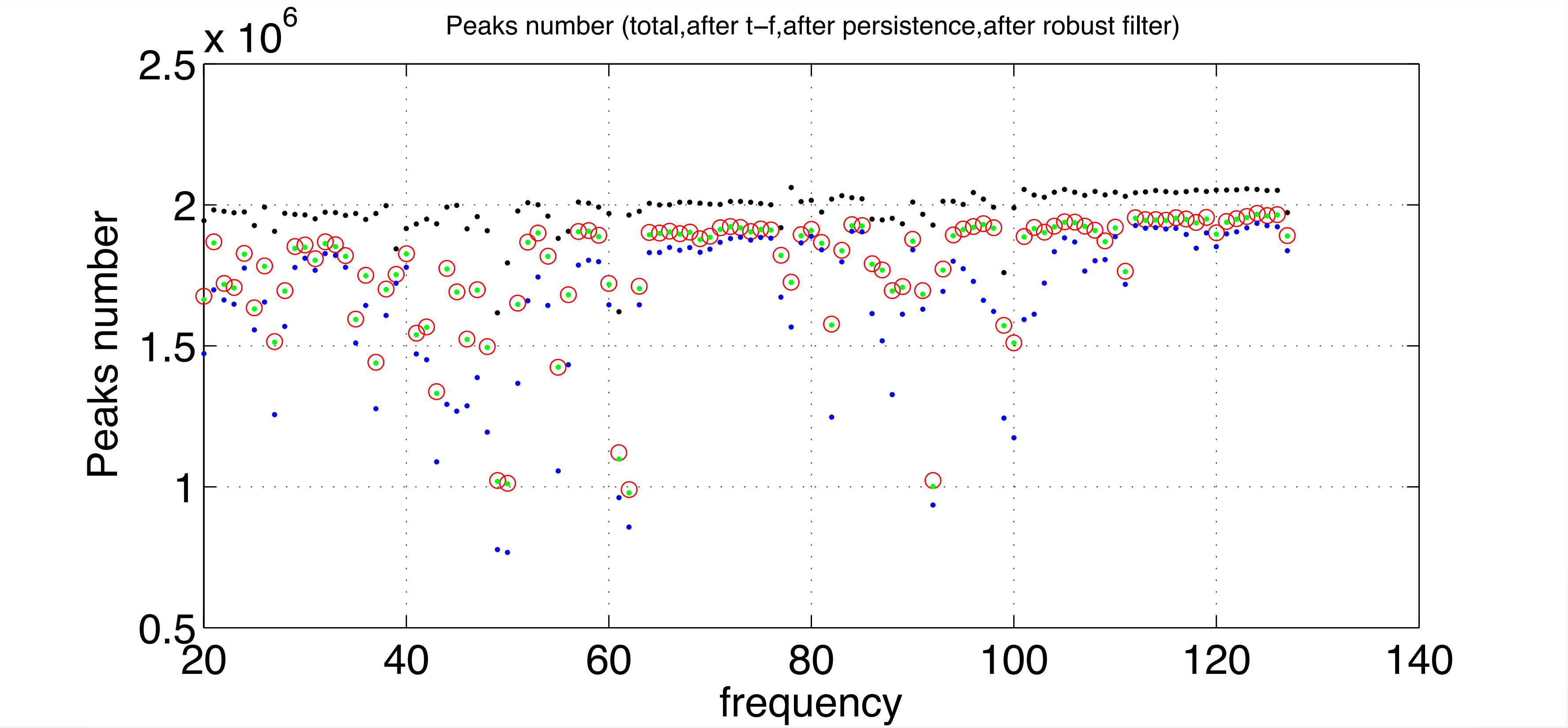
Persistency veto

	% after “gross histogram” veto	% after persistency veto	N. of peaks (after vetoes)
VSR2	89.7	89.5	191,771,835
VSR4	86.9	86.5	93,896,752

Function hfdf_peak.m

- Given a Hough map over 1 Hz and be N_0 the number of candidates to be selected in that band and for that Sky position (l, b).
- We divide the frequency axis of the map into N_0 small bands and we select the highest candidate in each of these bands.
- We end up with one vector with the maxima for that frequency band and that Sky position.
- In the actual search in each small band, after having excluded the maximum found and +- 4 frequency bins around it , we look for a second maximum and if it is farther than +-8 bins from the one already found we add it to the vector. We have thus selected 2 candidates per sub-band in almost all the cases: 97.2% in VSR2 and 96.9% in VSR4

