Particle Physics - Introduction
A.A. 2018 - 2019

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last mod.
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Contents

— Nostro figlio sta cambiando una lampadina... E' meraviglioso quello che insegnano all'università, al giorno d'oggi...

"Our son is changing a light bulb... What they teach at university nowadays is wonderful..."

slides / textbooks / original

- These slides have many sources (lectures in our + other Department(s), textbooks, seminars, …); many thanks to everybody, but all the mistakes are my own responsibility;
- download from http://www.roma1.infn.it/people/bagnaia/particle_physics.html
- comments and criticism to paolo.bagnaia@roma1.infn.it (**please !**)
- they are only meant to help you follow the lectures (and remember the items);
- i.e. NOT enough for the exam; students are also required to study on textbook(s) / original papers (see references);
- the original literature is always quoted; sometimes those papers offer a beautiful example of clarity; however, particularly in recent years, their technical level is difficult, probably more at PhD student level, than for an elementary presentation (i.e. **you**);
- however, students are strongly encouraged to attack the real stuff: these lectures are NOT meant for *amateurs* or interested public (which are welcome), but for future professionals !

Thanks !!!

Enjoy them !!!

PB

References

[BJ] **W. E. Burcham - M. Jobes – Nuclear and Particle Physics** – Wiley – 768 pag. [*clear, well-organized, old*];

[YN] **Yorikiyo Nagashima – Elementary Particle Physics** – Wiley VCH – 3 vol. [*clear, modern, complete, very expensive*];

[Bettini] **A.Bettini - Introduction to Elementary Particle Physics** [*another textbook*];

[MS] **B.R.Martin, G.Shaw – Particle Physics** [*ditto*];

[Perkins] **D.Perkins - Introduction to High Energy Physics**, 4th ed. [*ditto*];

[Povh] **Povh, Rith, Scholz, Zetsche - Particles and Nuclei** [*ditto, simpler*];

[Thoms] **M. Thomson – Modern Particle Physics** [*ditto*];

[CG] **R.Cahn, G.Goldhaber – The experimental foundation of particle physics** [*a collection of original papers + explanation, the main source for experiments*];

[FNSN1] **C.Dionisi, E.Longo - Fisica Nucleare e Subnucleare 1** – Dispense del corso [*in Italian, download it from our web – you are requested to know them*];

[MQR] **L.Maiani - O.Benhar – Meccanica Quantistica Relativistica** [*in Italian, the theory lectures of the previous semester*];

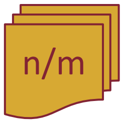
[IE] **L.Maiani – Interazioni elettrodeboli** [*ditto*];

[PDG] **The Review of Particle Physics –** latest: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) [*the bible; everything there, but more a reference, than a textbook, i.e. hard for newcomers*];

[original] **Original papers** are quoted in the slides [*try to read (some of) them* → help by [CG]].

quoted as [book, chapter] or [book, page]; *e.g.* [BJ, § 4] *: Burcham-Jobes, § 4*.

Symbols



(in the upper left corner) this is page *n* of a total of *m* pages : read them all together;

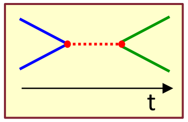
(in the upper right corner) optional material;

(in the upper right corner) tool, used also in other chapters;

summary;

animation (ppt/pptx only);

reference to a paper / textbook; [if textbook, you are requested to read it; if paper, try (at least some of) them];



in Feynman diagrams, time goes always left to right;

- "QM" : Quantum Mechanics;
- "SM" : Standard Model; here and there, the name and the history behind is explained;
- "bSM" : beyond Standard Model, i.e. the (until now unsuccessful) attempts to extend it, e.g. SUSY;
- ($\hbar$ = c = 1) whenever possible; i.e. mass, momentum and energy in MeV or GeV.
- m : scalar, E : component of a vector;
- $\mathbb{P}$: operator;
- $\vec{v}$: 3-vector, $\vec{v} = (x, y, z)$;
- p : 4-vector, $p = (E, p_x, p_y, p_z) = (E, \vec{p})$;
- if worth, the module is indicated $p = (E, p_x, p_y, p_z; m) = (E, \vec{p}; m)$;
- if irrelevant, the last component of a 3- or 4-vector is skipped : $p = (E, p_x, p_y) = (E, p_x, p_y; m)$.

room, time, …

Lecture time – aula Careri

- mon (lun) 12 - 14
- tue (mar) 11 - 13
- wed (mer) 11 - 13
- thu (gio) 12 - 14

[*not ideal but acceptable*]

We have also this room on tue 14-16:

- not for independent lectures (too much);
- problems, exercises, …
- long questions from you, e.g. if you feel you need something you should know, but actually don't (relativistic kinematics ?)

exam

- questions [by me] and answers [*possibly by you*];
- 1st question known few days in advance by email [*I'll choose randomly, with a little bias*];
- if theoretician or experimentalist, you may [or may not] tell me [*I'll use it*];
- let me also know curriculum type (e.g. phenomenology, electronics, medical physics) [*I'll apply a stronger bias*];
- other rules after discussion and experiment [*I'm an experimentalist*].

Nota Bene

- Starting with **<u>2017-2018</u>** (one year ago), these lectures are delivered in English.
- No problem, we all know and love the Shakespeare idiom [*needless to say, we love Italian and Roman too*].
- As a minor consequence, the name of the course has changed – it was "*Fisica Nucleare e SubNucleare 2*".
- Apart from name and language, no major change [*I would love to improve, come and discuss your ideas with me*].
- Past years' students don't have to worry: students are officially bound (really) to the rules of the year of their registration (*anno di immatricolazione*). They only have to be careful with the registration(s), i.e. the *INFOSTUD* stuff.
- The exam (both this and past years' students) will be in Italian or English, at **<u>your</u>** choice.
- During the lecture, questions and comments in the language as you like. I will start answering by translating them into English.

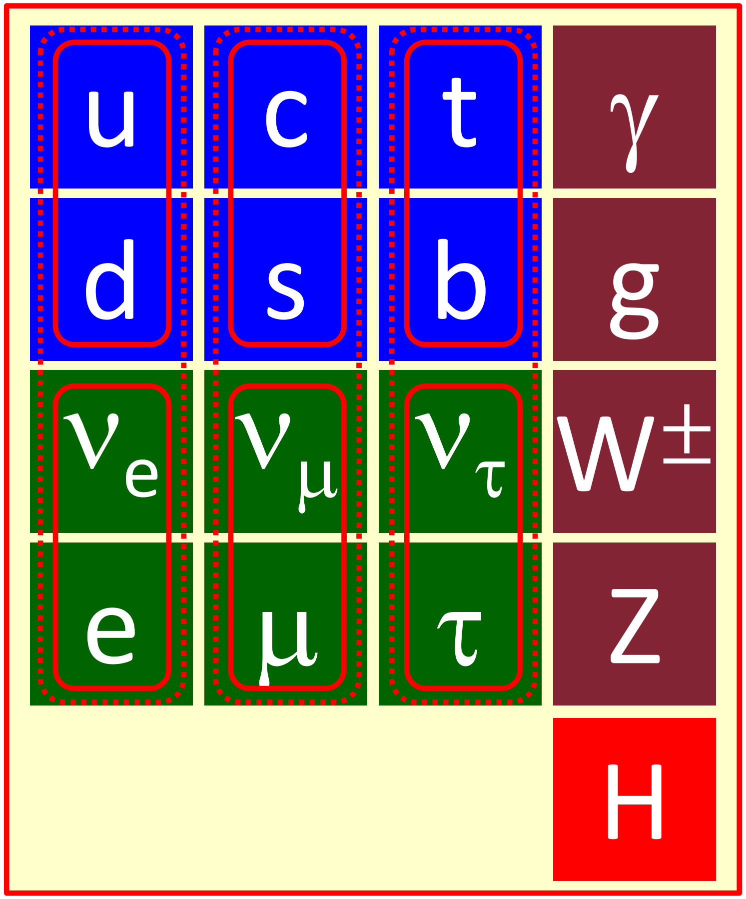
> *and please never forget the RULE:*
> *"If you are going to sin, sin against God, not the bureaucracy. God will forgive you, but the bureaucracy won't."*
> *Hyman G. Rickover*

Let's start

Prologue

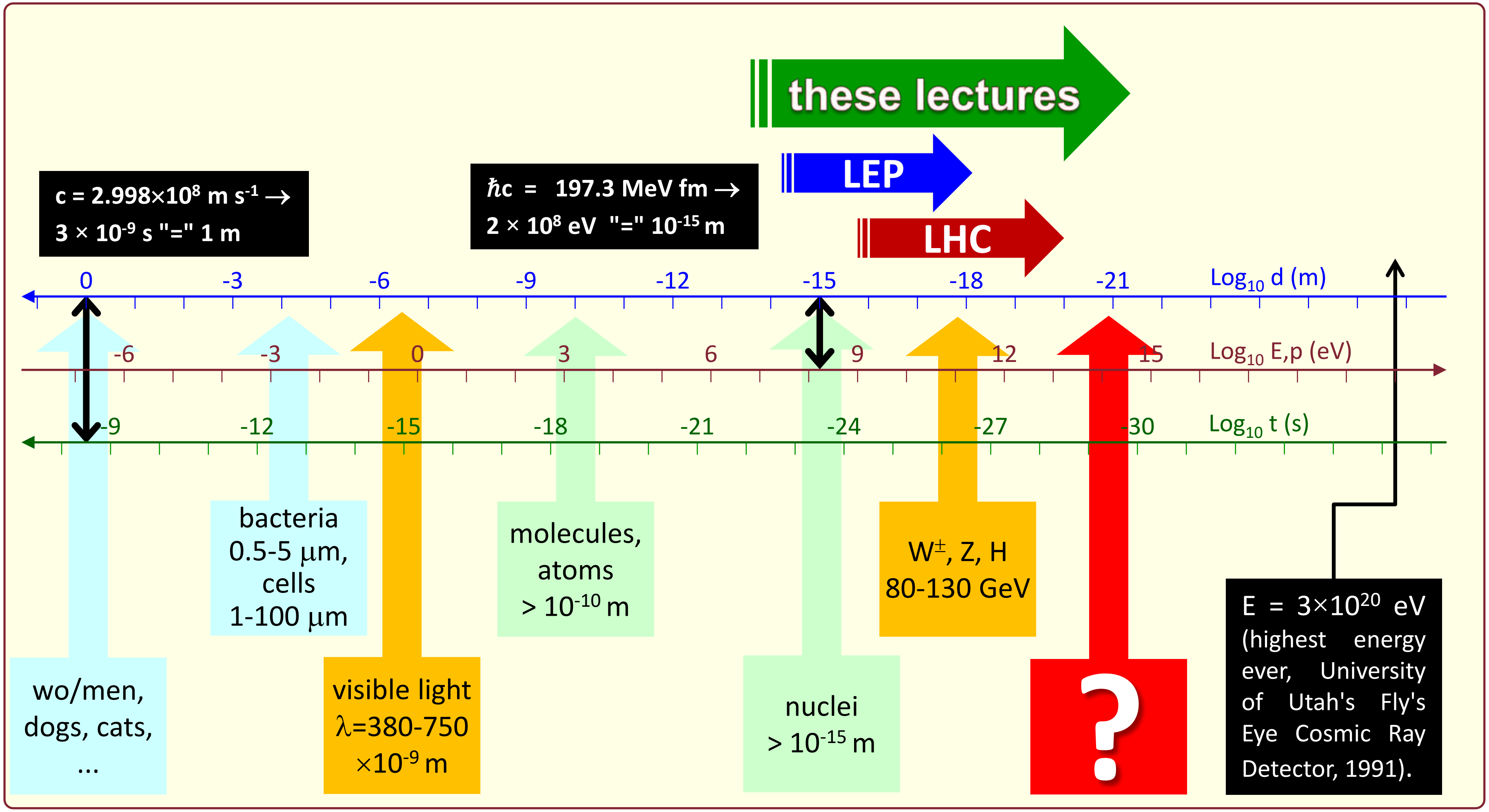
The present understanding of our world, in terms of its constituents and interactions, is much advanced:

- **fermions** (quarks/leptons) = matter:
  - "families" of doublets + antiparticles;
  - spin ½;
  - massive (large differences in mass);
  - charge $\pm⅔$, $\pm⅓$, 0, ±1;
- **bosons** = forces:
  - spin 1;
  - massless (γ, g) or massive ($W^\pm$, Z);
  - charged ($W^\pm$) or neutral (γ, g, Z);
  - some self-coupled;
- the mysterious **Higgs boson** carries the particle masses.



these lectures explain how

Prologue: twenty orders of magnitude

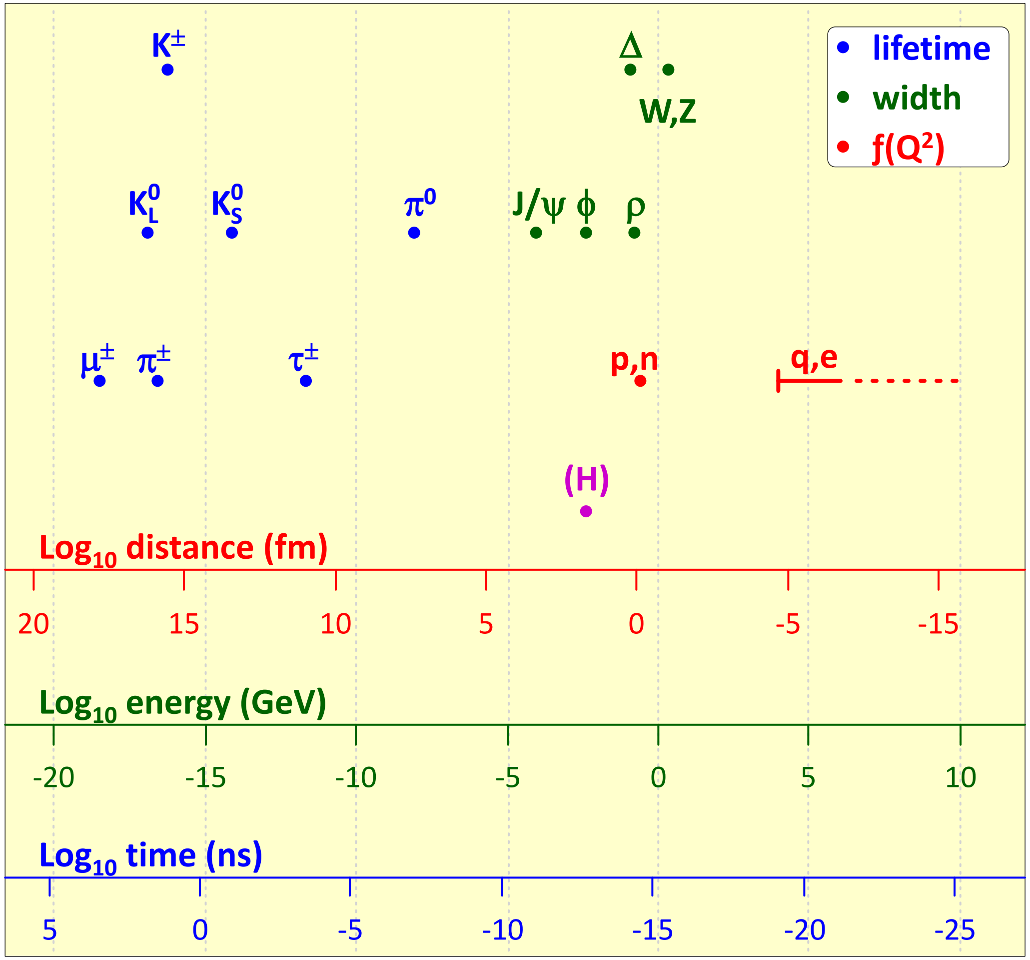


space / energy / time

Prologue: the realm of elementary particles

In these lectures, many phenomena. Consider the typical (rough) size/time/energy of the processes:

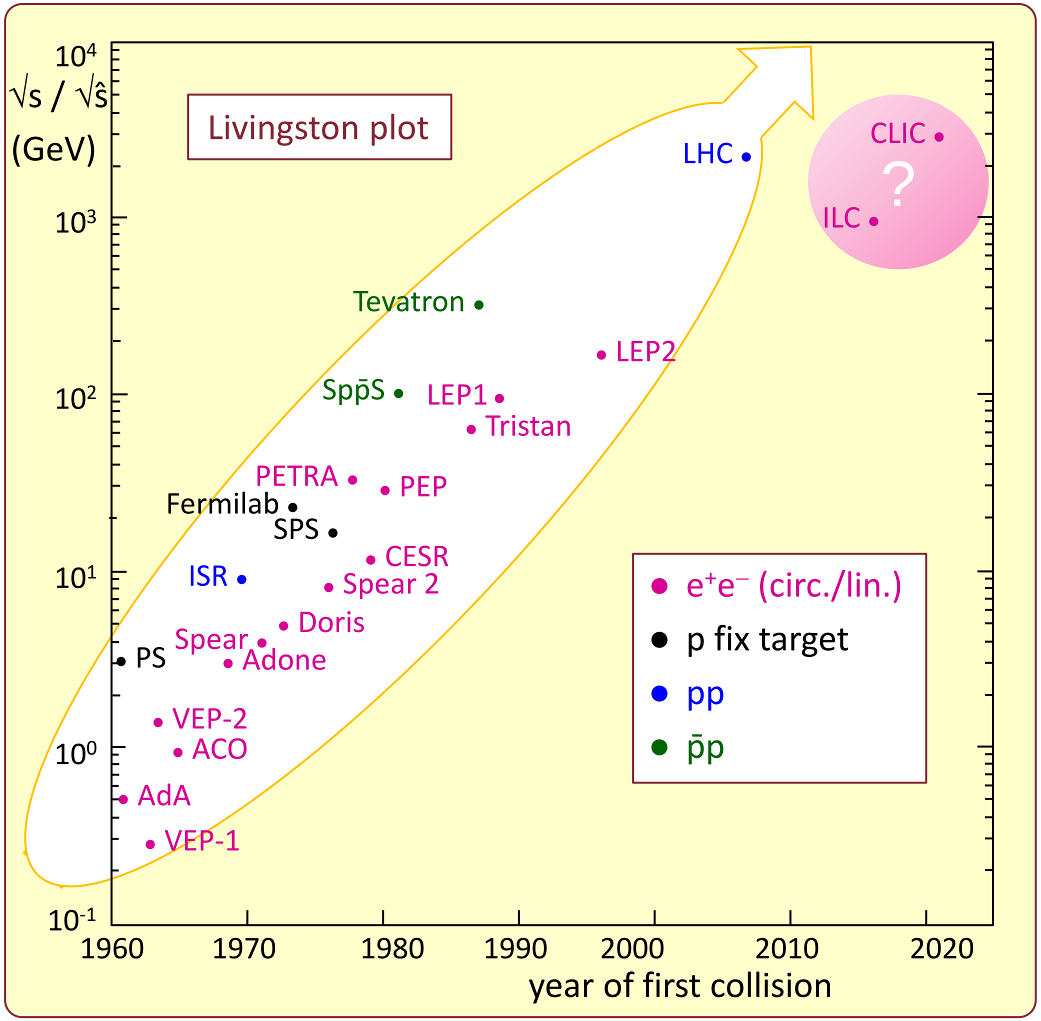
- **lifetimes** are measured in the rest system of the particles, i.e. in (nano-)s;
- the corresponding **distance** is the average space traveled by a particle with $\beta\gamma$=1 before decaying;
- the uncertainty principle relates a **width** to a lifetime: it is the fluctuation of the particle rest energy (= mass);
- $f(Q^2)$ deserves an explanation: sometimes the size of a particle is inferred "à la Rutherford", by a scattering experiment [*see chapter 2*] (only limits for q's and ℓ's: pointlike ?);
- the width of the Higgs boson (H) has not (yet ?) been measured and comes from theory.



Do NOT panic: you are supposed to fully understand this plot only at the end of the lectures. Every single point in the figure will be ***carefully*** *explained.*

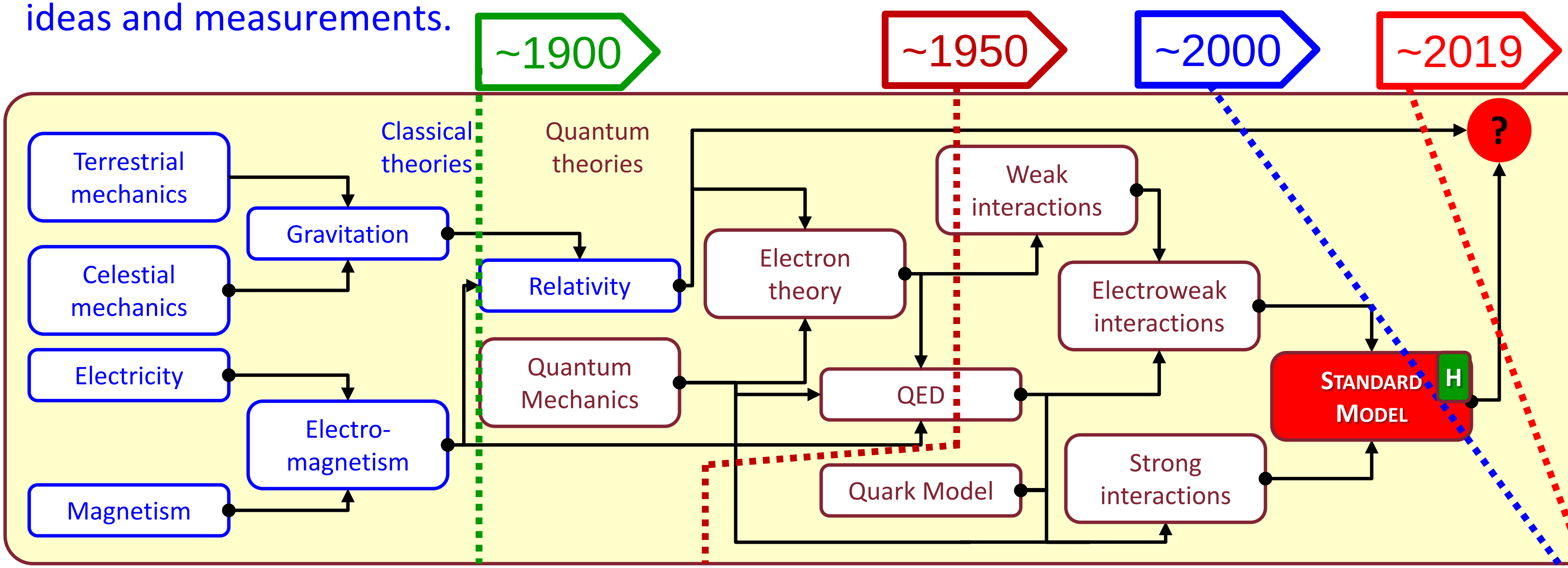
Prologue: the quest for higher energies

- Discovery range is limited by available data, i.e. by instruments and resources (an always improved microscope).
- The true variable is the resolving power [r.p.] of our microscope.
- From QM, r.p. $\propto \sqrt{Q^2}$ [i.e. $\propto \sqrt{s}$, the CM energy [*what ? why ? see § 2*].
- For non point-like objects, replace $\sqrt{s}$ with the CM energy at component level, called $\sqrt{\hat{s}}$ ($\sqrt{\hat{s}} < \sqrt{s}$).
- In the last half a century, the physicists have been able to gain a factor 10 in $\sqrt{s}$ (i.e. a factor ten in the quality of the microscope) <u>every 10 years</u> (see the "Livingston plot").
- Hope it will continue like that, but needs <u>IDEAS</u>, since not many $$$ (or €€€) will be available.



Prologue: the Standard Model

- The name **SM** (not a fancy name) designates the theory of the Electromagnetic, Weak and Strong interactions.
- The theory has grown in time, the name went together.
- The development of the SM is a complicated interplay between new ideas and measurements.
- Many theoreticians have contributed : since the G-S-W model is at the core of the SM, it is common to quote them as the main authors.
- The little scheme [BJ] of its time evolution may help (missing connections, approximations, …).

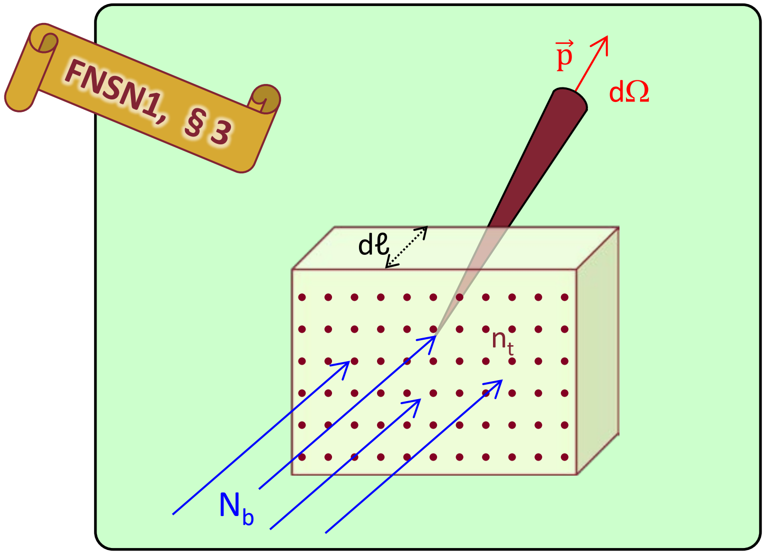


Repetita juvant

few subjects, well known, but …
[skip next pages, if you can afford it]:

- the cross section σ;
- excited states (resonances);
- Gauss distribution.
- measurements:
  - spectrometers;
  - calorimeters;
  - particle id;

the cross section σ



A beam of N_b particles is sent against a thin layer of thickness dℓ, containing dN_t scattering centers in a volume $\mathcal{V}$ ("target", density $n_t = dN_t/d\mathcal{V}$).

The number of scattered particles dN_b is:

$$dN_b \propto N_b \, n_t \, d\ell \quad \Rightarrow \quad dN_b = N_b \, n_t \, \sigma_T \, d\ell$$

the number of particles left after a finite length ℓ is

$$N_b(\ell) = N_b(0) \exp(- n_t \, \sigma_T \, \ell).$$

The parameter σ_T is the total **cross section** between the particles of the beam and those of the target; it can be interpreted as *the probability of an interaction when a single projectile enters in a region of unit volume containing a single target*.

If many exclusive processes may happen (simplest case : elastic or inelastic), σ_T is the sum of many σ_j, one for each process:

$\sigma_T = \Sigma_j \sigma_j$ [e.g. $\sigma_T = \sigma_{elastic} + \sigma_{inelastic}$];

in this case σ_j is proportional to the *probability of process j*.

Common differential dσ/d... 's:

$$\frac{d\sigma}{d\Omega} = \frac{d^2\sigma}{d\cos\theta \, d\varphi} \xrightarrow{\text{no } \varphi \text{ dependence}} \frac{1}{2\pi} \frac{d\sigma}{d\cos\theta};$$

$$\frac{d\sigma}{d\vec{p}} = \frac{d^3\sigma}{dp_x dp_y dp_z} = \frac{d^3\sigma}{p_T dp_T dp_\ell d\varphi} \rightarrow \frac{1}{\pi} \frac{d^2\sigma}{dp_T^2 dp_\ell};$$

+ others.

the cross section σ: $\sigma_{inclusive}$

In a process ($a\,b \to c\,X$), assume:

- we are only interested in "c" and not in the rest of the final state ["X"];
- "c" can be a single particle (e.g. $W^\pm$, Z, Higgs) or a system (e.g. $\pi^+\pi^-$).

Define:

$\sigma_{inclusive}(ab \to cX) = \sum_k \sigma_{exclusive}(ab \to cX_k)$,
where the sum runs on all the **exclusive** processes which in the final state contain "c" + anything else [define also $d\sigma_{inclusive}/d\Omega$ wrt angles of "c", etc.].

The word *inclusive* may be explicit or implicit from the context. E.g., "*the cross-section for Higgs production at LHC*" is obviously $\sigma_{inclusive}(pp \to HX)$.

From the definition, if $\sigma_{inclusive} << \sigma_{total}$:

$\mathcal{P}_c$ = probability of "c" in the final state =
= $\sigma_{inclusive}(ab \to cX) / \sigma_{total}(ab)$.

Instead, if "c" is common:

$\langle n_c \rangle$ = <number of "c" in the final state> =
= $\sigma_{inclusive}(ab \to cX) / \sigma_{total}(ab)$.

e.g.

σ_{Higgs}(LHC, 8 TeV) = $\sigma_{incl}(pp \to HX, \sqrt{s}=8\text{ TeV})$=
≈ 22.3 pb;

$\sigma_{total}(pp, \sqrt{s} = 8\text{ TeV})$ = 101.7 ± 2.9 mb;

→ $\mathcal{P}_{Higgs}$(LHC) ≈ 2×10^{-10};

[§ LHC]

$\sigma_{incl}(pp \to \pi^0 X, p_{LAB}=24\text{ GeV})$ = 53.5 ± 3.1 mb;

$\sigma_{total}(pp, p_{LAB}=24\text{ GeV})$ = 38.9 mb;

→ $\langle n_{\pi^\circ}(pp, p_{LAB}=24\text{ GeV})\rangle$ ≈ 1.37

[V.Blobel et al. - Nucl. Phys., B69 (1974) 454].

Mutatis mutandis, define
- "inclusive width" $\Gamma(A \to BX)$;
- "inclusive BR" $BR(A \to BX)$.

the cross section σ: Fermi 2nd golden rule

- N_b, N_t : particles in beam(b) / target(t);
- $\mathcal{V}$: volume element;
- n_b, n_t : density of particles [= $dN_{b,t}/d\mathcal{V}$];
- v_b : velocity of incident particles;
- ϕ : flux of incident particles [= $n_b\, v_b$];
- p', E' : 4-mom. of scattered particles;
- $\rho(E')$: density of final states;
- $\mathcal{M}_{fi}$: matrix element between i→f state;
- dN/dt : number of events / time [= $\phi N_t \sigma$];
- W : rate of process [= $(dN/dt) / (N_b N_t)$].

FNSN1, 78

Fermi second golden rule

$$\boxed{W = \frac{2\pi}{\hbar}\left|\mathcal{M}_{fi}\right|^2 \rho(E');}$$

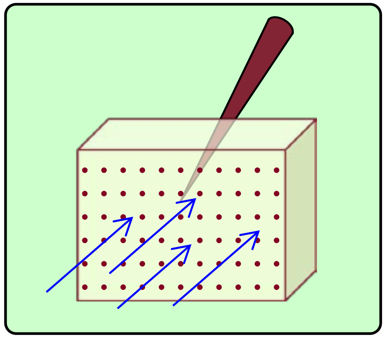
$$\rho(E') = \frac{dn(E')}{dE'} = \frac{\mathcal{V} 4\pi p'^2}{v'(2\pi\hbar)^3};$$

$$W = \frac{dN}{dt}\frac{1}{N_b N_t} = \frac{\phi N_t \sigma}{N_b N_t} = \frac{v_b \sigma}{\mathcal{V}}.$$

$$dn(p') = \frac{\mathcal{V} 4\pi p'^2}{(2\pi\hbar)^3} dp' = \frac{\mathcal{V} 4\pi p'^2}{(2\pi\hbar)^3}\frac{dE'}{v'}$$

$$\boxed{\sigma = \frac{W\mathcal{V}}{v_b} = \frac{2\pi}{\hbar}\left|\mathcal{M}_{fi}\right|^2 \rho(E')\frac{\mathcal{V}}{v_b}}$$

- the rule is THE essential connection (experiment ↔ theory);
- experiments measure event numbers → cross-sections;
- theories predict matrix elements → cross-sections;
- when we check a prediction, we are actually applying the rule;
- properly normalized, the rule is valid also for differential cases (i.e. dσ/dk, d$\mathcal{M}$/dk, dW/dk), where k is any kinematical variable, e.g. cosθ].



Excited states : decay pdf

Consider N (N large) unstable particles :

- independent decays;
- decay probability time-independent (e.g. no internal structure, like a timer);

Then :

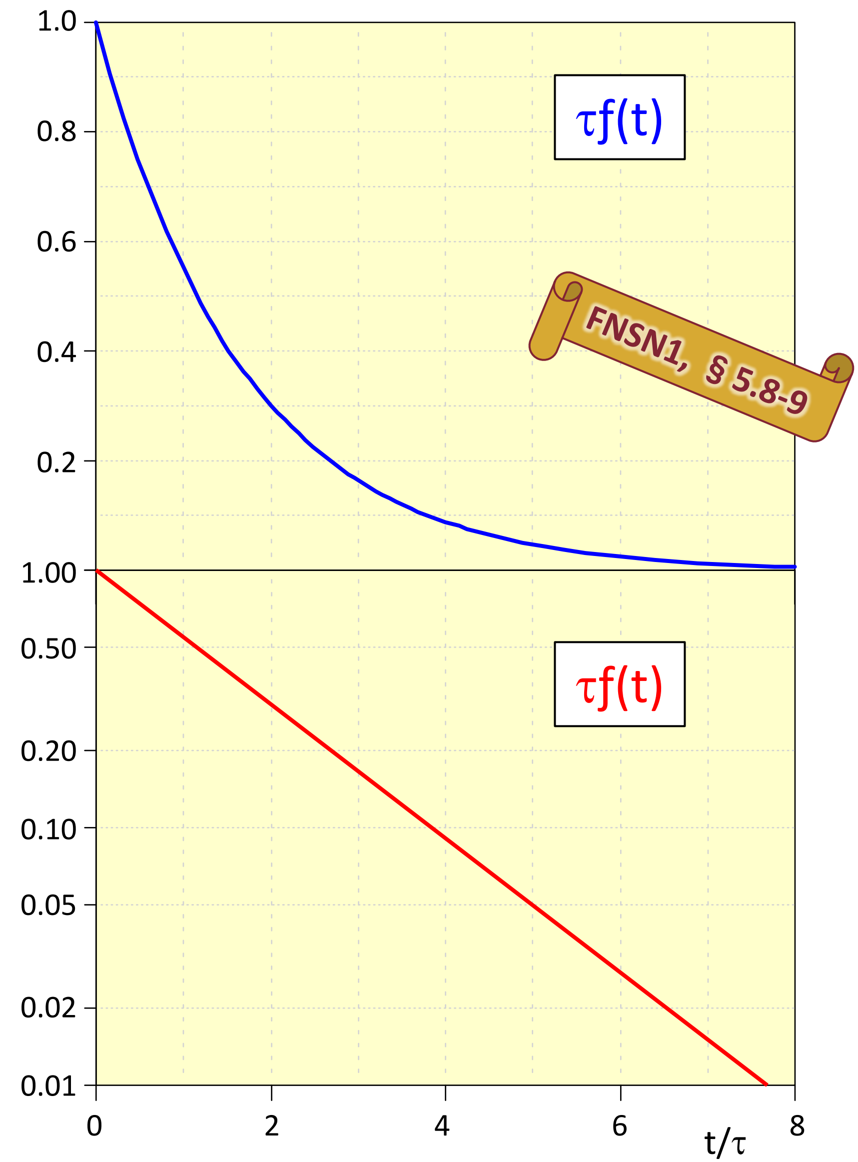
$$dN = -N\Gamma dt; \qquad \Gamma \equiv \frac{1}{\tau} = \text{const.} \qquad \Rightarrow$$

$$N(t) = N_0\, e^{-\Gamma t} = N_0 e^{-t/\tau}.$$

The pdf of the decay for a single particle is

$$\int_0^\infty f(t)dt = 1 \quad \Rightarrow \quad \boxed{f(t) = \frac{1}{\tau} e^{-t/\tau}.}$$

- average decay time : $\left(\sum t_j\right)/n$ = <t> = = τ;
- likelihood estimate of τ, after n decays observed : τ^* = <t>.



Excited states : Breit-Wigner

If τ is small, the energy at rest (= mass) of a state is not unique (= δ_{Dirac}), but may vary as $\tilde{f}(E)$ around the nominal value $E_0 = m$:

Define $\psi(t<0) = 0$; $\psi(t=0) = \psi_0$;
width Γ [unstable] ;

$$\psi(t) = \psi_0 e^{(-im-\Gamma/2)t};$$

$$|\psi(t)|^2 = |\psi_0|^2 e^{-\Gamma t} = |\psi_0|^2 e^{-t/\tau};$$

$$\tilde{f}(E) = |\tilde{\psi}(E)|^2 = \frac{|\psi_0|^2}{2\pi} \frac{1}{(E-E_0)^2 + \Gamma^2/4}.$$

YN1, 2.1

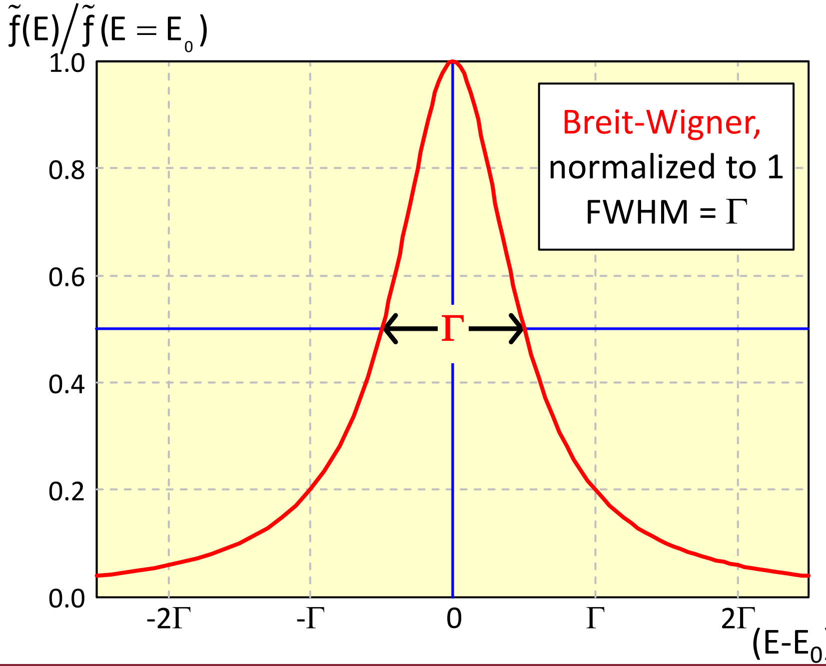
$$\tilde{\psi}(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{iEt} \psi(t) dt =$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{iEt} \psi_0 e^{-i(E_0 - i\Gamma/2)t} dt =$$

$$= \frac{\psi_0}{\sqrt{2\pi}} \frac{-1}{i(E-E_0) - \Gamma/2} = \frac{\psi_0}{\sqrt{2\pi}} \frac{i(E-E_0)+\Gamma/2}{(E-E_0)^2 + \Gamma^2/4}.$$

The curve $(1 + x^2)^{-1}$ is called "Lorentzian" or "Cauchy" in math and "**Breit-Wigner**" in physics; it describes a RESONANCE and appears in many other phenomena:

- forced mechanical oscillations;
- electric circuits;
- accelerators;
- …



Excited states : BW properties

Cauchy (or Lorentz, or BW) distribution :

$$\boxed{f(x) = BW\left(x\,|\,x_0,\gamma\right) = \frac{1}{\pi\gamma}\frac{\gamma^2}{\left(x - x_0\right)^2 + \gamma^2};}$$

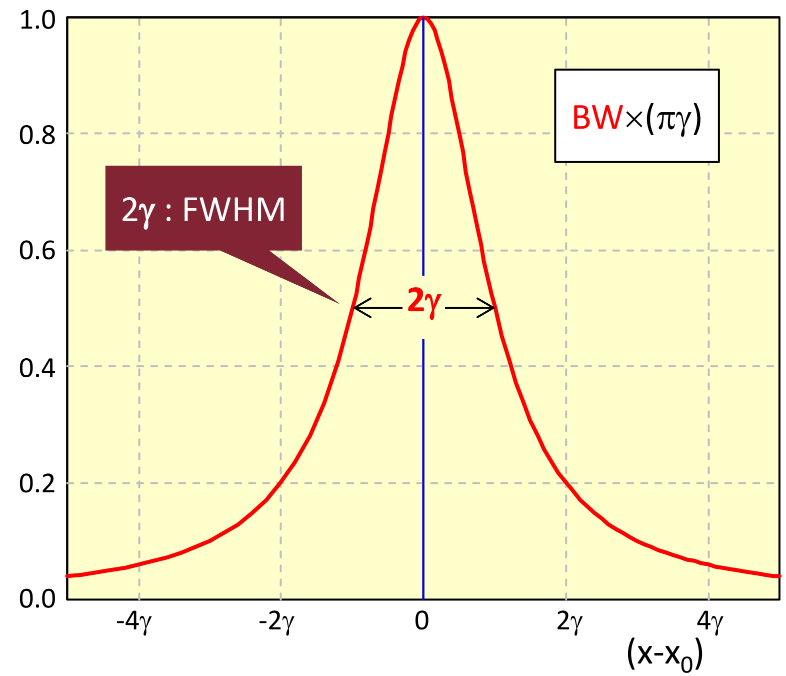
- median = mode = x_0;
- mean = math undefined [but use x_0];
- variance = really undefined [divergent]

This anomaly is due to

$$\langle x\rangle = \int_{-\infty}^{+\infty} x f(x)dx = \infty;$$

$$\langle x^2\rangle = \int_{-\infty}^{+\infty} x^2 f(x)dx = \infty$$

The anomaly does NOT conflict with physics : the BW is an approximation valid only if $\gamma << x_0$ and in the proximity of x_0, e.g. in case of an excited state (mass m, width Γ), for (Γ<<m) and ($|\sqrt{s}-m|$ < few Γ's).



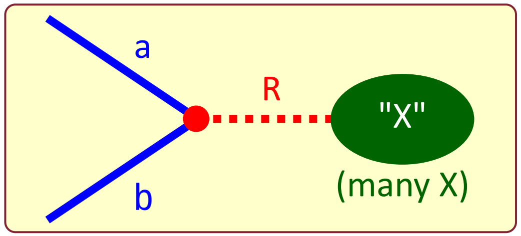
The "relativistic BW" is usually defined as

$$\boxed{BW_{rel}(x\,|\,x_0,\gamma) = \frac{x_0^2\gamma^2}{\left(x^2 - x_0^2\right)^2 + x_0^2\gamma^2}} \left[\begin{matrix}\text{properly}\\ \text{normalized}\end{matrix}\right].$$

The formula comes from the requirement to be Lorentz invariant [see *Berends et al., CERN 89-08, vol 1*].

Resonance : σ_R

From first principles of QM ([FNSN1], [BJ 9.2.3], [YN1 13.3.3], [PDG])



$(E, \vec{p})$: CM 4-mom.
Γ_R : constant width
$\Gamma_{ab,\,X}$: couplings
M_R : E_0, mass

$$\sigma_{ab\to R\to X}\left(E_{CM}=\sqrt{s}\right)=\frac{\pi}{\left|\vec{p}_{a,b}\right|^2}\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\frac{\Gamma_{ab}\Gamma_X}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\approx$$

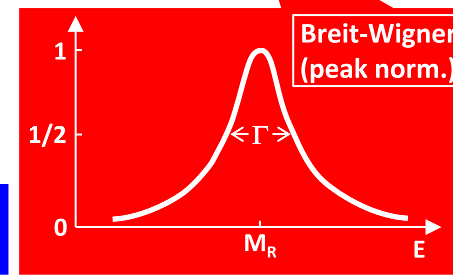
$$\approx\left[\frac{16\pi}{s}\right]\left[\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\right]\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_X}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

scale factor (1/s)

statistical factor (particle spins)

$= BR(R\to ab)$

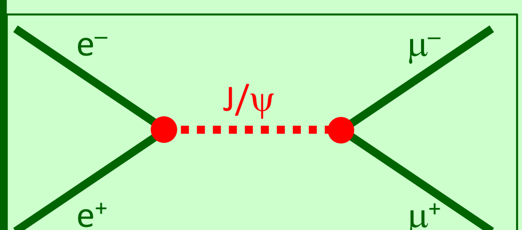
$= BR(R\to X)$



e.g.
$e^+e^- \to J/\psi \to \mu^+\mu^-$

$\sigma_{peak} \propto 1/s$ ($\approx M_R^{-2}$), independent from coupling strength.

$$\sigma(e^+e^-\to J/\psi\to\mu^+\mu^-)=\left[\frac{16\pi}{s}\right]\left[\frac{3}{4}\right]\left[\frac{\Gamma_{ee}}{\Gamma_{tot}}\right]\left[\frac{\Gamma_{\mu\mu}}{\Gamma_{tot}}\right]\left[\frac{(\Gamma_{tot}/2)^2}{\left(\sqrt{s}-M\right)^2+(\Gamma_{tot}/2)^2}\right]=$$



$$=\frac{12\pi}{s}BR_{J/\psi\to e^+e^-}BR_{J/\psi\to\mu^+\mu^-}\left[\frac{(\Gamma_{tot}/2)^2}{\left(\sqrt{s}-M\right)^2+(\Gamma_{tot}/2)^2}\right].$$

Resonance : different functions

Many more parameterizations used in literature (semi-empirical or *theory inspired*), e.g.:

$$\sigma_0 = \left[\frac{16\pi}{(2p)^2}\right]\left[\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\right]\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{final}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

original, non-relativistic

$$\sigma_1 = \left[\frac{16\pi}{s}\right]\left[\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\right]\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{final}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

m_a, m_b << p

$$\sigma_2 = \left[\frac{16\pi}{M_R^2}\right]\left[\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\right]\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{final}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

if $M_R \gg \Gamma_R$, neglect s-dependence

$$\sigma_3 = \left[\frac{16\pi}{M_Z^2}\right]\left[\frac{3}{4}\right]\left[\frac{\Gamma_{ee}}{\Gamma_Z}\right]\left[\frac{\Gamma_{f\bar{f}}}{\Gamma_Z}\right]\left[\frac{M_Z^2\Gamma_Z^2}{\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2}\right]$$

relativistic BW for $e^+e^- \to Z \to f\bar{f}$

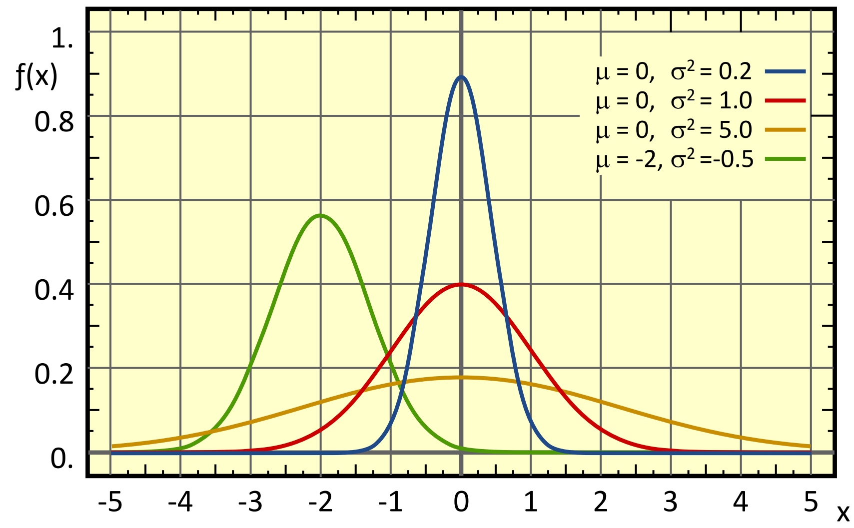
$$\sigma_4 = \left[\frac{16\pi}{M_Z^2}\right]\left[\frac{3}{4}\right]\left[\frac{\Gamma_{ee}}{\Gamma_Z}\right]\left[\frac{\Gamma_{f\bar{f}}}{\Gamma_Z}\right]\left[\frac{s\Gamma_Z^2}{\left(s-M_Z^2\right)^2+s^2\Gamma_Z^2/M_Z^2}\right]$$

"s-dependent Γ_Z" (used at LEP for the Z lineshape)

Gauss distribution

$$f(x) = G(x\,|\,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- mean = median = mode = μ;
- variance = σ^2;
- symmetric : $G(\mu+x) = G(\mu-x)$
- central limit theorem* : the limit of processes arising from multiple random fluctuations is a single G(x);
- similarly, in the large number limit, both the binomial and the Poisson distributions converge to a Gaussian;
- therefore $\boxed{G(x|\ \mu=x_{meas},\ \sigma=error_{meas})}$ is often used as the resolution function of a given experimental observation [*but as a good (?) first approx. only*].



* Consider n independent random variables $x = \{x_1, x_2, \ldots, x_n\}$, each with mean μ_i and variance σ^2_i; the variable

$$t = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{x_i - \mu_i}{\sigma_i}$$

can be shown to have a distribution that, in the large-n limit, converges to $G(t\,|\,\mu=0,\sigma=1)$.

Gauss distribution : hypothesis test

Given a measurement x with an expected value μ and an error σ, the value

$$F(x) = \int_x^{+\infty} G(t\,|\,\mu,\sigma)\,dt$$

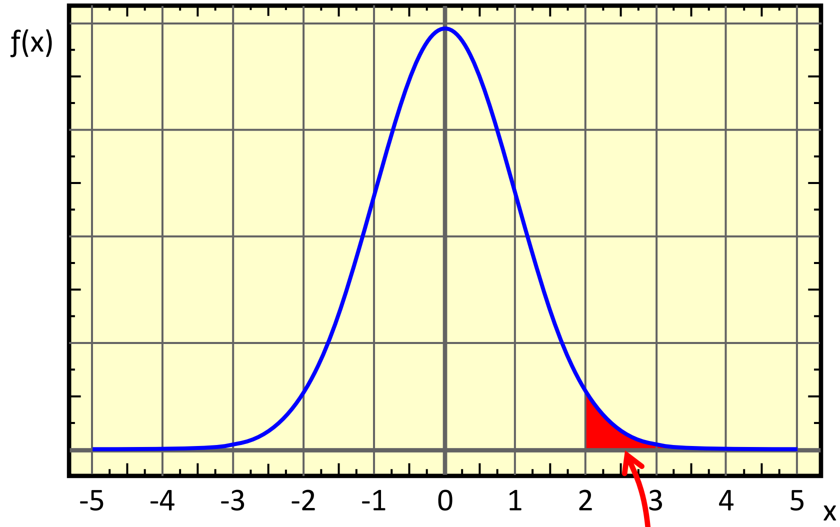
is often used as a "hypothesis test" of the expectation.

E.g. (see the ◣ plot): if the observation is at 2σ from the expectation, one speaks of a "2σ fluctuation" (not dramatic, it happens once every 44 trials – or 22 trials if both sides are considered).

The value of "5σ" * has assumed a special value in modern HEP [*see later*].

---

* if the expectation is not gaussian, one speaks of "5σ" when there is a fluctuation $\leq$ 2.87 E-7 in the tail of the probability, even in the non-gauss case.



x	G(x|0,1)	F(x)	$=1/n_{trial}$
0	3.989 E-01	5.000 E-01	2
1	2.420 E-01	1.587 E-01	6.3
2	5.399 E-02	**2.275 E-02**	44.0
3	4.432 E-03	1.350 E-03	741
4	1.338 E-04	3.167 E-05	31,500
5	**1.487 E-06**	**2.867 E-07**	**3.5 E+06**
6	6.076 E-09	9.866 E-10	1.0 E+09
7	9.135 E-12	1.280 E-12	7.8 E+11

Gauss distribution : the "Voigtian"

Assume :

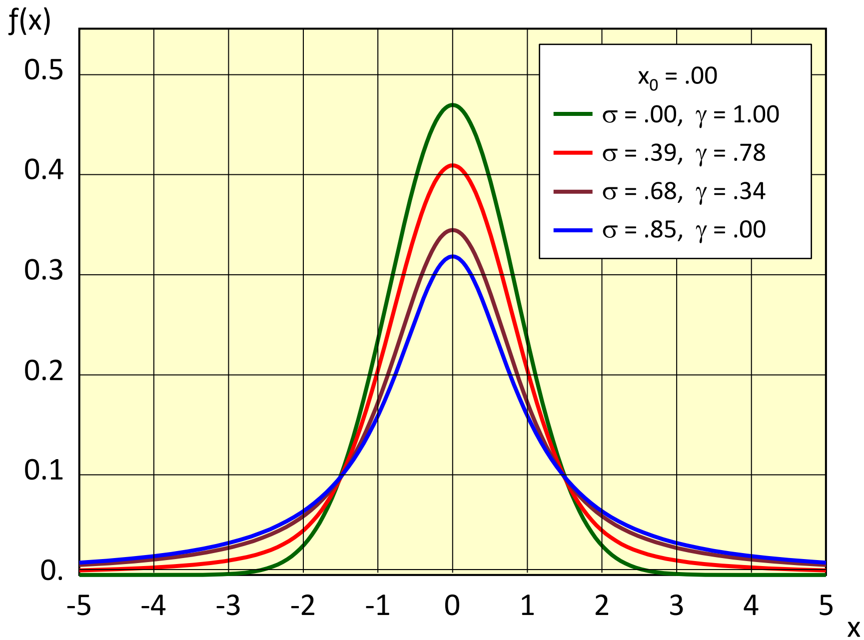
- a physical effect (e.g. a resonance) of intrinsic width described by a BW;
- a detector with a gaussian resolution;

→ the measured shape is a convolution "Voigtian" (after Woldemar Voigt).

- the V. is expressed by an integral and has no analytic form if $\gamma > 0$ AND $\sigma > 0$.
- however modern computers have all the stuff necessary for the numerical computations;
- mean = mathematically undefined [use x_0];
- variance = really undefined [divergent].

→ for real physicists : check carefully if resolution is gaussian, dynamics is BW, and γ and σ are uncorrelated .

$$f(x) = V(x|x_0,\gamma,\sigma) = \int_{-\infty}^{+\infty} dt\, G(t|0,\sigma)\, BW(x-t|x_0,\gamma) = \int_{-\infty}^{+\infty} dt \left[\frac{e^{\left(-\frac{t^2}{2\sigma^2}\right)}}{\sigma\sqrt{2\pi}}\right]\left[\frac{1}{\pi\gamma}\,\frac{\gamma^2}{(x-t-x_0)^2+\gamma^2}\right].$$



measurements

- Physics is an experimental science [*I would say "THE experimental science"*];
- therefore it is based on <u>experimental verification</u>;
- the "verification" is a sophisticated technique (see later & read Popper), but in essence it means that the theory has to be continuously confronted with experiments;
- … and when there are disagreements, the <u>experiment</u> wins(*);
- therefore, although this is NOT a course on experimental techniques, I find useful to remind a couple of formulæ about the main detectors of our science:
  - magnetic spectrometry;
  - calorimetry;
  - [do not forget Cherenkov's, scintillators, TRD's, …]
- although in real life the results do depend on experimental details and are obtained by complicated numerical evaluations, it is very instructive to study simple ideal cases.

(*) *remember the Brecht poem "The Solution" :*

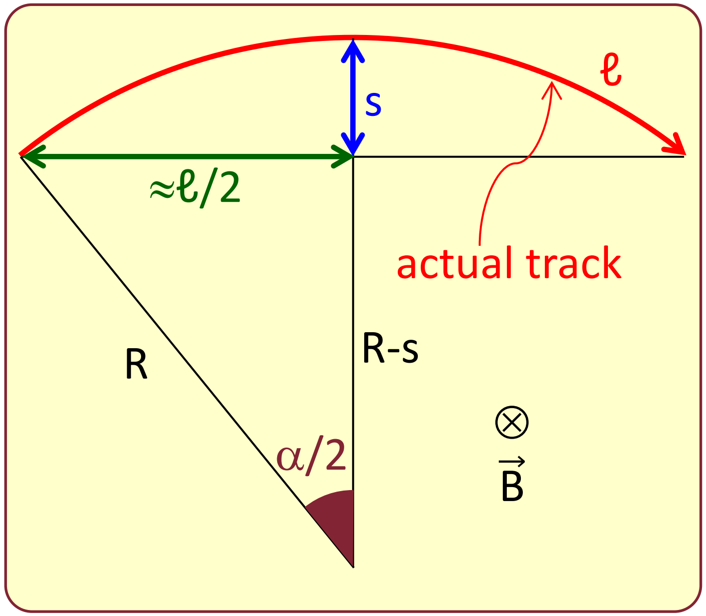
(…) das Volk
Das Vertrauen der Regierung verscherzt habe
Und es nur durch verdoppelte Arbeit
zurückerobern könne. Wäre es da
Nicht doch einfacher, die Regierung
Löste das Volk auf und
Wählte ein anderes ?

[… the people had forfeited the confidence of the government and could win it back only by redoubled efforts. Would it not be <u>easier</u> in that case for the government to dissolve the people and elect another ?]

particle measurement: spectrometers

The Lorentz force bends a charged particle in a magnetic field $\Rightarrow$ the particle momentum is computed from the measurement of a trajectory ℓ. Simple case:

- track $\perp \vec{B}$ (or ℓ = projected trajectory);
- $\vec{B}$ = constant (both mod. and dir.);
- $\ell \ll R$ (i.e. α small, $s \ll R$, arc ≈ chord);
- then (p in GeV, B in T, ℓ R s in m) :



$$R^2 = (R-s)^2 + \ell^2/4 \rightarrow (R, \ell \gg s)$$

$$0 = \cancel{s^2} - 2Rs + \ell^2/4 \rightarrow$$

$$s = \frac{\ell^2}{8R} \simeq \frac{R\alpha^2}{8};$$

$$p = 0.3BR = 0.3B\frac{\ell^2}{8s};$$

$$\frac{\Delta p}{p} = \left|\frac{\partial p}{\partial s}\right|\frac{\Delta s}{p} = \frac{\cancel{p}}{s}\frac{\Delta s}{\cancel{p}} = \frac{\Delta s}{s} = \left(\frac{8\Delta s}{0.3B\ell^2}\right)p.$$

- e.g. B = 1 T, ℓ = 1.7 m, Δs = 200 μm $\rightarrow$

  $\Delta p/p = 1.6 \times 10^{-3}$ p (GeV);

- in general, from N points at equal distance along ℓ, each with error ε :

$$\frac{\Delta p}{p} \simeq \frac{\varepsilon p}{0.3B\ell^2}\sqrt{\frac{720}{N+4}}$$

(Gluckstern formula [PDG]).

particle measurement: spectrometers

[small difference] A track displaced by δ respect to a straight trajectory after ℓ; compute its momentum in the same case:

- track $\perp$ $\vec{B}$ (or ℓ = projected trajectory);
- $\vec{B}$ = constant;
- $\ell \ll R$ (i.e. β small, $\delta \ll R$, arc ≈ chord);
- then (p in GeV, B in T, ℓ R s in m) :

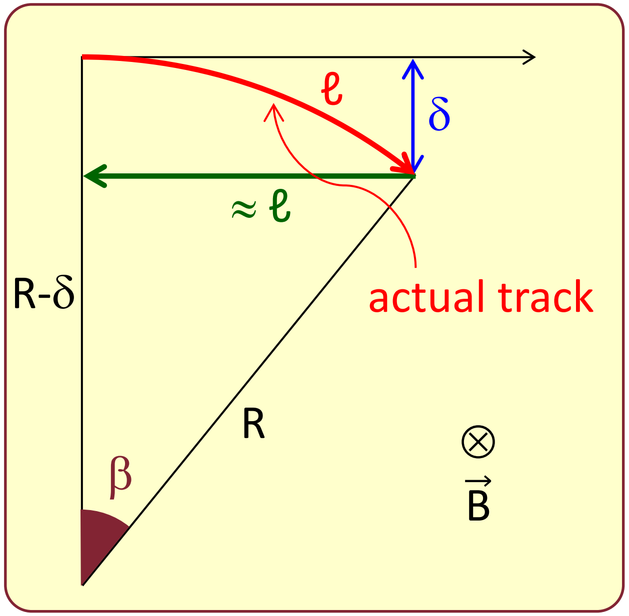
$$R^2 = (R-\delta)^2 + \ell^2 \to (R, \ell \gg \delta)$$

$$0 = \cancel{\delta^2} - 2R\delta + \ell^2 \to$$

$$\delta = \frac{\ell^2}{2R} = \frac{\ell\beta}{2};$$

$$p = 0.3BR = 0.3B\frac{\ell^2}{2\delta};$$

$$\frac{\Delta p}{p} = \left|\frac{\partial p}{\partial \delta}\right|\frac{\Delta\delta}{p} = \frac{\cancel{p}}{\delta}\frac{\Delta\delta}{\cancel{p}} = \frac{\Delta\delta}{\delta} = \left(\frac{2\Delta\delta}{0.3B\ell^2}\right)p.$$



- e.g. B = 1 T, ℓ = 1.8 m, $\Delta\delta$ = 200 μm $\to$

  $\Delta p/p = 4 \times 10^{-4}$ p (GeV);

- $\Delta p/p \propto p \to$ there exists a "maximum detectable momentum" (mdm), defined as the momentum with $\Delta p/p = 1$ (p_{mdm} = 2.5 TeV in the example);
- the mdm defines also the limit for charge identification.

particle measurement: spectrometers

- in presence of materials, the error depends also on the multiple scattering :

$$\Delta x = \frac{\ell}{\sqrt{3}} \frac{0.014}{\beta p(\mathrm{GeV})} \sqrt{\frac{\ell}{X_0}} \left[1 + 0.038 \ell n \left(\frac{\ell}{X_0} \right) \right];$$

$$\left.\frac{\Delta p}{p}\right|_{m.s.}^{\ell} \propto p\Delta x \propto \mathrm{constant};$$

e.g. ℓ = 1 m, air(X_0 = 300 m), p = 10 GeV :

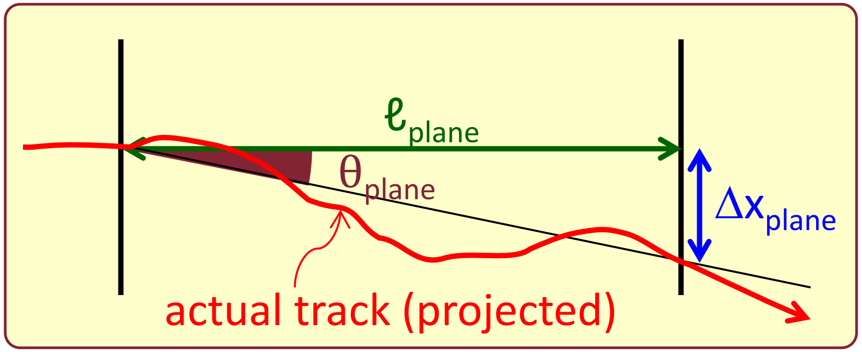
($\rightarrow \beta$ = 1, ℓn term negligible)

$$\Delta x \approx \frac{1}{\sqrt{3}} \frac{0.014}{10} \sqrt{\frac{1}{300}} = 47\ \mu\mathrm{m};$$

(comparable with meas. error).

- the overall error is obtained by the sum in quadrature of all the contributions :

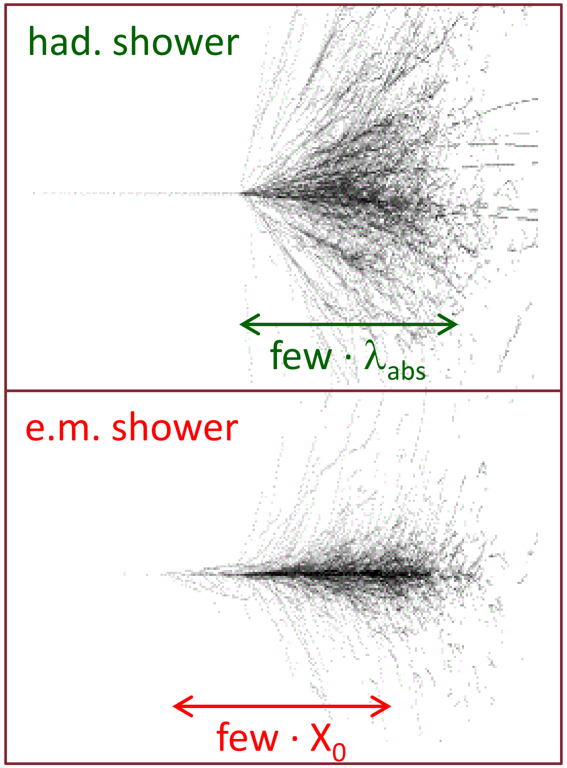
$$\left.\frac{\Delta p}{p}\right|_{tot} = \left(\left.\frac{\Delta p}{p}\right|_{meas} \right) \oplus \left(\left.\frac{\Delta p}{p}\right|_{m.s.} \right) = \sqrt{\left(\left.\frac{\Delta p}{p}\right|_{meas} \right)^2 + \left(\left.\frac{\Delta p}{p}\right|_{m.s.} \right)^2}.$$



particle measurement: calorimeters

Based on the interactions of the particles in a dense material; the total length of the trajectories of the particles in the shower (= the signal) is proportional the primary energy :

$$E = \text{calib} \times \text{track\_length} = \text{calib}' \times \text{signal}.$$



Errors depend on

- stochastic effects on shower development ;
- different response to different particles ($e^\pm \leftrightarrow \mu^\pm \leftrightarrow$ hadrons);
- shower physics [e.g. different amount of (γ+$e^\pm$) $\leftrightarrow$ (hadrons) in had showers];
- systematics of the detectors ("calibration" errors).

Formulas :

$$\lambda_{abs}(g/cm^2) \approx 35(g/cm^2)A^{1/3};$$

for solid heavy materials : $\lambda_{abs} = O(100\ cm)$;

discrimination (+ shape)

$$X_0(g/cm^2) \approx \frac{716(g/cm^2)A}{Z(Z+1)\ell n[287/\sqrt{Z}]};$$

for solid heavy materials : $X_0 = \text{few} \times 1\ cm.$

particle measurement: calorimeters

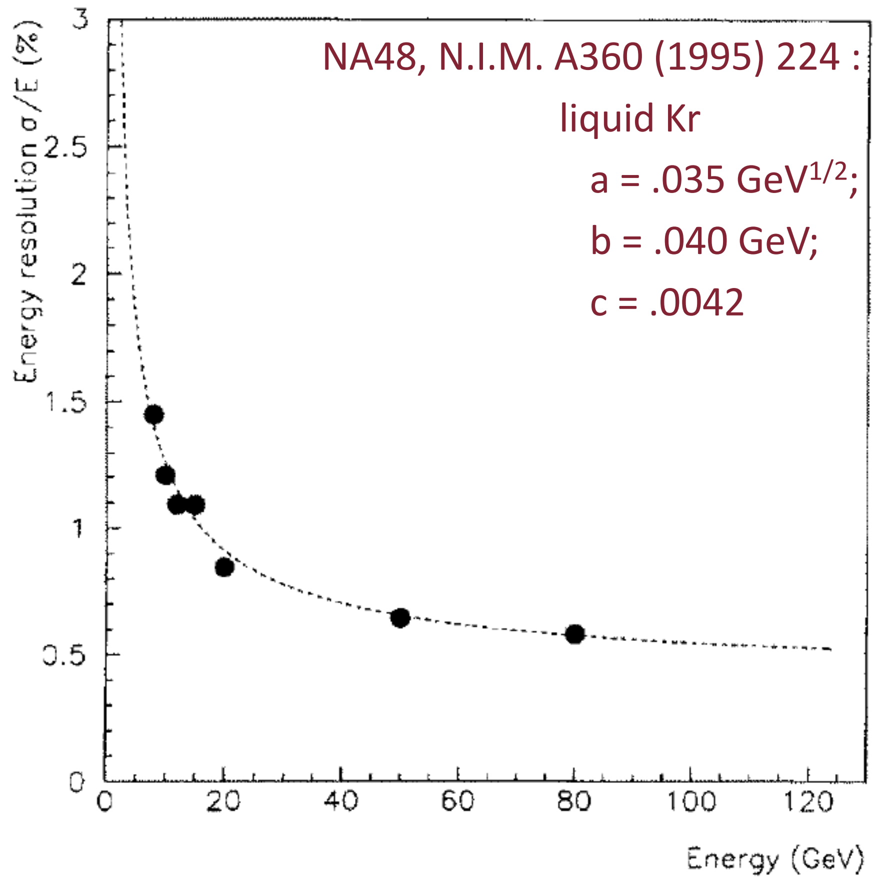
Energy errors, especially in e.m. calorimetry, are parametrized as :

$$\left.\frac{\Delta E}{E}\right|_{tot} = \left(\left.\frac{a}{\sqrt{E}}\right|_{stochastics}\right) \oplus \left(\left.\frac{b}{E}\right|_{noise}\right) \oplus \left(c|_{constant}\right).$$

- the <u>stochastic term</u> comes from the statistical fluctuations in the shower development;
- the <u>noise term</u> from the readout noise and pedestal fluctuations;
- the <u>constant term</u> from the non-uniformity and calibration error.

Other sources of error :

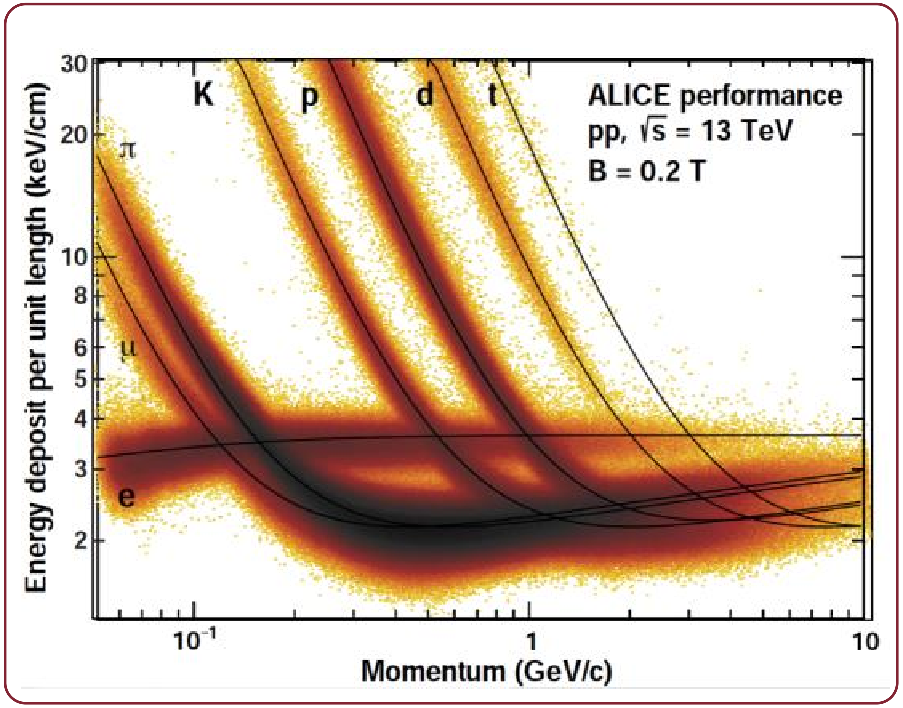
- shower leakage (longitudinal, lateral);
- upstream material;
- non-hermeticity;
- cluster algorithm (+ software approx.);
- e/π ratio [for hadr. non-compensating calos];
- non-linearity;
- nuclear effects;
- …



particle measurement: partid

The particle identification (*partid*) is a fundamental component of modern experiments; many algorithms are embedded in the event reconstruction [*no details*]:

- the gas detectors of the spectrometers detect the amount of ionization, which, for a given momentum, is a function of the particle mass (see fig.);
- the calorimeters select $e^{\pm}$ and γ from hadrons, thanks to the differences between e.m. and hadron showers;
- the $\mu^{\pm}$ are identified by their penetration through thick layers of material;
- the Cherenkov and TRD detectors measure the particle velocity (β and γ respectively), which allows for the determination of the mass;
- powerful kinematical algorithms put all the information together and combine it with known constraints (e.g. known decay modes);
- …



particle measurement: mass errors

Problem – For a given particle, assume independent measures of momentum ($p\pm\Delta p$) and velocity ($c\beta\pm c\Delta\beta$) [*e.g.* $|\vec{p}|$ *from magnetic bending and* β *from time-of-flight*]. Compute its mass ($m\pm\Delta m$).

$$m=\frac{p}{\beta\gamma}=p\frac{\sqrt{1-\beta^2}}{\beta};$$

$$\left(\frac{\Delta m}{m}\right)^2=\left(\frac{\Delta p}{p}\right)^2+\gamma^4\left(\frac{\Delta\beta}{\beta}\right)^2.$$

$$m=\sqrt{E^2-p^2}=\frac{p}{\beta\gamma}=p\frac{\sqrt{1-\beta^2}}{\beta};$$

$$\frac{\partial m}{\partial p}=\frac{1}{\beta\gamma};$$

$$\frac{\partial m}{\partial \beta}=p\left[-\frac{\sqrt{1-\beta^2}}{\beta^2}+\frac{1}{\cancel{\beta}}\frac{1}{\cancel{2}\sqrt{1-\beta^2}}\left(-\cancel{2}\cancel{\beta}\right)\right]=$$

$$=-p\left[\frac{\sqrt{1-\beta^2}}{\beta^2}+\frac{1}{\sqrt{1-\beta^2}}\right]=$$

$$=-p\left[\frac{1-\cancel{\beta^2}+\cancel{\beta^2}}{\beta^2\sqrt{1-\beta^2}}\right]=\frac{-p\gamma}{\beta^2}.$$

$$(\Delta m)^2=\left(\frac{\partial m}{\partial p}\right)^2(\Delta p)^2+\left(\frac{\partial m}{\partial \beta}\right)^2(\Delta\beta)^2=$$

$$=\left(\frac{\Delta p}{\beta\gamma}\right)^2+\left(\frac{p\gamma\Delta\beta}{\beta^2}\right)^2;$$

$$\left(\frac{\Delta m}{m}\right)^2=\left(\frac{\Delta p}{\cancel{\beta\gamma}}\frac{\cancel{\beta\gamma}}{p}\right)^2+\left(\frac{\cancel{p}\gamma\Delta\beta}{\beta^{\cancel{2}}}\frac{\cancel{\beta}\gamma}{\cancel{p}}\right)^2=\ldots$$

End - Introduction

Particle Physics - Chapter 1
The static quark model

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AA 18-19

last mod.
9-Mar-19

1 – The static quark model

1. Quantum numbers
2. Hadrons : elementary or composite ?
3. The eightfold way
4. The discovery of the Ω^-
5. The static quark model
6. The mesons
7. Meson quantum numbers
8. Meson mixing
9. The baryons
10. SU(3)
11. Color

Caveats for this chapter:

- *arguments are presented in historical order; some of the results are incomplete, e.g. heavy flavors are not mentioned here (wait a bit);*
- *large overlap with [FNSN1, MQR, IE].*

short summary

- <u>in this chapter</u> [*see § 6 for QCD*]:
  - no dynamics, only static classification, i.e. algebraic regularities of the states;
  - only hadrons, no photons / leptons;
- modest program, but impressive results:
  - all hadrons are (*may be classified as*) composites of the same elementary objects, called ***<u>quarks</u>***;
  - the quark dynamics, outside the scope of this chapter, follow simple conservation rules;
  - QM and **<u>group theory</u>** are enough to produce the hadron classification;
  - although quarks have not been observed, their **<u>static</u>** properties can be inferred from the particle spectra;
- does it mean that <u>quarks are "real"</u>? what *really* "real" means? [*???*]

The roadmap:

- operators associated with conserved quantum numbers;
- old attempts of classification;
- first successes (multiplets, Ω^-);
- modern classification:
  - quarks;
  - group theory: flavor-SU(3);
  - color: color-SU(3);
  - symmetries;
- "construction" of mesons and baryons;

$\rightarrow$ § 6, QCD.

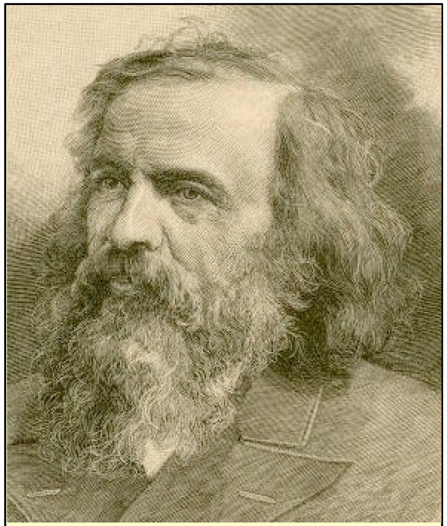
quantum numbers : the Mendeleev way

- Many hadrons exist, with different quantum numbers (qn).
- Some qn show regularities (spin, parity, …).
- Other qn are more intriguing (mass, …).
- A natural approach (à la D.I.M. (*)):
  - ➢ investigate in detail the qn:
    - ❑ the associated operators;
    - ❑ the qn conservation;
  - ➢ look for regularities;
  - ➢ classify the states.

(*) even if D.I.M. lived long before the advent of QM.

an example from many years ago [add antiparticles…]

the proliferation of hadrons started in the '50s – now they are few hundreds …



Dmitri Ivanovich Mendeleev (Дми́трий Ива́нович Менделе́ев)

Name	$\pi^\pm$	π^0	$K^\pm$	K^0	η	p	n	Λ	$\Sigma^{\pm,0}$	Δ
Mass (MeV)	140	135	494	498	548	938	940	1116	1190	1232
Charge	±1	0	±1	0	0	1	0	0	$\pm1,0$	$2,\pm1,0$
Parity	–	–	–	–	–	+	+	+	+	+
Baryon n.	0	0	0	0	0	1	1	1	1	1
Spin	0	0	0	0	0	½	½	½	½	$^3/_2$

many other hadrons

other qn …

quantum numbers : parity $\mathbb{P}$

for complete definitions and discussion, [FNSN1], [MQR], [BJ].

Definition* : $\mathbb{P}\ |\psi(q,\vec{x},t)\rangle = P|\psi(q,-\vec{x},t)\rangle$

- Particles at rest (= in their own ref.sys.) are parity eigenstates:
  $\mathbb{P}\ |\psi(q,\vec{x}=0,t)\rangle = P|\psi(q,\vec{x}=0,t)\rangle$.
- Eigenvalue P : intrinsic parity
  $\mathbb{P}^2 = \mathbb{1}$, P real $\rightarrow$ P = (± 1).
- Dirac equation $\rightarrow$ for spin ½ fermions, P(antiparticle) = -P(particle)
- Convention: P(quarks/leptons) = +1 $\rightarrow$
  $+1 = P_{e-} = P_{\mu-} = P_{\tau-} = P_u = P_d = P_s = \ldots$;
  $-1 = P_{e+} = P_{\mu+} = P_{\tau+} = P_{\bar{u}} = P_{\bar{d}} = P_{\bar{s}} = \ldots$
- Field theory: for spin-0 bosons $\rightarrow$ P(antiparticle) = +P(particle) :
  $P_{\pi+} = P_{\pi^\circ} = P_{\pi-}$, ...

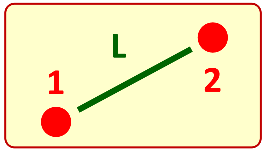
= +1 or −1 ?
... be patient ...

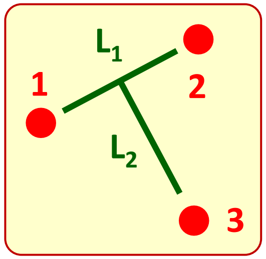
* here and in the following slides :
- q : charges + additive qn;
- $\vec{x},\vec{a}$: polar / axial vectors;
- t : time.

- Gauge theories $\rightarrow$ $P_\gamma = P_g = -1$.

$W^\pm$ and Z do NOT conserve parity in their interactions, so their intrinsic parity is not defined.

- For a many-body system, P is a multiplicative quantum number :
  $\mathbb{P}\psi(\vec{x}_1,\vec{x}_2\ldots\vec{x}_n,t) = P_1P_2\ldots P_n\psi(\vec{x}_1,\vec{x}_2\ldots\vec{x}_n,t)$.
- Particles in a state of orbital angular momentum are parity eigenstates :
  $Y_{km}(\theta,\phi) = (-1)^k\ Y_{km}(\pi-\theta,\phi+\pi) \rightarrow$
  $\mathbb{P}\ |\psi_{km}(\theta,\phi)\rangle = (-1)^k\ |\psi_{km}(\theta,\phi)\rangle$
- Therefore, for a two- or a three-particle system:
  $P_{sys(12)} = P_1P_2\,(-1)^L$;
  $P_{sys(123)} = P_1P_2P_3(-1)^{L1+L2}$.





quantum numbers : charge conjugation $\mathbb{C}$

Definition : $\mathbb{C}$ changes a particle p into its antiparticle $\bar{p}$, leaving untouched the space and time variables :

$\mathbb{C}\ |p,\psi(\vec{x},t)\rangle = C\ |\bar{p}, \psi(\vec{x},t)\rangle$.

- Therefore, under $\mathbb{C}$:

charge	$q \to -q$;	flip
baryon n.	$\mathscr{B} \to -\mathscr{B}$;	
lepton n.	$\mathscr{L} \to -\mathscr{L}$;	
strangeness	$S \to -S$;	
position	$\vec{x} \to \vec{x}$;	no-flip
momentum	$\vec{p} \to \vec{p}$;	
spin	$s \to s$.	

- $\mathbb{C}$ is hermitian; its eigenvalues are ± 1; they are multiplicatively conserved in strong and e.m. interactions. *see later*

- Only particles (like π^0, unlike K's) which are their own antiparticles, are eigenstates of $\mathbb{C}$, with values C = ($\pm$ 1) :

  C = +1 for π^0, η, η';

  C = –1 for ρ^0, ω, ϕ;

  C = –1 for γ. *for Z, $\mathbb{C}$ and $\mathbb{P}$ are not defined*

- However, few particles are an eigenstate of $\mathbb{C}$; e.g.

  $\mathbb{C}\ |\pi^+\rangle = -\ |\pi^-\rangle$.

- Why define $\mathbb{C}$? E.g. use C-conservation in e.-m. decays:

  $\pi^0 \to \gamma\gamma$: $+1 \to (-1)\ (-1)$ ok;

  $\pi^0 \to \gamma\gamma\gamma$: $+1 \to (-1)\ (-1)\ (-1)$ no.

  Br($\pi^0 \to \gamma\gamma\gamma$) measured to be ~$10^{-8}$.

quantum numbers : G-parity $\mathbb{G}$

proposed by Lee and Yang, 1956.

- charge conjugation $\mathbb{C}$ is defined as

  $\mathbb{C}\,|q, \mathscr{B}, L, S> = \pm\,|-q, -\mathscr{B}, -L, -S>$;

- therefore, only states $Q = \mathscr{B} = L = S = 0$ may be $\mathbb{C}$-eigenstates (e.g. π^0, η, γ, $[\pi^+\pi^-]$).

Generalization [**G-parity**]: $\mathbb{G} \equiv \mathbb{C}\,\mathbb{R}_2$,

where $\mathbb{R}_2$ = rotation in the isospin space:

$\mathbb{R}_2 \equiv \exp(-i\pi\tau_2)$;

$\mathbb{R}_2|I, I_3> = (-)^{I-I3}\,|I, -I_3>$

$\mathbb{R}_2|q, \vec{x}, t, I, I_3> = (-)^{I-I3}|-q, \vec{x}, t, I, -I_3>$;

- $\mathbb{G}$ has more eigenstates than $\mathbb{C}$; e.g.:

  $\mathbb{C}\;|\pi^\pm> \;= -\,|\pi^\mp>$;

  $\mathbb{C}\;|\pi^0> \;= +\,|\pi^0>$;

  $\mathbb{R}_2|\pi^\pm> \;= +\,|\pi^\mp>$;

  $\mathbb{R}_2|\pi^0> \;= -\,|\pi^0>$.

- therefore:

  $\mathbb{G}\,|\pi^{\pm,0}> = \mathbb{C}\,\mathbb{R}_2\,|\pi^{\pm,0}> = -\,|\pi^{\pm,0}>$.

- $\mathbb{G}$-parity is multiplicative :

  $\mathbb{G}\,|n\pi^+\, m\pi^-\, k\pi^0> =$

  $= (-)^{n+m+k}\,|n\pi^+\, m\pi^-\, k\pi^0>$;

  $\mathbb{G}\,|q\bar{q}> = (-)^{L+S+I}\,|q\bar{q}>$;

- $\mathbb{G}$ is useful:
  - $\mathbb{G}$-parity is conserved <u>only in strong interactions</u> ($\mathbb{C}$ and isospin are valid);
  - it produces selection rules (e.g. a decay in odd/even number of π's is allowed/forbidden).

- e.g. $\omega(782)$ is $I^G(J^{PC}) = 0^-(1^{--})$:

  BR $(\omega \to \pi^+\pi^-\pi^0)$ = $(89.2\pm0.7)\%$

  BR $(\omega \to \pi^+\pi^-)$ = $(\;1.5\pm0.1)\%$

  opposite to the obvious phase-space predictions (more room for 2π than 3π decay).

- [*see also* J/ψ *decay*].

quantum numbers : some proofs

- Be $|q, \vec{x}, \vec{a}>$ an eigenstate of a generic operator $\mathbb{K}$ ($\mathbb{K} = \mathbb{C}, \mathbb{G}, \mathbb{P}, \mathbb{S}$ [spin-flip], $\mathbb{T}$ [time-reversal]).

- From their definition:
  $\mathbb{K}^2 = \mathbb{1} \quad \rightarrow \boxed{\mathbb{K}^{-1} = \mathbb{K}.}$

- an eigenstate of $\mathbb{K}$ has eigenvalue K:
  $\mathbb{K}\ |q, \vec{x}, \vec{a}> = K\ |q, \vec{x}, \vec{a}>;$
  $\mathbb{K}^2\ |q, \vec{x}, \vec{a}> = K^2\ |q, \vec{x}, \vec{a}> = |q, \vec{x}, \vec{a}>;$
  $K^2 = 1 \rightarrow \boxed{K = \text{real} = \pm 1.}$

- $\boxed{P(\gamma) = -1}$ [from Maxwell equations];

- For a $q\bar{q}$ (or particle-antiparticle) state:
  $\mathbb{S}\ \mathbb{P}\ \mathbb{C}\ \ |q,\vec{x},\vec{s} \quad -q,-\vec{x},-\vec{s}> =$
  $= C\ \mathbb{S}\ \mathbb{P}\ |-q,\vec{x},\vec{s}, \ \ q,-\vec{x},-\vec{s}> =$
  $= C\ P\ \mathbb{S}\ \ |-q,-\vec{x},\vec{s}, \ \ q,\vec{x},-\vec{s}> =$
  $= C\ P\ S\ \ |-q,-\vec{x},-\vec{s}, q,\vec{x},\vec{s}> =$
  $= C\ P\ S\ \ |q,\vec{x},\vec{s}, \quad -q,-\vec{x},-\vec{s}>.$
  $\rightarrow \mathbb{S}\ \mathbb{P}\ \mathbb{C} = CPS = \pm\ 1;$
  $\rightarrow \boxed{\mathbb{C} = \pm\mathbb{S}^{-1}\ \mathbb{P}^{-1} = \pm\mathbb{S}\ \mathbb{P}.}$

- see also [FNSN1, §7]

the spin $\vec{s}$ is an axial vector ($\vec{a}$)

hadrons : "elementary" or composite ?

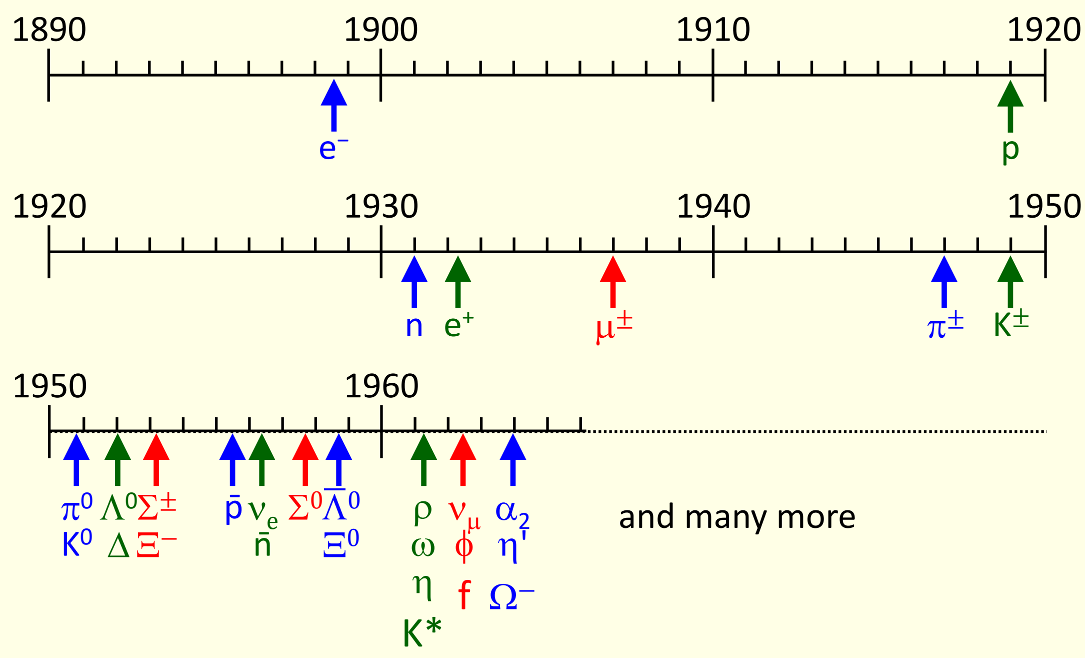
Over time the very notion of "*elementary* (???) particle" entered a deep crisis.

The existence of (too) many hadrons was seen as a contradiction with the elementary nature of the fundamental component of matter.

It was natural to interpret the hadrons as consecutive resonances of elementary components.

The main problem was then to measure the properties of the components and possibly to observe them.

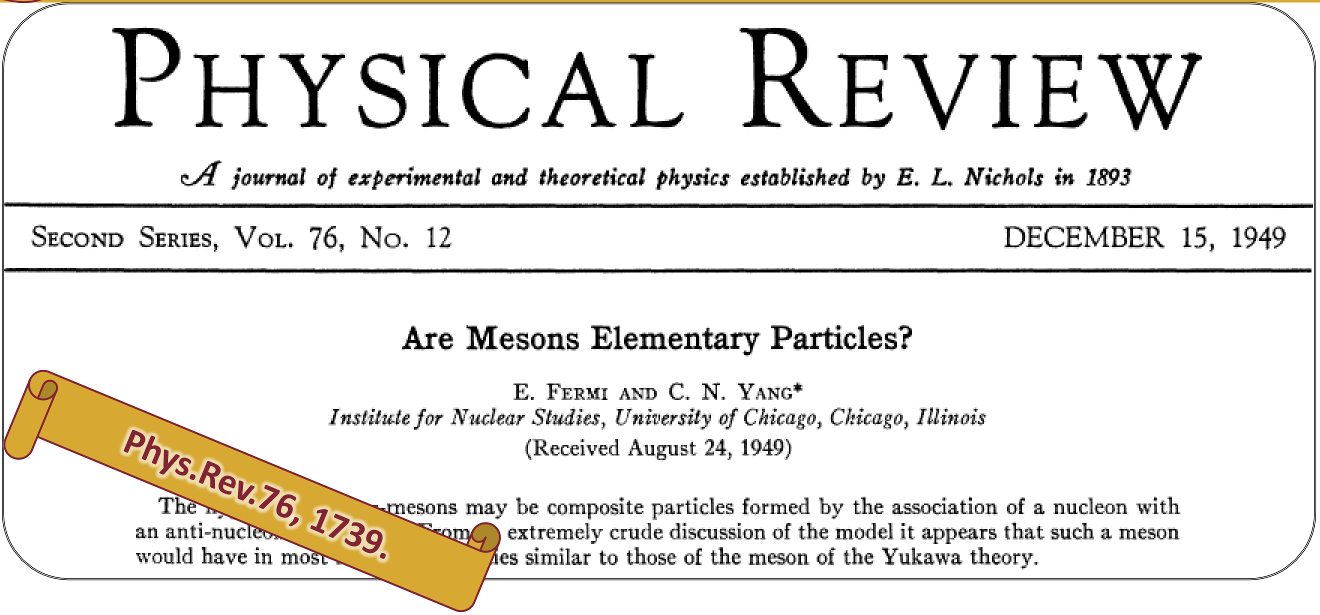
[... and the leptons ? ...]



too many hadronic states: resonances ?

the figure shows the particle discoveries from 1898 to the '60s; their abundance and regularity, as a function of quantum numbers like charge and strangeness, were suggesting a possible sequence, similar to the Mendeleev table [FNSN1].

hadrons : "elementary" or composite ?


PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 76, NO. 12 DECEMBER 15, 1949

Are Mesons Elementary Particles?

E. FERMI AND C. N. YANG*

Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

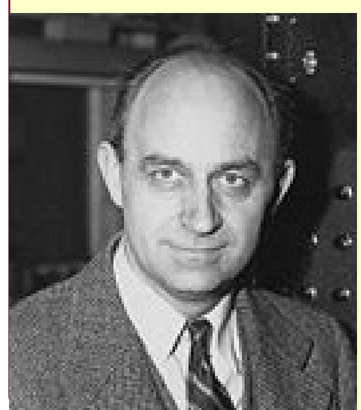
(Received August 24, 1949)

The ... -mesons may be composite particles formed by the association of a nucleon with an anti-nucle... From ... extremely crude discussion of the model it appears that such a meson would have in most ... ies similar to those of the meson of the Yukawa theory.

1949 : E.Fermi and C.N.Yang proposed that ALL the resonances were bound state p-n.

1956 : Sakata extended the Fermi-Yang model including the Λ, to account for strangeness : all hadronic states were then composed by (p, n, Λ) and their antiparticles.

Enrico Fermi

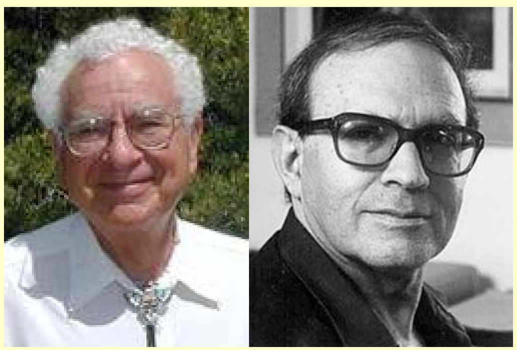


Chen-Ning Yang (杨振宁 - 楊振寧, *Yáng Zhènníng*)

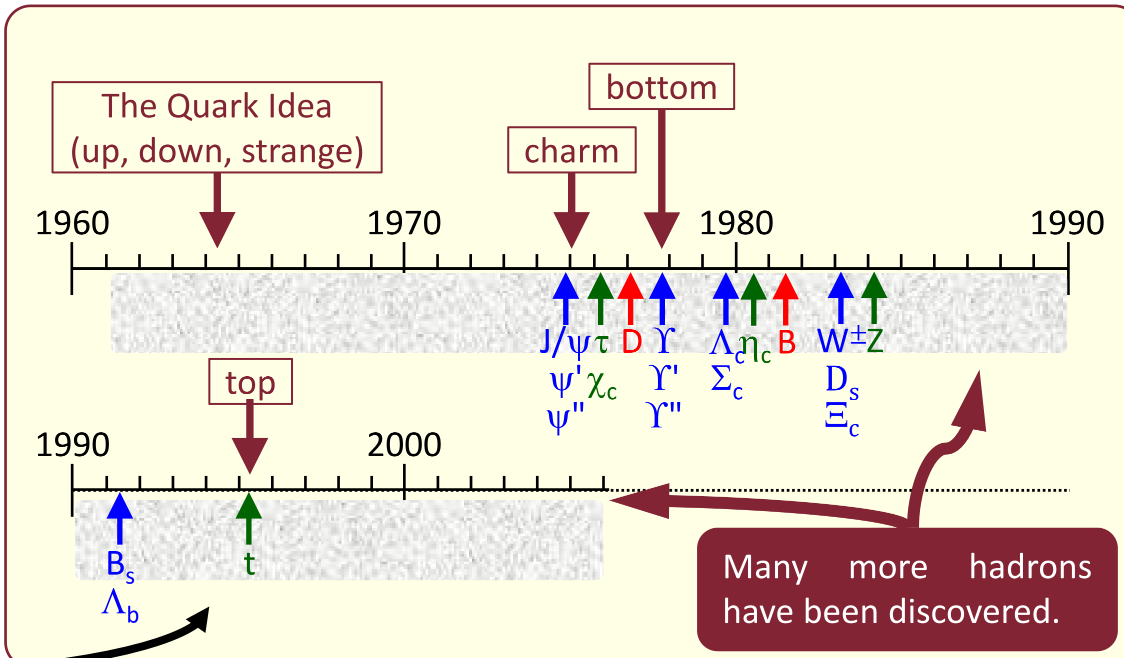
Shoiki Sakata (坂田 昌一, *Sakata Shōichi*)

hadrons : "elementary" or composite ?

1961 : M. Gell-Mann and Y. Ne'eman (independently) proposed a new classification, the **Eightfold Way**, based on the symmetry group SU(3). The classification did **NOT** explicitly mention an **internal structure**. The name was invented by Gell-Mann and comes from the "eight commandments" of the Buddhism.



Murray Gell-Mann | Yuval Ne'eman (יובל נאמן)

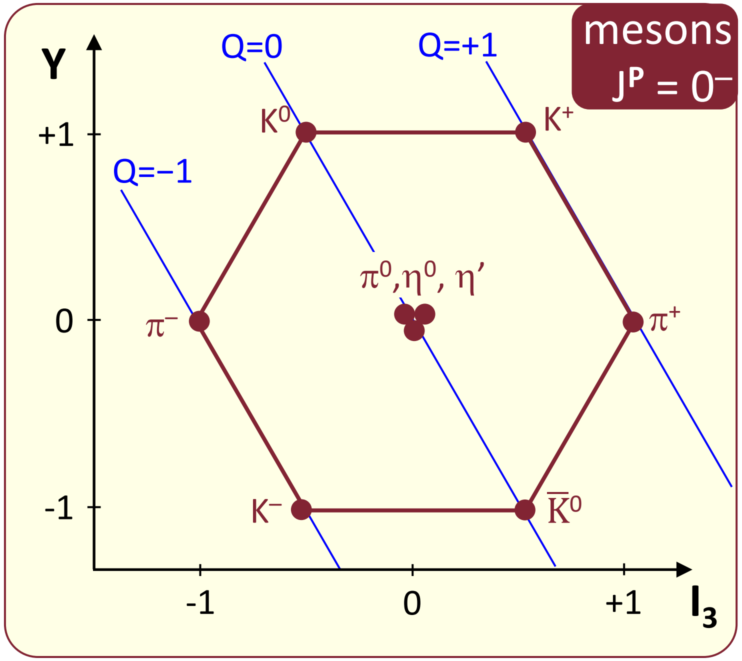


Warning : "*t*" is a quark, not a hadron (in modern language).

the Eightfold Way: 1961-64

FNSN1, 7

All hadrons (known in the '60s) are classified in the plane **(I_3 – Y)**, (<u>Y = strong hypercharge</u>):



$I_3 = I_z$ = third component of isospin;
$Y = \mathscr{B} + S$ **[baryon number + strangeness]**.

The strangeness ***S***, which contributes to ***Y***, had the effect to <u>enlarge the isospin symmetry</u> group **SU(2)** to the larger **SU(3): Special Unitarity group**, with dimension=3.

The Gell-Mann – Nishijima formula (1956) was :

$Q = I_3 + ½(\mathscr{B}+S)$

including heavy flavors [$\mathscr{B}$:baryon, B:bottom] :

$Q = I_3 + ½(\mathscr{B}+S+C+B+T)$

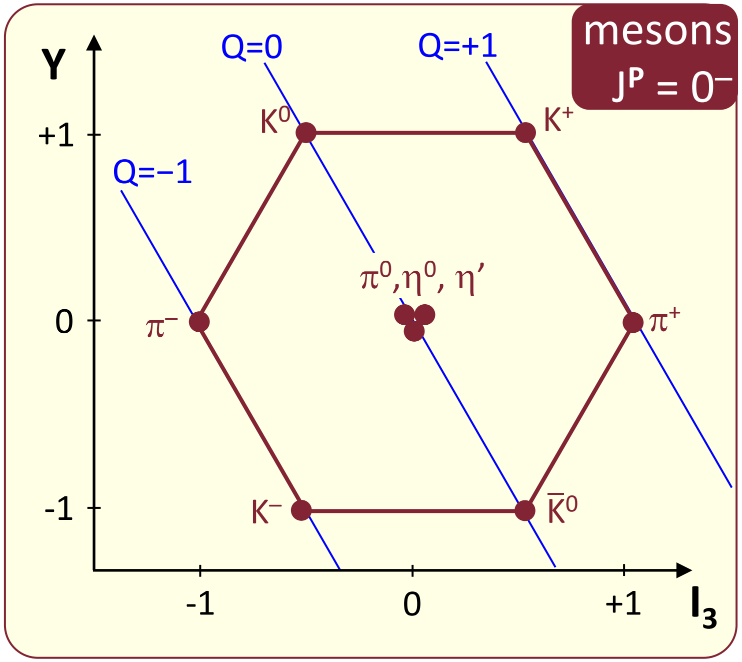
This symmetry is now called "**flavor SU(3) [$SU(3)_F$]**", to distinguish it from the "**color SU(3) [$SU(3)_C$]**", which is the exact symmetry of the strong interactions in QCD. *see later*

the Eightfold Way: SU(3)

The particles form the multiplets of **SU(3)$_F$**. Each multiplet contains particles that have the same spin and intrinsic parity. The basic multiplicity for mesons is nine ($3\times\bar{3}$), which splits in two SU(3) multiplets: (octet + singlet). For baryons there are octects + decuplets.

The gestation of SU(3) was long and difficult. It both explained the multiplets of known particles/resonances, and (more exciting) predicted new states, before they were actually discovered (*really a triumph*).

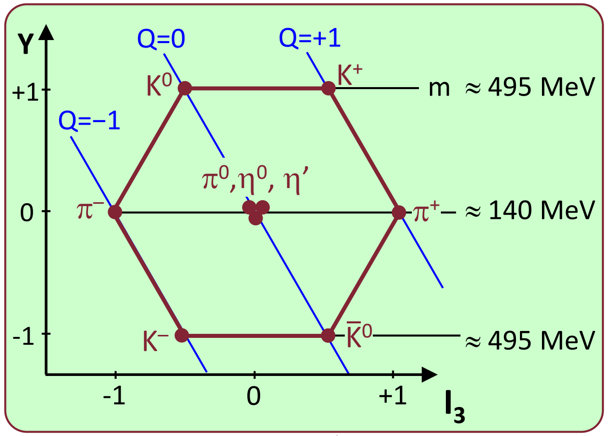
However, the mass difference p – n (or $\pi^\pm - \pi^0$) is < few MeV, while the π – K (or p – Λ) is much larger. Therefore, while the isospin symmetry **SU(2)** is almost exact, the symmetry **SU(3)$_F$**, grouping together strange and non-strange particles, is substantially violated.

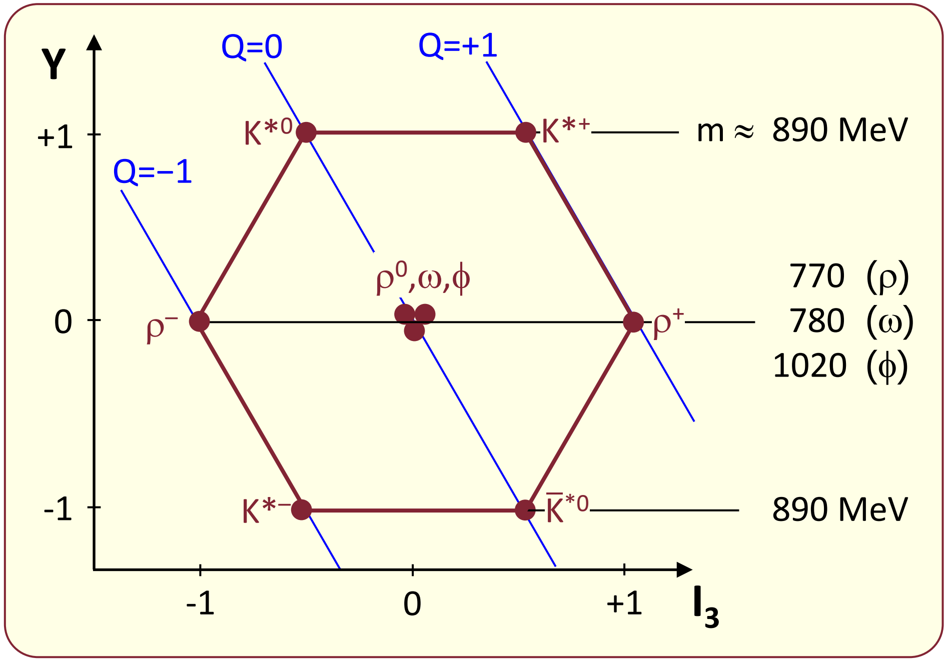


In principle, in a similar way, the discovery of heavier flavors could be interpreted with higher groups (e.g. **SU(4)$_F$** to incorporate the charm quark, and so on). However, these higher symmetries are broken even more, as demonstrated by the mass values. Therefore, **SU(6)$_F$** for all known mesons $J^P = 0^-$ is (almost) never used.

the Eightfold Way: mesons $J^P=1^-$

Another example of a multiplet: the octet of vector mesons :

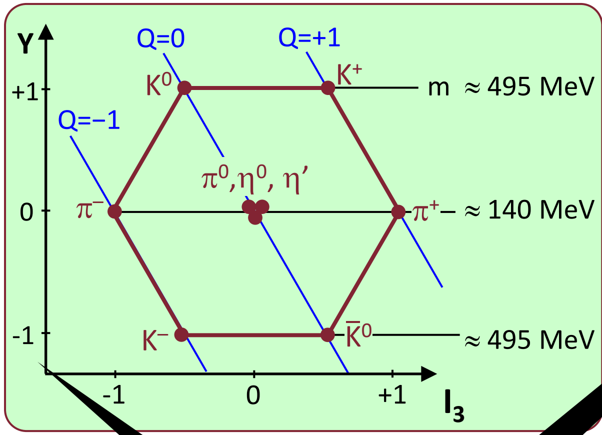


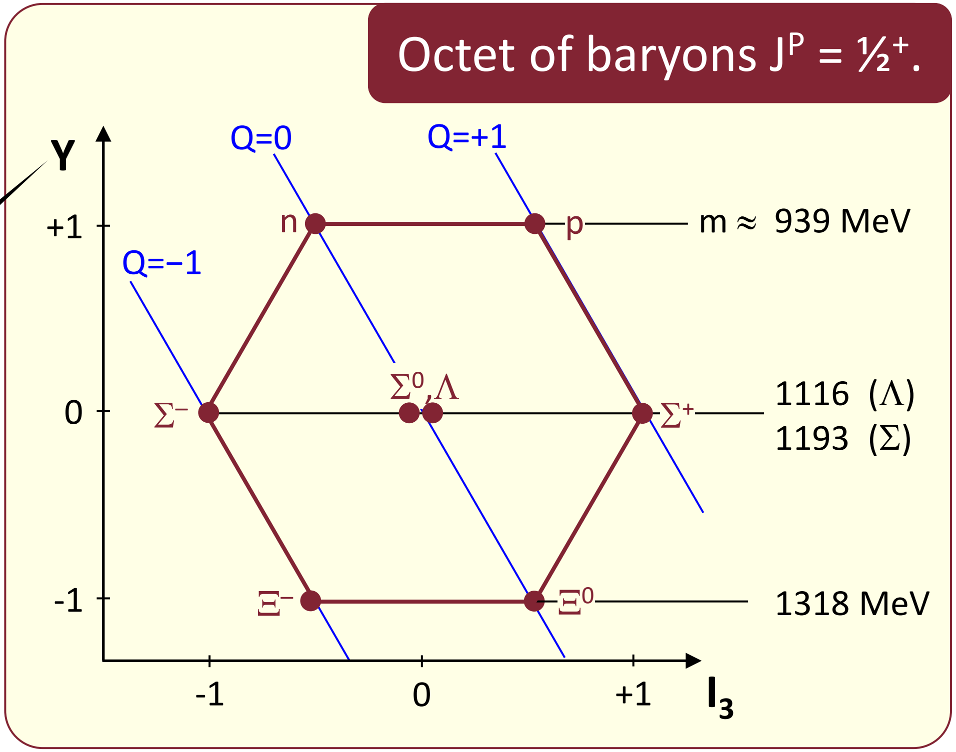


[mesons $J^P = 0^-$]

meson resonances $J^P = 1^-$ (all discovered by 1961).

the Eightfold Way: baryons $J^P=½^+$



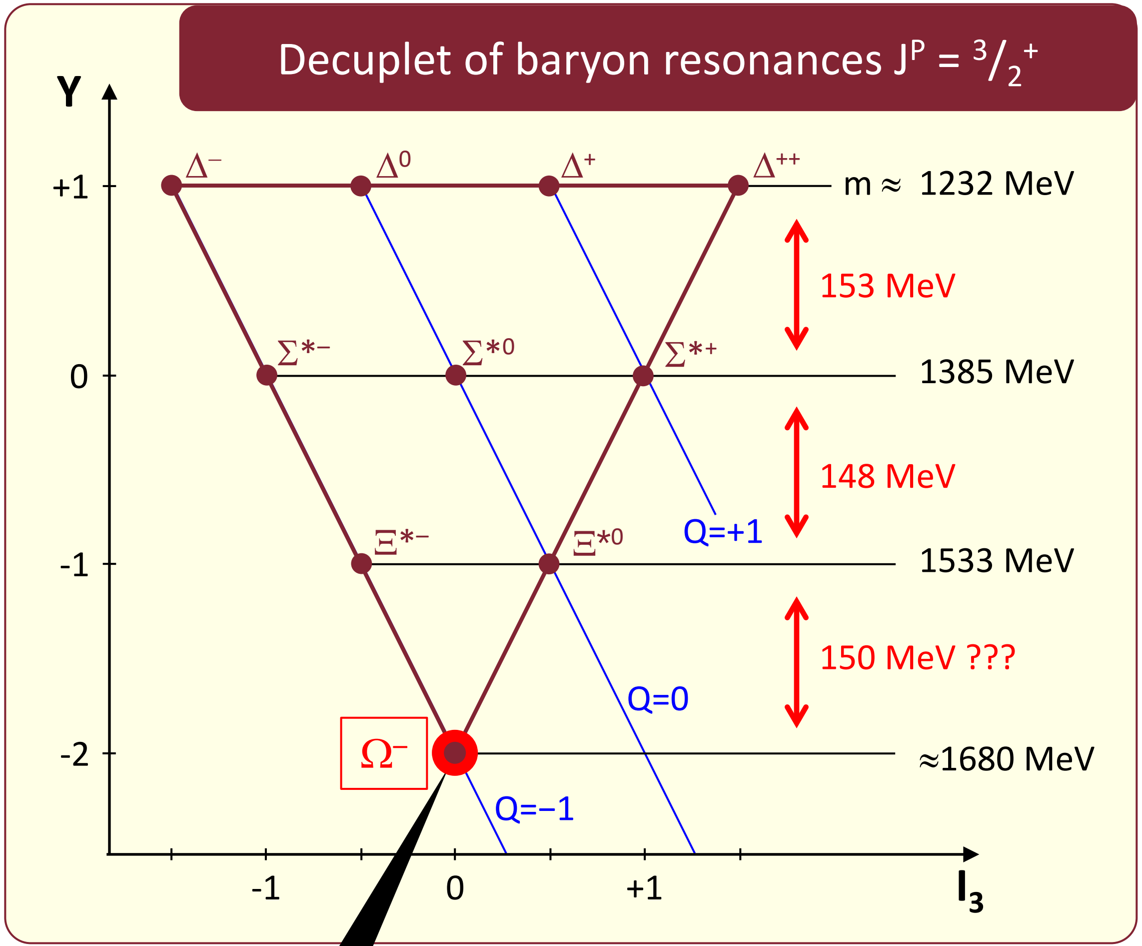


mesons: Y = S
baryons: Y = S + $\mathscr{B}$

notice the masses: for mesons, because of $\mathbb{CPT}$ (K ↔ K̄) the masses of an octet are symmetric wrt (S=0, I_3=0), while for baryons the mass increases as –S

[*because the s-quark (S= –1) is heavier than u/d, but they did not know it*]

the Eightfold Way: baryons $J^P={}^3/_2{}^+$



when the Eightfold Way was first proposed, this particle (now called Ω⁻) was not known → see next slide.

The next multiplet of baryons is a decuplet $J^P = {}^3/_2{}^+$.

When the E.W. was proposed, they knew only 9 members of the multiplet, but can predict the last member:

- it is a decuplet, because of E.W.;
- the state $Y = -2$, $I_3 = 0$ ($\rightarrow Q = -1$, $S = -3$, $\mathscr{B}=1$) must exists;
- call it Ω^-;
- look the mass differences vs Y:
- mass linear in Y $\rightarrow m_{\Omega^-} \approx 1680$ MeV (*NOT an E.W. requirement, but a reasonable assumption*);
- the conservation laws set the dynamics of production and decay of the Ω^-.

the discovery of the Ω^-

The particle Ω^-, predicted (★) in 1962, was discovered in 1964 by N.Samios et al., using the 80-inch hydrogen bubble chamber at Brookhaven (next slide).

The Ω^- can only decay weakly to an S = –2 final state [(1)] :

$\Omega^- \to \Xi^0\, \pi^-$; $\to \Xi^-\, \pi^0$; $\to \Lambda^0\, K^-$;

[a posteriori confirmed by the measurement $\tau_{\Omega^-} \cong 0.82 \times 10^{-10}$ s]

———————————

(1) Since the electromagnetic and strong interactions conserve the strangeness, the lightest (non-weak) S- and $\mathcal{B}$- conserving decay is :

$\Omega^- \to \Xi^0\, K^-$ [S : –3 → –2 –1, $\mathcal{B}$: +1 → +1 +0]

which is impossible, because

$m(\Omega) \approx 1700$ MeV < $m(\Xi) + m(K) \approx 1800$ MeV.

Therefore the Ω^- must decay via strangeness-violating weak interactions : the Ω^- lifetime reflects its weak (NOT strong NOR e.m.) decay.

(★) ***From a 1962 report:***

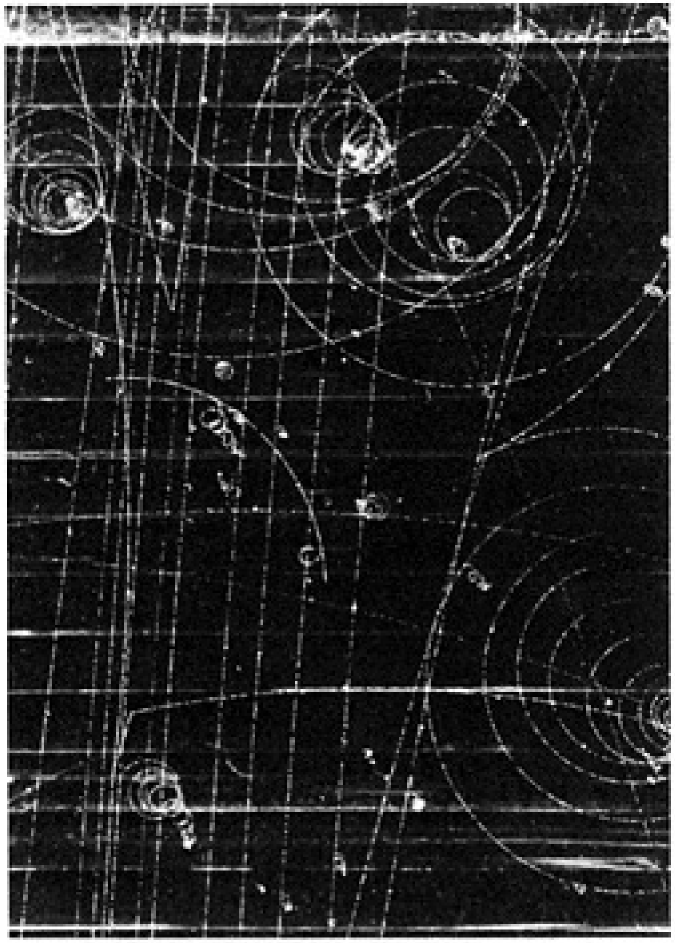
Discovery of Ξ^* resonance with mass ~1530 MeV is announced [...].

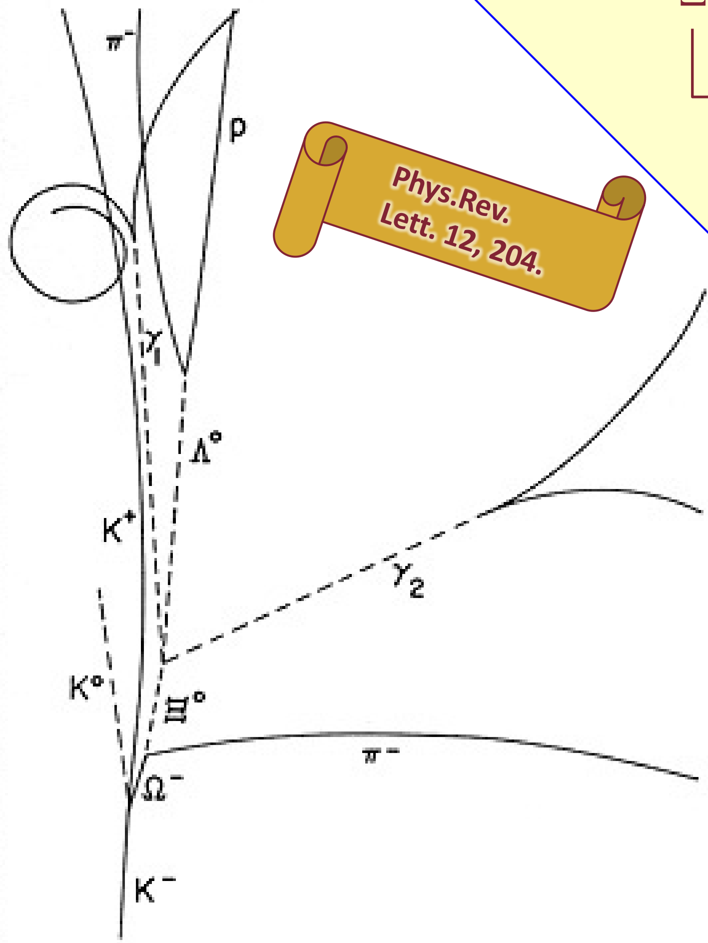
[*As a consequence,*] **Gell-Mann and Ne'eman** [...] predicted a new particle and all its properties:

- Name = Ω^- (*Omega* because this particle is the last in the decuplet);
- Mass ≈ 1680 MeV (the masses of Δ, Σ^* and Ξ^* are about equidistant ~150 MeV);
- Charge = –1;
- Spin = $^3/_2$;
- Strangeness = –3, Y = –2;
- Isospin = 0 (no charge-partners);
- Lifetime ~10^{-10} s, because of its weak decay, since strong decay is forbidden[(1)];
- Decay modes: $\Omega^- \to \Xi^0\, \pi^-$ or $\Omega^- \to \Xi^-\, \pi^0$.

the discovery of the Ω^-: the event

the Ω^- observation required both genius and luck (e.g. compute the probability of the two γ conversions in H_2):





Phys.Rev. Lett. 12, 204.

Brookhaven National Laboratory 80-inch hydrogen bubble chamber - 1964

$K^- + p \rightarrow \Omega^- + K^+ + K^0$
- $\hookrightarrow \Xi^0 + \pi^-$ ($\Delta S = 1$ weak decay)
  - $\hookrightarrow \pi^0 + \Lambda$ ($\Delta S = 1$ w.d.)
    - $\hookrightarrow \pi^- + p$ ($\Delta S = 1$ w.d.)
    - $\hookrightarrow \gamma + \gamma$ (e.m. decay)
      - $\hookrightarrow e^+e^-$
      - $\hookrightarrow e^+e^-$

Nick Samios

the static quark model

In 1964 M. Gell-Mann and G. Zweig proposed independently that all the hadrons are composed of three constituents, that Gell-Mann called[1] **quarks**.

This model, enriched by both extensions (other quarks) and dynamics (electroweak interactions and QCD) is still the basis of our understanding of the elementary particles, the **Standard Model**[2].

In this chapter we consider only the static properties of the three original quarks. Sometimes, in the literature, it is referred as the *naïve quark model*.

1969 : Gell-Mann is awarded Nobel Prize *"for his contributions and discoveries concerning the classification of elementary particles and their interactions"*.

---

[1] The name so whimsical was taken from the (now) famous quote "*Three quarks for Muster Mark !*", from James Joyce's novel "*Finnegans Wake*" (book 2, chapt. 4).

[2] At that time it was not clear whether the quark hypothesis was a mathematical convenience or reality. Today, as shown in the following, our understanding is clearer, but complicated: the quarks are real (to the extent that all QM particles are), but they cannot be seen as isolated single objects.

the static quark model: u d s

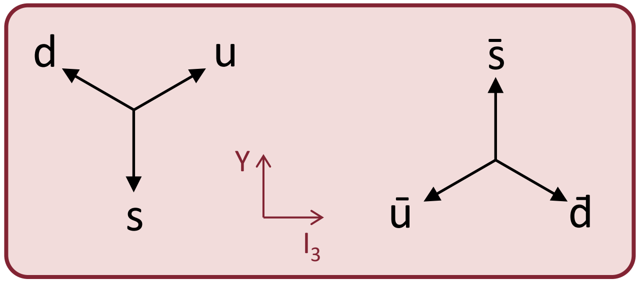
The hypothesis:

- three quarks **u**, **d**, and **s** (up, down, strange);
- quarks (q): standard Dirac fermions with spin ½ and fractional charge (±⅓e ±⅔e);
- antiquarks ($\bar{q}$): according to Dirac theory, the q-antiparticles;
- baryons: combinations qqq (e.g. uds, uud);
- antibaryons: three antiquarks (e.g $\bar{u}\bar{u}\bar{d}$);
- mesons: pairs $q\bar{q}$ (e.g $u\bar{u}$, $u\bar{d}$, $s\bar{u}$);
- "*antimesons*": a $\bar{q}q$ pair: the mesons are their own antiparticles, i.e. "anti-mesons" = mesons.

	u	d	s	c	b	t
$\mathscr{B}$ baryon	⅓	⅓	⅓			
J spin	½	½	½			
I isospin	½	½	0			
$\mathbf{I_3}$ 3rd i-spin	½	−½	0			
S strang.	0	0	−1			
Y $\mathscr{B}$+S	⅓	⅓	−⅔			
Q I_3 + ½Y	⅔	−⅓	−⅓			

c, **b**, **t** not yet discovered in the '60 !!! see § 3

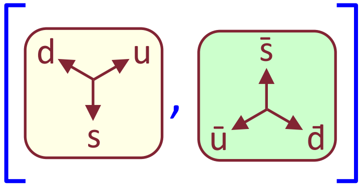
The quarks form a triplet, which is a basic representation of the group SU(3). Quarks may be represented in a vector shape in the plane I_3 – Y; their combinations (= hadrons) are the sums of such vectors.



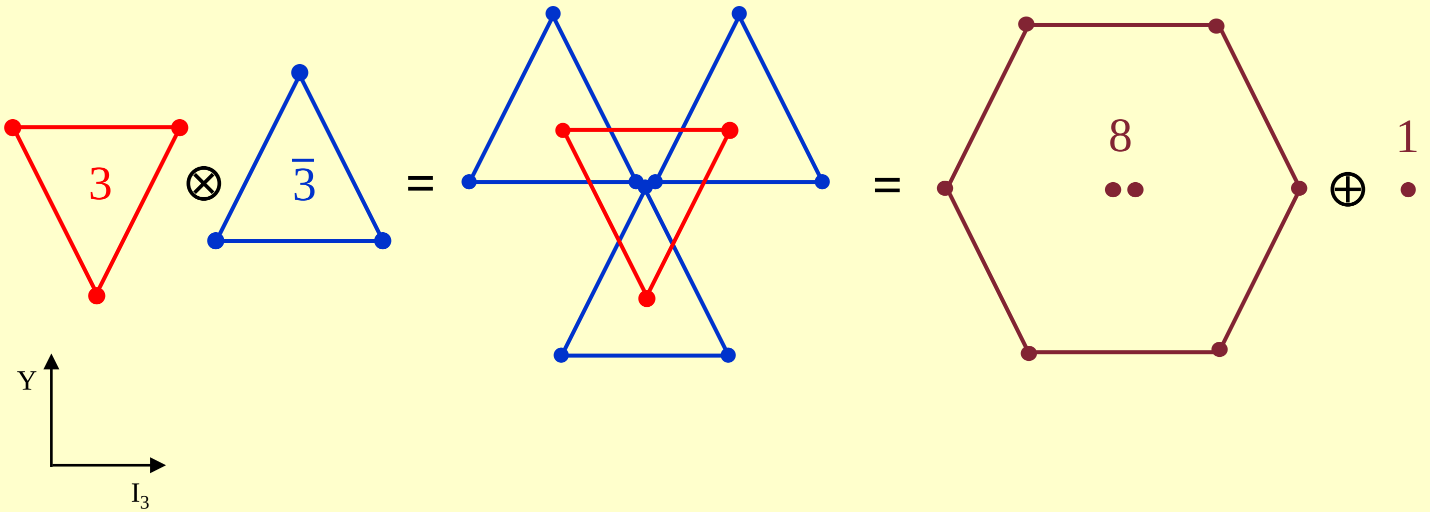
The mesons

"Build" the mesons $q\bar{q}$ with these rules :

- in the space I_3 - Y, sum "vectors" (i.e. quarks and antiquarks) to produce $q\bar{q}$ pairs, i.e. mesons;



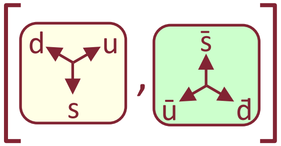
- all the combinations are allowed:



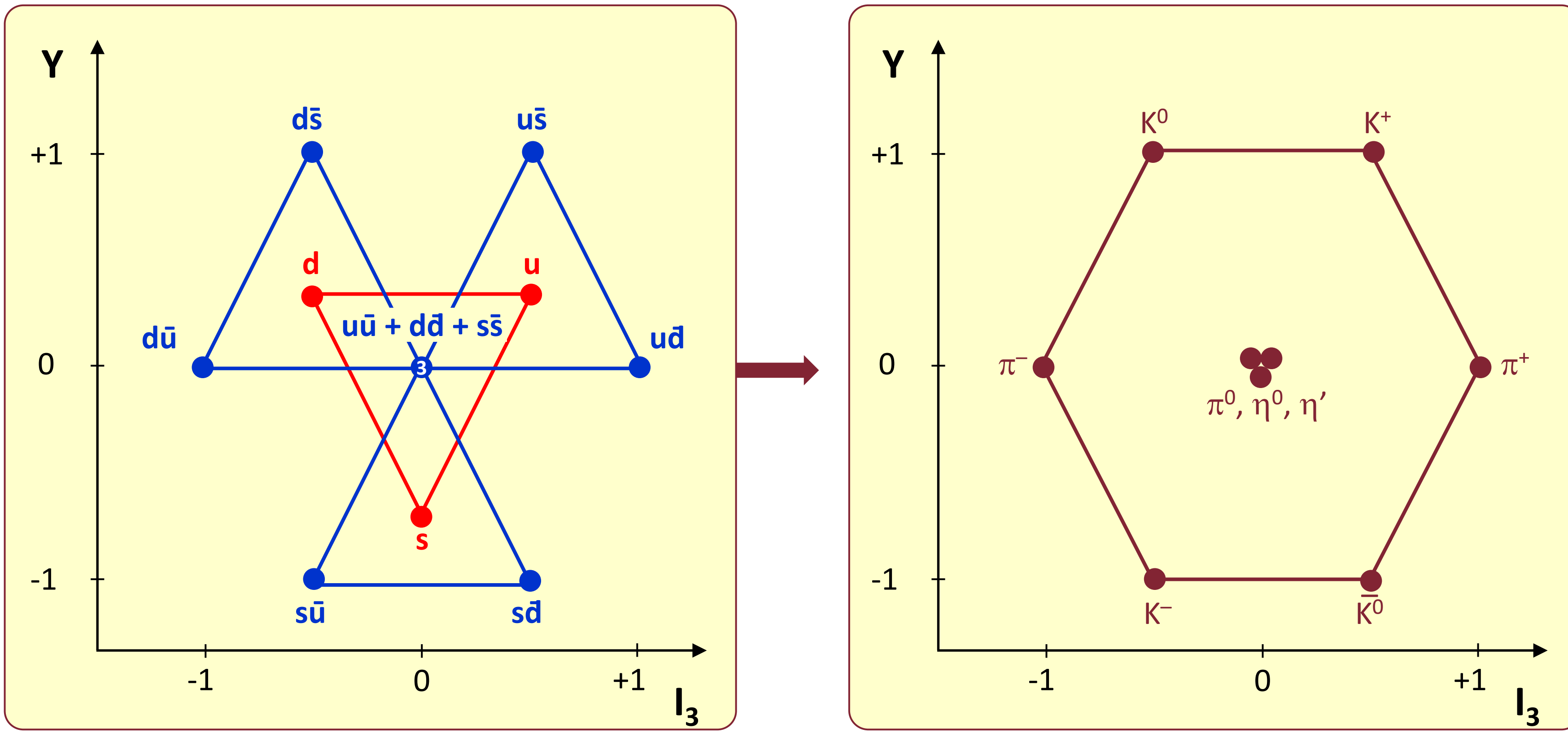
- the pseudoscalar mesons ($J^P=0^-$) are $q\bar{q}$ states in s-wave with opposite spins (⇑⇓).

The mesons: $J^{PC}=0^{-+}$

More specifically, with s-wave ($J^{PC}=0^{-+}$), we get the "pseudoscalar" nonet :



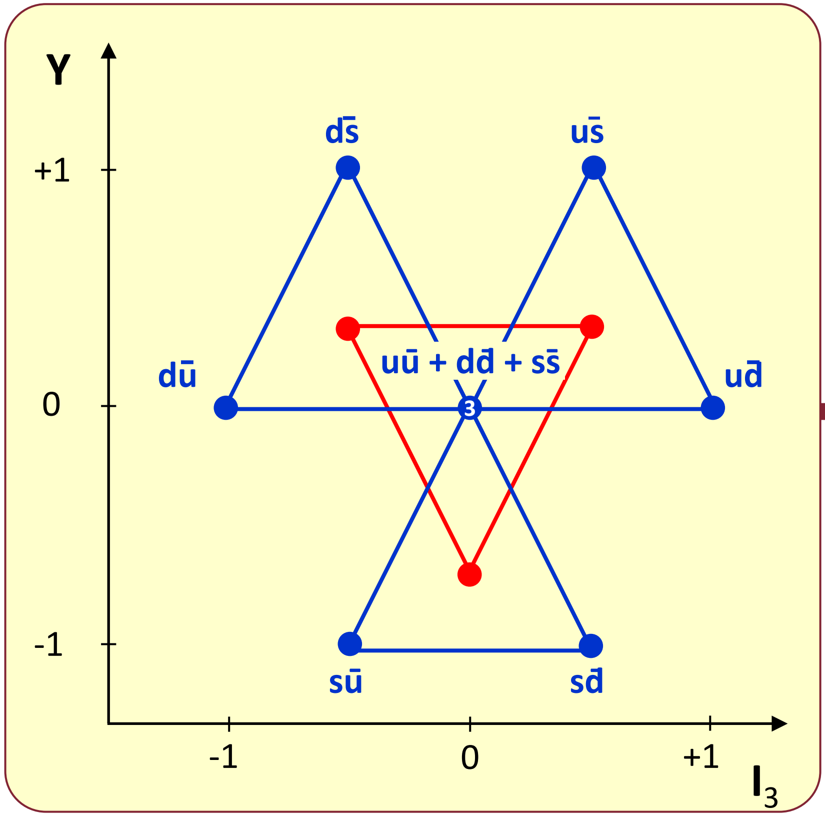
Notice that π^0, η, η' are combinations (mixing) of the three possible $q\bar{q}$ states (for the mixing parameters see later) :

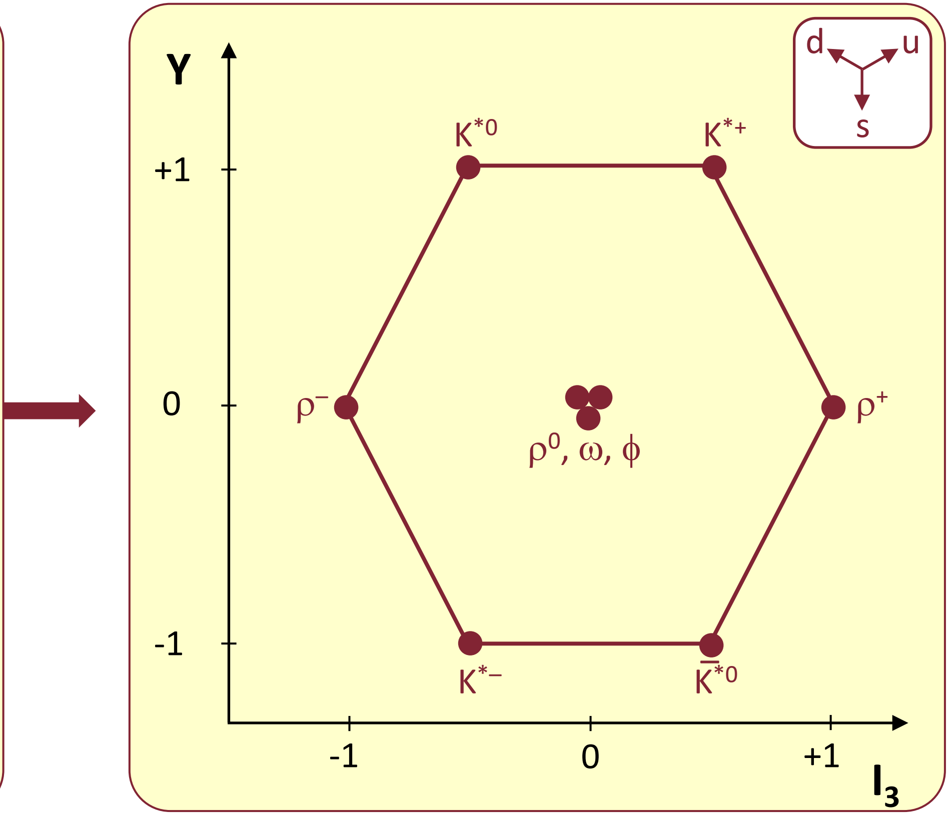


The mesons: $J^{PC}=1^{--}$

If $J^{PC} = 1^{--}$ (i.e. spin ⇑ ⇑), the "vector" nonet :

Notice that ρ^0, ω, φ are combinations (mixing) of the three possible $q\bar{q}$ states :





Meson quantum numbers: J^{PC}

- Parity : the quarks and the antiquarks have opposite P :
  $P_{q\bar{q}} = P_1 P_2 (-1)^L = -1\,(-1)^L = (-1)^{L+1}$.

- Charge conjugation : for mesons, which are also $\mathbb{C}$ eigenstates, $\mathbb{C} = \mathbb{P}\mathbb{S}$, parity followed by spin swap (see before).

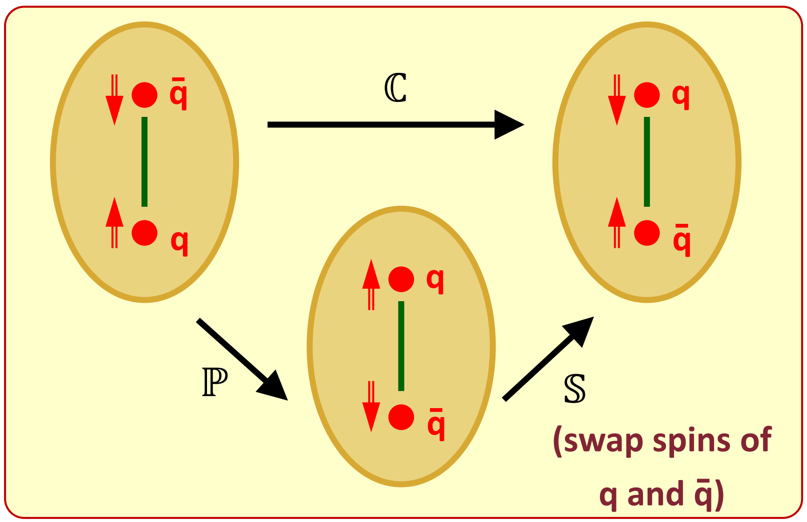
$P = (-1)^{L+1}$;

$S = (-1)^{S+1}$ (Pauli principle, [BJ, 263]);

$C = P \times S = (-1)^{L+S}$;

$G = (-1)^{L+S+I}$ (see before).

$$\underbrace{\frac{1}{\sqrt{2}}\left(\Uparrow\Downarrow - \Downarrow\Uparrow\right);}_{\substack{\mathbf{S=0}\\ \mathbf{antisymmetric}}} \qquad \underbrace{\Downarrow\Downarrow; \quad \frac{1}{\sqrt{2}}\left(\Uparrow\Downarrow + \Downarrow\Uparrow\right); \quad \Uparrow\Uparrow}_{\substack{\mathbf{S=1}\\ \mathbf{symmetric}}}$$



L	S	J=L⊕S	P	C	I	G
0	0	0	–	+	0	+
					1	–
	1	1	–	–	0	+
					1	–
1	0	1	+	–	0	–
					1	+
	1	0,1,2	+	+	0	–
					1	+

$J^{PC} = 0^{-+}, 1^{--}, 1^{+-}, 0^{++}, 1^{++}, 2^{++}, \ldots$

Meson quantum numbers : multiplets

- For the lowest state nonets, these are the quantum numbers :

L	S	J^{PC}	$^{2s+1}L_J$	I=1 state
0	0	0^{-+}	$^1S_0$	$\pi(140)$
	1	1^{--}	$^3S_1$	$\rho(770)$
1	0	1^{+-}	$^1P_1$	$b_1(1235)$
	1	0^{++}	$^3P_0$	$a_0(1450)$
		1^{++}	$^3P_1$	$a_1(1260)$
		2^{++}	$^3P_2$	$a_2(1320)$

- all these multiplets have main qn n = 1;
- as of today ~20 meson multiplets have been (partially) discovered [PDG].
- important activity from the ’50 to the ’70; still some addict;

- method (mainly bubble chambers) :
  - measure (zillions of) events; e.g. :

    $\bar{p}p \to \pi^+ \pi^+ \pi^- \pi^- \pi^0$;
  - look for "peaks" in final state combined mass, e.g. m($\pi^+ \pi^- \pi^0$);
  - the peaks are associated with high mass resonances, decaying via strong interactions (width $\to \Gamma \to$ strength);
  - the scattering properties (e.g. the angular distribution) and decay modes identify the other quantum numbers;
- result : an overall consistent picture;
- Great success !!!

“If I could remember the names of all these particles, I'd be a botanist.”
Enrico Fermi

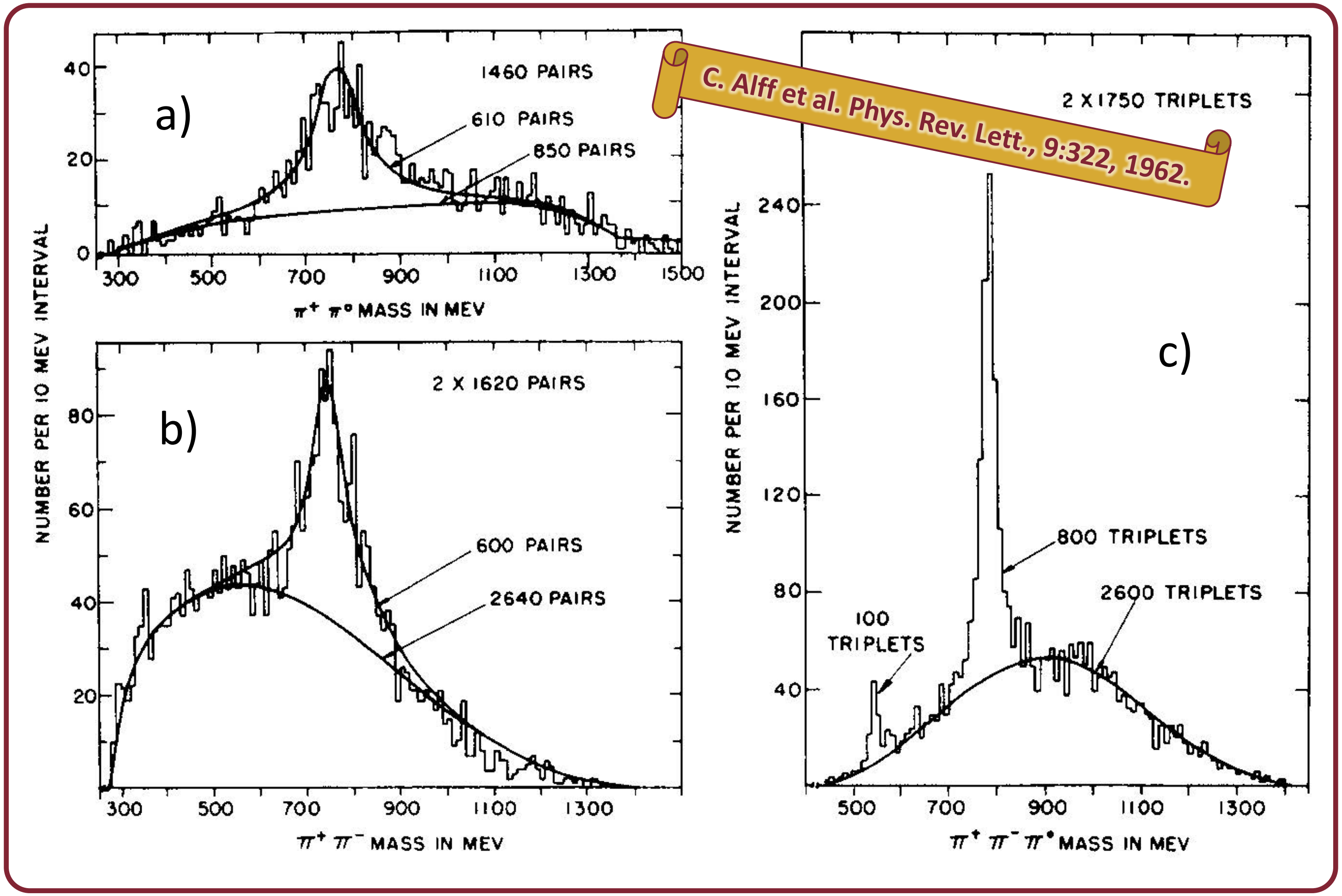
Meson quantum numbers : example

Three examples in $\pi^+ p \to X$

a) $m(\pi^+\pi^0)$ for $X = \pi^+\pi^0 p$

b) $m(\pi^+\pi^-)$ for $X = \pi^+ \pi^+ \pi^- p$

c) $m(\pi^+ \pi^- \pi^0)$ for $X = \pi^+ \pi^+ \pi^- \pi^0 p$



Q: which resonances ?

a) $\rho^+(770) \to \pi^+\pi^0$; b) $\rho^0(770) \to \pi^+\pi^-$; c) $\eta(548) \to \pi^+\pi^-\pi^0$; $\omega(782) \to \pi^+\pi^-\pi^0$.

why not the ρ^0 ?

Meson quantum numbers : $\rho^0 \not\to \pi^0\pi^0$

Problem: $\rho^0 \to \pi^0\pi^0$ is allowed ? **NO**, because of :

a) <u>ℂ-parity</u>

$C(\rho^0) = -1$; $C(\pi^0) = +1$

therefore, since the initial state is a C-eigenstate,

$-1 = (+1) \times (+1) \rightarrow$ **NO**

NB. A general rule : "a vector cannot decay into two equal (pseudo-)scalars".

But (a) and (b) do not hold for weak decays. Instead (c) is due to statistics + angular momentum conservation, and is valid for all interactions.

[(c) also forbids **Z → HH**]

b) <u>Clebsch-Gordan coeff. in isospin space</u>

$|\rho^0\rangle = |I=1, I_3=0\rangle$;

$|\pi^0\rangle = |1, 0\rangle$;

therefore the decay is

$\langle\pi^0\pi^0|\rho^0\rangle = \langle j_1 j_2 m_1 m_2 | J\ M\rangle =$
$= \langle 1\ 1\ 0\ 0\ |\ 1\ 0\ \rangle = 0$;

$\rightarrow$ **NO**.

[PDG, § 44 :

$1\otimes1$		…	1
		…	0
…	…	…	…
0	0	…	**0**

c) <u>Spin-statistics</u>

[Povh, problem 15-1]

- $S(\rho^0) = 1$, $S(\pi^0) = 0$ $\rightarrow L(\pi^0\pi^0) = 1$;
- ρ^0 is a boson → wave function symmetric;
- the π^0's are two equal bosons → space wave function symmetric;
- L=1 makes the wave function anti-symmetric

$\rightarrow$ **NO**.

Meson mixing

Light mesons	$q\bar{q}$	J^{PC} (1)	I	I_3	S	Q (1)	mass (MeV)	$q\bar{q}$ of I_3=0 (2)
π^+, π^0, π^-	$u\bar{d}$, $q\bar{q}^{(2)}$, $d\bar{u}$	0^{-+}	1	1, 0, -1	0	1, 0, -1	140	$\sim(u\bar{u}-d\bar{d})/\sqrt{2}$
η	$q\bar{q}^{(2)}$	0^{-+}	0	0	0	0	550	$\sim(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$
η'	$q\bar{q}^{(2)}$	0^{-+}	0	0	0	0	960	$\sim(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{6}$
K^+, K^0 (3)	$u\bar{s}$, $d\bar{s}$	0^-	½	½,-½	+1	1, 0	495	
$\bar{K}^0, K^-$ (3)	$s\bar{d}$, $s\bar{u}$	0^-	½	½, -½	-1	0, -1	495	
ρ^+, ρ^0, ρ^-	$u\bar{d}$, $q\bar{q}^{(2)}$, $d\bar{u}$	1^{--}	1	1, 0, -1	0	1, 0, -1	770	$\sim(u\bar{u}-d\bar{d})/\sqrt{2}$
ω	$q\bar{q}^{(2)}$	1^{--}	0	0	0	0	780	$\sim(u\bar{u}+d\bar{d})/\sqrt{2}$
ϕ	$q\bar{q}^{(2)}$	1^{--}	0	0	0	0	1020	$\sim s\bar{s}$
K^{*+}, K^{*0} (3)	$u\bar{s}$, $d\bar{s}$	1^-	½	½,-½	+1	1, 0	890	
$\bar{K}^{*0}, K^{*-}$ (3)	$s\bar{d}$, $s\bar{u}$	1^-	½	½, -½	-1	0, -1	890	

Notes :

(1) (L=0, $\mathscr{B}$=0) $\rightarrow$ P = $(-)^{L+1}$ = $-$; C = $(-)^{L+S}$ = $(-)^S$; Q = I_3 + ½Y = I_3 + ½S;

(2) The mesons π^0, η, η', ρ^0, ω, ϕ are mixing of $u\bar{u} \oplus d\bar{d} \oplus s\bar{s}$ (see next);

(3) States with strangeness $\neq 0$ are NOT eigenstates of C; since they have I=½, no I_3=0 exists.

Meson mixing: $J^P = 0^-, 1^-$

Mesons are bound states $q\bar{q}$. Consider only uds quarks (+ $\bar{u}\bar{d}\bar{s}$) in the nonets ($J^P = 0^-\ 1^-$, the *pseudo-scalar* and *vector* nonets) :

- the states ($\pi^+=u\bar{d}$, $\pi^-=d\bar{u}$, $K^+=u\bar{s}$, $K^0=d\bar{s}$, $K^-=s\bar{u}$, $\bar{K}^0=s\bar{d}$) have no quark ambiguity;
- but ($u\bar{u}$ $d\bar{d}$ $s\bar{s}$) have the same quantum numbers and the three states ($\psi_{8,0}$ $\psi_{8,1}$ ψ_1) mix together ($\rightarrow$ 2 angles per nonet);
- the physical particles (π^0, η, η' for 0^-, ρ^0, ω, ϕ for 1^-) are linear combinations $q\bar{q}$;
- ($\psi_{8,1}$) decuples (π^0 ρ^0) ($\rightarrow$ 1 angle only);
- θ_{ps} and θ_v are computed from the mass matrices* [PDG, §15.2];
- notice: the vector mixing $\theta_v \approx 36° \approx \tan^{-1}(1/\sqrt{2})$, i.e. the ϕ meson is almost $s\bar{s}$ only [i.e. $\phi \rightarrow K\bar{K}$, see KLOE exp.];

(... continue)

$$\left.\begin{aligned} \psi_{8,1}[\text{oct}, I=1] &= (u\bar{u} - d\bar{d})/\sqrt{2} \\ \psi_{8,0}[\text{oct}, I=0) &= (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6} \\ \psi_1[\text{sing}] &= (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \end{aligned}\right\} \begin{array}{c}\psi_{\text{multi},I} \\ \text{ideal case}\end{array}$$

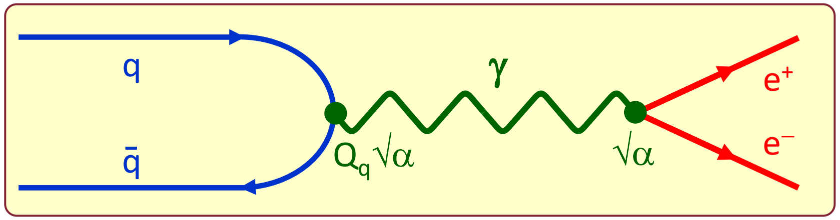
$$\left.\begin{aligned} \pi^0(140) &\approx \psi^{ps}_{8,1} = (u\bar{u} - d\bar{d})/\sqrt{2} \\ \eta(550) &= \psi^{ps}_{8,0}\cos\theta_{ps} - \psi^{ps}_1 \sin\theta_{ps} \\ \eta'(960) &= \psi^{ps}_{8,0}\sin\theta_{ps} + \psi^{ps}_1 \cos\theta_{ps} \end{aligned}\right\} \begin{array}{c} J^P = 0^-, \\ \theta_{\text{pseudo-scalar}} \approx -25°; \end{array}$$

$$\left.\begin{aligned} \rho^0(770) &\approx \psi^{v}_{8,1} = (u\bar{u} - d\bar{d})/\sqrt{2} \\ \phi(1020) &= \psi^{v}_{8,0}\cos\theta_{v} - \psi^{v}_1 \sin\theta_{v} \approx s\bar{s} \\ \omega(780) &= \psi^{v}_{8,0}\sin\theta_{v} + \psi^{v}_1 \cos\theta_{v} \approx \\ &\approx (u\bar{u} + d\bar{d})/\sqrt{2} \end{aligned}\right\} \begin{array}{c} J^P = 1^-, \\ \theta_{\text{vector}} \approx 36°. \end{array}$$

* in principle, both the mass spectra and the mixing angles can be computed from QCD lagrangian $\mathscr{L}_{QCD}$ ... waiting for substantial improvements in computation methods.

Meson mixing: $J^P = 1^-$

The decay amplitudes in the e.m. channels may be computed, up to a common factor, and compared to the experiment;



Few problems :

- the values are small*, e.g. BR($\rho^0 \to e^+e^-$) ≈ 4.7×10^{-5};
- the phase-space factor is important, especially for ϕ, which is very close to the $s\bar{s}$ threshold ($m_\phi - 2\, m_K$ = few MeV).

However, the overall picture is clear: the theory explains the data ***very well***.

$$\left.\begin{array}{l}\rho^0(770) = \dfrac{1}{\sqrt{2}}(u\bar{u} - d\bar{d});\\ \omega(780) = \dfrac{1}{\sqrt{2}}(u\bar{u} + d\bar{d});\\ \phi(1020) = s\bar{s};\end{array}\right\} \to \begin{array}{l}\mathcal{M}_{fi}(\rho^0\omega\phi \to e^+e^-)\\ \propto \alpha \sum_j Q_q^j;\end{array}$$

$$\to \left\{\begin{array}{l}\Gamma(\rho^0 \to e^+e^-) \propto \left[\dfrac{1}{\sqrt{2}}\left(\dfrac{2}{3} - \dfrac{-1}{3}\right)\right]^2 = \dfrac{1}{2};\\ \Gamma(\omega \to e^+e^-) \propto \left[\dfrac{1}{\sqrt{2}}\left(\dfrac{2}{3} + \dfrac{-1}{3}\right)\right]^2 = \dfrac{1}{18};\\ \Gamma(\phi \to e^+e^-) \propto \left[\dfrac{1}{3}\right]^2 = \dfrac{1}{9};\end{array}\right.$$

$$\to \Gamma_\rho : \Gamma_\omega : \Gamma_\phi = \left\{\begin{array}{lll}9 : 1 : 2 & & \text{(theo)}\\ 8.8\pm2.6 : 1 : 1.7\pm0.4 & & \text{(exp).}\end{array}\right.$$

---

* warning: the dominant $\rho^0\omega\phi$ decay modes are strong; however, the <u>e.m.</u> decays $\rho^0\omega\phi \to e^+e^-$, with a much smaller BR, are detectable → $\Gamma_{e.m.}$ measurable → quark charges compared.

The baryons

The construction looks complicated, but in fact is quite simple :

- add the three quarks one after the other;
- count the resultant multiplicity.

In group's theory language :

$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8' \oplus 1$

i.e. a decuplet, two octects and a singlet.

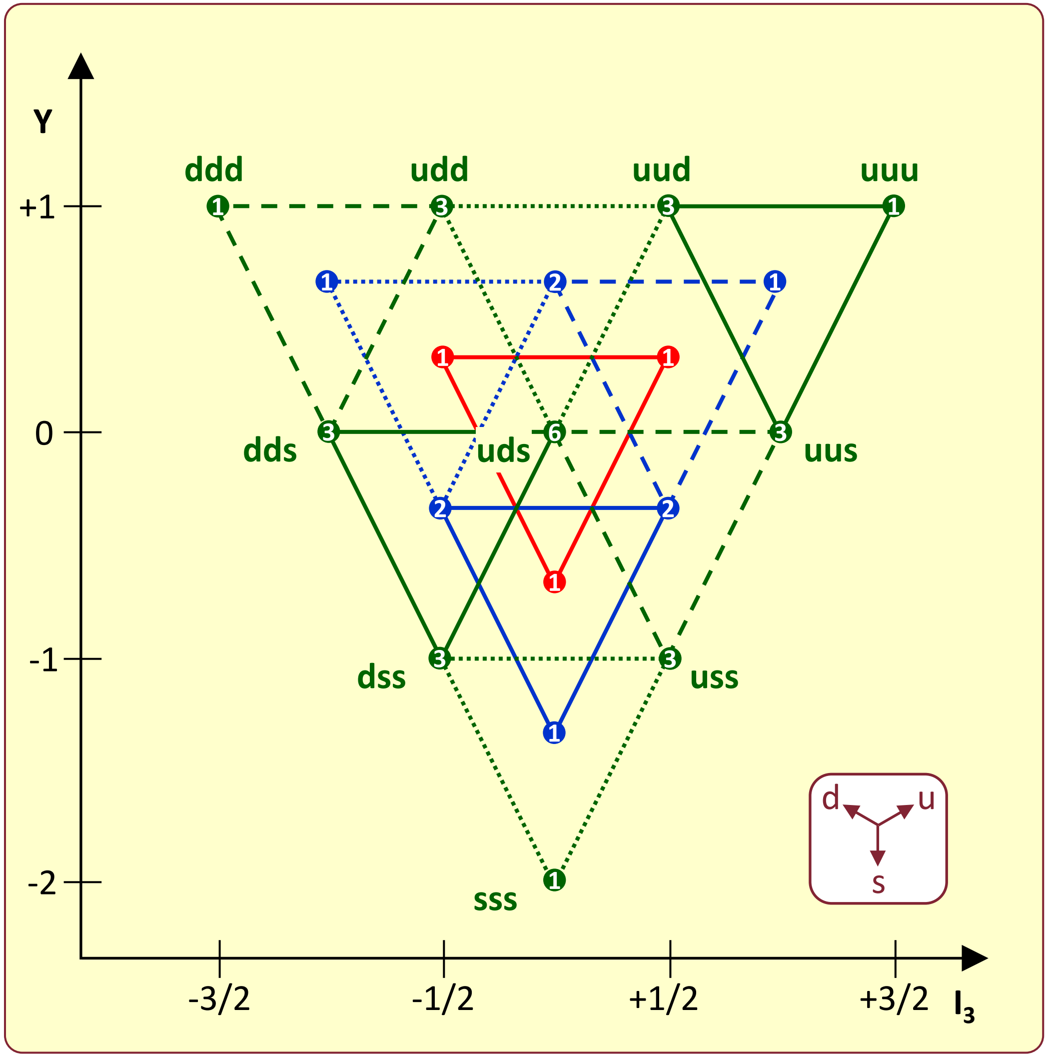
[proof. :

$3 \otimes 3 = \;\; 6 \oplus \bar{3};$

$6 \otimes 3 = 10 \oplus 8;$

$\bar{3} \otimes 3 = \;\; 8 \oplus 1.$ q.e.d.]

Both for 10, 8, 8' and 1 the three quarks have L = 0.



The baryons: quantum numbers

Baryons	qqq	J^P	I	I_3	S	$Q^{(1)}$	mass (MeV)
p, n	uud, udd	$½^+$	½	½, -½	0	1, 0	940
Λ	uds	$½^+$	0	0	-1	0	1115
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	$½^+$	1	1, 0, -1	-1	1, 0, -1	1190
Ξ^0, Ξ^-	uss, dss	$½^+$	½	½,-½	-2	1, 0	1320
$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	uuu, uud, udd, ddd	$^3/_2{}^+$	$^3/_2$	$^3/_2, {}^1/_2, -{}^1/_2, -{}^3/_2$	0	2, 1, 0, -1	1230
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	uus, uds, dds	$^3/_2{}^+$	1	1, 0, -1	-1	1, 0, -1	1385
Ξ^{*0}, Ξ^{*-}	uss, dss	$^3/_2{}^+$	½	½,-½	-2	1, 0	1530
Ω^-	sss	$^3/_2{}^+$	0	0	-3	-1	1670

10
$\Uparrow\Uparrow\Uparrow$

8
$\Uparrow\Uparrow\Downarrow$

Notes :

(1) $Q = I_3 + ½Y = I_3 + ½(\mathscr{B} + S)$; $\mathscr{B} = 1$.

The baryons: the octet $J^P = ½^+$

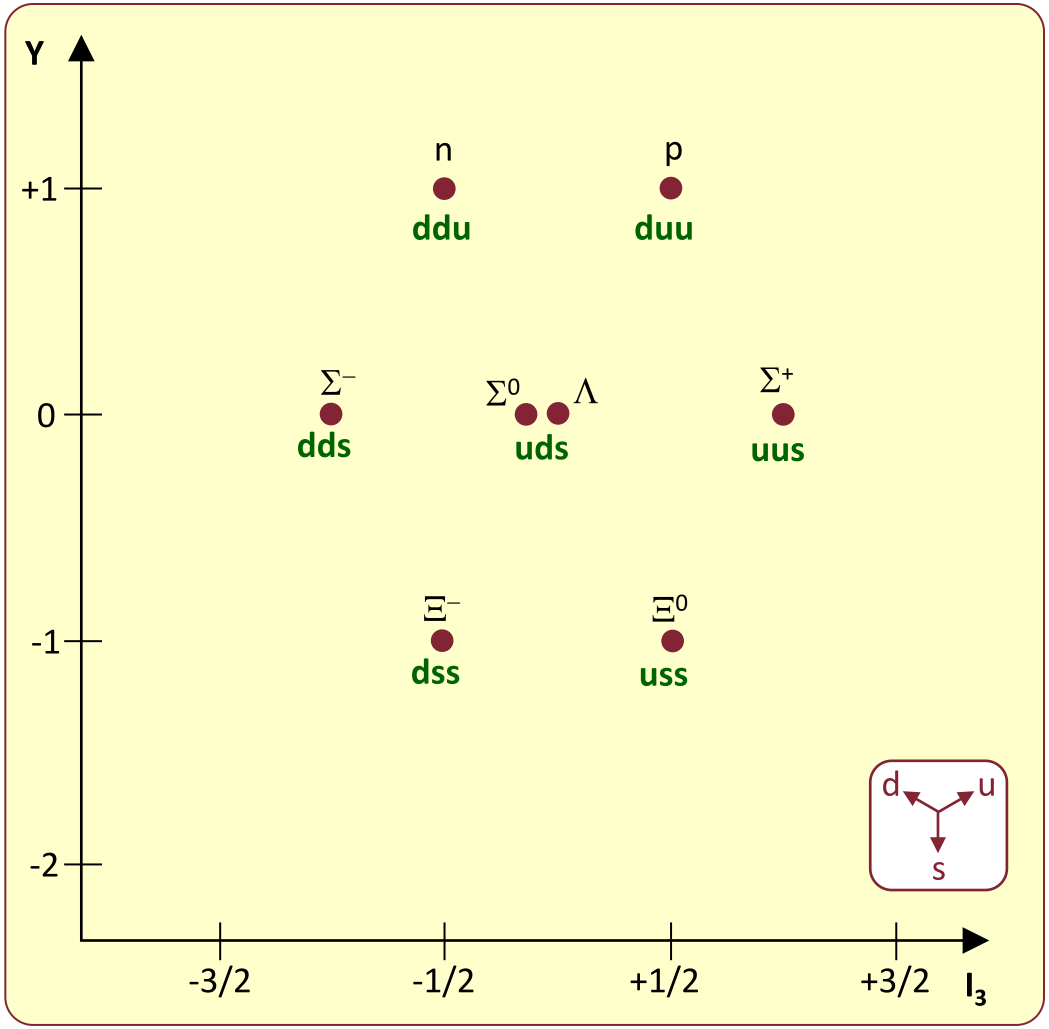
The lowest mass multiplet is an octet, which contains the familiar p and n, a triplet of S=-1 (the Σ's) a singlet S=-1 (the Λ) and a doublet of S=-2 (the Ξ's, sometimes called "cascade baryons").

The three quarks have ℓ = 0 and spin (⇑⇑⇓), i.e. a total spin of ½.

The masses are :

- ~ 940 MeV for p and n;
- ~1115 MeV for the Λ;
- ~1190 MeV for the Σ's;
- ~1320 MeV for the Ξ's;

(difference of < few MeV in the isospin multiplet, due to e-m interactions.)



The baryons: the decuplet $J^P = {}^3/_2{}^+$

The decuplet is rather simple (*but there is a spin/statistics problem, see later*). The spins are aligned (⇑ ⇑ ⇑), to produce an overall $J={}^3/_2$.

The masses, at percent level, are :

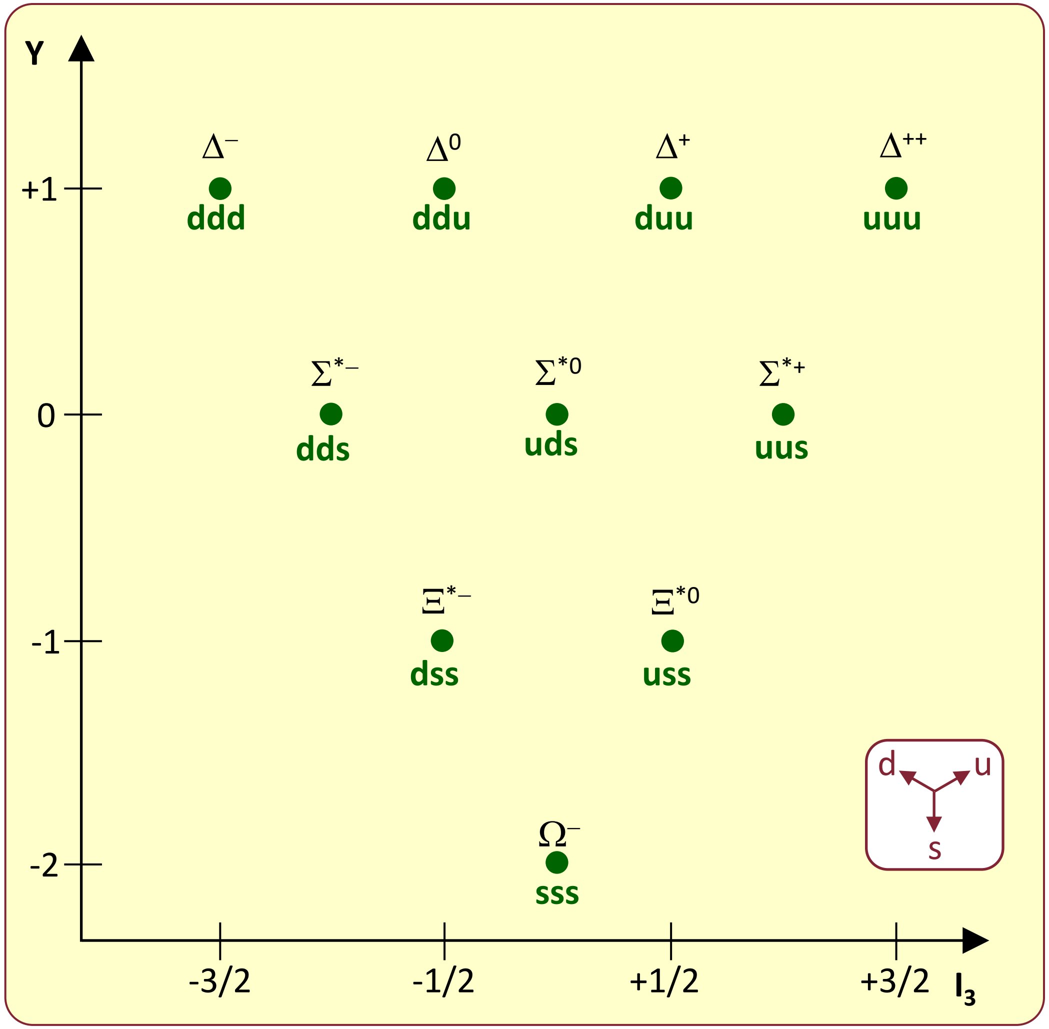
~ 1230 MeV for the Δ's;

~ 1385 MeV for the Σ^*'s,

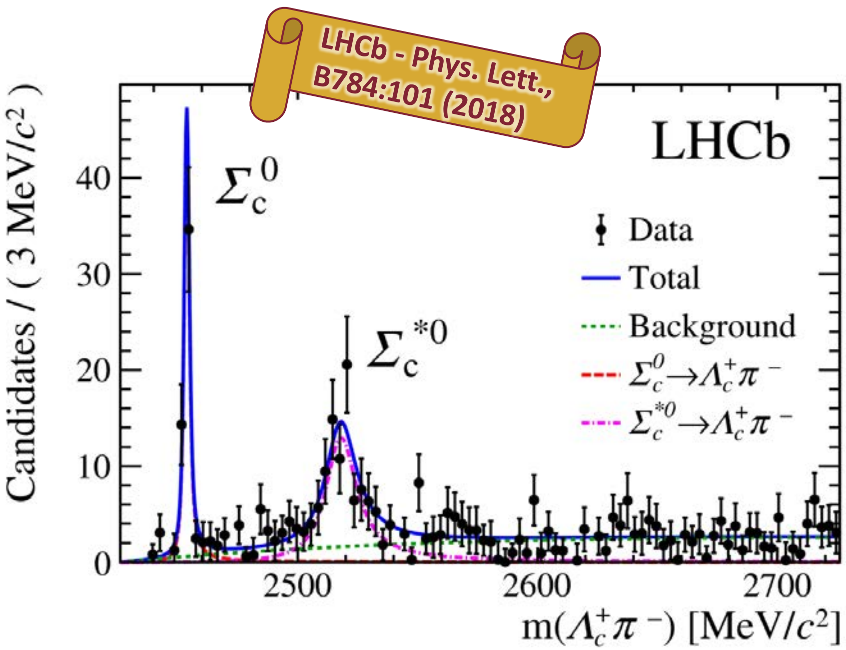
~ 1530 MeV for the Ξ^*'s

~ 1670 MeV for the Ω^-.

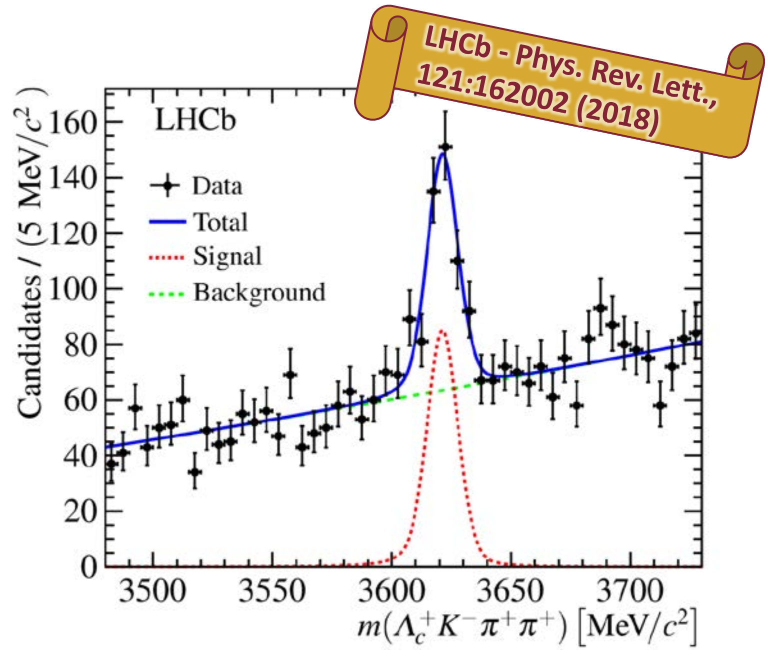
Notice that the mass split among multiplets is very similar, ~150 MeV (important for the Ω^- discovery, lot of speculations, no real explanation).



The baryons: example



A - $\boxed{\Sigma_c^0, \Sigma_c^{0*} \to \Lambda_c^+ \pi^-}$



B - $\boxed{\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+}$

Recently, the LHCb Collaboration at LHC has realized a nice search for baryons made with heavy quarks.

[these two examples should stay in § 3, because they contain the c quark]

quark content:

Ξ_{cc}^{++} : ucc;
$\Sigma_c^0, \Sigma_c^{0*}$: ddc;
Λ_c^+ : udc;
K^- : $\bar{u}$s;
$\pi^\pm$: u$\bar{d}$, $\bar{u}$d.

Check conservation of:
✓✓ a. baryon n.
✓✓ b. charge
✓✗ c. charm
–✗ d. strangeness
Write the Feynman diagrams of the decays
→ after § 6

SU(3)

- For the SU(2) symmetry, the generators are the Pauli matrices. The third one is associated to the conserved quantum number I_3.
- For SU(3), the Gell-Mann matrices T_j (j=1-8) are defined (next page).
- The two diagonal ones are associated to the operators of the third component of isospin (T_3) and hypercharge (T_8).
- The eigenvectors $|u\rangle$ $|d\rangle$ $|s\rangle$ are associated with the quarks (u, d, s).

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \psi_1^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \psi_1^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix};$$

$$\sigma_2 = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \quad \psi_2^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}; \quad \psi_2^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix};$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \psi_3^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \psi_3^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix};$$

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = -i\sigma_1\sigma_2\sigma_3 = I;$$

$$\left[\sigma_i, \sigma_j\right] = 2i\sum_k \varepsilon_{ijk}\sigma_{k.}$$

Pauli matrices and eigenvectors

in the following, some of the properties of SU(3) in group theory: no rigorous math, only results useful for our discussions.

Demonstrations (some trivial) are in [IE], [BJ 10] or [YK1 G]. A discussion of the group theory, applied to elementary particle physics, in [IE, app. C].

And we have separate – optional – courses.

SU(3) : Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \lambda_2 = i \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad \lambda_5 = i \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

$$\lambda_7 = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}; \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Gell-Mann matrices λ_i

$$T_i = \frac{1}{2}\lambda_i$$

$$\sum_{j=1}^{8} \lambda_j^2 = \frac{16}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ diagonal.}$$

$$U = 1 + \frac{i}{2} \sum_{j=1}^{8} \varepsilon_j \lambda_j \text{ unitary matrix, } \det U = 1.$$

SU(3) : eigenvectors

Definition of I_3, Y, quark eigenvectors and related relations :

$$\hat{T}_3 = \frac{1}{2}\lambda_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad Y = \frac{1}{\sqrt{3}}\lambda_8 = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix};$$

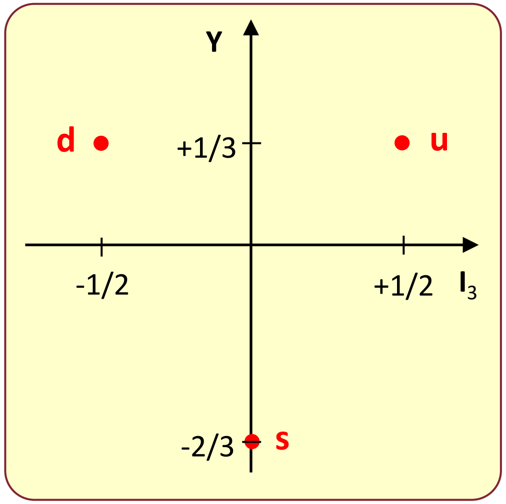
$$|u\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \qquad |d\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \qquad |s\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};$$

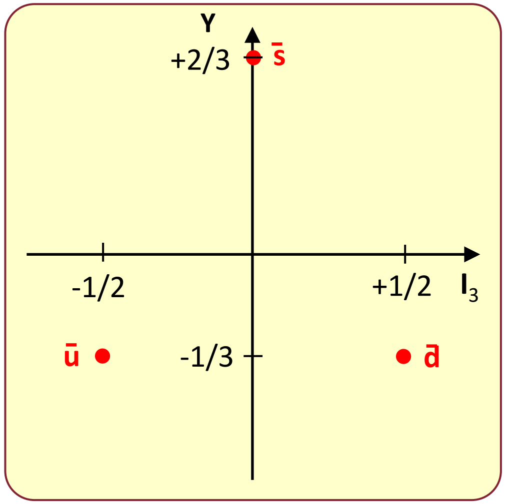
$$\hat{T}_3|u\rangle = +\frac{1}{2}|u\rangle; \qquad \hat{T}_3|d\rangle = -\frac{1}{2}|d\rangle; \qquad \hat{T}_3|s\rangle = 0;$$

$$\hat{Y}|u\rangle = +\frac{1}{3}|u\rangle; \qquad \hat{Y}|d\rangle = +\frac{1}{3}|d\rangle; \qquad \hat{Y}|s\rangle = -\frac{2}{3}|s\rangle;$$

$$\hat{T}_3|\bar{u}\rangle = -\frac{1}{2}|\bar{u}\rangle; \qquad \hat{T}_3|\bar{d}\rangle = +\frac{1}{2}|\bar{d}\rangle; \qquad \hat{T}_3|\bar{s}\rangle = 0;$$

$$\hat{Y}|\bar{u}\rangle = -\frac{1}{3}|\bar{u}\rangle; \qquad \hat{Y}|\bar{d}\rangle = -\frac{1}{3}|\bar{d}\rangle; \qquad \hat{Y}|\bar{s}\rangle = +\frac{2}{3}|\bar{s}\rangle.$$





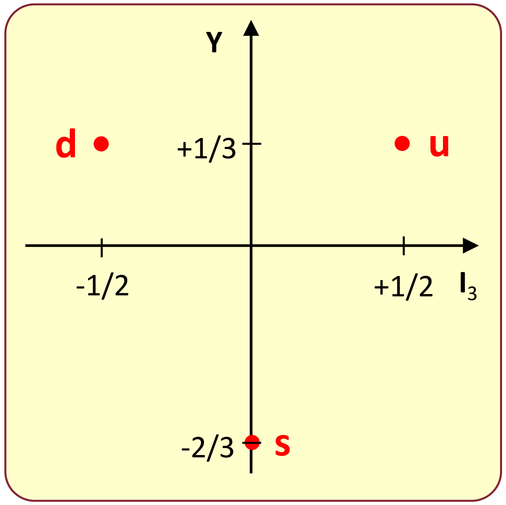
SU(3) : operators

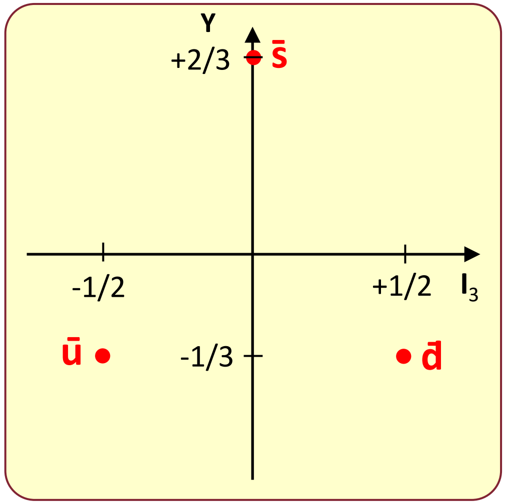
The ladder operators $T_\pm$, $U_\pm$, $V_\pm$:

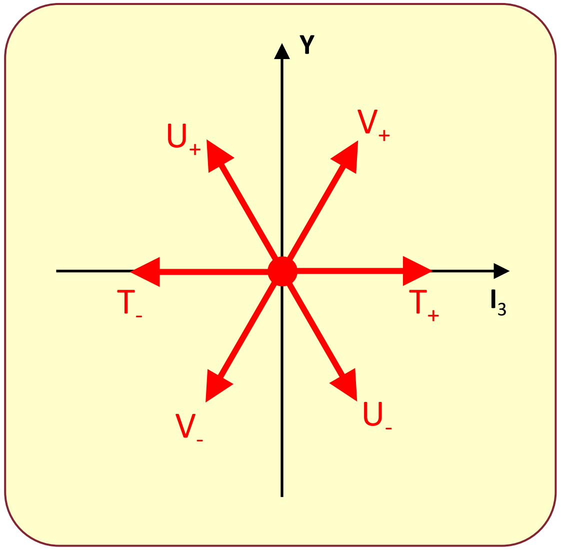
As an example, take V_+ :

$$\boxed{T_\pm = T_1 \pm iT_2; \quad U_\pm = T_6 \pm iT_7; \quad V_\pm = T_4 \pm iT_5;}$$

$$V_+ = T_4 + iT_5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \left\{ \begin{array}{r} V_+|\bar{u}\rangle = -|\bar{s}\rangle; \\ V_+|\bar{d}\rangle = 0; \\ V_+|\bar{s}\rangle = 0; \end{array} \right.$$





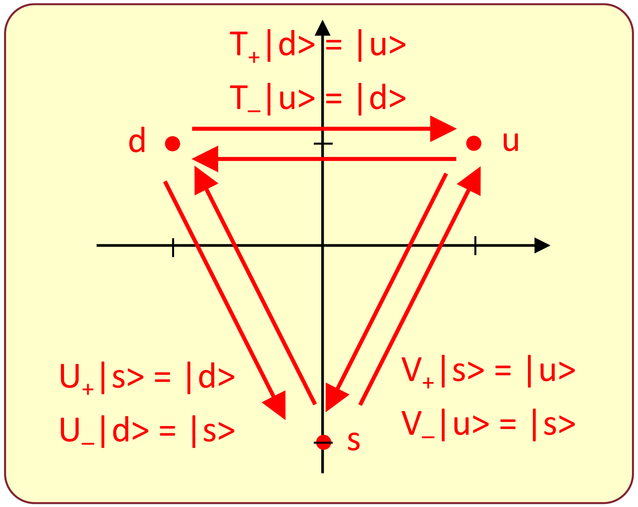


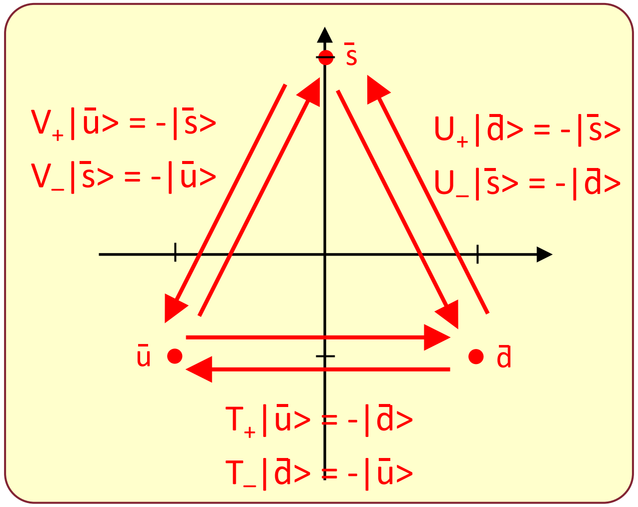
$$V_+|u\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0;$$

$$V_+|d\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0;$$

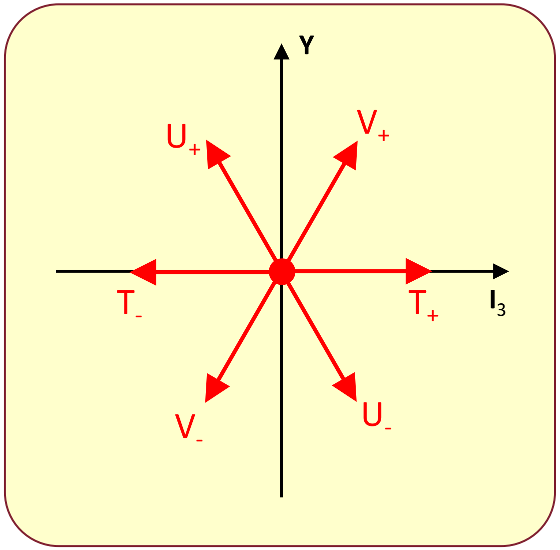
$$V_+|s\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = |u\rangle.$$

SU(3) : ladder operators





The ladder operators $T_{\pm}$, $U_{\pm}$, $V_{\pm}$.



§ QCD

Color : a new quantum number

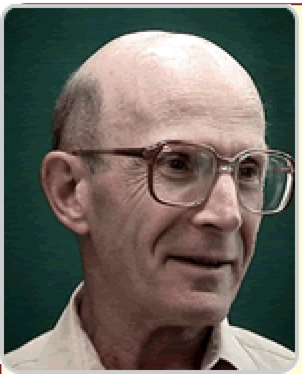
Consider the Δ^{++} resonance:

- $J^P=3/2^+$ (measured);

$\rightarrow$ quark/spin content [*no choice*]:
$|\Delta^{++}\rangle = |\, u\Uparrow u\Uparrow u\Uparrow \rangle$

- wave function :

$\psi(\Delta^{++}) = \psi_{space} \times \psi_{flavor} \times \psi_{spin}$ NO !!!



Oskar W. Greenberg

Moo-Young Han (한무영)

Yoichiro Nambu (南部 陽一郎, *Nambu Yōichirō*)

Why "NO" ?
Consider the symmetry of $\psi(\Delta^{++})$:

- it is lightest uuu state $\rightarrow$ L = 0

$\rightarrow$ ψ_{space} symmetric;

$\rightarrow$ ψ_{flavor} and ψ_{spin} symmetric;

$\rightarrow$ $\psi(\Delta^{++})$ = sym.$\times$sym.$\times$sym. = sym.

... but the Δ^{++} is a fermion ...NO

Anomaly : the Δ^{++} is a spin $^3/_2$ fermion and its function MUST be antisymmetric for the exchange of two quarks (Pauli principle). However, this function is the product of three symmetric functions, and therefore is symmetric $\rightarrow$ **???**.

The solution was suggested in 1964 by Greenberg, later also by Han and Nambu. They introduced a **new quantum number** for strongly interacting particles, composed by quarks : the **COLOR**.

Color : why's and how's

The idea [*see §6, the following is quite naïve*] :

1. quarks exist in three colors (say Red, Green and Blue, like the TV screen(*);
2. they sum like in a TV-screen : e.g. when RGB are all present, the screen is white;
3. the "anticolor" is such that, color + anticolor gives white (e.g. $\bar{R}$ = G + B);
4. anti-quarks bring ANTI-colors (see previous point);
5. Mesons and Baryons, which are made of quarks, are white and have no color: they are a "**color singlet**".

Therefore, we have to include the color in the complete wave function; e.g. for Δ^{++} :

$$\psi(\Delta^{++}) = \psi_{space} \times \psi_{flavor} \times \psi_{spin} \times \psi_{color}$$

$$\psi_{color} = (1/\sqrt{6})\,(u_r^1 u_g^2 u_b^3 + u_g^1 u_b^2 u_r^3 + u_b^1 u_r^2 u_g^3 - u_g^1 u_r^2 u_b^3 - u_r^1 u_b^2 u_g^3 - u_b^1 u_g^2 u_r^3)$$

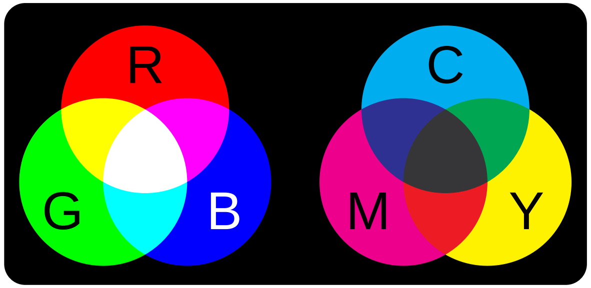
(where u_r, u_g, u_b are the color functions for u quarks of red, green, blue type).

Then ψ_{color} is antisymmetric for the exchange of two quarks and so is the global wave function.

The introduction of the color has many other experimental evidences and theoretical implications, *which we will discuss in the following*.

____

(*) *however, these colors are in no way similar to the real colors; therefore the names "red-green-blue" are totally irrelevant.*



Summary: Symmetries and Multiplets

for a complete discussion, [BJ 10].

1. Since the strong interactions conserve isotopic spin ("$\mathbb{I}$"), hadrons gather in I-multiplets. Within each multiplet, the states are identified by the value of I_3.
2. If no effect breaks the symmetry, the members of each multiplet would be mass-degenerate. The electromagnetic interactions, which do not respect the I-symmetry, split the mass degeneration (at few %) in I-multiplets.
3. Since the strong interactions conserve I, $\mathbb{I}$-operators must commute with the strong interactions Hamiltonian ("$\mathbb{H}_s$") and with all the operators which in turn commute with $\mathbb{H}_s$.
4. Among these operators, consider the angular momentum $\mathbb{J}$ and the parity $\mathbb{P}$. As a result, all the members of an isospin multiplet must have the same spin and the same parity.
5. $\mathbb{H}_s$ is also invariant with respect to unitary representations of SU(2). The quantum numbers which identify the components of the multiplets are as many as the number of generators, which can be diagonalized simultaneously, because are mutually commuting. This number is the *rank* of the Group. In the case of SU(2) the rank is 1 and the operator is $\mathbb{I}_3$.
6. Since $[\mathbb{I}_j , \mathbb{I}_k] = i\varepsilon_{jkm}\mathbb{I}_m$, each of the generators commutes with $\mathbb{I}^2$:

   $\mathbb{I}^2 = \mathbb{I}_1^2 + \mathbb{I}_2^2 + \mathbb{I}_3^2$.

   Therefore $\mathbb{I}^2$, obviously hermitian, can be diagonalized at the same time as $\mathbb{I}_3$.

(continue …)

Summary: Symmetries and Multiplets

7. The eigenvalues of $\mathbb{I}$ and $\mathbb{I}_3$, can "tag" the eigenvectors and the particles.
8. This fact gives the possibility to regroup the states into multiplets with a given value of *I*.
9. We can generalize this mechanism from the isospin case to any operator : if we can prove that $\mathbb{H}$ is invariant for a given kind of transformations, then:
   a. look for an appropriate symmetry group;
   b. identify its irreducible representations and derive the possible multiplets,
   c. verify that they describe physical states which actually exist.
10. This approach suggested the idea that Baryons and Mesons are grouped in two octets, composed of multiplets of isotopic spin.
11. In reality, since the differences in mass between the members of the same multiplet are ~20%, the symmetry is "broken" (i.e. approximated).
12. Since the octets are characterized by two quantum numbers (I_3 and Y), the symmetry group has rank = 2, i.e. two of the generators commute between them.
13. We are interested in the "irreducible representations" of the group, such that we get any member of a multiplet from everyone else, using the transformations.

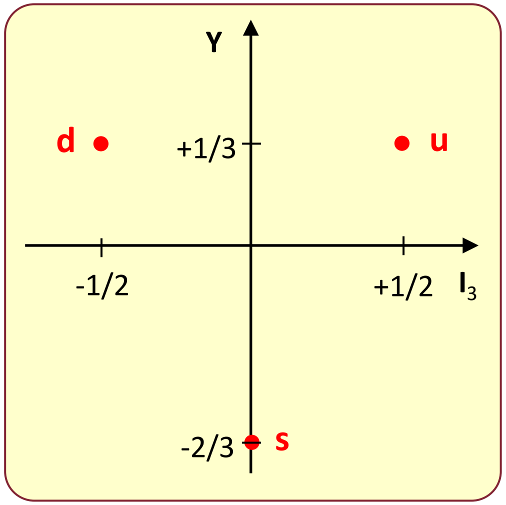
(… continue …)

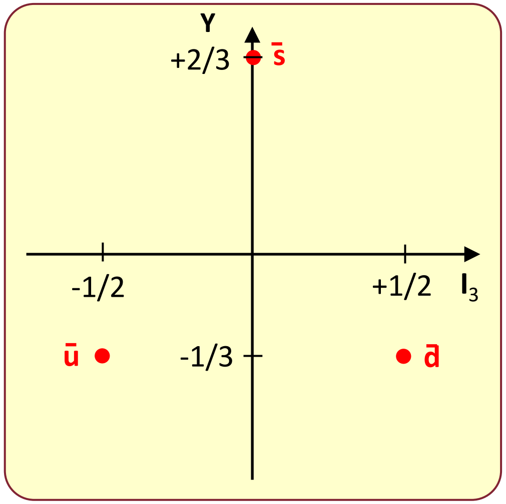
Summary: Symmetries and Multiplets

14. The non-trivial representation (non-trivial = other than the Singlet) of lower dimension is called "Fundamental representation".
15. In SU(3) there are eight symmetry generators. Two of them are diagonal and associated to I_3 and Y.
16. The fundamental representations are triplets ($\rightarrow$ quarks), from which higher multiplets ($\rightarrow$ hadrons) are derived :

    mesons: $3 \otimes \bar{3} \quad = 1 \oplus 8$;

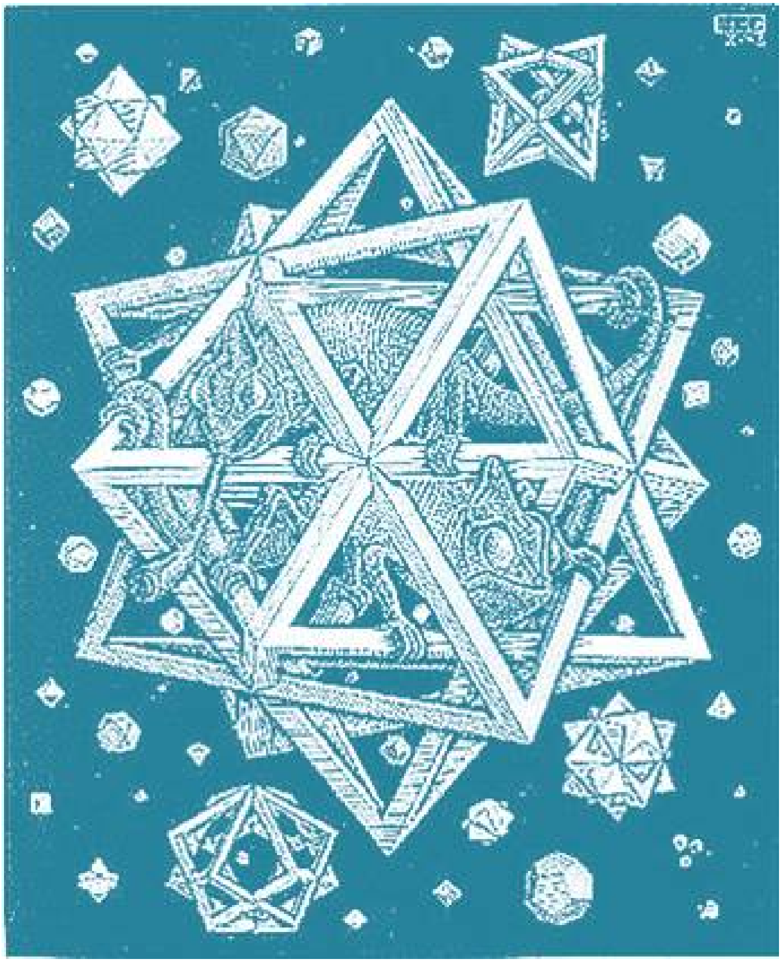
    baryons: $3 \otimes 3 \otimes 3 \quad = 1 \oplus 8 \oplus 8 \oplus 10$.
17. This purely mathematical scheme has two relevant applications:
    a. "flavour SU(3)", $SU(3)_F$ with Y_F and I_{3F} for the quarks uds – this symmetry is approximate (i.e. "broken");
    b. "color SU(3)", $SU(3)_C$ with Y_C and I_{3C} for the colors rgb; this symmetry is exact.





References

1. e.g. [BJ, 8];
2. large overlap with [FNSN1, 7]
3. isospin and SU(3) : [IE, 2];
4. group theory : [IE, app C];
5. color + eightfold way : [IE, 7-8]
6. G.Salmè – appunti.



End of chapter 1

Particle Physics - Chapter 2
Hadron structure

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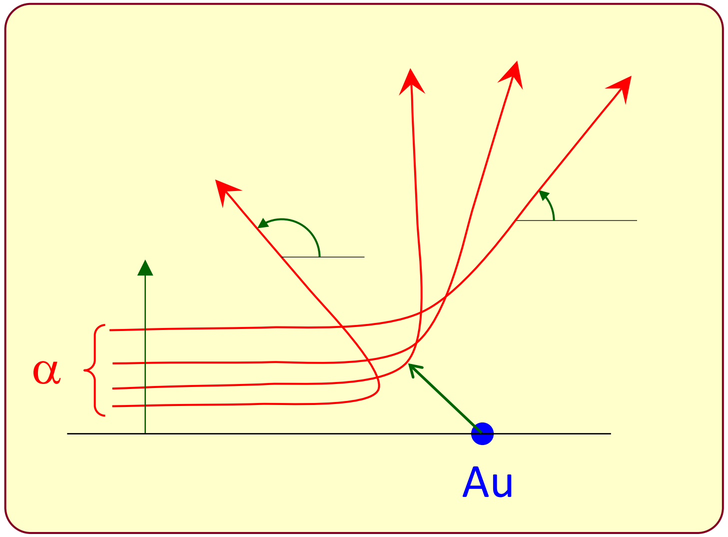
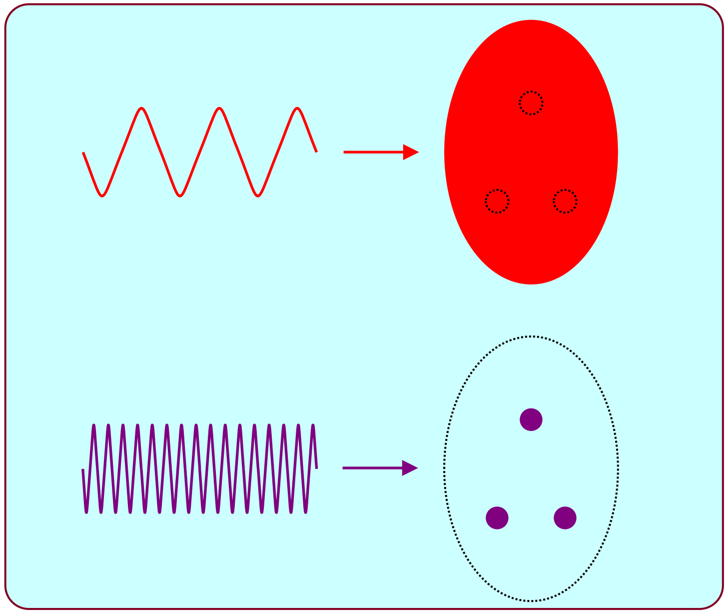
AA 18-19

last mod.
12-Mar-19

2 – Hadron structure





brief historical summary

"Hegel remarks somewhere that all great, world-historical facts and personages occur (…) twice. He has forgotten to add: the first time as tragedy, the second as farce." [Karl Marx, The 18th Brumaire of Louis Bonaparte]

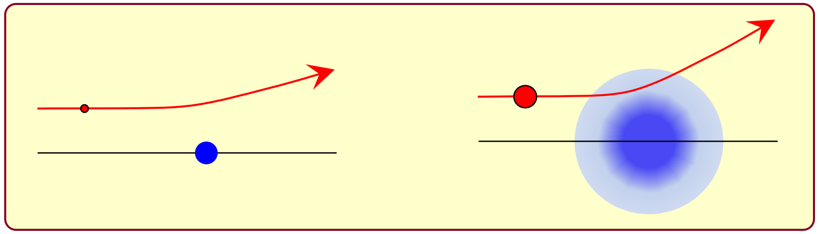
Despite this famous sentence, in this chapter a story is told, neither tragic nor farcical, which happened at least three times in the 20th century: *in a scattering experiment, a projectile probes the deep structure of the target; the scale of the observation depends on the energy of the probe:*

- 1911 (Rutherford) α particles $\rightarrow$ gold (nucleus) [$\rightarrow$ *FNSN1*];
- 1950-60 (Hofstadter) e^{-} $\rightarrow$ H/D/He (nuclear structure);
- 1965-80 (SLAC/CERN) e/ν $\rightarrow$ hadronic matter (quarks/partons)

The deep meaning of the mechanism resides in Quantum Mechanics, which relates the space scale of a phenomenon with the (transverse) momentum of the scattered particles.

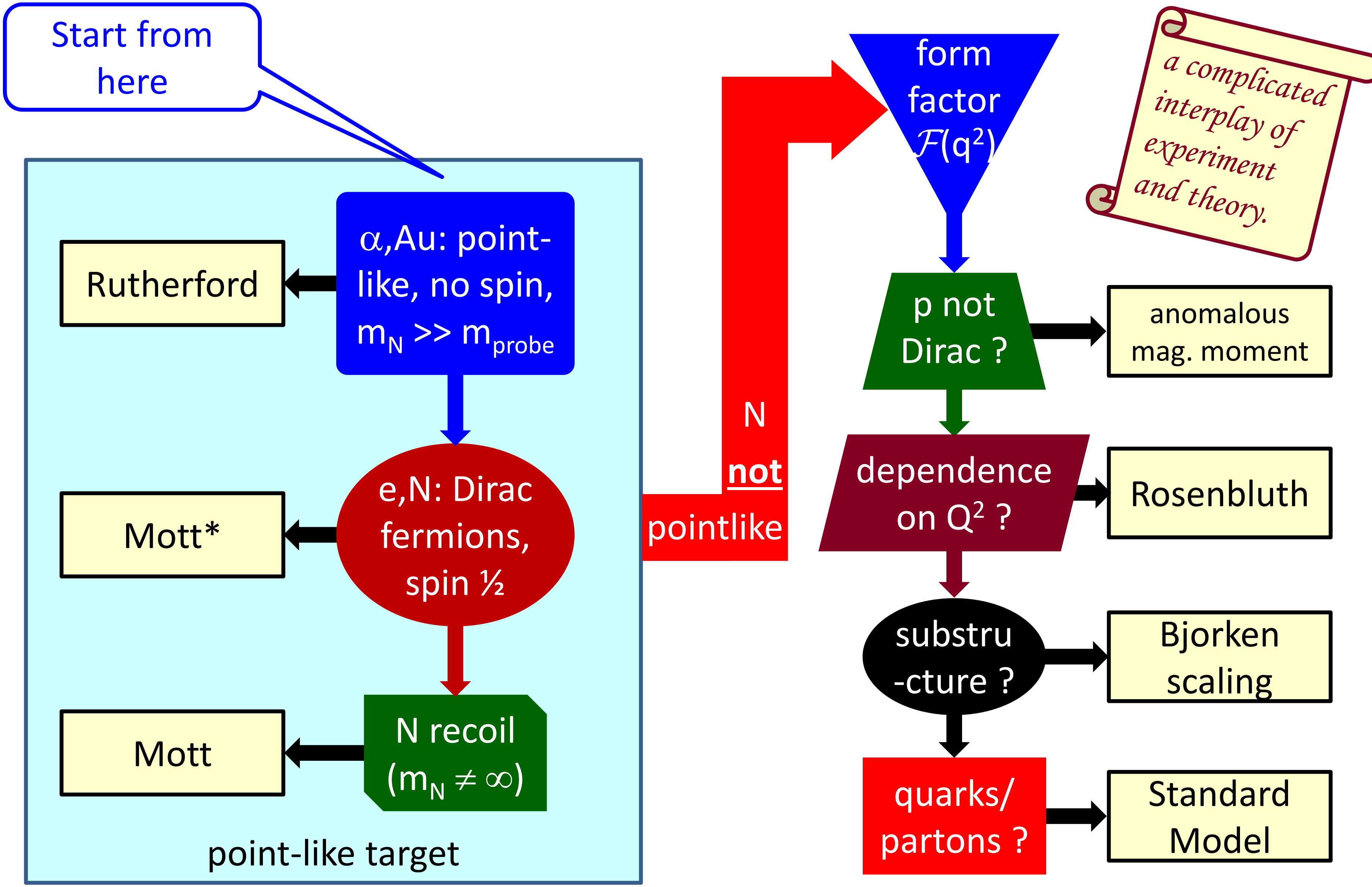
The role of technology is also important: the observation is possible because of powerful accelerators and detectors.