VSR2



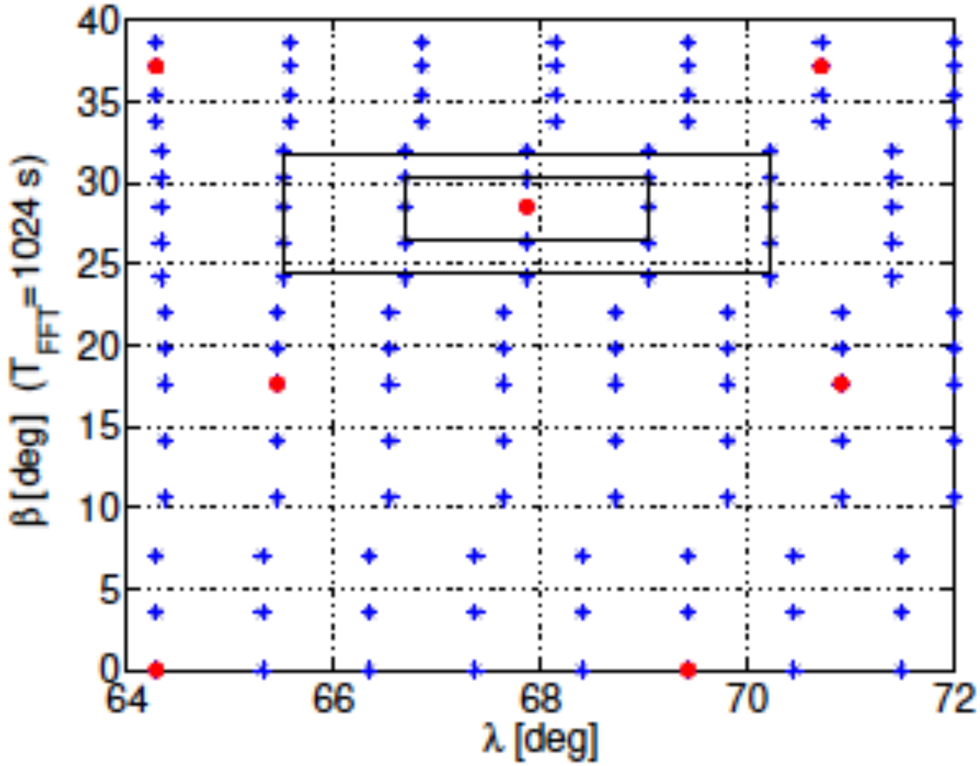
Black: total number of peaks in each 1 Hz band

Red open circles: after the gross histogram veto (time-frequency veto)

Green: after removal of lines of constant frequency

Blue: after removal of peakmaps with artificially low weights (robust filter)