We will follow the history and therefore will study phenomena of ever smaller size [*look the contents page*].



- 20xy (maybe <u>you</u>) new substructure emerging ???

the treasure map for scattering



the scattering experiment

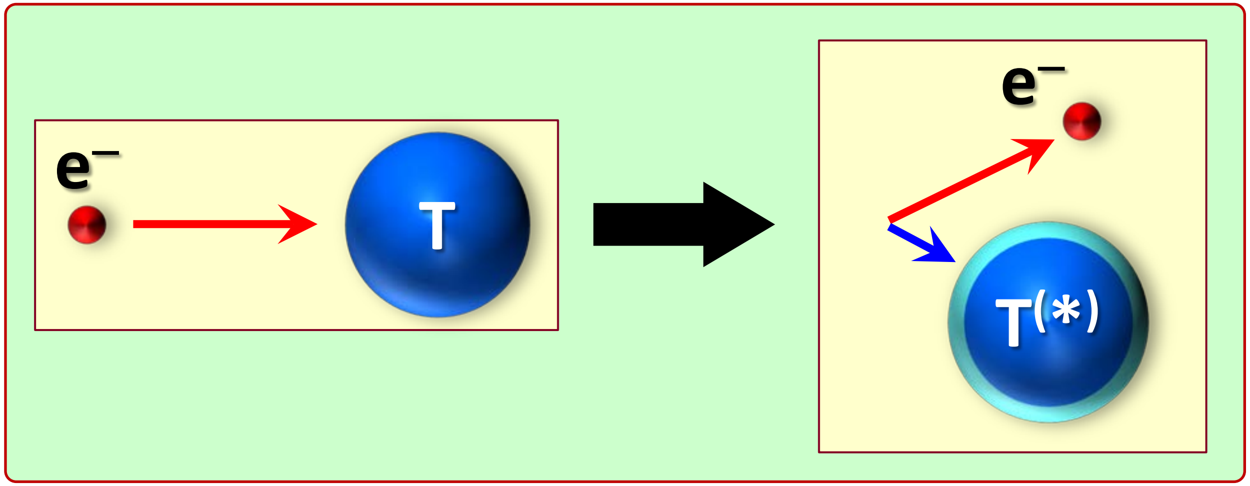
Q : is the target a <u>pointlike simple object</u> ? if not, how to probe its shape ?

A : *(à la Rutherford, but (a) he used α particles, (b) he did NOT see the nucleus size)*

- take a <u>probe</u>: e.g. an electron (e^-),
- study the <u>scattering e^-T</u>, [T=Nucl-eus/on]
- measure the <u>cross section</u> $\sigma(e^-T)$,
- … and the <u>angular distribution</u> of the e^-;
- … and detect the <u>excited states</u> or the final state hadronic system ("<u>inelastic interactions</u>").

<u>Path:</u>

1. study the <u>kinematics</u> (*);
2. compute $\sigma(e^-T)$ for pointlike nuclei in <u>classical electrodynamics</u> (Rutherford formula);
3. ditto in <u>QM</u> for spin ½ electrons and pointlike nuclei (Mott formula);
4. detect **<u>deviations</u>** from these models → derive informations on nuclear structure;
5. **new theory → smaller distance (i.e. higher Q^2) → deviations → newer theory → … → … → <u>(possibly ad infinitum)</u>**



*(*) We call "<u>kinematics</u>" the equations which follow from space / angular momentum conservation and mass. The game is to study the "**<u>dynamics</u>**" after imposing the "kinematical" constraints.*

Fermi gas model

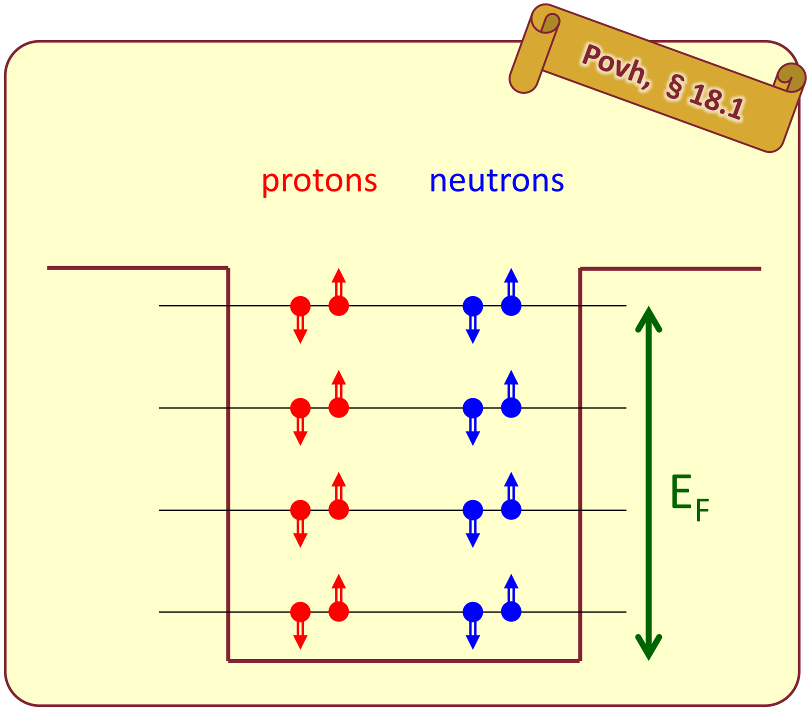
- Nuclei are bound states of protons (p) and neutrons (n).
- A simple model: the Fermi gas:

- p, n identical, but charge :
  - o little spheres r = r_0, mass = m;
  - o spin ½ fermions, pure Dirac-like;
  - o bound inside the nucleus, otherwise free to move;
- define:
  - o $n_{neutr.}$(= N), $n_{prot.}$ (= Z), A = N + Z,
  - o p_{Fermi} (= p_F), E_{Fermi} (= E_F);
  - $\rightarrow V_{Nucl}$ [$\propto$ A] = $4\pi r_0^3 A/3$;
- no e.m. interactions, only nuclear $\rightarrow$ N = Z = A/2, $p_F^p = p_F^n$, $E_F^p = E_F^n$ [better approx (not here):different interactions $\rightarrow p_F^p \neq p_F^n$];
- uncertainty principle $\rightarrow$ each p/n fills $V_{phase\ space} = [2\pi\hbar]^3$.

Therefore:

- well-shaped potential (⊔), identical for p/n, i.e. only interactions p↔p n↔n;
- Fermi statistics → two p/n per energy level (spin ⇑⇓);

[…*next page*…]



Fermi gas model: results

From those approximations, an elementary computation :

$$n^{n,\Uparrow} = n^{n,\Downarrow} = n^{p,\Uparrow} = n^{p,\Downarrow} = \frac{N}{2} = \frac{Z}{2} = \frac{A}{4} =$$

$$= \frac{\left[V_{space} V_{mom}\right]_{TOT}}{\left[V_{space} V_{mom}\right]_{each\ part}} = \frac{\frac{4}{3}\pi r_0^3 A \times \frac{4}{3}\pi p_F^3}{\left[2\pi\hbar\right]^3} =$$

$$= \frac{2 A r_0^3 p_F^3}{9\pi\hbar^3};$$

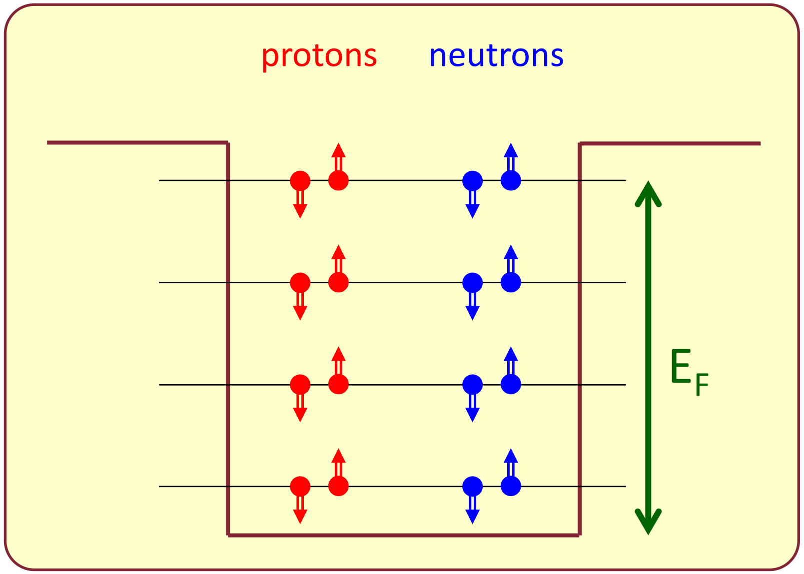
$$N = Z = \frac{\cancel{A}}{2} = \frac{4 \cancel{A} r_0^3 p_F^3}{9\pi\hbar^3}; \qquad p_F = \frac{\hbar}{r_0}\sqrt[3]{9\pi/8};$$

$$r_0 \approx 1.2 \text{ fm} \rightarrow \begin{cases} p_F \approx 250 \text{ MeV}; \\ E_F^{kin} = p_F^2/2m \approx 33 \text{ MeV}. \end{cases}$$

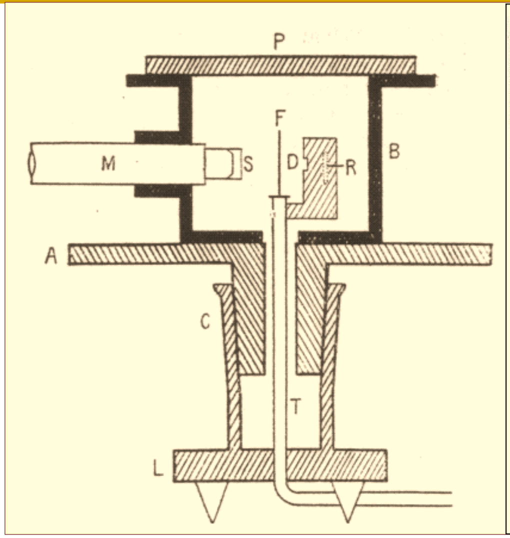
(*) fit from form factors (see later)

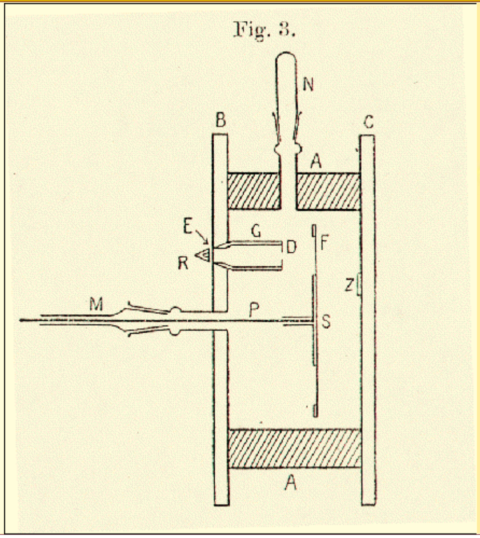
Conclusions :

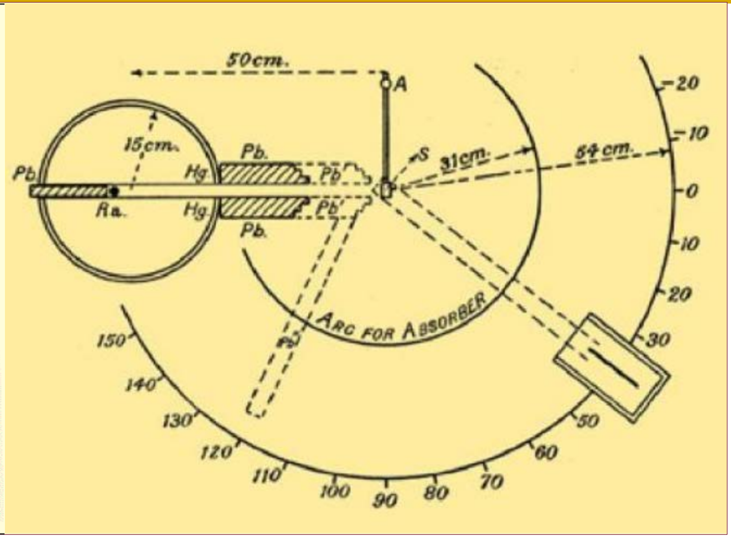
- $V_{space} \approx {}^4/_3 \pi r_0{}^3 A \rightarrow r_{nucl.} \propto A^{1/3}$;
- p_F, E_F not dependent on A (!!!);
- large p_F, small kin. energy;
- when p/n hit by probe ($e^{\pm}/\nu$), if E_{probe} >> 30 MeV $\rightarrow$ ignore Fermi motion.
- [*more elaborated model, e.g. add e.m. and spin interactions, etc. – see literature*]



Rutherford scattering







The birth of nuclear physics
(Manchester, 1908-13):

$\alpha(Z_\alpha=2, A_\alpha=4) \rightarrow Au(Z_{Au}=79, A_{Au}=197)$

- actually performed by H.Geiger and E.Marsden [*E.M. was 20 y.o. !*];
- alternative model by J.J.Thompson, with a diffused mass/charge ("soft matter");
- the first "fixed target" scattering experiment.

- *already discussed in FNSN1 (pag 25);*
- *do NOT repeat the math, simply recall the results;*
- *discussion of the physics;*
- *preparation for further steps.*

Lord Ernest Rutherford

modern simulation (look):
https://phet.colorado.edu/en/

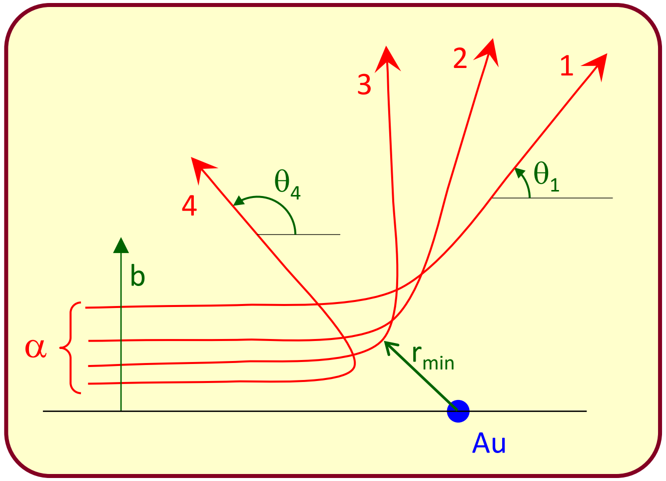
Rutherford scattering: in a nutshell

[*an incredible mix of genius, skill and luck*]

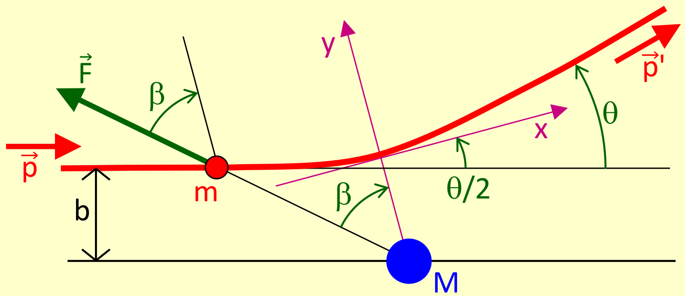
- α-particles (i.e. ionized He) $\rightarrow$ Au foil;
- $E_\alpha^{kin} \approx$ few MeV;
- sometimes, the α was scattered by $\theta > 90°$; *VERY* rare in reality, but impossible if matter were soft and homogeneous;
- only explanation: "matter" actually concentrated in small heavy bodies ("nuclei");

$\rightarrow$ the "matter" is essentially empty;

- how model the scattering ? Rutherford tried with a two-body scattering with Coulomb (electrostatic) force;
- **success !!!** [*within their limited observation capabilities*]
- a key point: the nucleus is small enough, that the α "sees" always its full charge;
- [*remember the Gauss' theorem: if impact parameter $b > r_{Nucleus}$, only see an effective point-like charge*]
- but the matter is neutral ! yes, but the electrons are so light, that they cannot stop/deflect the α ($m_e/m_\alpha \approx 1/8{,}000$).



Rutherford scattering: the math



α (m, z) → nucleus (M, Z):

- $\vec{v}_{\alpha,init} = \vec{v}$, $\vec{v}_{\alpha,final} = \vec{v}'$, $\vec{v}_{nucleus} = 0$;
- $\vec{p} = m\vec{v}$, $\vec{p}' = m\vec{v}'$, m << M;
- Coulomb force only ($\vec{F}$);
- v << c → non-relativistic;
- elastic → $|\vec{p}'| = |\vec{p}|$;
- conserve E, ang. mom $\vec{L}$;
- $\Delta p_x = 0$ because of symmetry, only Δp_y matters;
- integral over β, the angle wrt $\hat{y}$;
- if attractive force (e.g. +–), M → the other focus of the hyperbola.

FSNN1, 25

$$\Delta p = |\vec{p}' - \vec{p}| = 2p\sin(\theta/2);$$

$$|\vec{L}| = pb = |\vec{r} \times m\vec{v}| = |\vec{r} \times m(\frac{dr}{dt}\hat{r} + r\frac{d\beta}{dt}\hat{\beta})| = mr^2\frac{d\beta}{dt};$$

$$\Delta p_y = 2p\sin(\theta/2) = \int_{-\infty}^{+\infty} dt F_y = \int_{-\infty}^{+\infty} dt \frac{zZe^2}{4\pi\varepsilon_0}\frac{\cos\beta}{r(t)^2} =$$

$$= \int_{-(\pi-\theta)/2}^{(\pi-\theta)/2} \frac{zZe^2}{4\pi\varepsilon_0}\frac{\cos\beta}{\cancel{r}^2}\frac{m\cancel{r}^2}{pb} d\beta = \frac{zZe^2}{2\pi\varepsilon_0}\frac{m}{pb}\cos(\theta/2);$$

$$\tan(\theta/2) = \frac{zZe^2}{4\pi\varepsilon_0}\frac{m}{p^2 b} \rightarrow db = -\frac{zZe^2}{4\pi\varepsilon_0}\frac{m}{p^2}\frac{d\theta}{2\sin^2(\theta/2)}.$$

$$d\sigma = 2\pi b\, db = 2\pi\left(\frac{zZe^2 m}{4\pi\varepsilon_0 p^2}\right)^2 \frac{d\theta}{2\tan(\theta/2)\sin^2(\theta/2)};$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2 m}{4\pi\varepsilon_0}\right)^2 \frac{1}{4p^4\sin^4(\theta/2)} = \left(\frac{zZe^2 m}{2\pi\varepsilon_0}\right)^2 \frac{1}{|\vec{p}' - \vec{p}|^4}.$$

$$\boxed{d\Omega = 2\pi\sin\theta d\theta = 4\pi\sin(\theta/2)\cos(\theta/2)d\theta}$$

Rutherford scattering: more math

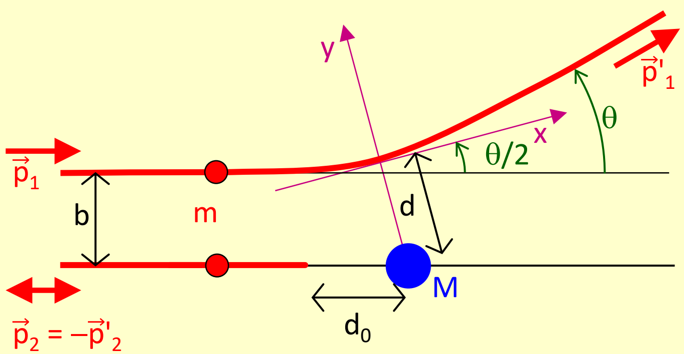
Useful formulas

$$d_0 = r_{min}(b=0) = \frac{zZe^2}{2\pi\varepsilon_0 mv^2};$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{d_0}{2b};$$

$$d = r_{min}(b) = \frac{d_0 + \sqrt{d_0^2 + 4b^2}}{2} = \frac{d_0}{2}\left(1 + \frac{1}{\sin(\theta/2)}\right);$$

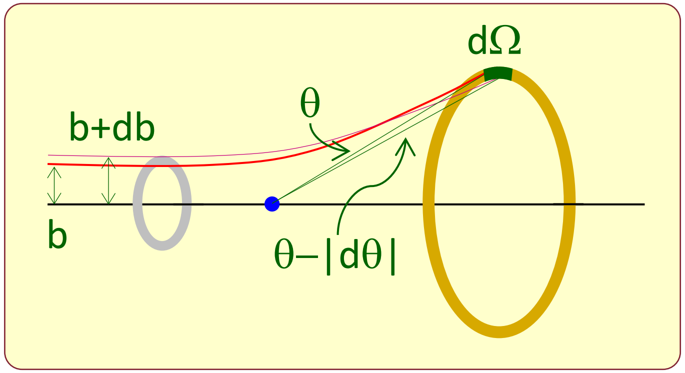
$$\boxed{\frac{d\sigma}{d\Omega} = \frac{d_0^2}{16\sin^4(\theta/2)}} \xrightarrow{\theta\to 0} \frac{d_0^2}{\theta^4}.$$

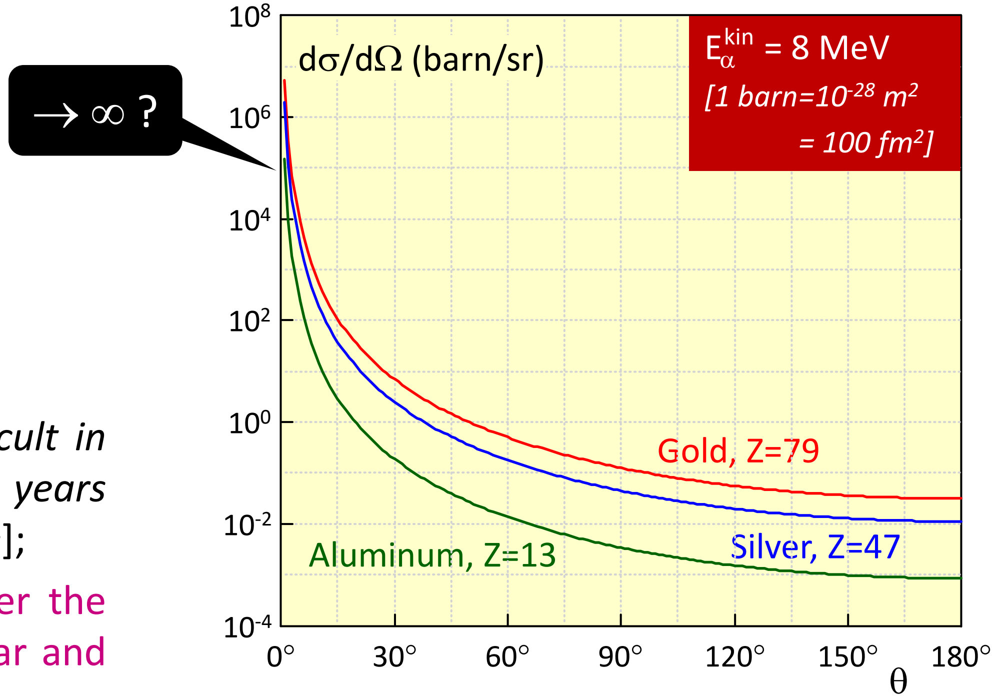


- [if force attractive (e.g. +–), $\vec{F} \to -\vec{F}$, then $\theta \to -\theta$, but everything else equal, e.g. same $d\sigma/d\Omega$;]
- consider a particle $\vec{p}_2$ with b=0 → θ_2 = 180°;
- define d_0 = "distance of closest approach" the r_{min} for it (when $r=d_0$, the particle is at rest);
- d_0 is easily computed from energy conservation;
- define $d_0 = (zZe^2)/(2\pi\varepsilon_0 mv^2)$ also for b≠0;
- write θ and $d\sigma/d\Omega$ as functions of d_0;½
- define d as r_{min}, when b≠0;
- d is computed from E and $\vec{L}$ conservation [*hint in the box, v_0 is the velocity in d*]:

$$\boxed{\begin{aligned}
&\vec{L}\text{ conserv} \to mbv = mdv_0 \to v_0/v = b/d \\
&E\text{ conserv} \to ½mv^2 = ½mv_0^2 + zZe^2/(4\pi\varepsilon_0 d) = \\
&\qquad = ½mv_0^2 + ½mv^2 d_0/d \\
&\to (v_0/v)^2 = (b/d)^2 = 1 - d_0/d \to \\
&\to d^2 - dd_0 - b^2 = 0 \to d = ...
\end{aligned}}$$

Rutherford scattering: dσ/dΩ



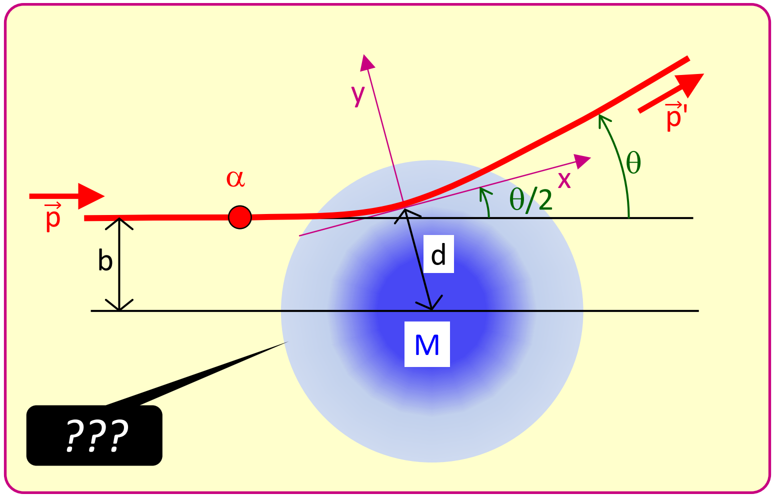


- [*the calculations above are *NOT* difficult in math: Newton could have done all 200 years earlier, had the correct model been made*];
- the real difficulty was to assess whether the matter is soft and continuous or granular and "empty";
- b large → θ small → dσ/dΩ → ∞ [*cutoff provided by other Au nuclei*].

A long and thorough investigation:

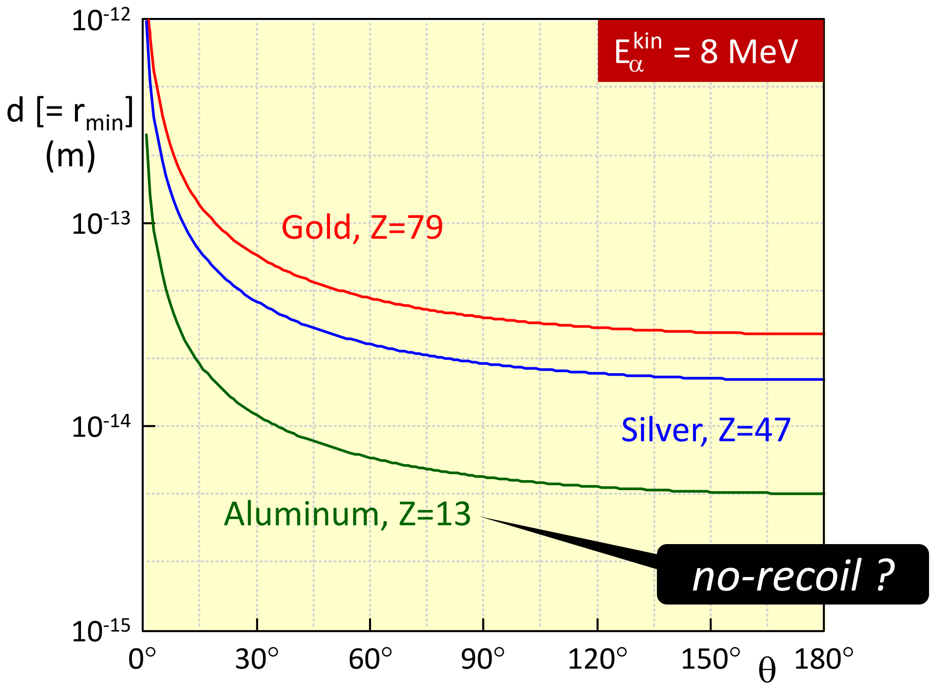
- 1909: found some events θ > 90°: big shock;
- 1911: falsification of the Thomson model, correct assumptions, check of dσ/dΩ in the range 30°–50°;
- 1913: check of dσ/dΩ in the range 5°–150°;
- check that yield ∝ thickness of Au foil;
- other nuclei : check that yield ∝ Z^2 [*roughly*];
- however Rutherford model clearly inconsistent in its "planetary" part: acceleration of charged electrons → radiation → collapse;
- after birth of QM, Rutherford computation redone in Born approx : → same dσ/dΩ [*big luck !*] + no more inconsistency [*next slides*].

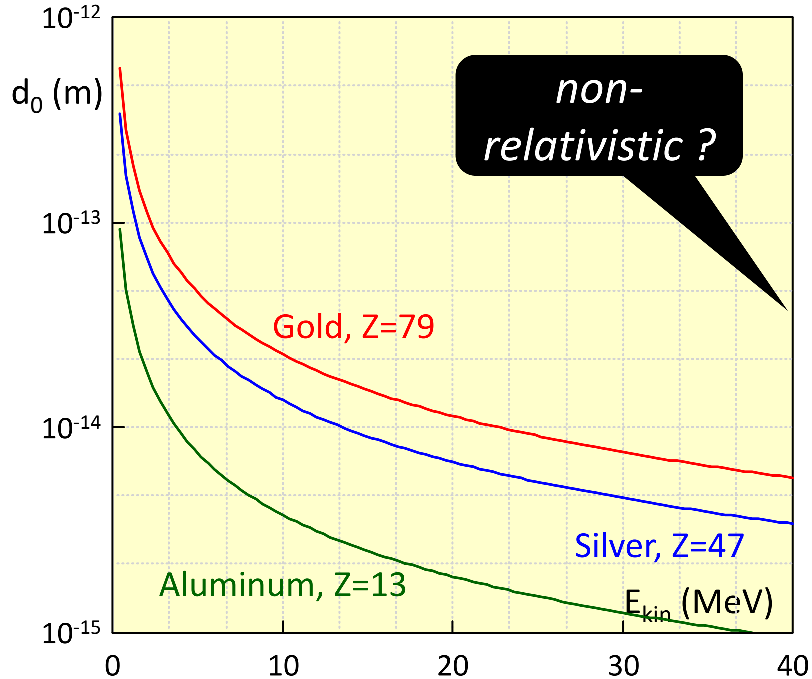
Rutherford scattering: $R_{nucleus}$



How large is the nucleus ?

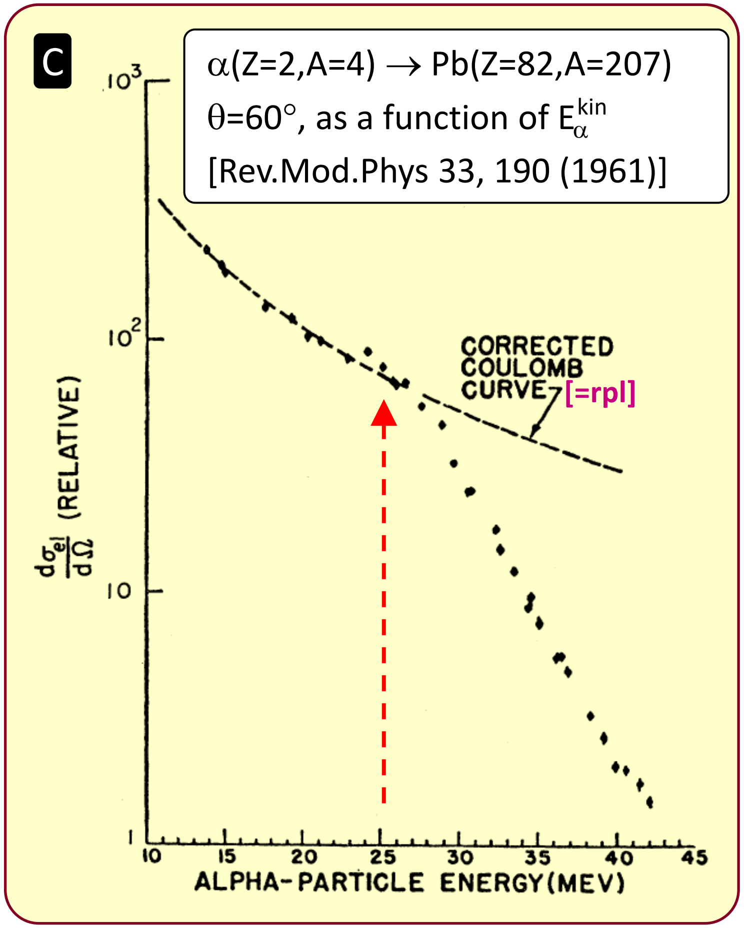
- [*remember the Gauss' theorem*]
- if the α trajectory is completely external to the nucleus, it does *NOT* probe its (*possible*) structure;
- the Rutherford experiment could only limit $R_{nucleus} < 10^{-14}$ m [*still an important result !*];
- to "see" 10^{-15} m → probes with $E_{kin} >$ 20÷30 MeV.

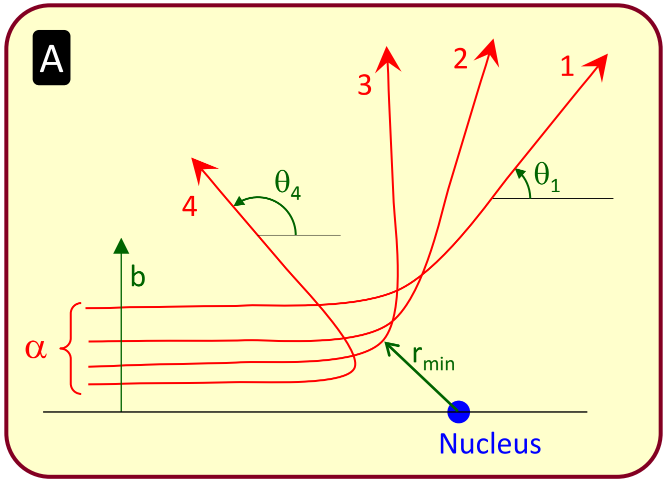


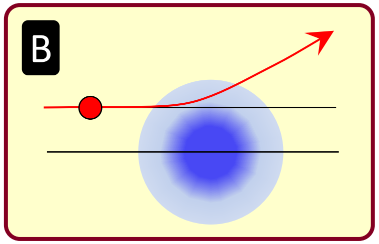


Rutherford scattering: measure $R_{nucleus}$

- plot [A]: b and r_{min} could *NOT* be measured directly for each event, but Rutherford point-like law (*rpl*) relates b $\leftrightarrow \theta$; in fact $b_{small} \leftrightarrow \theta_{large}$;
- plot [B]: the Gauss' theorem predicts a deviation from rpl, when (E_α^{kin} large) $\rightarrow$ ($r_{min} < R_{nucleus}$) $\rightarrow$ shielding $\rightarrow$ "smaller θ";
- plot [C] (**1961** !!!): a "Rutherford-like" scattering α-Pb; at θ=60°, deviation for E_α^{kin} > 25 MeV;
- at high θ, point-like target $\rightarrow$ larger σ, soft target $\rightarrow$ smaller σ (*deviations from rpl related to size of target*) [*please, remember*].







Q. find r_{min} for Pb, θ = 60°, E_α^{kin} = 25 MeV

A. r_{min} = [☞ formula] = 14 fm.

interlude: a funny question

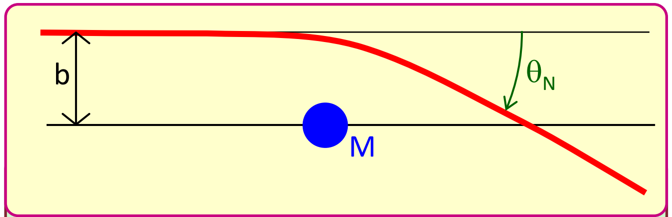
Define a "Newton photon" (γ_N) as a very light classic corpuscle with speed "c", which carries light [*remember Newton theory of light*].

Q : is γ_N deflected in a gravitational field ? how much ?

- [*careful: γ_N is *NOT* a classic e.m. wave, which follows Maxwell equations in vacuum*];
- the answer is "yes" (!!!): in a gravitational field, all bodies are accelerated, independent of their mass (Galileo experiment);
- compute the scattering à la Rutherford, then send $m_\gamma \to 0$ [see box];
- the question is almost meaningless, but the answer is interesting; in general relativity:

  $\theta_{GR} = 4GM/bc^2 = 2\theta_N$ (!!!).

[see Perkins, Particle Astrophysics, pag. 159].



Rutherford scattering :

$$\tan\frac{\theta}{2} = \frac{zZe^2}{4\pi\varepsilon_0}\frac{m}{p^2 b};$$

$$\left[\frac{zZe^2}{4\pi\varepsilon_0}\right]_{\text{Rutherford/Coulomb}} \to [GMm]_{\text{Newton}}$$

$$\tan\frac{\theta_N}{2} = GMm\frac{m}{p^2 b} = \frac{GM}{b}\frac{m^2}{m^2c^2} = \frac{GM}{bc^2} \left| \begin{array}{l}\text{not dependent} \\ \text{on m, if } m \neq 0\end{array}\right. ;$$

$$\theta_N \text{ small} \to \boxed{\theta_N = \frac{2GM}{bc^2}}.$$

kinematics

This is a collection of kinematical computations. It is probably useful to have all in the same place. Notice that here we work in the LAB sys (= N at rest), not in the CM.

This chapter (and many others) deals with scattering. A "probe", usually assumed point-like (e.g. $e^\pm$) hits a hadronic complex system (a nucleus) [*see box*].

In the final state, the probe emerges unchanged, while the nucleus may or may not survive intact:

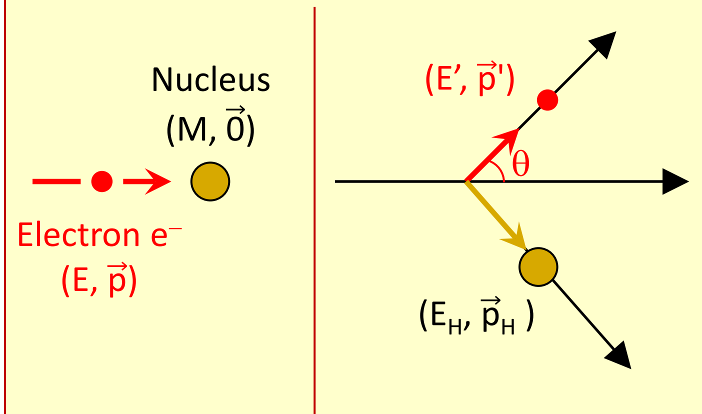
- elastic scattering, when the nucleus is unchanged, i.e. *identical initial and final state particles* (W=M);
- excitation, when the nucleus in the final state is excited, i.e. heavier (W = M* > M);
- a new hadronic system, with n particles (i=1…n):

$$E_H = \sum_{i=1}^{n} E_i; \qquad \vec{p}_H = \sum_{i=1}^{n} \vec{p}_i;$$

$$W = \sqrt{(E_H)^2 - (p_H)^2} = M_{\text{had. sys.}} > M.$$

The underlying idea is to study (*understand ?*) the structure of the hadrons by observing the scattering.

$$e^- N \rightarrow e^- H$$



$$\text{electron } e^\pm : \begin{cases} (E, & \vec{p}\,; & m) & \text{[init.]} \\ (E', & \vec{p}\,'; & m) & \text{[fin.]} \end{cases}$$

$$\text{had. sys.} : \begin{cases} (M, & \vec{0}; & M) & \text{[init.]} \\ (E_H, & \vec{p}_H; & W) & \text{[fin.]} \end{cases}$$

$$\text{4-mom cons.} \rightarrow \begin{cases} \vec{p} + \vec{0} = \vec{p}\,' + \vec{p}_H; \\ E + M = E' + E_H. \end{cases}$$

kinematics: elastic scattering

- To begin with, assume elastic scattering, i.e. "H" = N;
- Define, in the target nucleus ref.sys. :

$$\text{electron } e^{\pm} : \begin{cases} (E, & \vec{p}\,; & m) & [\text{init.}] \\ (E', & \vec{p}\,'; & m) & [\text{fin.}] \end{cases}$$

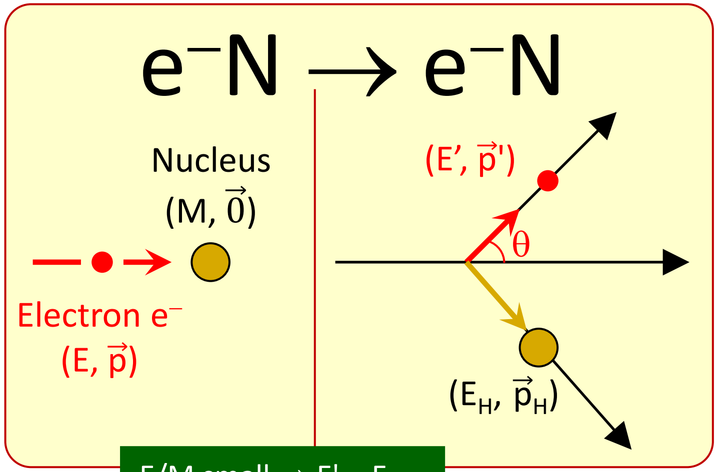
$$\text{nucleus} : \begin{cases} (M, & \vec{0}; & M) & [\text{init.}] \\ (E_H, & \vec{p}_H; & M) & [\text{fin.}] \end{cases}$$

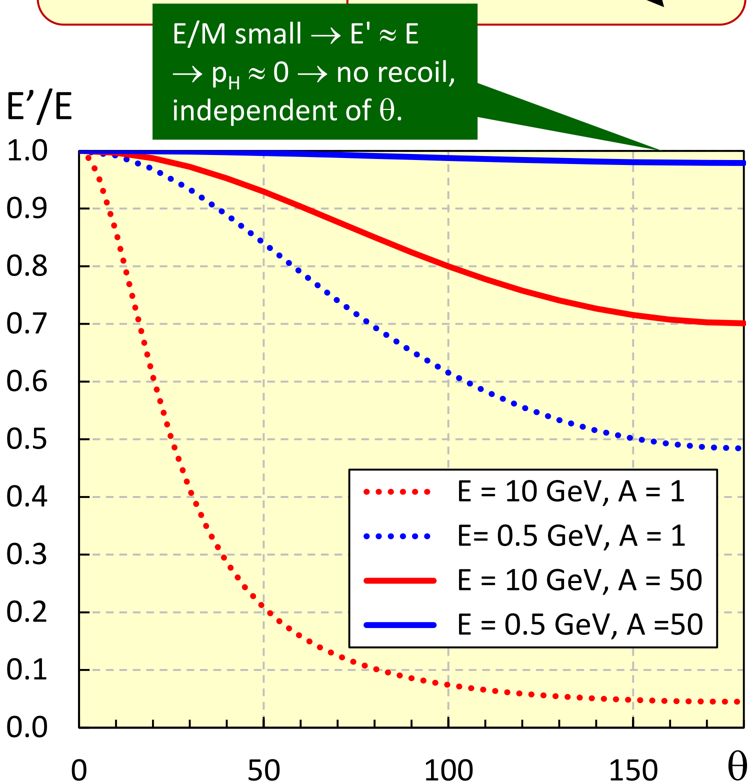
- 4-mom cons. $\rightarrow \begin{cases} \vec{p} + \vec{0} = \vec{p}\,' + \vec{p}_H; \\ E + M = E' + E_H. \end{cases}$

- The relation between the observed quantities (E, E', θ) is [*next slide*] :

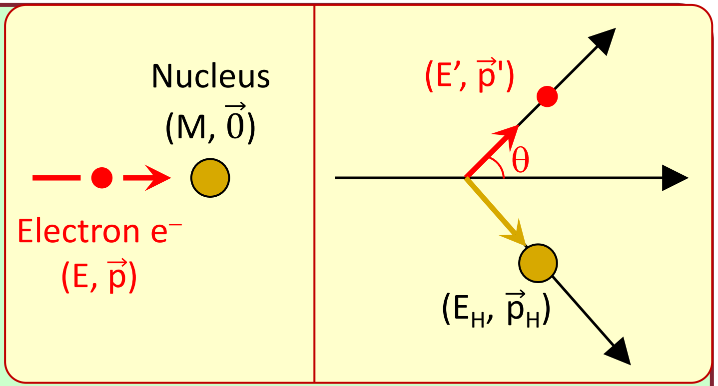
$$E' = \frac{E}{1 + \frac{E}{M}(1 - \cos\theta)} = \frac{E}{1 + \frac{2E}{M}\sin^2(\theta/2)} \approx |\vec{p}\,'|;$$

- Therefore, for known initial energy E and fixed M, the final state is defined by one independent variable (E' or θ).





kinematics: elastic scattering - E' vs θ



$$\begin{cases} e^-_{init} & (E, \vec{p}; m); \\ N_{init} & (M, \vec{0}; M); \end{cases} \qquad \begin{cases} e^-_{fin} & (E', \vec{p}'; m); \\ H_{fin} & (E_H, \vec{p}_H; M); \end{cases}$$

4-momentum conservation
$$\begin{cases} E + M = E' + E_H \to E_H = E + M - E'; \\ \vec{p} + \vec{0} = \vec{p}' + \vec{p}_H \to \vec{p}_H = \vec{p} - \vec{p}'; \end{cases}$$

Square and subtract
$$\left\{ (E_H)^2 - (\vec{p}_H)^2 = M^2 = \left(E^2 + M^2 + E'^2 + 2EM - 2EE' - 2ME'\right) - \left(p^2 + p'^2 - 2pp'\cos\theta\right); \right.$$

Ultra-relativistic approx.
$(m_e << E, E') \to (p \approx E, p' \approx E')$
$$\begin{cases} \cancel{M}^2 = \cancel{E}^2 + \cancel{M}^2 + \cancel{E}'^2 + 2EM - 2EE' - 2ME' - \cancel{E}^2 - \cancel{E}'^2 + 2EE'\cos\theta; \\ 0 = EM - EE' - ME' + EE'\cos\theta = EM - E'\left[E(1 - \cos\theta) + M\right]; \end{cases}$$

$$E' = \frac{EM}{M + E(1 - \cos\theta)} = \frac{E}{1 + \frac{2E}{M}\sin^2\left(\frac{\theta}{2}\right)}$$ q.e.d.

NB – The reaction is planar (why?). The final state is defined by 6 variables. There are 3 $(E, \vec{p})$ conservations and 2 $(m^2=E^2-p^2)$ rules. Therefore: 6-5=1 independent variable.

kinematics: Q^2 in elastic scattering

- in the following, $(E, \vec{p}, E', \vec{p}', m, M, \theta)$;

  $[m = m_e \text{ small} \to E \approx |\vec{p}|,\ E' \approx |\vec{p}'|]$
- new (not independent) variable:

  $\boxed{\vec{q} \equiv \vec{p} - \vec{p}'}$ "momentum transfer";

  $\left[E/M \text{ small} \to p' = p \to |\vec{q}| = 2|\vec{p}|\sin(\theta/2)\right]$
- relativistic equivalent (p and p' are 4-mom):

  $\boxed{q \equiv p - p'} \quad \left[= \left(E - E', \vec{p} - \vec{p}'\right)\right]$;

  $Q^2 \equiv -q^2 = -2m_e^2 + 2EE' - 2|\vec{p}||\vec{p}'|\cos\theta$

  $\approx 4EE'\sin^2(\theta/2)$ [defined $\to Q^2 > 0$];
- $\cancel{E}' = \dfrac{EM}{M + 2E\sin^2(\theta/2)} = \dfrac{EM}{M + Q^2/(2E')} =$

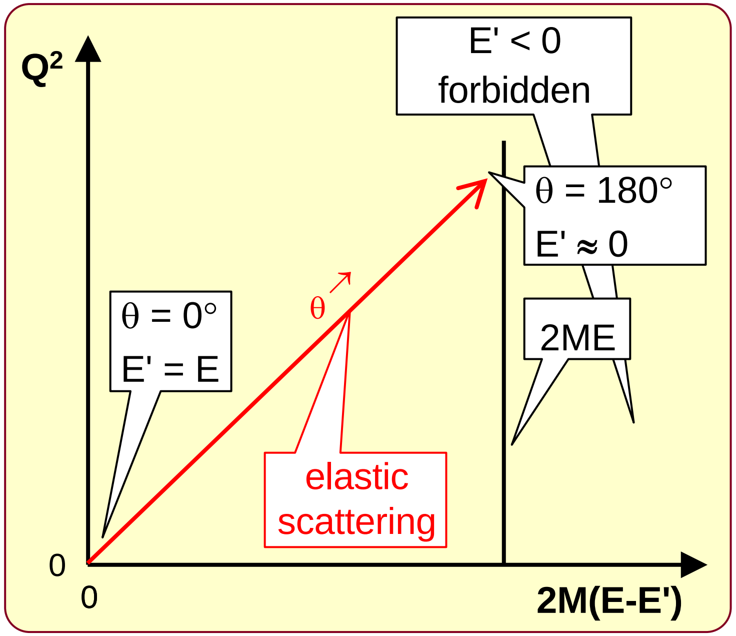
  $= \dfrac{2\cancel{E}'EM}{2E'M + Q^2} \to 2EM = 2E'M + Q^2$

  $\to \boxed{Q^2 = 2M(E - E')}$
- [*for elastic scattering one independent variable* $\to E' = E'(\theta),\ Q^2 = Q^2(E')$];

Study the kinematical limits:

- $\theta = 0°$: $E' = E$; $Q^2 = 0$;
- $\theta = 180°$: $E - E' = E\dfrac{M+2E}{M+2E} - \dfrac{EM}{M+2E} = \dfrac{2E^2}{M+2E}$

  $(E \gg M)$: $E - E' = E \to E' \approx 0$;
- in conclusion $\underline{E > E' > \text{"}0\text{"}}$.
- Plot Q^2 vs $2M(E-E')$: only a segment allowed [*useless for elastic scatt., but ...*]:



kinematics: why $|\vec{q}|$, Q^2

an advance of dynamics

The variable $\vec{\mathbf{q}}$ is *very* important:

- [if relativistic, use Q^2 or its root $\sqrt{Q^2}$];
- it is related to the deBroglie wavelength of the probe: $\lambda\!\!\!^{-} = \hbar/|\vec{q}|$;
- it represents the "scale" of the scattering;
- i.e. structures smaller than $\lambda\!\!\!^{-} \sim 1/|\vec{q}|$ are not "visible" to the probe;
- [the uncertainty principle $\Delta p \Delta x \geq \hbar/2$ leads to the same conclusion – *actually it is exactly the same argument*];

Comments:

- large $|\vec{q}| \to$ large E, but not necessarily the opposite: high-energy & large distance processes do exist;
- the quest for smaller scales leads inevitably to larger Q^2 and therefore to larger E [$\to$ money and resources...]

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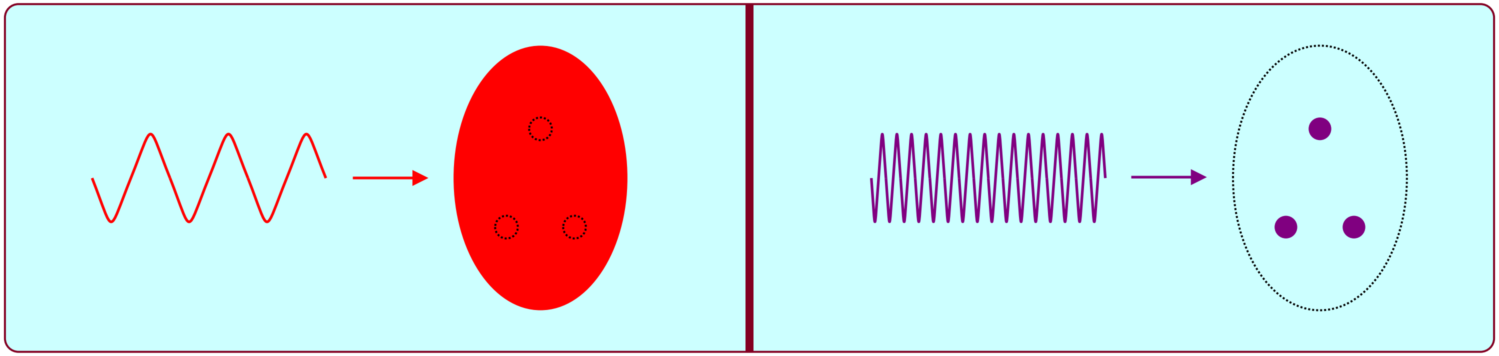
[as usual] sometimes in the literature the notation is confusing: $Q^2 = -t$, see later;



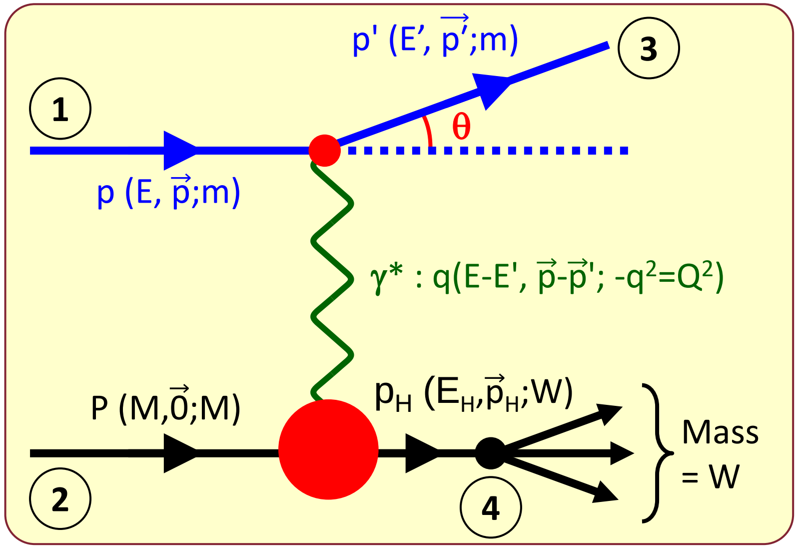
- popular understanding:
  higher Q^2 $\to$ smaller distance $\to$
  $\to$ "better microscope".
- conclusion:
  Q^2 is an important variable, possibly the most important in modern particle physics.

kinematics: the inelastic case

[in general, ℓN → ℓ'H (ℓ,ℓ' generic leptons); the kinematics is the same, if E_ℓ, $E_{\ell'}$ >> m_ℓ, $m_{\ell'}$]

Kinematical variables (**ℓ N → ℓ' H**) :

- [ℓ'=ℓ, H=N → elastic];
- 4-mom. in LAB sys (≡ had CM);
- $p_1 = p$, $p_2 = P$, $p_3 = p'$, $p_4 = p_H$;
- q = p' – p [as in previous slides];



Lorentz – invariant variables:

- $\nu = q \cdot P/M = E - E'$ [= energy lost by e^-];
- $Q^2 = -q^2 = 2(EE' - pp'\cos\theta) - m^2 - m^2 \approx 4\ EE' \sin^2(\theta/2)$ [= – module of the 4-momentum transfer];
- $x = Q^2 / (2M\nu)$ [later : x-Bjorken x_B, the fraction of the hadron 4-momentum carried by the interacting parton];
- $y = (q \cdot P) / (p \cdot P) = \nu / E$ [= the fraction of the energy lost by the lepton in the target frame];
- $W^2 = (p_H)^2 = (P + q)^2 = M^2 - Q^2 + 2M\nu$ [=(mass)² of the hadron system in the final state] : W = M if elastic;
- [with these variables, the (energy)² in the CM is $s = (p+P)^2 = (p'+p_H)^2$]

[*next slide*]

kinematics: Q^2, ν, x, y, W^2

$$\begin{cases} e^-_{init} & p\ \ (E,\vec{p};m_\ell); \\ N_{init} & P\ (M,\vec{0};M); \end{cases} \qquad \begin{cases} e^-_{fin} & p'\ (E',\vec{p}\,';m_\ell); \\ N_{fin} & p_H\ (E_H,\vec{p}_H;W); \end{cases}$$

p, p', P, P_H, q, Q^2, M, ν, x, y, W^2 Lorentz invariant;

E,E', … Lab sys (= P at rest).

m << M (safe approx);

$$q = p - p' = (E-E', \vec{p}-\vec{p}\,');$$

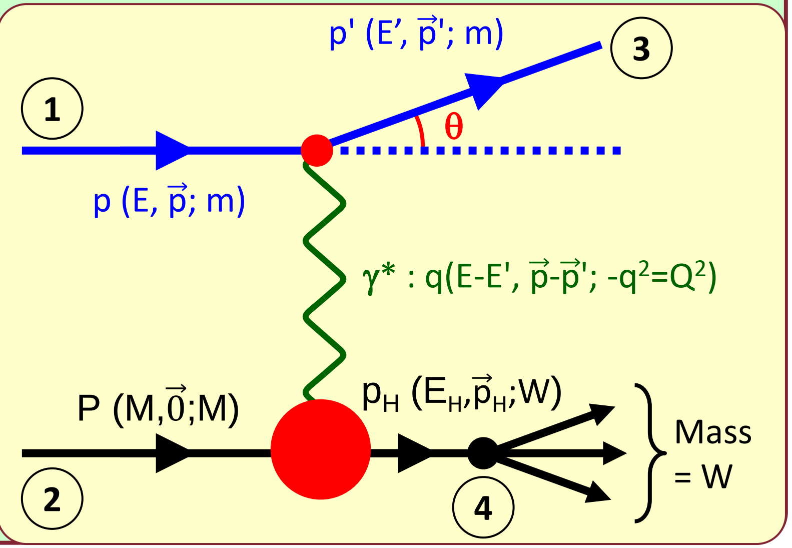
$$q^2 = m^2 + m^2 - 2EE' + 2pp'\cos\theta \approx -2EE'(1-\cos\theta) = -4EE'\sin^2\left(\frac{\theta}{2}\right) \equiv -Q^2;$$

$$\nu \equiv \frac{q\cdot P}{M} = \frac{(E-E')M}{M} = (E-E');$$

warning: x_B is very interesting, *see later*

$$x \equiv \frac{Q^2}{2M\nu}; \qquad y \equiv \frac{q\cdot P}{p\cdot P} = \frac{(E-E')M}{EM} = \frac{E-E'}{E} = \frac{\nu}{E};$$

$$W^2 = p_H^2 = (P+q)^2 = M^2 - Q^2 + 2M\nu.$$



kinematics: the inelastic case - remarks

Remarks :

- a lot of kinematical relations, e.g.

$$W^2 = M^2 + 2MEy(1-x);$$
$$Q^2 = 2MExy;$$
$$s = M^2 + m^2 + Q^2/(xy);$$

- in the <u>elastic case</u> $eN \to eN$ [$ep \to ep$], ν and Q^2 are NOT independent :

$$W^2 = M^2 = (P + q)^2 = M^2 - Q^2 + 2M\nu$$
$$\to \quad Q^2 = 2M\nu \to Q^2 / (2M\nu) = x = 1;$$

- therefore (obviously) in the elastic case, there is only <u>one</u> independent parameter (E' or θ, choice according to the meas.);

- instead, in the <u>inelastic scattering</u> :

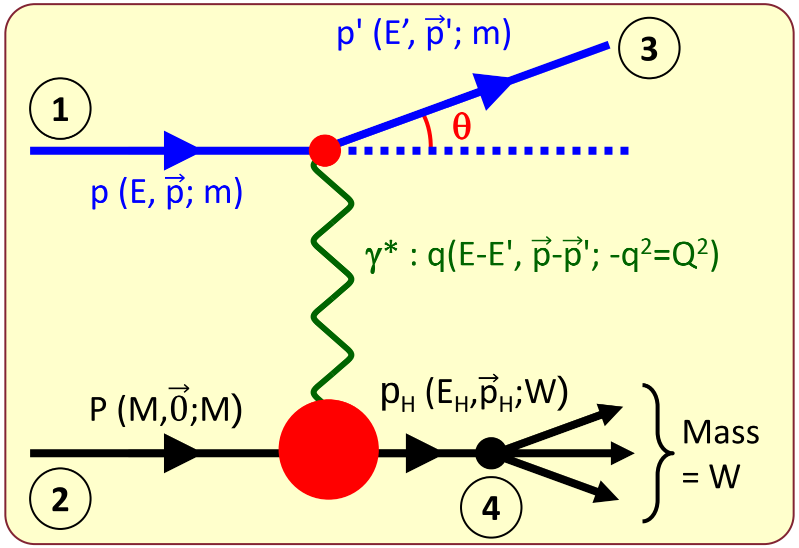
$$Q^2 = M^2 + 2M\nu - W^2 =$$
$$= 2M\nu - (W^2 - M^2) \leq 2M\nu \to x \leq 1;$$

if W not fixed, Q^2 and ν are independent;

- therefore, in the inelastic case, there are <u>two</u> independent variables;

- in the analysis, choose two among all variables, according to convenience, e.g.:

$$(E', \theta), (Q^2, \nu), (x, y).$$

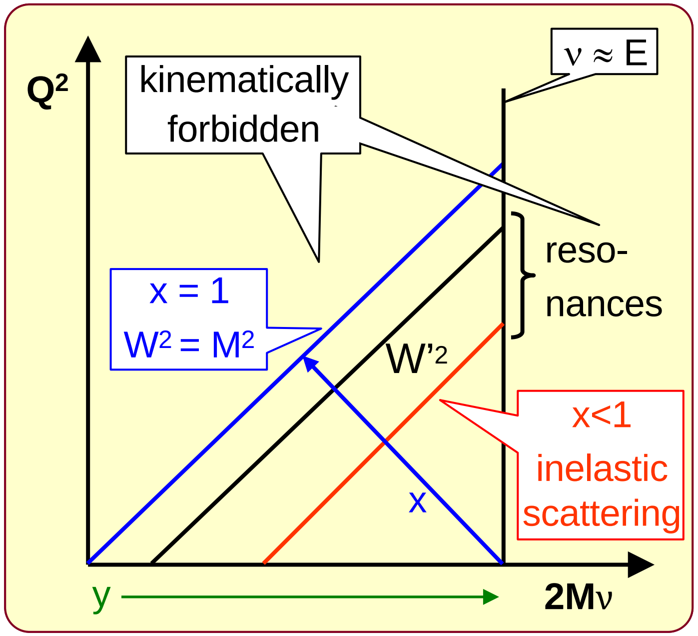


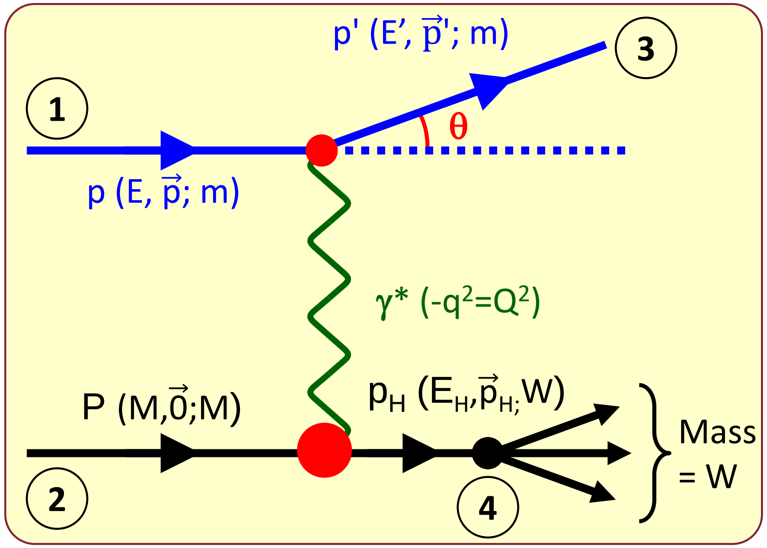
kinematics: deep inelastic scattering

Redefine the kinematics of the scattering process in the plane (Q^2 vs ν) [more precisely (Q^2 vs $2M\nu$)]:

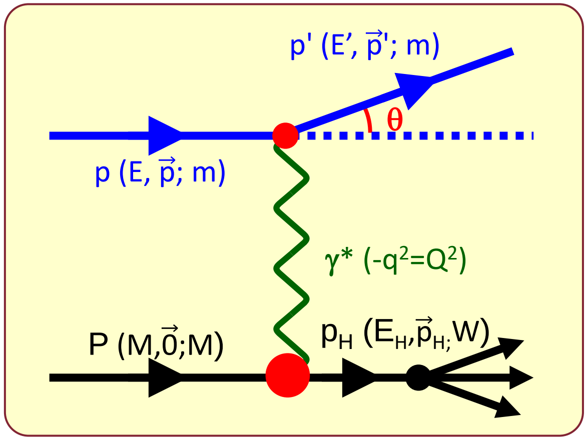
- both are Lorentz-invariant [*but usually used in the lab. frame, where the initial state hadron is at rest*] ;
- $\nu = E - E' \rightarrow 0 \leq \nu \leq E \rightarrow$ only a band is allowed;
- $Q^2 = 4\,EE' \sin^2(\theta/2) \geq 0 \rightarrow$ only the 1st quadrant;
- $x = Q^2 / (2M\nu) \leq 1 \rightarrow 0 \leq x \leq 1 \rightarrow$ only "lower triangle";
- $y = (q \cdot P) / (p \cdot P) = \nu / E \rightarrow 0 \leq y \leq 1$;
- $W^2 = M^2 + 2M\nu - Q^2 \rightarrow$ the bisector x=1 ("/") defines the elastic scattering, where $W^2 = M^2$;
- on the bisector, only θ varies : $\theta = 0 \rightarrow Q^2 = \nu = 0$;
- the loci W'^2 = constant are lines parallel to the bisector → some of them define the excited states (one shown in fig.);
- at higher distance from the bisector we have the deep inelastic scattering (*DIS*) and (possibly) new physics.

[*see next slide*]



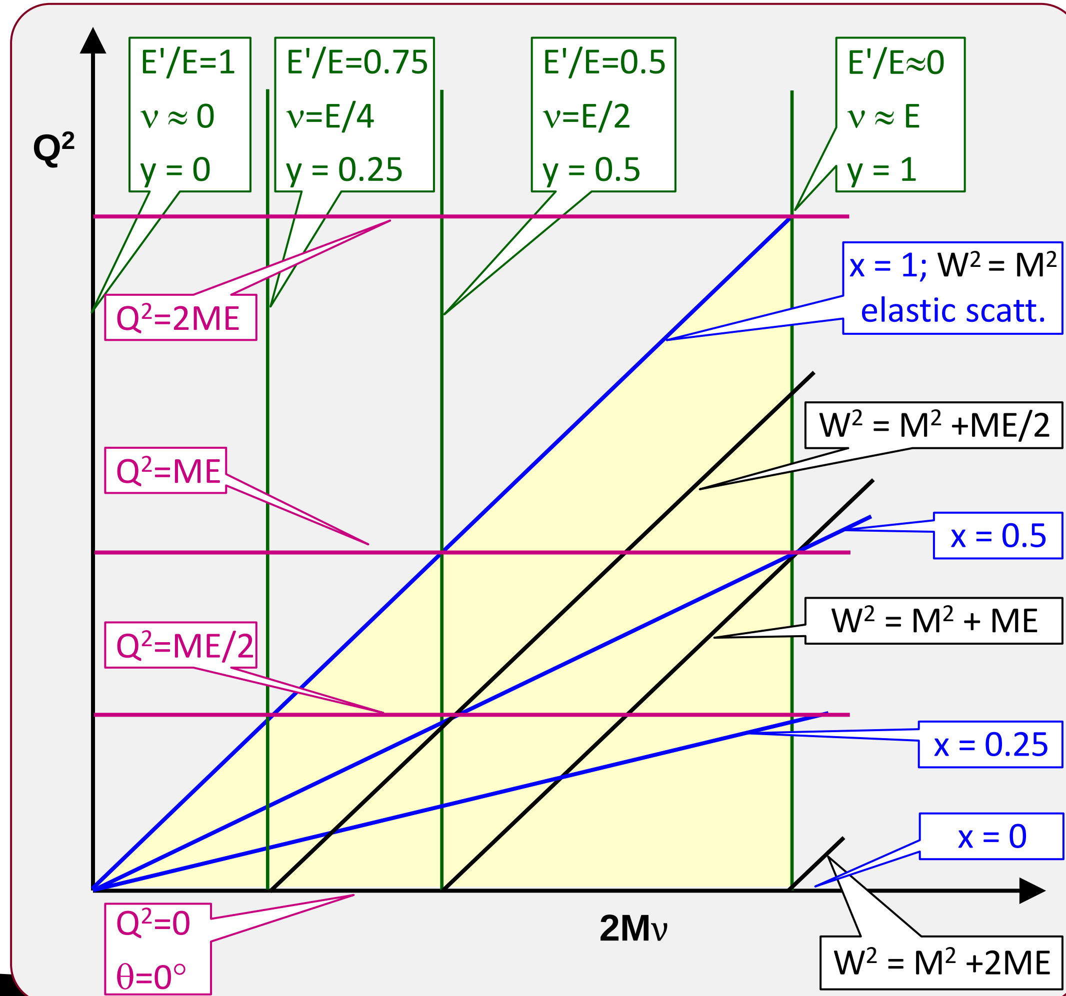


kinematics: a summary



0	<	x	< 1
0	<	y	< 1
0	<	ν	< E
M^2	<	W^2	< M^2+2ME
0	<	Q^2	< 2ME
0	<	E'	< E
0°	<	θ	< 180°

limits (some only if E >> M).



You are kindly (but strongly) requested to look carefully to this slide and get used to these variables.

THANKS.

"higher Q²"

elastic scattering e-N : $\sigma_{\text{Rutherford + Mott}}$

- The scattering α-Nucleus actually takes place between two nuclei (e.g. He^{++}-Au);
- not suitable for measuring a (possible) nucleus structure → replace the α with a more (?) point-like probe: electron (e^-);
- if the process is e.m., the dynamics of the eN scattering can be described by the Rutherford formula (use the momentum transfer $\vec{q}=\vec{p}-\vec{p}\,'$) [*next slide*]:

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Rutherford}} = \frac{4Z^2\alpha^2E'^2}{|\vec{q}|^4}; \quad |\vec{q}| = 2|\vec{p}|\sin\frac{\theta}{2}.$$

- in relativistic quantum mechanics the elastic scattering cross-section is described by a formula, due to Mott :

$$\left[\frac{d\sigma}{d\Omega}\right]^*_{\text{Mott}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Rutherford}} \times \left(1-\beta^2\sin^2\frac{\theta}{2}\right) \rightarrow$$

$$\xrightarrow{\beta=|\vec{p}|/E\rightarrow 1} \left[\frac{d\sigma}{d\Omega}\right]_{\text{Rutherford}} \cos^2\frac{\theta}{2} = \frac{4Z^2\alpha^2E'^2}{|\vec{q}|^4}\cos^2\frac{\theta}{2}.$$

- similar to the Rutherford formula, the Mott* cross-section neglects (a) the nucleus dimension and (b) its recoil*.
- unlike Rutherford, Mott takes into account the e^- spin (=½) [*next slide*].

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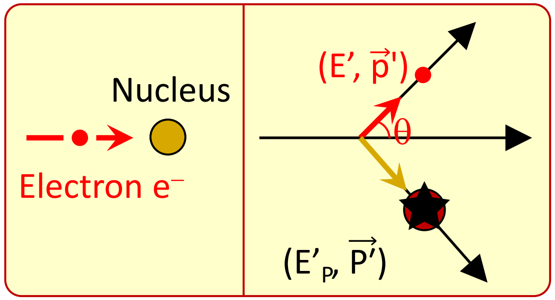
NB The "*" in the definition of Mott* means that the "no-recoil" approximation is used → leave it out when the recoil is considered ("Mott*" → "Mott"].



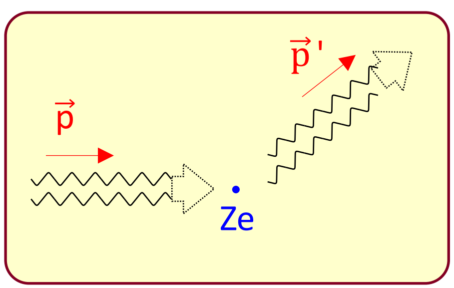
Sir Nevill Francis Mott

elastic scattering e-N : Rutherford + q.m.

FNSN1, 58

q.m. calculation

- already computed in classical approx.
- non-relativistic q.m. + Born approx.;
- Coulomb potential;
- negligible recoil;
- initial (i) and final (f) particle as plane waves [*see introduction + box*];
- $\vec{q}=\Delta\vec{p}$ (as usual);
- $\hbar$ and c for the last time;
- $V(r=\infty)$ does NOT contribute, because of other nuclei → in the last integration, do not use the value at $r=\infty$ [YN1, 135 has a cutoff "μ"].



$$V(r)=-\frac{Z\alpha\hbar c}{r};\quad \vec{q}=\Delta\vec{p}=\vec{p}-\vec{p}\,';\quad q=|\vec{q}|=2p\sin(\theta/2);$$

$$\psi_{i,f}=e^{i\vec{p}\cdot\vec{r}/\hbar}/\sqrt{\mathcal{V}};\quad \psi_f=e^{i\vec{p}\,'\cdot\vec{r}/\hbar}/\sqrt{\mathcal{V}};\quad \frac{dn}{dE'}=\frac{\mathcal{V}4\pi p'^2}{v'(2\pi\hbar)^3};$$

$$\mathcal{M}_{fi}=\langle\psi_f\,|V(\vec{r})|\,\psi_i\rangle=\frac{1}{\mathcal{V}}\int e^{-i\vec{p}\,'\cdot\vec{r}/\hbar}V(\vec{r})e^{i\vec{p}\cdot\vec{r}/\hbar}d^3\vec{r}=$$

$$=-\frac{1}{\mathcal{V}}\iiint\frac{Z\alpha\hbar c}{r}e^{i\vec{q}\cdot\vec{r}/\hbar}r^2dr\sin\theta d\theta d\varphi=-\frac{4\pi}{\mathcal{V}}\frac{Z\alpha\hbar^3c}{q^2};$$

$$\boxed{\frac{d\sigma}{d\Omega}}=\frac{1}{4\pi}\left[\frac{2\pi}{\hbar}|\mathcal{M}_{fi}|^2\frac{dn}{dE'}\frac{\mathcal{V}}{v'}\right]\xrightarrow{v'=c,p'=E'/c}$$

$$=\frac{1}{2\hbar}\left|\frac{4\pi}{\mathcal{V}}\frac{Z\alpha\hbar^3c}{q^2}\right|^2\frac{\mathcal{V}E'^2}{2\pi^2c^3\hbar^3}\frac{\mathcal{V}}{c}=\boxed{\frac{4Z^2\alpha^2\hbar^2E'^2}{q^4c^2}}$$

$$\int_0^{2\pi}d\varphi\int_0^\infty rdr\int_{-1}^{1}d\cos\theta\, e^{iqr\cos\theta/\hbar}=2\pi\int_0^\infty dr\int_{-r}^{r}e^{iqt/\hbar}dt\quad[t=r\cos\theta]$$

$$=\frac{2\pi\hbar}{iq}\int_0^\infty dr\left(e^{iqr/\hbar}-e^{-iqr/\hbar}\right)=\frac{2\pi\hbar}{iq}\frac{\hbar}{iq}\left[e^{iqr/\hbar}+e^{-iqr/\hbar}\right]^{r=0}=-\frac{4\pi\hbar^2}{q^2}.$$

elastic scattering e-N : helicity

The $\cos^2(\theta/2)$ factor in $[d\sigma/d\Omega]_{Mott}$ comes from Dirac equation; it is understood by considering the extreme case of θ~180°.

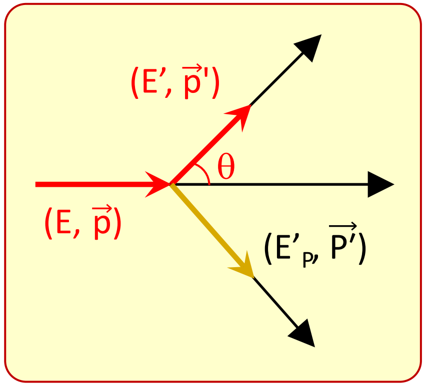
For relativistic particles ($\beta \to 1$), the helicity h (the projection of spin along momentum) is conserved :

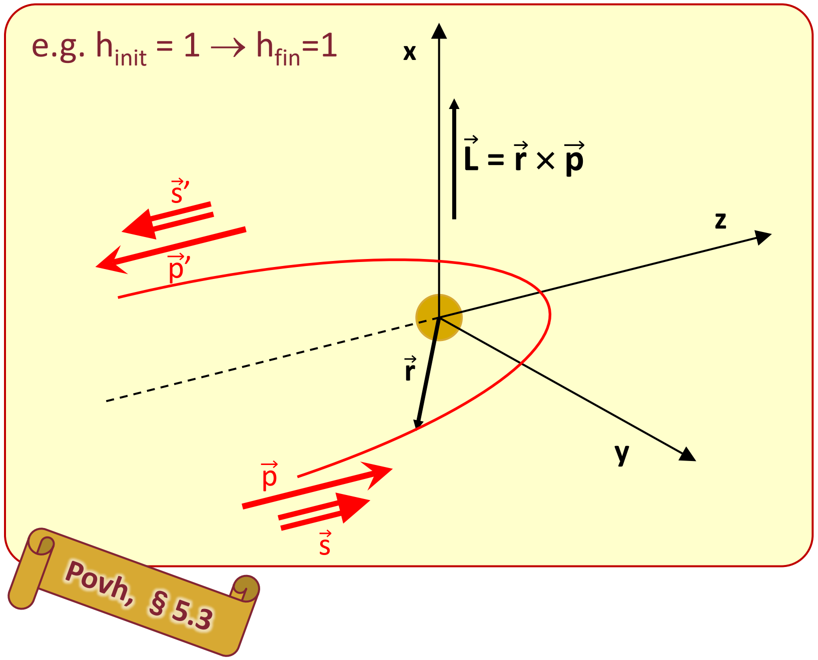
$$h = \frac{\vec{s} \cdot \vec{p}}{|\vec{s}| \cdot |\vec{p}|}.$$

The conservation requires the "spin flip" of the electron between initial and final state, because the momentum also flips at θ=180°.

In this condition, the angular momentum is NOT conserved, if the nucleus does NOT absorb the spin variation (e.g. because it is spinless). Therefore the scattering for $\theta \approx 180°$ is forbidden.

The factor $\cos^2(\theta/2)$ in the Mott formula is connected to the spin and describes the magnetic part of the interaction.





elastic scattering e-N : experiment

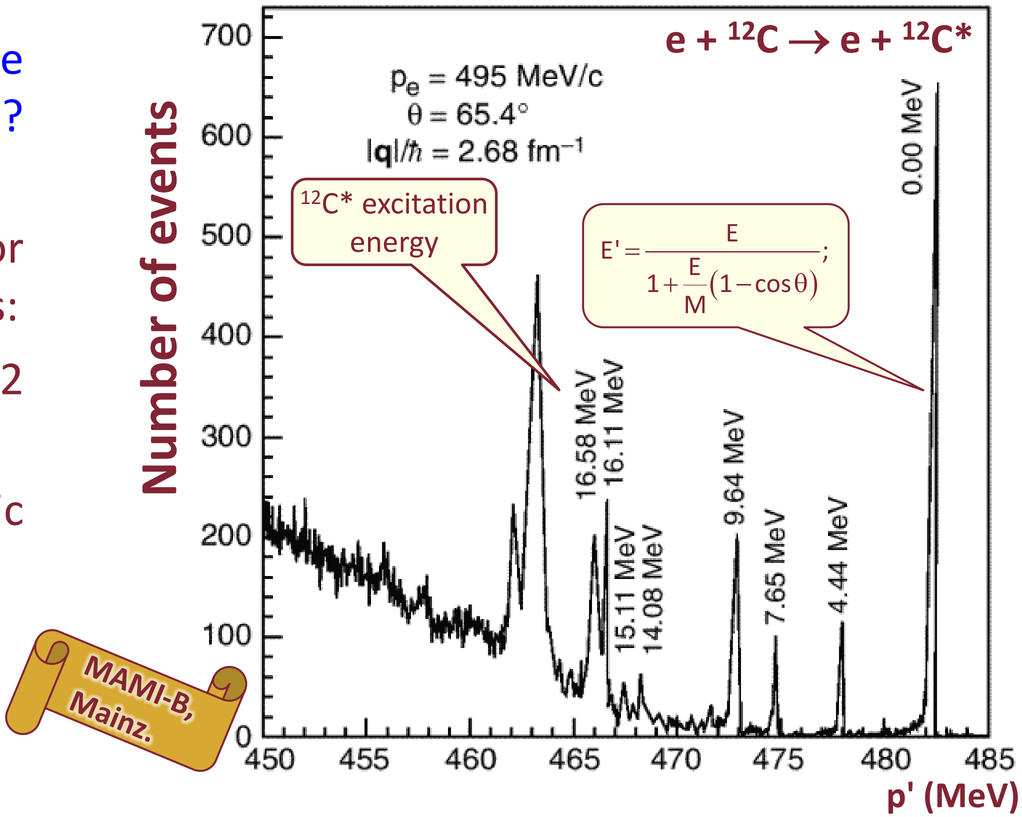
Is the experiment consistent with the kinematics of the elastic scattering ? Get e + ^{12}C data.

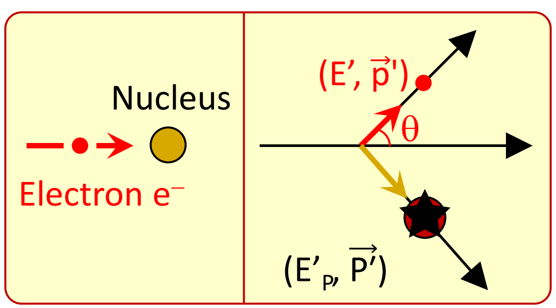
The plot of the number of events, for fixed E_{init} at fixed θ, shows many peaks:

- the expected elastic ($E' \approx p'$ = 482 MeV),
- a rich structure, due to inelastic scattering:

$$e + {}^{12}C \to e + {}^{12}C^*$$

[$^{12}C^*$ = excited carbon, mass M*].





- the expected elastic [e + ^{12}C → e + ^{12}C] is there;
- but "*more things in heaven, than in your philosophy*";
- **back to elastic scattering !**
- kinematics ok, dynamics ?

→ measure $d\sigma/d\Omega$ vs θ !!!

form factors: definition

- The experimental $d\sigma/d\Omega$ agrees with the Mott cross-section only for small $|\vec{q}|$;
- otherwise, the cross section is smaller;
- possibly the reason is the structure of the nucleus, which results in a smaller effective charge, as seen by the projectile (Gauss' theorem);

→ define the **form factor** [$\mathcal{F}(\vec{q})$], as the **Fourier transform of the charge distribution function ρ**:

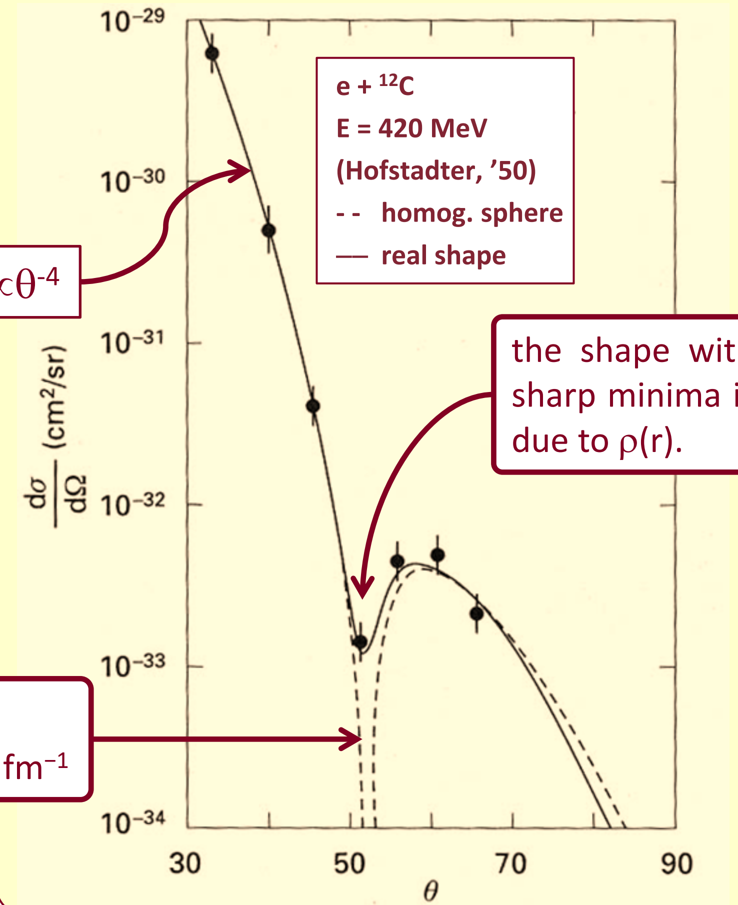
$$\rho(\vec{x}) = Zef(\vec{x}), \quad \int f(\vec{x})d^3x = 1; \quad \vec{q} = \vec{p} - \vec{p}\,';$$

$$\boxed{\mathcal{F}(\vec{q}) = \int e^{\left(i\frac{\vec{q}\cdot\vec{x}}{\hbar}\right)} f(\vec{x})d^3x};$$

- if $f(\vec{x}) = \delta(\vec{x}) \rightarrow \mathcal{F}(\vec{q}) = 1$.
- if $\rho(\vec{x})$ depends only on $|\vec{x}|$ [*next slides*]:

$$\left[\frac{d\sigma}{d\Omega}\right]_{exp} = \left[\frac{d\sigma}{d\Omega}\right]^*_{Mott} \times \left|\mathcal{F}(q^2)\right|^2.$$

form factors are measurable, at least in principle.



$\propto\theta^{-4}$

the shape with sharp minima is due to ρ(r).

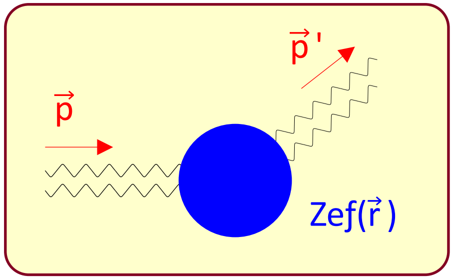
$\theta \approx 51°$,
$r^{-1} = q/\hbar \approx 1.8\ \text{fm}^{-1}$

[*in the following, we will discuss only the case with spherical symmetry* ρ(r), *when* $\mathcal{F}(\vec{q})$ depends on $q=|\vec{q}|$].

form factors: q.m. definition

q.m. calculation [Thomson, 166]

- non-relativistic q.m. + Born approx.;
- Coulomb potential;
- negligible recoil;
- initial (i) and final (f) particle as plane waves with ƛ << nucleus size [*see little box*];
- charge distribution $f(\vec{r})$, normalized to 1;
- $\vec{q} = \vec{p} - \vec{p}\,'$ and $\mathcal{F}(q^2)$ as defined before.



$$V(\vec{r}) = -\int d^3\vec{r}\,' \frac{Z\alpha f(\vec{r}\,')}{4\pi|\vec{r}-\vec{r}\,'|};$$

$$\psi_i = e^{i(\vec{p}\cdot\vec{x}-Et)}/\sqrt{\mathcal{V}}; \qquad \psi_f = e^{i(\vec{p}\,'\cdot\vec{x}-Et)}/\sqrt{\mathcal{V}};$$

$$\mathcal{M}_{fi} = \langle\psi_f\,|V(\vec{r})|\,\psi_i\rangle = \frac{1}{\mathcal{V}}\int e^{-i\vec{p}\,'\cdot\vec{r}}V(\vec{r})e^{i\vec{p}\cdot\vec{r}}d^3\vec{r} =$$

$$= -\frac{1}{\mathcal{V}}\int\!\!\int e^{i\vec{q}\cdot\vec{r}}\frac{Z\alpha f(\vec{r}\,')}{4\pi|\vec{r}-\vec{r}\,'|}d^3\vec{r}\,'d^3\vec{r} =$$

$$= -\frac{1}{\mathcal{V}}\int\!\!\int e^{i\vec{q}\cdot(\vec{r}-\vec{r}\,')}e^{i\vec{q}\cdot\vec{r}\,'}\frac{Z\alpha f(\vec{r}\,')}{4\pi|\vec{r}-\vec{r}\,'|}d^3\vec{r}\,'d^3\vec{r} =$$

$$= \left[-\frac{1}{\mathcal{V}}\int e^{i\vec{q}\cdot\vec{R}}\frac{Z\alpha}{4\pi|\vec{R}|}d^3|\vec{R}|\right]\times\left[\int f(\vec{r}\,')e^{i\vec{q}\cdot\vec{r}\,'}d^3\vec{r}\,'\right] =$$

$$= \mathcal{M}_{fi}^{point}\times\mathcal{F}(q^2)$$

$$\vec{R} = \vec{r} - \vec{r}\,'$$

$$\rightarrow \boxed{\left[\frac{d\sigma}{d\Omega}\right]_{\text{non-point}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{point}} \times \left|\mathcal{F}(q^2)\right|^2}.$$

$\mathcal{V}$ = volume

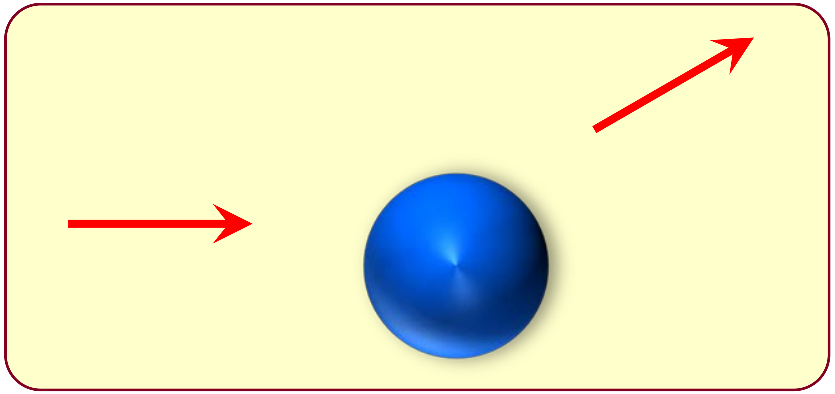
form factors: radial symmetry

In principle, the function $\rho(r)$ may be computed by measuring $\mathcal{F}(q^2)$ and then, e.g. numerically:

$$\rho(r) = \frac{Ze}{(2\pi)^3}\int \mathcal{F}(q^2)\, e^{-i\frac{qr}{\hbar}}\, d^3q$$

However, the range of q accessible to experiments is limited; therefore, the behavior of $\mathcal{F}(q^2)$ for q^2 large (i.e. r small, *the interesting region*) has to be extrapolated with reasonable assumptions.

In the next slides, examples of $\rho(r)$ and $\mathcal{F}(q^2)$ are computed (e.g. the case of a homogeneous sphere of radius R).



Compute the symmetrical case[(1)]; neglect the nuclear recoil :

$$\mathcal{F}(q^2) = \frac{1}{S}\int e^{i\frac{\vec{q}\cdot\vec{x}}{\hbar}} f(\vec{x})\, d^3x = \qquad [f(\vec{x}) = f(r) \rightarrow]$$

$$= \frac{2\pi}{S}\int_0^\infty f(r)\, r^2 dr \int_{-1}^{1} e^{i\frac{qr\cos\theta}{\hbar}}\, d\cos\theta =$$

$$= \frac{2\pi}{S}\int_0^\infty f(r)\, r^2\, \frac{2}{2}\frac{\hbar}{iqr}\left[e^{i\frac{qr}{\hbar}} - e^{-i\frac{qr}{\hbar}}\right] dr =$$

$$= \frac{4\pi}{S}\int_0^\infty f(r)\, r^2\, \frac{\sin(qr/\hbar)}{qr/\hbar}\, dr;$$

$$S = 4\pi\int_0^\infty f(r)\, r^2 dr \qquad [=1 \text{ if normalized}];$$

---

(1) $d\sigma/d\Omega$, both Rutherford and Mott, is scale-independent. However, if $\rho(r)$ depends on a scale (e.g. by a sphere radius), form factors break the scale invariance of the dynamics.

form factors: examples

$f(\vec{r}) = f(|\vec{r}|)$

$$f(r) = \frac{1}{(2\pi)^3}\int \mathcal{F}(q^2)\, e^{-i\frac{qr}{\hbar}}\, d^3q$$

$$\mathcal{F}(q^2) = 4\pi\int_0^\infty f(r)\, r^2 \frac{\sin(qr/\hbar)}{qr/\hbar}\, dr$$

Charge distribution	f(r)	form factor	$\mathcal{F}(q^2)$	~ example
point-like	$\delta(r)/(4\pi)$	constant	1	$e^\pm$
exponential	$(a^3/8\pi)$ $\exp(-ar)$	dipolar	$(1+q^2/a^2\hbar^2)^{-2}$	p
gaussian	$[a^2/(2\pi)^{3/2}]$ $\exp(-a^2r^2/2)$	gaussian	$\exp[-q^2/(2a^2\hbar^2)]$	^{6}Li
homog. sphere	$3/(4\pi R^3)$ $r \le R$; 0 $r > R$	oscill.	$3\alpha^{-3}(\sin\alpha - \alpha\cos\alpha)$ $\alpha = \lvert q\rvert R/\hbar$	– (see)
sphere with soft surface	$\rho_0 / [1 + e^{(r-c)/a}]$	oscill.		^{40}Ca

Fermi (Woods-Saxon) function

form factors: homogeneous sphere

Homogeneous sphere with unit charge :

$$\rho(r) = f(r) = \begin{cases} \rho_0 = \dfrac{3}{4\pi R^3} & r \le R \\ 0 & r > R \end{cases}$$

$$\mathcal{F}(q^2) = 4\pi \int_0^\infty f(r)\, r^2 \frac{\sin(qr/\hbar)}{qr/\hbar} dr =$$

$$= \frac{4\pi\hbar\rho_0}{q} \int_0^R r \sin\left(\frac{qr}{\hbar}\right) dr = \qquad \left[w = \frac{qr}{\hbar}; \bar{W} = \frac{qR}{\hbar} \right]$$

$$= \frac{4\pi\hbar^3\rho_0}{q^3} \int_0^{\bar{W}} w \sin w\, dw = \frac{4\pi\hbar^3\rho_0}{q^3} \left[\sin w - w \cos w\right]_0^{\bar{W}} =$$

$$= \frac{4\pi\hbar^3\rho_0}{q^3} \left[\sin\left(\frac{qR}{\hbar}\right) - \frac{qR}{\hbar} \cos\left(\frac{qR}{\hbar}\right) \right] =$$

$$= \frac{3\hbar^3}{q^3 R^3} \left[\sin\left(\frac{qR}{\hbar}\right) - \frac{qR}{\hbar} \cos\left(\frac{qR}{\hbar}\right) \right]$$

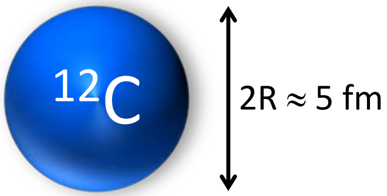
if $qR/\hbar$ [= t] $\to 0$
$\mathcal{F} \approx 3/t^3\ [(t - t^3/6) - - t(1-t^2/2)] = 1.$

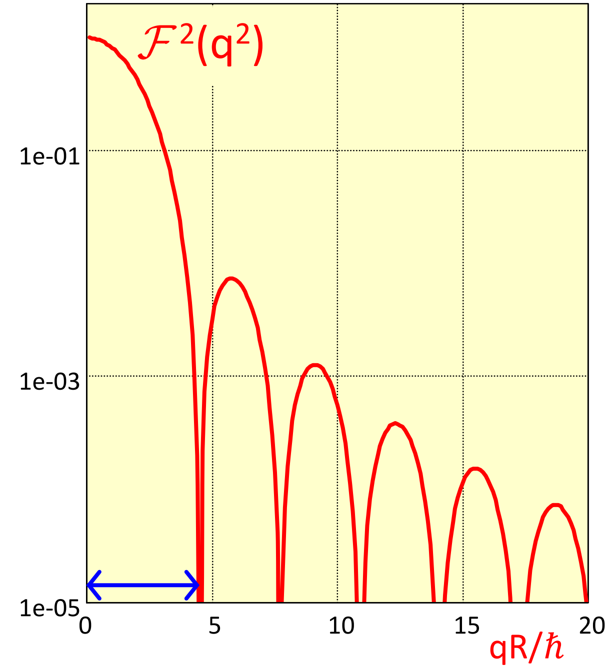
first minimum :
$qR/\hbar = \tan(qR/\hbar)$
$\to qR/\hbar \approx 4.5$

By comparing the first minimum with the experiment of ^{12}C ($q/\hbar \approx 1.8\ fm^{-1}$), we get :

$$R \approx 4.5\ r_{min} = 4.5/1.8 \approx 2.5\ fm$$

i.e. ^{12}C is approximately a hard sphere with radius of 2.5 fm.





form factors: <r²>

Study the behavior for $q \to 0$:

$$\mathcal{F}(q^2) = \iiint e^{i\frac{qr\cos\theta}{\hbar}} f(r) r^2 dr d\cos\theta d\varphi =$$

$$= 2\pi \int_0^\infty f(r) r^2 dr \int_{-1}^{1} \left[\begin{array}{l} 1 + i\dfrac{qr}{\hbar}\cos\theta - \\ -\dfrac{1}{2}\left(\dfrac{qr}{\hbar}\right)^2 \cos^2\theta + \ldots \end{array} \right] d\cos\theta =$$

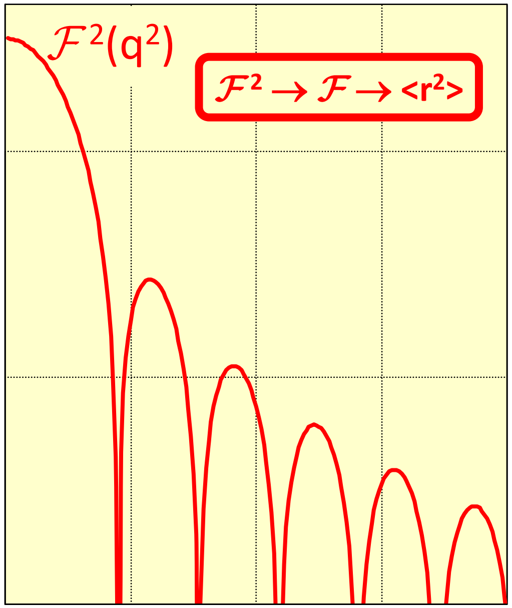
$$= 4\pi \int_0^\infty f(r) r^2 dr + 0 - \frac{4\pi}{6}\frac{q^2}{\hbar^2}\int_0^\infty f(r) r^4 dr + \ldots =$$

$$= 1 - \frac{1}{6}\frac{q^2 < r^2 >}{\hbar^2} + \ldots$$

with $\quad < r^2 > \equiv \iiint r^2 f(\vec{x}) d^3x = 4\pi \int_0^\infty r^2 f(r) r^2 dr.$

i.e. $\quad < r^2 > = -6\hbar^2 \left. \dfrac{d\mathcal{F}(q^2)}{dq^2} \right|_{q^2=0} .$

The parameter <r²> contains the information of the charge distribution.



Simple problem : check that for the homogeneous sphere, both directly and from the definition :
$<r^2> = 3R^2/5$.

form factors: solution

Simple problem : check that for the homogeneous sphere, both directly and from the definition :
$\langle r^2 \rangle = 3R^2/5$.

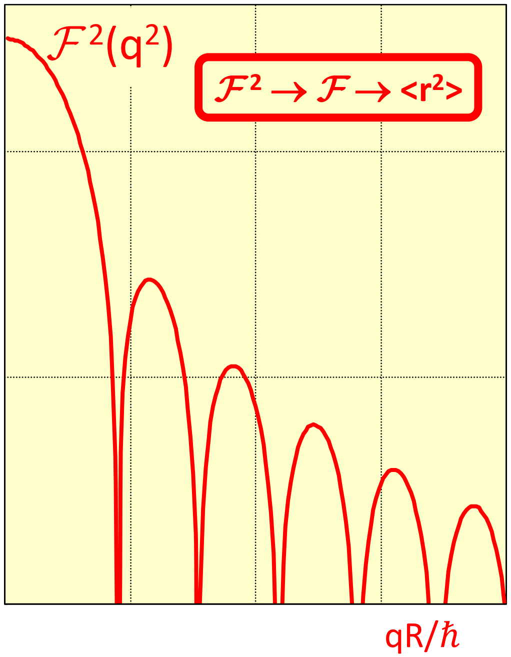
$$\langle r^n \rangle = \frac{1}{V}\iiint r^n d^3x = \frac{4\pi}{V}\int_0^R r^n r^2 dr =$$

$$= \frac{4\pi}{V}\frac{R^{n+3}}{n+3} = \frac{4\pi R^{n+3}}{n+3}\frac{3}{4\pi R^3} =$$

$$= \frac{3}{n+3}R^n$$

$$\xrightarrow{n=2} \langle r^2 \rangle = \frac{3}{5}R^2$$

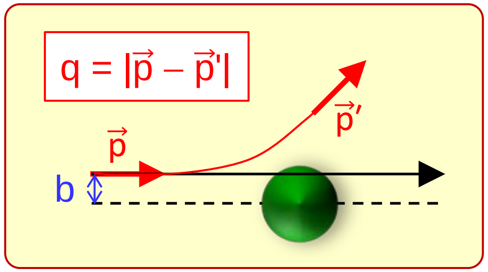
[qed, too easy to enjoy]



form factors: $q \to 0$ vs $q \to \infty$

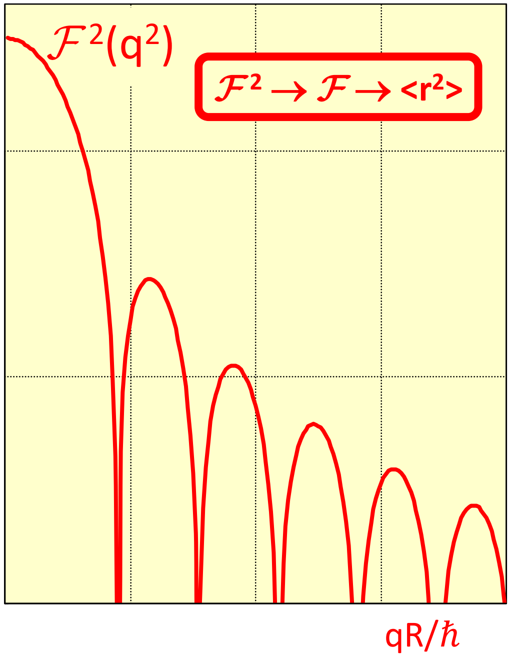
The limits $q \to 0$, $\to \infty$ have a deep meaning:

- q is (approximately) the conjugate variable of b, the impact parameter of the projectile wrt the target center:
  - $\to$ for q very small (i.e. b very large), the target behave as a point-like object;
  - $\to$ for q quite small (i.e. b quite large) it behaves as a coherent homogeneous charged sphere with radius $\sqrt{\langle r^2 \rangle}$;
  - $\to$ large q probes the nucleus at small b;
- "new physics" (a substructure emerging at very small distance) requires very large q, which in turn is only possible if a large projectile energy is available.



The same story has repeated many times, from Rutherford to the LHC, but at smaller b (i.e. larger q). This fact is the main justification for higher energy accelerators …

… and (unfortunately) larger experiments, larger groups, more expensive detectors, politics, troubles, ... [*the usual "laudatio temporis acti", forgive me*]



form factors: shape of nuclei

Summary of systematic study of the form factors for nuclei [just results, no details]:

- heavy nuclei :
  - NOT "homogeneous spheres" with a sharp edge;
  - similar to **spheres with a soft edge**;
  - charge distribution is well reproduced by a standard Fermi function :

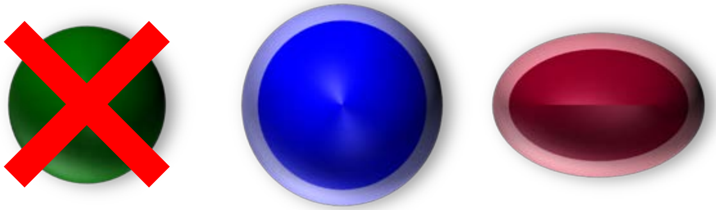
    $\rho_{charge}(r) = \rho_0 / [1 + e^{(r-c)/a}]$;
  - for large A (see figure) :

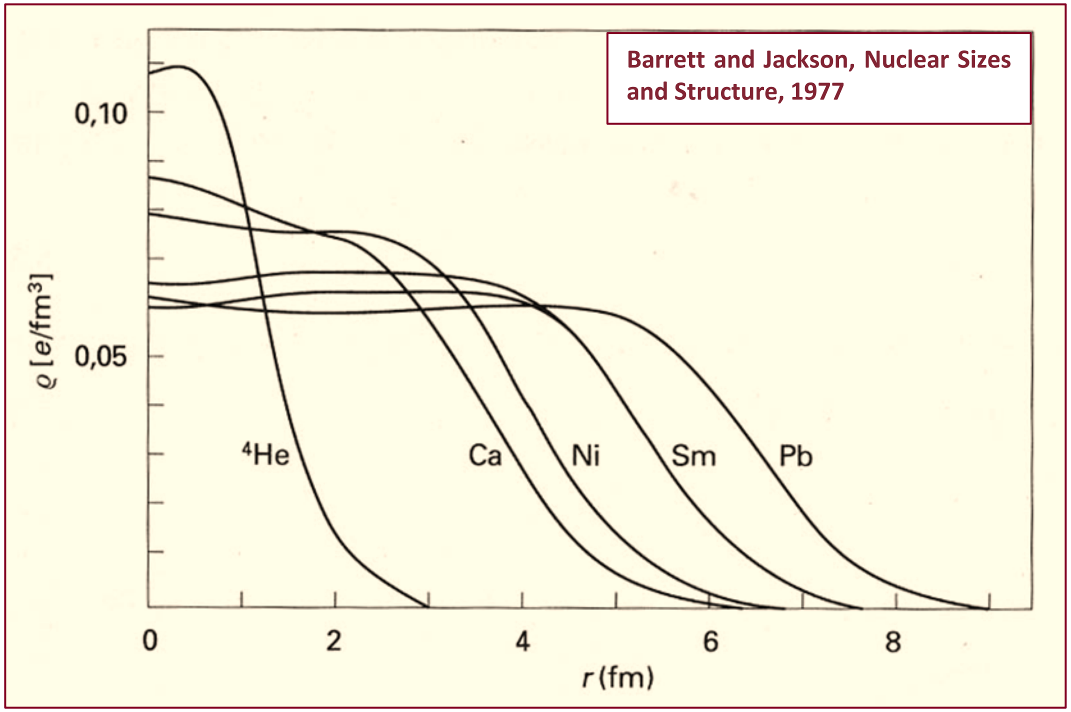
    $c \approx 1.07 \text{ fm} \times A^{1/3}$ ["radius"]

    $a \approx 0.54 \text{ fm}$ ["skin"];

$V_{nucleus} \propto A \rightarrow c \approx r_{nucleus} \propto A^{1/3}$

- light nuclei (^{4}He, ^{6,7}Li, ^{9}Be) more Gaussian-like;
- all these nuclei have spherical symmetry;
- lanthanides (rare earths) are more like ellipsoids [*think to an experiment to show it*].





form factors: nuclear density

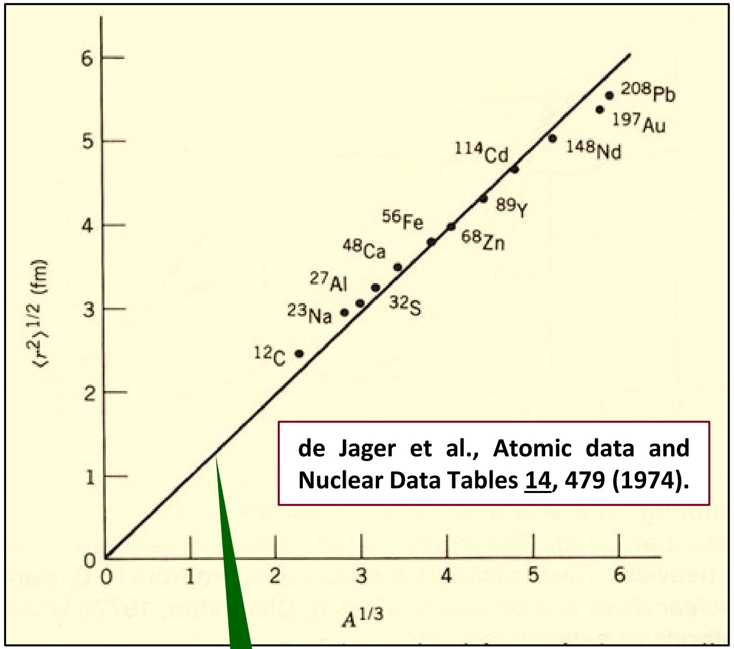
Compute the nuclear densities of p and n
$[q_p\,\rho_Q = dq/dV\ ,\ m_p\,\rho_p = dm_p/dV]$:

- assume in the nucleus homogeneous and equal distribution of p and n;
- then:
  - $\rho_Q = \rho_p$ = proton density;
  - ρ_n = neutron density = ρ_p;
  - ρ_T = nuclear density = $\rho_p + \rho_n$;
- compute :
  - $\rho_T = \rho_p + \rho_n = \rho_p + N\,\rho_p / Z = A\,\rho_Q/Z$;
  - $A = V\,\rho_T = 4\pi/3\ R^3\rho_T$;
  - ρ_T = 0.17 nucleons / fm³ (from ρ_0 of previous slide);
- $\dfrac{4\pi}{3}R^3 = \dfrac{4\pi}{3}R_0^3 A \rightarrow$

$$R_0 = \frac{R}{\sqrt[3]{A}} = \sqrt[3]{\frac{3}{4\pi\rho_T}} \approx 1.12\ \text{fm}.$$

- in fair agreement with "c" [*previous slide*] and with the slope of the fig.:

$R_0^{exp} = 1.23$ fm.



for light nuclei, the model is NOT valid: do NOT plot them.

e-N scattering: higher energy

Probing smaller space scales requires larger energies, both in the initial and final state [*today experiments work at the TeV scale → ~10^{-18} m = 10^{-3} fm*].

High-energy + q.m. corrections to the Rutherford formula [1st already discussed]:

- consider the electron spin [*Rutherford had only bosons !!!*];
- include the target recoil in the Mott cross section [Perkins-1971, 197];
- use 4-vectors p and p′ to describe the scattering [instead of $\vec{p}$ and $\vec{p}'$]:

$$q^2 = (p-p')^2 = 2m^2 - 2\left(EE' - |\vec{p}||\vec{p}'|\cos\theta\right)$$
$$\approx -4EE'\sin^2(\theta/2);$$
$$Q^2 \equiv -q^2 \approx 4EE'\sin^2(\theta/2).$$

- for scattering eN, consider the magnetic moment of the nucleons, by introducing the parameter $\tau = Q^2/(4M^2)$ [*next slide*].

Description of the scattering

↓ no electron spin, no recoil, no magn. moment

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Rutherford}} = \frac{4Z^2\alpha^2 E'^2}{|\vec{q}|^4};$$

↓ + electron spin

$$\left[\frac{d\sigma}{d\Omega}\right]^*_{\text{Mott}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Rutherford}} \times \boxed{1-\beta^2\sin^2\frac{\theta}{2}};$$

$$\boxed{\approx \cos^2(\theta/2)}$$

↓ + recoil

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} = \left[\frac{d\sigma}{d\Omega}\right]^*_{\text{Mott}} \times \frac{E'}{E};$$

$$\boxed{"\tau"}$$

↓ + magn. moment

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{point, spin}\frac{1}{2}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} \times \left(1 + 2\boxed{\frac{Q^2}{4M^2}}\tan^2\frac{\theta}{2}\right).$$

e-N scattering: magnetic moments

For particles of mass m, charge e:

- point-like,
- spin ½;

the Dirac equation assigns an intrinsic magnetic dipole moment

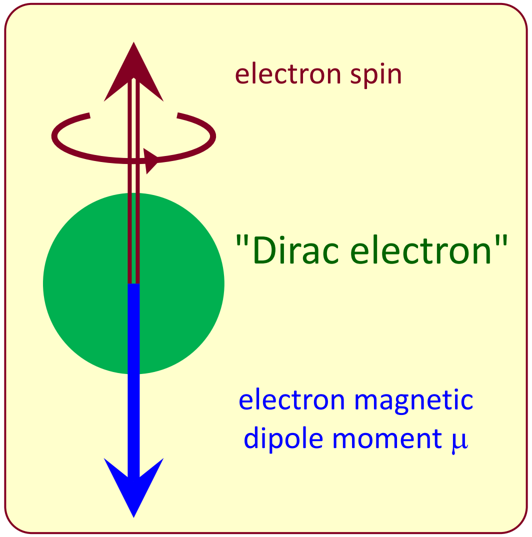
$\mu_C = g\, e\, \hbar\, /\, (4\, m)$;

g = "gyromagnetic ratio" = 2;

- an ideal "Dirac-electron" has a magnetic dipole moment

$\mu_e = e\hbar/(2m_e) \approx 5.79 \times 10^{-5}$ eV/T;

- the first measurements roughly confirmed this value.
- for neutral particles (neutron ?) $\mu_N = 0$;
- this effect adds to the cross-section a term, corresponding to the "spin flip" probability, proportional to [Povh § 6.1]:
  - $\sin^2(\theta/2)$ [*cfr. the "Mott* factor"*];
  - $1/\cos^2(\theta/2)$ (to remove the non-flip dependence);
  - $\mu_N{}^2$ ($\propto 1/M^2$);
  - Q^2 (mag field induced by the e)^2;
  - $$\left[\frac{d\sigma}{d\Omega}\right]_{\substack{\text{point,}\\ \text{spin½}}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} \times \left(1 + 2\frac{Q^2}{4M^2}\tan^2\frac{\theta}{2}\right).$$
- Therefore the spin-flip is particularly relevant for large Q^2 and large θ.



e-N scattering: anomalous magnetic moments

In the nuclei and nucleons sector the experiments measured the following quantities :

☺ nuclear magnetism is a combination of the intrinsic magnetic moments of the nucleons and their relative orbital motions;

☺ all nuclei with Z=even and N=even have $\mu_{nuclei} = 0$;

➢ define for the nucleons (proton and neutron) the Dirac value

$$\mu_N = e\hbar/(4m_N) \approx 3.1525 \times 10^{-14} \text{ MeV/T};$$

➢ if p and n were ideal Dirac particles, they should have

$$\mu_p = 2\mu_N, \qquad \mu_n = 0,$$

i.e. in conventional notation

$$g_p/2 = \mu_p/\mu_N = 1, \qquad g_n/2 = 0;$$

☹ instead, experiments found *anomalies*

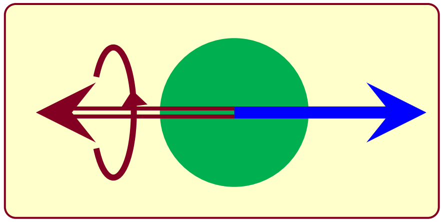
$$g_p/2 = +(2.7928473508 \pm 0.000000085),$$

$$g_n/2 = -(1.91304273 \pm 0.00000045);$$

☺ therefore, there are other effects which contribute to the magnetic moments, i.e. p and n are NOT ideal spin-½ point-like Dirac particles;

☺ [maybe] they are NOT point-like;

☺ in this case, their "g" is due to their (possibly complicated) internal structure, in analogy with the nuclear case.



e-N scattering: Rosenbluth cross-section

In the eN scattering, the main contribution is from single photon exchange [see fig.].

The **eeγ^* vertex** is well under control, with three point-like, well-understood particles.

Instead, the **NN'γ^* vertex** is the unknown, due to the internal structure of the proton.

Strategy : assume a simpler process (N = Dirac fermion), compare it with exp., then modify the theory, inserting parameters which model the nucleon structure.

Take also into account the spin and magnetic moment, both of the electron and the nucleon.

"Generalize" the cross section by defining the **Rosenbluth cross-section**, function of TWO form factors, both dependent on Q^2:

- $G_e(Q^2)$ for the electric part (no spin-flip);
- $G_M(Q^2)$ for the magnetic one (spin-flip).

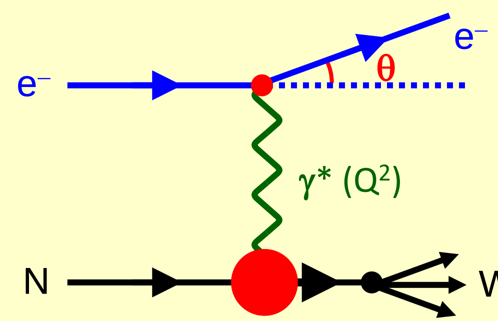
[formerly : $G_e(Q^2) = \mathcal{F}(Q^2)$, no G_M].

For a charged Dirac fermion f_D, proton, neutron :

- f_D : $G_E^f(\text{any } Q^2) = 1$, $G_M^f(\text{any } Q^2) = 1$;
- p : $G_E^p(Q^2 = 0) = 1$, $G_M^p(Q^2=0) \approx 2.79$;
- n : $G_E^n(Q^2 = 0) = 0$, $G_M^n(Q^2=0) \approx -1.91$.

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Rosenbluth}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} \times \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right);$$

$$\tau = \frac{Q^2}{4M^2}; \qquad G_E = G_E\left(Q^2\right); \qquad G_M = G_M\left(Q^2\right).$$



e-N scattering: remarks on $\sigma_{\text{Rosenbluth}}$

A non-exhaustive personal classification(*) of "physics formulae":

1. "principles" [$\vec{F} = m\vec{a}$] – They require the a-priori knowledge of all entities involved; not direct empirical laws;
2. "natural laws" [the gravitational/Hooke law] – (semi-)empirical descriptions of the behavior of the Nature;
3. "positions" [$K = ½mv^2$] – They define a new entity, using other well-known entities;
4. "theorems" [the Gauss law] – Relations among well-known entities, math derived from other laws;
5. … other types (???) …

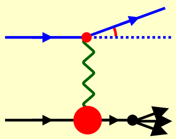
(*) This is too simplistic [I know], e.g. one of the Gauss and Coulomb law may be chosen as a law and the other as a derived theorem; the same for the positions; but in a given approach, the above classification is approximately correct.

The "Rosenbluth formula" is another type of math-logical relation:

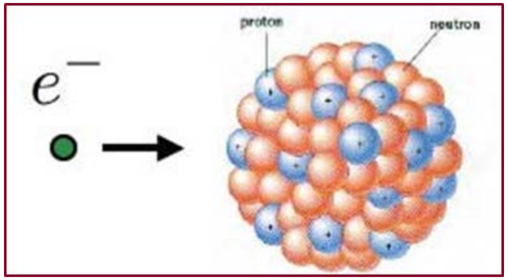
- it is a model, which includes some constraints (e.g. the θ dependence cannot be modified);
- but it is "open" (e.g. G_E and G_M depends on the unknown Nucleon structure);
- it contains in-se no full predictive power;
- but it is a powerful working tool to study the phenomena and incorporate new knowledge in a (quasi-)formal theory.

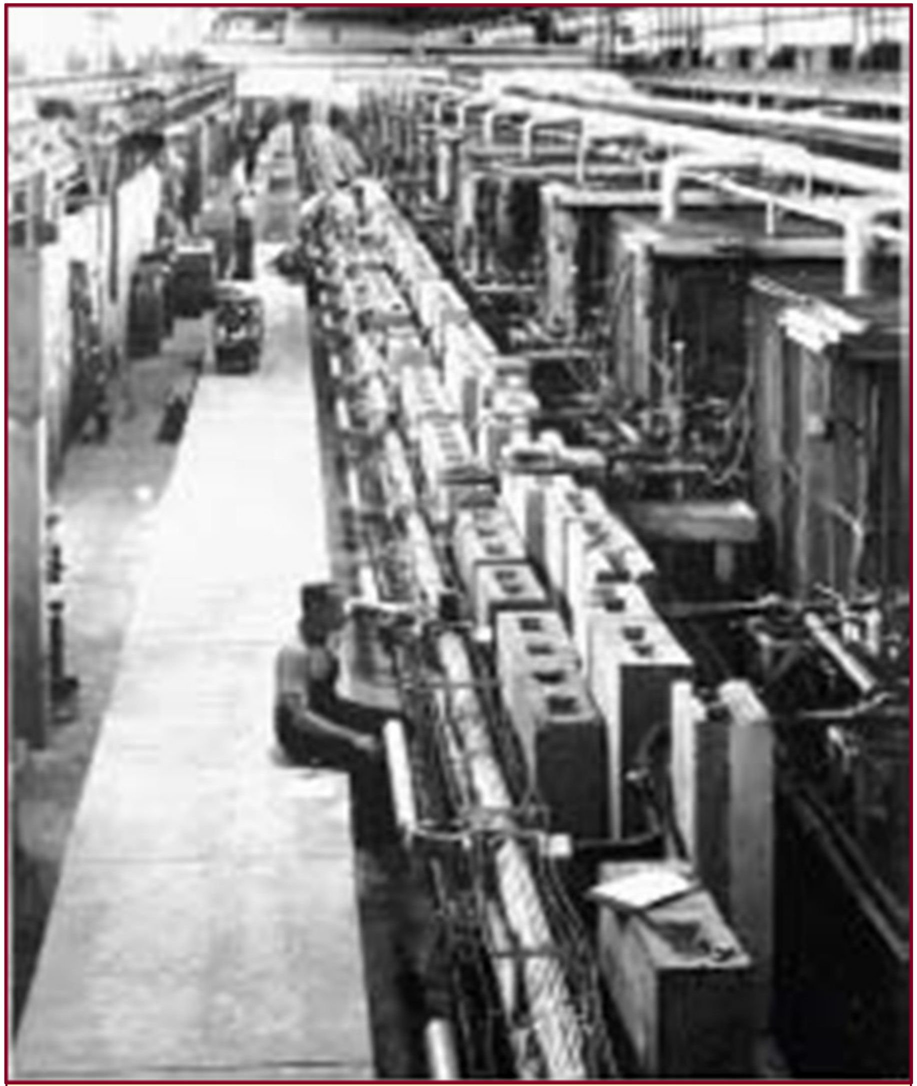
A "frontier" approach, quite common in modern research, which requires some care by the users/students.

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Rosenbluth}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} \times \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right).$$



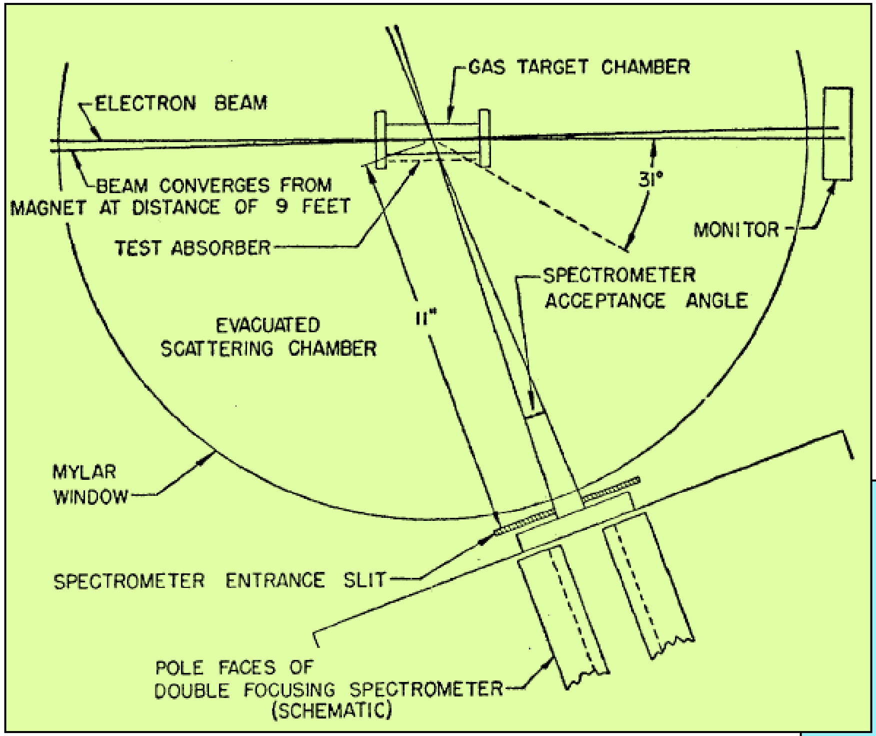
Proton structure: Mark 3 Linac





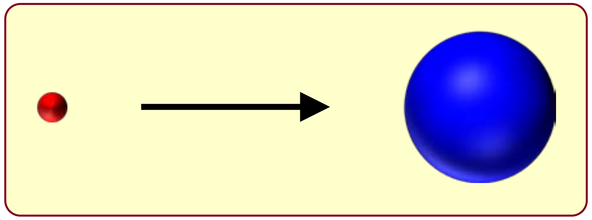
Mark 3 electron Linac – Stanford University – 1953

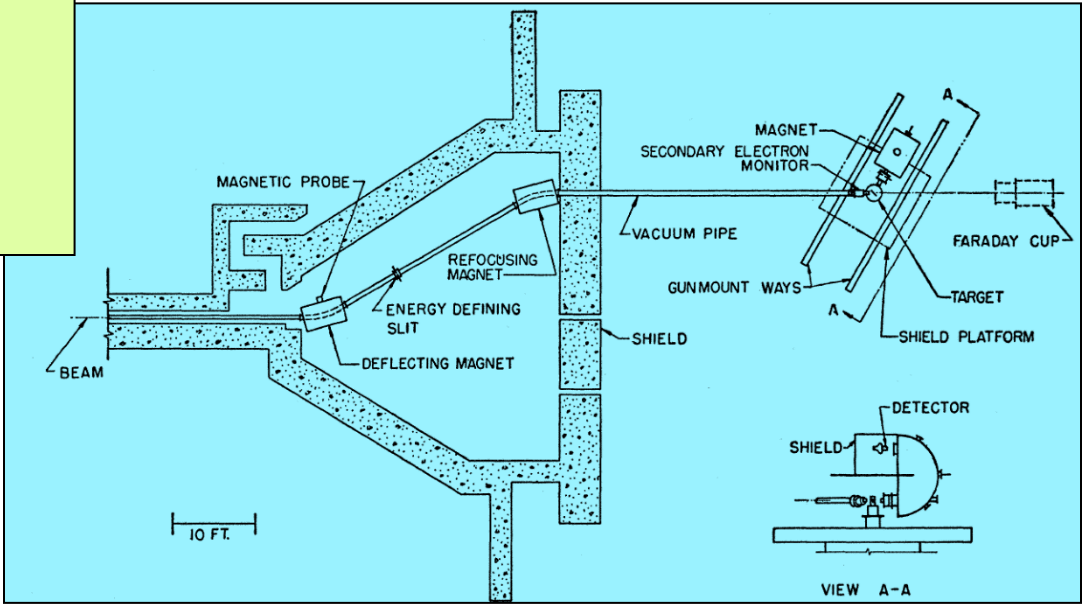
Proton structure: setup



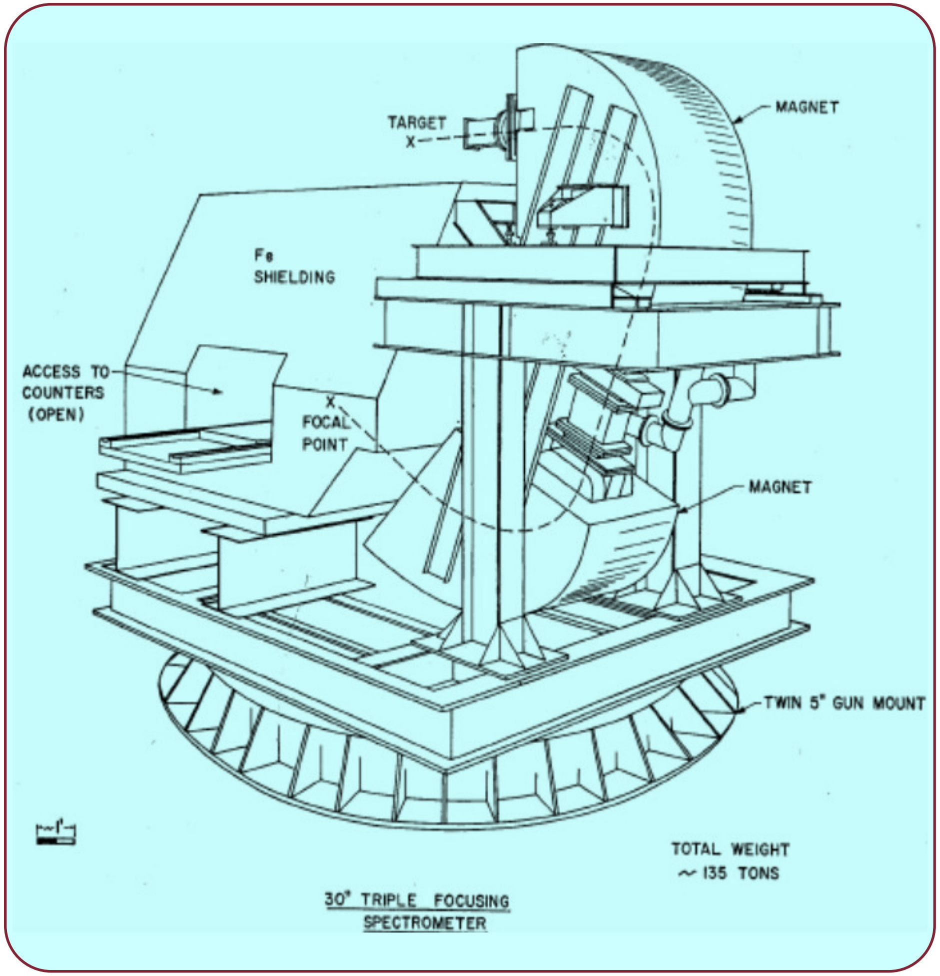
Stanford - 1956

Robert Hofstadter



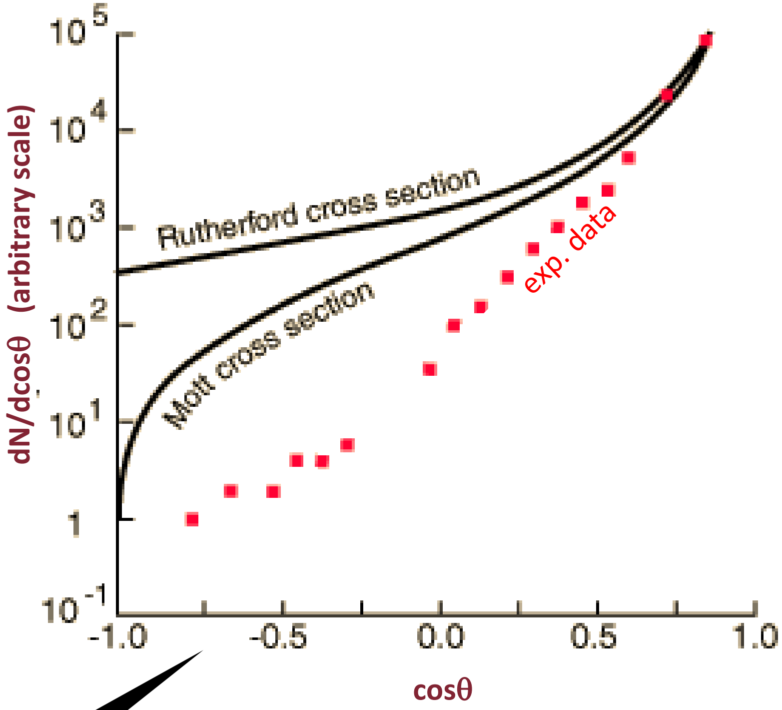


Proton structure: Mark 3 detector



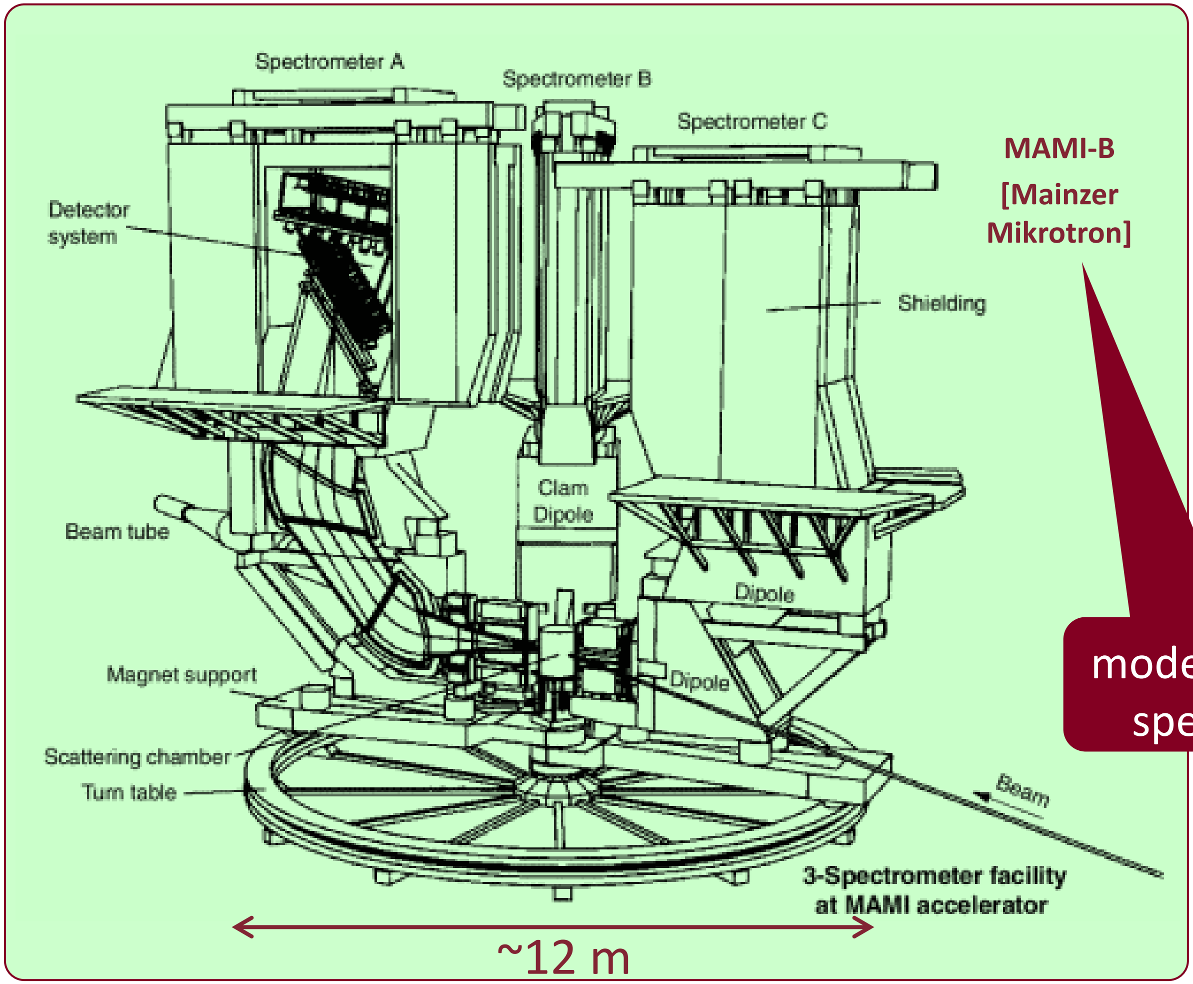
Hofstadter et al., Phys. Rev. 92, 978 (1953)

p(e⁻) = 125 MeV



A summary of Hofstadter experiments, see later

Proton structure: MAMI-B



MAMI-B [Mainzer Mikrotron]

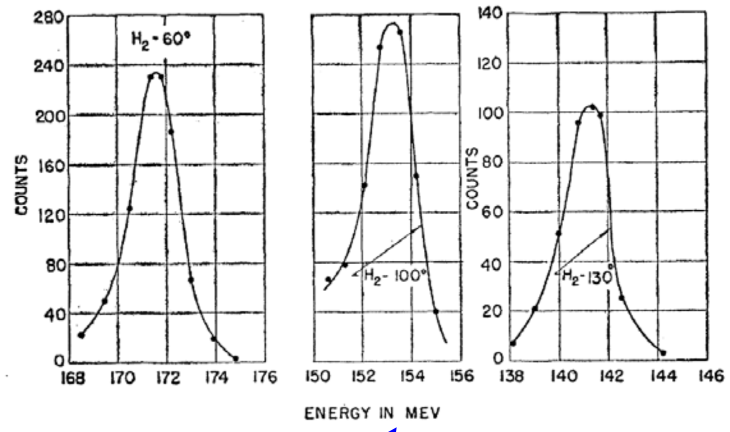
modern magnetic spectrometer

Proton structure: quality check

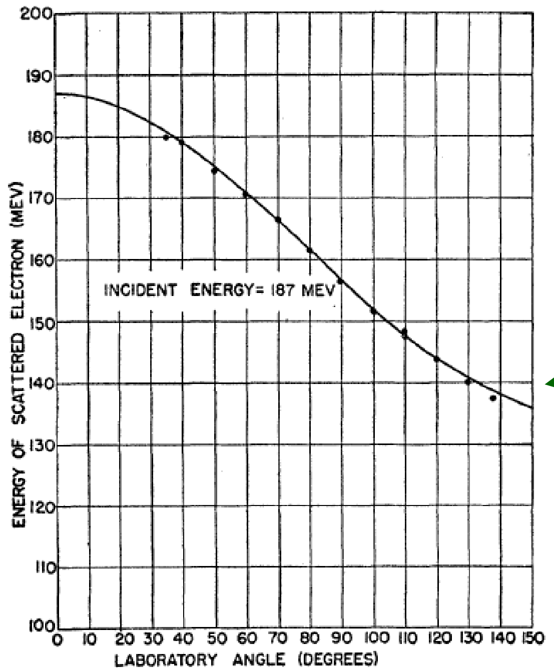
In 1956 the Hofstadter spectrometer measured the elastic $ep \to ep$. It measured θ in the range 35°-138°, and therefore Q^2, using the relations :

$$E' = \frac{E}{1+E(1-\cos\theta)/M};$$

$$Q^2 = 2EE'(1-\cos\theta).$$



Plot E' for E = 185 MeV at fixed θ (60°, 100°, 130°) [*in a perfect experiment, expect* δ_{Dirac}].



Show the plot $\theta = \theta(E')$.

Result:

Kinematics ok. Experiment under control. Study the dynamics.

Proton structure: results

Show the measured cross section:

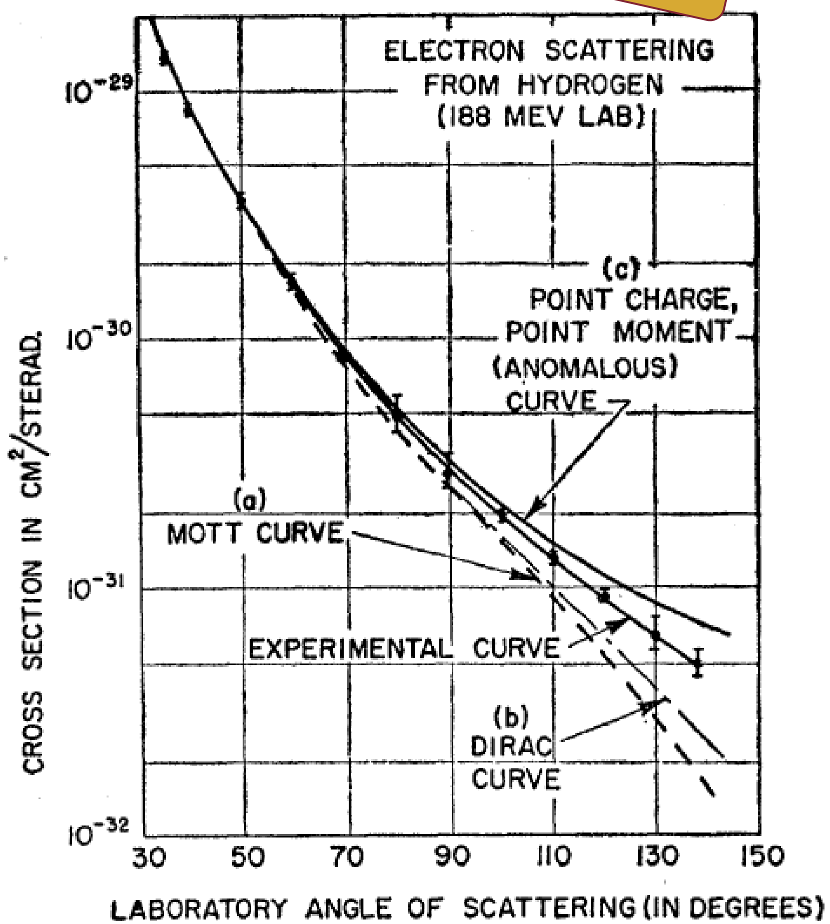
- at small θ, Mott (a), Dirac (b), Rosenbluth with fixed G_E,G_M (c) and data ("exp. curve") all agree;
- however, for large θ (i.e. large Q^2, small distance), the data do NOT agree with ANY theoretical prediction : they are larger than (a) and (b), but smaller than (c);
- the disagreement with (a) and (b) was foreseen (proton $g_p \neq 2$);
- the one with (c) is more interesting : it shows a dependence on Q^2 (i.e. on scale) $\rightarrow$ the proton is NOT point-like;
- Hofstadter measured ($r_{rms} \equiv \sqrt{<r^2>}$, see) :

$$r^p_{rms} = (0.77 \pm 0.10) \times 10^{-15}\text{ m};$$

$$r^\alpha_{rms} = (1.61 \pm 0.03) \times 10^{-15}\text{ m}.$$

… and received the 1961 Nobel Prize in Physics.

Rev.Mod.Phys.,28, 214.



Proton structure: $G_{E,M}^{p,n}$ vs Q^2

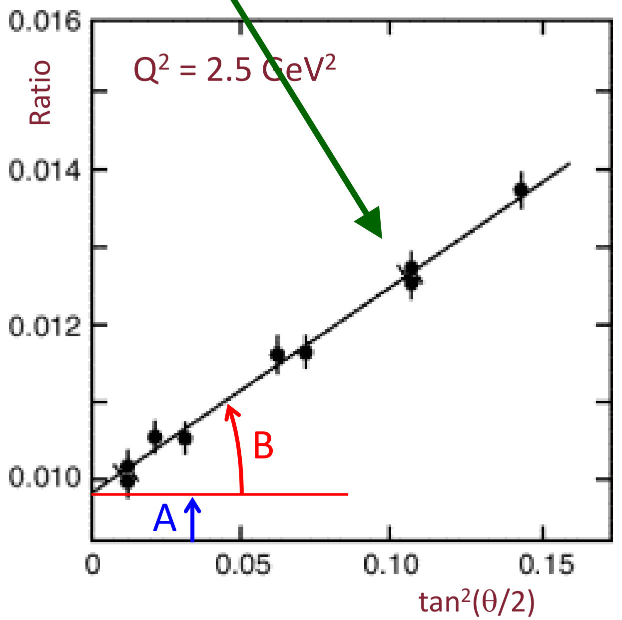
Write the Rosenbluth formula, at fixed Q^2, :

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Rosenbluth}} \Big/ \left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right).$$

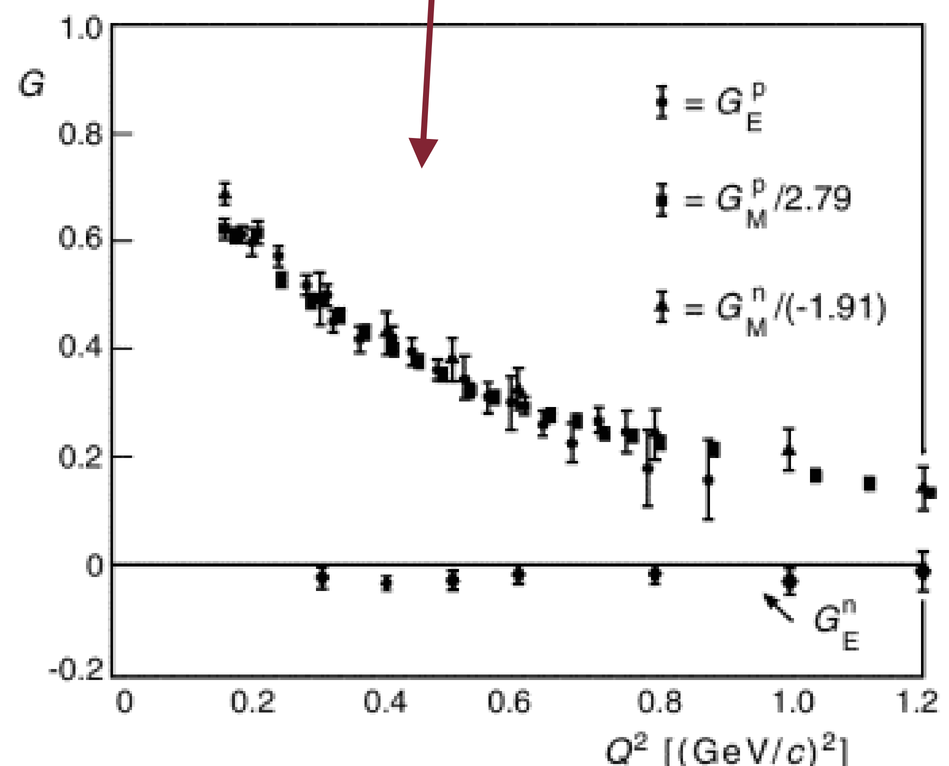
→ Ratio(E, θ, fixed Q^2) = A + B $\tan^2(\theta/2)$;

→ measure (A, B at fixed Q^2) vs $\tan^2(\theta/2)$;

→ get G_E^p, G_M^p, (G_E^n, G_M^n) at fixed Q^2 (example shown)



By repeating it at many Q^2, the full dependence can be measured (SLAC, '60s).



Proton structure: $G^{p,n}_{E,M}$ - remarks

- The fig. shows that the electric and magnetic form factors tend to a "universal" function of Q^2, with a **dipolar** shape :

$$G^p_E(Q^2) \approx \frac{G^p_M(Q^2)}{2.79} \approx \frac{G^n_M(Q^2)}{-1.91} \approx G(Q^2) =$$

$$= \left(1 + \frac{Q^2}{A^2}\right)^{-2}; \qquad A^2 = 0.71\ \mathrm{GeV}^2$$

- From the curve, it is possible to derive the function $\rho(r)$, at least where the 3- and 4-momentum coincide, i.e. at small Q^2. It turns out :

  $\rho(r) \approx \rho_0\, e^{-ar}$, $a \approx 4.27\ \mathrm{fm}^{-1}$.

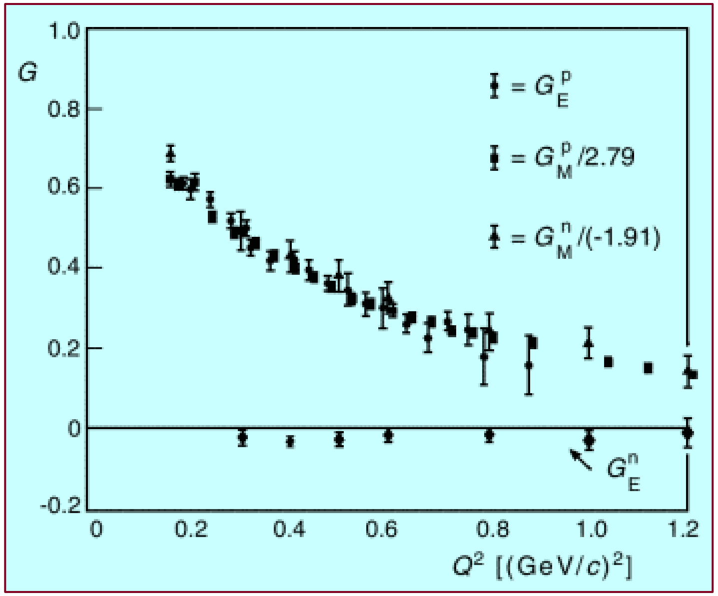
- The nucleons do NOT look like ~~point-like particles~~, nor ~~homogeneous spheres~~, but like diffused non-homogeneous systems.

- From the values at $Q^2=0$:

$$\langle r^2 \rangle_{dipole} = -6\hbar^2 \left.\frac{dG(q^2)}{dq^2}\right|_{q^2=0} =$$

$$= \frac{12}{a^2} \approx 0.66\ \mathrm{fm}^2;$$

$$\sqrt{\langle r^2 \rangle_{dipole}} \approx 0.81\ \mathrm{fm}.$$



Proton structure: comments

$$\left[\frac{d\sigma}{d\Omega}\right]_{\substack{\text{Rosen}\\\text{bluth}}} \Bigg/ \left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} =$$

$$= \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2} \right); \quad \left[\tau = \frac{Q^2}{4M^2} \right].$$

therefore $\displaystyle \lim_{Q^2\to 0} \left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{Rosen}\\\text{bluth}}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}.$

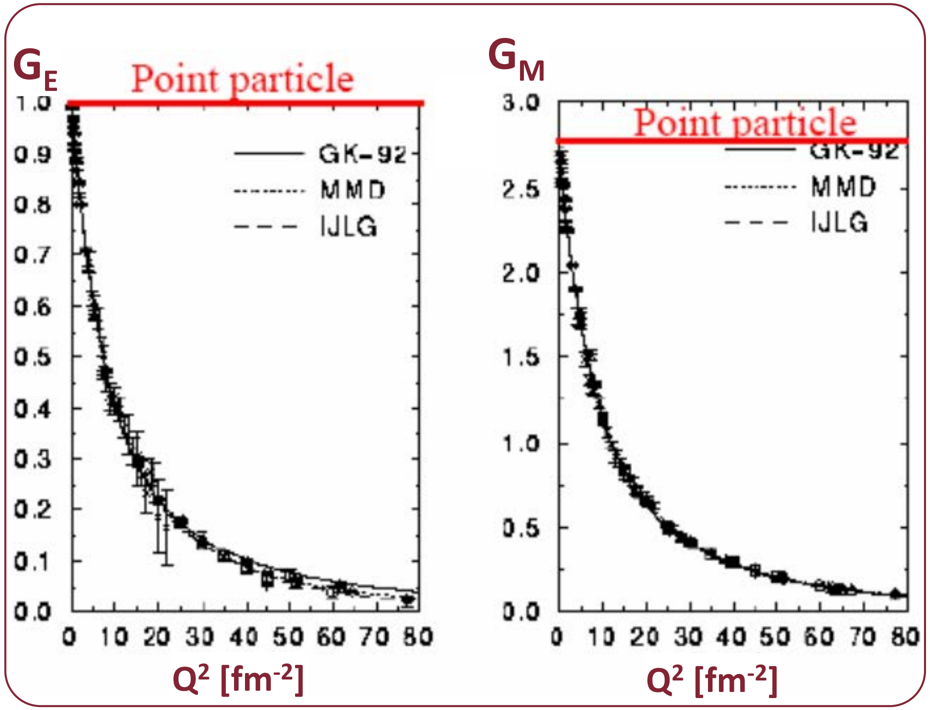
The form factors of the nucleons show three different ranges :

1. $Q^2 \ll m_p^2$: τ small, G_E dominates the cross section; in this range we measure the average radius of the electric charge : $\langle r_E \rangle = 0.85 \pm 0.02$ fm;
2. $0.02 \leq Q^2 \leq 3$ GeV^2 : G_E and G_M are equally important;
3. $Q^2 > 3$ GeV^2 : G_M dominates.

Notice also that, if the proton were point-like, one would find :

$G_E^p(Q^2) = G_M^p(Q^2) = 1$, <u>independent of Q^2</u>

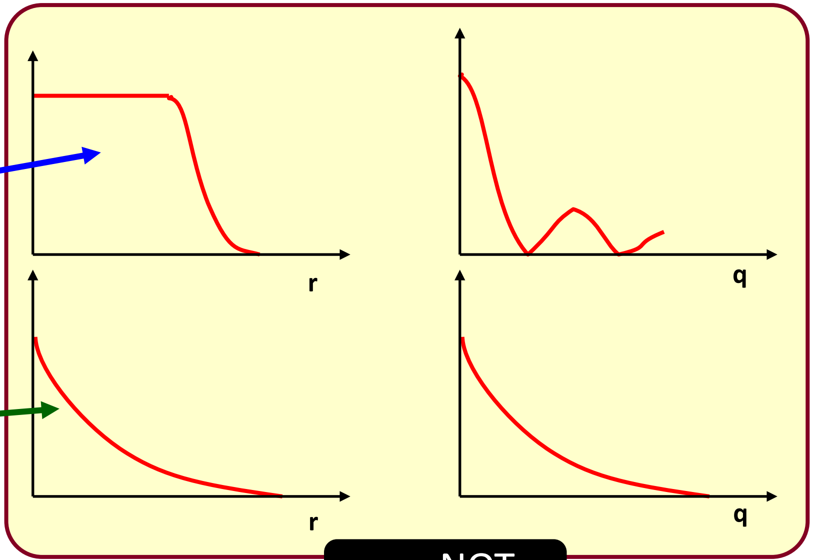
[*and in addition would not understand why "2.79"*].



Proton structure: interpretation

Differences between nuclei and nucleons :

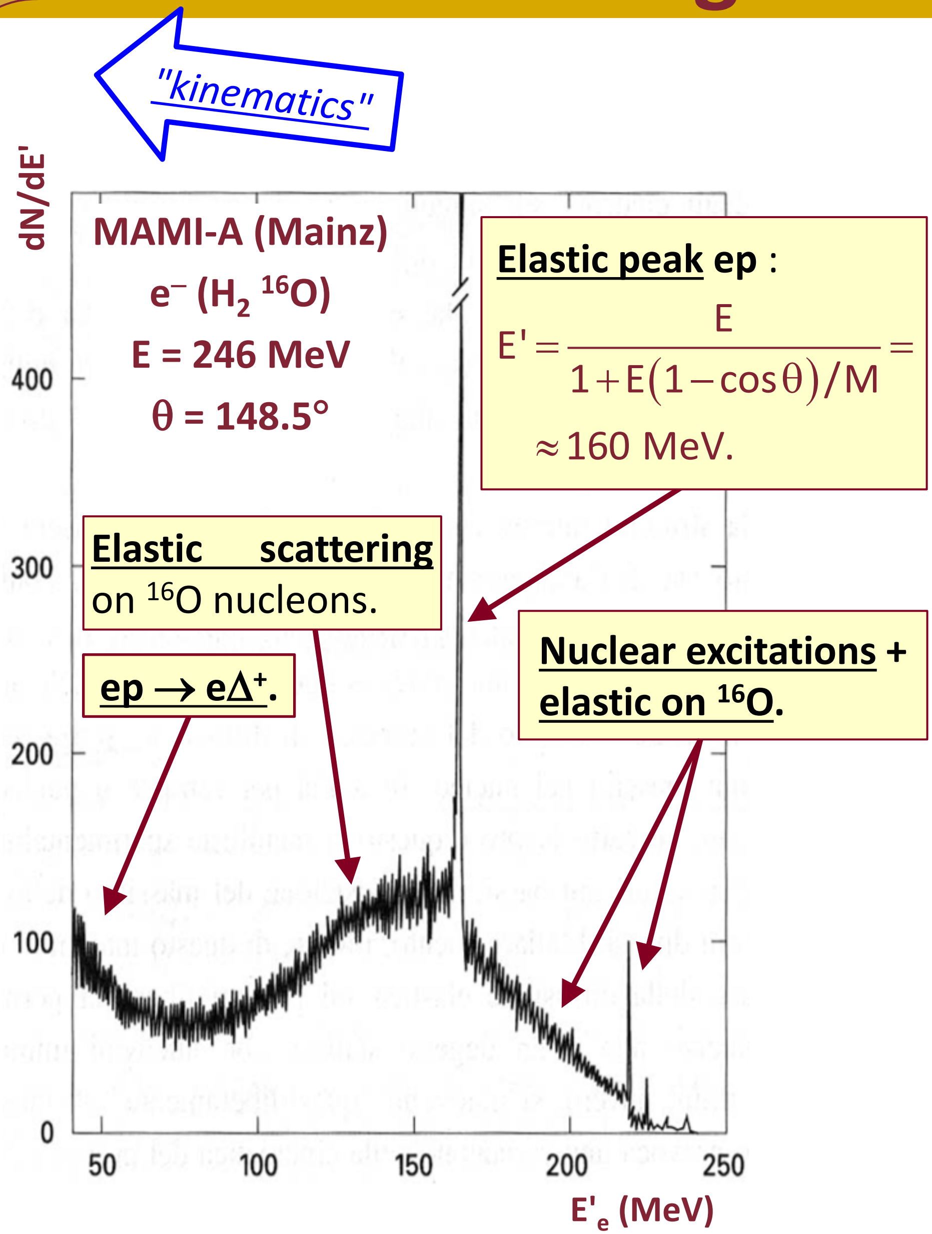
1. nuclei exhibit diffraction maxima/minima; this fact corresponds to charge distributions similar to homogeneous spheres with thin skin;
2. nucleons have diffused, dipolarly distributed form factors → exp. charge;
3. at this level, it is unclear whether the nucleons have substructure(s) → need experiments at smaller value of distances (i.e. larger values of Q^2);
4. [*hope that*] the structure of the nucleons in the elastic scattering, described by the Rosenbluth formula, is an average with insufficient resolution;
5. at higher Q^2, one can expect a wider variety of phenomena :



r, q NOT same scale

a. elastic scattering : ep → ep;
b. excitation : ep → e "p*" (e.g. ep → $e\Delta^+$, $\Delta^+ \to p\pi^0$);
c. new states : ep → eX^+ (X^+ = system of many particles).

higher Q^2: H_2O



Send 246 MeV electrons → water vapor.

The scattering shows a complex distribution, with different phenomena in the same plot. At fixed θ of the electron in the final state, with increasing E' :

- e p → e Δ^+ (excitation of p from H);
- e p/n → e p/n (elastic on ^{16}O nucleons);
- e p → e p (elastic on H, E'≈160 MeV);
- e p → e X^+ (nuclear excitations);
- e ^{16}O → e ^{16}O (nucl. exc. / elastic)

The distribution depends also on the electron energy E and the final state angle θ.

[Problem: the Δ^+ has m ≈ 1230 MeV, Γ ≈ 120 MeV. In the plot only the tail of ep→eΔ^+ is shown. "Compute" the effect of the Breit-Wigner in mass in the E' variable. Is it sufficient to predict the E' plot ?]

higher Q^2: He^4, $\theta = 45°$

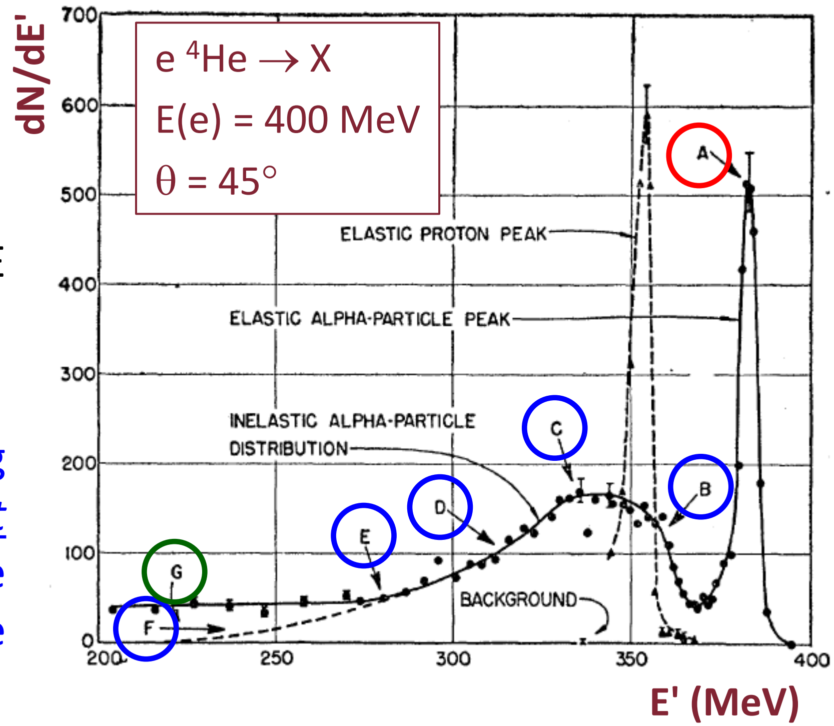
Another of these experiments (Hofstadter 1956, see fig.). Observe :

A. the elastic scattering e ^{4}He [*expected*];

-- the elastic peak for ep → ep at the same E and θ, shown for comparison [*no problem*];

BCDEF. the elastic scattering ep / en (p/n acting like free particles) [*maybe unexpected, but understandable*]; notice the peak width, due to the Fermi motion of nucleons inside the nucleus;

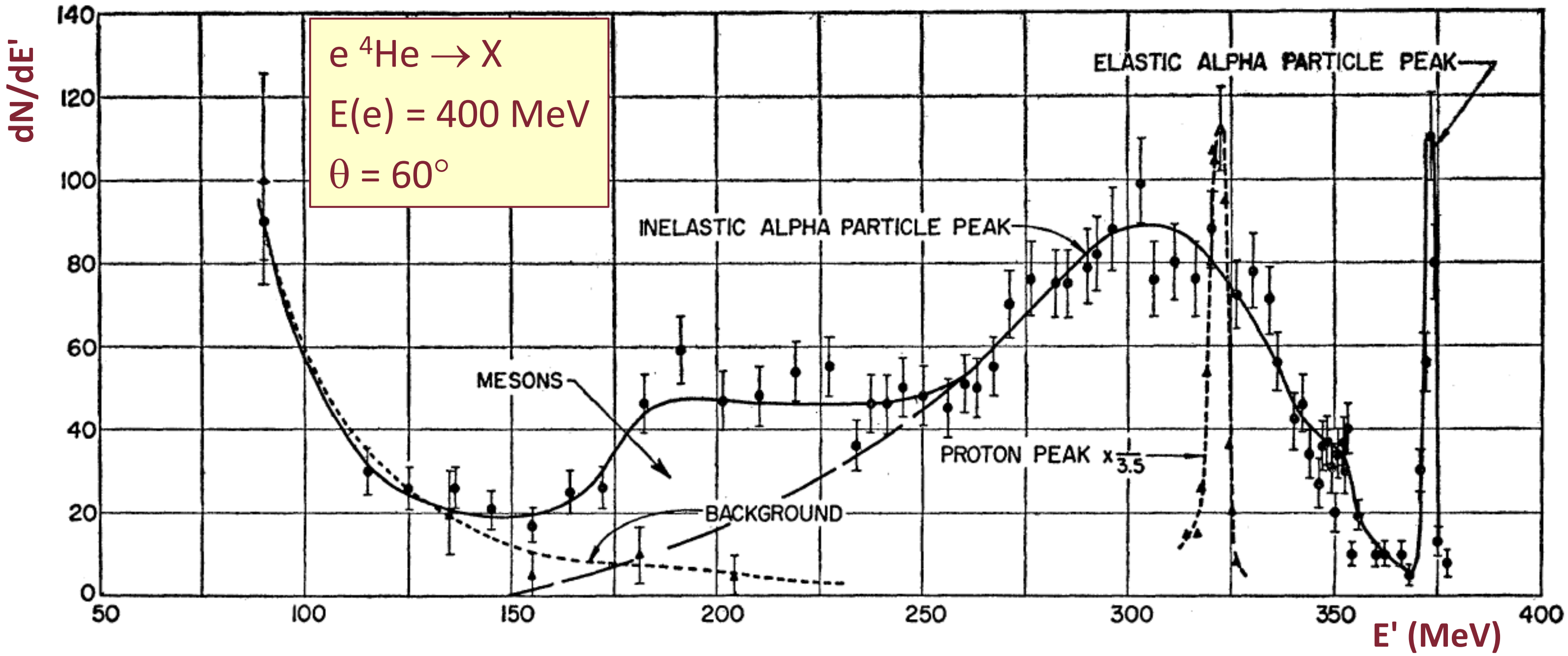
G. the production of π^- (i.e. of Δ's), which enhances the cross section (otherwise F.); notice : smaller E' → larger energy transfer [*the new entry in the game*].



$$E' = \frac{E}{1 + E(1-\cos\theta)/M}$$

$$M\uparrow \Rightarrow E/M\downarrow \Rightarrow E'\uparrow$$

higher Q^2: He^4, θ = 60°



Same as before, but θ = 60°, i.e. larger Q^2 [$Q^2 \approx 4EE'\sin^2(\theta/2)$]. Notice :

- smaller elastic peak, both for (e^- ^{4}He) and (e^-p);
- wider ep/en (p/n inside ^{4}He) peak;
- (roughly) constant π production (seems independent from Q^2, as expected for point-like (?) particles;

Possible conclusions [*possibly wrong*] :

- everything under control for elastic and quasi-elastic data;
- the high-Q^2 part shows no evidence for sub-structures;
- maybe Q^2 is still too small (or maybe there are no substructures … !?);

→ go to even higher Q^2 !!!

higher Q^2: summary

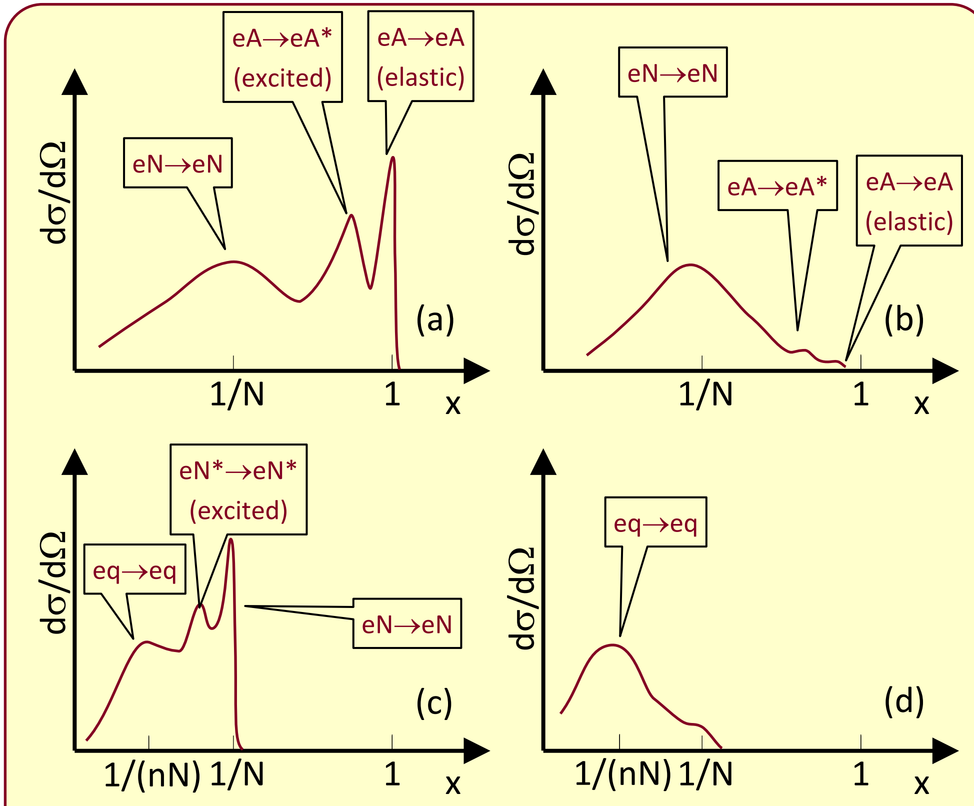
Follow [BJ 444] to understand the dependence of $d\sigma/d\Omega$ on Q^2 for electron on a nucleus A:

- choose the adimensional variable $x = Q^2/(2M\nu)$;
- from (a) to (d), Q^2 increases;
- "N" includes p(roton) and n(eutron);
- "q" = *hypothetical component* of the nucleons (maybe quarks, but we are far from conclusive argument).

a) At small Q^2, there are both scatterings with A and N (see);

b) increasing Q^2, the eA scattering disappears, while the eN scattering stays constant;

c) increasing Q^2, the constituents (if any) appears as eq $\to$ eq;



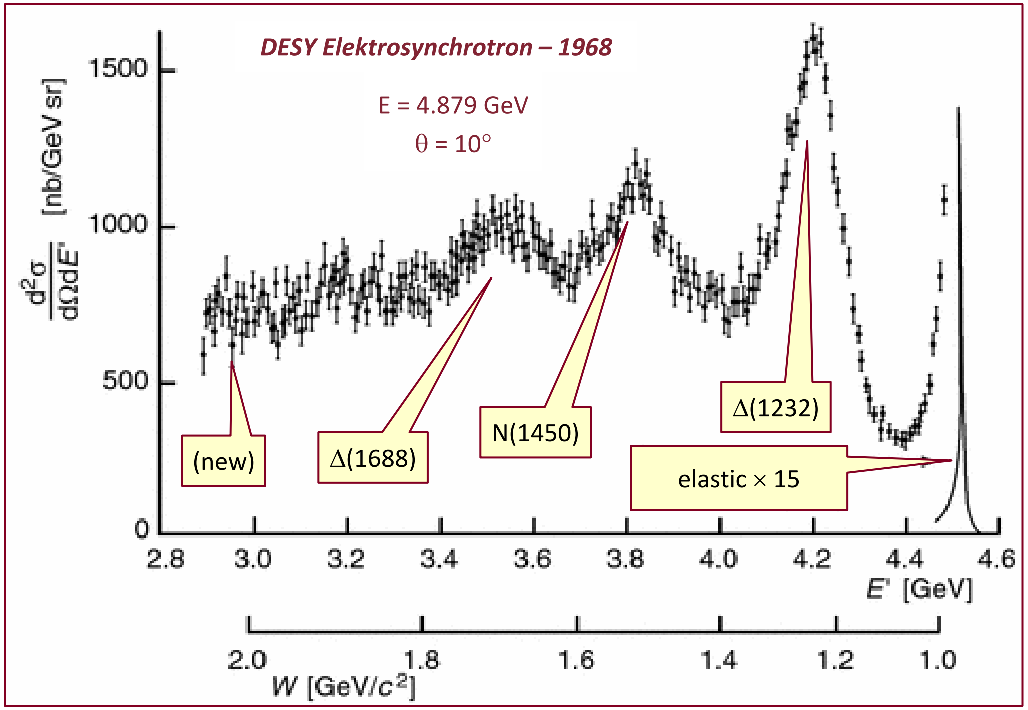
[just a sketch, not a reproduction of real experiments]

d) finally, at very large Q^2, the most important process is eq $\to$ eq (with all the possible inelastic companions).

higher Q^2: constituents show up

Scattering ep → eX (DESY 1968) :

- Electron energy ≈ 5 GeV (higher than SLAC);
- resonances (R) production ep → eR clearly visible;
- new region at small E' (= high W);
- in this "new" region :
  - continuum (NO peaks);
  - rich production of hadrons;
  - NO new particles, only (p n π's); i.e. the proton breaks, but (different from the nucleus) NO constituent appears;
  - the constituents, if any, do not show up as free particles;

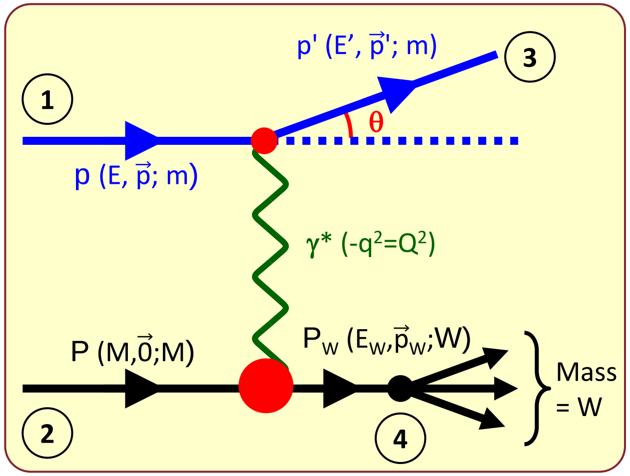


→ **Do quarks exist ???**
are they confined ??? why ???

[NB in 1968 color was proposed but not really understood, QCD did not exist]

Deep inelastic scattering : functions $W_{1,2}$

The usual parameterization of the cross section in the DIS region is the formula (Z=1 for a proton) :



$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{DIS} = \left[\frac{d\sigma}{d\Omega}\right]^*_{Mott}\left[W_2(Q^2,\nu)+2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right] =$$

$$= \frac{4Z^2\alpha^2(\hbar c)^2E'^2}{|qc|^4}\cos^2\frac{\theta}{2}\times\left[W_2(Q^2,\nu)+2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right] =$$

$$= \frac{4\alpha^2E'^2}{Q^4}\times\left[W_2(Q^2,\nu)\cos^2\frac{\theta}{2}+2W_1(Q^2,\nu)\sin^2\frac{\theta}{2}\right].$$

Remarks :

- the inelastic cross section requires 2 final-state variables, e.g. θ and E'; other choices are equivalent; Q^2 and ν are L-invariant, so more convenient;
- W_1 and W_2 are the equivalent of G_E and G_M for DIS (with dimension 1/E, *see next slide*) : they are called <u>*structure functions*</u> [later they will be a sum of "PDF"];
- W_1 and W_2 reflect the structure of the particles; the formula is general, but contains little information until $W_{1,2}$ are explicitly measured (and/or computed from a deeper theory);
- the dynamics of the scattering depends on the structure of the target; W_1 and W_2 are the real "containers" of this information.

Deep inelastic scattering : $G_{E,M}$ vs $W_{1,2}$

Some algebra, quite boring, to show for the ep (Z=1, M_p):

- the explicit values of Mott and Rosenbluth cross-sections;
- the relation $G_{E,M}$ vs $W_{1,2}$.

Enjoy !!!

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} = \left[\frac{4\alpha^2 E'^2}{Q^4}\right]_{\text{Rutherford}} \left[\cos^2\frac{\theta}{2}\right]_{\to\text{Mott*}} \left[\frac{E'}{E}\right]_{\to\text{Mott}} = \frac{4\alpha^2 E'^3}{EQ^4}\cos^2\frac{\theta}{2};$$

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Rosenbluth}} = \left[\frac{4\alpha^2 E'^3}{EQ^4}\cos^2\frac{\theta}{2}\right]_{\text{Mott}} \left[\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]_{\to\text{Rosenbluth}};$$

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{\text{Rosenbluth}} = \frac{12\alpha^2 E'^2}{EQ^4}\left(\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right);$$

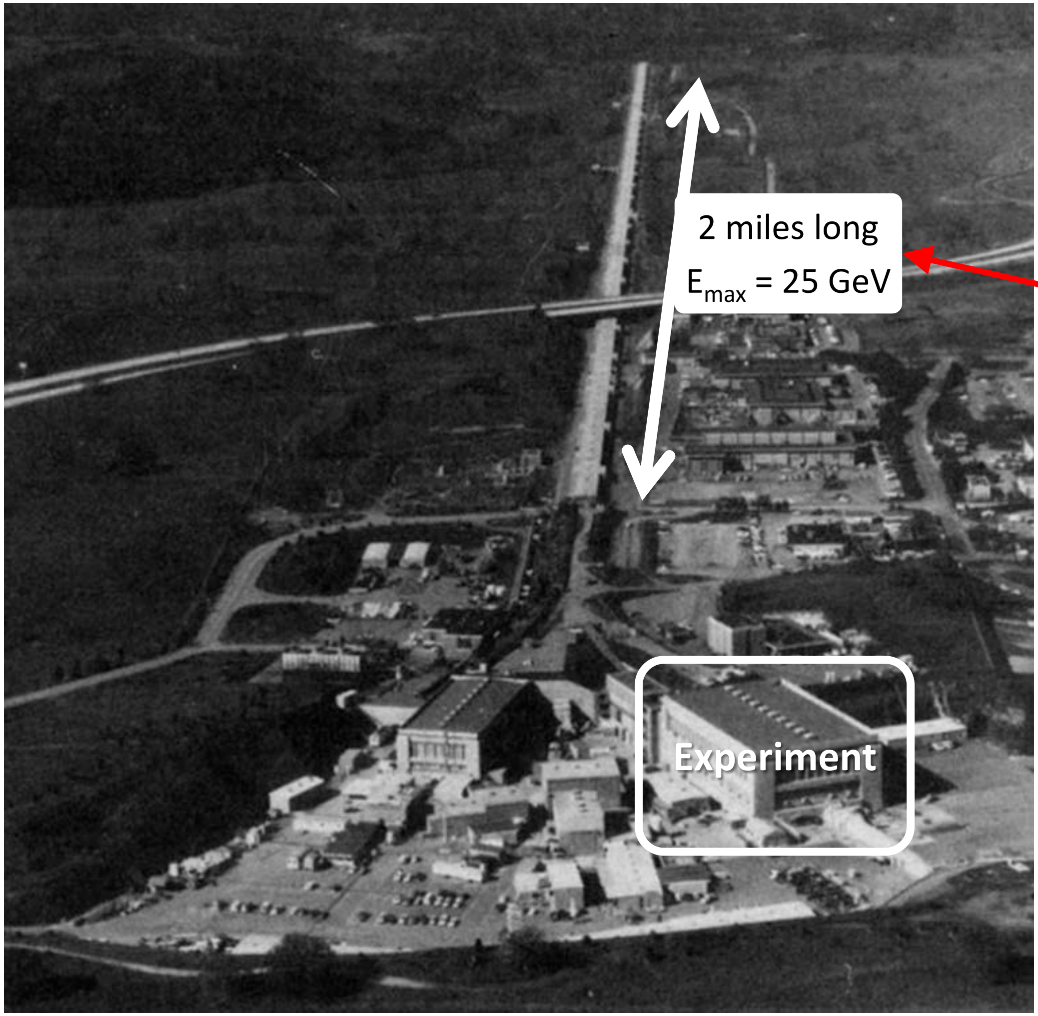
$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{\text{DIS}} = \frac{4\alpha^2 E'^2}{Q^4}\times\left[W_2\left(Q^2,\nu\right)\cos^2\frac{\theta}{2} + 2W_1\left(Q^2,\nu\right)\sin^2\frac{\theta}{2}\right];$$

$$W_1\left(Q^2,\nu\right) = \frac{3}{E}\tau G_M^2 = \frac{3Q^2}{4EM_p^2}G_M^2;$$

$$W_2\left(Q^2,\nu\right) = \frac{3}{E}\left(\frac{G_E^2+\tau G_M^2}{1+\tau}\right) = \frac{3}{E}\left(\frac{4M_p^2 G_E^2 + Q^2 G_M^2}{4M_p^2+Q^2}\right).$$

An interesting question. Do you understand why ?	Rutherford, Mott* and Mott $d\sigma/d\Omega$'s do NOT depend on the proton mass. Rosenbluth $d\sigma/d\Omega$ depends on τ ($Q^2/4M^2$) + any hidden dependence in $G_{E,M}$. $W_{1,2}$ have *NO* dependence: wait'n see.

Deep inelastic scattering : SLAC

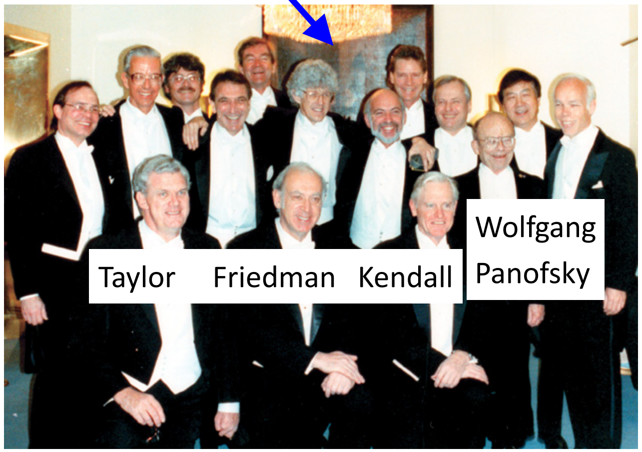


SLAC

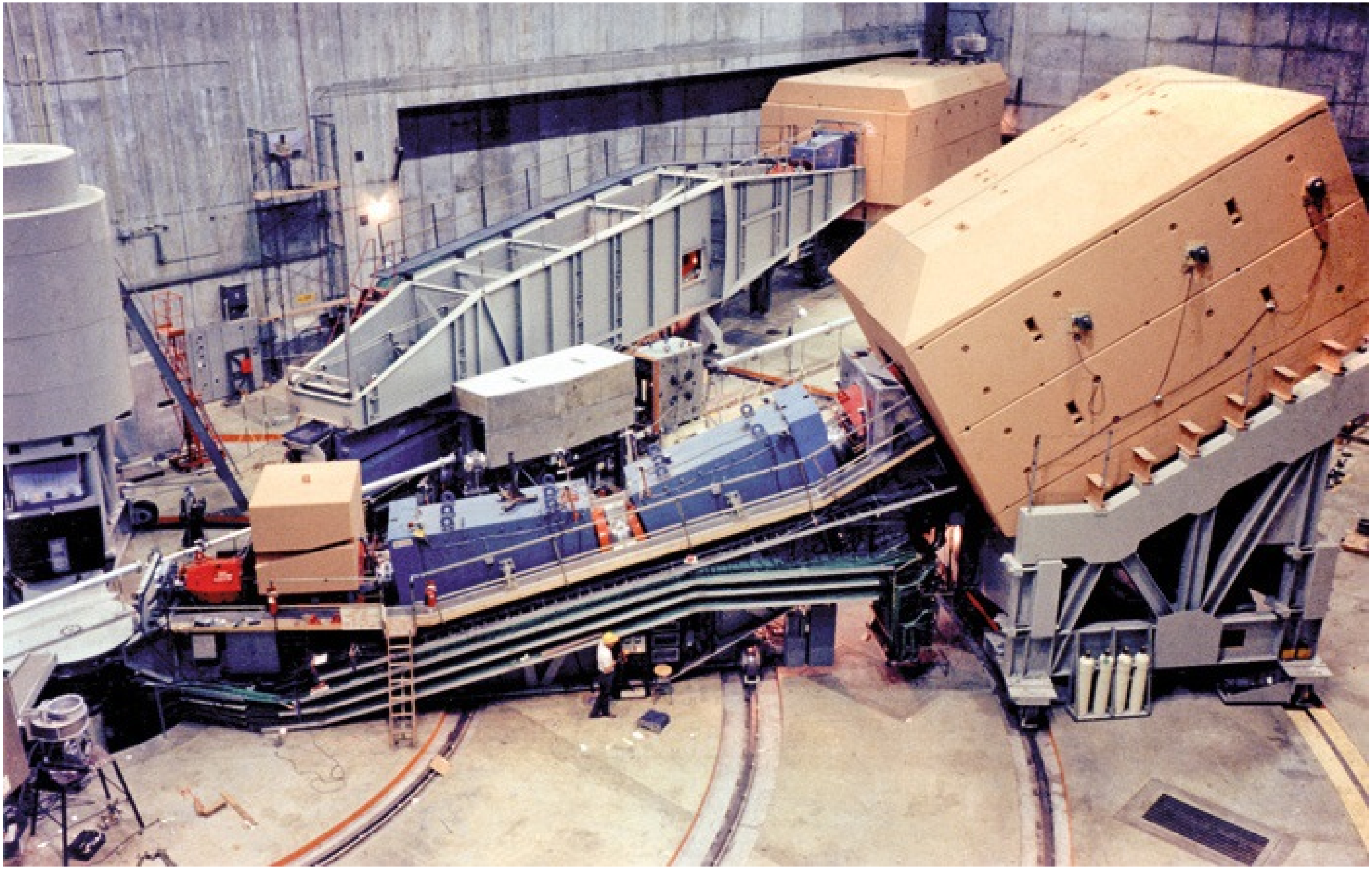
Stanford Linear Accelerator Center

the beginning of the story (1960)

… and this is NOT the end (1990)

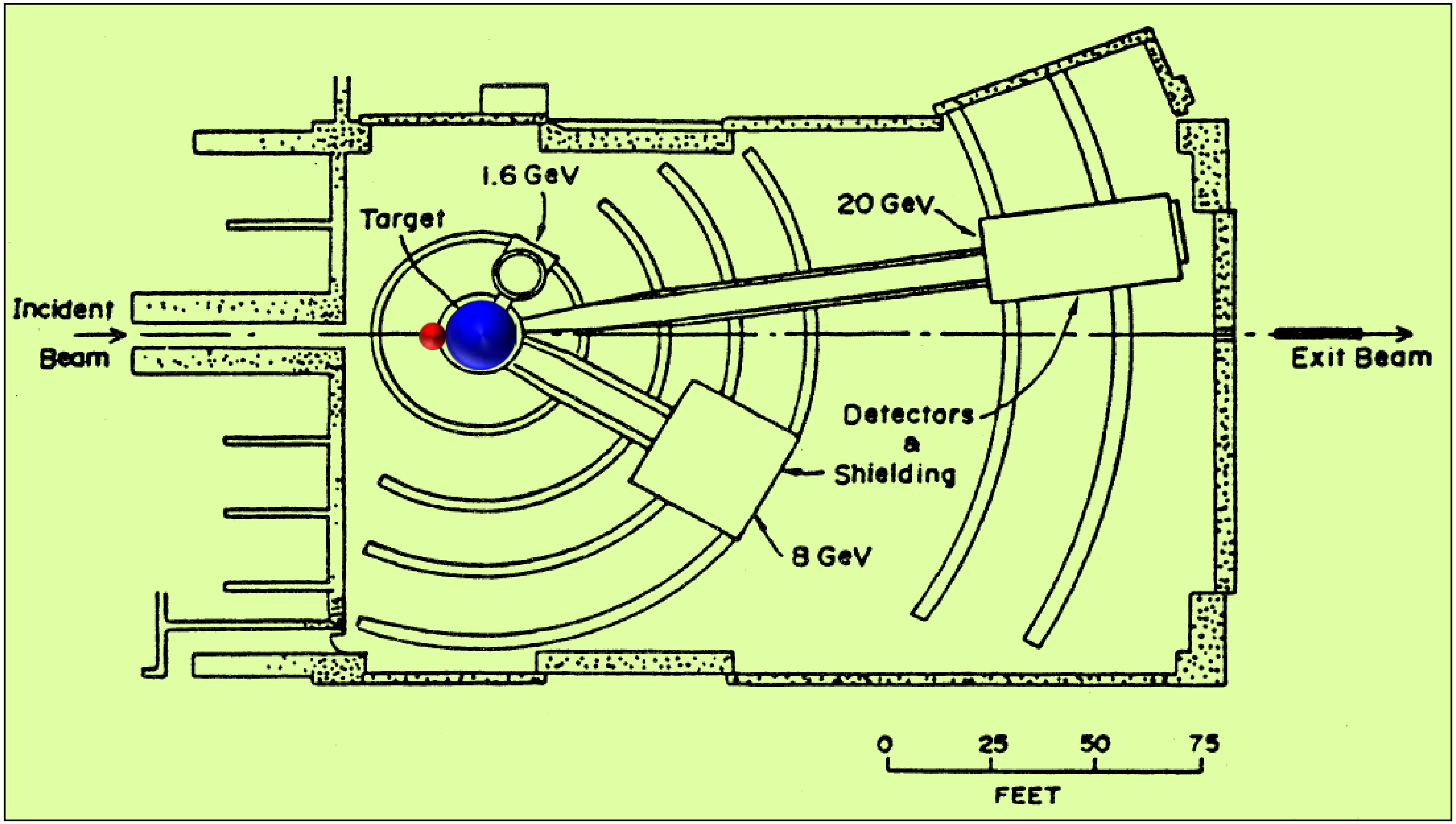


Deep inelastic scattering : SLAC experiment



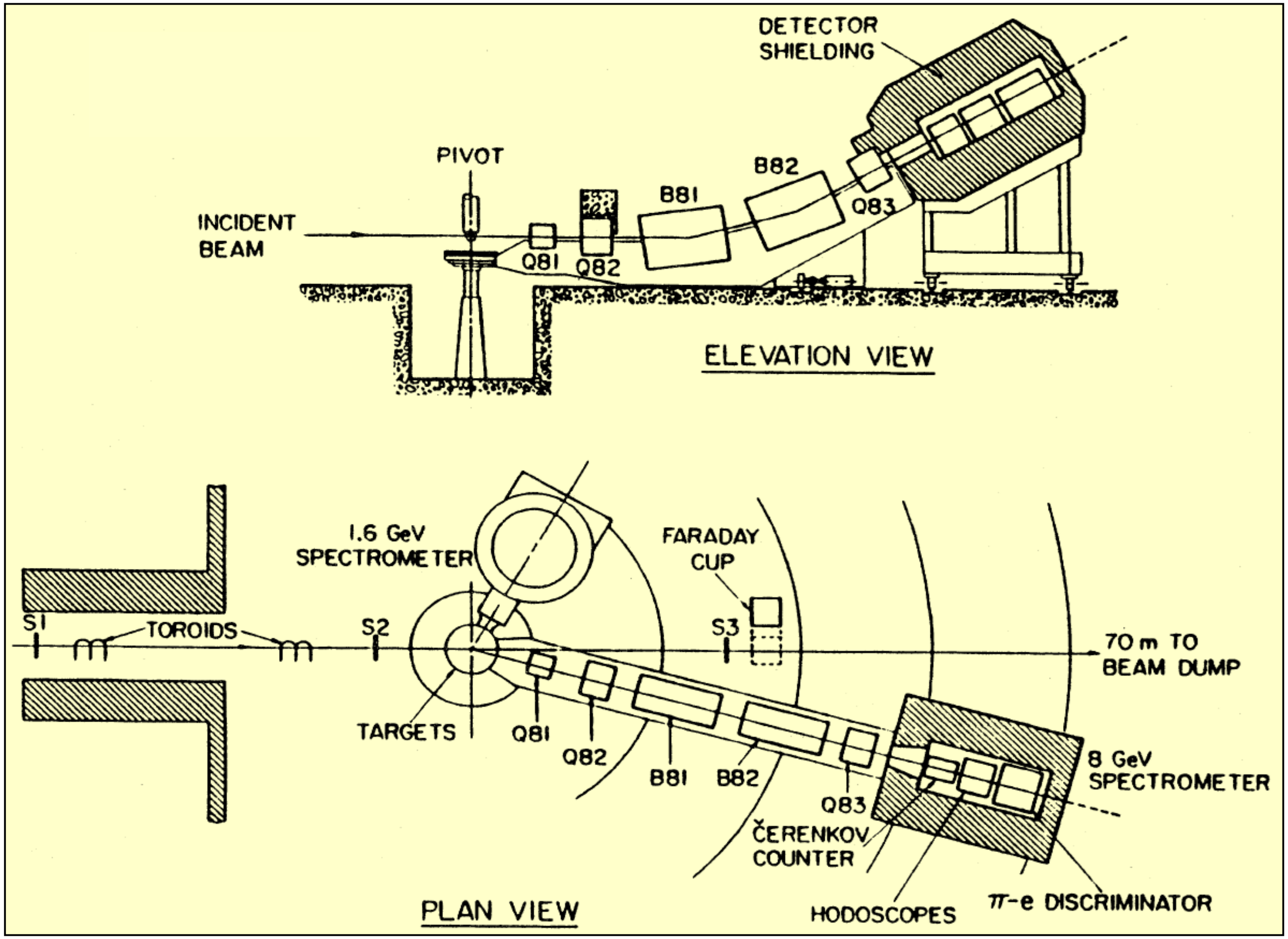
The 8 GeV spectrometer – 1968 (notice the men at the bottom)

Deep inelastic scattering : layout



Layout of the three spectrometers : they can be rotated about their pivot, as shown in the figure. [75 ft $\approx$ 23 m]

Deep inelastic scattering : layout details



Draw of the 8 GeV spectrometer [the 20 GeV is NOT shown]:

B : bending magnets (dipoles);

Q : quadrupoles;

Čerenkov counters;

scintillation hodoscopes,

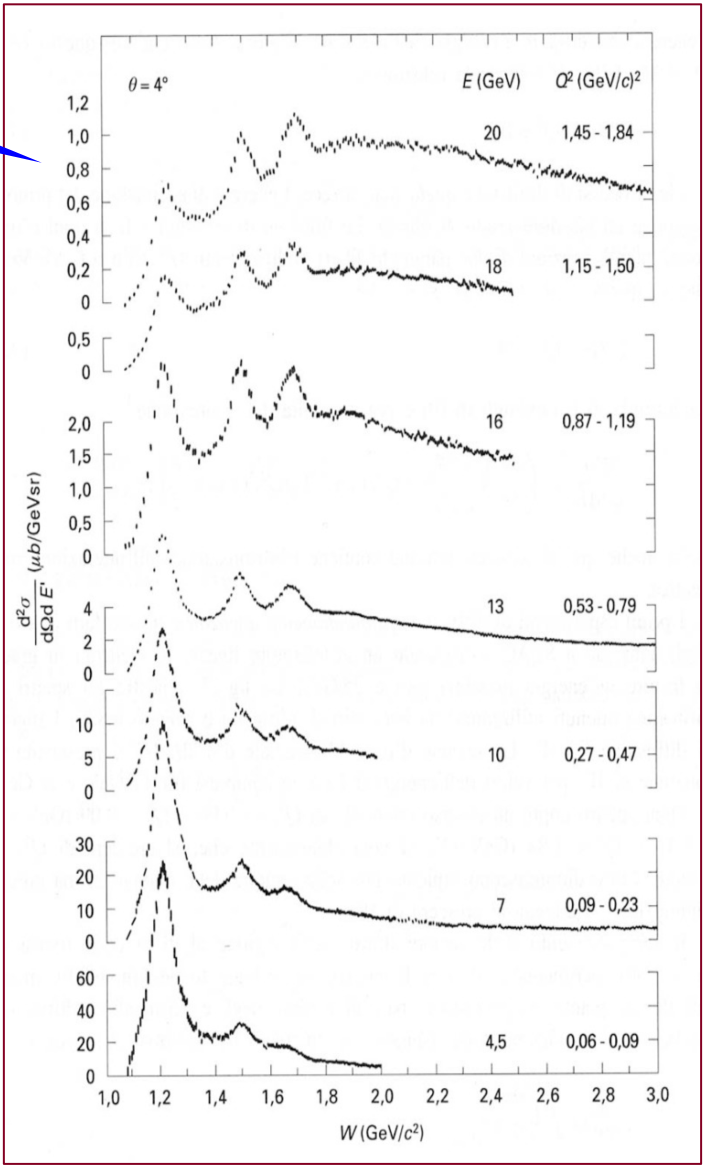
shower counters for e-π discrimination;

dE/dx counters.

a big effort for physics and engineering of 50 years ago !!!
not to be compared with modern experiments …

Deep inelastic scattering : $d^2\sigma/d\Omega dE'$

ep $\rightarrow$ eX, θ = 4°, $d^2\sigma/d\Omega dE'$ vs W (= hadr. mass)



Notice :

- the intervals in W and Q^2, due to fixed E and θ;
- the elastic scattering (W = M_p) is out of scale;
- the decrease in cross section (the vertical scale) when E increases;
- the presence of excited states of the nucleon (resonances $\rightarrow$ peaks), e.g. $\Delta^+(1232)$;
- the "fading out" of resonances, when W increases at fixed E and θ;
- the continuum at high W, with ~const σ (1-2 μb / GeV sr, independent from E and Q^2).

Deep inelastic scattering : $d\sigma/d\theta$ vs $d\sigma/d\theta_{Mott}$

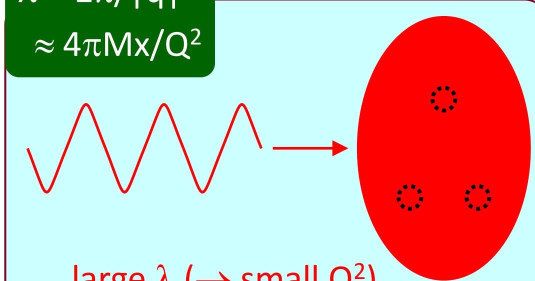
Ratio R = exp./Mott = $W_2 + 2\, W_1 \tan^2 \theta/2 = R(Q^2)$.

Notice that the structure functions appear to be nearly independent of Q^2. Instead, the elastic scattering for a non-pointlike target has a strong Q^2 dependence !!!

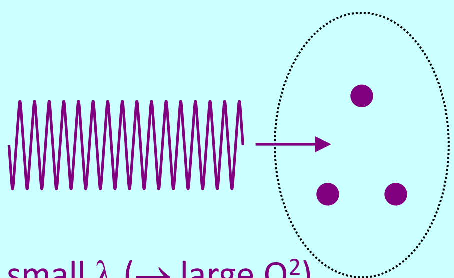
I.e., for DIS, the target (whatever it be), behaves like a point-like particle [$\mathcal{F}(Q^2)$=const] , cfr the Rutherford formula] !!! [NB constant, but << 1 → charge < 1]

This Q^2 independence is another confirmation that the DIS "breaks" the proton : the scattering happens with one of its constituents. The constituents looks "quasi-free" and "quasi-pointlike", at least at this scale of Q^2.

$\lambda \approx 2\pi/|q|$
$\approx 4\pi M x/Q^2$



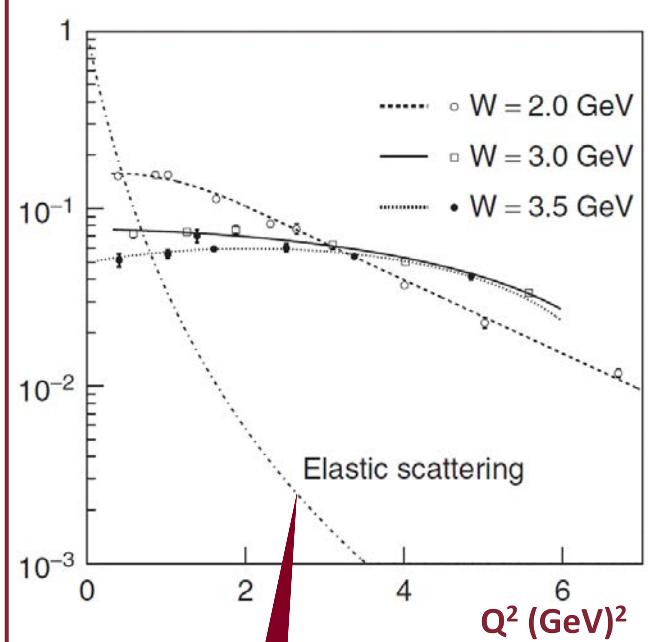
large λ (→ small Q^2)
coherent scattering ep



small λ (→ large Q^2)
scattering eq

M.Breidenbach et al., PRL 23, 935

$R=(d\sigma_{exp}/d\theta)/(d\sigma_{Mott^*}/d\theta)|_{\theta=10°}$



dipole form factor:
- $R(Q^2=0) = 1$;
- $R(Q^2) \propto Q^{-8}$

Bjorken scaling: structure functions F_1, F_2

Define two dimensionless functions F_1 and F_2, instead of W_1 and W_2 [for $d^2\sigma/dxdy$ see later]:

$$F_1(x,Q^2) = MW_1(Q^2,\nu);$$

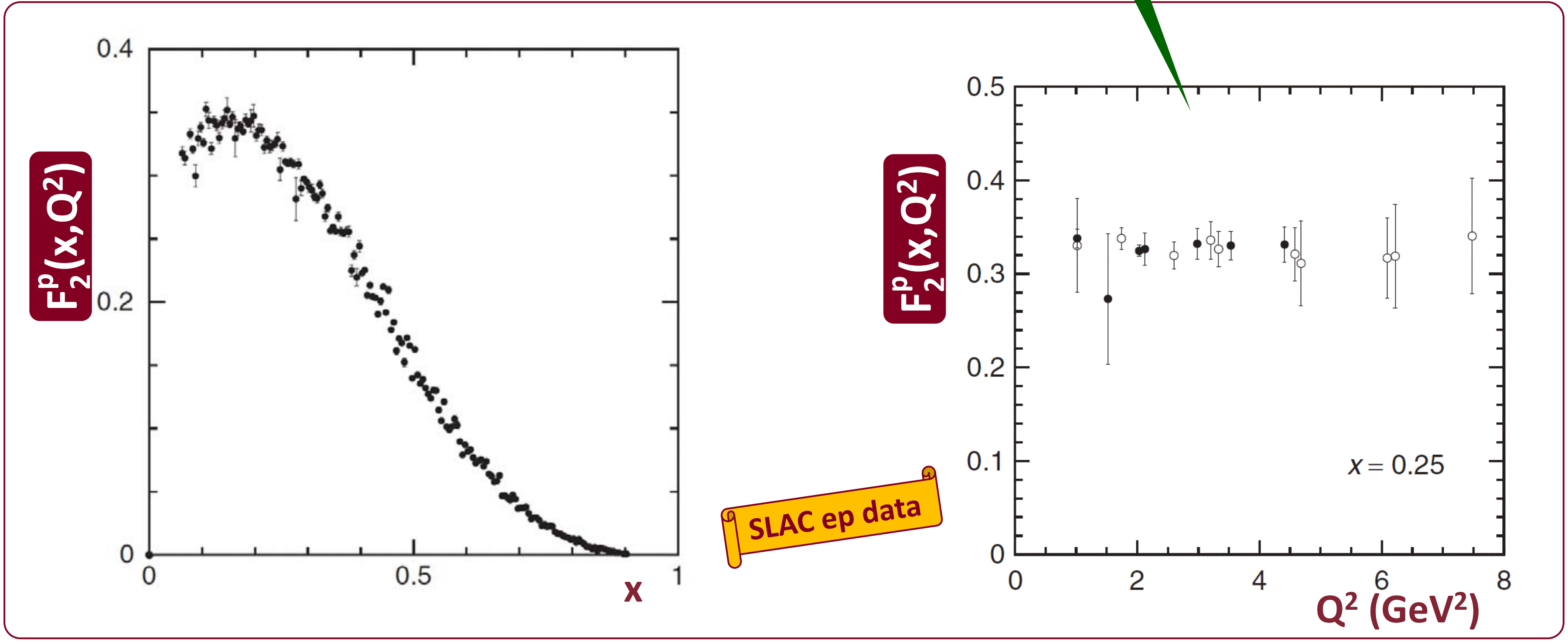
$$F_2(x,Q^2) = \nu W_2(Q^2,\nu).$$

$F_1(x,Q^2)$ and $F_2(x,Q^2)$ are called *structure functions*.