The refined Sky Grid

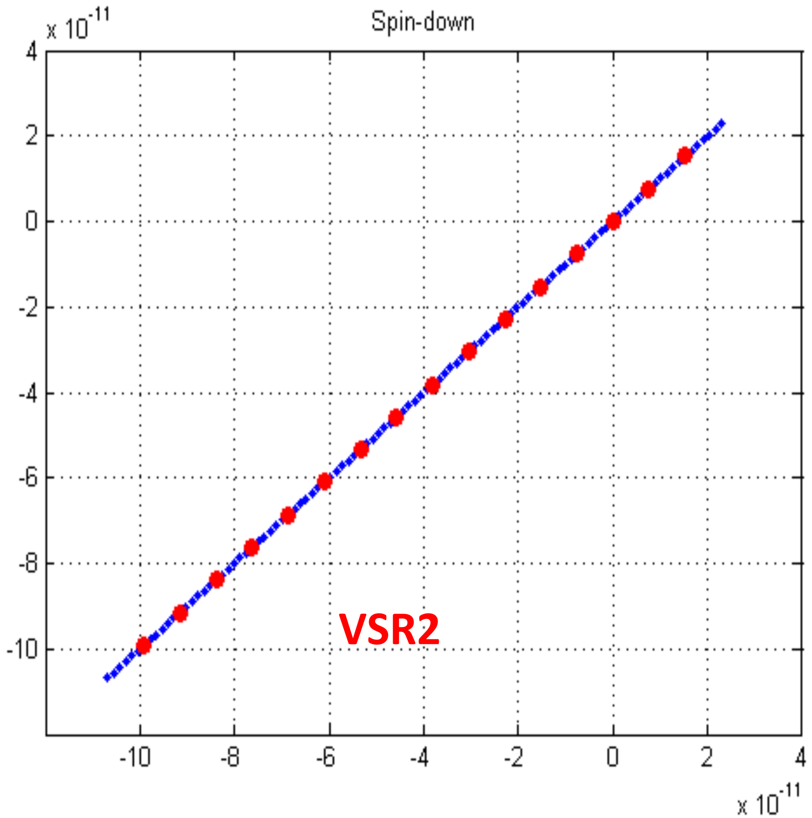


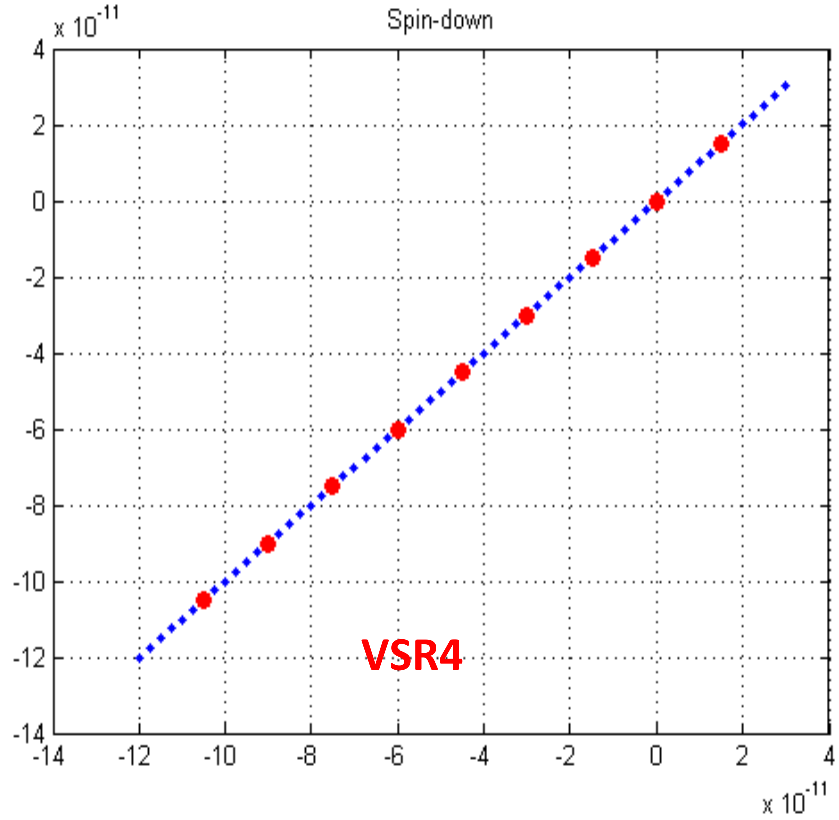
11: An example of sky grid. Red dots define points of the coarse grid, black asterisks are points of the refined gr
rectangles defines the two "layers" that identify the refinement range around an hypothetical candidate.

N layers =2 mean an over-resolution factor of
Ksky = 2 N layers +1 = 5.

Note that the over-resolution is symmetric around the coarse candidate

Spin-down range and refinement

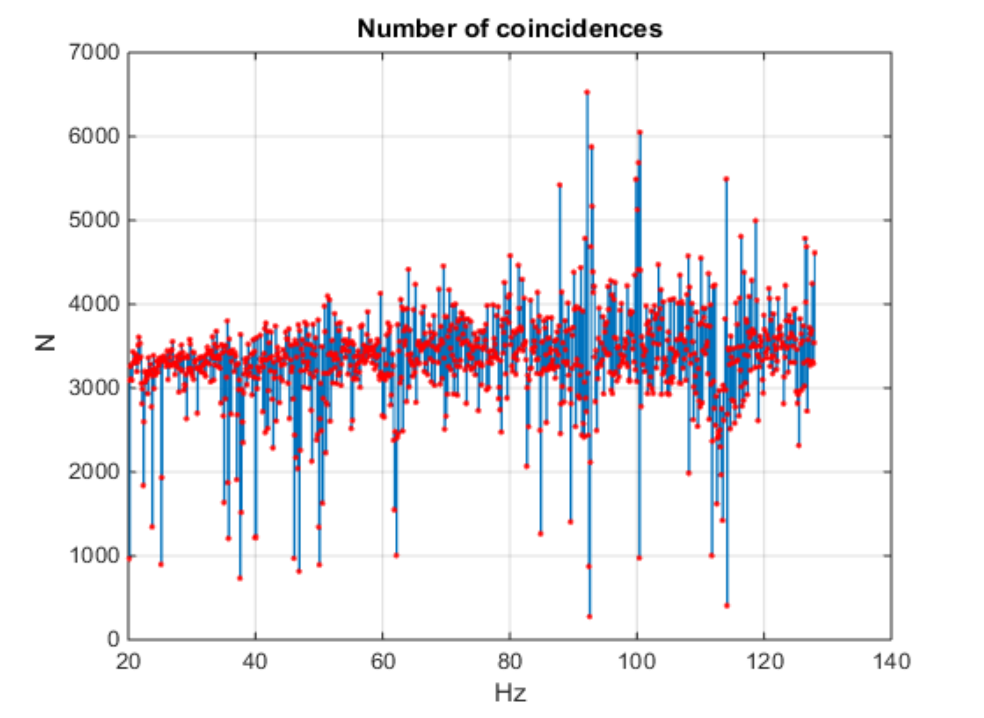




A factor 6 for the refinement, see next slide

Ranking

- Coincident candidates are divided in bands of 0.1Hz



- In each band (and for each detector): order candidates in descending order of the Hough amplitude

- Assign a rank to each of them: from 1/N to the highest to 1 to the smallest (N: number of candidates in the band)