If the nucleons are made by point-like, spin ½ objects, from the DIS formula the **Callan-Gross relation** can be derived [next slide] :

$$2xF_1(x) = F_2(x)$$

Seen as functions of x and Q^2, $F_{1,2}$ appear NOT to depend on Q^2 for a large range of it.



Bjorken scaling : Callan-Gross formula

a) the cross sections of pointlike spin ½ particle of mass m (à la Rosenbluth with $G_E=G_M=1$) :

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{\text{point-like, spin1/2}} = \frac{12\alpha^2 E'^2}{EQ^4}\left[\cos^2\frac{\theta}{2} + 2\tau\sin^2\frac{\theta}{2}\right];$$

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{\text{DIS}} = \frac{4\alpha^2 E'^2}{Q^4}\left[W_2\cos^2\frac{\theta}{2} + 2W_1\sin^2\frac{\theta}{2}\right];$$

$$W_2\cos^2\frac{\theta}{2} + 2W_1\sin^2\frac{\theta}{2} = \frac{3}{E}\left[\cos^2\frac{\theta}{2} + 2\tau\sin^2\frac{\theta}{2}\right];$$

$$W_1 = \frac{3\tau}{E}; \qquad W_2 = \frac{3}{E}; \qquad \frac{W_1}{W_2} = \frac{F_1(x)}{F_2(x)}\frac{\nu}{M} = \tau = \frac{Q^2}{4m^2};$$

b) from the kinematics of elastic scattering of point-like constituents of mass m :

$$Q^2 = 2m\nu = 2M\nu x \rightarrow m = xM;$$

$$\frac{F_1(x)}{F_2(x)} = \frac{Q^2}{4m^2}\frac{M}{\nu} = \frac{2m\nu}{4m^2}\frac{M}{\nu} = \frac{M}{2m} = \frac{1}{2x}; \quad \rightarrow$$

$$\boxed{2xF_1(x) = F_2(x).}$$

Warnings :

- don't confuse M (the nucleon) with m (the constituent);
- don't confuse the inelastic scattering ep with the elastic scattering eq;
- x refers to the inelastic case;
- an hypothetical [*nobody uses it*] variable ξ, analogous to x but for the constituent scattering; in this case, $Q^2=2m\nu\xi$, **ξ = 1;**
- we learn that x = m/M [REMEMBER].

Bjorken scaling : parton model

Assume that the nucleon be made of ***partons*** (point-like, spin ½, mass m_i), which scatter elastically in the ep process.

Then the DIS cross section

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left[W_2 \cos^2\frac{\theta}{2} + 2W_1 \sin^2\frac{\theta}{2}\right];$$

reduces to an incoherent sum of constituent cross sections, $q_{electron}e_i$ being the charge of each of them :

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{m_i} = \frac{4\alpha^2 E'^2}{Q^4}\sum_i \left[e_i^2 \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_i^2}\sin^2\frac{\theta}{2}\right) \delta\left(\nu - \frac{Q^2}{2m_i}\right)\right];$$

where the $\delta()$ means that, at the constituent level, the scattering is elastic, i.e. $Q^2 = 2m_i\nu$.

For such partons [*next 2 slides*]:

$$\begin{cases} F_1\left[x = \dfrac{Q^2}{2m\nu}\right] = MW_1(Q^2,\nu) = \dfrac{1}{2}\sum_j e_j^2 f_j(x) \\ F_2\left[x = \dfrac{Q^2}{2m\nu}\right] = \nu W_2(Q^2,\nu) = x\sum_j e_j^2 f_j(x) \end{cases}$$

i.e. F_1 and F_2 do NOT depend on Q^2 and ν separately, but only on their ratio. F_1 and F_2 are also related by the Callan-Gross equation.

This mechanism (the **Bjorken scaling**) was interpreted by Feynman in 1969 as the dominance of partons in the nucleon dynamics (the **parton model**).

Richard Feynman James Bjorken

Bjorken scaling : $\sigma_{DIS} \to W_{1,2}$

[BJ, 446-460]

2

if $B(x=x_0)=0 \to$
$$\int A(x)\delta[B(x)]dx = A(x_0)/|B'(x_0)|,$$
$$B(x) = \nu - \frac{Q^2}{2Mx} \to x_0 = \frac{Q^2}{2M\nu};$$
$$\to B'(x_0) = \left.\frac{Q^2}{2Mx^2}\right|_{x=x_0} = \frac{2M\nu^2}{Q^2}.$$

3

DIS formula for ep, p NOT pointlike, mass=M:
$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{DIS} = \frac{4\alpha^2E'^2}{Q^4}\left[W_2(Q^2,\nu)\cos^2\left(\frac{\theta}{2}\right) + 2W_1(Q^2,\nu)\sin^2\left(\frac{\theta}{2}\right)\right]$$

Elastic scattering e"q", pointlike, spin ½, charge e, mass m=Mx:
$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{e"q"} = \frac{4\alpha^2E'^2}{Q^4}\left[e^2\cos^2\left(\frac{\theta}{2}\right) + e^2\frac{Q^2}{2m^2}\sin^2\left(\frac{\theta}{2}\right)\right]\delta\left(\nu - \frac{Q^2}{2m}\right)$$

1

here ONLY ONE parton "q", with m, e, x=m/M.

4

$$\left.W_1\right|_x = \frac{e^2Q^2}{4m^2}\delta\left(\nu - \frac{Q^2}{2m}\right) = \frac{e^2Q^2}{4M^2x^2}\delta\left(\nu - \frac{Q^2}{2Mx}\right);$$ [at fixed x]

f(x) : x-distribution of (a single) substructure;

$$W_1 = \int \frac{e^2Q^2}{4M^2x^2}\delta\left(\nu - \frac{Q^2}{2Mx}\right)f(x)dx = \frac{e^2Q^2}{4M^2}\int\frac{f(x)dx}{x^2}\delta\left(\nu - \frac{Q^2}{2Mx}\right) =$$
$$= \frac{e^2Q^2}{4M^2}\left.f(x)\right|_{x=\frac{Q^2}{2M\nu}}\left(\frac{Q^2}{2M\nu}\right)^{-2}\frac{Q^2}{2M\nu^2} =$$
$$= e^2 f(x)\left(2^{-2+2-1}\right)\left(M^{-2+2-1}\right)\left(Q^{2-4+2}\right)\left(\nu^{2-2}\right) = \frac{e^2 f(x)}{2M}.$$

5

[similarly:]
$$\left.W_2\right|_x = e^2\delta\left(\nu - \frac{Q^2}{2Mx}\right);$$
$$W_2 = \int e^2\delta\left(\nu - \frac{Q^2}{2Mx}\right)f(x)dx =$$
$$= e^2\left.f(x)\right|_{x=\frac{Q^2}{2M\nu}}\frac{Q^2}{2M\nu^2} = \frac{e^2xf(x)}{\nu}.$$

Bjorken scaling : $W_{1,2} \to F_{1,2}$

previous page

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{DIS} = \frac{4\alpha^2 E'^2}{Q^4}\left[W_2\left(Q^2,\nu\right)\cos^2\left(\frac{\theta}{2}\right) + 2W_1\left(Q^2,\nu\right)\sin^2\left(\frac{\theta}{2}\right)\right];$$

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{e"q"} = \frac{4\alpha^2 E'^2}{Q^4}\left[e^2\cos^2\left(\frac{\theta}{2}\right) + e^2\frac{Q^2}{2m^2}\sin^2\left(\frac{\theta}{2}\right)\right]\delta\left(\nu - \frac{Q^2}{2m}\right);$$

a single substructure $\{e, m{=}Mx\} \to W_1 = \frac{e^2 f(x)}{2M}; \quad W_2 = \frac{e^2 x f(x)}{\nu}.$

this form ("Σ...") is actually very important (why ?)

Many sub-structures: for each $\{e, x, f(x)\} \to \{e_j, x_j, f_j(x)\}$:

$$W_1 = \frac{e^2 f(x)}{2M} \quad \to W_1 = \sum_j \frac{e_j^2 f_j(x)}{2M} \quad \to \boxed{MW_1 = F_1(x) = \frac{1}{2}\sum_j e_j^2 f_j(x)};$$

$$W_2 = \frac{e^2 x f(x)}{\nu} \quad \to W_2 = \sum_j \frac{e_j^2 x f_j(x)}{\nu} \quad \to \boxed{\nu W_2 = F_2(x) = x\sum_j e_j^2 f_j(x)};$$

$$\to \boxed{\text{Callan} - \text{Gross} \;:\; 2xF_1(x) = F_2(x)}.$$

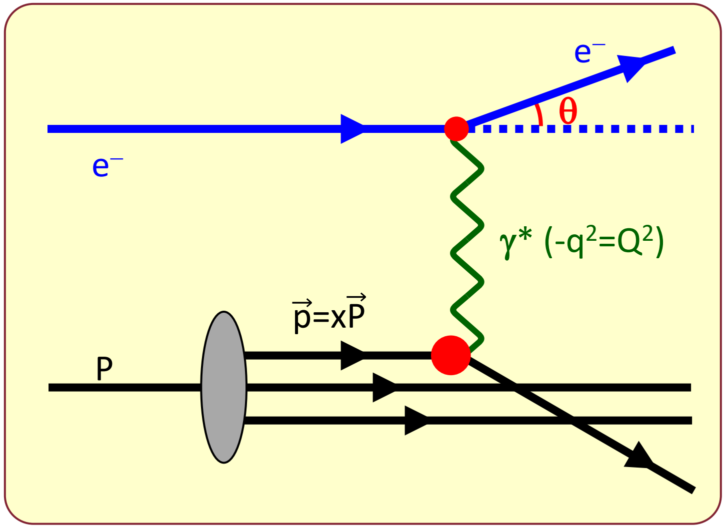
The parton model

Summary: the nucleons are made by partons (later identified with quarks, but at the time there was no reason) :

- point-like (at least at the scale of Q^2 accessible to the experiments, *both then and now*);
- spin ½ fermions;
- define the ratio $|\vec{p}(\text{parton})| / |\vec{p}(\text{nucleon})|$:

  $$x_{\text{Feynman}} = x_F = |\vec{p}_{\text{parton}}| / |\vec{p}_{\text{nucleon}}|$$

  (cfr. $x_{\text{Bjorken}} = x_B = m/M$);
- the interaction e-parton is so fast, that they behave like free particles (similar, *mutatis mutandis*, to the collision approximation in classical mechanics);
- the other partons [*at least in 1st approx.*] do NOT take part in the interaction ("*spectators*");
- it follows $x_F = x_B$ [next slide];
- the DIS is an incoherent sum of the processes on the partons; at high Q^2 the nucleons as such are mere containers, with no role [$F_{1,2} = \Sigma$...].



Despite the formal identity between x_F and x_B, they have a different dynamical origin :

- x_F is defined in the hadronic system (= fraction of the proton momentum);
- x_B comes from the lepton part (momentum transfer and lepton energies).

The parton model : $x_F \leftrightarrow x_B$

Show : $x_{Feynman} \equiv x_F = x_{Bjorken} \equiv x_B$

In the "infinite momentum frame" (IMF), where all the masses are negligible :

$$p^{init}_{proton}\Big|_{IMF} = (p,\vec{p});$$

$$p^{init}_{parton}\Big|_{IMF} = x_F p^{init}_{proton} = (x_F p, x_F \vec{p});$$

$$p^{fin}_{parton}\Big|_{IMF} = p^{init}_{parton} + q_{transf};$$

$$\left(p^{fin}_{parton}\right)^2 = 0 = \left(p^{init}_{parton} + q_{transf}\right)^2 =$$

$$= 0 + q^2_{transf} + 2\left(p^{init}_{parton} \cdot q_{transf}\right);$$

$(p^{init}_{parton} \cdot q_{transf})$ is Lorentz-invariant; let's compute it in the lab frame:

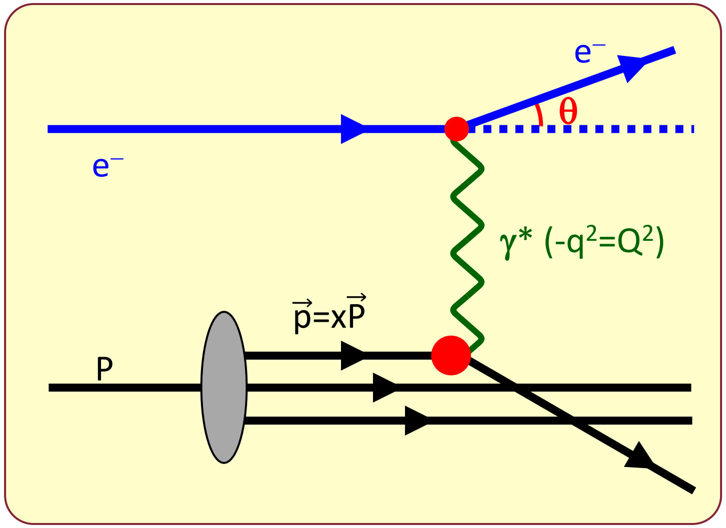
$$p^{init}_{proton}\Big|_{LAB} = (M,\vec{0}); \quad p^{init}_{parton}\Big|_{LAB} = (Mx_F,\vec{0});$$

$$q_{transf}\Big|_{LAB} = (E - E' = \nu, q_x, q_y, q_z);$$

$$2Mx_F\nu = -q^2_{transf} = Q^2 \rightarrow$$

$$x_F = Q^2/(2M\nu) \equiv x_B.$$

Warning : the equality holds only in the IMF. It is also a reasonable approx. in the "ultra-relativistic" case, when the masses are negligible wrt momenta.

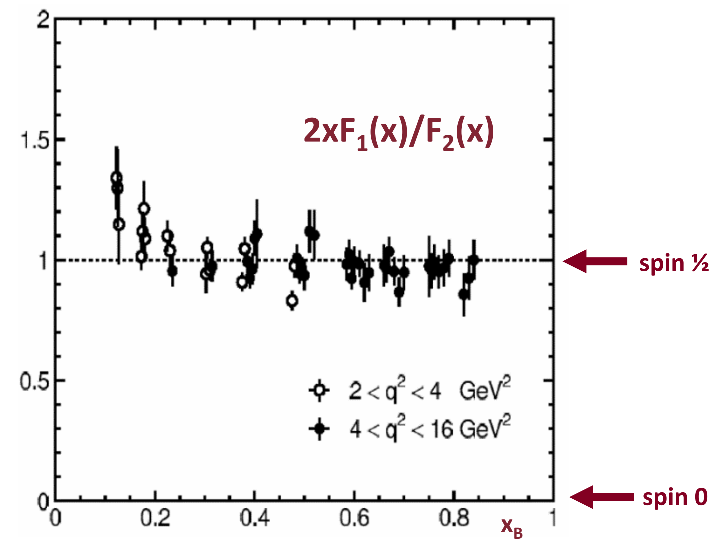


The parton model : sum rules

Remarks and comments (discuss the proton, the neutron is similar):

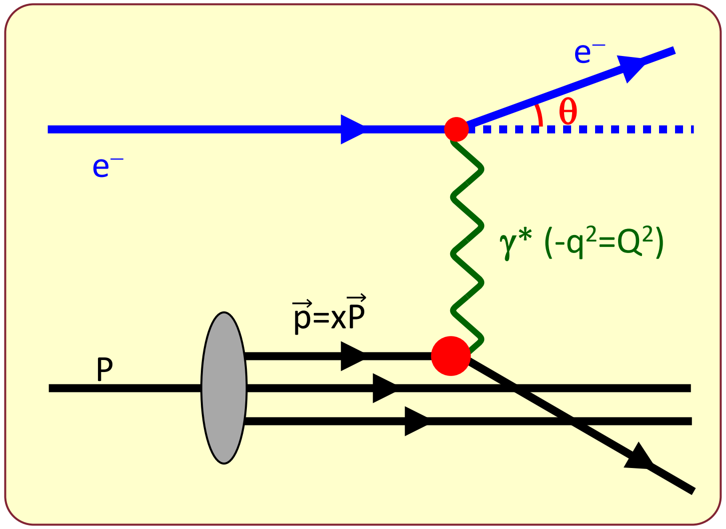
- experimentally, it is enough to control the initial state (E_{e-}, M_p) + measure the leptonic final state (E', θ);
- the model seems to imply that
  $$\Sigma_i \; x_i = 1,$$
  when the sum runs over ALL the partons;
- at the time there was no clue about the nature of the partons, nor if they are charged or neutral (i.e. not interacting with the electrons); therefore:
  $$\Sigma'_i \; x_i \leq 1$$
  (the sum is only over those partons, which interact with the electron);
- given the intrinsic q.m. structure of the nucleon, the values x_i are not fixed, but described by a distribution $f_j^p(x)$ for partons of type "j" in the proton:
  $$f_j^p(x) = dP / dx; \quad \Sigma_j \int dx \, [x f_j^p(x)] \leq 1,$$
  with the same caveats over the sum.
- if partons are spin ½, then the Callan-Gross relation $2xF_1(x) = F_2(x)$ holds;
- instead, spin = 0 $\rightarrow \tau = 0 \rightarrow F_1(x) = 0$;
- but … can we measure it ? YES, it's OK !!!

A.Bodek et al., Phys.Rev. D20, 1471 (1979).



The parton model : summary

A summary of the model, with final formulæ (shown below and in the next slide):



- at high Q^2, a hadron (p/n) behaves as a mixture of small components, the partons.
- partons are pointlike, spin ½;
- each parton in each interaction is described by its fraction x_i of the 4-momentum of the hadron;
- the x_i are qm variables, described by their distribution functions $f_i^p(x)$ [called "***PDF***"];
- in principle the PDF are different for each parton and each hadron;
- $\sum_j \int dx\, x\, f_j^p(x) \leq 1$;
- parton spin → Callan-Gross $2xF_1(x) = F_2(x)$.

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left[W_2\left(Q^2,\nu\right)\cos^2\frac{\theta}{2} + 2W_1\left(Q^2,\nu\right)\sin^2\frac{\theta}{2}\right];$$

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}\left[xy^2F_1\left(x,Q^2\right) + (1-y)F_2\left(x,Q^2\right)\right];$$

$$F_1\left(x,Q^2\right) = MW_1(Q^2,\nu) = \frac{1}{2}\sum_j e_j^2 f_j(x);$$

$$F_2\left(x,Q^2\right) = \nu W_2(Q^2,\nu) = x\sum_j e_j^2 f_j(x).$$

The parton model : $d^2\sigma/dxdy$

$$s = 2EM; \quad \nu = E - E'; \quad y = \frac{\nu}{E} = 1 - \frac{E'}{E}; \quad E' = E(1-y) = \frac{s}{M}\frac{1-y}{2}; \quad Q^2 = 4EE'\sin^2\frac{\theta}{2} = \left(\frac{s}{M}\right)^2 (1-y)\sin^2\frac{\theta}{2}.$$

$$x = \frac{Q^2}{2M\nu} = \left(\frac{s}{M}\right)^2 (1-y)\sin^2\left(\frac{\theta}{2}\right)\frac{1}{\boxed{2M}}\frac{E}{\boxed{E-E'}} = \frac{s}{M^2}\frac{1-y}{y}\sin^2\left(\frac{\theta}{2}\right);$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{M^2xy}{s(1-y)}; \quad \cos^2\left(\frac{\theta}{2}\right) = 1 - \sin^2\left(\frac{\theta}{2}\right) \approx 1.$$

Kinematics

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left(W_2\cos^2\frac{\theta}{2} + 2W_1\sin^2\frac{\theta}{2}\right).$$

$$\frac{\partial x}{\partial\cos\theta} = \frac{\partial x}{\partial\sin^2(\theta/2)}\frac{\partial\sin^2(\theta/2)}{\partial\cos\theta} = \frac{s}{M^2}\frac{1-y}{y}\left(-\frac{1}{2}\right) = -\frac{E}{M}\frac{1-y}{y}; \quad \frac{\partial y}{\partial E'} = -\frac{1}{E};$$

$$\frac{\partial}{\partial\cos\theta}\left(\sin^2\frac{\theta}{2}\right) = \frac{\partial}{\partial\cos\theta}\left(\frac{1-\cos\theta}{2}\right) = -\frac{1}{2}$$

$$J = \begin{vmatrix} \dfrac{\partial x}{\partial\cos\theta} & \dfrac{\partial x}{\partial E'} \\ \dfrac{\partial y}{\partial\cos\theta} & \dfrac{\partial y}{\partial E'} \end{vmatrix} = \frac{1-y}{My}.$$

Jacobian
$\cos\theta, E' \rightarrow x, y$

[YN1], probl. 17.7 : page 697, 698, 911.

L-inv : s, M, ν, x, y, Q^2.
Labo : E, E', θ, Ω.

result

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi}{|J|}\frac{d^2\sigma}{d\cos\theta dE'} = \frac{2\pi My}{1-y}\frac{4\alpha^2E^2(1-y)^2}{Q^4}\times\left[\frac{F_2(x,y)}{\nu}\cos^2\left(\frac{\theta}{2}\right) + \frac{2F_1(x,y)}{M}\frac{M^2xy}{s(1-y)}\right] =$$

$$= \frac{s\pi y 4\alpha^2 \cancel{E}(1-y)}{Q^4}\left[\frac{F_2(x,y)}{\cancel{E}y} + \cancel{2}F_1(x,y)\frac{\cancel{M}xy}{\cancel{s}(1-y)}\frac{\cancel{E}}{\cancel{E}}\right] = \frac{4\pi\alpha^2}{Q^4}s\left[(1-y)F_2(x,y) + xy^2F_1(x,y)\right].$$

The quark-parton model

partons = quarks ???

In the SM the answer is **YES** :

the **quark-parton model**;

Which is the <u>dynamical meaning</u> of $F_{1,2}$? Can we measure them ? [*yes, of course*]

- in principle the proton and the neutron have different structure functions;
- also a given process could result in a different structure [*e.g. the electron scattering could "see" different* $F_{1,2}$ *from neutrino- or hadron-hadron interactions*];
- in this picture, e.g. we will refer to "$F_1^{ep}(x)$", meaning <u>$F_1(x)$ for the proton, when probed in DIS by an electron</u>;
- similarly "$F_2^{ep}(x)$", "$F_2^{en}(x)$", "$F_2^{\nu p}(x)$", …
- however, these functions are NOT really independent : if they reflect the true dynamics, they must be correlated.

- assume that the nucleons are made by three quarks [*Nature is much more complicated, but wait …*];
- call them "**valence quarks**" [*why ???*];
- each of them is described by a x distribution, identified with "$f_j^p(x)$" [e.g. "$u^p(x)$" = <u>the x distribution for u-quarks in the proton</u>];
- e.g. $u^p(x)dx$ = <u>number</u> of u quarks in the proton, with x in the interval (x, x+dx);
- then $d^p(x)$, $\bar{u}^p(x)$, $\bar{u}^{\bar{p}}(x)$, $u^n(x)$, $\bar{u}^{\bar{n}}(x)$,…;
- (already defined) the functions $q^N(x)$ [q=u,d,ū,…; N=p,n] are called <u>parton distribution functions</u> (**PDF**);

(*continue …*)

The q-p model: u^p, u^n, d^p, d^n, …

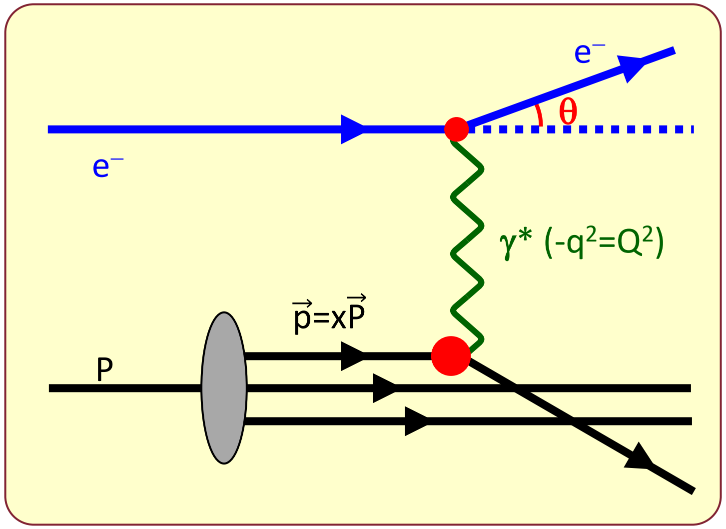
(… *continue*)

Some obvious relations hold [*the green ones with a* (*) *are provisional, we'll modify them*] :

- from charge conjugation : $u^p(x) = \bar{u}^{\bar{p}}(x)$;
- from quark model and isospin invariance : $u^p(x) \approx d^n(x)$;
- from quark model + isospin $u^p(x) \approx 2\ u^n(x)$;
- from quark model + isospin $d^n(x) \approx 2\ d^p(x)$;
- (*) for valence quarks only, $\bar{u}^p(x) = 0$;
- (*) for valence quarks only, $s^p(x) = 0$;
- (*) therefore, e.g.

$$F_2^{ep}(x) = x \sum_j e_j^2 f_j(x) = x \left(\frac{4u^p(x) + d^p(x)}{9} \right);$$

… many more formulæ, all quite intuitive.



A leitmotiv of these lectures. An incomplete list :

§2 : The parton model in Hadron structure;

§7 : Structure functions in ν DIS;

§8 : The quark parton model in Hadron Colliders.

The q-p model : valence and sea

- According to the uncertainty principle, for short intervals q.m. allows quark-antiquark pairs to exist in the nucleons;
- in the hadrons some neutral particles exist, called gluons [*??? ... wait*].

Therefore, let us modify the scheme:

- in the nucleons, 3 types of particles :
  - **valence quarks** [already seen] with distribution $q_V(x)$ *[e.g* $u^p_V(x)$ *[already defined with the simpler notation* $u^p(x)$*]*;
  - **sea quarks**, i.e. the quark-antiquark pairs, described by distributions $q_S(x)$ [e.g $u^p_S(x)$, $s^p_S(x)$, $\bar{u}^p_S(x)$, $\bar{s}^p_S(x)$];
  - **gluons**, described by the distributions $g^p(x)$ and $g^n(x)$.

Obviously only sums can be measured:

$$u^p(x) \equiv u^p_V(x) + u^p_S(x);$$
$$d^p(x) \equiv d^p_V(x) + d^p_S(x);$$
$$\bar{u}^p(x) \equiv \bar{u}^p_V(x) + \bar{u}^p_S(x) = \bar{u}^p_S(x);$$
$$s^p(x) \equiv s^p_V(x) + s^p_S(x) = s^p_S(x);$$

Relations (*final, no further refinement*) :

- charge conjugation constraint :
  $$u^p(x) = \bar{u}^{\bar{p}}(x);$$
- from quark model + isospin invariance :
  $$u^p_V(x) \approx d^n_V(x) \equiv u_V(x);$$
  $$d^p_V(x) \approx u^n_V(x) \equiv d_V(x);$$
- from quark model : $u^p_V(x) \approx 2\, u^n_V(x)$;
- from quark model : $d^n_V(x) \approx 2\, d^p_V(x)$;
- from quantum mechanics and isospin invariance [*but neglecting quark masses*] :
  $$u^p_S(x) = \bar{u}^p_S(x) \approx d^p_S(x) = \bar{d}^p_S(x) \approx$$
  $$\approx s^p_S(x) = \bar{s}^p_S(x) \equiv q^p_S(x) \approx q^n_S(x);$$
- ... many more, all quite intuitive.

the "valence-ness" is not an observable, i.e. a u-quark "does not know" whether (s)he is v or s.

The q-p model : $F^{proton}(x)$ vs $F^{neutron}(x)$

Putting everything together, we have [neglecting heavier quarks] :

$$F_2^{ep}(x) = x\left\{\frac{4}{9}\left[u^p(x)+\bar{u}^p(x)\right]+\frac{1}{9}\left[d^p(x)+\bar{d}^p(x)\right]+\frac{1}{9}\left[s^p(x)+\bar{s}^p(x)\right]\right\} =$$

$$= x\left\{\frac{4}{9}\left[u_V(x)+2q_S(x)\right]+\frac{1}{9}\left[d_V(x)+2q_S(x)\right]+\frac{1}{9}\left[2q_S(x)\right]\right\} =$$

$$= x\left\{\frac{4}{9}u_V(x)+\frac{1}{9}d_V(x)+\frac{4}{3}q_S(x)\right\};$$

$$F_2^{en}(x) = x\left\{\frac{1}{9}u_V(x)+\frac{4}{9}d_V(x)+\frac{4}{3}q_S(x)\right\};$$

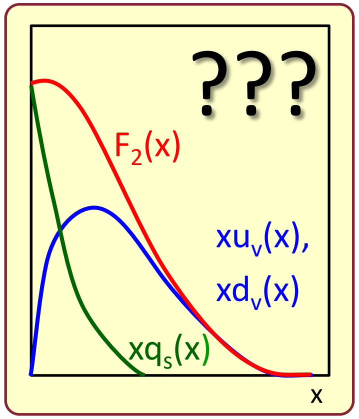
$$F_2^{en}/F_2^{ep} = R_{np} = \begin{cases} 1 & \text{(a);} \\ \left[4d_V(x)+u_V(x)\right]/\left[4u_V(x)+d_V(x)\right] & \text{(b).} \end{cases}$$

(a) if sea dominates (see little sketch);

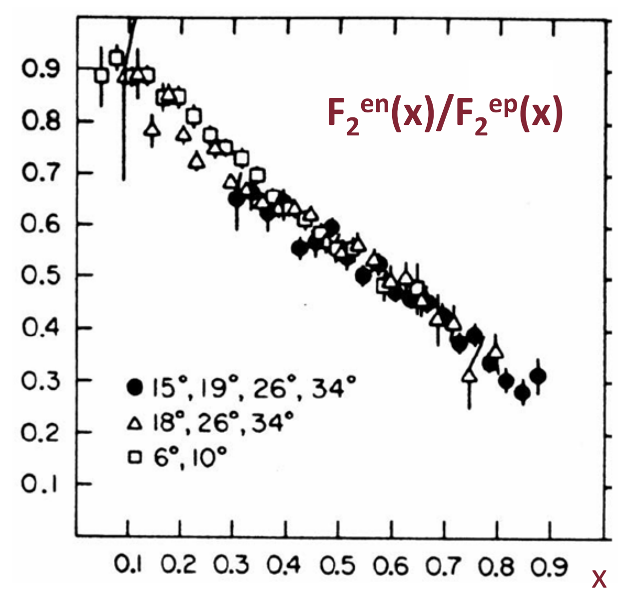
(b) if valence dominates [if (u_V>>d_V) $\rightarrow R_{np} \approx$ ¼].

The measurement shows that case (a) happens at low x, while (b) dominates at high x.

In other words, there are plenty of $q\bar{q}$ pairs at small momentum, while valence is important at high x….



A.Bodek et al., PL 51B (1974) 417.



The q-p model : toy models for $F_2(x)$

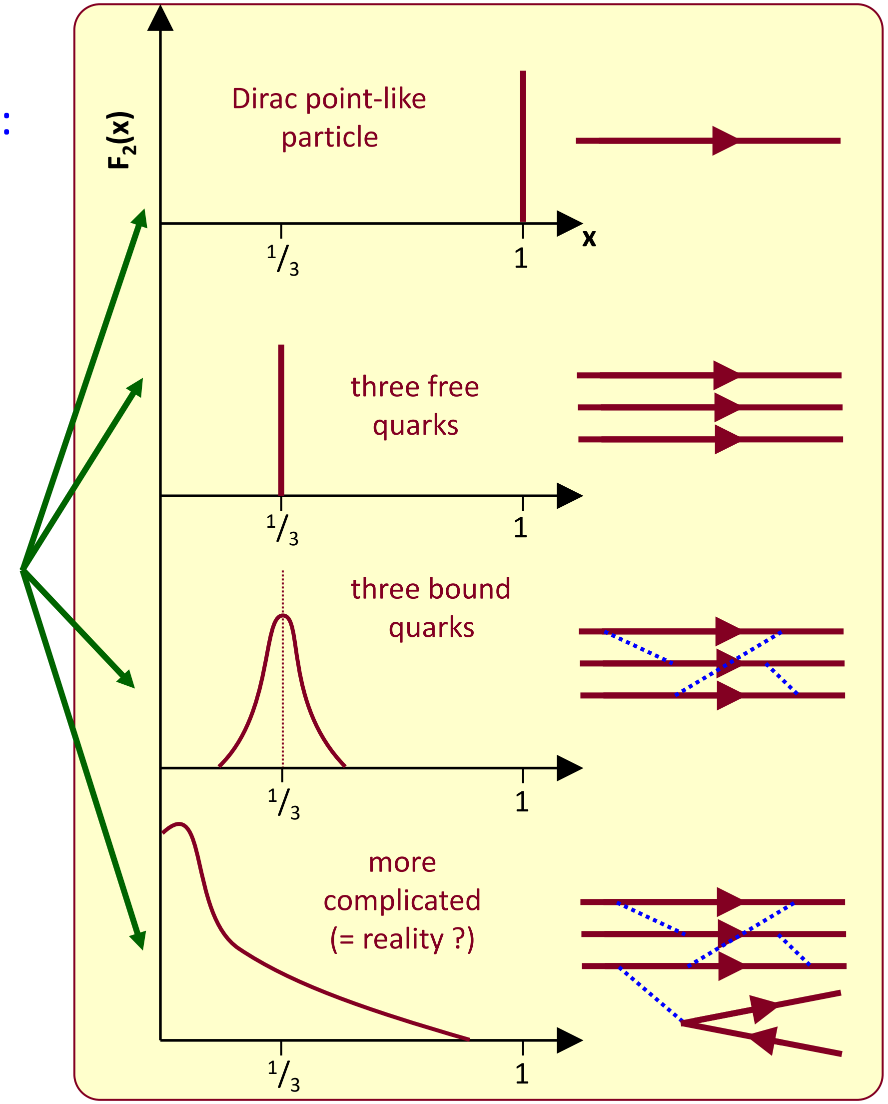
Sum rules (from momentum conservation) :

$$\int_0^1 dx\left[u^p(x)-\overline{u}^p(x)\right]=\int_0^1 dx u_V^p(x)=2;$$

$$\int_0^1 dx\left[d^p(x)-\overline{d}^p(x)\right]=\int_0^1 dx d_V^p(x)=1;$$

$$\int_0^1 dx\left[s^p(x)-\overline{s}^p(x)\right]=0.$$

Hypothetical (**NOT CORRECT**) shapes of $F_2(x)$ from naïve dynamical models :



The q-p model : $F_2^{ep}(x) - F_2^{en}(x)$

From :

$F_2^{ep}(x) = x\,[4u_V(x) + d_V(x) + 12\,q_S(x)] / 9;$

$F_2^{en}(x) = x\,[u_V(x) + 4d_V(x) + 12\,q_S(x)] / 9;$

we get

$F_2^{ep}(x) - F_2^{en}(x) = x\,[u_V(x) - d_V(x)] / 3;$

If, moreover, from the naïve quark model

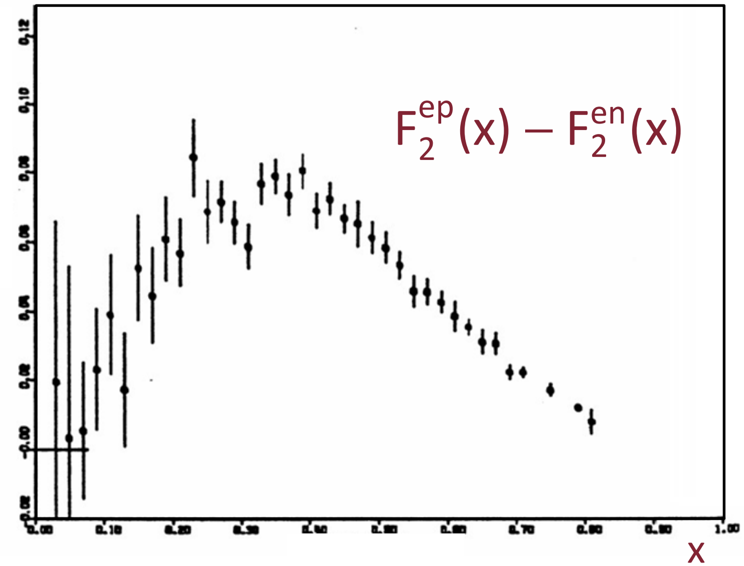
$u_V(x) \approx 2\,d_V(x)$

we get

$F_2^{ep}(x) - F_2^{en}(x) = x\,d_V(x) / 3;$

i.e. this difference, which is an observable, roughly corresponds to the x-distribution of the "lone" valence quark (d_V^p or u_V^n).

Friedman, Kendall - Ann.Rev.Nucl.Sci. 22, 203 (1972)



The q-p model : the gluon

The integrals of $F_2(x)$ are both calculable and measurable. By neglecting the small contribution of $s\bar{s}$:

$$\int_0^1 dx F_2^{ep}(x) = \frac{4}{9}\int_0^1 x\left[u^p(x)+\bar{u}^p(x)\right]dx + \frac{1}{9}\int_0^1 x\left[d^p(x)+\bar{d}^p(x)\right]dx = \frac{4}{9}f_u + \frac{1}{9}f_d;$$

$$\int_0^1 dx F_2^{en}(x) = \frac{4}{9}\int_0^1 x\left[d^p(x)+\bar{d}^p(x)\right]dx + \frac{1}{9}\int_0^1 x\left[u^p(x)+\bar{u}^p(x)\right]dx = \frac{4}{9}f_d + \frac{1}{9}f_u;$$

where $f_{u,d}$ are the fractions of the proton momentum carried by the quark u,d (and the respective $\bar{q}$).

From direct measurement, we get :

$$\left.\begin{array}{l}\int_0^1 dx F_2^{ep}(x) = \frac{4}{9}f_u + \frac{1}{9}f_d \approx \boxed{0.18;} \\ \int_0^1 dx F_2^{en}(x) = \frac{4}{9}f_d + \frac{1}{9}f_u \approx \boxed{0.12;}\end{array}\right\} \rightarrow \left\{\begin{array}{l} f_u \approx 0.36; \\ f_d \approx 0.18; \\ f_u + f_d \approx \boxed{0.54.}\end{array}\right.$$

meas.

Result (important) :

$$f_u + f_d \approx 50\ \%.$$

Only ≈ ½ of the nucleon momentum is carried by quarks and antiquarks.

The rest is "invisible" in the DIS by a charged lepton.

This was one of the first (and VERY convincing) evidences for the existence of the **gluons**, the carriers of the hadronic force.

The gluons are neutral and do not "see" the e.m. interactions.

The q-p model : e⁻p vs νp DIS

Compute $F_2^{eN}(x)$ for an *isoscalar target* ***N***, i.e. a target with $n_{protons} = n_{neutrons}$, both *quasi-free* (*Fermi-gas approx*) :

$$F_2^{ep}(x) = x\left\{\frac{4}{9}\left[u^p(x)+\bar{u}^p(x)\right]+\frac{1}{9}\left[d^p(x)+\bar{d}^p(x)\right]+\frac{1}{9}\left[s^p(x)+\bar{s}^p(x)\right]\right\};$$

$$F_2^{en}(x) = x\left\{\frac{4}{9}\left[d^p(x)+\bar{d}^p(x)\right]+\frac{1}{9}\left[u^p(x)+\bar{u}^p(x)\right]+\frac{1}{9}\left[s^p(x)+\bar{s}^p(x)\right]\right\};$$

$$F_2^{eN}(x) \equiv \frac{F_2^{ep}(x)+F_2^{en}(x)}{2} =$$

$$= x\left\{\frac{5}{18}\left[u^p(x)+\bar{u}^p(x)+d^p(x)+\bar{d}^p(x)\right]+\frac{1}{9}\left[s^p(x)+\bar{s}^p(x)\right]\right\}\xrightarrow[\text{neglect s}]{}$$

$$\rightarrow \frac{5x}{18}\left[u^p(x)+\bar{u}^p(x)+d^p(x)+\bar{d}^p(x)\right].$$

why "isoscalar" ?

because (especially in ν scattering) the target has to be heavy, i.e. made of heavy nuclei, well reproduced by this approximation.

Notice that in neutrino DIS (see) the dynamics is different, but the effective structure function for an isoscalar target **turns out to be very similar**, up to a factor, as in the purely e.m. case :

$$F_2^{\nu N}(x) = x\left[u^p(x)+\bar{u}^p(x)+d^p(x)+\bar{d}^p(x)\right] = F_2^{eN}(x)\Big/\frac{5}{18}.$$

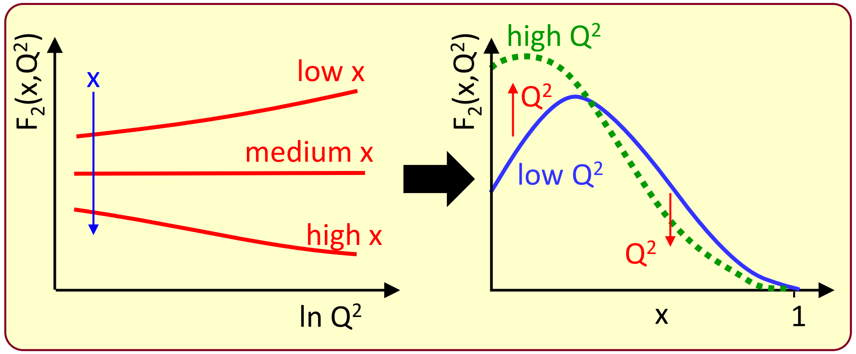
i.e. the structure functions depend on real properties of the nucleon structure, and are not dependent on the interaction.

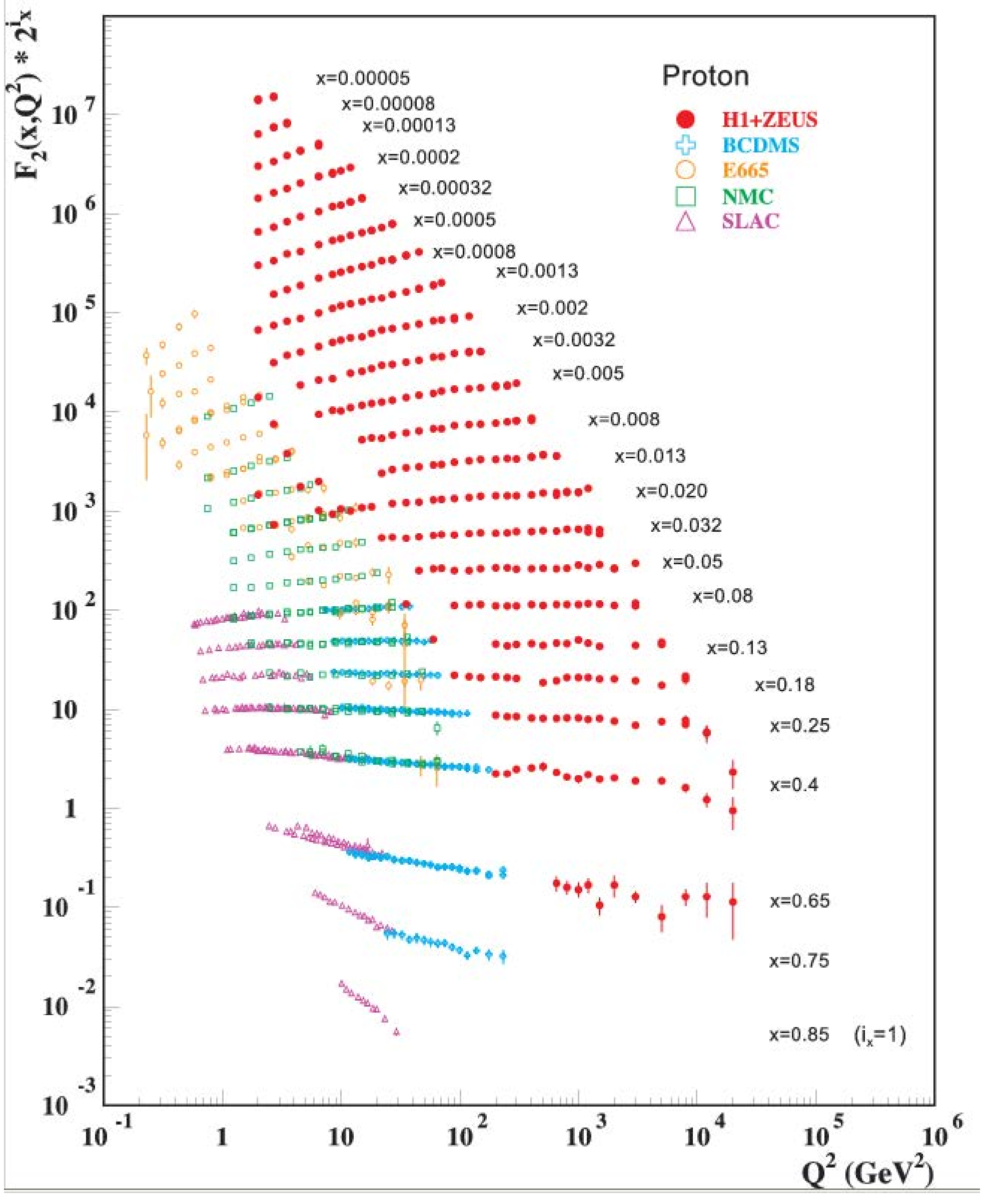
The experimental value (see) is $F_2^{eN} / F_2^{\nu N} = 0.29 \pm 0.02$, very compatible with this prediction (5/18 = 0.278).

$F_2(x,Q^2)$: Scaling violations

Modern experiments have probed the nucleon to very high values of Q^2. Now electrons are often replaced with muons, which have the advantage of intense beams of higher momenta. Or, even better, the experiments are carried out at e^-p Colliders (HERA).

There are data up to $Q^2 \approx 10^5$ GeV^2: when plotting F_2 as function of Q^2 at fixed x, some Q^2-dependence appears, incompatible with Bjorken scaling [*see plot and sketch, and the next slides*].





$F_2(x,Q^2)$: Q^2 evolution

However, such an effect, known as *scaling violations*, is NOT due to sub-structures or other novel effects, but to a dynamical change in F_2, well understood in QCD.

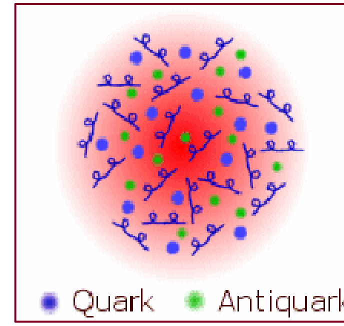
In QCD :

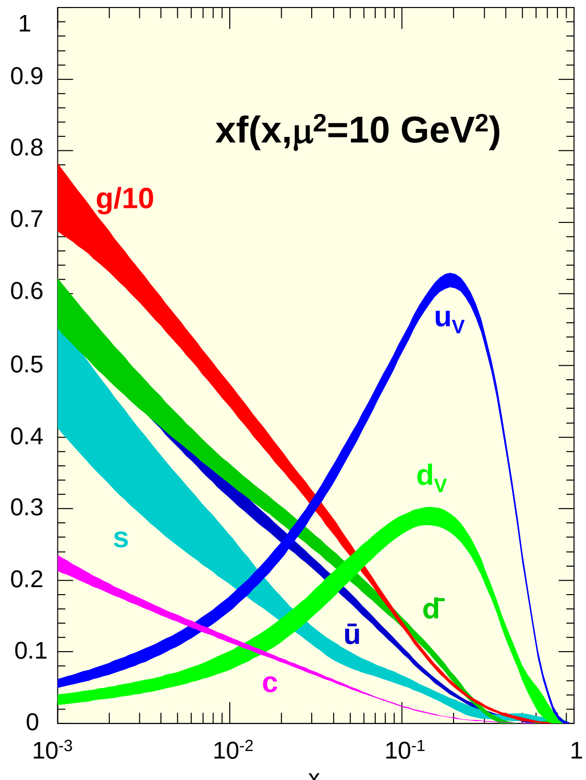
- higher Q^2

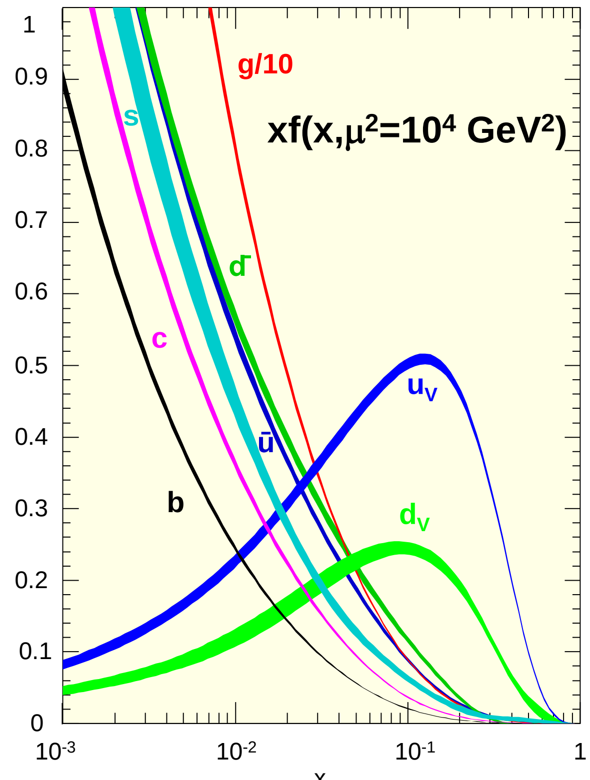
→ smaller size probed

→ more $q\bar{q}$ and gluons

→ less valence quarks.







a modern parameterization of the PDF [*NNPDF3.0-(NNLO)*] shows clearly the difference in the PDF when $Q^2 = 10 \div 10^4$ GeV2:

- u_V, d_V → down;
- $\bar{u}$, $\bar{d}$, [= u_S, d_S,] g → up;
- s, c, b → up (more phase space)

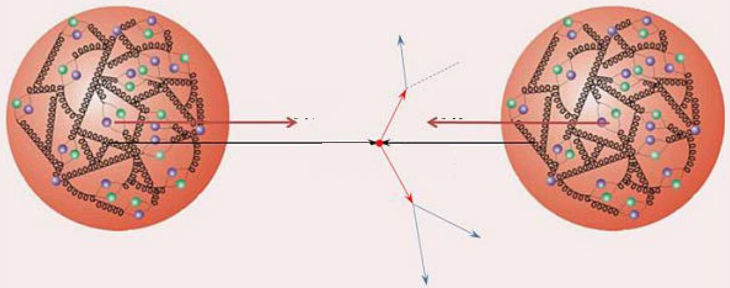
$F_2(x,Q^2)$: parton distribution functions

For modern experiments with hadrons the knowledge of $F_2^{p,n}(x)$ is a necessary ingredient of the data analysis.

- The structure functions are an effect of the hadronic forces. However, being a complicated result of an ill-defined number of bodies in non-perturbative regime, they cannot be reliably computed with today's technology (lattice QCD is still a hope).
- *Similar to the chemistry of complicated molecules, which is a difficult subject, although the fundamental interactions are* [*supposed to be*] *well understood*.
- When studying hadron interactions at large Q^2, the initial state is parameterized by its structure function, as an incoherent sum of all the PDF's, including the gluon.

- In practice, all the computations (e.g. the Higgs production) must use a numerical parameterization of the PDF's, and take into account their uncertainties.

- the PDF's are probabilistic, i.e. the value of x is different for each event !!!
- *consequence: the 4-mom conservation at parton level is a difficult constraint in the computation !!! (see later*)



An artist's view of the pp interaction [from the CERN ATLAS www site]

Summary of cross-sections

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Rutherford}} = \frac{4Z^2\alpha^2E'^2}{|q|^4};$$

$$\left[\frac{d\sigma}{d\Omega}\right]^*_{\text{Mott}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Rutherford}} \times \left(1-\beta^2\sin^2\frac{\theta}{2}\right) \rightarrow \left[\frac{d\sigma}{d\Omega}\right]_{\text{Rutherford}} \times \cos^2\frac{\theta}{2};$$

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} = \left[\frac{d\sigma}{d\Omega}\right]^*_{\text{Mott}} \times \frac{E'}{E}; \qquad \left[\frac{d\sigma}{d\Omega}\right]_{\text{non-point.}} = \left[\frac{d\sigma}{d\Omega}\right]^{(*)}_{\text{Mott}} \times \left|F(q^2)\right|^2.$$

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{point-like spin1/2}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} \times \left(1+2\tau\tan^2\frac{\theta}{2}\right); \qquad \left[\tau = \frac{Q^2}{4M^2c^2}\right];$$

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{Rosenbluth}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{Mott}} \times \left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\right);$$

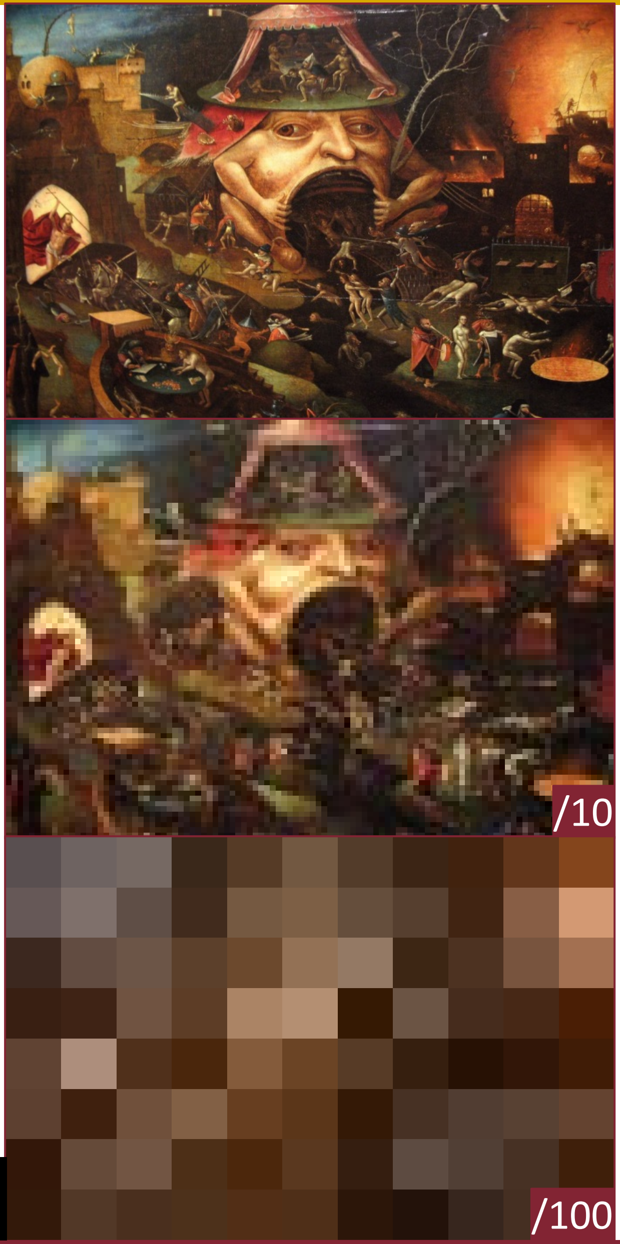
$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{\text{DIS}} = \frac{4\alpha^2E'^2}{Q^4} \times \left[W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2}\right];$$

$$\left[\frac{d^2\sigma}{dxdy}\right]_{\text{DIS}} = \frac{4\pi\alpha^2 s}{Q^4} \times \left[xy^2F_1(x,Q^2) + (1-y)F_2(x,Q^2)\right].$$

Q^2 scale

References

1. [Povh, 6,7,8]
2. [BJ, 12]
3. M. N. Rosenbluth, Phys.Rev. 79 (1950) 615.
4. R. Hofstadter, Rev. Mod. Phys. 28 (1956) 214.
5. J.Friedman, H.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203.



Follower of Hieronymus Bosch – Christ in Limbo (particular) – Indianapolis Museum of Art

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End of chapter 2

Particle Physics - Chapter 3

Heavy flavors – e^+e^- low energy

Paolo Bagnaia

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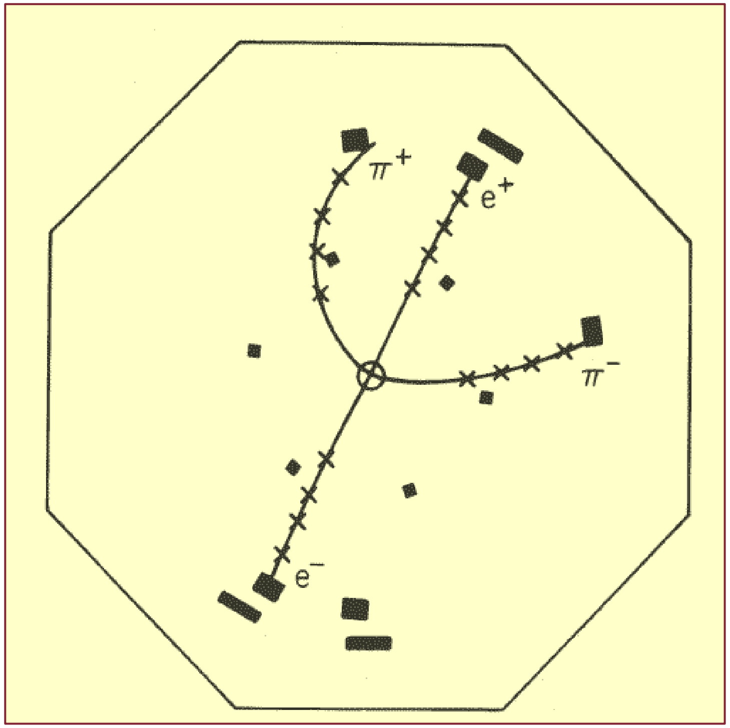
AA 18-19

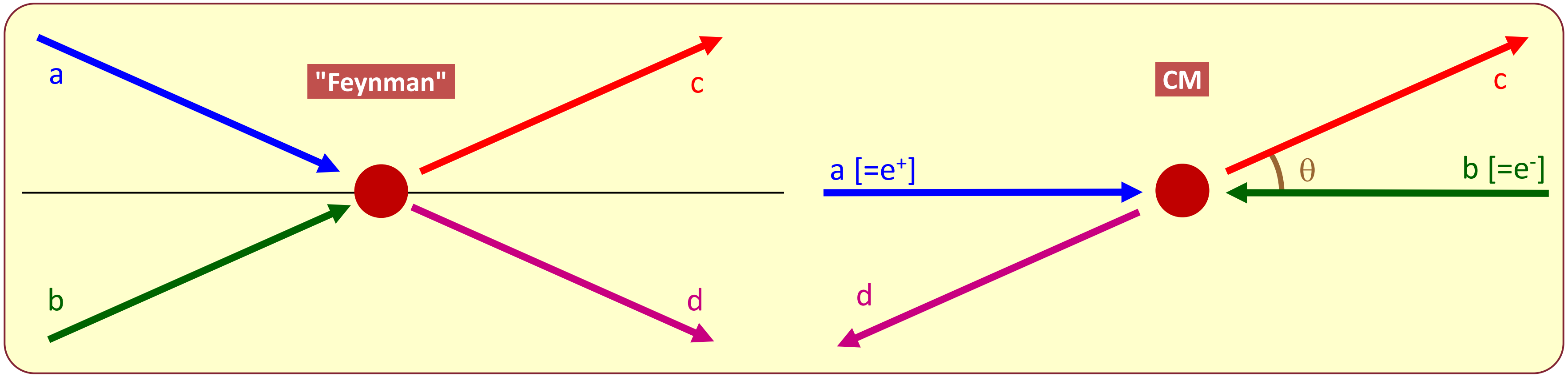
last mod.
17-Mar-19

3 – Heavy flavors – e^+e^- low energy



much of h.f. studies have been performed in e^+e^- collisions; therefore this chapter contains also a discussion of this subject.

Mandelstam variables(*)



The Mandelstam variables $\boxed{s, t, u}$:

CM system

- $p_a = [E, \quad p, \quad 0, \quad 0];$
- $p_b = [E, \quad -p, \quad 0, \quad 0];$
- $p_c = [E, p\cos\theta, p\sin\theta, \quad 0];$
- $p_d = [E, -p\cos\theta, -p\sin\theta, 0];$

s,t,u L-invariant

- $s \equiv (p_a + p_b)^2 = (p_c + p_d)^2 = 4E^2;$
- $t \equiv (p_a - p_c)^2 = (p_b - p_d)^2 = -\tfrac{1}{2}\, s\,(1 - \cos\theta) = -s\sin^2(\theta/2);$
- $u \equiv (p_a - p_d)^2 = (p_b - p_c)^2 = -\tfrac{1}{2}\, s\,(1 + \cos\theta) = -s\cos^2(\theta/2);$
- $s + t + u = 0$ ($\rightarrow$ 2 independent variables, e.g. $[E,\theta]$, $[s, t]$, $[\sqrt{s},\theta]$).

Lorentz-invariant variables for $2\rightarrow2$ processes.

Assume $E \gg m_i$, for the masses of all 4 bodies (otherwise, look for the formulas in [PDG]).

Q. : what about φ (the azimuth) ?
A. : if nothing in the dynamics is φ-dependent (e.g. the spin direction), then the cross-section must be φ-symmetric.

(*) **NOT** specific of h.f. or e^+e^-; here just for convenience.

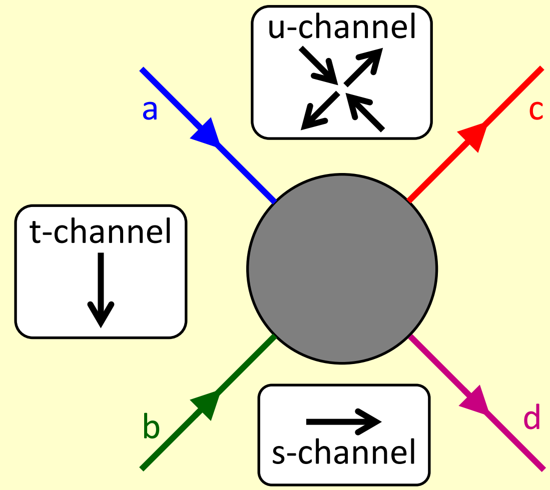
Mandelstam variables: $m_i \neq 0$

General case $ab \to cd$, masses NOT negligible:

[p_i and p_j are 4-mom, p_ip_j = dot product]

- $s \equiv (p_a + p_b)^2 = (p_c + p_d)^2 = m_a^2 + m_b^2 + 2p_ap_b;$
- $t \equiv (p_a - p_c)^2 = (p_b - p_d)^2 = p_a^2 + m_c^2 - 2p_ap_c;$
- $u \equiv (p_a - p_d)^2 = (p_b - p_c)^2 = p_a^2 + m_d^2 - 2\ p_ap_d;$
- $s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2 +$
  $+ 2p_a(p_a + p_b - p_c - p_d) =$
  $= m_a^2 + m_b^2 + m_c^2 + m_d^2 = \Sigma_i\ m_i^2.$

In addition, the <u>crossing symmetry</u> correlates the processes which are symmetric wrt time (s-, t-, and u-channels [see box]). If the c.s. is conserved in the interaction, the same amplitude is valid for all the channels, in their appropriate physical domains (an example on next page).



s-channel	$ab \to cd$	$(\bar{p}p \to \bar{n}n)$
t-channel	$a\bar{c} \to \bar{b}d$	$(\bar{p}n \to \bar{p}n)$
u-channel	$a\bar{d} \to \bar{b}c$	$(\bar{p}\bar{n} \to \bar{p}\bar{n})$

an old approach (1950-80), now almost forgotten, especially important for strong interactions at low energies (see the example $\bar{p}p \to \bar{n}n$), where the dynamics was not calculable (still is not).

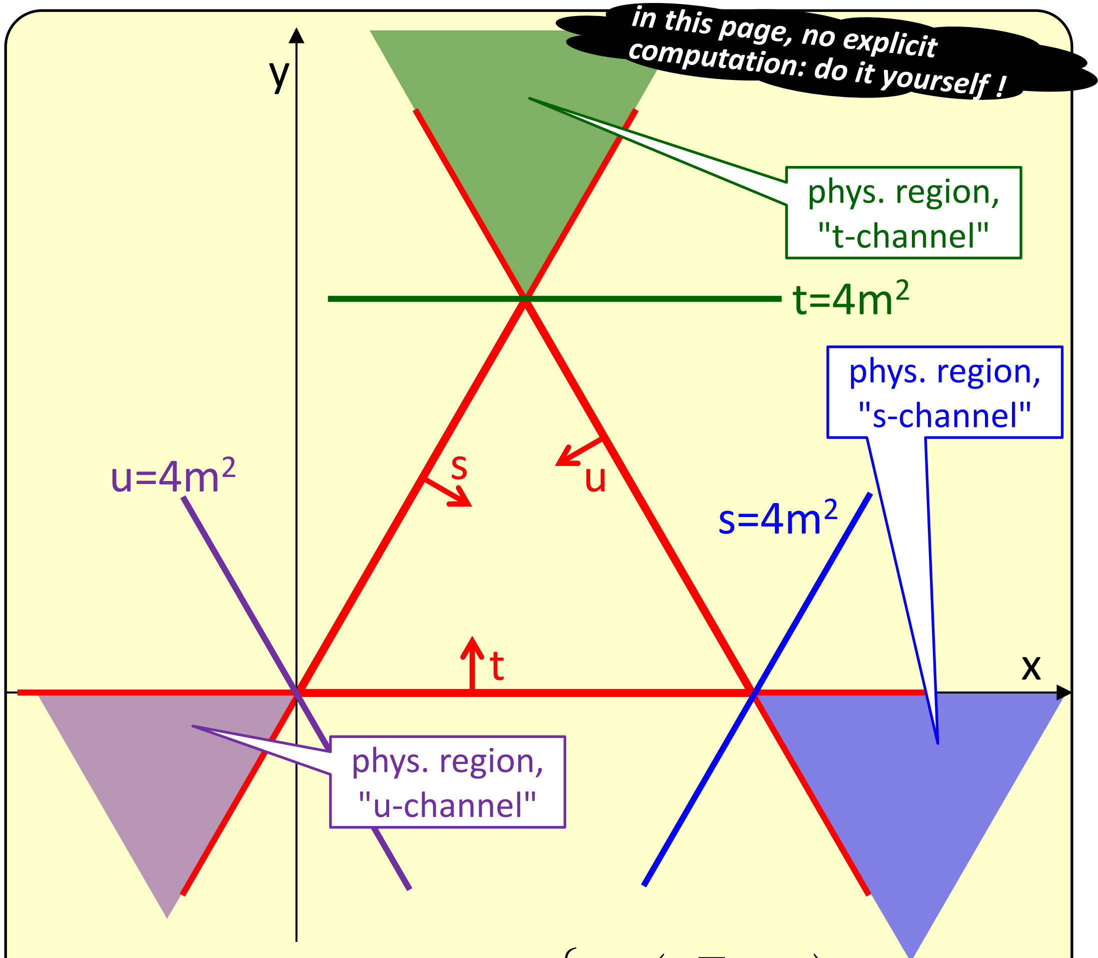
Mandelstam variables: example

Example : $m_a = m_b = m_c = m_d = m$;

- $s = 4E^2 \geq 4m^2$;
- $t = -4p^2\sin^2(\theta/2)$;
- $u = -4p^2\cos^2(\theta/2)$;

$$\left.\begin{array}{l} s = 4E^2 \geq 4m^2; \\ t = -4p^2\sin^2(\theta/2); \\ u = -4p^2\cos^2(\theta/2); \end{array}\right\} s + t + u = 4m^2;$$

- in a xy plane draw an equilateral triangle of height $4m^2$, and label s-t-u the three sides and the lines through them (drawn in red);
- remember Viviani's theorem and its extension ("*the sum of the signed distances between a point and the lines of a triangle is a constant*");
- find the physical regions (i.e. the allowed values of s-t-u) for the given process (i.e. the "s-channel") and for the t and u channels;
- among s-t-u, only two variables are independent $\rightarrow$ the "space of the parameters" is 2D.

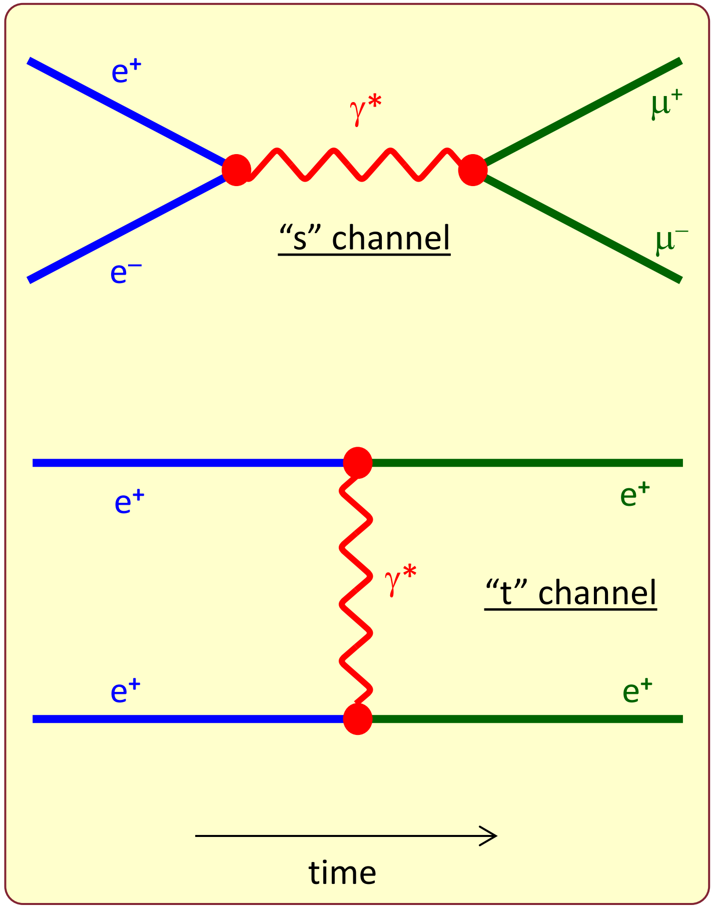


$$\begin{cases} x = \dfrac{4m^2 + s - u}{\sqrt{3}} \\ y = t = 4m^2 - s - u \end{cases} \rightarrow \begin{cases} s = \left(\sqrt{3}x - y\right)/2 \\ t = y \\ u = 4m^2 - \left(\sqrt{3}x + y\right)/2. \end{cases}$$

Mandelstam variables: s vs t

- in a "s-channel" process (e.g. $e^+e^- \to \mu^+\mu^-$), the $|4\text{-momentum}|^2$ of the mediator γ^* is exactly s [i.e. $m(\gamma^*) = \sqrt{s}$, $\sqrt{s} > 0$];
- in a "t-channel" process (e.g. $e^+e^+ \to e^+e^+$), the $|4\text{-momentum}|^2$ of the mediator (γ^* also in this case) is t [$t < 0$!!!] ;
- some processes (e.g. $e^+e^- \to e^+e^-$, called "Bhabha scattering") have more than one Feynman diagrams; some of them are of type s and some others of type t; in such a case we say it is a sum of "s-type diagrams" and "t-type diagrams" + the interference,

… although, *needless to say*, on an event-by-event basis, the observer does **NOT** know whether the event was *s* or *t*.



This discussion is over-simplified, e.g. "the u-channel" is not even mentioned. However, it is sufficient for the experimental results of this chapter.

Mandelstam variables: 1/s

➢ in absence of polarization, the cross sections of a process "X" does NOT depend on the azimuth φ :

$$\frac{d\sigma_{"X"}}{d\Omega} = \frac{1}{2\pi}\frac{d\sigma_{"X"}}{d\cos\theta} = \frac{s}{4\pi}\frac{d\sigma_{"X"}}{dt}.$$

➢ for $m^2 << s$, if $\mathcal{M}_{"X"}$ is the matrix element of the process$^{(*)}$:

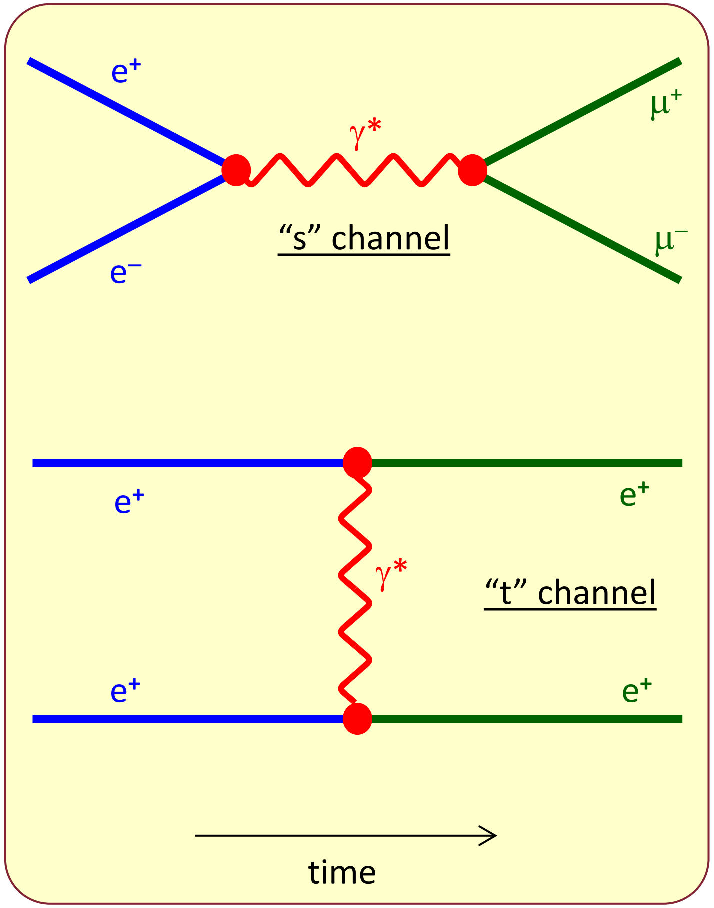
$$\frac{d\sigma_{"X"}}{dt} = \frac{\left|\mathcal{M}_{"X"}\right|^2}{16\pi s^2}.$$

➢ in lowest order QED, if $m^2 << s$:

$$\frac{d\sigma_{"X"}}{d\cos\theta} = \frac{\left|\mathcal{M}_{"X"}\right|^2}{32\pi s} = \frac{\alpha^2}{s} f(\cos\theta).$$

➢ when $\theta \to 0$, $\cos\theta \to 1$:

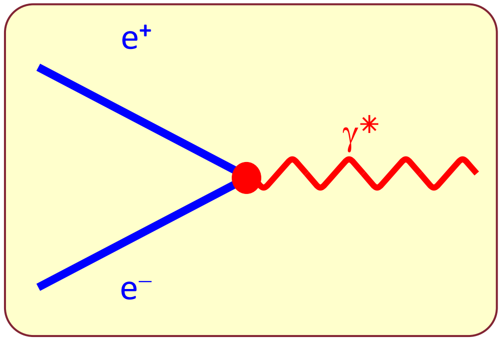
- s-channel : $f(\cos\theta) \to$ constant;
- t-channel : $f(\cos\theta) \to \infty$.



(*) also by dimensional analysis :
$[c = \hbar = 1]$, $[\sigma] = [\ell^2]$; $[t] = [s] = [\ell^{-2}]$;
therefore, in absence of any other dimensional scale,
σ [and $d\sigma/d\Omega$] = [number] × 1/s.

Collisions e^+e^- : initial state

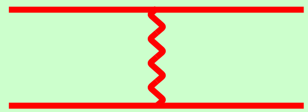
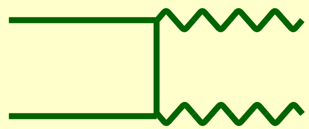
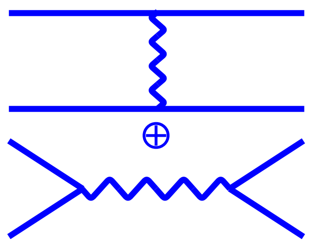
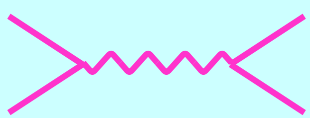
- At low energy(*), the main processes happen with annihilation into a virtual γ^*.
- The initial state is :
  - ➢ charge = 0;
  - ➢ lepton (+ baryon + other additive) number = 0;
  - ➢ spin = 1 ("γ^*");
- CM kinematics :
  - ➢ e^+ [E, p, 0, 0];
  - ➢ e^- [E, −p, 0, 0];
  - ➢ γ^* [2E,0, 0, 0];
  - ➢ $m(\gamma^*) = \sqrt{s} = 2E$ [virtual photon, short lived].



(*) "low energy" ($m_f \ll \sqrt{s} = E_{CM} = 2E = m_{\gamma*} \ll m_Z$), where m_f are the masses of all (initial+final) fermions. When $E_{CM} \sim m_Z$, a $Z^{(*)}$ may also be formed; the process $e^+e^- \rightarrow Z$ resonates at $\sqrt{s} = m_Z$ and becomes dominant (see § LEP).

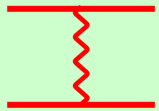
Collisions e^+e^- : QED cross sections

Consider some QED processes in lowest order [$\sqrt{s} << m_Z$, only γ^* exchange] :

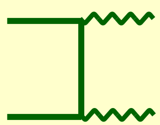
➢ $e^\pm e^\pm \to e^\pm e^\pm$		$\dfrac{d\sigma(e^\pm e^\pm \to e^\pm e^\pm)}{d\cos\theta} = \dfrac{2\pi\alpha^2}{s} \times \left(\dfrac{3+\cos^2\theta}{1-\cos^2\theta}\right)^2;$
➢ $e^+e^- \to \gamma\gamma$		$\dfrac{d\sigma(e^+e^- \to \gamma\gamma)}{d\cos\theta} = \dfrac{2\pi\alpha^2}{s} \times \dfrac{1+\cos^2\theta}{1-\cos^2\theta};$
➢ $e^+e^- \to e^+e^-$		$\dfrac{d\sigma(e^+e^- \to e^+e^-)}{d\cos\theta} = \dfrac{\pi\alpha^2}{2s} \times \left(\dfrac{3+\cos^2\theta}{1-\cos\theta}\right)^2;$
➢ $e^+e^- \to \mu^+\mu^-$		$\dfrac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\cos\theta} = \dfrac{\pi\alpha^2}{2s} \times \left(1+\cos^2\theta\right);$

Collisions e^+e^- : QED $d\sigma/d\cos\theta$

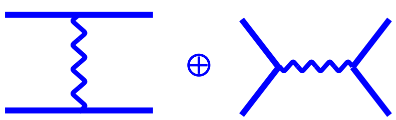
$$\frac{d\sigma(e^{\pm}e^{\pm}\to e^{\pm}e^{\pm})}{d\cos\theta}=\frac{2\pi\alpha^2}{s}\times\left(\frac{3+\cos^2\theta}{1-\cos^2\theta}\right)^2;$$



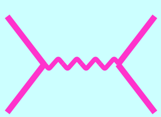
$$\frac{d\sigma(e^{+}e^{-}\to \gamma\gamma)}{d\cos\theta}=\frac{2\pi\alpha^2}{s}\times\frac{1+\cos^2\theta}{1-\cos^2\theta};$$

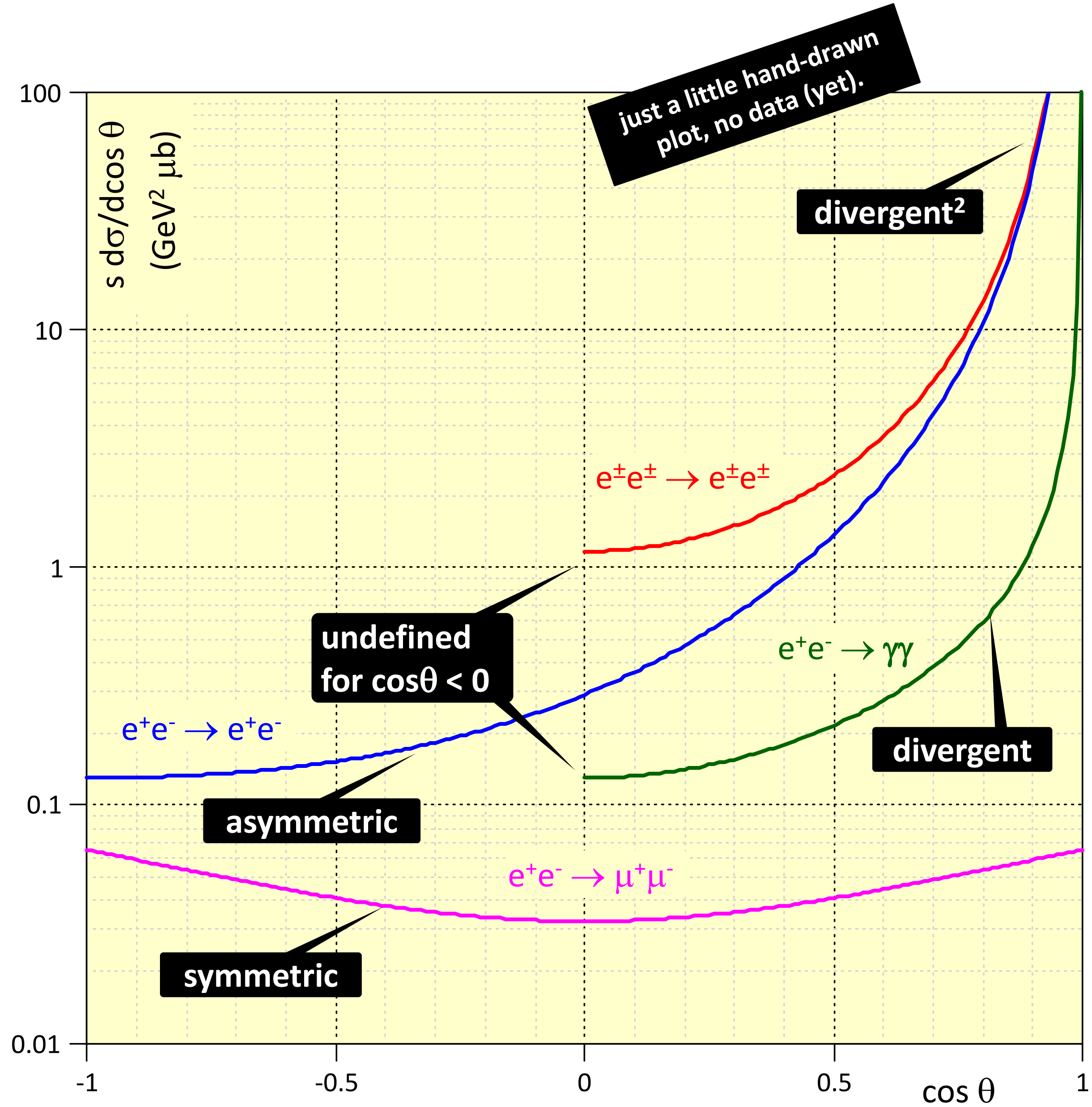


$$\frac{d\sigma(e^{+}e^{-}\to e^{+}e^{-})}{d\cos\theta}=\frac{\pi\alpha^2}{2s}\times\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2;$$



$$\frac{d\sigma(e^{+}e^{-}\to \mu^{+}\mu^{-})}{d\cos\theta}=\frac{\pi\alpha^2}{2s}\times\left(1+\cos^2\theta\right);$$



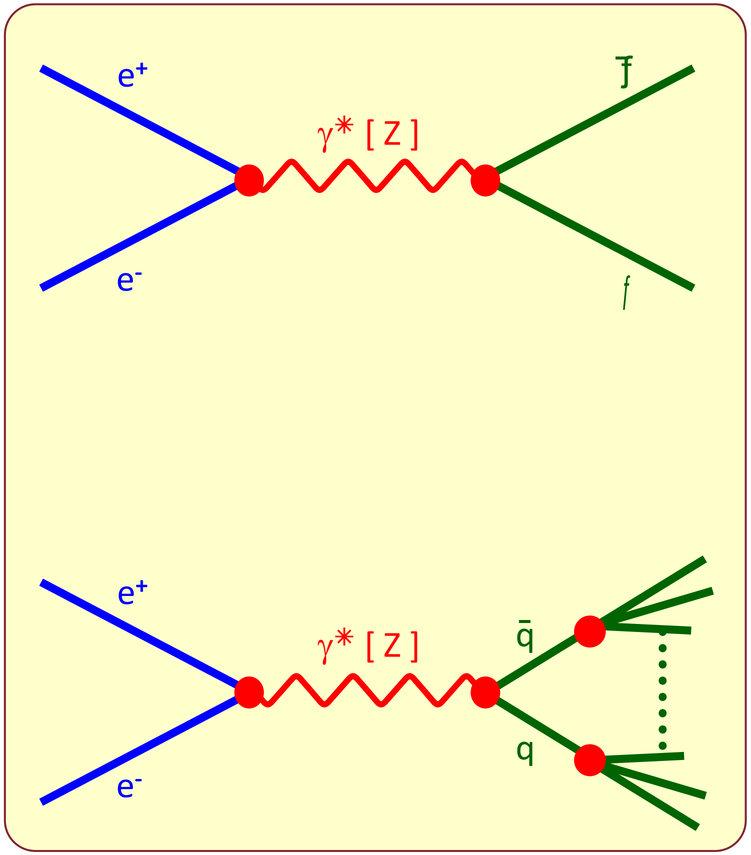


Collisions e^+e^- : $e^+e^- \to \mu^+\mu^-, q\bar{q}$

- kinematics, computed in CM sys, $\sqrt{s} >> m_e, m_\mu$:

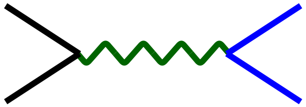
$$e^+ \quad (E, \; p, \; 0, \; 0);$$
$$e^- \quad (E, \; -p, \; 0, \; 0);$$
$$\mu^+ \quad (E, \; p\cos\theta, \; p\sin\theta, \; 0);$$
$$\mu^- \quad (E, \; -p\cos\theta, \; -p\sin\theta, \; 0);$$
$$p \approx E = \sqrt{s}/2;$$
$$\vec{p}(e^+) \cdot \vec{p}(\mu^+) \approx E^2 \cos\theta \approx s \cos\theta / 4;$$
$$p(e^+)\, p(\mu^+) \approx E^2 (1 - \cos\theta) = s \sin^2(\theta/2) = -t;$$

- the case $e^+e^- \to q\bar{q}$ is similar at parton level; however free (anti-)quarks do NOT exist → quarks hadronize, producing collimated jets of hadrons [+ *subtleties due to the fact that hadrons and leptons, unlike quarks, are color singlets with integer charge*] .



Collisions e^+e^- : $\sigma(e^+e^- \to \mu^+\mu^-, q\bar{q})$

- $e^+e^- \to \mu^+\mu^-$



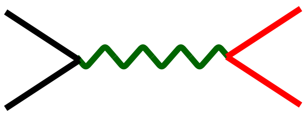
$$\sigma_{\mu\mu} = \int_{-1}^{1} d\cos\theta \left[\frac{d\sigma_{\mu\mu}}{d\cos\theta} \right] = \frac{\pi\alpha^2}{2s} \int_{-1}^{1} d\cos\theta \left(1 + \cos^2\theta\right) =$$

$$= \frac{4\pi\alpha^2}{3s} = \frac{86.8 \text{ nb}}{s[\text{GeV}^2]} = \frac{21.7 \text{ nb}}{E^2_{beam}[\text{GeV}^2]}.$$

$[1+\cos^2\theta] = P_1^{Legendre}(\cos\theta)$

[spin 1 $\to$ 2 spin ½]

- $e^+e^- \to q\bar{q}$



$$\frac{d\sigma_{q\bar{q}}}{d\cos\theta} = \frac{d\sigma_{\mu\mu}}{d\cos\theta} \times c_f e_f^2 = \frac{\pi\alpha^2}{2s} c_f e_f^2 \left(1 + \cos^2\theta\right); \quad c_f = \begin{Bmatrix} 3 & \text{quarks} \\ 1 & \text{leptons} \end{Bmatrix} \quad \text{[color]}$$

$$\sigma_{q\bar{q}} = \sigma_{\mu\mu} c_f e_f^2 = \frac{4\pi\alpha^2}{3s} c_f e_f^2; \quad e_f = \begin{Bmatrix} 1 & \text{leptons} \\ 2/3 & u\ c\ t \\ -1/3 & d\ s\ b \end{Bmatrix} \quad \text{[charge].}$$

If $m_e \ll E_{beam}$, but m_f (the mass of the the final-state fermion) is NOT negligible, the complete formula ($m_f > 0$) must be used [see next slide].

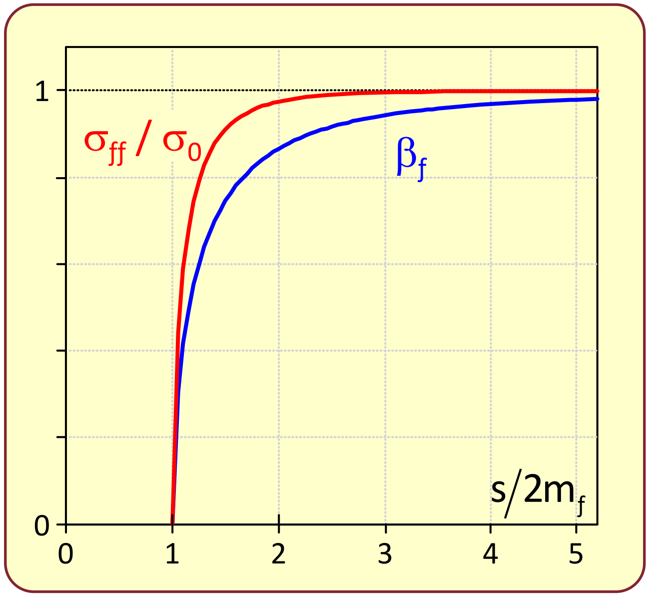
Collisions e^+e^- : $m_f > 0$

Previous formulæ NOT correct if m_f NOT negligible, e.g. near the threshold for the production of heavy quarks/leptons, $\sqrt{s} \approx 2m_f$.

$\rightarrow$ list (no proof) the formulæ for $e^+e^- \rightarrow f\bar{f}$

$(2m_e \ll \sqrt{s} \approx 2m_f)$:

- $\beta_f = \sqrt{1 - \dfrac{4m_f^2}{s}}$ (see blue curve);
- $\dfrac{d\sigma_{f\bar{f}}}{d\cos\theta} = \dfrac{\pi\alpha^2 c_f e_f^2}{2s}\beta_f\left[\left(1+\cos^2\theta\right)+\left(1-\beta_f^2\right)\sin^2\theta\right];$
- $\sigma_{f\bar{f}} = \boxed{\dfrac{4\pi\alpha^2}{3s}}\ \beta_f\,\dfrac{3-\beta_f^2}{2} = \boxed{\sigma_0}\ \beta_f\,\dfrac{3-\beta_f^2}{2}$ (see red curve).

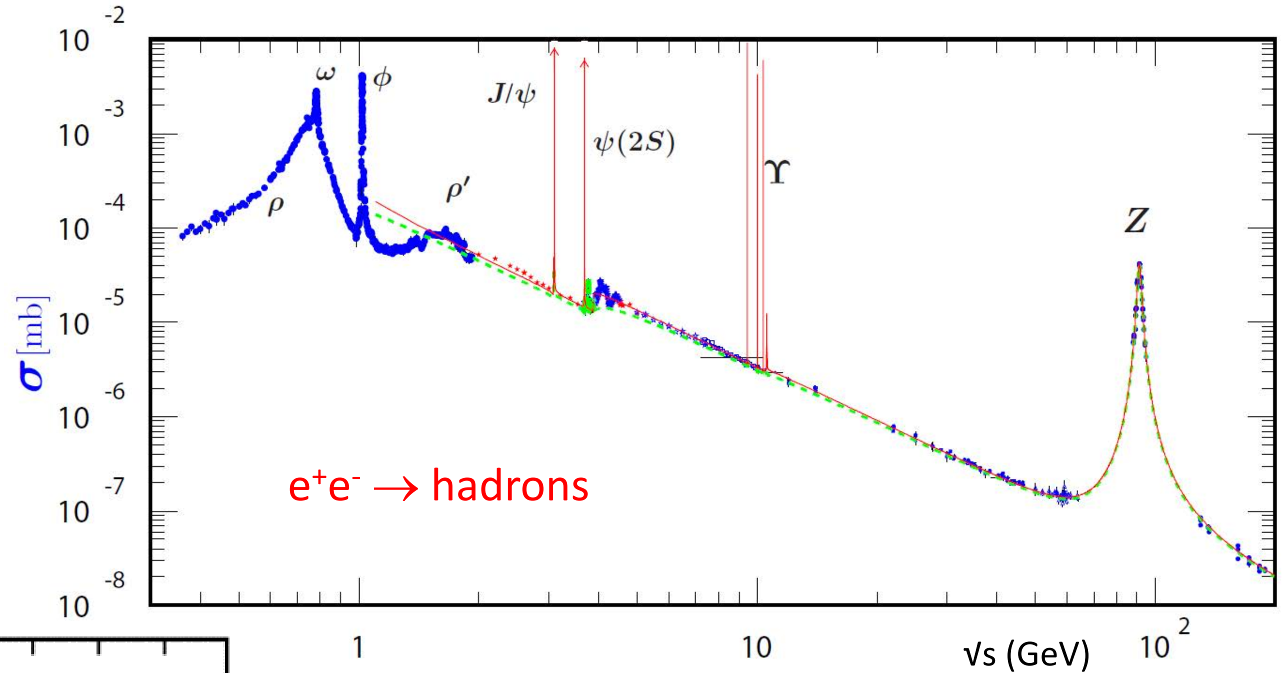


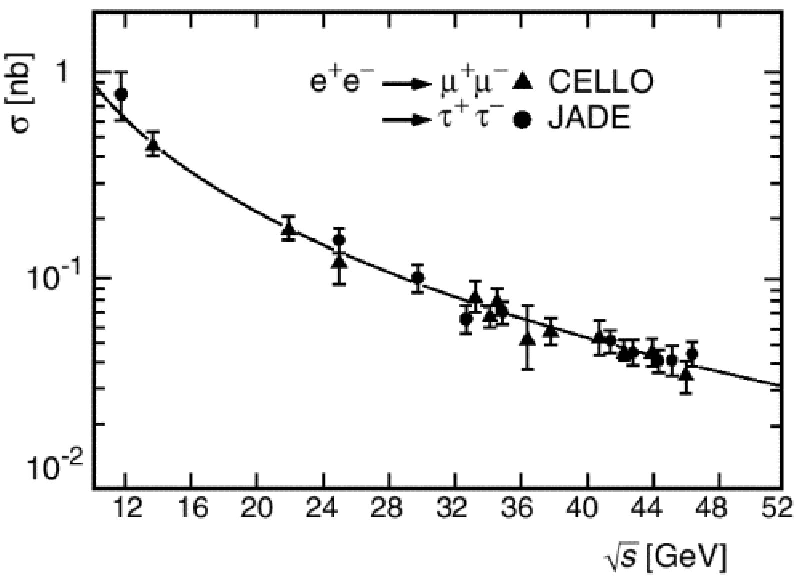
Clearly:

- $\sqrt{s} < 2m_f \rightarrow$ no f production;
- $\sqrt{s} \gg 2m_f \rightarrow 2m_f/\sqrt{s} \rightarrow 0,\ \ \beta_f \rightarrow 1,\ \ \sigma_{f\bar{f}} \rightarrow \sigma_0.$

Collisions e^+e^- : $\sigma_{large\sqrt{s}}(e^+e^- \to \mu^+\mu^-, q\bar{q})$

$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{3s} = $$
$$= \frac{86.8 \text{ nb}}{s[\text{GeV}^2]} = \frac{21.7 \text{ nb}}{E^2[\text{GeV}^2]}.$$



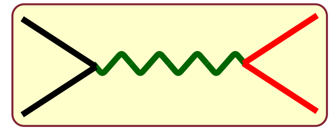


- the continuum, for $0.5 \le \sqrt{s} \le 50$ GeV, agrees well with the predicted 1/s [the line in log-log scale];
- + resonances $q\bar{q}$ [the bumps];
- for $\sqrt{s} > 50$ GeV [§ LEP] it is dominated by the Z formation in the s-channel.

Collisions e^+e^- : $R = \sigma(q\bar{q})/\sigma(\mu^+\mu^-)$

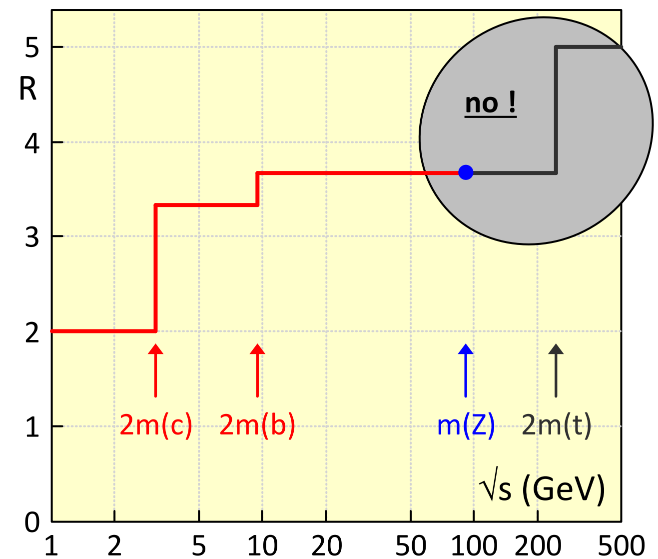
- define the quantity, both simple conceptually and easy to measure:

$$R = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = 3\sum_{\text{quarks}} e_i^2 = R\left(\sqrt{s}\right);$$



en passant, a powerful test of the existence of the color quantum number

- sum over all the quarks, produced at energy $\sqrt{s}$ (i.e. $2m_q < \sqrt{s}$) :
  - $0 < \sqrt{s} < 2\,m_c$: $R = R_{uds} = 3 \times [\,(^2/_3)^2 + (^{-1}/_3)^2 + (^{-1}/_3)^2\,] = 2;$
  - $2\,m_c < \sqrt{s} < 2\,m_b$: $R = R_{udsc} = R_{uds} + 3 \times (^2/_3)^2 = 3 + {^1/_3};$
  - $2\,m_b < \sqrt{s} < 2\,m_t$: $R = R_{udscb} = R_{udsc} + 3 \times (^{-1}/_3)^2 = 3 + {^2/_3};$
  - ~~$2\,m_t < \sqrt{s} < \infty$: $R = R_{udscbt} = R_{udscb} + 3 \times (^2/_3)^2 = 5$~~;
- but reality is more complicated :
  - the step at $\sqrt{s} = 2m_q$ is rounded [*see before*];
  - $q\bar{q}$ resonances are formed at $\sqrt{s} \approx 2m_q$; their decay modes affects the measurement of R;
  - at $\sqrt{s} \approx m_Z$ [and $\sqrt{s} \approx 2m_W$] the weak interactions change completely the scenario $\to$ for $\sqrt{s} \geq 50$ GeV, R has a different explanation [*see § LEP*];
  - also notice that $m_Z < 2m_t$; therefore the "t step" happens at higher $\sqrt{s}$ than the Z resonance.

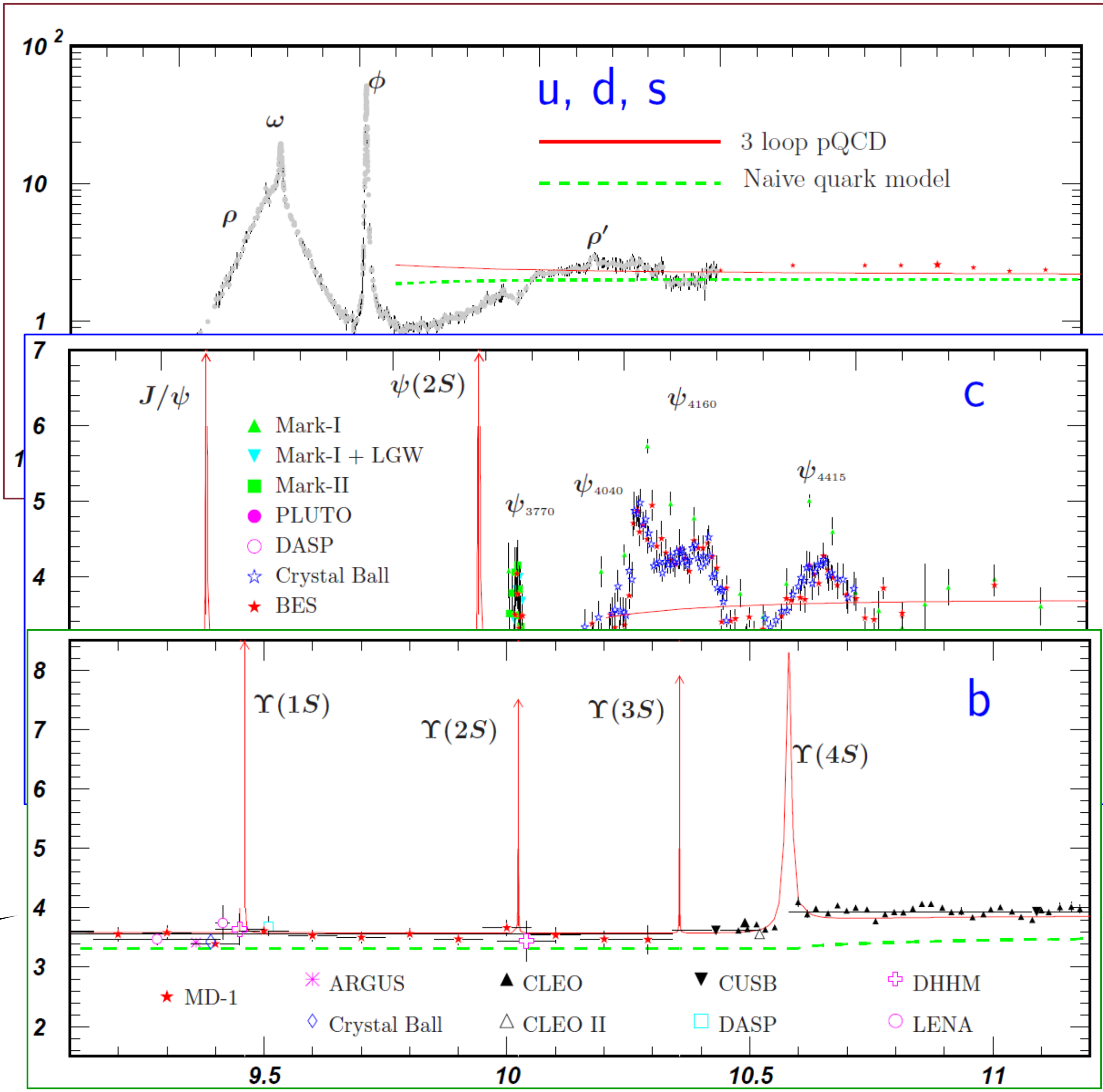


Collisions e⁺e⁻ : R vs √s (small √s)

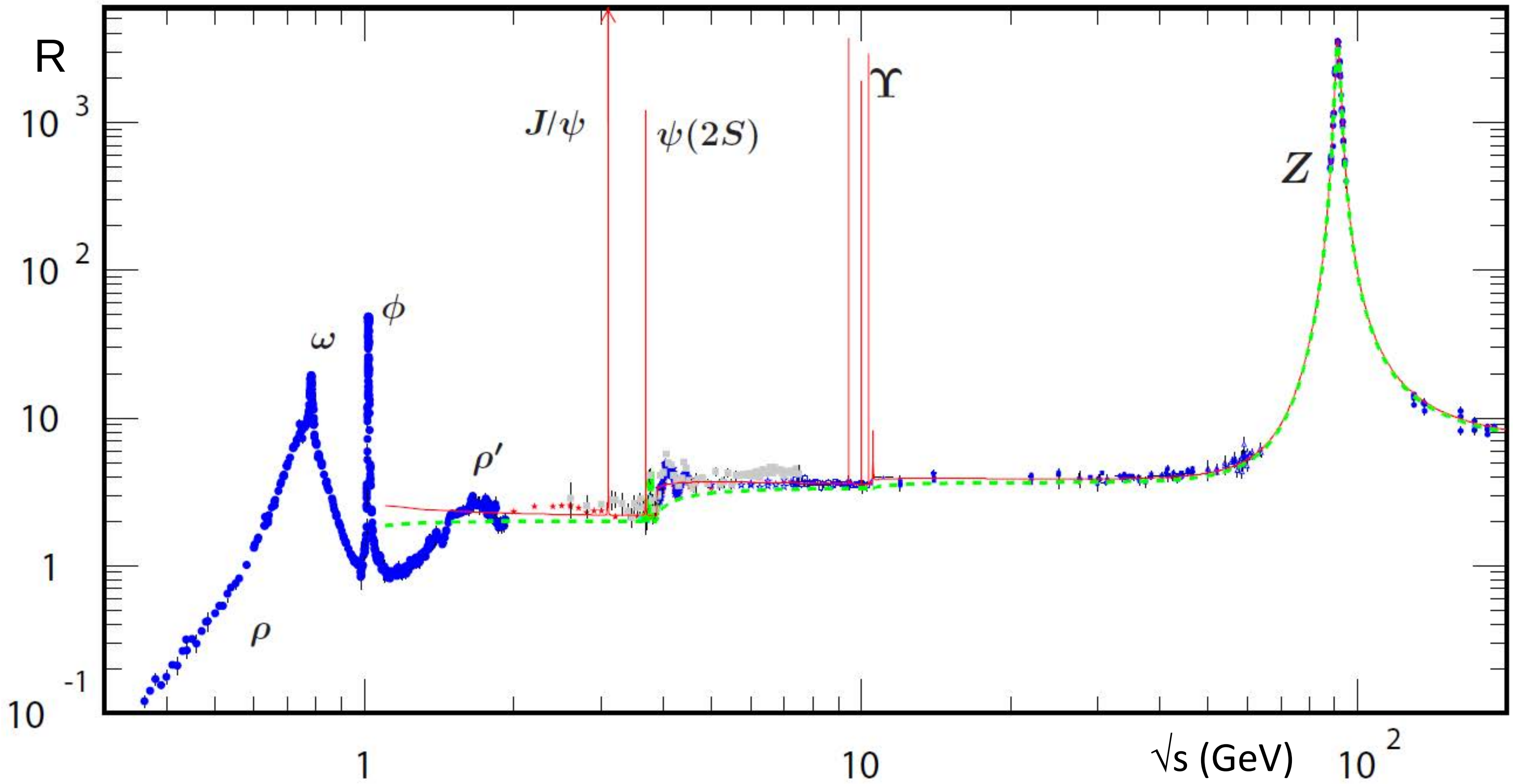
Plot R vs √s (=2E):

- resonances uū, dđ, ss̄ at 1-2 GeV (only those with $J^P=1^-$) (→"vector dominance");
- step at $2m_c$ (J/ψ);
- step at $2m_b$ (ϒ);
- slow increase at √s > 50 GeV (Z, next slide);
- [lot of effort required, as demonstrated by the number of detectors and accelerators];
- strong evidence for the color (factor 3 necessary).

plots from [PDG, 588]



Collisions e^+e^- : R vs $\sqrt{s}$ (large $\sqrt{s}$)

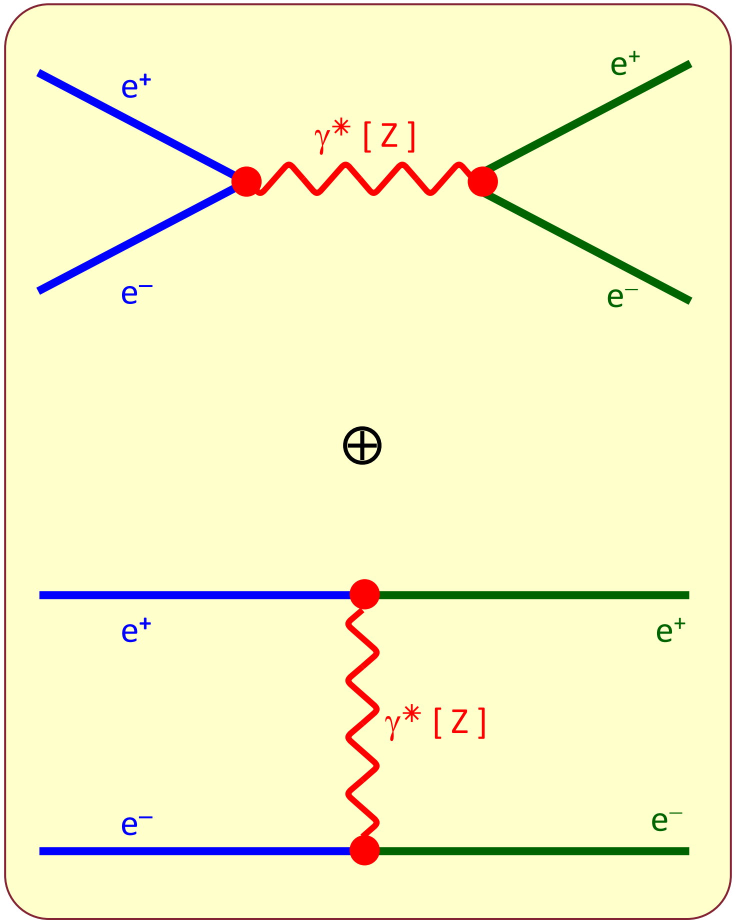


- The full range 200 MeV < $\sqrt{s}$ < 200 GeV (3 orders of magnitude !!!).
- For $\sqrt{s}$ > 50 GeV new phenomenon: electroweak interactions and the Z pole.

Collisions e^+e^- : $e^+e^- \rightarrow e^+e^-$

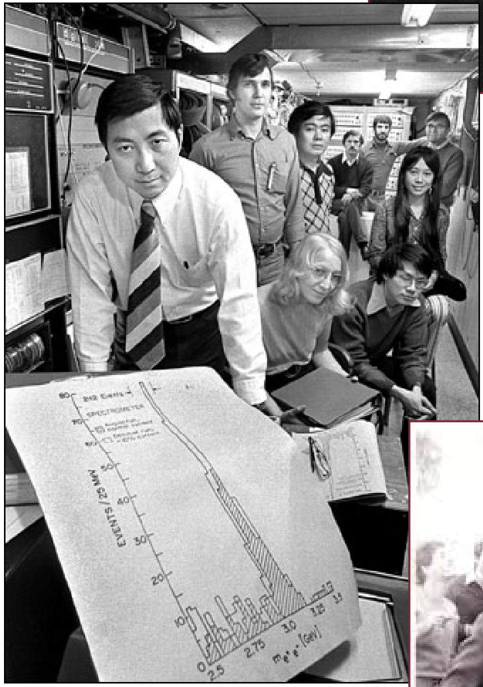
The case $e^+e^- \rightarrow e^+e^-$ (Bhabha scattering) is different, as seen before:

- two Feynman diagrams with a spin-1 boson exchange (γ^* [+ Z at higher energy]) :
  - s-channel, similar to $\mu^+\mu^-$;
  - t-channel, like e^+e^+;
  - interference between the two diagrams [four at higher energies];
- the angular distribution (see before) reflects these differences;
- [*il va sans dire que*] on an event-by-event basis it is NOT possible to determine whether an event belongs to s- or t-channel; however, different regions of the final state parameter space are actually dominated by s- or t-channel [*therefore physicists speak of "s-channel" physics (e.g. the formation of resonances) or t-channel physics (e.g. Bhabha at small* θ)].



The November Revolution

- The u,d,s quarks have not been predicted; in fact the mesons and baryons have been discovered, and later interpreted in terms of their quark content [§ 1];
- ***Some theoreticians had foreseen another quark, based on* (no $K^0 \to \mu^+\mu^-$)*, but people did not believe it.***
- In November 1974, the groups of Burton Richter (SLAC) and Samuel Ting (Brookhaven) discovered simultaneously a new state with a mass of $\approx$ 3.1 GeV and a tiny width, much smaller than their respective mass resolution.
- Ting & coll. had the name "J", while Richter & coll. called it "ψ". Today's name is "J/ψ".
- We split the discussion : start with the hadronic experiment.
- The width was measured, after some time, to be 0.087 MeV, a surprisingly small value for a resonance of 3 GeV mass.



the two experiments are quite different: we will review first the "J" and then the "ψ".

The November Revolution : J

- The group of Ting at the AGS proton accelerator measured the inclusive production of e^+e^- pairs in interactions of 30 GeV protons on a plate of beryllium :

  $$p\,Be \to e^+e^-\,X.$$

- The detector was designed to search for high mass resonances with $J^P = 1^-$ (= γ), decaying into (e^+e^-) pairs.

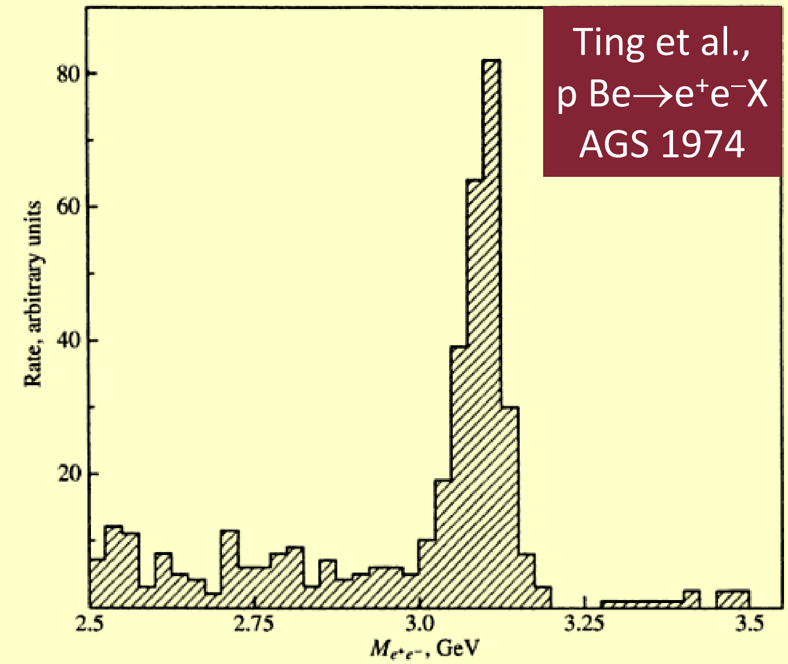
- They were very clever in minimizing the multiple scattering $\to$ the resolution for the invariant mass was good:

  $$\Delta m(e^+e^-) \approx 20 \text{ MeV}.$$

- This resolution allowed for a much higher sensitivity wrt another previous exp. (Leon Lederman), which studied $\mu^+\mu^-$ pairs in the same range. Lederman had a "shoulder" in $d\sigma/dm(\mu^+\mu^-)$, but no conclusive evidence [next slide].

- Ting called the new particle "J", because of the e.m. current.

Measured quantum numbers of the J:
- mass ~3.1 GeV;
- width << 20 MeV (upper limit, not meas.);
- charge = 0;
- $J^P = 1^-$;
- no isospin, Γ, other decay modes ...

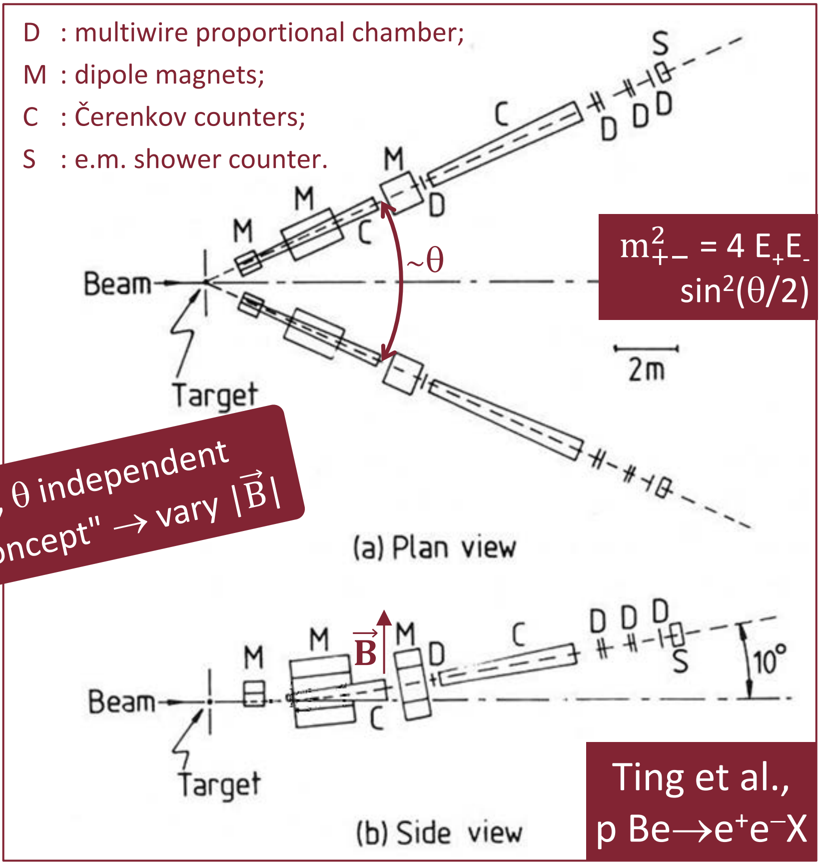


The November Revolution : the J experiment

- The Ting experiment used a two arm magnetic spectrometer, to measure separately the electron and the positron.
- Ting (and also Lederman) studied the Drell-Yan process [§$\bar{p}p$]: hadron collisions $\to \gamma^* \to \ell^+\ell^-$ (Ting: e^+e^- / Lederman: $\mu^+\mu^-$).
- Leptonic events are rare $\to$ very intense beams (2×10^{12} ppp $^{(*)}$) $\to$ high rejection power (~10^8) to discard hadrons, that can fake e^+e^- or $\mu^+\mu^-$.
- Advantage in the $\mu^+\mu^-$ case: μ penetration $\to$ select leptons from hadrons with a thick absorber in a large solid angle $\to$ larger acceptance, higher counting rate.
- Disadvantage : thick absorber $\to$ multiple scattering $\to$ worst mass resolution.

$^{(*)}$ "ppp" : "particles (or protons) per pulse", i.e. once per accelerator cycle every few seconds; it is the typical figure of merit of a beam from an accelerator.

- Benefit in the e^+e^- case: electron identification with Čerenkov counter(s) + calorimeters $\to$ simpler setup.
  Disadvantage : small instrumented solid angle $\to$ smaller yield.



$m^2_{+-} = 4\, E_+ E_- \sin^2(\theta/2)$

"p, θ independent concept" $\to$ vary $|\vec{B}|$

Ting et al., p Be$\to e^+e^-$X

The November Revolution : $\Delta m_{c\bar{c}}$

Problem (see previous slides)

Three similar exp. distributions:

$d\sigma(\text{hadron Nucleus} \rightarrow \ell^+ \ell^- X) / dm_{\ell\ell}$.

Similar dynamics:

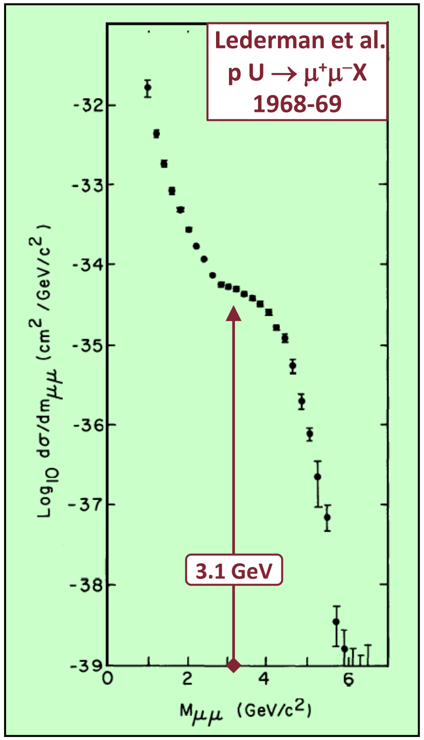
- continuum, exponentially falling [*yes, even in Ting's plot*];
- resonance(s) on top [*how many/plot ?*].

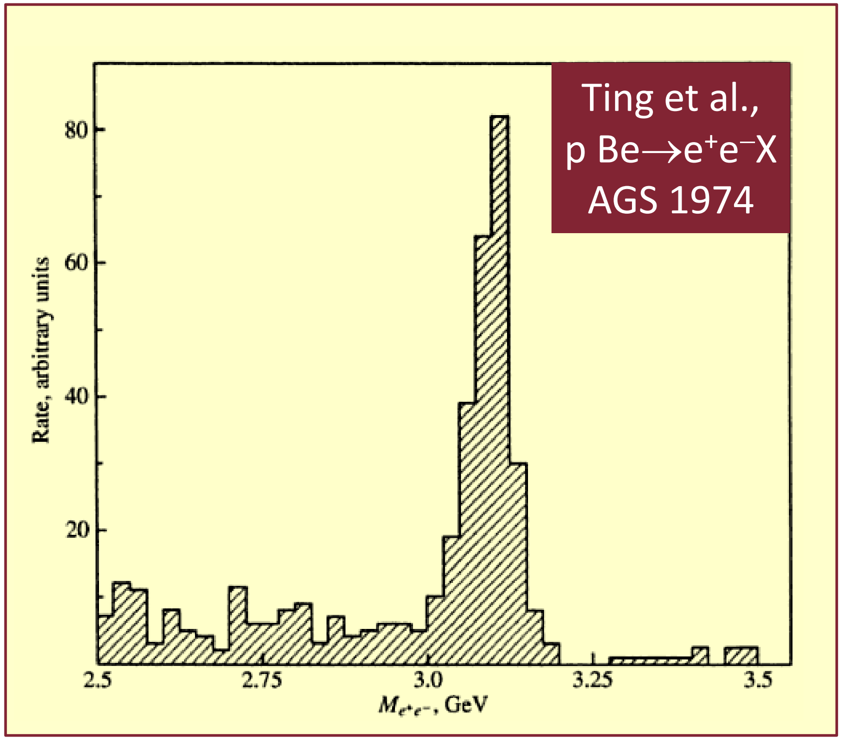
Differences:

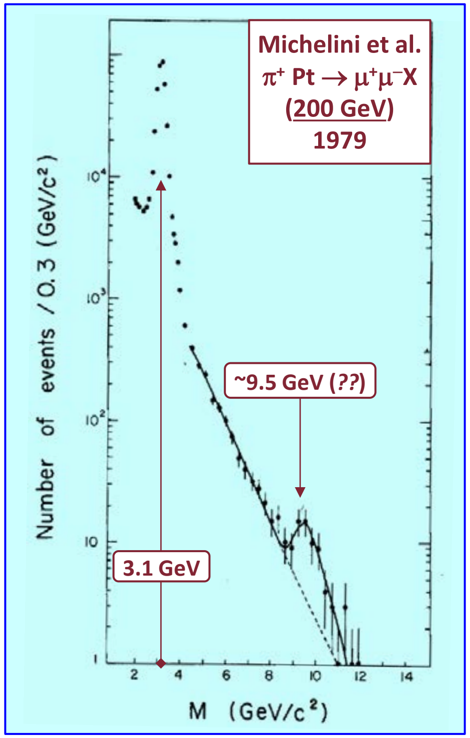
- $m_{\ell\ell}$ resolution [*!!! why ?*];
- horizontal scale (i.e. mass interval);
- vertical scale (i.e. resonance size)

Please comment on:

- effect of these differences on ratio resonance/continuum ($\rightarrow$ discovery ?);
- "quality" of the experiments.



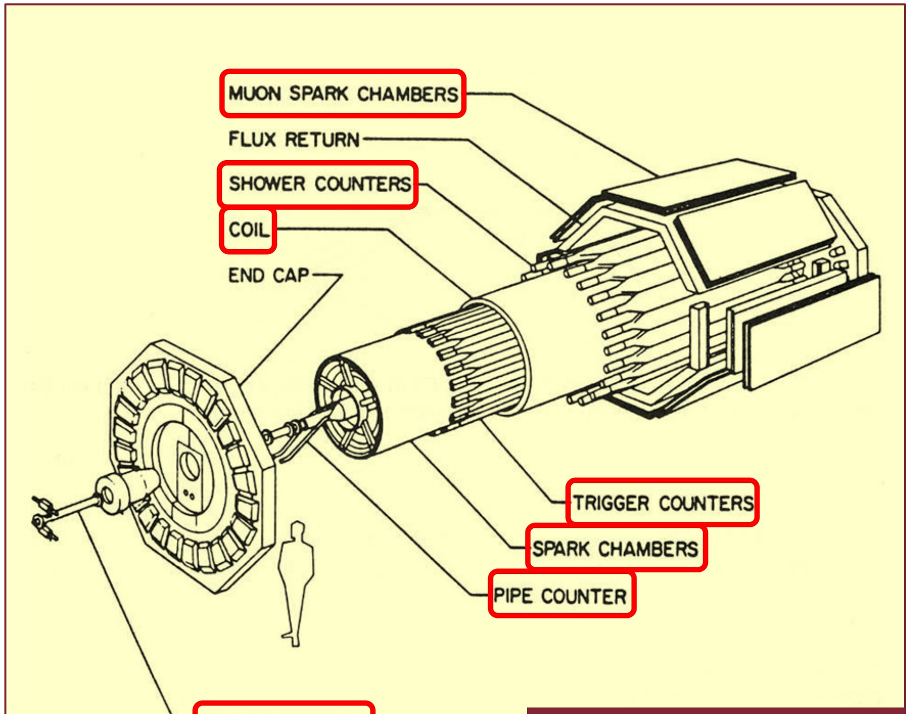




The November Revolution : Mark I

[back to 1974 : they did not know]

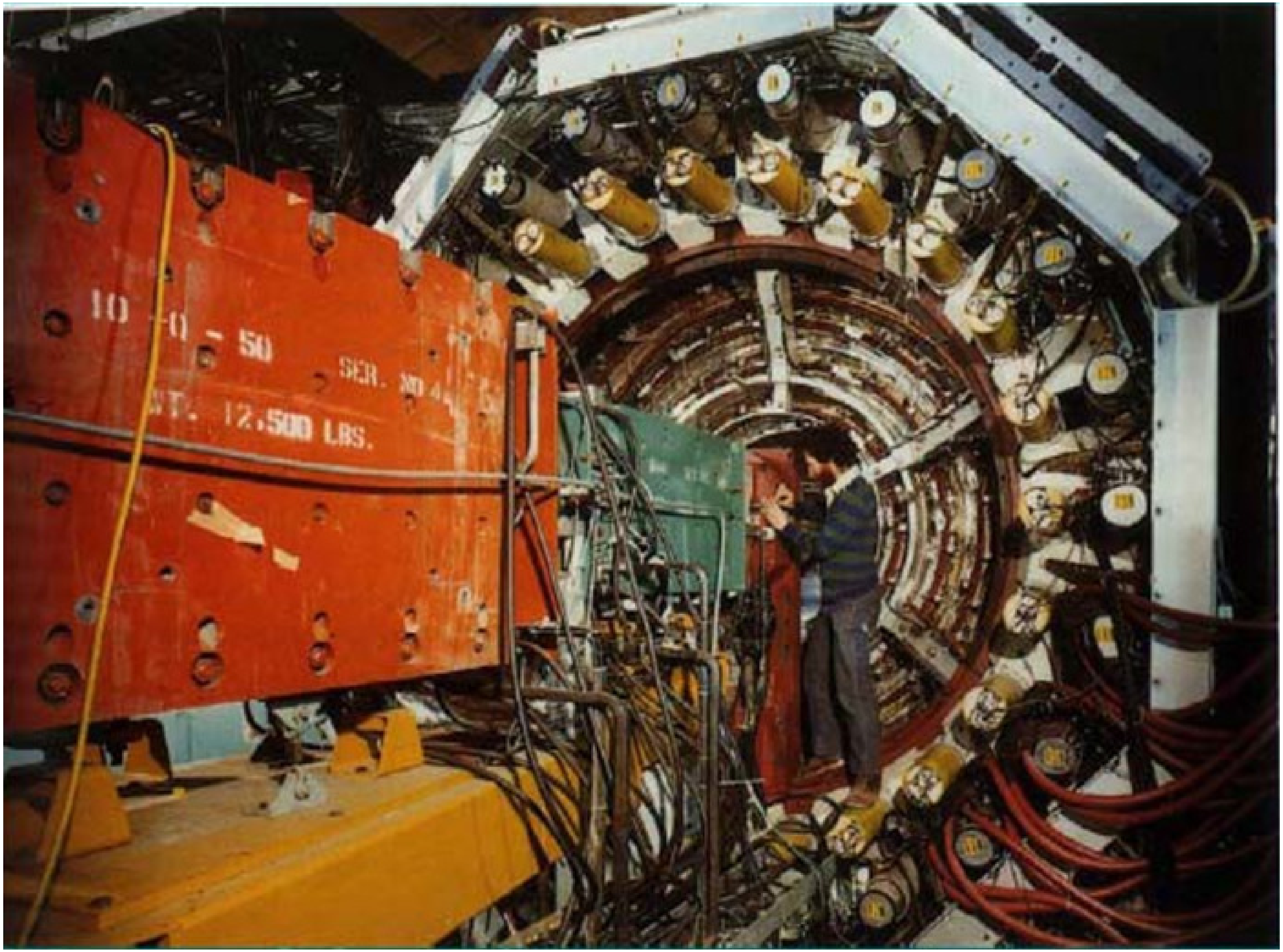
- Mark I at the e^+e^- collider SPEAR was studying collisions at $\sqrt{s}$ = 2.5 ÷ 7.5 GeV.
- The detector was made by a series of concentrical layers ("onion shaped").
- Starting from the beam pipe :
  - magnetostrictive spark chamber (tracking),
  - time-of-flight counters (particles' speed + trigger),
  - coil (solenoidal magnetic field, 4.6 kG),
  - electromagnetic calorimeter (energy and identification of γ's and $e^\pm$'s),
  - proportional chambers interlayered with iron plates (identification of $\mu^\pm$'s).



Mark I (SLAC –LBL)

- [*Notice the strong similarity among all the Collider detectors : CMS – 40 years later – has the same "onion" structure, with a scale factor > 10, i.e. a volume ~1000 times larger. However, ATLAS is different*].

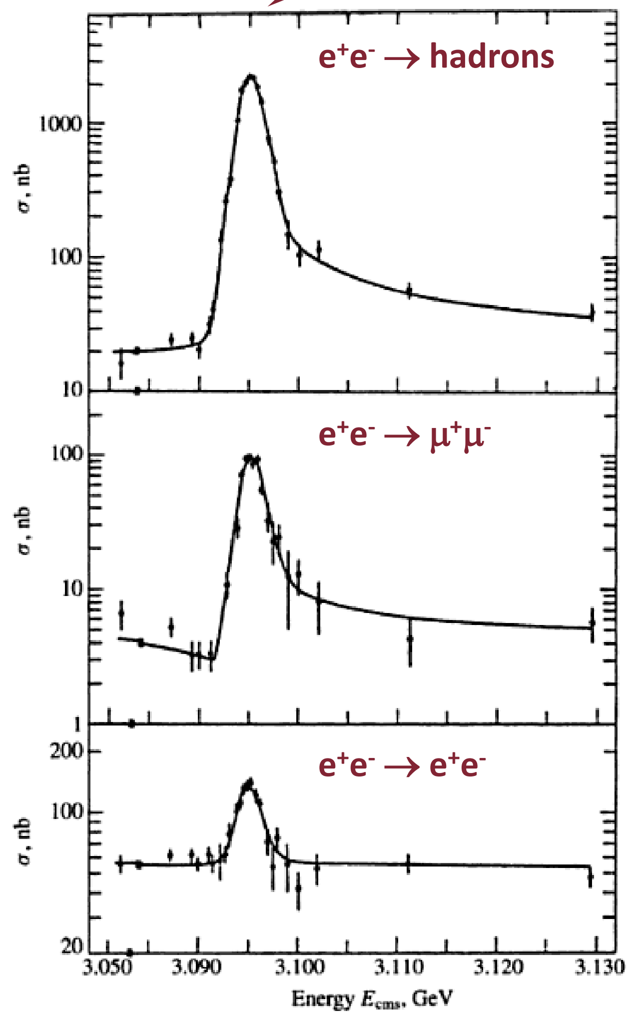
The November Revolution : Mark I at SLAC



The November Revolution : ψ

- In 1974, up to the highest available energies, $R = \sigma(e^+e^- \to \text{hadrons}) / \sigma(e^+e^- \to \mu^+\mu^-) \approx 2$.
- Measurements at the Cambridge Electron Accelerator (CEA, Harvard) in the region of energies of SPEAR had found $R \cong 6$ (a mixture of continuum and resonances). Also ADONE at LNF, which could reach an energy just sufficient, was not pushed to its max energy [*At the time the large amount of information carried by R was not completely clear*].
- At the novel Collider SPEAR, the scanning in energy was performed in steps of 200 MeV.
- The measured cross-section appeared to be a constant, NOT with expected trend $\propto 1/s$.
- When a drastic reduction in the step (200 $\to$ 2.5 MeV) increased the "resolving power", a resonance appeared, with width compatible with the beam dispersion (even compatible with a δ-Dirac).
- The particle was called "ψ" (see fig. on page 2).

inside Mark I acceptance and normalized to Bhabha.



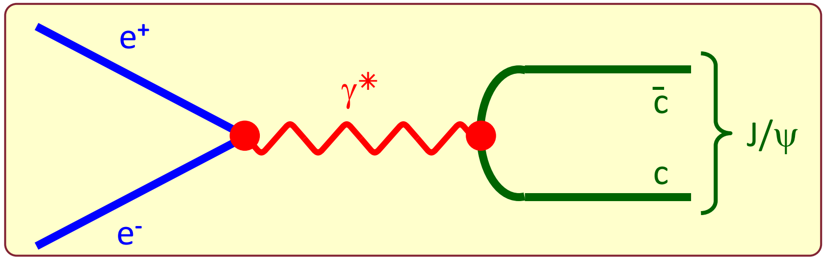
Charmonium: J/ψ properties

- After some discussion, the correct interpretation emerged :
  - the resonance, now called J/ψ, is a bound state of a new quark, called **charm** (c), and its antiquark;
  - the c had been proposed in 1970 to exclude FCNC [GIM mechanism, § *4*];
  - the J/ψ has $J^P = 1^-$ [*next slide*];
  - the name "charmonium" is an analogy with positronium ("onium" : bound state particle-antiparticle);
- The cross-section (Breit-Wigner) for the formation of a state (J_R = 1) from e^+e^- ($S_a = S_b$ = ½), followed by a decay into a final state, shows that [*see* § *intro.*]:

$$\sigma(ab \to J/\psi \to f\bar{f}, \sqrt{s}) = \frac{16\pi}{s} \frac{(2J_R+1)}{(2S_a+1)(2S_b+1)} \left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{f\bar{f}}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

- $\sigma(e^+e^- \to J/\psi \to f\bar{f}, \sqrt{s}) = \frac{12\pi}{s}\left[\frac{\Gamma_e}{\Gamma_{tot}}\right]\left[\frac{\Gamma_f}{\Gamma_{tot}}\right]\frac{\Gamma_{tot}^2/4}{\left(m_{J/\psi}-\sqrt{s}\right)^2+\Gamma_{tot}^2/4};$
- Γ_f = width for the $\left(J/\psi \leftrightarrow f\bar{f}\right)$ coupling;
- $\Gamma_{tot} = \Gamma_e + \Gamma_\mu + \Gamma_{had}$ = full width of J/ψ;
- $\Gamma_f/\Gamma_{tot} = BR(J/\psi \to f\bar{f})$ [very useful].

- After 1974, many exclusive decays have been precisely measured, all confirming the above picture; the last PDG has 227 decay modes; the present most precise value of the mass and width is
  m(J/ψ) = 3097 MeV, Γ_{tot}(J/ψ) = 93 keV.



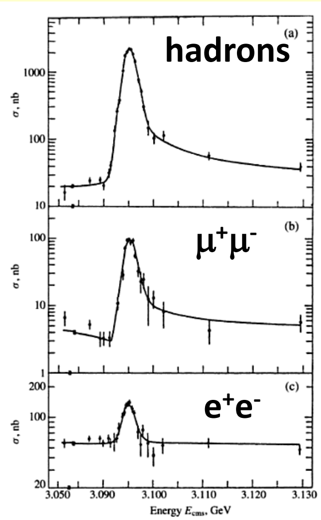
Charmonium : J/ψ quantum numbers

At SPEAR they were able to measure many of the J/ψ quantum numbers :

- the resonance is asymmetric (the right shoulder is higher); therefore there is interference between J/ψ formation and the usual γ^* exchange in the s-channel; therefore the J/ψ and the γ have the same $\boxed{J^P = 1^-;}$
- from the cross section, by measuring σ_{had}, σ_μ and σ_e, they have 3 equations + a constraint (see the box, three $\sigma_f + \Gamma_{tot}$) for the 4 unknowns (three $\Gamma_f + \Gamma_{tot}$); therefore they measured everything, obtaining a $\boxed{\Gamma_{tot}}$ very small (~90 keV, a puzzling results, see next slides);
- the equality of the BR ($J/\psi \to \rho^0\pi^0$) and ($\to \rho^\pm\pi^\mp$) implies isospin $\boxed{I = 0;}$
- the J/ψ decays into an odd (3, 5) number of π, not ~~in an even (2, 4) number~~; this fact has two important consequences :
  - ➢ the G-parity is conserved in the decay (so the $\boxed{\text{J/ψ decays via strong inter.}}$).
  - ➢ $\boxed{\text{G-parity = -1}}$ [also $(-1)^{I+\ell+s} = -1$].

$$\sigma(e^+e^- \to J/\psi \to f\bar{f}) =$$
$$= \frac{3\pi}{s} \frac{\Gamma_e \Gamma_f}{\left(m_{q\bar{q}} - \sqrt{s}\right)^2 + \Gamma_{tot}^2/4}$$
$$= \sigma_f(\Gamma_e, \Gamma_f, \Gamma_{tot}, \sqrt{s});$$
$$\Gamma_{tot} = \Gamma_e + \Gamma_\mu + \Gamma_{had}$$

[see previous slide].

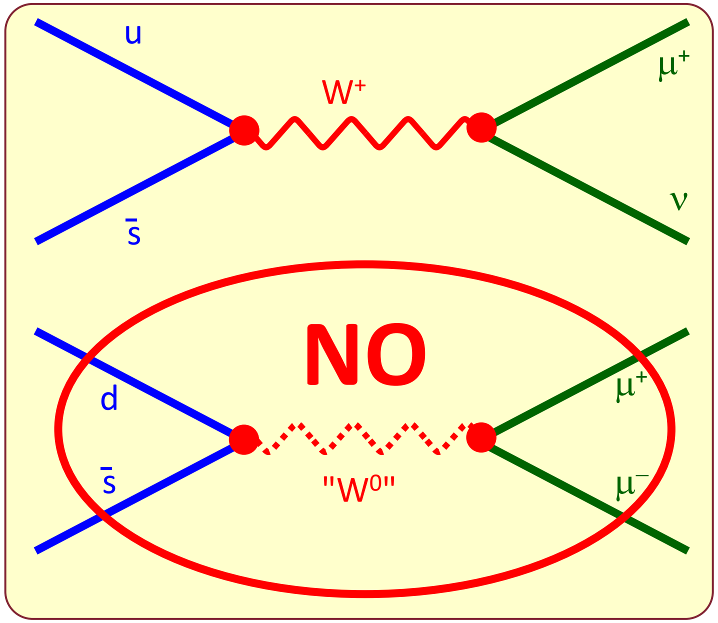


4 equations (f=e,μ,had + Γ_{tot}), 4 unknowns;
NO direct measurement of "width" required.

Charmonium : the GIM mechanism

- The weak neutral current processes between quarks of different flavor (FCNC, "Flavor Changing Neutral Current") are strongly suppressed [e.g. $\Gamma(K^0{}_L \to \mu^+\mu^-) << \Gamma(K^\pm \to \mu^\pm\nu)$].
- This fact was explained in 1970 by S. Glashow, J. Iliopoulos and L. Maiani by introducing the **charm quark** (*Phys. Rev. D2, 1285*);
- they predicted:
  - a fourth quark (c), identical to the u quark, apart from its mass, carrying a new quantum number C, "charm";
  - as for the strangeness, C is conserved in strong and electromagnetic interactions and violated in weak interactions;
  - the lightest charmed mesons are $c\bar{q}$ or $\bar{c}q$ pairs (q = uds), and have a mass of 1500 - 2000 MeV and $J^P = 0^-$;
  - these mesons decay weakly; because of their larger mass, their lifetimes are O(ps), an order of magnitude shorter than those of the K mesons;
  - the positive meson with open charm ($c\bar{d}$, now called D^+) decays preferably in final states with negative strangeness ($c \to sf\bar{f}$, $\Delta S = \Delta C$).

[*see* § *4 for more details*]



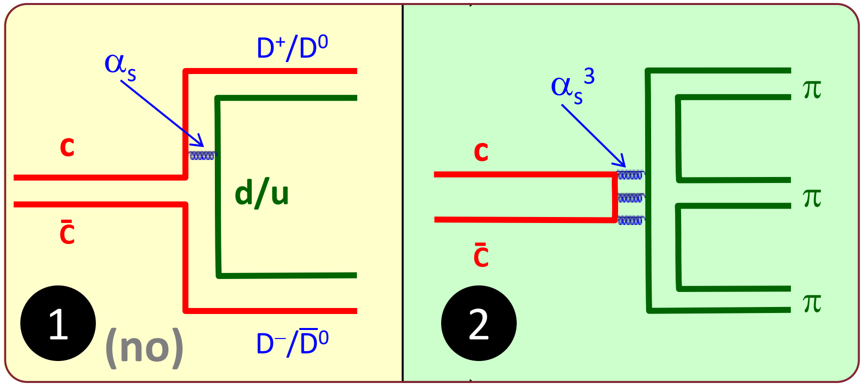
Charmonium : QCD decay

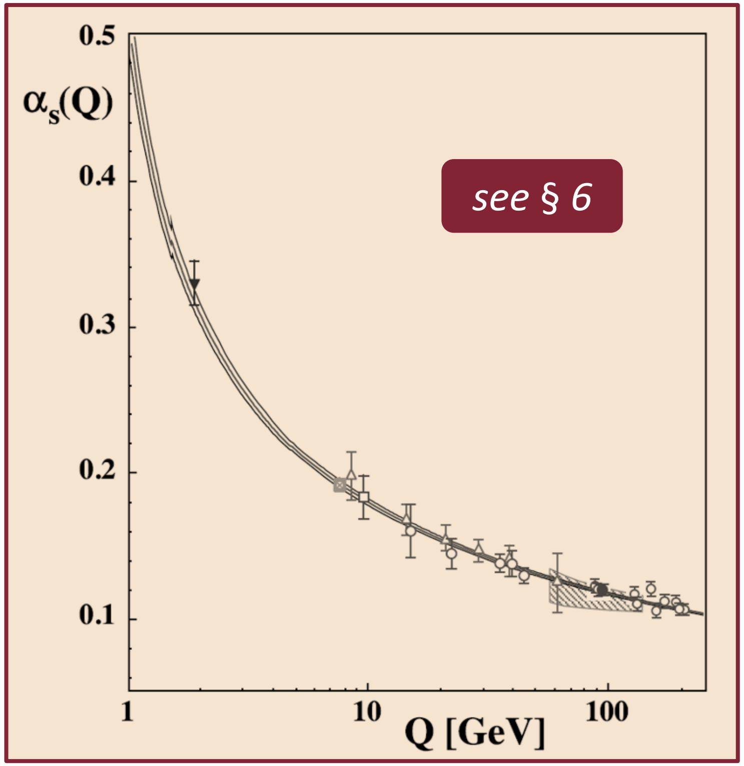
$Q\bar{Q}$ states(*) [e.g. ϕ ($s\bar{s}$), J/ψ ($c\bar{c}$), Υ ($b\bar{b}$)] :

- decay preferentially ❶ [$(Q\bar{Q}) \to (Q\bar{q})\,(\bar{Q}q)$], e.g. $\phi \to \bar{K}K$, i.e. [$(s\bar{s}) \to (\bar{d}s)\,(d\bar{s})$];
- $J/\psi \to D^+D^-$ (or $D^0\bar{D}^0$) [$(c\bar{c}) \to (\bar{d}c)\,(d\bar{c})$ or $(\bar{u}c)\,(u\bar{c})$] forbidden ($m_{J/\psi} < 2m_D$);
- then $c\bar{c}$ annihilate into gluons ($J/\psi \to \pi$'s ❷):
  - ➢ 1 gluon forbidden by color;
  - ➢ 2 gluons forbidden by C-parity [$C_{2g} = +1$; $C_{J/\psi} = C_\gamma = -1$];
  - ➢ 3 gluons allowed :

$$\Gamma(Q\bar{Q} \to 3g \to \pi's) = \frac{160(\pi^2 - 9)}{81 m^2_{Q\bar{Q}}} \boxed{\alpha_s^3} |\psi(0)|^2 ;$$

- The value α_s^3 (and its "running" [§ *6*]) produces a smaller width for larger masses :
  - ➢ $\alpha_s^3(m^2_\phi) \approx 0.5^3 = .125$;
  - ➢ $\alpha_s^3(m^2_{J/\psi}) \approx 0.3^3 = .027$;
  - ➢ $\alpha_s^3(m^2_\Upsilon) \approx 0.2^3 = .008$.





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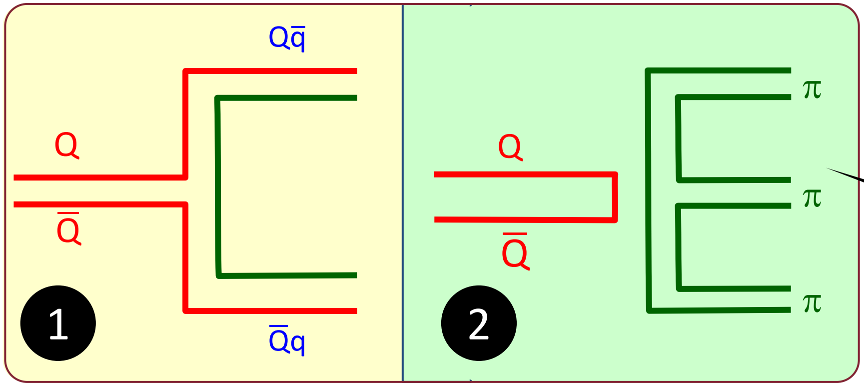
(*) in these slides: q = u/d, Q = s/c".

Charmonium : the Zweig rule (OZI)

The "Zweig rule" was set out empirically in a qualitative way before the advent of QCD :

- compare $(\phi \to 3\pi) \leftrightarrow (\phi \to KK) \leftrightarrow (\omega \to 3\pi)$;
- in the decay of a bound state of heavy quarks Q, the final states without Q's ("decays with disconnected diagrams" 2) have suppressed amplitude wrt "connected decays" 1;
- if only the decays 2 are kinematically allowed (ex. J/ψ or Υ), the total width is small and the bound state is "narrow";

1963-1966 :
Susumu Okubo
(大久保 進
Ōkubo Susumu),
George Zweig,
Jugoro Iizuka (飯塚)



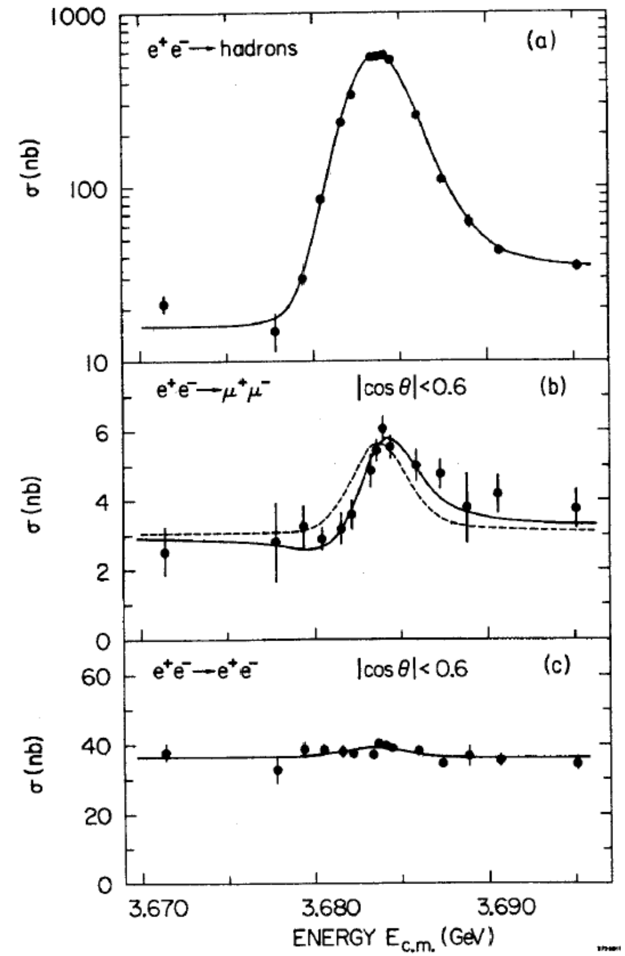
before the QCD advent, gluons were not considered.

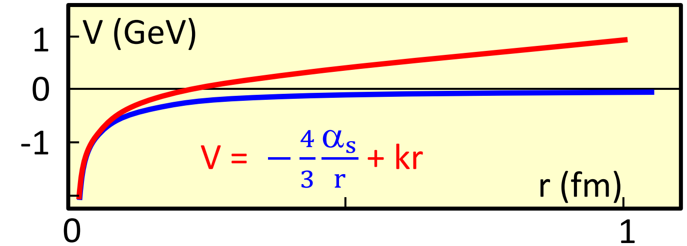
Charmonium: ψ'

- After the discovery of the J/ψ, at SPEAR they performed a systematic energy scanning with a very small step. After ten more days a second narrow resonance was found, called ψ', with the same quantum numbers of the J/ψ.
- The analysis shows that the J/ψ was the 1S state of $c\bar{c}$, while the ψ' is the 2S.
- Both particles have $J^P = 1^-$, I=0.
- The next page gives a scheme of the $c\bar{c}$ levels.
- They offer a reasonable agreement with the solution of the Schrödinger equation of a hypothetical QCD potential [*see § Standard Model*]

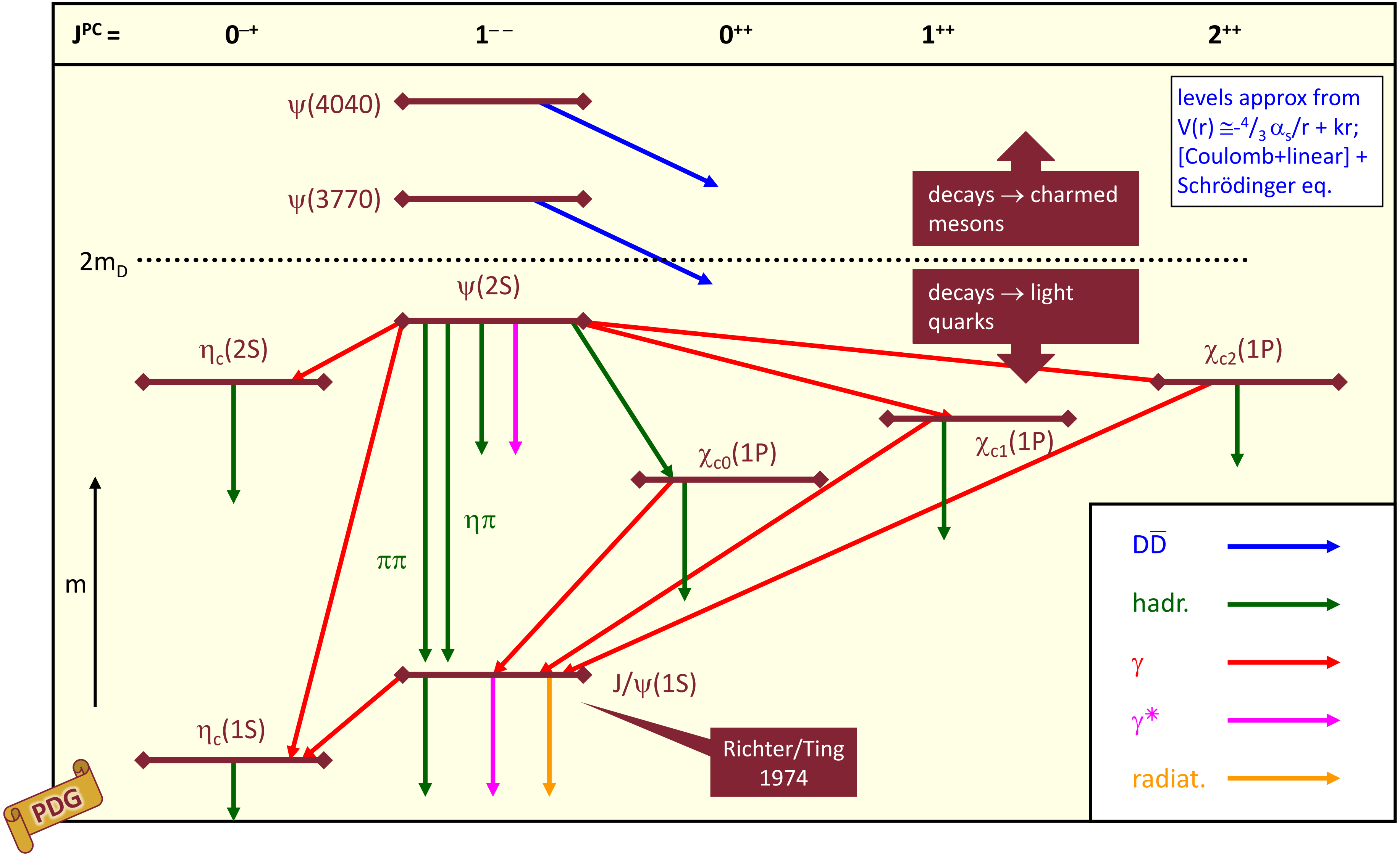
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr = \frac{A}{r} + Br.$$

- Notice that this approximation should become more realistic for heavier quarks, when the non-relativistic limit gets better.





Charmonium : $c\bar{c}$ levels

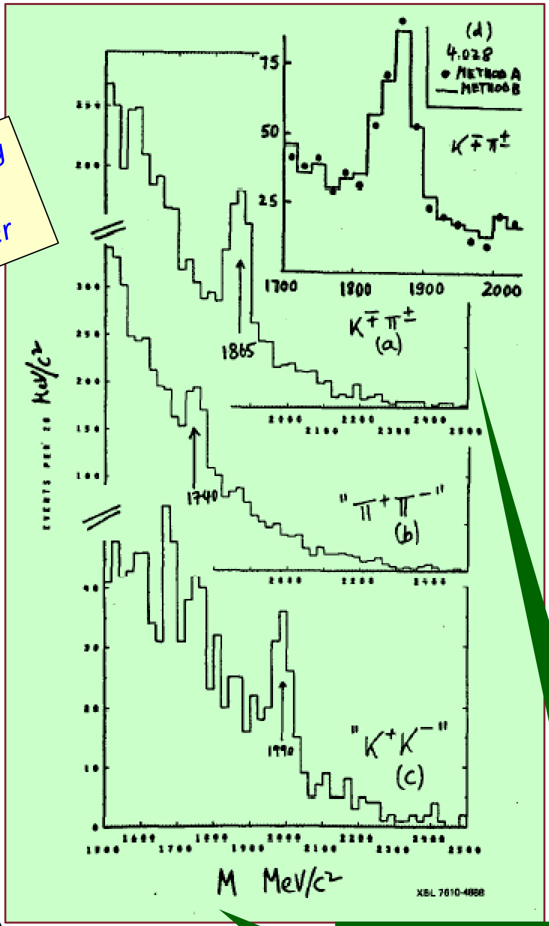


Open charm : discovery

- If the J/ψ is a bound $c\bar{c}$ state, then mesons $c\bar{q}$ and $\bar{c}q$ must exist, with a mass $m_{J/\psi}/2$ + 100÷200 MeV [$3690/2 < m_D < 3770/2$ MeV].
- In 1976, the Mark I detector started the search for charmed pseudoscalar mesons, the companions of π's and K's.
- They looked at $\sqrt{s}$ = 4.02 GeV in the channels

  $e^+e^- \to D^0\,\bar{D}^0\,X^0$; $\quad \to D^+\,D^-\,X^0$.
- According to theory, D-mesons lifetimes are small, with a decay vertex not resolved (*with 1976 detectors*) wrt the e^+e^- one.
- Therefore the strategy of selection was the presence of "narrow peaks" in the combined mass of the decay products.
- A first bump at 1865 MeV with a width compatible with the experimental resolution was observed in the combined mass ($K^\pm\pi^\mp$), corresponding to the D^0 and $\bar{D}^0$ decay.

handwriting by Gerson Goldhaber

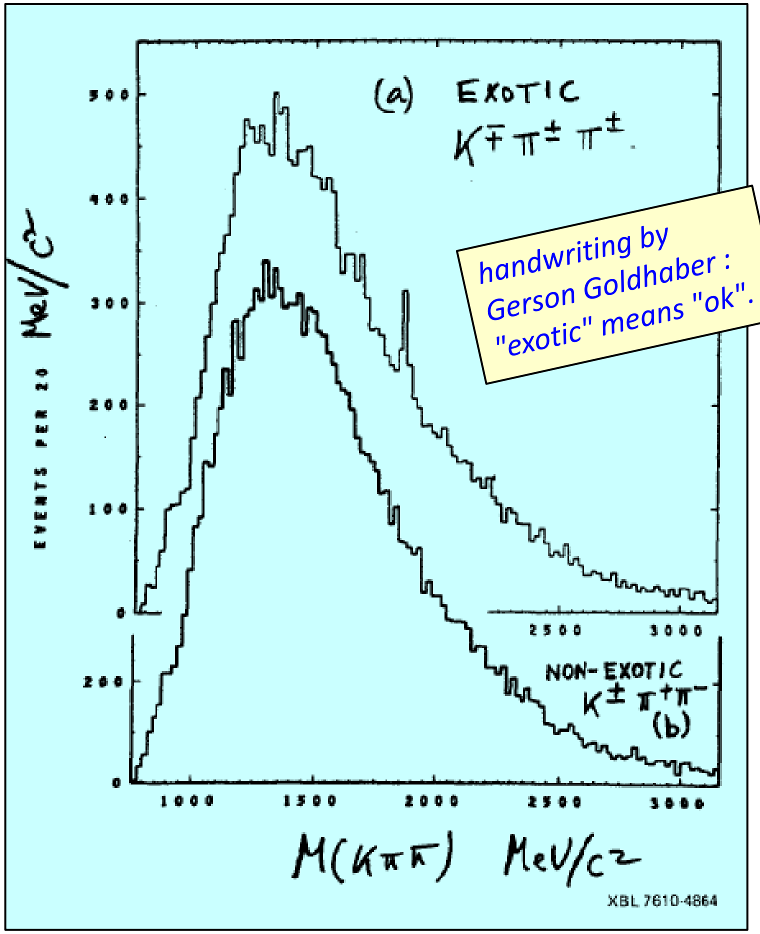


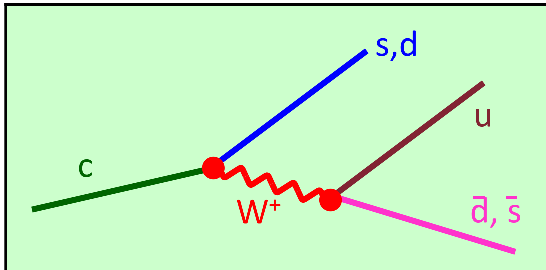
Today's technology allows for it !!!

They were afraid of K/π exp. misidentification → they computed the mass twice, also with the other particle hypothesis

Open charm: "C-allowed, suppressed"

- Also the mass ($K^{\mp}\pi^{\pm}\pi^{\pm}$) had a bump at 1875 MeV, corresponding to the D^+ and D^- decays.
- Moreover, in perfect agreement with the GIM predictions, no bump was found in ($K^{\pm}\pi^{+}\pi^{-}$), which is forbidden ("Cabibbo doubly suppressed", in this language).





the c quark decays through its Cabibbo couplings (see):
[c↔s, u↔d] ∝ cos θ_c = "big"
[c↔d, u↔s] ∝ sin θ_c = "small"

⅔	-⅓ ⅔ ⅓	K/π	"Cabibbo" dependence	
c →	s u d̄	$\bar{K}(n\pi)$	$\propto \cos^2\theta_c$	"allowed"
	s u s̄	$\bar{K}K(n\pi)$	$\propto \sin\theta_c \cos\theta_c$	"suppressed"
	d u d̄	$(n\pi)$	$\propto \sin\theta_c \cos\theta_c$	"suppressed"
	d u s̄	$K(n\pi)$	$\propto \sin^2\theta_c$	("suppressed")^2

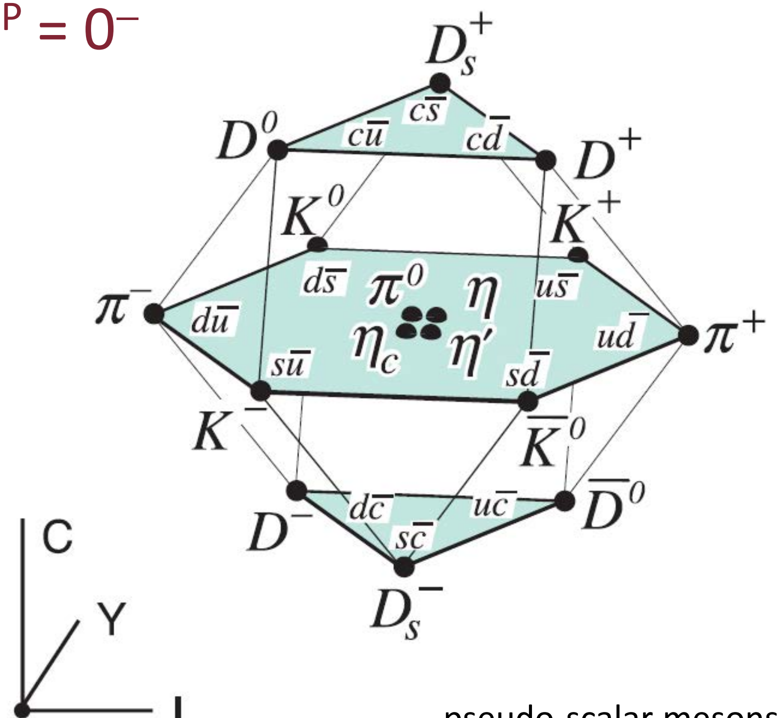
the so-called "$\Delta S = \Delta C$" rule :

$c \to \bar{K}$: $(C: +1 \to 0) \leftrightarrow (S: 0 \to -1)$

$\bar{c} \to K$: $(C: -1 \to 0) \leftrightarrow (S: 0 \to +1)$

Open charm: meson multiplets

$J^P = 0^-$

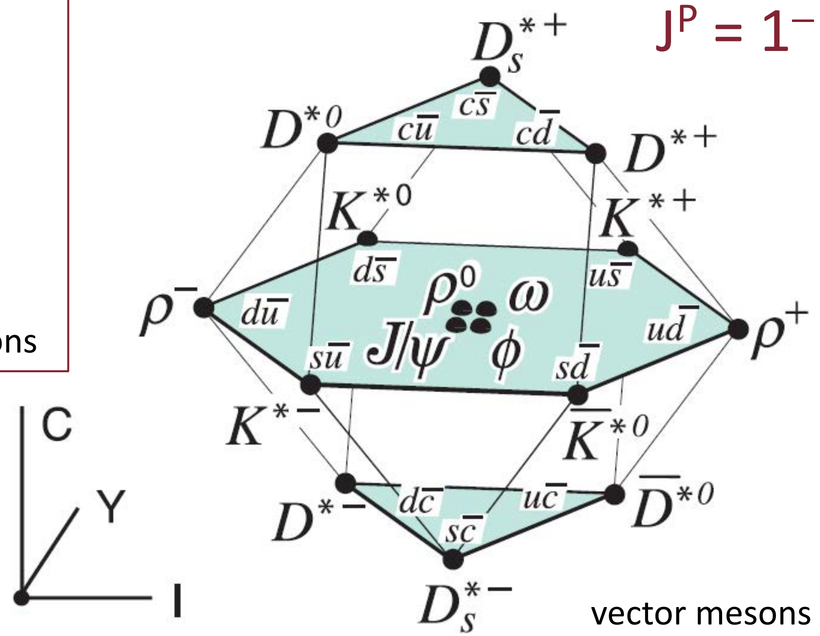


pseudo-scalar mesons

$SU(3)_{flavor} \rightarrow SU(4)_{flavor}$

With 4 quarks, the SU(3) nonets become multiplets in a 3-D space. However, the c quark has a large mass, so $SU(4)_{flavor}$ is much more broken that $SU(3)_{flavor}$.

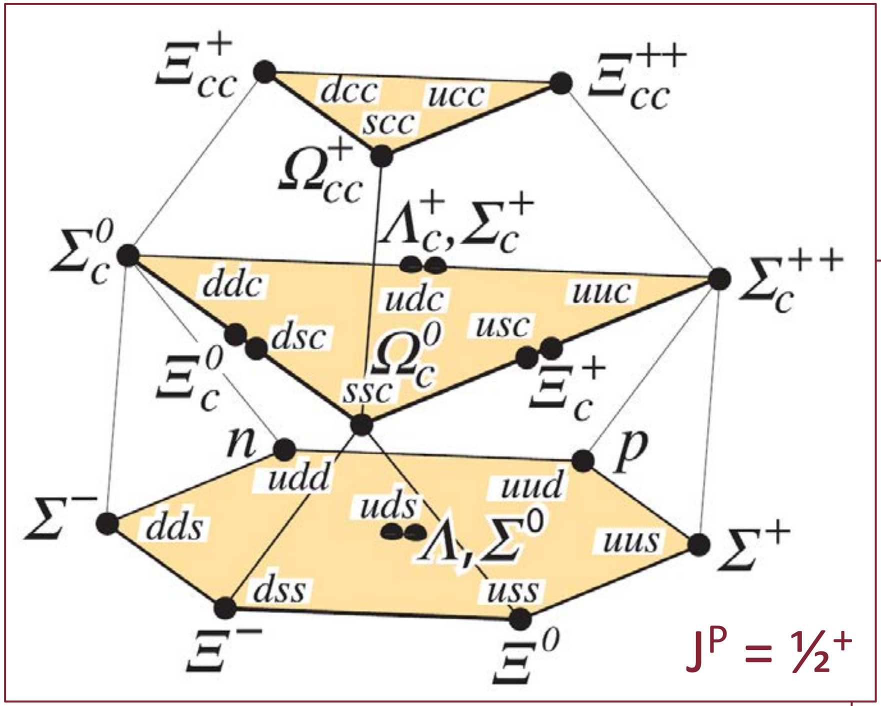
$J^P = 1^-$



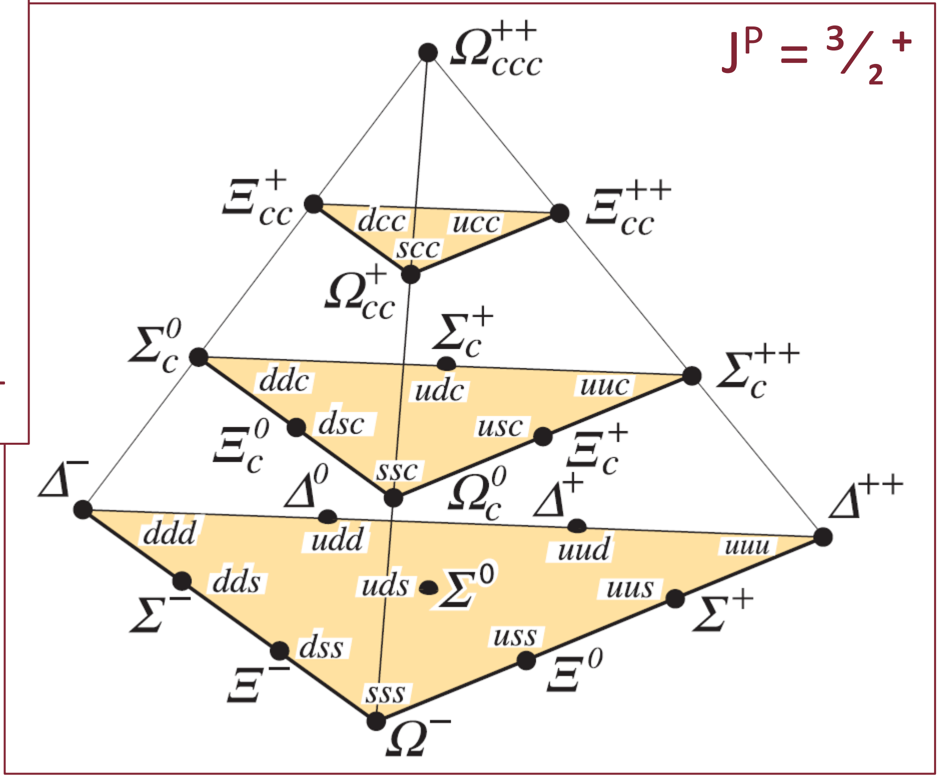
vector mesons

$4 \otimes \bar{4} = 15 \oplus 1.$

Open charm : baryon multiplets



SU(4)$_{flavor}$ baryons

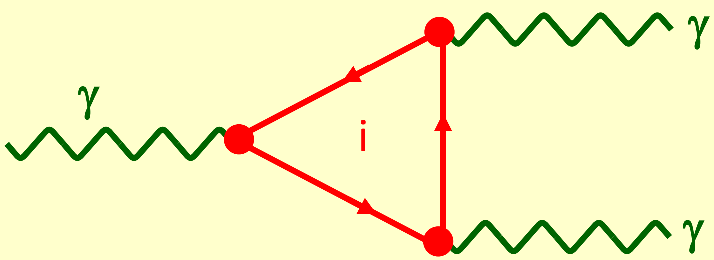


The 3rd family

- "who ordered that ?" [*I.I.Rabi about the* μ];
- in modern terms : "why consecutive families of quarks/leptons, differing only in mass ? why/how they mix ?" [*see* § *4-5*]
- as of today, nobody knows : the number of families and the mixing matrix are free parameters of the SM [*maybe one day some theory bSM will constrain it*];
- "non-QCD" constraints in the SM:
  - families must be complete : the existence of a single member (e.g. the ν or the ℓ⁻) implies the existence of all the others, to avoid **anomalies** (Adler-Bell-Jackiw); it requires $\Sigma_i\ e_i = 0$, where the sum runs on all members i and colors c of the family F [*see red box*];
  - the Z full width Γ^Z_{tot} constrains the number of "light ν's" [*see* § *LEP*] ;
  - in the SM, (at least) three families are necessary to generate a natural mechanism of CP violation in the quark decays [*see* § K^0];
  - in the SM, n_F is free, but n_c must be 3.

$$\begin{cases} \begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix} & \begin{matrix} e_i = -1 \quad ,c=1 \\ e_i = 0 \quad ,c=1 \end{matrix} \\ \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} & \begin{matrix} e_i = 2/3 \quad ,c=3 \\ e_i = -1/3, c=3 \end{matrix} \end{cases}$$

$$\boxed{\sum_F\left(\sum_i e_i\right) = n_F \times \left\{ \begin{matrix} (-1)+(0)+ \\ +3_c \times \left[\left(\frac{2}{3}\right)+\left(\frac{-1}{3}\right)\right] \end{matrix} \right\} = 0}.$$



The τ lepton : discovery

The analysis of Mark I data produced another beautiful discovery : the τ lepton (M. Perl won the 1995 Nobel Prize):

- the selection followed a method well known, pioneered at LNF-Frascati : the "unbalanced pairs $e^{\pm}\mu^{\mp}$" :

$$e^+e^- \to \tau^+\tau^-$$
$$\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau, \quad \tau^+ \to e^+\nu_e\bar{\nu}_\tau \quad \Big\} \to \mu^-e^+ \text{ (\underline{unbalanced})}$$

(+ CC μ^+e^-).

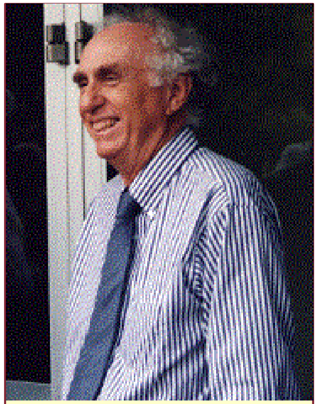
- events from this process are extremely clean and free from background [*see fig.*];
- the e^+e^- / $\mu^+\mu^-$ unbalanced pairs, which have to be present in the correct number

$N_{unb}(e^+e^-) = N_{unb}(\mu^+\mu^-) =$

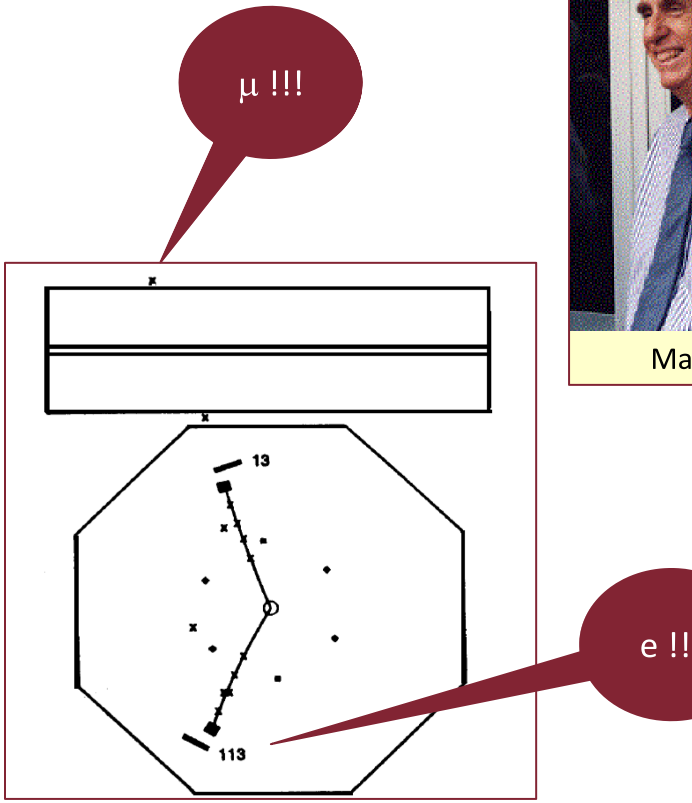
$= N(e^+\mu^-) = N(e^-\mu^+)$,

are only used to cross-check the sample.

In principle the τ lepton has very little to do with the c quark. However collider, detector, energy, selection and analysis are closely linked. Therefore, in experimental reviews, the τ lepton is usually treated together with the charm quark.



Martin Perl

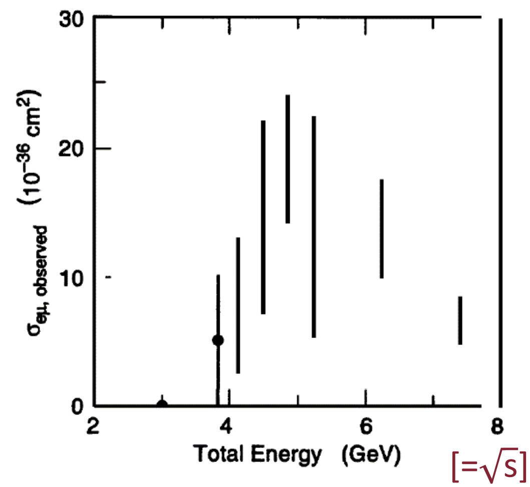


The τ lepton : identification

Simple method: the yield of $e^{\pm}\mu^{\mp}$ pairs vs $\sqrt{s}$: it immediately points to the threshold $\sqrt{s} = 2m_\tau$.

- therefore : $m_\tau \approx 1780$ MeV.
  [best present value 1776.8 MeV]
- why is the $\tau^{\pm}$ a lepton ?
  - at the time, the evidence came from the lack of any other plausible explanation;
  - today, the evidence is solid :
    - the Z and W decays into (e μ τ) with the same BR and angular distribution;
    - the lifetime has been measured and found in agreement with predictions …
- the discovery of the τ started the hunt for the particles of a new (3rd) family, still unknown:
  - the ν_τ (possibly mixed with the others);
  - the pair of quarks q_{up} q_{down}, similar to ud (now called **t**op and **b**ottom).

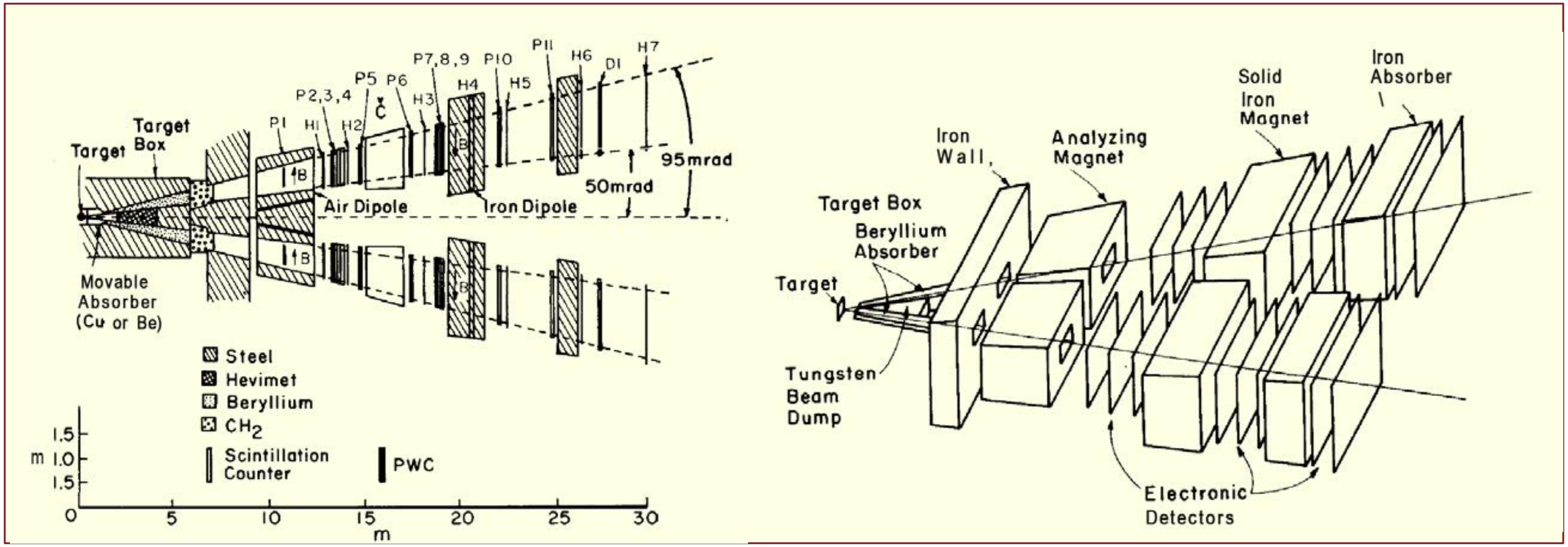
$$e^+e^- \rightarrow \tau^+\tau^-$$
$$\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau, \quad \tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau \quad \Big\} \rightarrow \mu^-e^+$$
(unb.)



The b quark : discovery

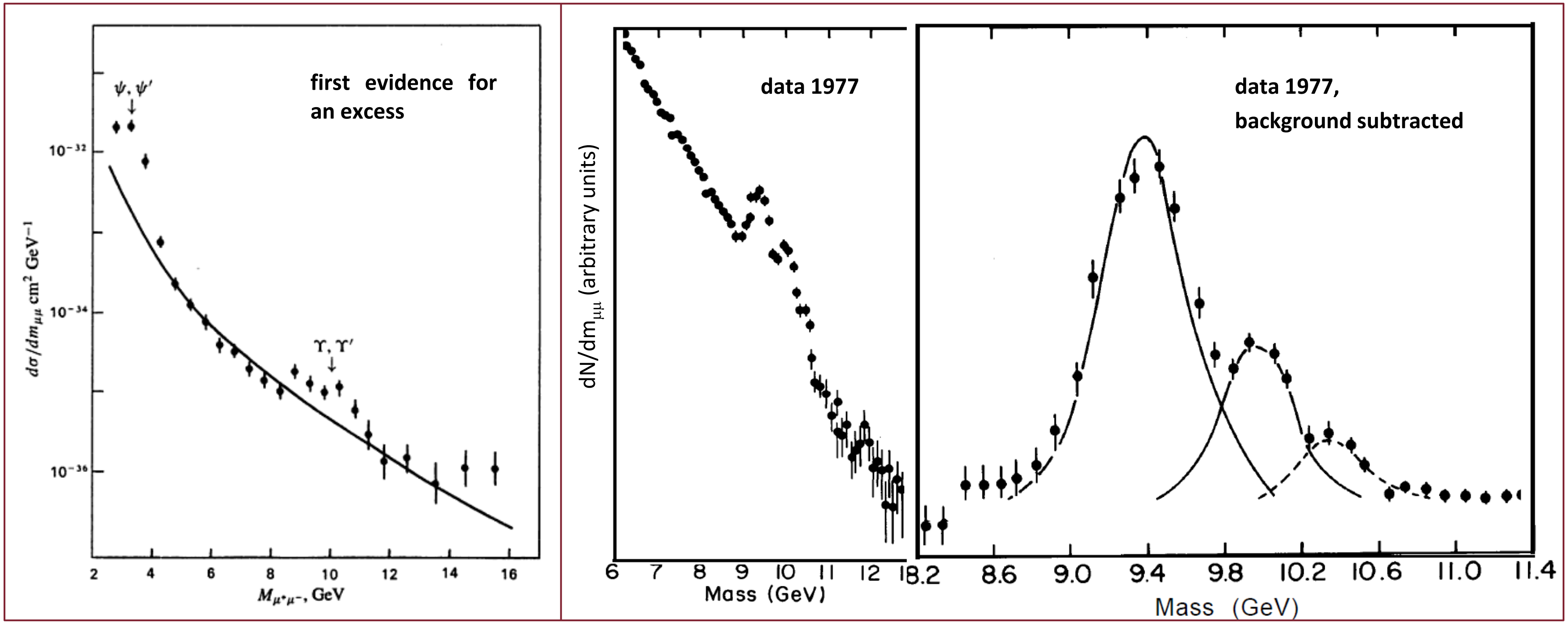
- The down quark of the 3rd family was called b (= beauty, bottom).
- In 1977 Leon Lederman and collaborators built at Fermilab a spectrometer with two arms, designed to study $\mu^+\mu^-$ pairs produced by interactions of 400 GeV protons on a copper (or platinum) target.
- The reaction under study was again the Drell-Yan process. As already pointed out, this type of events is rare, therefore requiring intense beams (in this case 10^{11} ppp) and high rejection power against charged hadrons.

Leon Lederman



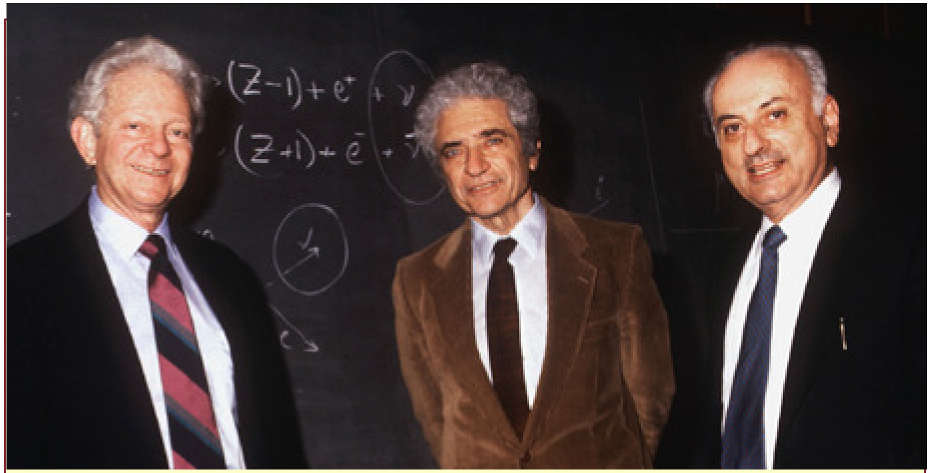
The b quark : dσ/dm

- The usual price of the absorber technique is a loss of resolution in the muon momenta, which was $\Delta m_{\mu\mu} / m_{\mu\mu} \approx 2\%$.
- The figures show the distribution of $m_{\mu\mu}$. Between 9 and 10 GeV : there is a clearly visible excess.
- When the $\mu\mu$ continuum is subtracted, the excess appears as the superimposition of three separate states.
- The states, called $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ are bound states $b\bar{b}$.



The b quark : open b

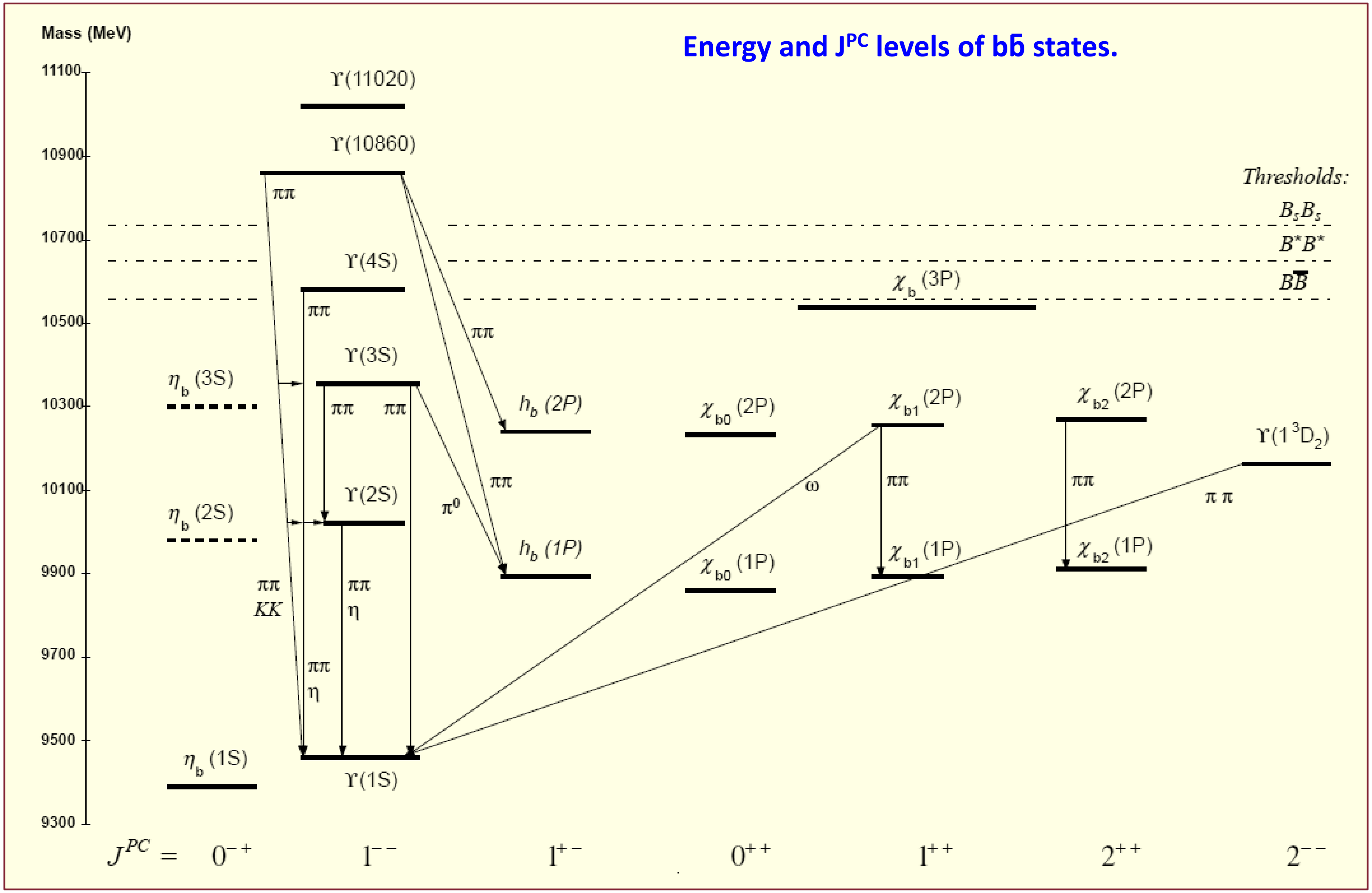
- Precision measurement, carried out at DESY and Cornell with e^+e^- Colliders, soon confirmed the results. After two years, also "open beauty", i.e. bound states $b\bar{q}$, was identified and called $B^{0,\pm}$.
- The figure in the next page shows an updated compilation of the $b\bar{b}$ states.
- Bottomonium (beauty in not used anymore, *don't know why*) is a very interesting system. Recently, a lot of studies (BABAR) have been performed on the $\mathbb{CP}$ violation in the $B^0\bar{B}^0$ system (similar to the K^0's, but different from the charms) [*see* § K^0].
- *Leon Lederman together with Mel Schwartz and Jack Steinberger got the 1988 Nobel Prize, NOT for his $b\bar{b}$ discovery, but for his neutrino studies (the "two neutrino experiment" in 1962).*



Leon Lederman Jack Steinberger Mel Schwartz

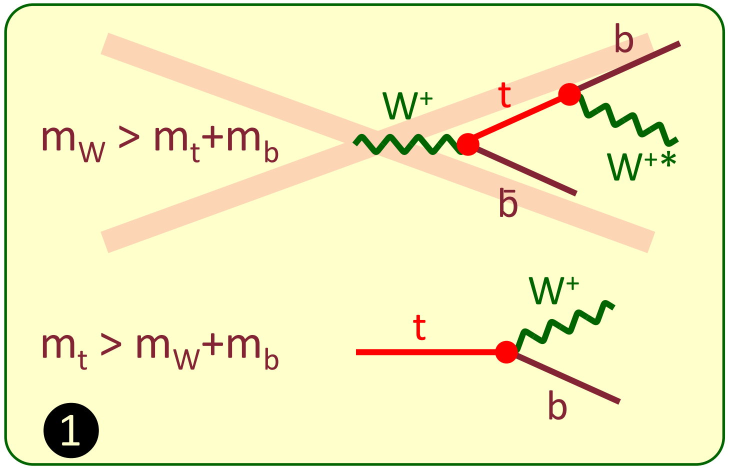
Nobel laureates 1988

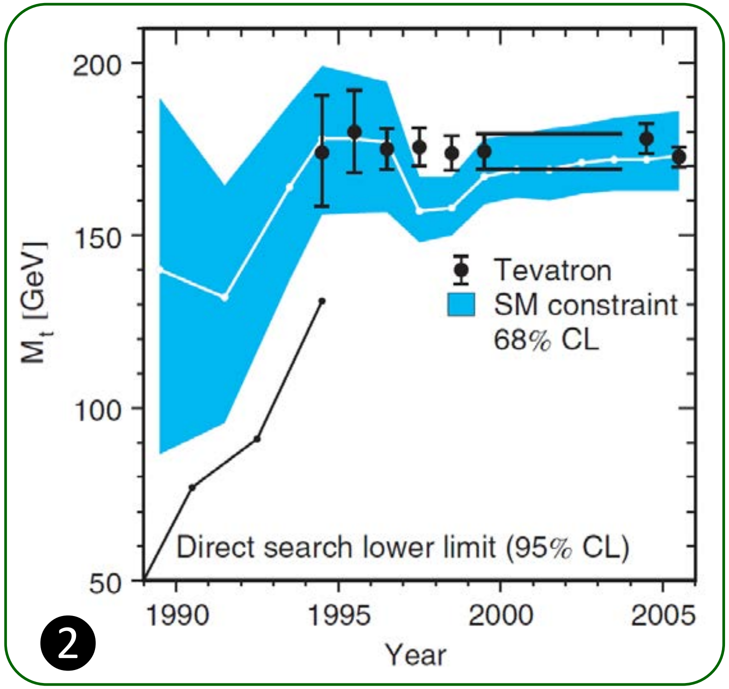
The b quark : bottomonia



The t quark : search

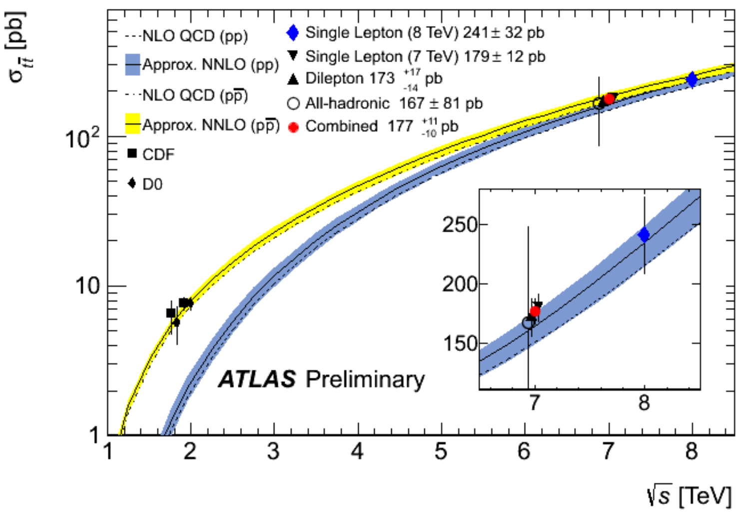
- The top quark was directly searched in hadron ($Sp\bar{p}S$, Fermilab) and lepton (Tristan, LEP) colliders, but was NOT found until 1990's;
- at the time the mass limit was $m_t \geq 90$ GeV;
- at $m_t \approx m_W - m_b$ ($\approx$ 75 GeV), the search changes: the "golden discovery channel" moves from ($W^+ \to t\bar{b} \to W^{+*}b\bar{b}$) to ($t \to W^+b$) [fig. ❶];
- the mass was first *computed* from the radiative corrections for m_W and m_Z [*see* § *LEP*];
- the LEP data, together with all other e.w. measurements, allowed for a prediction of $m_t \approx 175$ GeV [fig. ❷];
- in the 1990's the search was finally concluded at the Tevatron, by the CDF and D0 experiments.
- At present, we measure $m_t = 173 \pm 0.4$ GeV.

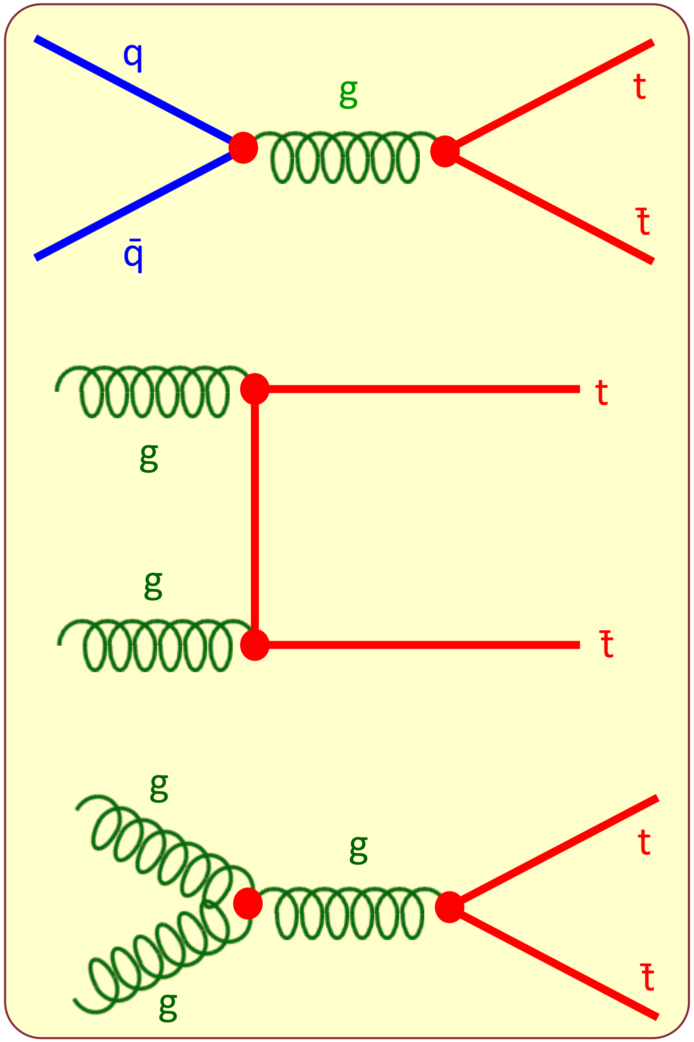




The t quark : production

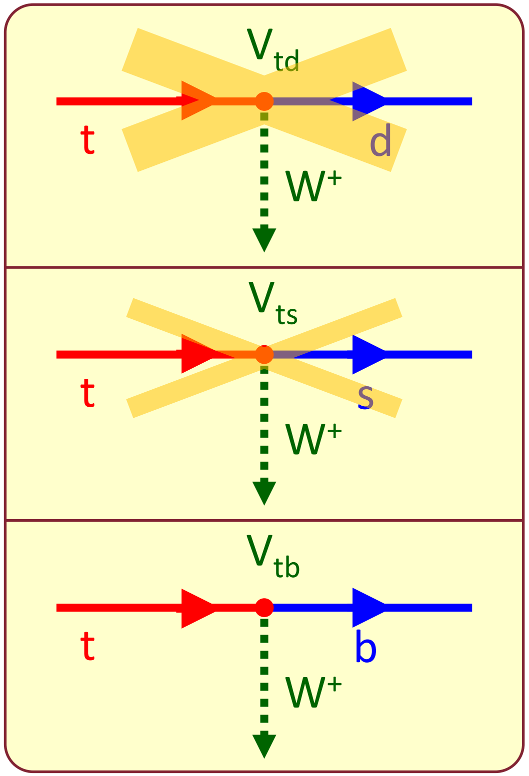
- in a hadronic collider [*see* § *Colliders*], the top is produced in pairs, via hadronic interactions;
- in pp and p̄p the PDF of initial state partons are different (valence / sea) [*see* § *Colliders*]: the qq̄ channel decreases from 90% (p̄p at Tevatron, √s=1.8 TeV) to 5% (pp at LHC, √s=14 TeV) [*qualitatively understandable*];
- in the same range, the total cross section increases from 5 to 600 pb [*also quite understandable*].

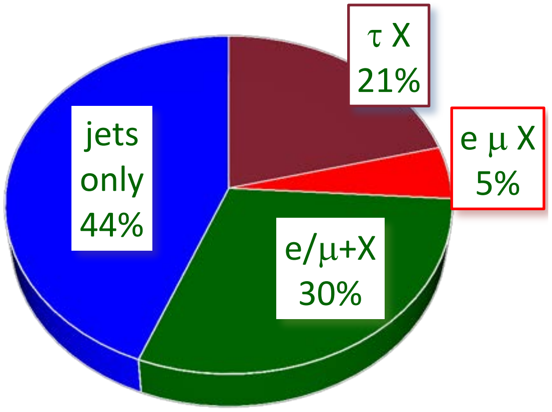




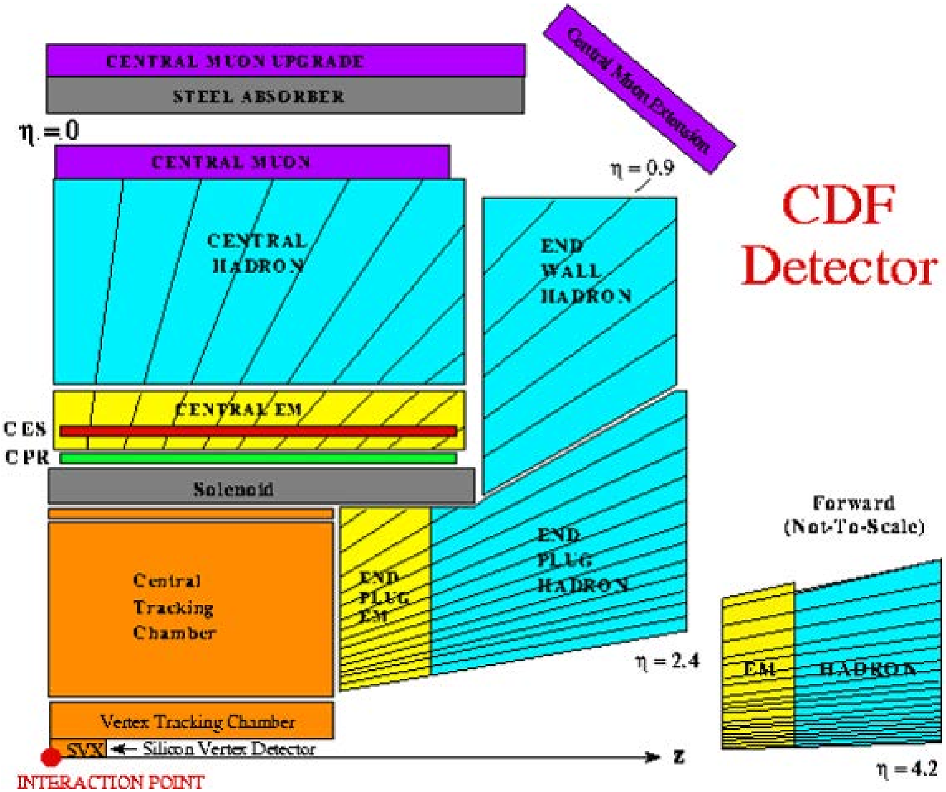
The t quark : decay

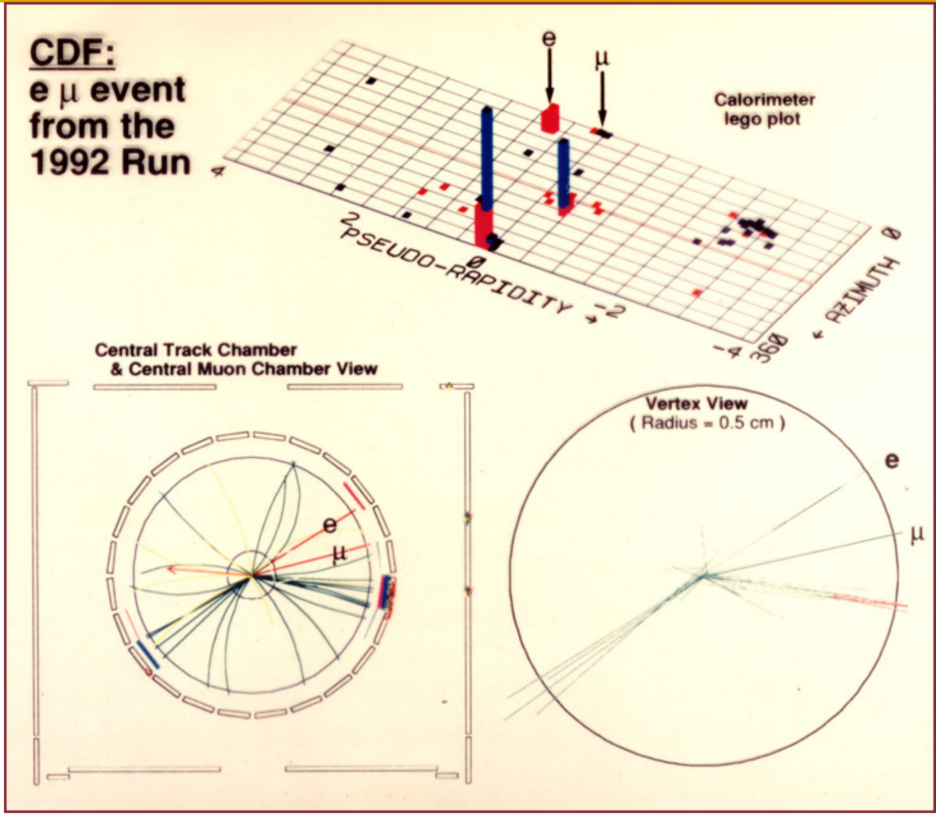
- the top quark decays weakly in a (real) W and a "down-type" quark (q=d/s/b), with a coupling $\propto V_{tq}$ [CKM, see § 5];
- therefore the most common decay is $t \to bW^+$ ($\bar{t} \to \bar{b}W^-$);
- since $\Gamma \approx G_F m_t^3 / (8\pi\sqrt{2})$ ~ 2 GeV, $\tau_t \sim 4 \times 10^{-25}$ s [¿ *"m^3"* ?];
- therefore the top decays ***before*** any hadronic process (hadronization, toponium formation) may happen;
- in turn the W decays "democratically" [*see § LEP*] into all the ($\ell\nu$) ($q\bar{q}$) pairs (hadrons × 3 because of color);
- putting all together, the main decays for a $t\bar{t}$ pair are :
  - both W's into e/μ : the golden channel, but rare;
  - only one W into e/μ : more common, less easy;
  - both W into quarks (i.e. jets) : difficult;
  - (one or more) $\tau^\pm$ in the final state : ν's → almost impossible with present technology.





The t quark : discovery (1992-4)





main tools for $t\bar{t}$ events at Tevatron (1992-4) :

- multibody final states;
- lepton id ($e^\pm$, $\mu^\pm$);
- secondary b vertices;
- mass fits.

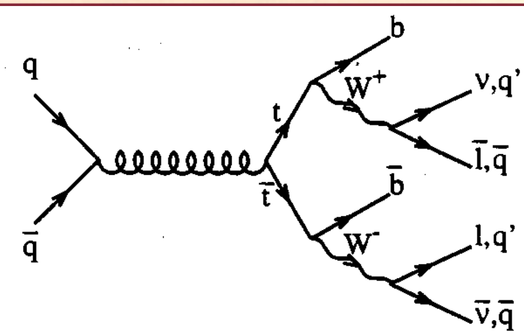
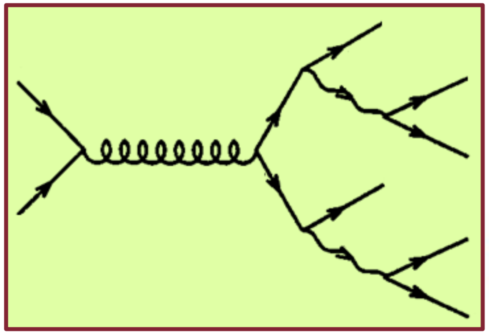


Figure 28: Tree level top quark production by $q\bar{q}$ annihilation followed by the Standard Model top quark decay chain.

The t quark : results (1992-4)

- in may 1994, with 20 pb^{-1} of data, the CDF collaboration was able to claim the top "evidence" (3σ) and, one year after, its "discovery" (5σ);
- [for the latest results on top, see § LHC].



- data before b-tag
- data after b-tag
- [t-signal]
- computed background

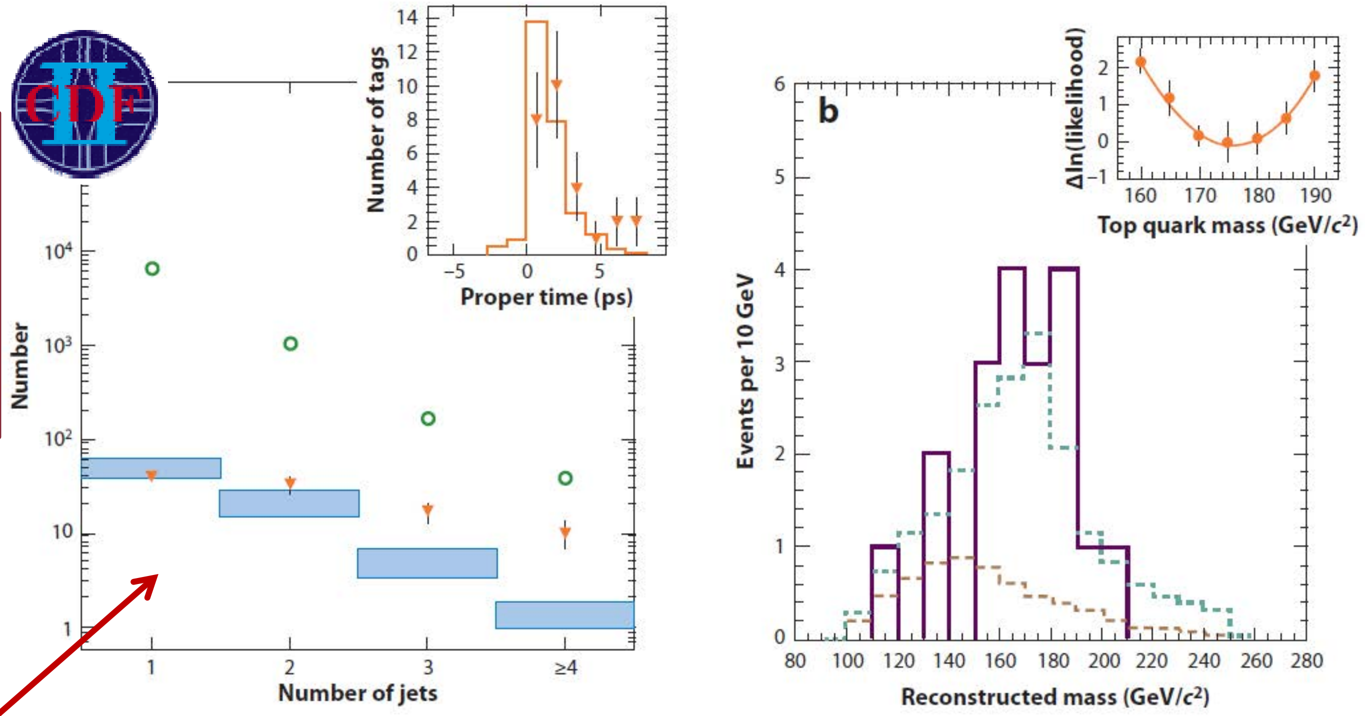


Figure 10

(*a*) Number of CDF events before secondary vertex *b* tagging (*circles*), number of tags observed (*triangles*), and expected number of background tags (*hatch marks*) versus jet multiplicity. (*Inset*) The secondary vertex proper time distribution for events with three or more jets (*triangles*) compared with the expectation for *b* quark jets in top quark decay. (*b*) CDF reconstructed mass distribution for *b*-tagged events with at least four jets (*solid line*). Also shown are the background shape (*dashed purple line*) normalized to the expected number of background events and the sum of the background and top quark contributions (*dotted green line*). (*Inset*) The likelihood fit used to determine the top quark mass. Modified from Abe et al. 1995 (115) with permission.

Summary

Finally, a simple table with all the quarks and their quantum numbers [antiquarks have same I and opposite $\mathcal{B}$, Q, I_3, S, C, B, T]:

	d	u	s	c	b	t
$\mathcal{B}$: baryon number	⅓	⅓	⅓	⅓	⅓	⅓
Q : electric charge	–⅓	+⅔	–⅓	+⅔	–⅓	+⅔
I : Isospin	½	½	0	0	0	0
I_3 : Isospin 3-component	–½	+½	0	0	0	0
S : strangeness	0	0	–1	0	0	0
C : charm	0	0	0	+1	0	0
B : bottomness	0	0	0	0	–1	0
T : topness	0	0	0	0	0	+1
Gell-Mann – Nishijima formula : $Q = I_3 + ½ (\mathcal{B} + S + C + B + T)$.						

conventional rules:
- **in Gell-Mann–Nishijima all +ve;**
- **I_3 –ve for d / +ve for u;**
- **S/B –ve for s/b;**
- **C/T +ve for c/t;**

(could use a different rule, but stay consistent).

Is this the REAL end of the story, i.e. no other quark exists ?
- the SM does not answer: discoveries or mass limits are left to the experiments;
- LEP measurement of n_ν [see];
- present mass limits, see §LHC;
- a bSM theory could predict the number of families (or any other constraint).

References

1. [BJ, 10];
2. [Bettini, 4];
3. [YN1 14], [YN2 11.9]
4. the process $e^+e^- \to f\bar{f}$: [MQR 14];
5. the CKM mixing and the GIM mechanism : [§ 4] and refs. therein;
6. the LEP fit to m_t : [§ 6];
7. Tevatron results : Ann. Rev. Nucl. Part. Sci. 2013. 63:467–502 [*notice that the LEP fit to m_t is NOT mentioned*].

Giuseppe Arcimboldo - Rudolf II as Vertumnus ~1590-1 - Skokloster Castle, Sweden

End of chapter 3

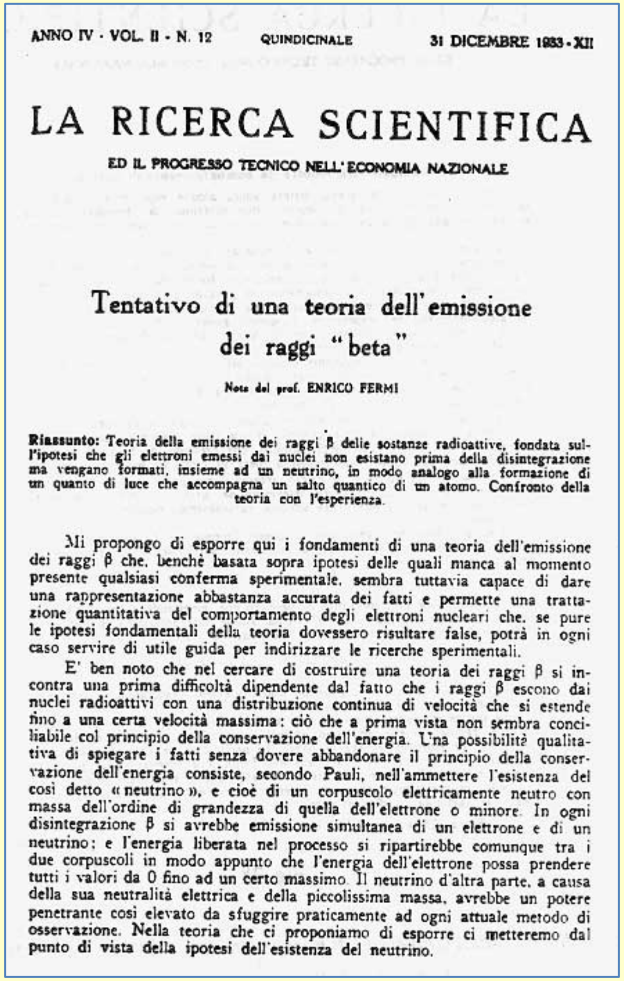
Particle Physics - Chapter 4
Weak Interactions

Paolo Bagnaia
SAPIENZA
UNIVERSITÀ DI ROMA

AA 18-19

last mod.
21-Mar-19

4 – Weak interactions



ANNO IV · VOL. II · N. 12 QUINDICINALE 31 DICEMBRE 1933 · XII

LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL'ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi "beta"

Nota del prof. ENRICO FERMI

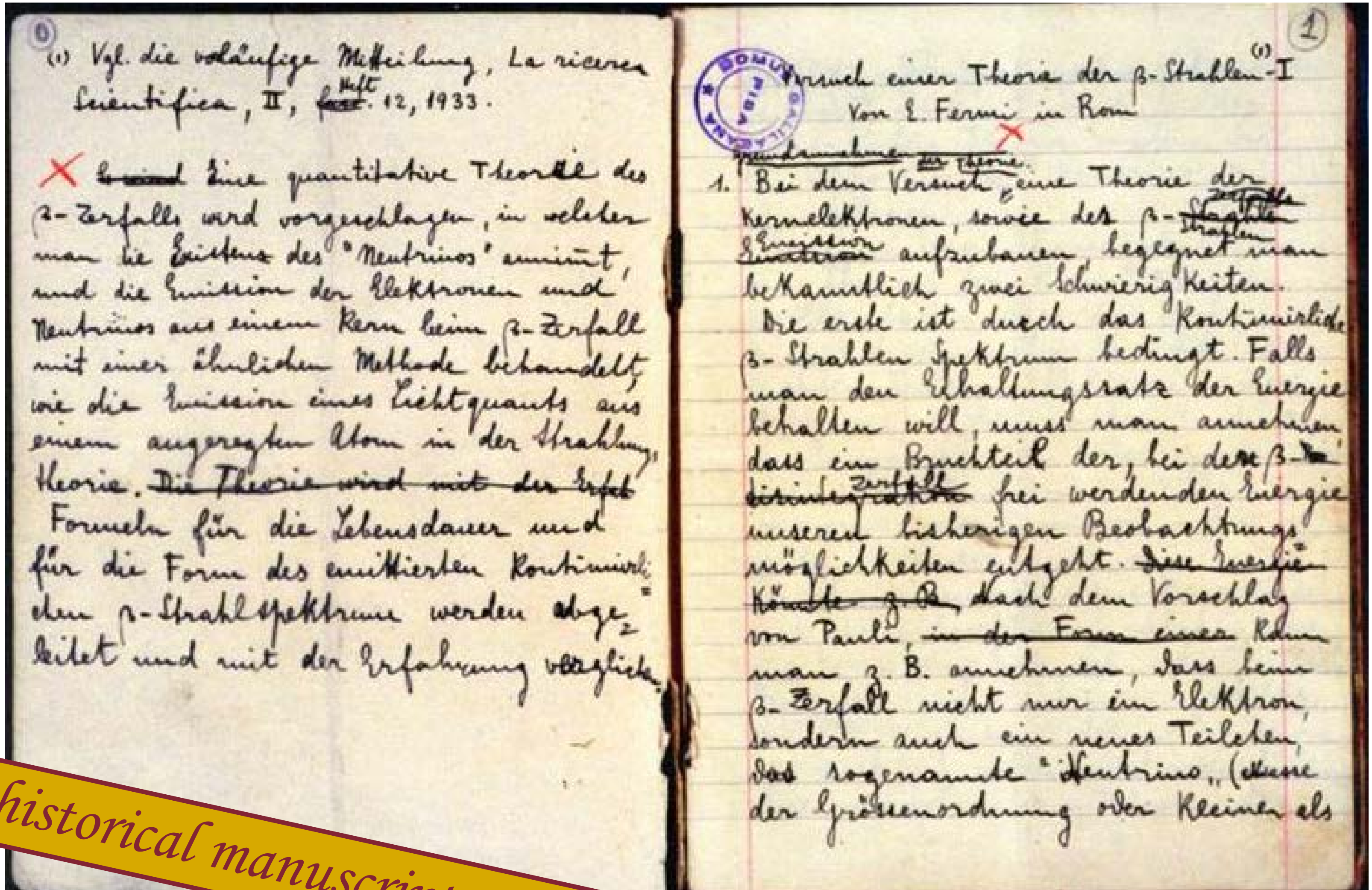
Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

Mi propongo di esporre qui i fondamenti di una teoria dell'emissione dei raggi β che, benchè basata sopra ipotesi delle quali manca al momento presente qualsiasi conferma sperimentale, sembra tuttavia capace di dare una rappresentazione abbastanza accurata dei fatti e permette una trattazione quantitativa del comportamento degli elettroni nucleari che, se pure le ipotesi fondamentali della teoria dovessero risultare false, potrà in ogni caso servire di utile guida per indirizzare le ricerche sperimentali.

E' ben noto che nel cercare di costruire una teoria dei raggi β si incontra una prima difficoltà dipendente dal fatto che i raggi β escono dai nuclei radioattivi con una distribuzione continua di velocità che si estende fino a una certa velocità massima: ciò che a prima vista non sembra conciliabile col principio della conservazione dell'energia. Una possibilità qualitativa di spiegare i fatti senza dovere abbandonare il principio della conservazione dell'energia consiste, secondo Pauli, nell'ammettere l'esistenza del così detto «neutrino», e cioè di un corpuscolo elettricamente neutro con massa dell'ordine di grandezza di quella dell'elettrone o minore. In ogni disintegrazione β si avrebbe emissione simultanea di un elettrone e di un neutrino: e l'energia liberata nel processo si ripartirebbe comunque tra i due corpuscoli in modo appunto che l'energia dell'elettrone possa prendere tutti i valori da 0 fino ad un certo massimo. Il neutrino d'altra parte, a causa della sua neutralità elettrica e della piccolissima massa, avrebbe un potere penetrante così elevato da sfuggire praticamente ad ogni attuale metodo di osservazione. Nella teoria che ci proponiamo di esporre ci metteremo dal punto di vista della ipotesi dell'esistenza del neutrino.

This chapter is just the preamble of our discussion on w.i.; also § K^0 and § ν are mainly dedicated to w.i., while § $\bar{p}p$, § LEP and § LHC contain a good fraction of w.i.

the weak interactions : the origins


(1) Vgl. die vorläufige Mitteilung, La ricerca Scientifica, II, ~~fasc.~~ Heft 12, 1933.

~~Es wird~~ Eine quantitative Theorie des β-Zerfalls wird vorgeschlagen, in welcher man die Existenz des "Neutrinos" annimmt, und die Emission der Elektronen und Neutrinos aus einem Kern beim β-Zerfall mit einer ähnlichen Methode behandelt, wie die Emission eines Lichtquants aus einem angeregten Atom in der Strahlungstheorie. ~~Die Theorie wird mit der Erfah~~ Formeln für die Lebensdauer und für die Form des emittierten kontinuierlichen β-Strahlspektrums werden abgeleitet und mit der Erfahrung verglichen.

Versuch einer Theorie der β-Strahlen. I (1)

Von E. Fermi in Rom

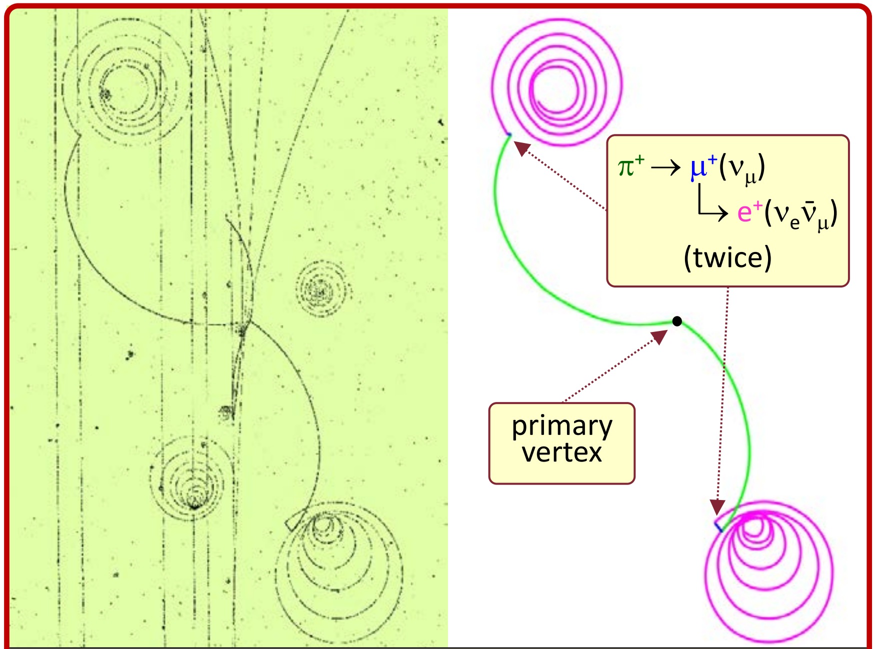
~~Grundannahmen der Theorie.~~

1. Bei dem Versuch, eine Theorie der Kernelektronen, sowie der β-~~Strahlen~~ Emission aufzubauen, begegnet man bekanntlich zwei Schwierigkeiten. Die erste ist durch das kontinuierliche β-Strahlen Spektrum bedingt. Falls man den Erhaltungssatz der Energie behalten will, muss man annehmen, dass ein Bruchteil der, bei dem β-Zerfall frei werdenden Energie unseren bisherigen Beobachtungsmöglichkeiten entgeht. ~~Diese Energie könnte z.B.~~ Nach dem Vorschlag von Pauli ~~in der Form eines~~ kann man z. B. annehmen, dass beim β-Zerfall nicht nur ein Elektron, sondern auch ein neues Teilchen, das sogenannte "Neutrino" (Masse der Grössenordnung oder kleiner als

a historical manuscript [thanks to F. Guerra]

the weak interactions : introduction

- Some rare processes, i.e. small coupling, violate the conservation laws, valid for strong and electromagnetic interactions.
- In ordinary matter the **weak interactions** (w.i.) have a negligible effect, except in cases otherwise forbidden (e.g. β decay).
- The w.i. are responsible for the fact that STABLE matter contains only u and d quarks and electrons. Other quarks and leptons are UNSTABLE because of w.i..
- Therefore, in spite of their "weakness" (small range of interaction $\approx 10^{-3}$ fm, tiny cross sections $\approx 10^{-47}$ m^2), the w.i. play a crucial role in the features of our world.
- ALL elementary particles, but ~~gluons~~ and ~~photons~~ (carriers of other interactions), are affected by w.i. : quarks and charged leptons have w.i., ν's have ONLY them.
- In the scattering processes of charged hadrons and leptons, the effects due to the strong and electromagnetic interactions "obscure" those of the w.i..
- Therefore most of our knowledge on this subject, at least until the '70s, has been obtained from the study of the decays of particles and from ν beams.



CERN 2m hydrogen bubble chamber: $K^+p \to \pi^+\pi^+X$

the weak interactions : some history

1930 Pauli : ν existence to explain β–decay.

1933 Fermi : first theory of β–decay.

1934 Bethe and Peierls : νN and $\bar{\nu}$N cross sections.

1936 Gamow and Teller : G.-T. transitions.

1947 Powell + Occhialini : decay $\pi^+ \to \mu^+ \to e^+$.

1956 Reines and Cowan : ν's detection from a reactor.

1956 Landè, Lederman and coll. : K^0_L.

1956 Lee and Yang : parity non-conservation.

1957 Feynman and Gell-Mann, Marshak and Sudarshan : V–A theory.

1958 Goldhaber, Grodzins and Sunyar : ν helicity.

1960 (ca) Pontecorvo and Schwarz : ν beams.

1961 Pais and Piccioni : $K_L \leftrightarrow K_S$ regeneration.

1962 First ν beam from accelerator : Lederman, Schwarz, Steinberger : ν_μ.

1963 Cabibbo theory.

1964 Cronin and Fitch : CP violation in K^0 decay.

1964 Brout, Englert, Higgs : Higgs mechanism.

1968 Weinberg–Salam model.

1968 Bjorken scaling, quark-parton model.

1970 GIM mechanism.

1972 Kobayashi, Maskawa : CKM matrix.

1973-90 ν DIS experiments : Fermilab, CERN.

1973 CERN Gargamelle : neutral currents.

1983 CERN Sp$\bar{p}$S : $W^\pm$ and Z.

1987 CERN Sp$\bar{p}$S : B^0 mixing discovery.

1989-95 CERN LEP : Z production + decay.

1997-2000 CERN LEP : W^+W^- production.

1998-2000 ν oscillations.

1999-20xx B^0 mixing detailed studies.

2012 CERN LHC : Higgs boson.

- **only major facts ≥ 1930 considered;**

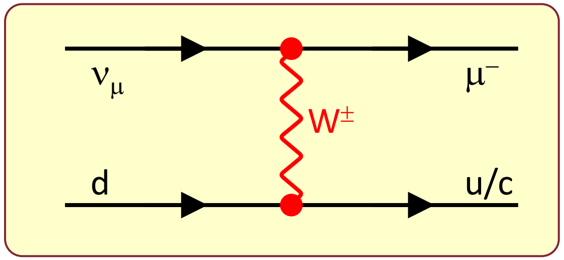
- **this chapter;**
- **other chapters of these lectures;**
- **other lectures in our CdL.**

the weak interactions : CC, NC

In the SM, weak interactions (w.i.) are classified in two types, according to the charge of their carriers :

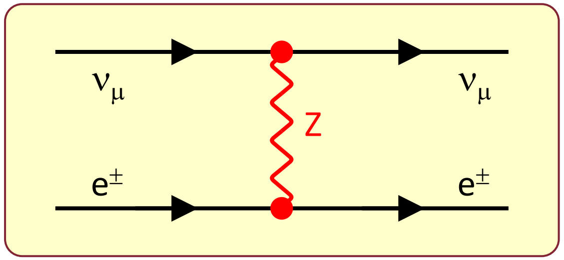
- Charged currents (**CC**), **W**$^\pm$ exchange:
  - in the CC processes, the charge of quark and leptons CHANGES by ±1; at the same time there is a variation of their IDENTITY, including FLAVOR, according to the Cabibbo theory (today Cabibbo-Kobayashi-Maskawa)

e.g.



- Neutral currents (**NC**), **Z** exchange:
  - in the NC case, quarks and leptons remain unchanged (no **FCNC**);
  - until 1973 no NC weak process was observed [*but another example of NC was well known, i.e. the e.m. current: γ's carry no charge !*]

e.g.

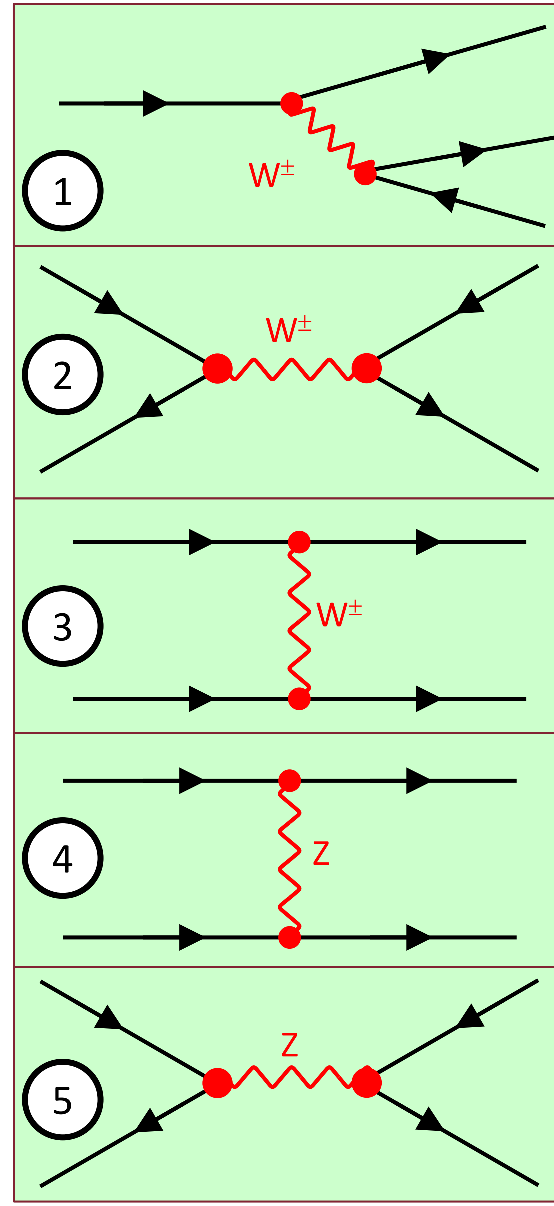


- In the 60's Glashow, Salam and Weinberg (+ many other theoreticians) developed a theory (today known as the "Standard Model", SM), that unifies the w.i. (both CC and NC) and the electromagnetism.

The SM was conceived BEFORE the discovery of NC. So the existence of NC and its carrier (the Z boson), predicted by the SM and observed at CERN in 1973 and 1983 respectively, were among the first great successes of the SM.

the weak interactions : classification

weak interactions	CC	leptonic	$\Delta S = 0$	$\mu \to e\, \nu_e\, \nu_\mu$	1
		semi-leptonic		$\pi^\pm \to \mu^\pm\, \nu_\mu$	2
				$n \to p\, e\, \nu_e$	1 *
				$\nu_e\, d \to e^-\, u$	3 *
				$d\bar{u} \to W^- \to e^-\bar{\nu}_e$	2 *
			$\Delta S = \pm 1$	$K^\pm \to \mu^\pm\, \nu_\mu$	2
				$\Lambda \to p\, e\, \nu_e$	1 *
		hadronic		$K^\pm \to \pi^\pm\, \pi^0$	2 *
				$\Lambda \to p\, \pi^-,\, n\, \pi^0$	1 *
	NC	leptonic	$\Delta S = 0$ (only)	$\nu_\mu\, e^\pm \to \nu_\mu\, e^\pm$	4
		semi-leptonic		$\nu\, N \to \nu\, N'$	4 *
		hadronic		$u\, \bar{u} \to Z \to q\, \bar{q}$	5 *



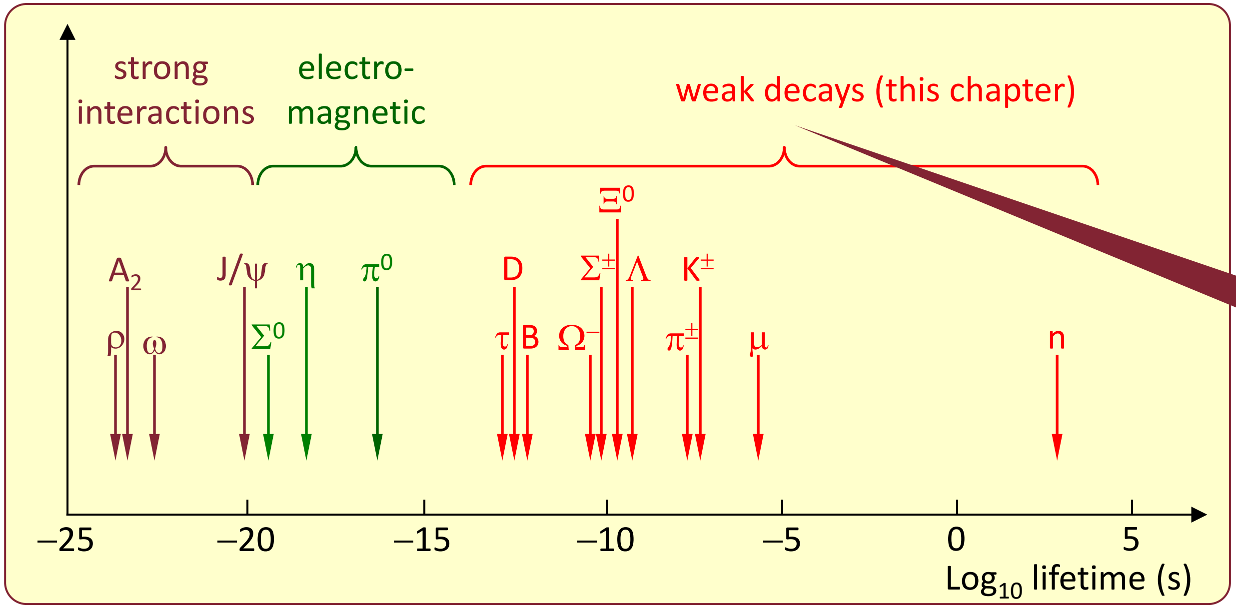
Some processes (list NOT exhaustive), classified in terms of general characteristics and Feynman diagrams.

A "*" in the last column means that the interacting hadron is composite; the diagrams shows only the interacting quark(s); the other partons (the "spectators") do not participate in the interaction, at least in 1st approximation.

In the table, ν means both ν and $\bar{\nu}$ [*only the correct one !*].

charged currents : decays

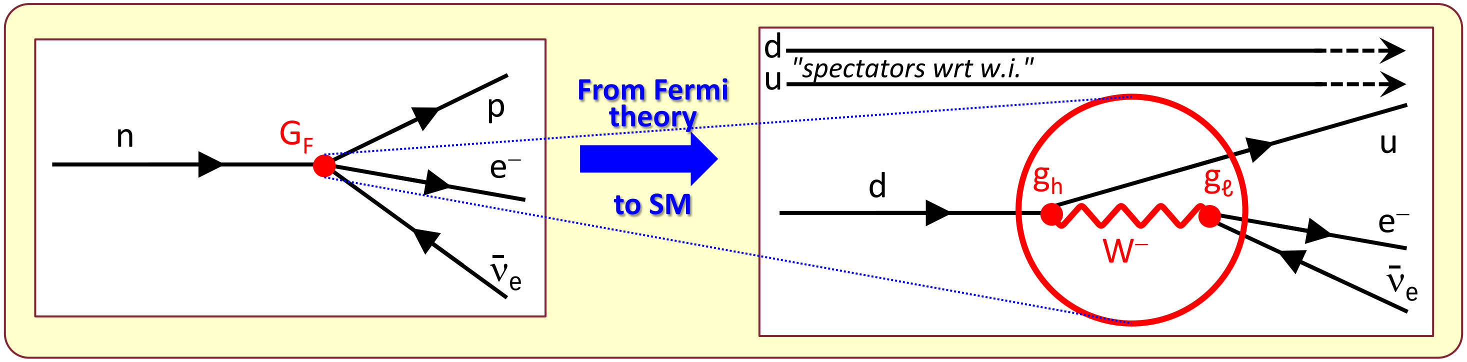
process	Lifetime (s)	comment
$\bar{\nu}_e\, p \to n\, e^+$	(none)	Neutrinos have only weak interactions (not a decay).
$n \to p\, e^-\, \bar{\nu}_e$	$\mathcal{O}(10^3)$	Long lifetime because of small mass difference (p-n).
$\pi^+ \to \mu^+\, \nu_\mu$	$\mathcal{O}(10^{-8})$	The $\pi^\pm$ is the lightest hadron, so it decays $\to$ leptons.
$\Lambda \to p\, \pi^-$	$\mathcal{O}(10^{-10})$	The decay of Λ violates strangeness conservation.



Some of the most interesting weak decays are the neutral heavy mesons of type $Q\bar{Q}$ (K^0, B^0) [see § 5].

charged currents : Fermi theory

- The modern theory of the CC interactions (i.e. this part of the SM) is a successor of the Fermi theory of β decay.
- The Fermi theory describes a point-like interaction, proportional to the coupling G_F; <u>the theory had intrinsic problems</u> ("not renormalizable" in modern terms, i.e. cross-sections violate unitarity at high energy);
- the SM "expands" the point-like interaction, introducing a heavy charged mediator, called $W^{\pm}$.
- <u>the SM is mathematically consistent</u> (it is "renormalizable");
- *(more important)* it reproduces the experimental data with unprecedented accuracy.

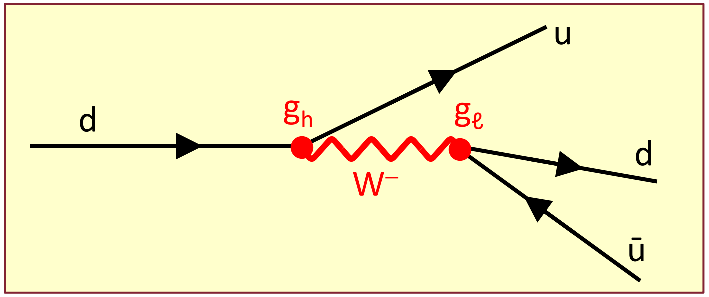


usual comment : to see a smaller scale requires higher $Q^2 \rightarrow$ higher energy

charged currents : simple problem

Q. why is the decay $n \to p\pi^-$ (similar to $\Delta^0 \to p\pi^-$) forbidden ?

A. write the Feynman diagram



- possible ? forbidden ?

  yes, possible

- then ?

  $m(n) - m(p) \approx 1.3$ MeV

  The only possible pair ff' with q = −1 and baryon/lepton number = 0 is clearly $e^-\bar{\nu}_e$, since $m(e^-) + m(\bar{\nu}_e) \approx m(e^-) \approx 0.5$ MeV.

Q. why $n \to pe^-\bar{\nu}_e$ and not $p \to ne^+\nu_e$?

A. [... left to the reader]

charged currents : coupling

A simple comparison between the couplings (g is the "charge" of the w.i. and plays a similar role as e):

- Electromagnetism :

  $\alpha \quad \propto e^2;$

  amplitude $\propto \alpha \propto e^2;$

  rate $\propto \alpha^2 \propto e^4.$

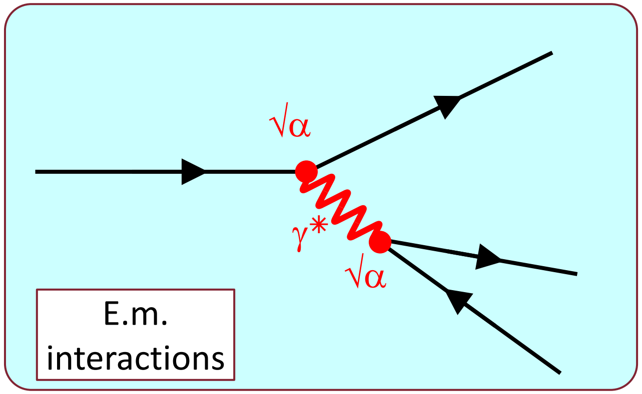
- Weak interactions :

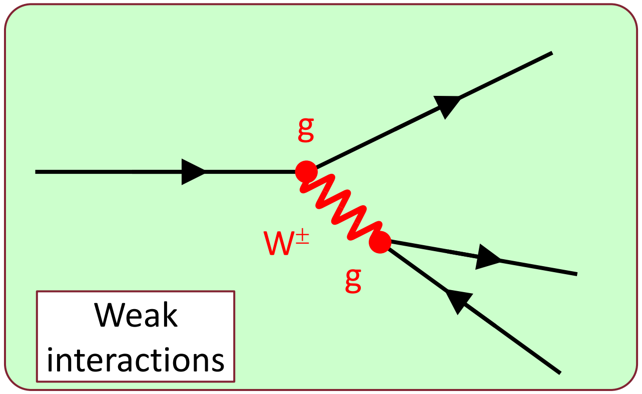
  $G_F \quad \propto g^2;$

  amplitude $\propto G_F \propto g^2;$

  rate $\propto G_F^2 \propto g^4;$

NB. unlike α, G_F is not adimensional (next slide); the similarity electromagnetism ↔ weak interactions is hidden.





charged currents : effect of m_W on coupling

- The e.m. coupling constant α is proportional to the square of the electric charge e :

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}.$$

- In a similar way, the intensity of the CC is G_F (Fermi constant), proportional to the square of the "weak charge" g.

- The matrix elements of the transitions are proportional to the square of the "weak charge" g and to the propagator :

$$\mathcal{M}_{fi} \propto g \frac{1}{Q^2 + m_W^2} g \xrightarrow{Q^2 \ll m_W^2} \frac{g^2}{m_W^2} \equiv G_F.$$

- The difference respect to the e.m. case is the mass of the carrier: while the γ is massless, the CC carrier is the $W^\pm$, a massive particle of spin 1. Therefore the range of CC turns out to be small ($1/m_W$).

- Unlike the case of the massless photon, for small Q^2 the propagator term "stays constant".

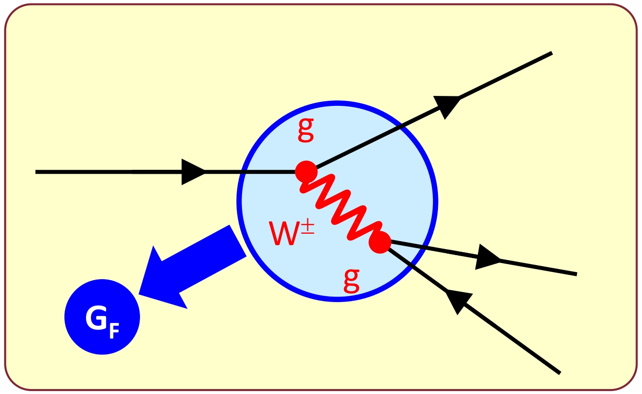
- Therefore the Fermi constant G_F has dimensions :

$$[G_F] = [m_W^{-2}] = [m^{-2}] = [\ell^2],$$

- and a small value, due to m_W :

$$\frac{G_F}{(\hbar c)^3} = O\left(10^{-5}\,\mathrm{GeV}^{-2}\right) = O\left[(10^{-3}\,\mathrm{fm})^2\right].$$

- This effect obscures the similarity of the e.m. and weak charges ($e \leftrightarrow g$), which are indeed of the same order [see § 6].



charged currents : G_F

- the most precise value of the Fermi constant G_F is measured by considering the muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$:
  - low energy process ($\sqrt{Q^2} \approx m_\mu << m_W$);
  - approximated by a four-fermion point-like process, determined by the Fermi constant ($\approx g^2/m_W^2$);
  - only leptons $\to$ free from hadronic interactions which affect other processes, e.g. the nuclear β decays.
- if $m_e \approx 0$, m_μ is the only scale of the decay $\to$ dimensional analysis:

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = 1/\tau_\mu \propto G_F^2\, m_\mu^5,$$

- while the correct computation gives :

$$\Gamma\left(\mu^- \to e^- \bar{\nu}_e \nu_\mu\right) = \frac{G_F^2 m_\mu^5}{192\pi^3}(1+\varepsilon),$$

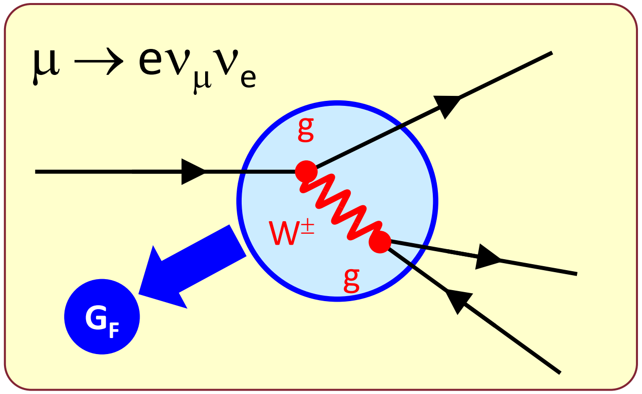
where ε is small and depends on the radiative corrections and on the electron mass.

- the mass of the muon and its average lifetime were measured with great precision:

$$m_\mu = (105.658389 \pm 0.000034)\ \text{MeV};$$
$$\tau_\mu = (2.197035 \pm 0.000040) \times 10^{-6}\ \text{s}.$$

- then the value of the Fermi constant is

$$G_F = (1.16637 \pm 0.00001) \times 10^{-5}\ \text{GeV}^{-2}.$$



lepton universality : $(\tau\to e) \leftrightarrow (\tau\to\mu)$

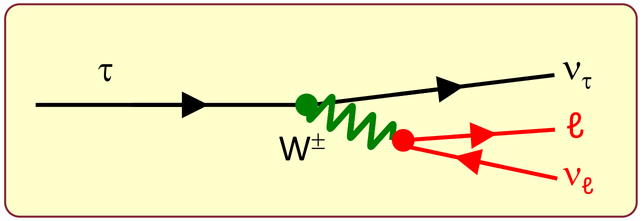
Q. Is the weak CC the same for all leptons and quarks ? Do they share the same coupling constant G_F for all the processes ?

- the **CC universality** has received extensive tests.
- [*absolutely true for leptons, some further refinement* – CKM – *for quarks*]
- The **e–μ universality** is measured by analyzing the leptonic decays of the $\tau^\pm$ (ℓ^- is the appropriate lepton, e^- / μ^-) :

$$\Gamma\left(\tau^- \to \ell^-\overline{\nu}_\ell\nu_\tau\right) \equiv \Gamma_\ell^\tau = \frac{g_\tau^2 g_\ell^2}{m_W^2 m_W^2} m_\tau^5 \rho_\ell;$$

[where ρ_ℓ is the phase space factor]

$$BR\left(\tau^- \to \ell^-\overline{\nu}_\ell\nu_\tau\right) \equiv BR_\ell^\tau = \frac{\Gamma_\ell^\tau}{\Gamma_{tot}^\tau};$$



- it follows that :

$$\frac{\Gamma_\mu^\tau}{\Gamma_e^\tau} = \frac{BR_\mu^\tau}{BR_e^\tau} = \frac{g_\mu^2 \rho_\mu}{g_e^2 \rho_e} \rightarrow$$

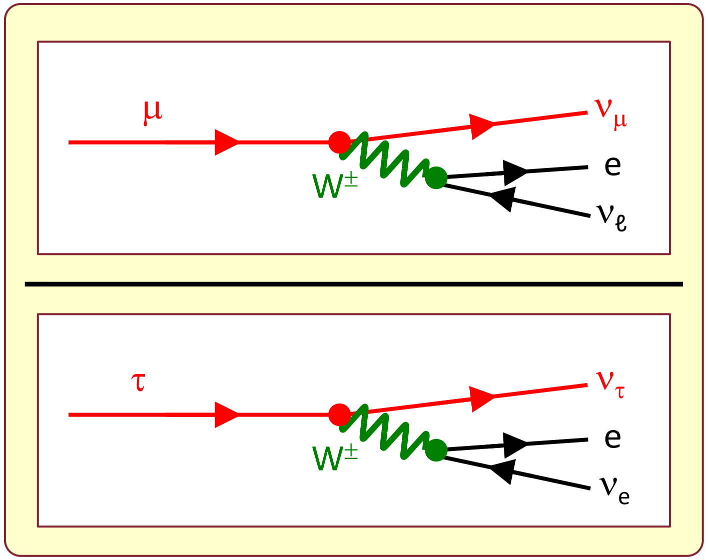
$$\left.\frac{BR_\mu^\tau}{BR_e^\tau}\right|_{meas.} = \frac{(17.36 \pm .05)\%}{(17.84 \pm .05)\%} = 0.974 \pm .004,$$

and, taking into account the values of ρ_μ and ρ_e :

$$\boxed{g_\mu / g_e\Big|_{meas.} = 1.001 \pm .002.}$$ **!!!**

lepton universality : (μ→e) ↔ (τ→e)

The measurement of the **μ–τ universality** is similar [$BR_x = \Gamma_x / \Gamma_{tot} = \tau\, \Gamma_x$] :



$$BR\left(\mu^- \to e^- \overline{\nu}_e \nu_\mu\right) \approx 100\% \text{ (experimentally)};$$

$$\frac{\Gamma\left(\mu^- \to e^- \overline{\nu}_e \nu_\mu\right)}{\Gamma\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau\right)} = \frac{\tau_\tau}{\tau_\mu} \frac{BR\left(\mu^- \to e^- \overline{\nu}_e \nu_\mu\right)}{BR\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau\right)};$$

the prediction is :

$$\frac{\Gamma\left(\mu^- \to e^- \overline{\nu}_e \nu_\mu\right)}{\Gamma\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau\right)} = \frac{g_e^2}{g_e^2}\frac{g_\mu^2}{g_\tau^2}\frac{m_\mu^5}{m_\tau^5}\frac{\rho_\mu}{\rho_\tau} = \frac{g_\mu^2}{g_\tau^2}\frac{m_\mu^5}{m_\tau^5}\frac{\rho_\mu}{\rho_\tau},$$

$$\to \frac{g_\mu^2}{g_\tau^2} = \frac{\tau_\tau}{\tau_\mu} \frac{1}{BR\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau\right)} \frac{m_\tau^5 \rho_\tau}{m_\mu^5 \rho_\mu},$$

- from the measured values of m_μ, m_τ, τ_μ, τ_τ,

and $BR\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau\right)$, we finally get :

$$\boxed{\left.\frac{g_\mu}{g_\tau}\right|_{\text{meas.}} = 1.001 \pm .003.}$$

!!!

"τ_τ" ?

lepton universality : τ decays

More ambitious test: extend universality to **τ hadronic decays** :

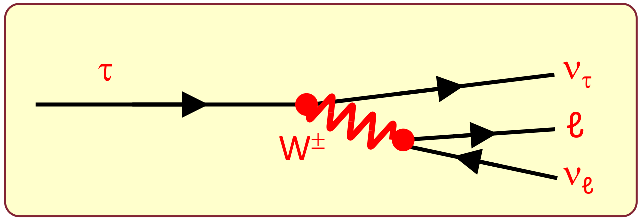
- consider again the leptonic decays of the τ lepton: mainly the following three decay modes :

$$\tau^- \to e^- \bar{\nu}_e \nu_\tau; \quad \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau; \quad \tau^- \to \bar{u} d \nu_\tau.$$

- from the BR_i ratio, expect (3 for color) :

$$\Gamma^{\text{meas.}}_{\tau\to e} \approx \Gamma^{\text{meas.}}_{\tau\to\mu} \approx \Gamma^{\text{meas.}}_{\tau\to\bar{u}d}/3,$$

in excellent agreement with universality and presence of color in the hadronic sector [*it is the first time we see the color appear in the weak interactions sector*].



Another test is the **τ lifetime** :

$$\Gamma_{\tau\to\mu} \approx \frac{\Gamma^{tot}_\tau}{5} = \frac{m^5_\tau}{m^5_\mu}\Gamma_{\mu\to e} = \frac{m^5_\tau}{m^5_\mu}\frac{1}{\tau_\mu};$$

$$\tau_\tau = 1/\Gamma^{tot}_\tau \approx \frac{\tau_\mu m^5_\mu}{5 m^5_\tau} \approx \boxed{3.1\times 10^{-13}\ \text{s};} \ !$$

experimentally it is found :

$$\tau^{exp}_\tau = \boxed{(2.956 \pm .031)\times 10^{-13}\ \text{s.}} \ !$$

- Many other experimental tests [*... but I suppose that you are convinced*].
- At least for CC weak interactions (but also in e.m., and in NC, as in the Z decay) all three leptons have exactly the same interactions.
- The only differences are due to their different mass.
- *Isidor Isaac Rabi said in the 30's about the muon: "who ordered that ?".*

lepton universality : Z decays

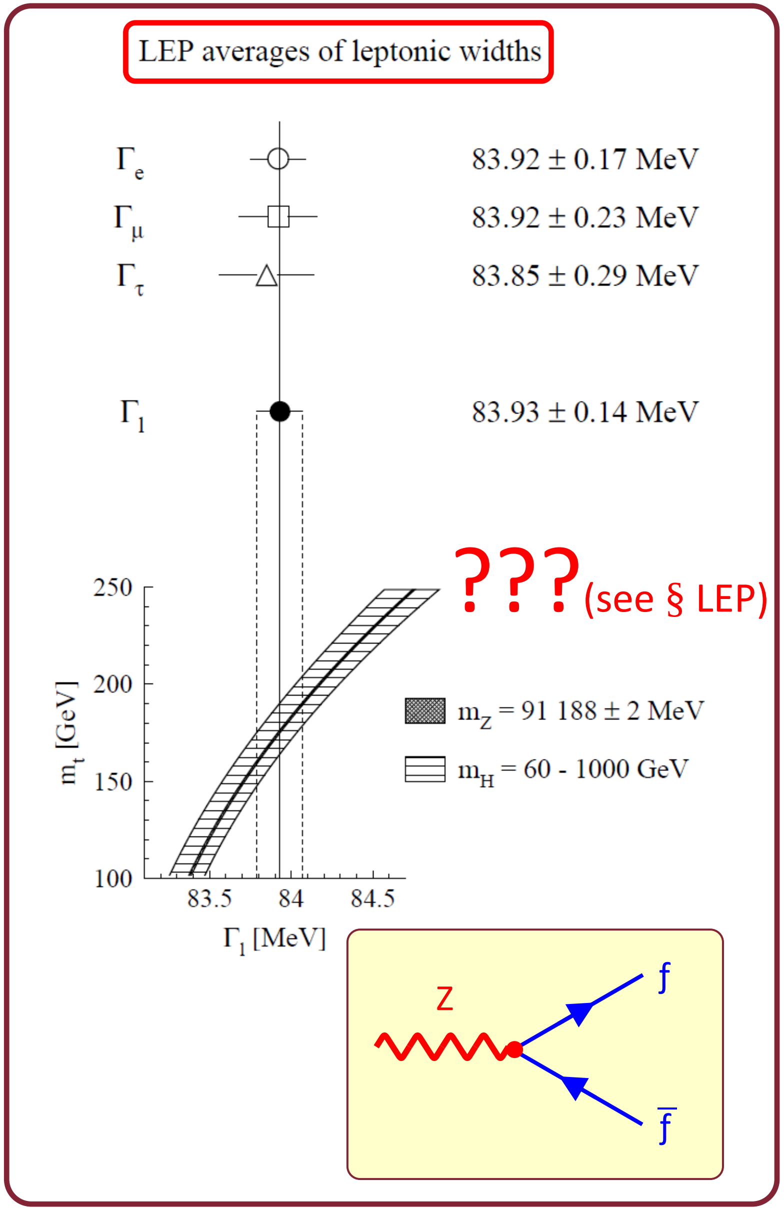
- A similar test on lepton universality has been performed at LEP, in the decay of the Z (a NC process).
- The experiments [*see* § *LEP*] have measured the decay of the Z into fermion-antifermion pairs.
- They [*well, WE*] have found :

$$Z \rightarrow \; e^+e^- \; : \; \mu^+\mu^- \; : \; \tau^+\tau^-$$

$$1. \; : 1.000 \pm .004 \; : .999 \pm .005.$$

- Similar – more qualitative – tests can be carried with angular distributions, higher orders, … [see § LEP].
- The total amount of information is impressive and essentially no margin is left to any alternative theory.

warning – in these pages we mix measurements of different ages, e.g. μ-decay in the '50s, τ-decay in the '80s, Z-decay in the '90s.



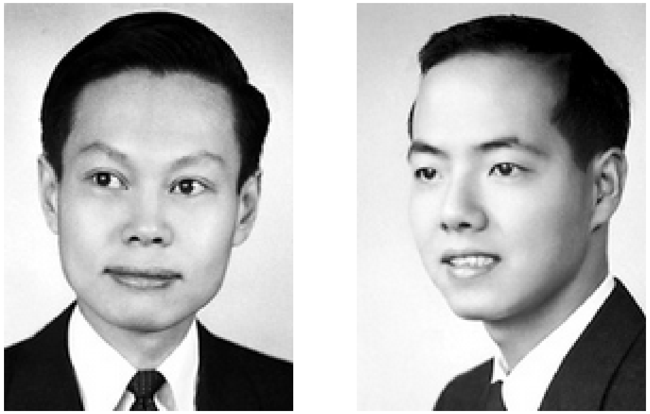
parity violation : history

- The effect was proposed in 1956 by two young theoreticians in a classical paper and immediately verified in a famous experiment (Mme Wu) [*FNSN 1*] and in the $\pi^{\pm}$- and $\mu^{\pm}$-decays by Lederman and coll.
- The historical reason was a review of weak interaction processes and the explanation of the "θ-τ puzzle", i.e. the K^0 decay into 2π or 3π systems.

particle (ν)	anti-particle ($\bar{\nu}$)
h = −1	h = +1

- ν only h=−1;
- $\bar{\nu}$ only h=+1;

→ **PARITY VIOLATION**



Nobel Prize 1957
Tsung-Dao Lee (*Lǐ Zhèngdào*, 李政道)

Chen-Ning Franklin Yang (Yáng Zhènníng, 杨振宁 or 楊振寧)

for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles.

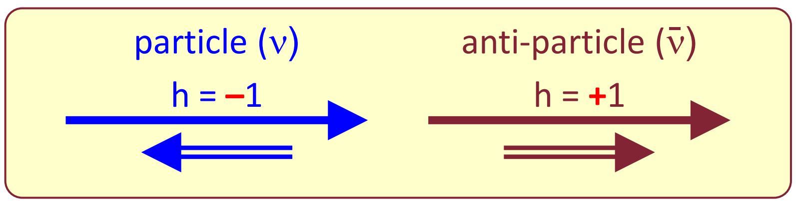
$\vec{\ }$ vectors & co.

parity violation : mechanism

- The two authors found that parity conservation in weak decays was NOT really supported by measurements.

[then experiment, and then a new theory]

- The CC current is "V – A", which is an acronym for the factor $\gamma_\mu(1 - \gamma_5)$ in the current; it shows that the CC have a "preference" for <u>left-handed particles</u> and <u>right-handed anti-particles</u>.



- These effects clearly violates the parity : the parity operator $\mathbb{P}$ flips the helicity:

$$\mathbb{P}\,|\nu, h = -1> = |\nu, h = +1>$$

$\rightarrow$ it changes ν's with a –ve helicity into ν's with +ve helicity, which DO NOT EXIST (or do not interact).

- Few comments :
  - V or A alone would NOT violate the parity. The violation is produced by the simultaneous presence of the two, technically by their interference.
  - The conservation is restored, applying also $\mathbb{C}$, the charge conjugation:

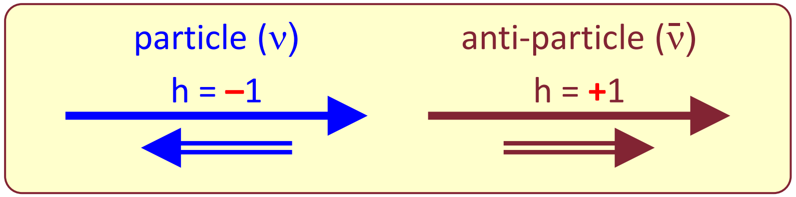
$$\mathbb{CP}|\nu,h=-1> = \mathbb{C}|\nu, h=+1> = |\bar{\nu}, h=+1>,$$

i.e. $\nu_{h=-1} \rightarrow \bar{\nu}_{h=+1}$, which <u>does exist</u>. Therefore, "<u>$\mathbb{CP}$ is not violated</u>" [*not by ν's in these experiments, at least*].

- *the above discussion holds only if* m_ν = 0 *(NOT TRUE), or* $m_\nu \ll E_\nu$ *(ultra-relativistic approximation -* ***<u>u.r.a.</u>****); the u.r.a. for* ν*'s is used in this chapter.*

parity violation : the ν helicity

- For massless ν's or in the u.r.a. approximation(*), V–A implies :



- Therefore in the "forbidden" amplitudes, there is a factor [$\propto$ (1 − β)] for massive particles, which vanishes when $\beta \to 1$.

- If we assume a factor (1 $\pm$ β) for the production of (h = $\mp$ 1) particles (the opposite for anti-particles), we get :

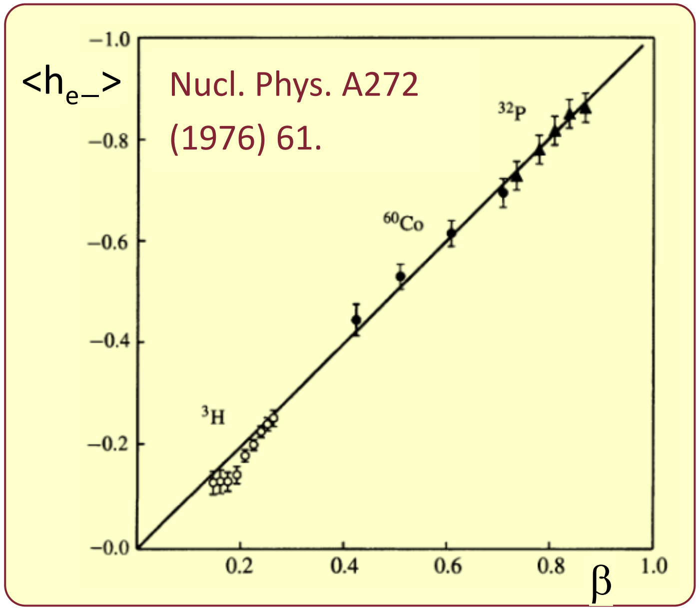
$$\langle h \rangle_{part} = ½ \, [(1 + \beta)\,(-1) + (1 - \beta)(+1)] = -\beta;$$

$$\langle h \rangle_{\overline{part}} = ½ \, [(1 + \beta)\,(+1) + (1 - \beta)(-1)] = +\beta;$$

i.e., when produced in CC interactions, particles in average have –ve helicity, while anti-particles have +ve helicity.

- The effect is maximal for ν's ($\beta_\nu \approx 1$), which also have no other interactions.

- For e^-, it is also well confirmed by data in β decays [YN1, 570] :



(*) *If $m_\nu > 0 \to \beta_\nu < 1$; a L-transformation can reverse the sign of the momentum, and hence the ν helicity, so the following argument is NOT L-invariant for massive particles [previous slide].*

parity violation : the Feynman's view

[... *I*]magine that we were talking to a Martian, or someone very far away, by telephone. We are not allowed to send him any actual samples to inspect; for instance, if we could send light, we could send him right-hand circularly polarized light. [...] But we cannot give him anything, we can only talk to him.

[*Feynman explains how to communicate: math, classical physics, chemistry, biology are simple*]

[...] "Now put the heart on the left side." He says, "Duhhh - the left side?" [...] We can tell a Martian where to put the heart: we say, "Listen, build yourself a magnet, and put the coils in, and put the current on, and [...] then the direction in which the current goes through the coils is the direction that goes in on what we call the right.

[... *However,*] does the right-handed matter behave the same way as the right-handed antimatter? Or does the right-handed matter behave the same as the left-handed antimatter? Beta-decay experiments, using positron decay instead of electron decay, indicate that this is the interconnection: matter to the "right" works the same way as antimatter to the "left."

[... *We then*] make a new rule, which says that matter to the right is symmetrical with antimatter to the left.

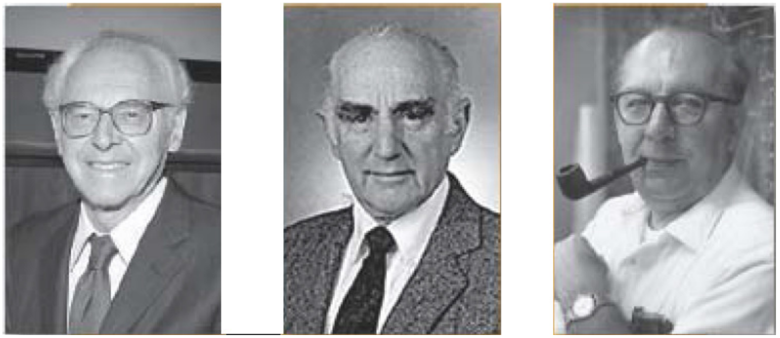
So if our Martian is made of antimatter and we give him instructions to make this "right" handed model like us, it will, of course, come out the other way around. What would happen when, after much conversation back and forth, we each have taught the other to make space ships and we meet halfway in empty space? [...] Well, if he puts out his left hand, watch out!

From Feynman Lectures on Physics, 1, 52: "Symmetry in Physical Laws".

Quite amusing and great physics :

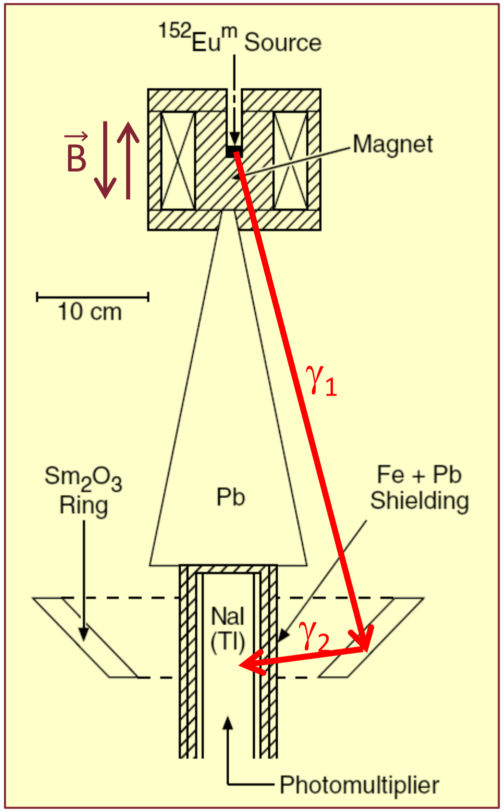
- the symmetry he is talking about is "$\mathbb{CP}$" and NOT simply "$\mathbb{P}$" or "$\mathbb{C}$" !!!
- but $\mathbb{CP}$ is also violated [see § K^0].

the ν_e helicity



In 1958, Goldhaber, Grodzins and Sunyar measured the **helicity of the electron neutrino** ν_e with an ingenious experiment.

- A crucial confirmation of the V−A theory; pure V or A had been ruled out, but V+A was still in agreement with data.
- Metastable Europium (Eu) decays via K-capture → excited Samarium (Sm*) + $\boldsymbol{\nu_e}$, whose helicity is the result of the exp.;
- the Sm* decays again into more stable Samarium (Sm), emitting a γ [γ_1 in fig.].
- For such a γ the transmission in matter depends on the e^- spins; therefore a large B-field is applied to polarize the iron.
- The γ's are used to excite again another Sm; only γ's from the previous chain may do it; another γ is produced [γ_2 in fig.].
- The resultant γ's are detected.



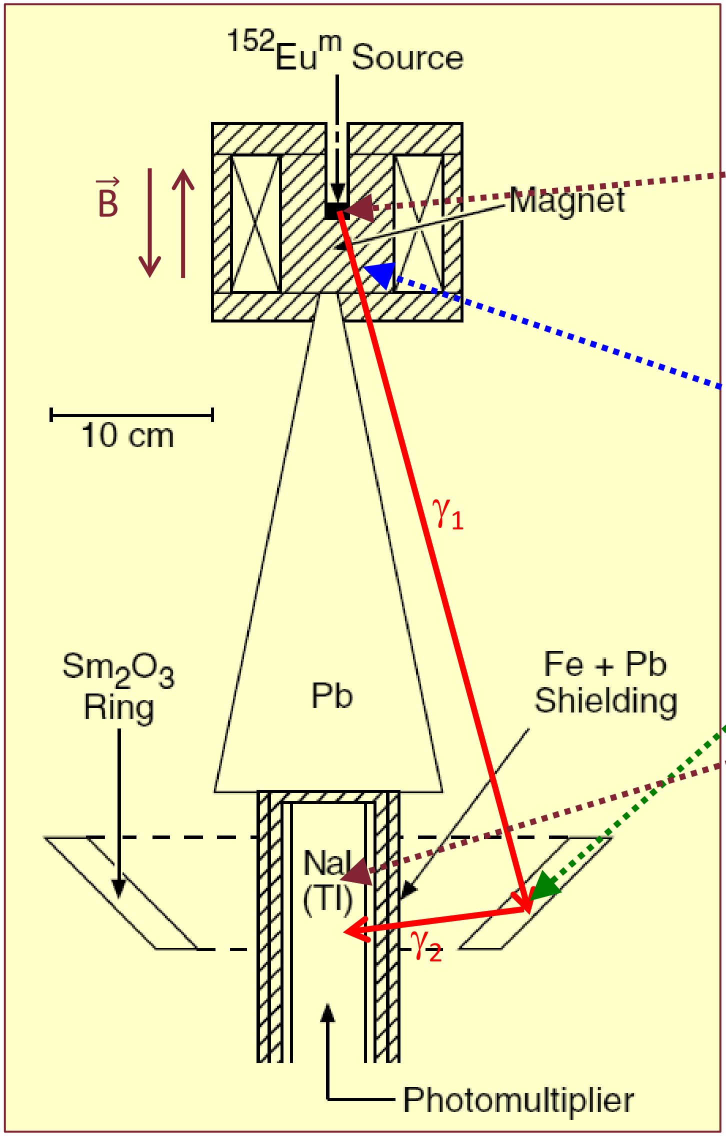
- Final result :

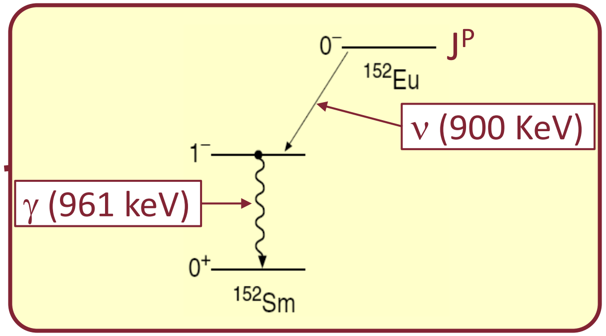
$$\boxed{h(\nu_e) = -1.0 \pm 0.3}$$

consistent with V–A only.

[the experiment is ingenuous and complex: it is discussed step by step.]

the ν_e helicity : summary of the experiment





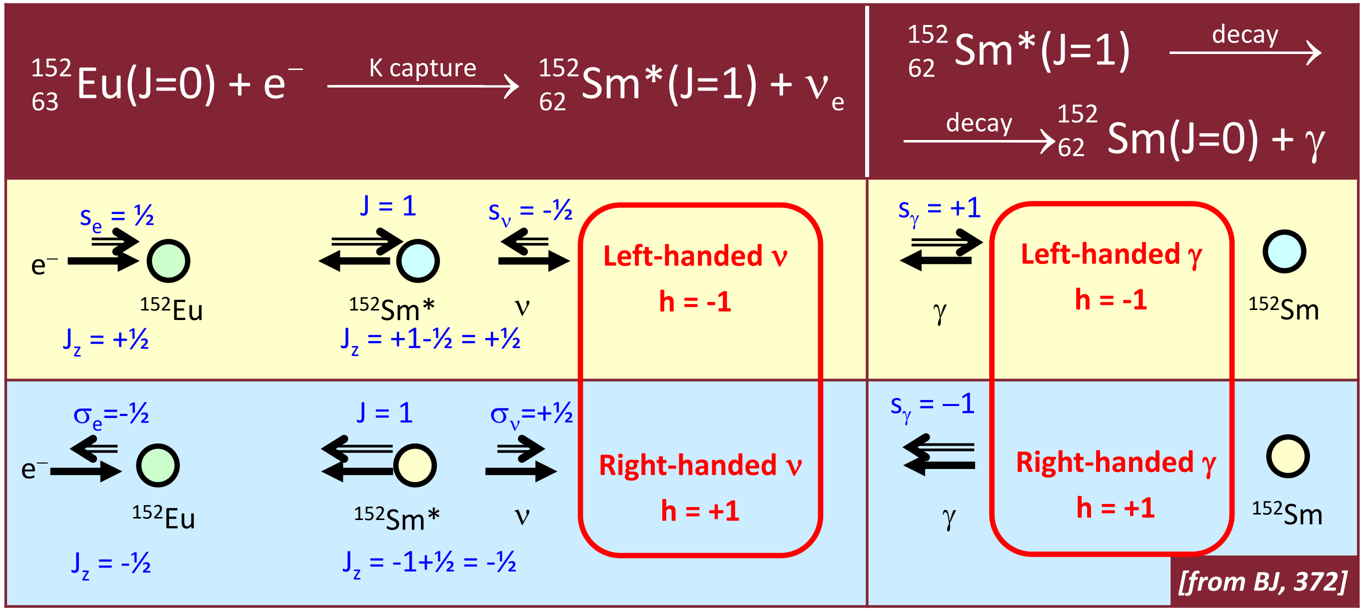
Compton effect does depend on the γ_1-spin wrt $\vec{B}$ (NB γ_1 in the figure escapes Compton effect).

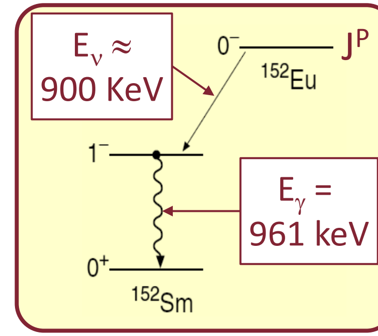
$\gamma_1 + {}^{152}Sm \rightarrow {}^{152}Sm^* \rightarrow {}^{152}Sm + \gamma_2$.

γ_2 detection via photomultiplier.

The experiment detects the number of γ_2 when $\vec{B}$ is (anti-)parallel to γ_1. The asymmetry depends on the (ν_e-helicity $\rightarrow$) γ_1-spin.

the ν_e helicity : Europium → Samarium → γ





- ν_e monochromatic, $E_\nu \approx 900$ keV;
- Sm* lifetime = ~10^{-14} s, short enough to neglect all other interactions;
- Sm* excitation energy = 961 KeV ($\approx E_\nu$);
- *only* for γ in the direction of Sm* recoil, angular momentum conservation implies Sm* helicity = ν_e helicity = γ helicity = ±1 [*see box with 2 alternative hypotheses*].

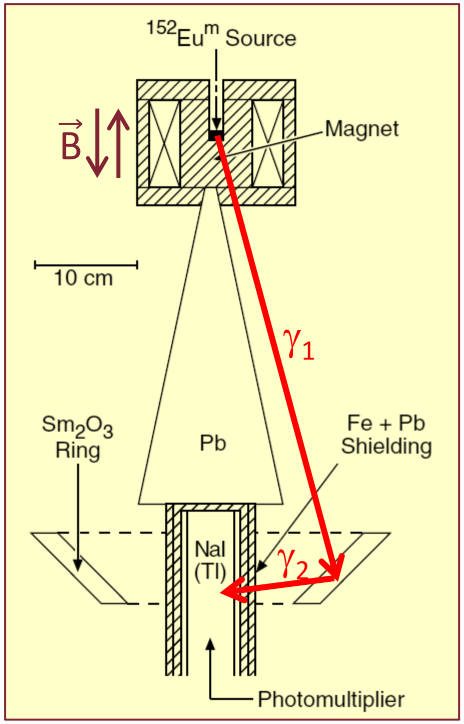
- Therefore, the method is:
  - [cannot measure directly the ν_e spin]
  - select and measure the γ's emitted anti-parallel to the ν_e's, i.e. in the same direction of the ($^{152}Sm^*$);
  - measure their spin;
  - reconstruct the ν_e **helicity**.

the ν_e helicity : resonant scattering

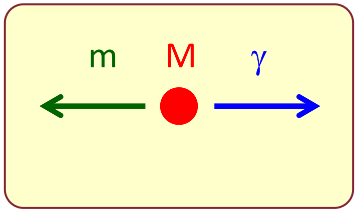
- For γ of 961 keV, the dominant interaction with matter is the Compton effect; the Compton cross section is spin-dependent: the transmission is larger when the γ and e^- spin are parallel.
- Therefore, a strong and reversible $\vec{B}$ (saturated iron) selects the polarized γ's, producing an asimmetry between the two $\vec{B}$ orientations.
- Need also to select only the γ's polarized according to the ν_e spin, i.e. produced opposite to the ν_e's → use the method of *resonant scattering* in the Sm_2O_3 ring:

$$\gamma_1 + {}^{152}Sm \rightarrow {}^{152}Sm^* \rightarrow {}^{152}Sm + \gamma_2.$$

- [kinematics (next slide) : a nucleus at rest, excited by an energy E_0, decays with a γ emission; the γ energy in the lab. is reduced by a factor $E_0/(2M)$].
- In general, γ_1 energy is degraded and NOT sufficient for Sm excitation (i.e. to produce γ_2).
- But, if γ_1 is anti-parallel to ν_e, the Sm* recoils against ν_e. The resultant Doppler effect in the correct direction provides γ_1 of the necessary amount of extra energy ($E_\nu \approx E_\gamma$).
- In conclusion, only the γ's anti-parallel to ν_e's are detected, but those γ's carry the information about ν_e helicity.



the ν_e helicity : kinematics



Kinematics

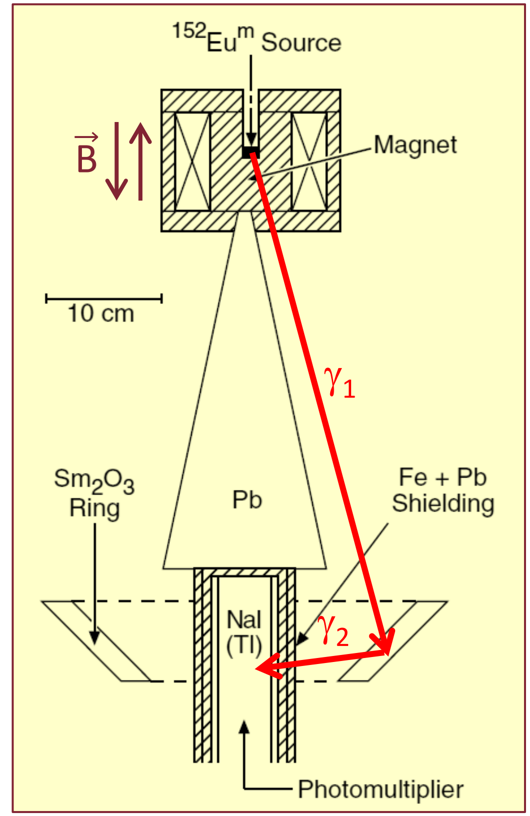
$$M \rightarrow m\,\gamma; \qquad E_0 = M - m;$$

$$\text{M sys.}\ \begin{cases} M = [M, & 0, & 0,0]; \\ \gamma = [E_\gamma, & E_\gamma, & 0,0]; \\ m = [M - E_\gamma, & -E_\gamma, & 0,0]; \end{cases}$$

$$m^2 = (M - E_\gamma)^2 - E_\gamma^2 = M^2 + \cancel{E_\gamma^2} - 2ME_\gamma - \cancel{E_\gamma^2};$$

$$E_\gamma = \frac{M^2 - m^2}{2M} = \frac{M + m}{2M} E_0 =$$

$$= \frac{M + M - E_0}{2M} E_0 = E_0 \left(1 - \frac{E_0}{2M} \right).$$



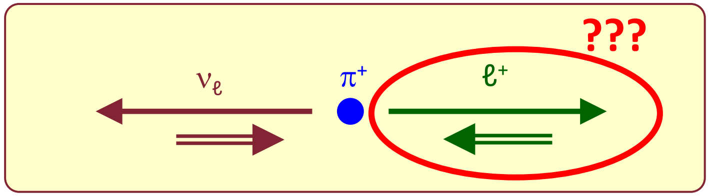
$\rightarrow$ if the excited nucleus (M) is at rest, the energy of the γ in the lab. is smaller than the excitation energy E_0; therefore it is insufficient to excite another nucleus at rest; for this to happen, the excited nucleus has to move in the right direction with the appropriate energy.

weak decays : $\pi^\pm$

- The $\pi^\pm$ is the lightest hadron; therefore it may only decay through semileptonic CC weak processes, like (consider only the +ve case, the −ve is similar) :

$$\pi^+ \to \mu^+ \nu_\mu; \quad \pi^+ \to e^+ \nu_e.$$

- In reality, it almost decays only into μ's: the electron decay is suppressed by a factor ≈ 8,000, NOT understandable, also because the $\pi \to e$ decay is favored by space phase.
- The reason is the **helicity**:
  - in the π^+ reference frame, the momenta of the ℓ^+ and the ν_ℓ must be opposite;
  - since the π^+ has spin 0, the spins of the ℓ^+ and the ν must also be opposite;
  - therefore the two particles must have the same helicity;
  - since the ν (a ~massless particle) must have negative helicity, the ℓ^+ (a non-massless antiparticle) is also forced to have negative helicity;
  - therefore the transition is suppressed by a factor $(1 - \beta_\ell)$;
  - the e^+ is ultrarelativistic ($p_e \approx m_\pi / 2 >> m_e$), while the μ^+ has small β [*compute it !!!*];
  - therefore the decay $\pi \to e$ is strongly suppressed respect to $\pi \to \mu$.



Kinematics (next slide) :

- $p_\ell = [(m_\pi^2 - m_\ell^2) / (2\, m_\pi)]$;
- $\beta_e = (1 - 2.6 \times 10^{-5})$;
- $\beta_\mu = 0.38$.

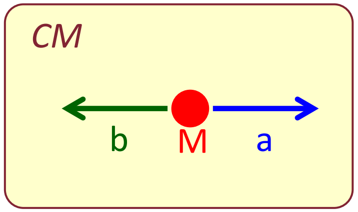
weak decays : kinematics

SOLUTION : (more general)

Decay M $\to$ a b. Compute p = $|\vec{p}_a|$ = $|\vec{p}_b|$ in the CM system, i.e. the system of M:

$$\text{CM}\begin{cases}(M, & 0, & 0,0) \\ (\sqrt{m_a^2+p^2}, & p, & 0,0); \\ (\sqrt{m_b^2+p^2}, & -p, & 0,0)\end{cases}$$

$$p^2 = \frac{\left[M^2-(m_a-m_b)^2\right]\left[M^2-(m_a+m_b)^2\right]}{4M^2}.$$



energy conservation : $M = \sqrt{m_a^2+p^2} + \sqrt{m_b^2+p^2}$;

$2\sqrt{m_a^2+p^2}\sqrt{m_b^2+p^2} = M^2 - m_a^2 - m_b^2 - 2p^2$;

$4\left[m_a^2 m_b^2 + p^2\left(m_a^2+m_b^2\right) + \cancel{p^4}\right] = \left(M^2-m_a^2-m_b^2\right)^2 + 4\cancel{p^4} - 4p^2\left(M^2-m_a^2-m_b^2\right)$;

$4p^2\left[\left(\cancel{m_a^2}+\cancel{m_b^2}\right)+\left(M^2-\cancel{m_a^2}-\cancel{m_b^2}\right)\right] = -4m_a^2 m_b^2 + \left(M^2-m_a^2-m_b^2\right)^2$;

$4p^2M^2 = \left[\left(M^2-m_a^2-m_b^2\right)+2m_a m_b\right]\left[\left(M^2-m_a^2-m_b^2\right)-2m_a m_b\right] = \text{(see above)}$

a) $m_a = m_b = m$; e.g. $K^0 \to \pi^0\pi^0$;

$$p^2 = \frac{M^2-4m^2}{4} = \frac{(M+2m)(M-2m)}{4};$$

§ LHC

b) $m_a = m_b = 0$; e.g. $\pi^0 \to \gamma\gamma$, $H \to \gamma\gamma$;

$$p^2 = \frac{M^2}{4};\quad p = \frac{M}{2};$$

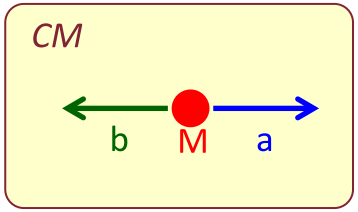
§ LEP2

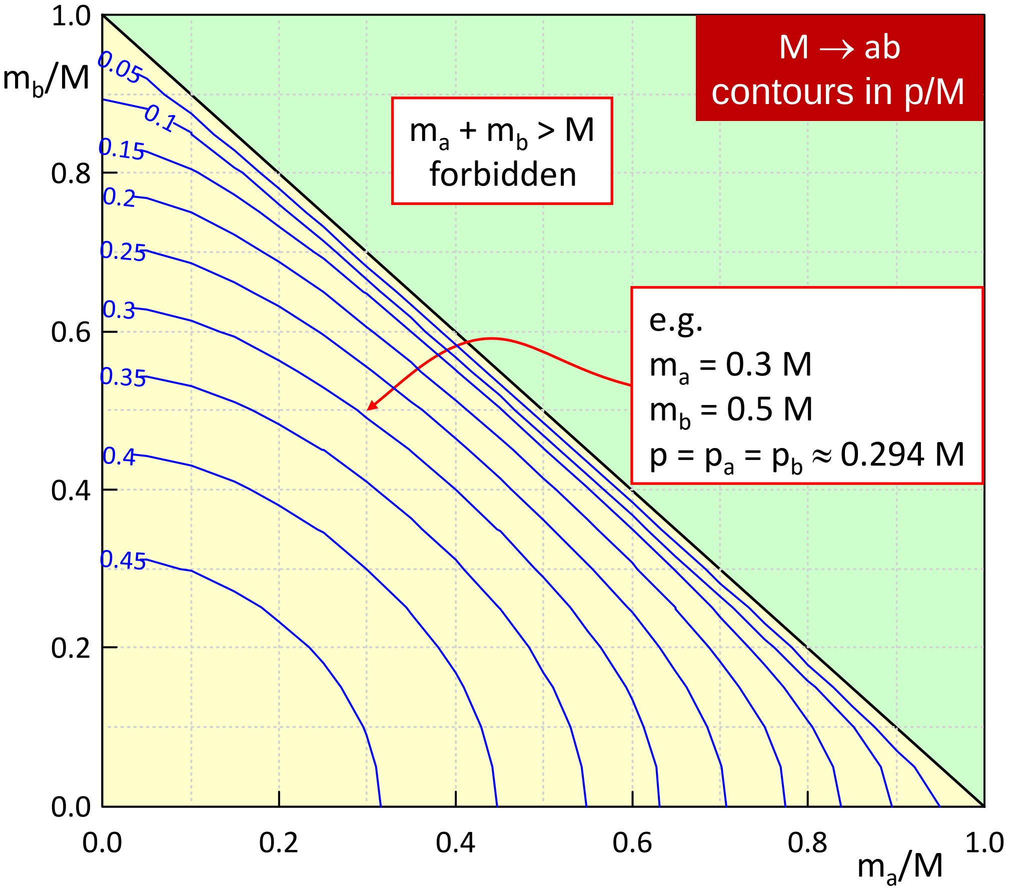
c) $m_a = m$; $m_b = 0$; e.g. $\pi^+ \to \mu^+\nu_\mu$, $Z^* \to Z\gamma$;

$$p = \frac{M^2-m^2}{2M} = \frac{M}{2}\left[1-\left(\frac{m}{M}\right)^2\right].$$

weak decays : contour plot

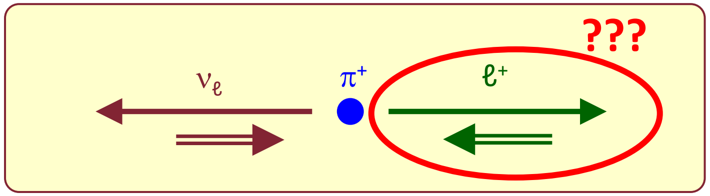
same info as in previous slide, only "easier" to see





the plot is only here to show you how easy it is to produce an apparently sophisticated and professional plot.

weak decays : $\pi^\pm \to (e^\pm \leftrightarrow \mu^\pm)$



Problem: compute the factor in the $\pi^\pm$ decay between μ and e.

Assume for the decay $\pi \to \ell$ [$\ell = \mu$ or e] :

p = decay product momentum;

ρ_ℓ = dN/dE_{tot} = phase space factor;

dN = $Vp^2dpd\Omega/(2\pi)^3$;

$(1 - \beta_\ell)$ = helicity suppression;

BR_ℓ = const $\times\ \rho_\ell \times (1 - \beta_\ell)$.

In this case the decay is isotropic. Then :

$\rho_\ell \propto p^2dp/dE_{tot}$;

4-momentum conservation [use previous slide and save only terms ℓ-dependent]:

$$p_\ell = p_\nu = E_\nu \equiv p; \qquad E_{tot} = m_\pi; \qquad E_\ell = m_\pi - E_\nu = m_\pi - p;$$

$$p = \frac{m_\pi^2 - m_\ell^2}{2m_\pi} = \frac{E_{tot}}{2} - \frac{m_\ell^2}{2E_{tot}}; \quad \frac{dp}{dE_{tot}} = \frac{1}{2} + \frac{m_\ell^2}{2m_\pi^2} = \frac{m_\pi^2 + m_\ell^2}{2m_\pi^2};$$

$$\rho_\ell \propto \left(\frac{m_\pi^2 - m_\ell^2}{2m_\pi}\right)^2 \frac{m_\pi^2 + m_\ell^2}{2m_\pi^2} = \frac{\left(m_\pi^2 + m_\ell^2\right)\left(m_\pi^2 - m_\ell^2\right)^2}{\cancel{8}\,\cancel{m_\pi^4}};$$

$\rho_e > \rho_\mu$

$$1 - \beta_\ell = 1 - \frac{p_\ell}{E_\ell} = 1 - \frac{p}{m_\pi - p} = \frac{m_\pi - 2p}{m_\pi - p} =$$

$$= \frac{m_\pi - 2(m_\pi^2 - m_\ell^2)/(2m_\pi)}{m_\pi - (m_\pi^2 - m_\ell^2)/(2m_\pi)} = \frac{2m_\ell^2}{m_\pi^2 + m_\ell^2};$$

$$BR_\ell \propto \left(m_\pi^2 + m_\ell^2\right)\left(m_\pi^2 - m_\ell^2\right)^2 \frac{\cancel{2}m_\ell^2}{m_\pi^2 + m_\ell^2} =$$

$$\propto m_\ell^2\left(m_\pi^2 - m_\ell^2\right)^2.$$

$1 - \beta_e <<< 1 - \beta_\mu$

For electrons, $m_e << m_\pi$, so :

$$\frac{BR\left(\pi^+ \to e^+\nu_e\right)}{BR\left(\pi^+ \to \mu^+\nu_\mu\right)} = \left(\frac{m_e}{m_\mu}\frac{m_\pi^2 - \cancel{m_e^2}}{m_\pi^2 - m_\mu^2}\right)^2 \approx 1.28 \times 10^{-4}.$$

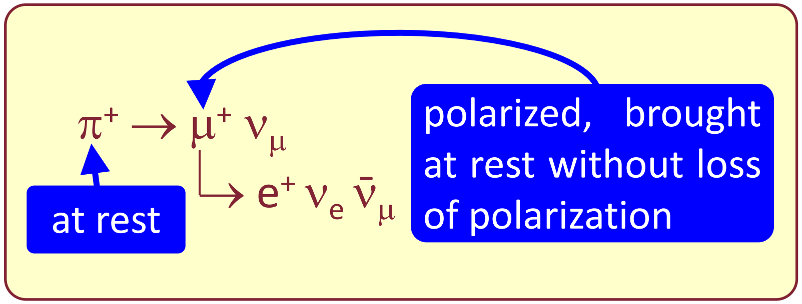
Experimentally, it is measured

$$\frac{BR\left(\pi^+ \to e^+\nu_e\right)}{BR\left(\pi^+ \to \mu^+\nu_\mu\right)} = 1.23 \times 10^{-4}.$$

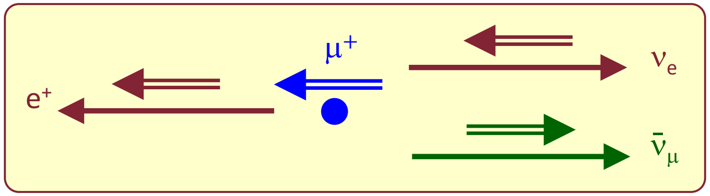
i.e. $N(\pi\to\mu) \approx$ 8,000 $N(\pi\to e)$

weak decays : $\mu^\pm$

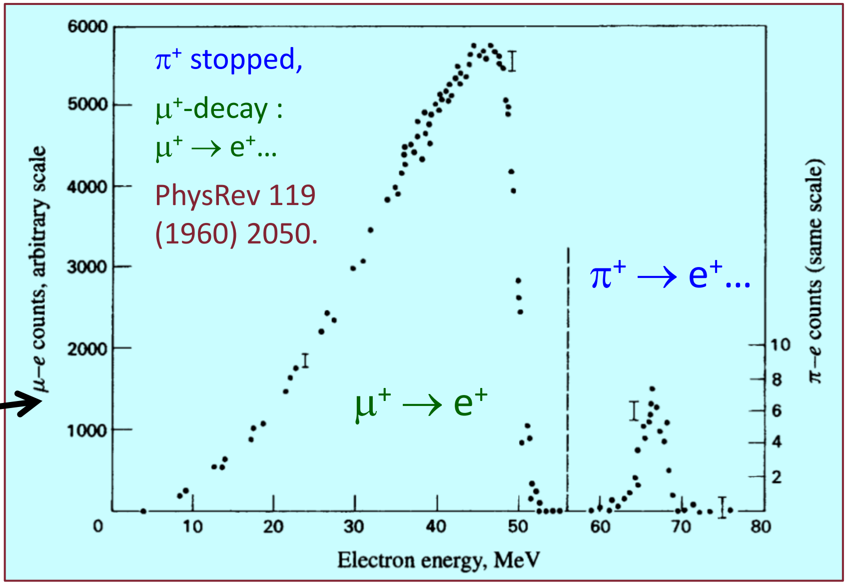
- Consider a famous experiment (Anderson et al., 1960) :



- In the μ^+ ref. frame (=LAB), this configuration is clearly preferred :

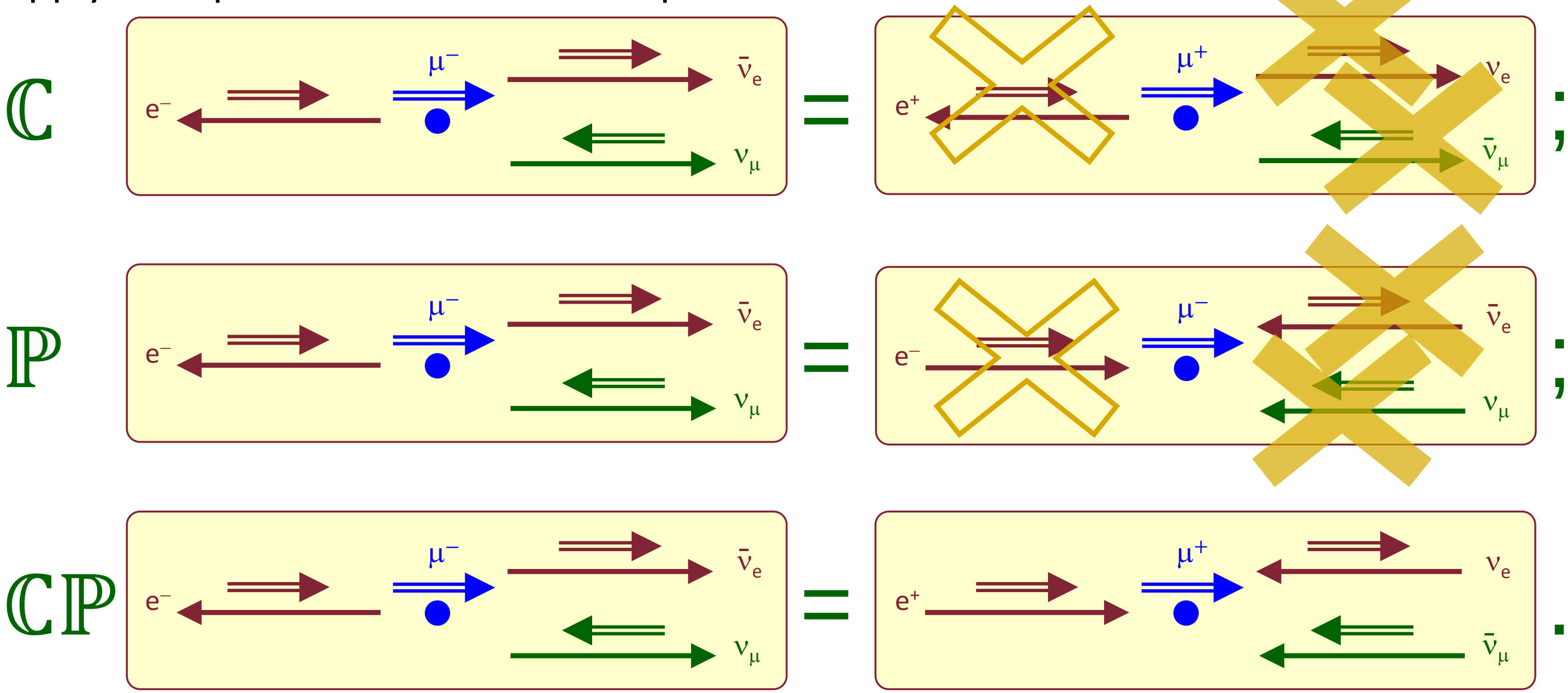


- In this angular configuration, both space and angular momentum are conserved, the particles are left- and the anti-particles right-handed.
- From the figure :
  - few e^+ directly from π^+ decay, shown in the right part ($\int\mu$ / $\int e \approx 8{,}000$);
  - the electron energy is the only measurable variable;
  - kinematical considerations show that it is correlated with the angular variables, and that the value $E_e \approx m_\mu$ / 2 is possible only for parallel ν's.
  - the distribution clearly shows the parity violation in muon decay.



weak decays : $\mathbb{C}$, $\mathbb{P}$ in μ decay

Apply the operators $\mathbb{C}$ and $\mathbb{P}$ to the previous cases :



- [the "×" shows the forbidden – not existent – particles]
- both $\mathbb{C}$ and $\mathbb{P}$ alone transforms the decay into non-existent processes (we say "**both $\mathbb{C}$ and $\mathbb{P}$ separately are not conserved in this process**");
- instead, the application of $\mathbb{CP}$ turns a μ^- decay (**which does exist**) into a μ^+ decay (**which also exists**) $\rightarrow$ "$\mathbb{CP}$ is conserved in this process".

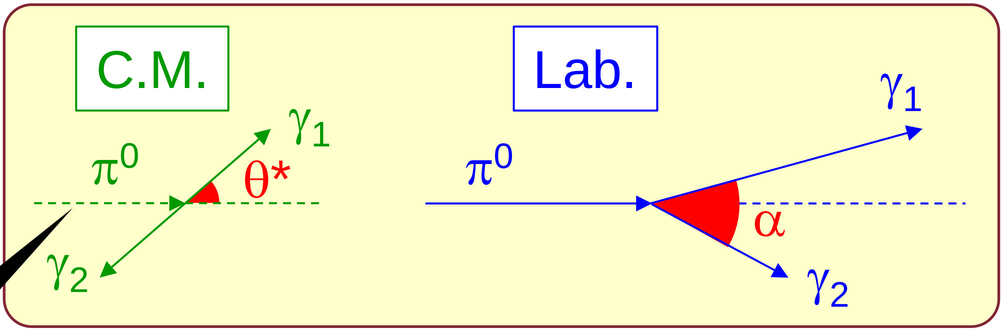
decay $\pi^0 \to \gamma\gamma$: L-transf.

L-transf
$$\begin{cases} E = \gamma(E^* + \beta p_\ell^*); \\ p_\ell = \gamma(p_\ell^* + \beta E^*); \\ p_T = p_T^*; \end{cases}$$

NB: L-transf. CM → Lab.

$$m \equiv m_{\pi^0}; \qquad \beta \equiv \frac{p_{\pi^0}}{E_{\pi^0}}; \qquad \gamma \equiv \frac{E_{\pi^0}}{m_{\pi^0}}.$$

In CM, π^0 at rest.



$$\begin{cases} & \text{C.M.} & \text{Lab.} \\ \pi^0 & m\{1,0,0,0\} & m\{\gamma,\beta\gamma,0,0\} \\ \gamma_1 & \frac{m}{2}\{1,\cos\theta^*,\sin\theta^*,0\} & \frac{m}{2}\{\gamma(1+\beta\cos\theta^*),\gamma(\cos\theta^*+\beta),\sin\theta^*,0\} \\ \gamma_2 & \frac{m}{2}\{1,-\cos\theta^*,-\sin\theta^*,0\} & \frac{m}{2}\{\gamma(1-\beta\cos\theta^*),\gamma(-\cos\theta^*+\beta),-\sin\theta^*,0\} \end{cases}$$

- the $\pi^0 \to \gamma\gamma$ decay is an e.m. process; it is here just for convenience;
- see also FNSN1, § Cinematica, 22-26.

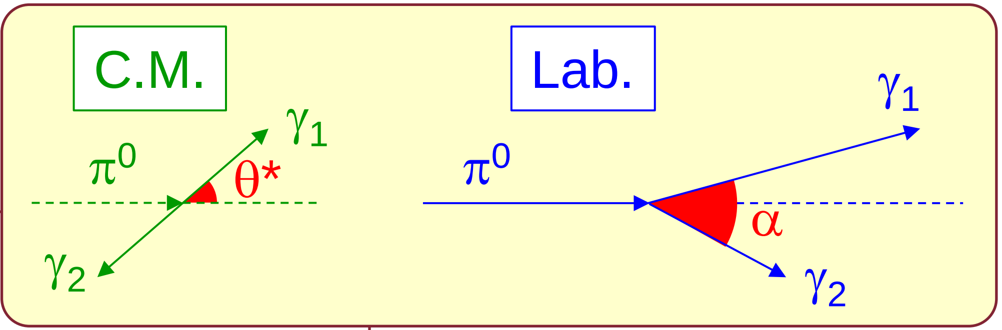
[...] = 1

$$\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2} = \frac{\vec{p}_1^{Lab}\cdot\vec{p}_2^{Lab}}{E_1^{Lab}E_2^{Lab}} = \frac{\cancel{\gamma}^2\left(\beta^2 - \cos^2\theta^*\right) - \sin^2\theta^*\left[\cancel{\gamma}^2(1-\beta^2)\right]}{\cancel{\gamma}^2\left(1-\beta^2\cos^2\theta^*\right)} = \frac{\beta^2\left(1+\sin^2\theta^*\right)-1}{1-\beta^2\cos^2\theta^*};$$

for a γ : $|\vec{p}| = E$

$$\sin^2\frac{\alpha}{2} = -\frac{1}{2}\left(\frac{\beta^2\left(1+\sin^2\theta^*\right)-1}{1-\beta^2\cos^2\theta^*} - \frac{1-\beta^2\cos^2\theta^*}{1-\beta^2\cos^2\theta^*}\right) = \frac{\boxed{\beta^2+\beta^2-2}}{\boxed{-2}\left(1-\beta^2\cos^2\theta^*\right)} = \frac{1}{\boxed{\gamma^2}\left(1-\beta^2\cos^2\theta^*\right)}.$$

decay $\pi^0 \to \gamma\gamma$: angle α



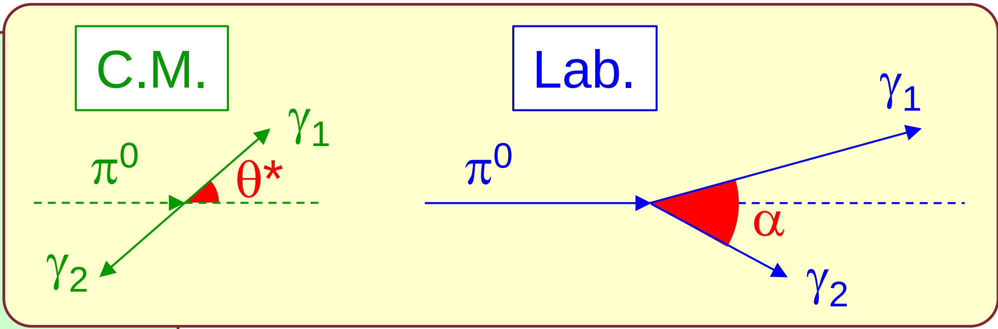
$$\sin^2\frac{\alpha}{2} = \frac{1}{\gamma^2\left(1-\beta^2\cos^2\theta^*\right)};$$

$$\xrightarrow[\substack{\theta^*=90°,\\ \cos\theta^*=0}]{} \quad [\mapsto\updownarrow] \quad \sin^2\frac{\alpha}{2}\bigg|_{min} = \frac{1}{\gamma^2} = \left(\frac{m_{\pi^0}}{E_{\pi^0}}\right)^2 \quad \to \alpha_{min} \cong \frac{2m_{\pi^0}}{E_{\pi^0}};$$

$$\xrightarrow[\substack{\theta^*=0°,\\ \cos\theta^*=1}]{} \quad [\mapsto\leftrightarrow] \quad \sin^2\frac{\alpha}{2}\bigg|_{max} = \frac{1}{\gamma^2\left(1-\beta^2\right)} = 1 \quad \to \alpha_{max} = 180°;$$

	$f(\theta^*)$	$\alpha\vert_{min}\,[\cos\theta^*=0]$	$\alpha\vert_{max}\,[\cos\theta^*=1]$
π^0	$m\{\gamma,\beta\gamma,0;1\}$	$m\{\gamma,\beta\gamma,0;1\}$	$m\{\gamma,\beta\gamma,0;1\}$
γ_1	$\frac{m}{2}\{\gamma(1+\beta\cos\theta^*),\gamma(\cos\theta^*+\beta),\sin\theta^*;0\}$	$\frac{m}{2}\{\gamma,\beta\gamma,1;0\}$	$\frac{m}{2}\{\gamma(1+\beta),\gamma(1+\beta),0;0\}$
γ_2	$\frac{m}{2}\{\gamma(1-\beta\cos\theta^*),\gamma(-\cos\theta^*+\beta),-\sin\theta^*;0\}$	$\frac{m}{2}\{\gamma,\beta\gamma,-1;0\}$	$\frac{m}{2}\{\gamma(1-\beta),\gamma(-1+\beta),0;0\}$

decay $\pi^0 \to \gamma\gamma$: $\mathcal{P}(\alpha)$



$$\mathrm{spin}(\pi^0) = 0 \rightarrow \mathcal{P}(\cos\theta^*) = \mathrm{flat} = 1/2.$$

Therefore :

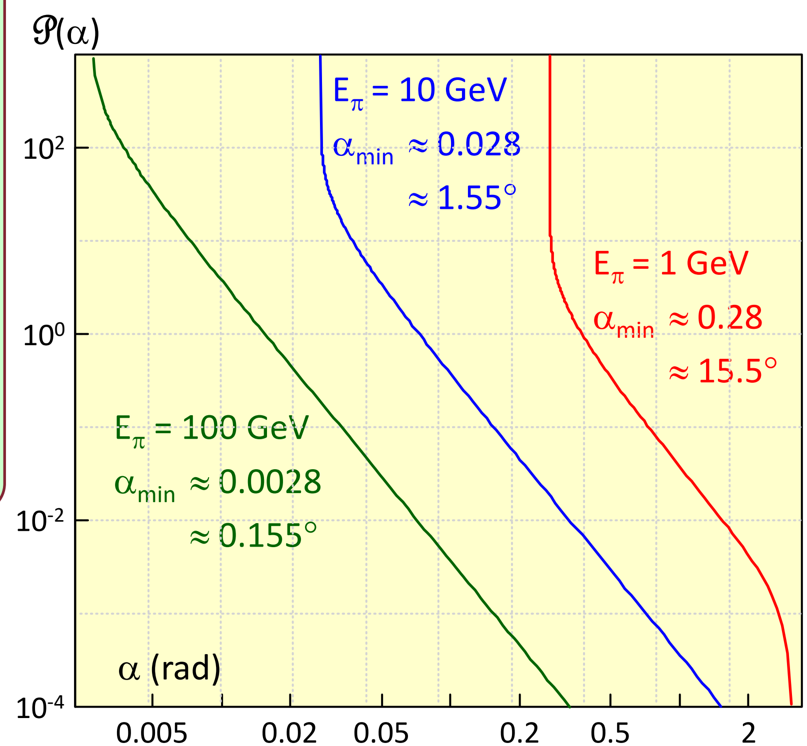
$$E_\gamma^{1,2} = \frac{m\gamma}{2}(1 \pm \beta\cos\theta^*) \rightarrow \frac{dE_\gamma^{1,2}}{d\cos\theta^*} = \pm\frac{m\beta\gamma}{2} \rightarrow$$

$$\boxed{\mathcal{P}\left(E_\gamma^{1,2}\right) = \mathcal{P}(\cos\theta^*) \Big/ \left|\frac{dE_\gamma^{1,2}}{d\cos\theta^*}\right| = \frac{1}{2}\frac{2}{m\beta\gamma} = \frac{1}{m\beta\gamma} = \frac{1}{p_{\pi^0}}}$$

flat in $\left[\frac{m\gamma}{2}(1-\beta), \frac{m\gamma}{2}(1+\beta)\right]$.

$$\boxed{\mathcal{P}(\alpha) = \frac{1}{4\beta\gamma}\frac{\cos(\alpha/2)}{\sin^2(\alpha/2)\sqrt{\gamma^2\sin^2(\alpha/2)-1}}}$$

[no proof, $\rightarrow$ FNSN1, §cinematica, 26].

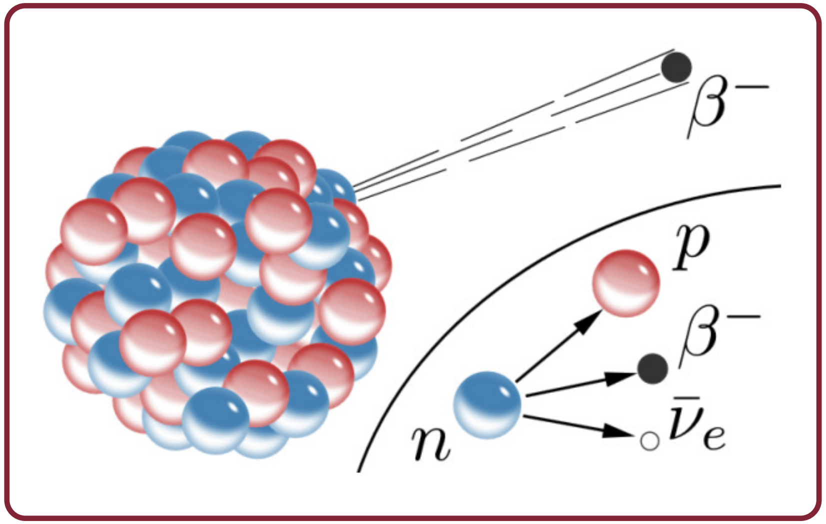


nota bene –
mutatis mutandis, similar kinematics also for $H \rightarrow \gamma\gamma$ [spin(π^0) = spin(H) = 0].

β decay : introduction

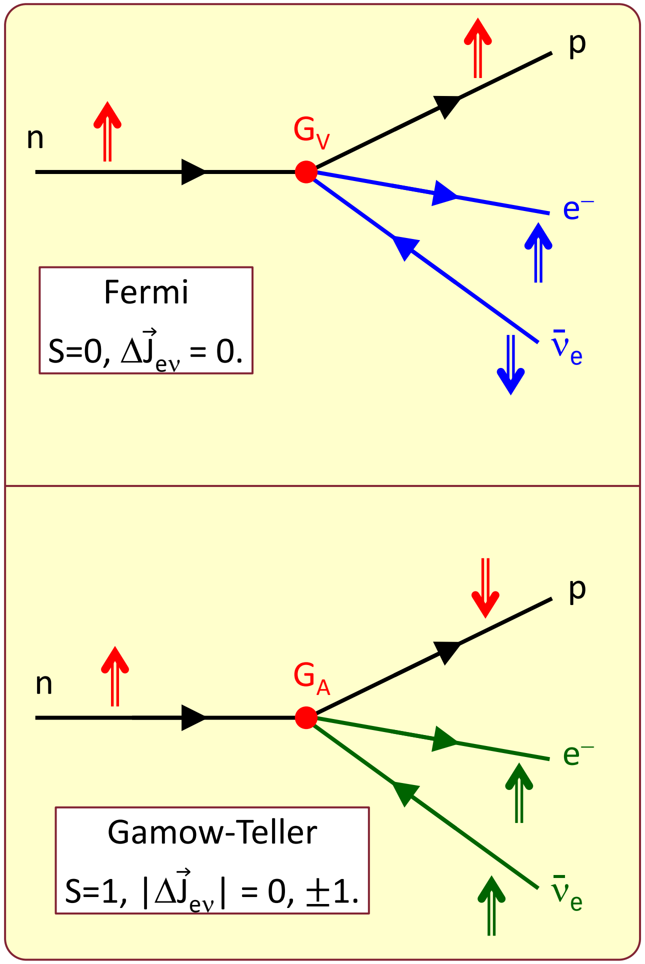
- For **point-like fermions**, CC is "V – A", both for leptons and quarks [*the only difference for hadrons being the CKM "rotation", see later*];
- however, nucleons and hyperons (p, n, Λ, Σ, Ξ, Ω) are **bound states of non-free quarks**;
- for low Q^2 processes, the "spectator model" (in this case the free quark decay) is an **unrealistic** approximation;
- **strong interaction corrections** are important → modify V – A dynamics;
- the standard approach, due to Fermi, is to produce a parameterization, based on the vector properties of the current (S-P-V-A-T, see) and then compute ↔ measure the coefficients;
- pros : quantitative theory, which reproduces the experiments well;
- cons : lack of deep understanding of the parameters.

the simple and successful approach, used for point-like decays, is not valid here, because of strong interaction corrections; those are (possibly understood, but) non-perturbative and impossible to master with present-day math; same as chemistry ↔ electromagnetism.



β decay : Fermi $\leftrightarrow$ Gamow-Teller

- In Fermi theory, CC currents were classified according to the properties of the transition operator.
- In neutron β-decay, the e-ν pair may be created as a spin singlet (S=0) or triplet (S=1). In case of NO orbital angular momentum, there are two possibilities to conserve the total angular momentum :
  - Fermi transitions [F], S=0, $\Delta J_{e\nu}=0$: the direction of the spin of the nucleon remains unchanged; in modern language, [*it can be shown that*] the interaction takes place with vector coupling G_V;
  - Gamow-Teller transitions [G-T], S=1, $\Delta J_{e\nu}= 0, \pm 1$: the direction of the spin of the nucleon is turned upside down (it "flips"); [...] the transition happens with axial-vector coupling G_A.
- In principle, F and G-T processes are completely different : there is no a-priori reason why the coupling should be similar or even related.



β decay : S, P, V, A, T

- Study the **neutron β decay**; assume :
  - p and n are spin-½ fermions;
  - $e^{\pm}$ and ν are spin-½ fermions, but the ν exists only with helicity = −1.

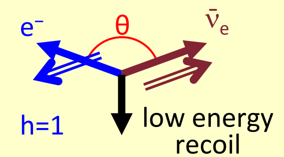
- Then, the most general matrix element for the four-body interaction is

$$\mathcal{M}_{fi} = \frac{G_F}{\sqrt{2}} \sum_j C_j \left[\bar{u}_p O_j u_n \right] \left[\bar{u}_e O_j \left(1 - \gamma_5 \right) u_\nu \right],$$

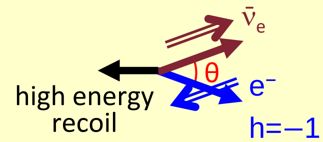
  - G_F : the overall coupling;
  - $\bar{u}_{p,n,e,\nu}$ ($u_{p,n,e,\nu}$) : creation (destruction) operators for p, n, e, ν;
  - $(1-\gamma_5)$: projector of −ve ν helicity;
  - C_j : sum coefficients (adimensional free parameters, *possibly of order 1*);
  - O_j : current operators with given vector properties : **S** = scalar, **P** = pseudo-scalar, **V** = vector, **A** = axial-vector, **T** = tensor.

- For β-decay, the pseudo-scalar term is irrelevant : P can only be built from the proton velocity v_p in the neutron rest frame, which are depressed by v_p/c;

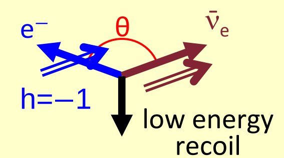
- For the other four terms, the angular distributions are [*BJ 399, YN1 561*] (1, ⅓ for singlet and triplet, β=electron velocity) :

- **S** ΔJ=0 

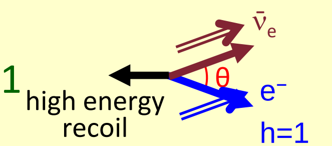
$1-\beta\cos\theta$

- **V** ΔJ=0 

$1+\beta\cos\theta$

- **A** |ΔJ|=1 

$1-⅓\beta\cos\theta$

- **T** |ΔJ|=1 

$1+⅓\beta\cos\theta$

β decay : V, A

- From comparison with data, some terms can be excluded:
  - (S and V) are Fermi transitions : they cannot be both present, due to the lack of observed interference between them;
  - (A and T) are G-T transitions : same argument holds;
  - the angular distributions of the electrons are only consistent with V for F and A for G-T.
- So the matrix element becomes :

$$\mathcal{M}_{fi} = \frac{G_F}{\sqrt{2}}\left[\bar{u}_p\gamma^\mu\left(C_V + C_A\gamma_5\right)u_n\right]\left[\bar{u}_e\gamma^\mu\left(1-\gamma_5\right)u_\nu\right],$$

- the value of C_V can be measured by comparing (composite) hadrons with (free, pure V–A) leptons; it turns out

$$C_V \approx 1.$$

- The value of C_A^2 can be measured from the relative strength of F and G-T, by comparing neutron β–decay with a pure Fermi ($^{14}O \to {}^{14}N\, e^+\nu$); for β decay:

$$|C_A| \cong 1.267.$$

- The sign of C_A could be measured from the polarization of the protons (a very difficult measurement); in practice from the interference between F and G-T in polarized neutrons decays :

$$C_A \cong -1.267.$$

Fermi did not know about parity violation, and would have written different matrix elements for his ("Fermi") transitions.

However, the final result for leptons and free quarks is very similar to his original proposal, but the factor (1-γ_5) :

$$\mathcal{M}_{fi} = \frac{G}{\sqrt{2}}\left[\bar{u}_p\gamma^\mu\left(1-\gamma_5\right)u_n\right]\left[\bar{u}_e\gamma^\mu\left(1-\gamma_5\right)u_\nu\right].$$

β decay : CVC, PCAC

- For the leptonic current, $C_A = -C_V$. These processes are much simpler, because leptons, unlike quarks, exist as free particles.
- The hadrons can be treated similarly when their partons (= quarks) interact as "quasi-free" particles, (e.g. DIS + the "spectator approximation" [§ν, § *Collider*]).
- In this case (e.g. in ν DIS), the CC exhibits for hadrons the same "V−A" structure as for leptons.
- However, **at low Q^2**, when hadrons behave as coherent particles and not as parton containers, the similarity appears to be broken.

$$\mathcal{M}_{fi} \propto \left[\bar{u}_p \gamma^\mu \left(1 - \frac{C_A}{C_V}\gamma_5\right) u_n\right]\left[\bar{u}_e \gamma^\mu (1-\gamma_5) u_\nu\right]$$

- In low Q^2 processes, [*it can be shown that*] the vector part of the hadronic current stays constant (**CVC**, conserved vector current), while the axial part is broken (**PCAC**(*), *"partially conserved axial current"*).
- In baryon β-decays, it is measured :
  - $n \to p\, e\, \bar{\nu}_e$, $\quad -C_A/C_V = 1.267$
  - $\Lambda \to p\, \pi^-, n\, \pi^0$ $\quad = +.718$
  - $\Sigma^- \to n\, e\, \bar{\nu}_e$ $\quad = -0.340$
  - $\Xi^- \to \Lambda e^- \bar{\nu}_e$ $\quad = +0.25$
  - [high Q^2 (free quarks) $\quad = 1$].

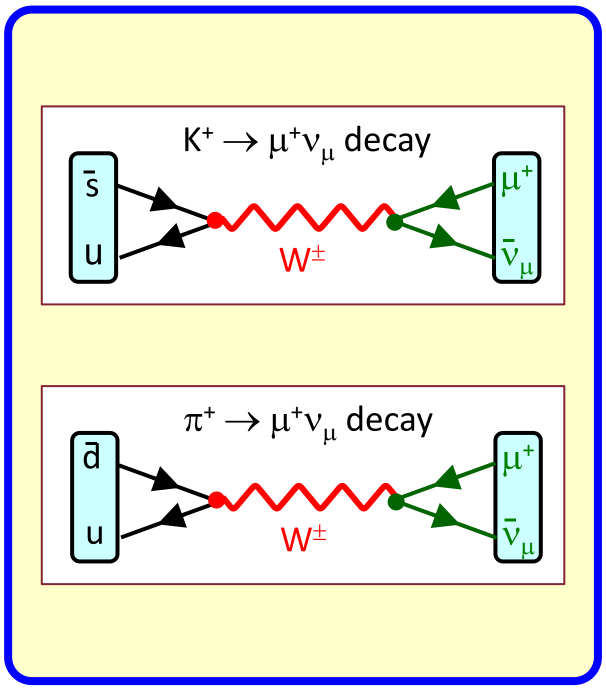
(*) *at the time, they preferred to say "partially conserved" instead of "badly broken"; it now seems that the acronym "PCAC" is slowly disappearing from the texts : you are kindly requested to forget the term "PCAC" forever.*

quark decays

- At quark level and high Q^2, the beautiful structure "V−A" seems restored: quarks behave as free, point-like particles, exactly like the leptons [§ *Collider*] .

- However, with more accurate data, some discrepancies appear, not due to strong interactions (see boxes).
- An apparent violation of CC universality ? A mistake ?

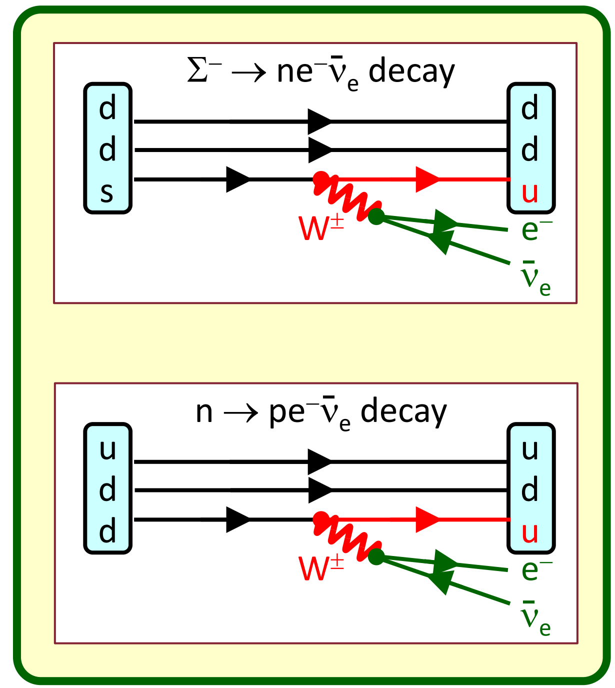
(*continue…*)



It is measured :

$$\frac{G_F^{2'}\begin{bmatrix}K^+ \to \mu^+\nu_\mu, \\ \Delta S = 1\end{bmatrix}}{G_F^{2''}\begin{bmatrix}\pi^+ \to \mu^+\nu_\mu, \\ \Delta S = 0\end{bmatrix}} \approx 0.05;$$

$$\frac{\Gamma\begin{bmatrix}\Sigma^- \to n e^- \bar{\nu}_e, \\ \Delta S = 1\end{bmatrix}}{\Gamma\begin{bmatrix}n \to p e^- \bar{\nu}_e, \\ \Delta S = 0\end{bmatrix}} \approx 0.05.$$



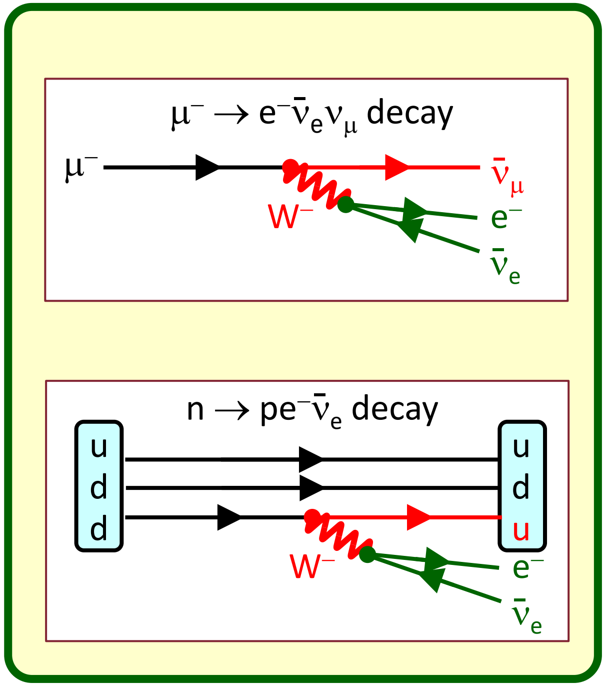
quark decays : Cabibbo theory

(*… continue …*)

Even tiny, but well measured effects seem to contradict the universality; "G_F" is slightly larger for leptons :

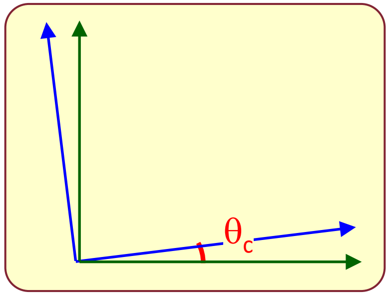
$$G_F\left[\mu^- \to e^- \bar{\nu}_e \nu_\mu\right] \approx 1.166 \times 10^{-5}\ \text{GeV}^{-2};$$

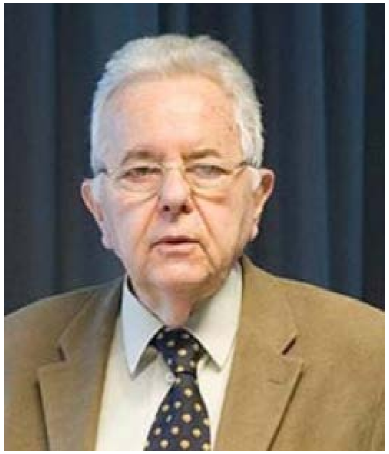
$$G_F\left[\begin{array}{r} n \to p e^- \bar{\nu}_e, \\ \text{i.e. } d \to u e^- \bar{\nu}_e \end{array}\right] \approx 1.136 \times 10^{-5}\ \text{GeV}^{-2}.$$



In 1963 N. Cabibbo [*at the time much younger than in the image*], invented a theory to explain the effect : the "Cabibbo angle" θ_c :

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}.$$





quark decays : Cabibbo "rotation"

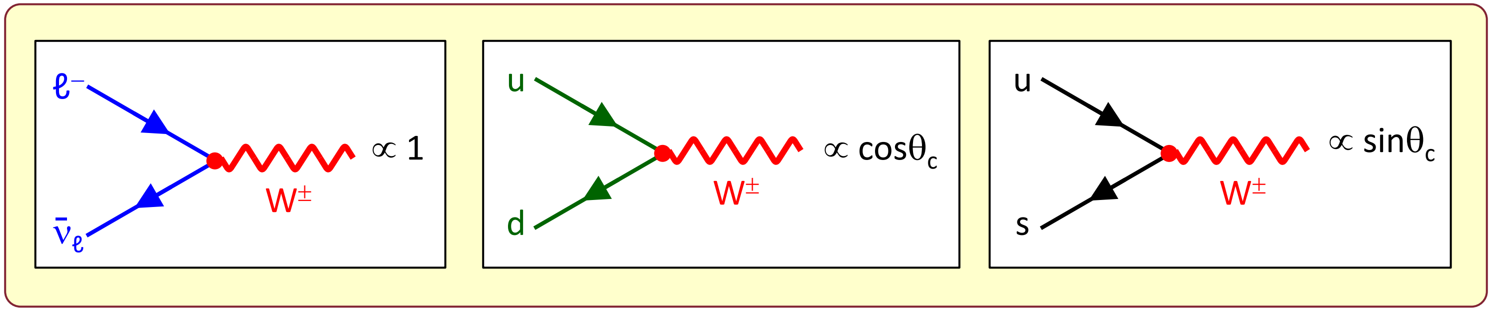
The idea was the following :

- the hadrons are built up with quarks **u d s** (**c b t** not yet discovered);
- however, in the CC processes, the quarks (d s) – same quantum numbers but S – mix together (= "rotate" by an angle θ_c), in such a way that the CC processes see "rotated" quarks (d' s') :

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}.$$

- therefore, respect to the strength of the leptonic processes (no mix), the ud coupling (actually ud') is decreased by a factor cos θ_c and the us coupling (actually us') by a factor sin θ_c;
- therefore the processes with $\Delta S = 0$ happen $\propto \cos^2\theta_c$ and those with $\Delta S = 1$ $\propto \sin^2\theta_c$;
- even processes $\propto \sin^4\theta_c$ may happen (e.g. in the charm sector, see §3), when two "Cabibbo suppressed" couplings are present in the same process;
- all the anomalies come back under control if

$$\sin^2\theta_c \approx .03, \cos^2\theta_c \approx .97.$$



quark decays : GIM mechanism

In this context the GIM mechanism was invented to explain the absence of FCNC:

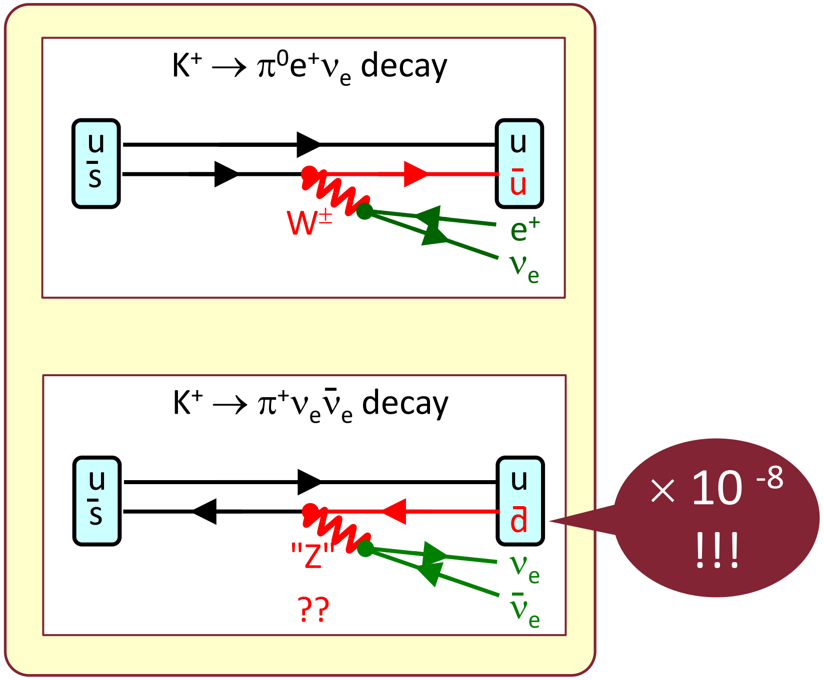
- data, at the time not understandable :

$$\left.\begin{array}{l} BR(K^0 \to \mu^+\mu^-) = 7\times 10^{-9} \\ BR(K^+ \to \mu^+\nu_\mu) = 0.64 \end{array}\right\} \left[\begin{array}{c}\text{already} \\ \text{mentioned}\end{array}\right];$$

$$\left.\begin{array}{l} BR(K^+ \to \pi^+\nu\bar{\nu}) \quad = (1.5^{+1.3}_{-0.9})\times 10^{-10} \\ BR(K^+ \to \pi^0 e^+\nu_e) = (4.98\pm 0.07)\times 10^{-2} \end{array}\right\}.$$

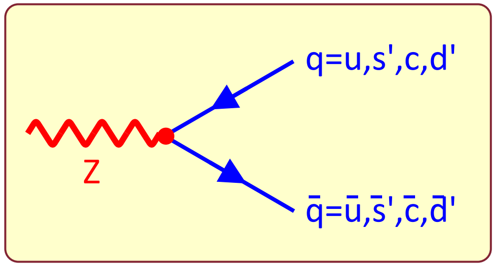
  i.e. a factor ~10^{-8} between NC and CC decays;

- if the Z, carrier of NC, see the same quark mixture as the $W^\pm$ in CC, then the NC decay would be suppressed only by a factor 5%;
- the idea was to introduce a fourth quark, called c (charm), with charge ⅔, as the u quark; this solves the FCNC problem;
- the c quark was discovered in 1974 [*see* § *3*].



quark decays : no FCNC

In the GIM mechanism, NC contain four hadronic terms, coupled with the Z.



Assume Cabibbo theory and sum all terms:

$$u\bar{u} + d'\bar{d}' + c\bar{c} + s'\bar{s}' =$$
$$= u\bar{u} + (d\cos\theta_c + s\sin\theta_c)(\bar{d}\cos\theta_c + \bar{s}\sin\theta_c) +$$
$$+ c\bar{c} + (s\cos\theta_c - d\sin\theta_c)(\bar{s}\cos\theta_c - \bar{d}\sin\theta_c) =$$
$$= u\bar{u} + c\bar{c} + d\bar{d} + s\bar{s} + \text{"0"}. \qquad (!!!)$$

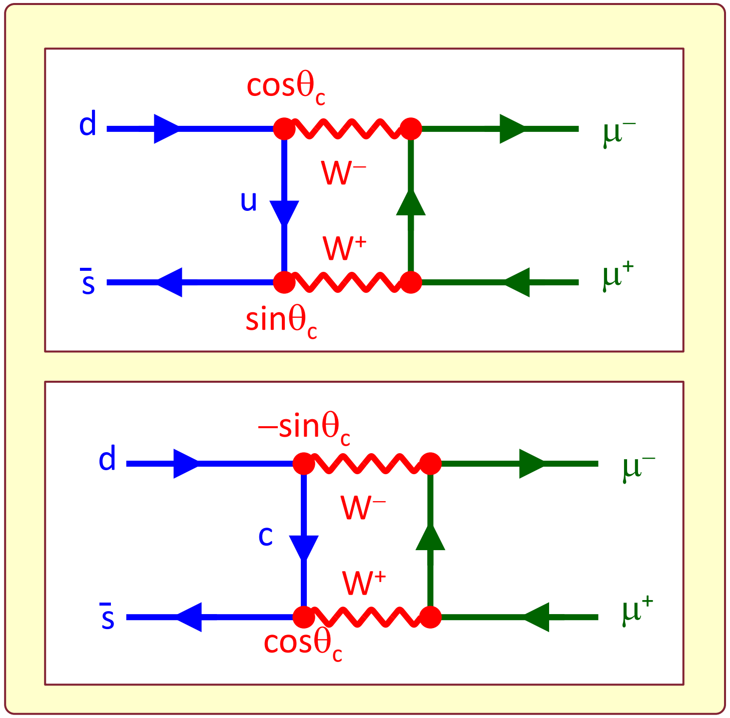
the "non-diagonal" terms, which induce FCNC, **disappear**.

Why $(K^0 \to \mu^+\mu^-)$ is small, but NOT = 0 ?

Look at the 1st "box diagram":

- technically a 2nd order ($\propto g^4 \sin\theta_c \cos\theta_c$) CC;
- same final state as a 1st order FCNC;
- incompatible with data (BR too large);
- cured by the 2nd diagram with a c quark, whose contribution cancels the first in the limit $m_c \to m_u$.

The cancellation depends on m_c. The decay $(K^0 \to \mu^+\mu^-)$ puts limits on m_c between 1 and 3 GeV [$J/\psi \to 2m_c \approx 3.1$ GeV, *see*].



quark decays : the third generation

In 1973, Kobayashi and Maskawa extended the Cabibbo scheme to a new generation of quarks : the new mixing matrix (analogous to the Euler matrix in ordinary space) is a three-dimension unitary matrix, with three real parameters ("Euler angles") and one imaginary phase :

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad \updownarrow W^{\pm}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The matrix is known as **CKM** (*Cabibbo-Kobayashi-Maskawa*) matrix.

K-M observed that the $\mathbb{CP}$ violation, already discovered, is automatically generated by the matrix, when the imaginary phase is non-zero.

In addition to the $\mathbb{CP}$-violation, the nine elements of the CKM matrix govern the flavor changes in CC processes.

The measurement of the elements and the check of the unitarity relations is an important subject of physics studies : e.g. if some element is too small, this could be an indication of term(s) missing in the sum, i.e. the presence of a next generation of quarks.

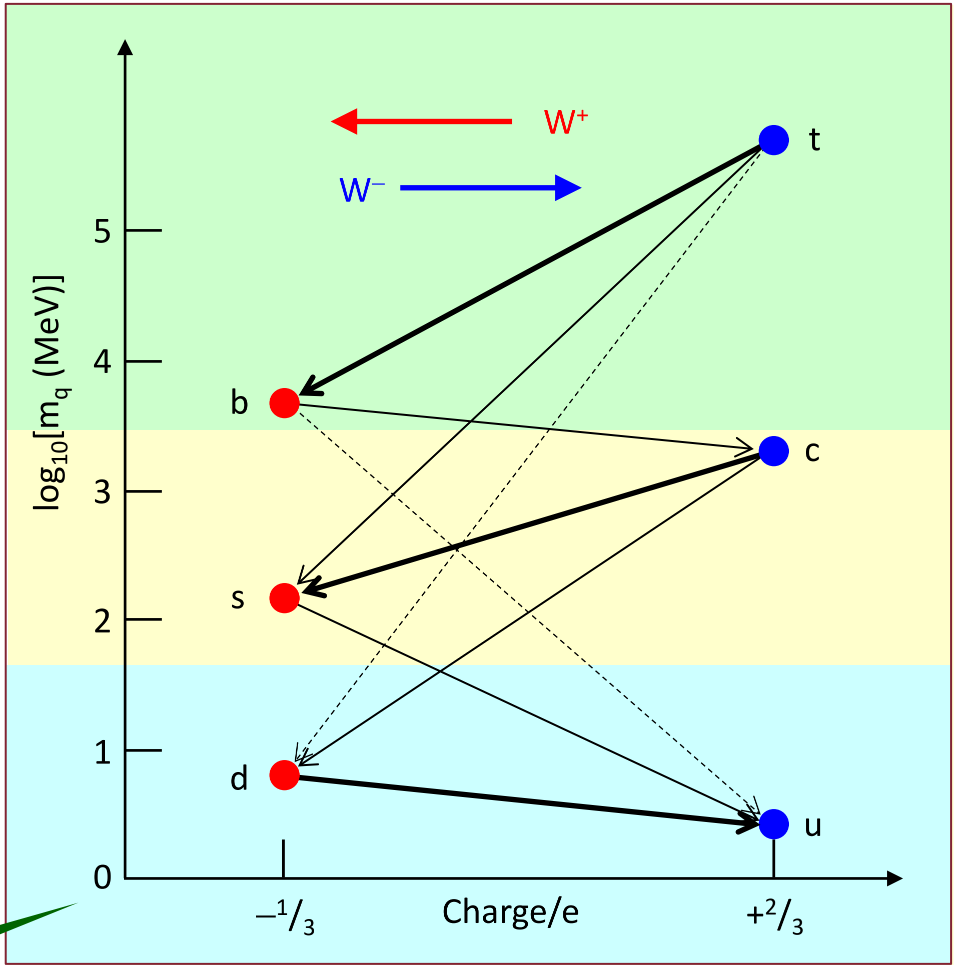
[*A discussion of the CKM matrix in* §5.]

Makoto Kobayashi

Toshihide Maskawa

summary : CC decays

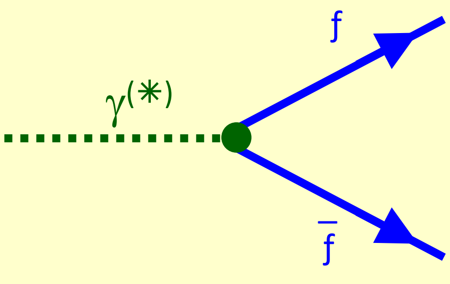
- The quark flavor changes only as a consequence of a weak CC interaction (*).
- Each type of quark can convert into each other with charge ±1, emitting or absorbing a W boson.
- The coupling is modulated by the strength of the mixing (the width of the line in fig.); in the SM it is described by the V_{CKM} matrix [§5].

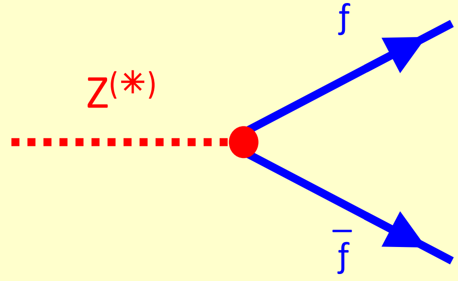


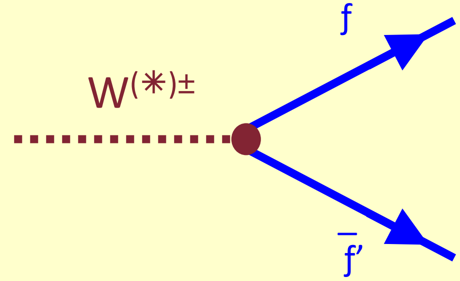
+ the equivalent table for q̄'s.

(*) since FCNC do NOT [seem to] exist, NC processes – with Z mediators – do NOT play any role in flavor decays.

summary : e.m., NC, CC







photon (γ)
(electromagnetism)

$$\mathscr{L}_F = -eJ^\mu_{e.m.}A_\mu;$$
$$J^\mu_{e.m.} = Q_f \bar{\Psi}_f \gamma^\mu \Psi.$$

[V]

neutral IVB (Z)
(neutral current)

$$\mathscr{L}_F = \frac{-e}{\sin\theta_W \cos\theta_W} J^\mu_{nc} Z_\mu;$$
$$J^\mu_{nc} = \bar{\Psi}_f \gamma^\mu \frac{g^f_V - g^f_A \gamma^5}{2} \Psi_f.$$

[combination g^f_V V + g^f_A A]

charged IVB ($W^\pm$)
(charged current)

$$\mathscr{L}_F = \frac{-e}{\sqrt{2}\sin\theta_W} J^\mu_{cc} \tau^\pm W^\pm_\mu;$$
$$J^\mu_{cc} = \bar{\Psi}_f \gamma^\mu \frac{1-\gamma^5}{2} \Psi_f.$$

[V - A]

Vectors & co.

vector properties of physical quantities :

- a 4-vector $\vec{\mathbf{v}}$ is the well-known quantity, which transforms canonically under a L-transformation $\mathbb{L}$ (both boosts and rotations), and Parity $\mathbb{P}$ in space :
  - space-time, 4-momentum, electric field, …
- an axial vector $\vec{\mathbf{a}}$ transforms like a vector under $\mathbb{L}$, but gains an additional sign flip under $\mathbb{P}$:
  - cross-products $\vec{v} \times \vec{v}$, magnetic field, angular momentum, spin, …
- a scalar **s** is invariant both under $\mathbb{L}$ and $\mathbb{P}$:
  - [4-]dot-products $\vec{v} \cdot \vec{v}$ or $\vec{a} \cdot \vec{a}$, module of a vector, mass, charge, …
- a pseudoscalar **p** is invariant under $\mathbb{L}$, but changes its sign under $\mathbb{P}$:
  - a triple product $\vec{v} \cdot \vec{v} \times \vec{v}$;
  - a scalar product $\vec{a} \cdot \vec{v}$ between a vector and an axial vector, e.g. the helicity(*);
- a tensor **t** is a quantity which also transforms canonically under $\mathbb{L}$ and $\mathbb{P}$, with ≥ 2 dimensions :
  - the electro-magnetic tensor $F^{\mu\nu}$.

____

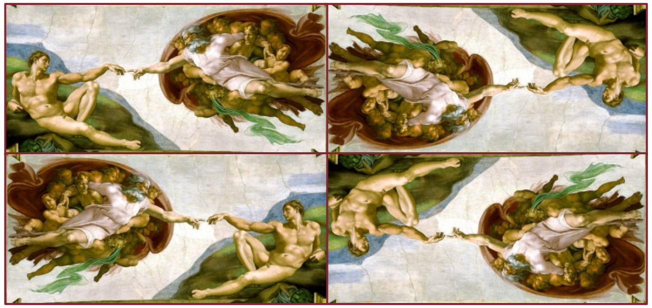
(*) the helicity h is the projection of the spin $\vec{s}$ along the momentum $\vec{p}$:

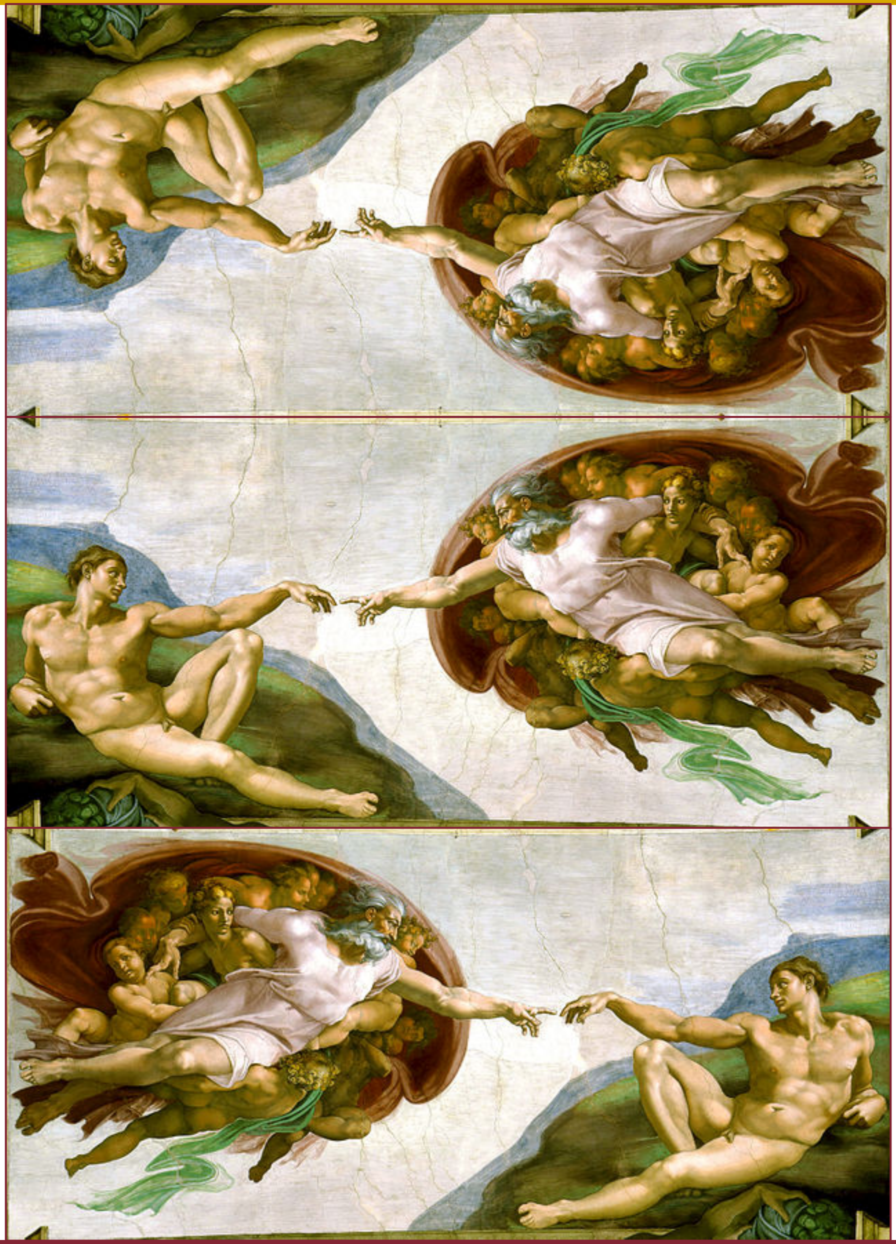
$$h = \frac{\vec{s} \cdot \vec{p}}{|\vec{s}| \cdot |\vec{p}|}.$$

Q. : this "parity violation" does NOT happen. Why ?

References

1. [BJ, 11], [YN1, 15], [YN2, 6.1-6.2];
2. Fermi theory : [FNSN1, 6];
3. the weak interactions : [MQR, 15] and [IE, 9-10];
4. π and μ decay : Garwin et al. (Lederman) Phys.Rev. 105 (1957) 1415, Anderson et al, Phys.Rev. 119 (1960) 2050.
5. modern β-decays : Severijns et al., Rev.Mod.Phys. 78 (2006) 991.
6. modern hyperon decays : Cabibbo et al., Ann.Rev.Nucl.Part.Sci. 53 (2003) 39.





Sapienza Università di Roma

End of chapter 4

Particle Physics - Chapter 5
K^0 mesons - CKM matrix

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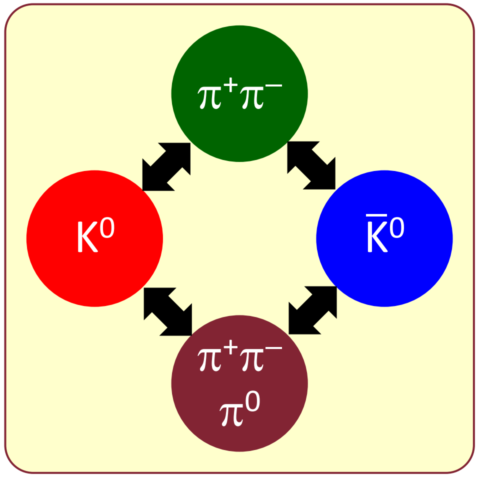
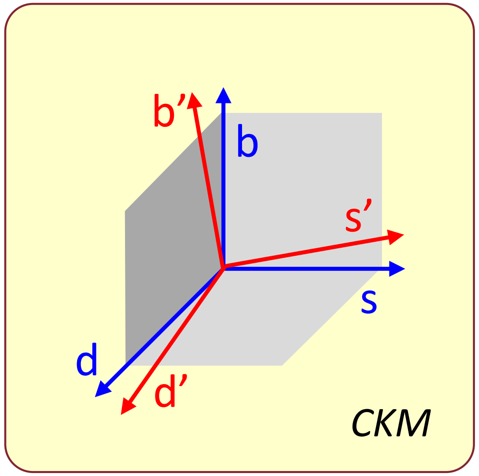
AA 18-19

last mod. 28-Mar-19

5 – K^0 mesons – CKM matrix

this section belongs to another chapter: It is here because of the similarity between ν and K^0 oscillations.





introduction

- The neutral mesons $\boxed{K^0 \text{ and } \bar{K}^0}$ are special quark systems, in which unusual and surprising phenomena are generated.
- The mathematical interpretation of these phenomena is based almost exclusively on the application of the fundamental principles of q.m., in particular the principle of quantum superposition.
- The experimental observation of the effects of oscillation and regeneration is a further elegant confirmation of the validity of these principles.
- The successes of the experimental physics of the '50s and '60s have been based both on the confirmation of accurate theoretical predictions (like oscillations) and to new and unexpected phenomena (like $\mathbb{CP}$ violation).
- They have been possible thanks to new techniques (e.g. regeneration), and to new experimental methods (e.g. the new accelerators, bubble / spark chambers) and by data analysis via computer.
- The study of these particles is possible only by analyzing the symmetry of Nature; K^0 physics emerges from the analysis of CPT symmetries, strangeness and isospin.
- In successive years, the K^0 meson system has been replicated by the $\boxed{B^0 \text{ mesons,}}$ with further fundamental studies.
- The interpretation in the SM of the flavor and $\mathbb{CP}$ violations requires the weak interactions theory and the CKM matrix.
- *… but we hope that experiments show also physics **bSM** !!!*

introduction : quantum states

- Quarks and antiquarks of the u and d type can form two <u>different</u> neutral mesons : $(u\bar{u})$ $(d\bar{d})$, or linear combinations like π^0 or η [*see <u>§ quark model</u>*].
- The same mechanism holds when heavier families, like (cs) (tb), are considered. Each heavy flavor has a quantum number which identifies it and its $\bar{q}$.
- These states make sense in a quantum basis of distinct <u>conserved flavors</u>, as in strong interactions.
- In different quantum bases (e.g. the one where $\mathbb{CP}$ is conserved, but not $\mathbb{C}$ and $\mathbb{P}$ separately), different states appear, which are linear superposition of the above.
- These states may offer a more natural description of the phenomena.

	K^0	$\bar{K}^0$	D^0	$\bar{D}^0$	B^0_d	$\bar{B}^0_d$	B^0_s	$\bar{B}^0_s$
$q\bar{q}$	$d\bar{s}$	$s\bar{d}$	$c\bar{u}$	$u\bar{c}$	$d\bar{b}$	$b\bar{d}$	$s\bar{b}$	$b\bar{s}$
S	+1	−1	0	0	0	0	−1	+1
C	0	0	+1	−1	0	0	0	0
B	0	0	0	0	+1	−1	+1	−1

quantum numbers of $q\bar{q}$ neutral mesons.

Warning: K^0 and K^+ are in the same doublet and contain $\bar{s}$; B^0/B^+ contain $\bar{b}$, while D^0 and D^+ contain c (not $\bar{c}$).

Questions (simple):

- other neutral mesons with heavy quarks ? [yes, D^0_s and $\bar{D}^0_s$ → *write their q.n.*;
- why states like $t\bar{u}$, $t\bar{c}$, ..., are not listed ?

production of K^0 mesons: the problem

- The K^0-mesons are produced by strong interactions with a fixed strangeness S :

  $|K^0\rangle = |d\bar{s}\rangle$, S = +1; $|\bar{K}^0\rangle = |s\bar{d}\rangle$, S = −1.

- Problem : get a pure sample of K^0's.

- A K^0 sample is created, e.g. ($\pi^- p \to \Lambda K^0$), with a "threshold energy" [*next slide*] :

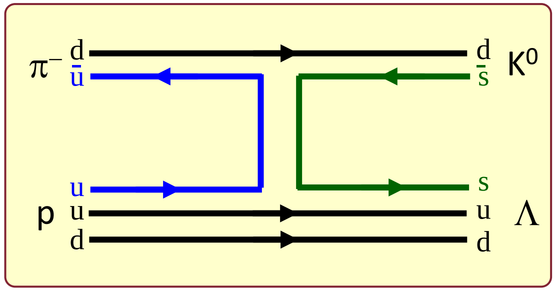
$$E_{\pi^-}^{min} = \frac{(m_\Lambda + m_K)^2 - (m_\pi^2 + m_N^2)}{2m_N} = 0.91 \text{ GeV},$$

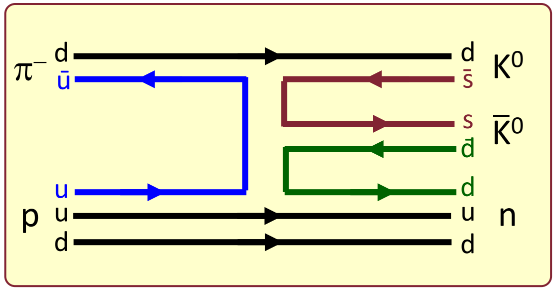
to be compared with ($\pi^- p \to K^0 \bar{K}^0 n$):

$$E_{\pi^-}^{min} = \frac{(2m_K + m_N)^2 - (m_\pi^2 + m_N^2)}{2m_N} = 1.50 \text{ GeV},$$

- Since these processes are the simplest for K^0 / $\bar{K}^0$ respectively, with $0.91 < E_\pi < 1.50$ GeV only K^0's are produced [*the observation of the products of the interaction confirms the conservation of S*]

- However, even when selecting pure K^0's, some unexpected $\bar{K}^0$ mesons show up among the final state particles;





- this effect demonstrates that production and "life" (i.e. decay) of K^0 / $\bar{K}^0$ mesons follow different rules.

- [*the weak interactions do NOT conserve S, therefore they do NOT distinguish* K^0 *from* $\bar{K}^0$ $\to$ *once produced, their S is "forgotten" and they behave as the same particle, a superposition of different states*]

production of K^0 mesons : kinematics

general case

Study the reaction a b → c d (e.g. $\pi^- p \to \Lambda K^0$).

If $(m_c + m_d) > (m_a + m_b)$, it requires some kinetic energy to happen.

Study the process in the LAB system, i.e. the system where b (the proton) is at rest:

- the projectile a hits the target b, producing c and d :
- define E_a^{min} = the minimum energy of a <u>IN THE LAB</u>, such that the process happens
- in this case, c and d are at rest in the CM frame.

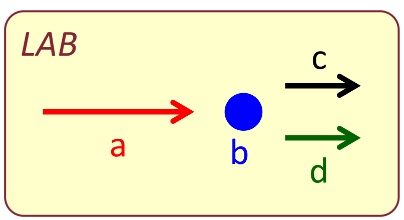
$$\text{LAB system} \begin{cases} a & (E_a, & p_a, & 0) \\ b & (m_b, & 0, & 0) \end{cases}$$

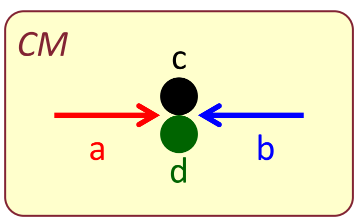
$$\text{CM system} \begin{cases} c & (m_c, & 0, & 0) \\ d & (m_d, & 0, & 0) \end{cases}$$

$$s\big|_{LAB}^{ini} = m_a^2 + m_b^2 + 2E_a^{min} m_b = s\big|_{CM}^{fin} = (m_c + m_d)^2;$$

$$E_a^{min} = \frac{(m_c + m_d)^2 - (m_a^2 + m_b^2)}{2m_b}.$$

- what, if $E_a^{min} < m_a$??? (an easy question);
- the result does NOT depend on the dynamics, but only on general kinematical constraints : it will be used in similar cases.





production of K^0 mesons : comments

To be specific, these strong interactions are allowed, because they conserve S :

a. $K^+ n \to K^0 p$;

b. $K^- p \to \bar{K}^0 n$;

c. $K^0 p \to K^+ n$;

d. $\bar{K}^0 p \to \pi^0 \Sigma^+$;

- instead, the following s.i. are forbidden :

e. $K^+ n \to \bar{K}^0 p$;

f. $K^- p \to K^0 n$;

g. $\bar{K}^0 p \to K^+ n$;

h. $K^0 p \to \pi^0 \Sigma^+$.

- Reactions (e-h) are only forbidden by S conservation;
- for a particle-antiparticle pair, because of the $\mathbb{CPT}$ symmetry, all the intrinsic properties are exactly correlated (equal or opposite mass, spin, charge, baryon-lepton number, decay channels, BR's).

- However, sometimes, the K^0 particle, generated via reaction (a), re-interacts as a $\bar{K}^0$ via reaction (d), or (b) $\to$ (c) :

i. $K^+ n \to$ "X^0" p, "X^0" p $\to \pi^0 \Sigma^+$;

ii. $K^- p \to$ "Y^0" n, "Y^0" p $\to K^+ n$;

[$X^0/Y^0 = K^0$ or $X^0/Y^0 = \bar{K}^0$?]

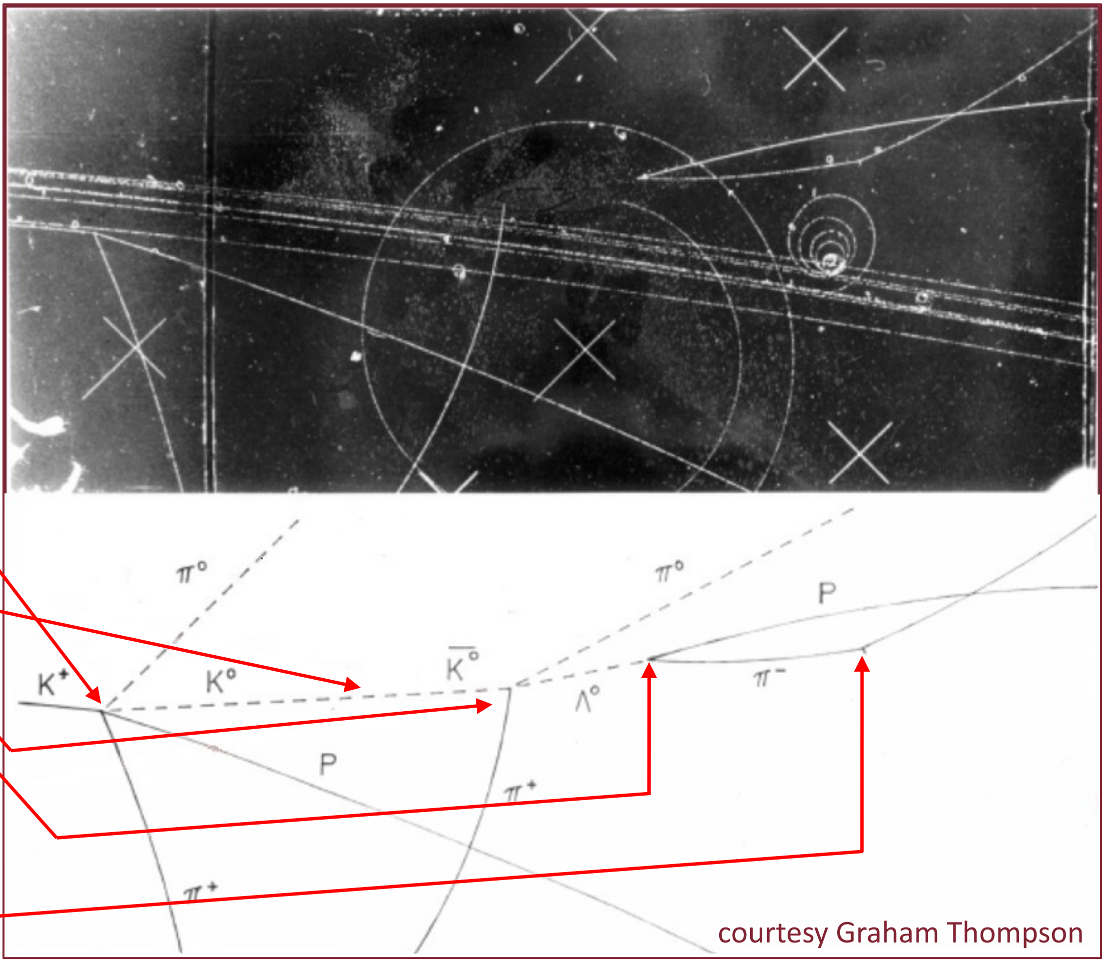
- it seems that there are transitions "in flight" (i.e. oscillations) $K^0 \leftrightarrow \bar{K}^0$.
- Can this effect show up also in their decay ?

NB Transitions ($n \leftrightarrow \bar{n}$) *are forbidden because of baryon number,* ($e^+ \leftrightarrow e^-$) *because of electric charge and lepton number. All these "charges" are conserved by all known interactions. Instead the oscillations* ($K^0 \leftrightarrow \bar{K}^0$) *are only forbidden by S conservation.*

the $K^0 \leftrightarrow \bar{K}^0$ puzzle : an event

A nice oscillation $K^0 \to \bar{K}^0$:

1. beam of K^+;
5. main vertex $K^+ p \to K^0 p \pi^+ \pi^0$;
6. $\boxed{K^0 \to \bar{K}^0 \text{ (???)};}$
4. $\bar{K}^0 p \to \Lambda\pi^+\pi^0$;
3. $\Lambda \to p\pi^-$ (decay);
2. $\pi^- p \to X$;



courtesy Graham Thompson

[end/right $\to$ start/left] K^0 and $\bar{K}^0$ unambiguously identified, no other explanation.

???

the $K^0 \leftrightarrow \bar{K}^0$ puzzle : solution

In addition, the decay of K^0 and $\bar{K}^0$ was not understood and created a puzzle.

- Both K^0 and $\bar{K}^0$ can decay into $(\pi^+\pi^-)$ and $(\pi^+\pi^-\pi^0)$ [2π *and* 3π *states have different G-parity, but G is NOT conserved in w.i.*].
- The explanation was provided by Gell-Mann and Pais [Phys. Rev. 97, 1387 (1955)], before the discovery that w.i. violate parity:
  - K^0 and $\bar{K}^0$ are eigenstates of the strong interactions;
  - each is the antiparticle of the other, the $\mathbb{C}$ operator transforms $(K^0 \leftrightarrow \bar{K}^0)$;
  - they have opposite strangeness S;
  - if S were not there, they would mix (like in π^0 and η);
  - w.i. do not conserve S;
  - … and see **a mixture of K^0 and $\bar{K}^0$**.

Consequences:

- the mixture is interpreted as two new states, quantum superpositions of $K^0/\bar{K}^0$;
- if w.i. conserve $\mathbb{CP}$, the two new states must be $\mathbb{CP}$ eigenstates(*);
- since the new states are NOT a particle-antiparticle pair, they may have different properties (masses, lifetimes, decays);
- if the mass difference allows for that, the states oscillate between themselves;
- the only known decay was $(\text{"}K^0\text{"} \to \pi^+\pi^-)$; a possible transition, generated via w.i., is then $[K^0 \leftrightarrow (\pi^+\pi^-) \leftrightarrow \bar{K}^0]$;
- another "K^0" must exist, "K^0" $\to \pi\pi\pi$.

(*) Today we know that the w.i. violate also $\mathbb{CP}$, but this violation is small, so provisionally we do not take it into account.

the $K^0 \leftrightarrow \bar{K}^0$ puzzle: predictions

(more formally ...)

TWO "K^0" STATES:

- different values of CP → CP = ± 1;
- one with CP=+1 and decay →(ππ), another with CP=–1 and decay →(πππ);
- other decays are allowed for both states, but they have to conserve ℂℙ (e.g. no → ππ for the state CP=–1);
- the state (πππ) is near the kinematical threshold ($m_K \approx 3m_\pi$ + 70 MeV) → the lifetime of the (πππ) state is <u>much longer</u> than the lifetime of the (ππ) one.
- the obvious proposal was to call "short" the CP=+1 state and "long" the CP=–1;
- so, two new particles have born:
  - they have been discovered;
  - their lifetimes and properties have been measured and found in agreement with the predictions :

1) K^0_S : CP = +1, $\tau = 0.90 \times 10^{-10}$ s, decay $\rightarrow \pi^+\pi^-, \rightarrow \pi^0\pi^0$;

2) K^0_L : CP = –1, $\tau = 0.51 \times 10^{-7}$ s, decay $\rightarrow \pi^+\pi^-\pi^0, \pi^0\pi^0\pi^0$.

J.W. Cronin and M.S. Greenwood, Physics Today (July 1982) :

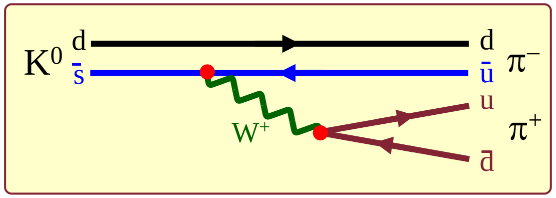
"So these gentlemen, Gell-Mann and Pais, predicted that in addition to the short-lived K mesons, there should be long-lived K mesons. They did it beautifully, elegantly and simply.

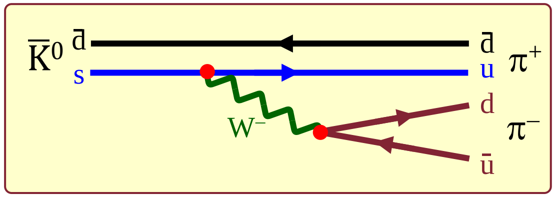
I think theirs is a paper one should read sometime just for its pure beauty of reasoning. It was published in Physical Review in 1955. A very lovely thing ! You get shivers up and down your spine, especially when you find you understand it. At the time many of the most distinguished theoreticians thought this prediction was really baloney."

the $K^0 \leftrightarrow \bar{K}^0$ puzzle : oscillations

In q.m. or quark model language:

- Both the K^0 and $\bar{K}^0$ decay via w.i. in the same final states; the $\pi^+\pi^-$ diagram is shown in the figure, while the others ($\pi^0\pi^0$; $\pi^+\pi^-\pi^0$; $\pi\ell\nu$) are similar :

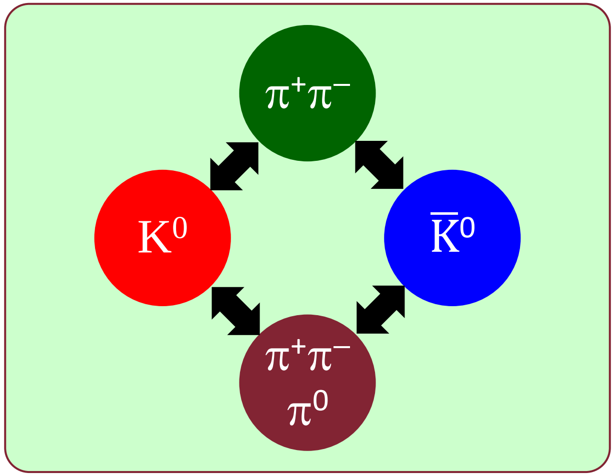


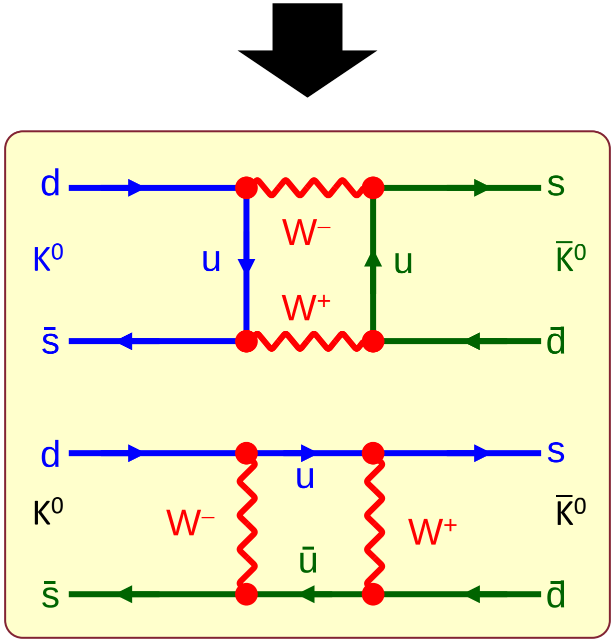


- The oscillations can be understood as a continuous transformation between the K^0 and $\bar{K}^0$ themselves, via the second order box-diagrams, or as a mixture, with time-dependent coefficients $\alpha(t)$, $\beta(t)$:

$$|K(t)\rangle = \alpha(t)\ |K^0\rangle + \beta(t)\ |\bar{K}^0\rangle\ ;$$

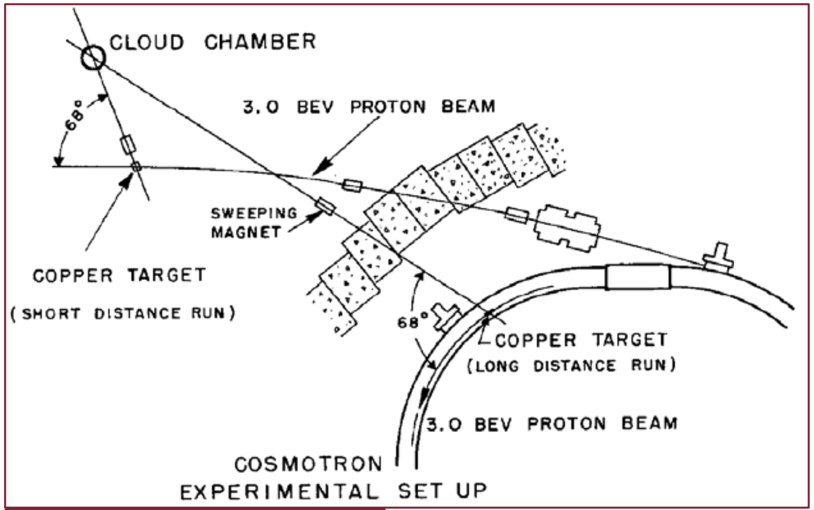
$$\alpha(t)^2 + \beta(t)^2 = 1$$ [× *a decreasing function of t, to account for their decay*]





the $K^0 \leftrightarrow \bar{K}^0$ puzzle : K^0_L

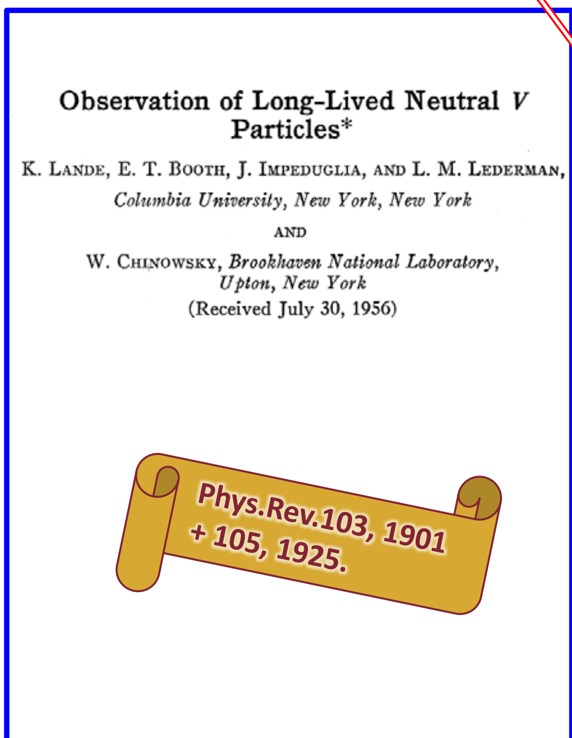
- The K^0_L was first observed in 1956 by Lande and coll. with a cloud chamber.
- Brookhaven Cosmotron (3 GeV protons).
- Path between the beam and the cloud chamber (6 meters) is ~100 K^0_S / Λ lifetimes.
- This path is therefore sufficient for the decay of all strange particles known at the time.
- A few months later the same authors confirmed the result. They also observed in the cloud chamber interactions of these particles with the nuclei of He, producing final states with total $S \neq 0$, like ($\bar{K}^0\ {}^4He \rightarrow \Sigma^- ppn\pi^+$).
- These states cannot be generated by a K^0, because of the value of S.
- However, no $\bar{K}^0$ should be present, because the primary proton energy was chosen to be below the energy threshold for $\bar{K}^0$ production, which is higher than for K^0 [*same argument as before*] .
- For some reason, $\bar{K}^0$ mesons have "appeared" $\rightarrow$ oscillation.



modern : "BEV" = GeV

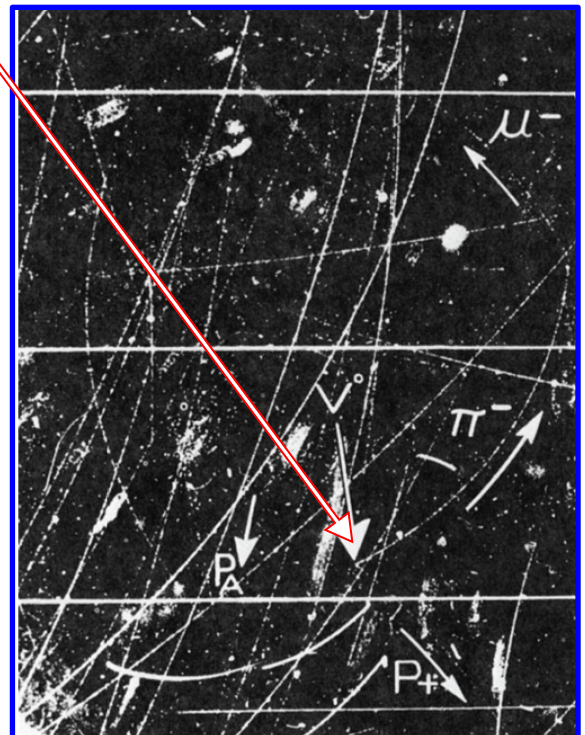
the $K^0 \leftrightarrow \bar{K}^0$ puzzle : K^0_L results

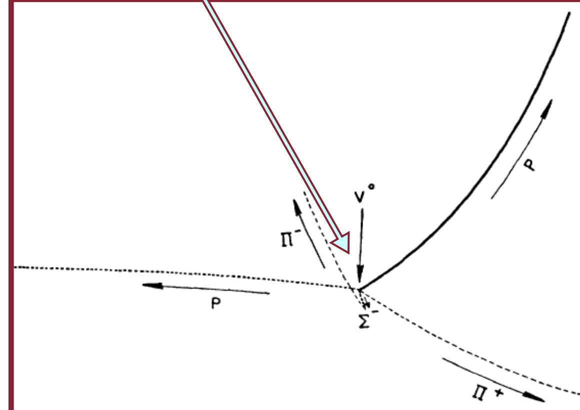
- *The K^0_L was first observed in 1956 by Lande and coll. with a cloud chamber.*
- They found 26 events with a "V-zero", incompatible to be $(\pi^+\pi^-)$ because of their Q^2 (one shown on the right).
- [*today we interpret these events as decays* $(\pi^\pm e^\mp \nu_e)$, $(\pi^\pm \mu^\mp \nu_\mu)$, $(\pi^\pm \pi^\mp \pi^0)$].
- Events consistent with 3 body decays of neutral mesons of mass ~ 500 MeV.
- First estimate of the lifetime : 10^{-9} s < τ < 10^{-6} s, now τ = 0.53×10^{-7} s.
- Another beautiful and "impossible" event (no $\bar{K}^0$ in the beam, see previous pages).


Observation of Long-Lived Neutral *V* Particles*

K. Lande, E. T. Booth, J. Impeduglia, and L. M. Lederman,
Columbia University, New York, New York
and
W. Chinowsky, *Brookhaven National Laboratory, Upton, New York*
(Received July 30, 1956)

Phys.Rev.103, 1901 + 105, 1925.





$\bar{K}^0\,{}^4He \rightarrow \Sigma^- p p n \pi^+$;
i.e. $\bar{K}^0\, n[ppn] \rightarrow \Sigma^- \pi^+ [ppn]$
$\Sigma^- \rightarrow n\pi^-$;

[modern : $V^0 = K^0$; $\Pi^\pm = \pi^\pm$]

K^0 decays in $\mathbb{CP}$ eigenstates : caveat

- In the following slides we assume that the K^0 decay conserve $\mathbb{CP}$, i.e. that both K^0_S and K^0_L are $\mathbb{CP}$ eigenstates with eigenvalues = ± 1.
- Although this is not true (see later), the violation is small and therefore the results obtained with this approximation are in fair agreement with (almost) all observations.
- To remember that, the next pages are marked by a little sign "$\mathbb{CP}$" in the upper right corner.

warning : the sign of ζ in
$\mathbb{C}\ |K^0\rangle = \zeta\ |\bar{K}^0\rangle$; $\mathbb{C}\ |\bar{K}^0\rangle = \zeta\ |K^0\rangle$;
is non-physical; in literature both $\zeta=\pm 1$;
here we (try to) stick to $\zeta = -1$.

K^0 decays in $\mathbb{CP}$ eigenstates : K^0_S and K^0_L

- The states $|K^0\rangle$ and $|\bar{K}^0\rangle$ are strong interactions (s.i.) eigenstates:

$$\mathbb{C}\,|K^0\rangle = -\,|\bar{K}^0\rangle; \qquad \mathbb{C}\,|\bar{K}^0\rangle = -\,|K^0\rangle;$$
$$\mathbb{P}\,|K^0\rangle = -\,|K^0\rangle; \qquad \mathbb{P}\,|\bar{K}^0\rangle = -\,|\bar{K}^0\rangle;$$
$$\mathbb{CP}\,|K^0\rangle = +\,|\bar{K}^0\rangle; \qquad \mathbb{CP}\,|\bar{K}^0\rangle = +\,|K^0\rangle;$$

- these equations show that the s.i. states K^0 / $\bar{K}^0$ are NOT $\mathbb{CP}$ eigenstates;
- $|K^0_1\rangle$ and $|K^0_2\rangle$ are linear combinations of $|K^0\rangle$ and $|\bar{K}^0\rangle$, which are $\mathbb{CP}$ eigenstates :

$$|K^0_1\rangle = 1/\sqrt{2}\,[\,|K^0\rangle + |\bar{K}^0\rangle\,];$$
$$|K^0_2\rangle = 1/\sqrt{2}\,[\,|K^0\rangle - |\bar{K}^0\rangle\,];$$
$$|K^0\rangle = 1/\sqrt{2}\,[\,|K^0_1\rangle + |K^0_2\rangle\,];$$
$$|\bar{K}^0\rangle = 1/\sqrt{2}\,[\,|K^0_1\rangle - |K^0_2\rangle\,].$$

$$\mathbb{CP}\,|K^0_1\rangle = +\,|K^0_1\rangle; \quad \mathbb{CP}\,|K^0_2\rangle = -\,|K^0_2\rangle.$$

- The $(\pi\pi)$ and $(\pi\pi\pi)$ give (next slide) :

$$\mathbb{CP}\,|2\pi\rangle = +\,|2\pi\rangle\,;$$
$$\mathbb{CP}\,|3\pi\rangle = -\,|3\pi\rangle\,;$$

$\Rightarrow$ $K^0_1 \to 2\pi$, $K^0_2 \to 3\pi$

- Therefore :
  $K^0_S \equiv K^0_1$; $\quad K^0_L \equiv K^0_2$.

if $\mathbb{CP}$ not conserved, **NOT** true !!!

- K^0 and $\bar{K}^0$ are eigenstates of the strong interactions;
- therefore, the creation process generates one of them [NOT the other];
- but, as soon as they are created, they behave as a linear combination of K^0_S and K^0_L;
- therefore they "live" (i.e. decay) as them;
- then $K^0_S \to 2\pi$ (lot of phase space, small τ);
- and $K^0_L \to 3\pi$ (small phase space, long τ);
- if $K^0_{S,L}$ interact via strong interactions, they come back to the s.i. eigenstates, as K^0 or $\bar{K}^0$ with a given probability each.

K^0 decays in $\mathbb{CP}$ eigenstates : eigenvalues

Compute the eigenvalues of $\mathbb{CP}$.

For 2π systems :

- Since $J^{PC}(\pi^0) = 0^{-+}$:

  $\mathbb{P}\ |\pi^0\pi^0> = (-)^2 (-)^L |\pi^0\pi^0> = +|\pi^0\pi^0>$;

  $\mathbb{C}\ |\pi^0\pi^0> = (+)^2 |\pi^0\pi^0> = +|\pi^0\pi^0>$;

  $\mathbb{CP}\ |\pi^0\pi^0> = +\ |\pi^0\pi^0>$;

- if $L = S_1 = S_2 = 0$:

  $\mathbb{PC}\ |\pi^+\pi^-> = \mathbb{P}\ |\pi^-\pi^+> = +|\pi^+\pi^->$;

- i.e. $CP(2\pi) = +1$, both for the $(\pi^0\pi^0)$ and $(\pi^+\pi^-)$ systems.

For 3π systems :

- $P(\pi^0\,\pi^0\,\pi^0) = (-)^3 (-)^{L1} (-)^{L2} = -1$;

  $C(\pi^0\,\pi^0\,\pi^0) = (+)^3 = +1$;

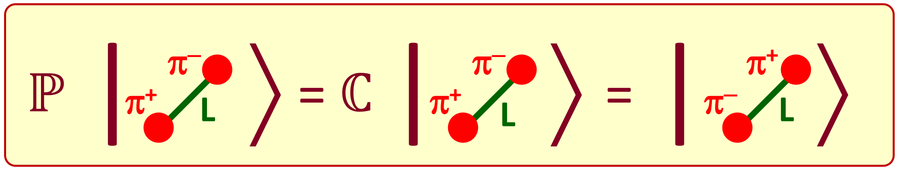
  $CP(\pi^0\,\pi^0\,\pi^0) = -1$;

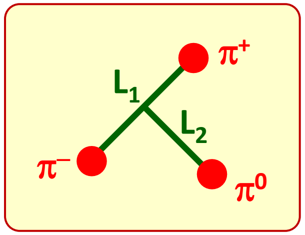
- $P(\pi^+\,\pi^-\,\pi^0) = (-)^3 (-)^{L1} (-)^{L2} = -1$;

  $C(\pi^+\,\pi^-\,\pi^0) = (+)\,(-)^{L1} = +1$;

  $CP(\pi^+\,\pi^-\,\pi^0) = -1$;

- i.e. $CP(3\pi) = -1$, both for the $(\pi^0\pi^0\pi^0)$ and $(\pi^+\pi^-\pi^0)$ systems.





K^0 decays in $\mathbb{CP}$ eigenstates : Γ and τ

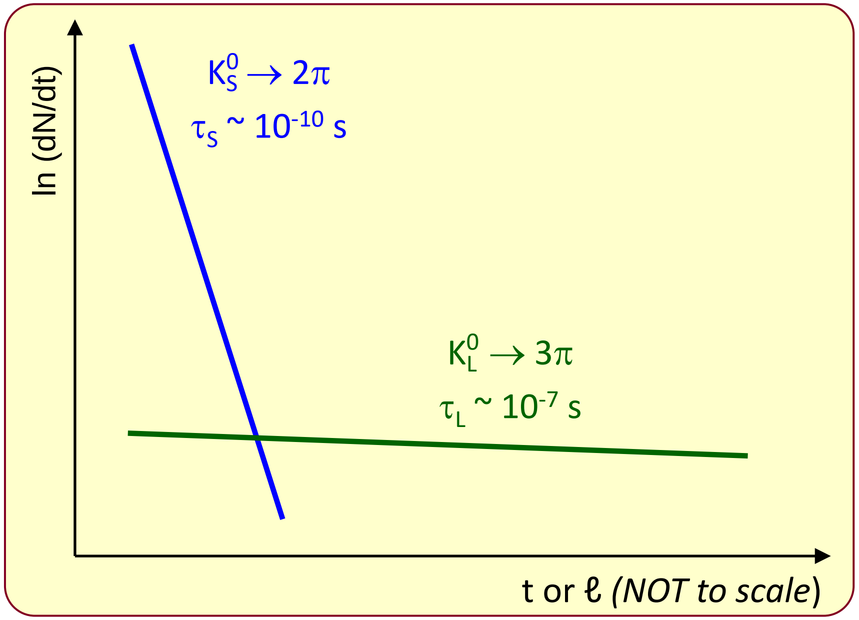
Conclusion : after strange particle production, expect two neutral particles of (not exactly, but almost) equal mass [*actually 498 MeV*] :

- the shorter (K^0_S) with
  - CP = +1;
  - decay into 2π;
  - "short" lifetime;
  - [τ_S = 0.90 × 10^{-10} s = 7.4 μeV^{-1}, ℓ_S= cτ_S = 2.68 cm];
- the longer (K^0_L) with
  - CP = –1;
  - decay into 3π;
  - "long" lifetime [580 ×τ_S];
  - [τ_L = 0.51 × 10^{-7} s = 0.013 μeV^{-1}, ℓ_L= 15.5 m]

- therefore :
  - $\Delta\Gamma_K \equiv \Gamma_L - \Gamma_S \approx -\Gamma_S$ = –7.4 μeV = = –11.2 ns^{-1}.

1 μeV = 1.52 ns^{-1};

1 ns^{-1} = 0.66 μeV.



K^0 oscillations

- While the K^0 and $\bar{K}^0$ masses are equal because of $\mathbb{CPT}$, no symmetry equalizes the masses and lifetimes of K^0_S and K^0_L;
- the measurement gives [*see later*] :

  $\Delta m_K = m(K^0_L) - m(K^0_S) = 3.51 \pm 0.018\ \mu eV$

  $= 5.303 \pm 0.009\ ns^{-1}$;
- $\Delta m_K \approx -\frac{1}{2}\Delta\Gamma_K$ [*no explanation, but deep phenomenological consequences*];
- the mass difference means that the two states [K^0_L *and* K^0_S] evolve with <u>different time constants</u>;
- following the evolution on the basis (K^0, $\bar{K}^0$), a "desynchronization" is observed between the K^0_S and K^0_L components, interpreted as oscillations ($K^0 \leftrightarrow \bar{K}^0$);
- a little algebra shows that, instead of a pure evolution of a particle of width Γ, which would give rise to an intensity N(t) $\propto$ exp $(-\Gamma t)$ = exp $(-t/\tau)$, we have a different phenomenon :

$$\psi_S(t) = \psi_S^0 \exp\left[-\left(\Gamma_S/2 + i m_S\right)t\right];$$

$$\psi_L(t) = \psi_L^0 \exp\left[-\left(\Gamma_L/2 + i m_L\right)t\right];$$

- take a pure K^0 beam at t=0 : then, in case of no decay (Γ = 0, τ = ∞), the probability $\mathcal{P}$ to find a K^0 or a $\bar{K}^0$, function of t, is:

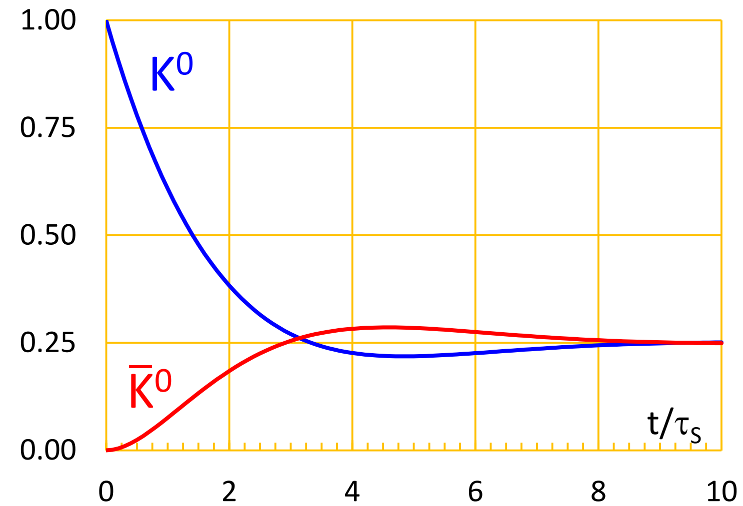
$$\mathcal{P}_{K^0}(t) = \frac{1}{4}\left|e^{(-im_S t)} + e^{(-im_L t)}\right|^2 = \cos^2\left(\frac{\Delta m_K}{2}t\right);$$

$$\mathcal{P}_{\bar{K}^0}(t) = \frac{1}{4}\left|e^{(-im_S t)} - e^{(-im_L t)}\right|^2 = \sin^2\left(\frac{\Delta m_K}{2}t\right).$$

- In addition, the oscillations are damped by the occurrence of the decays ($\tau_L = 1/\Gamma_L$ >> $\tau_S = 1/\Gamma_S$); Γ_S dominates, because of the shorter lifetime [*next slide*].

K^0 oscillations: formulæ

- The amount of K^0 and $\bar{K}^0$ can be computed as a function of (proper) time, by simple considerations of quantum mechanics.
- E.g. starting with pure K^0 (fig.), there is an "oscillation" between the two states, according to τ_S, τ_L, Δm (=$|m_S - m_L|$).
- The figure is made with $\tau_S \ll \tau_L$ and $\Delta m = 1/(2\tau_S)$ (not exact, but realistic and simple).
- For the computations, see next page.



$$R(K^0)(t) = \left|\langle K^0 | \psi(t) \rangle\right|^2 = \frac{1}{4}\left[\exp\left(-\frac{t}{\tau_S}\right) + \exp\left(-\frac{t}{\tau_L}\right) + 2\exp\left(-\frac{\tau_L + \tau_S}{2\tau_L\tau_S}t\right)\cos(\Delta m_K t)\right];$$

$$R(\bar{K}^0)(t) = \left|\langle \bar{K}^0 | \psi(t) \rangle\right|^2 = \frac{1}{4}\left[\exp\left(-\frac{t}{\tau_S}\right) + \exp\left(-\frac{t}{\tau_L}\right) - 2\exp\left(-\frac{\tau_L + \tau_S}{2\tau_L\tau_S}t\right)\cos(\Delta m_K t)\right].$$

K^0 oscillations: math

CP

Some (simple and tedious) algebra. Start with f K^0 and (1–f) $\bar{K}^0$. Then put f=1:

$$\left|\psi(t=0)\right\rangle = f\left|K^0\right\rangle + (1-f)\left|\bar{K}^0\right\rangle = \frac{f}{\sqrt{2}}\left(\left|K^0_S\right\rangle + \left|K^0_L\right\rangle\right) + \frac{1-f}{\sqrt{2}}\left(\left|K^0_L\right\rangle - \left|K^0_S\right\rangle\right) = \frac{2f-1}{\sqrt{2}}\left|K^0_S\right\rangle + \frac{1}{\sqrt{2}}\left|K^0_L\right\rangle;$$

$$\left|\psi(t)\right\rangle = \frac{2f-1}{\sqrt{2}} e^{-\left(\frac{\Gamma_S}{2}+im_S\right)t}\left|K^0_S\right\rangle + \frac{1}{\sqrt{2}} e^{-\left(\frac{\Gamma_L}{2}+im_L\right)t}\left|K^0_L\right\rangle \xrightarrow{\boxed{f=1}} \frac{1}{\sqrt{2}} e^{-\left(\frac{\Gamma_S}{2}+im_S\right)t}\left|K^0_S\right\rangle + \frac{1}{\sqrt{2}} e^{-\left(\frac{\Gamma_L}{2}+im_L\right)t}\left|K^0_L\right\rangle;$$

$$R(K^0)(t) = \left|\left\langle K^0 \middle| \psi(t)\right\rangle\right|^2 = \frac{1}{4}\left|\left[\left\langle K^0_S\right| + \left\langle K^0_L\right|\right]\left[e^{-\left(\frac{1}{2\tau_S}+im_S\right)t}\left|K^0_S\right\rangle + e^{-\left(\frac{1}{2\tau_L}+im_L\right)t}\left|K^0_L\right\rangle\right]\right|^2 =$$

$$\boxed{\left|x\cdot\exp(iy)\right|^2 = (x\cdot\cos y)^2 + (x\cdot\sin y)^2}$$

$$\boxed{\left\langle K^0_i \middle| K^0_j\right\rangle = \delta_{ij}}$$

$$= \frac{1}{4}\left[e^{\frac{-t}{2\tau_S}}\cos(m_S t) + e^{\frac{-t}{2\tau_L}}\cos(m_L t)\right]^2 + \frac{1}{4}\left[e^{\frac{-t}{2\tau_S}}\sin(m_S t) + e^{\frac{-t}{2\tau_L}}\sin(m_L t)\right]^2 =$$

$$\boxed{\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)}$$

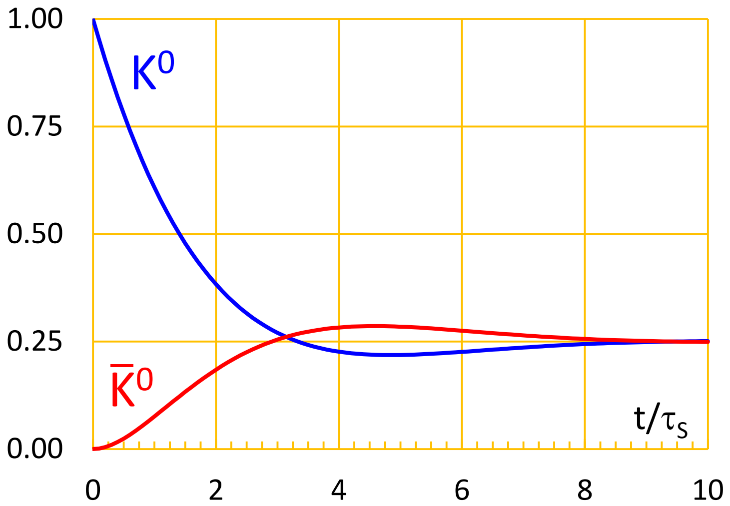
$$= \boxed{\frac{1}{4}\left[e^{\frac{-t}{\tau_S}} + e^{\frac{-t}{\tau_L}} + 2e^{-\frac{(\tau_L+\tau_S)t}{2\tau_L\tau_S}}\cos(\Delta m_K t)\right]} \boxed{\xrightarrow[\tau_S\to\infty,\tau_L\to\infty]{} \cos^2\left(\frac{\Delta m_K t}{2}\right)}.$$

$$\boxed{1+\cos\alpha = 2\cos^2(\alpha/2)}$$

Damped oscillation (previous slide). If both τ_L and τ_S >> 1/Δm_K (not true) → simple oscillation.

The computations for R($\bar{K}^0$)(t) and for f≠1 are left to the (patient) reader.

K^0 oscillations: semileptonic decays



- To test this prediction, the experimental problem [Bettini] is to distinguish $K^0 \leftrightarrow \bar{K}^0$ when they decay. It is not possible from the 2π or 3π states, because these channels have definite CP, not definite strangeness.

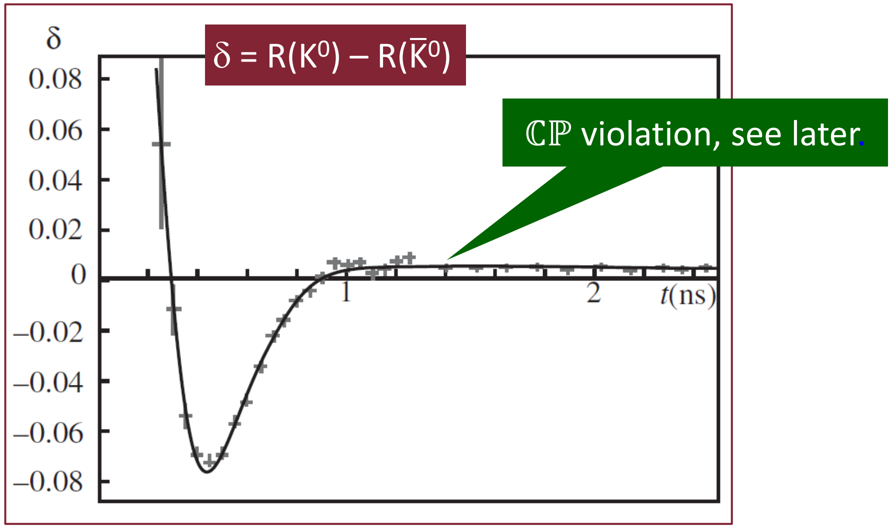
- To select definite strangeness states, select semileptonic decays of K^0_L. These decays obey the "$\Delta S = \Delta Q$ rule": *the difference between the strangeness of the hadrons in the final and initial states is equal to the difference of their electric charges*. The rule is a consequence of the quark contents of the states [$K^0 = \bar{s}d$] :

$$\bar{s} \to \bar{u}\ell^+\nu_\ell \Rightarrow K^0 \to \pi^-\ell^+\nu_\ell;\ K^0 \nrightarrow \pi^+\ell^-\bar{\nu}_\ell;$$
$$s \to u\ell^-\bar{\nu}_\ell \Rightarrow \bar{K}^0 \to \pi^+\ell^-\bar{\nu}_\ell;\ \bar{K}^0 \nrightarrow \pi^-\ell^+\nu_\ell.$$

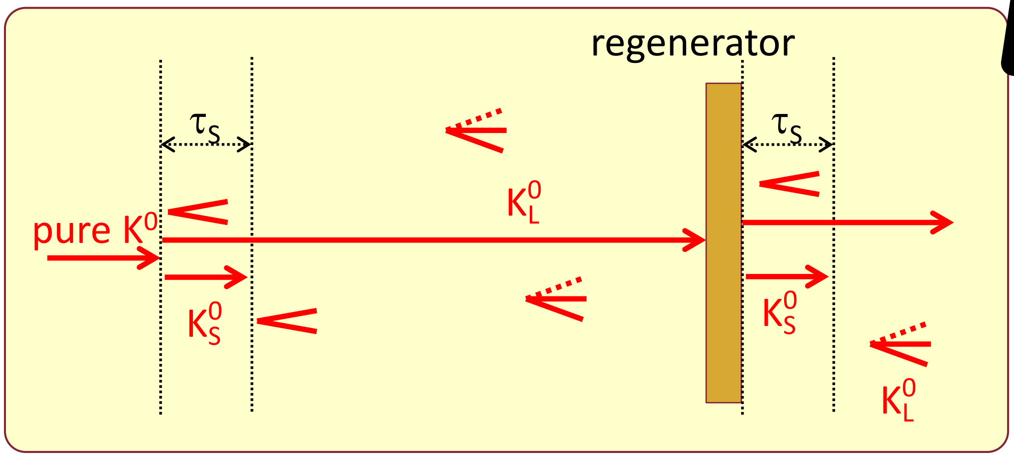
- The sign of the charged lepton flags the strangeness of the $K^0/\bar{K}^0$. The semileptonic decays are called K^0_{e3} and $K^0_{\mu 3}$ depending on the lepton. Their branching ratios are large:

  $BR(K^0_{e3}) = 41\%$, $BR(K^0_{\mu 3}) = 27\%$.

- The experimental measure regards the charge asymmetry δ, i.e. the difference between +ve and –ve leptons, which is directly related to the oscillations. The results agree very well with the expectations, but the tail.



K^0 regeneration



[the correct computation takes into account the L-factor $\beta\gamma$]

The regeneration (Pais and Piccioni, 1956) consisted in a clever use of an absorber (the "regenerator"), positioned at a distance determined by τ_S and τ_L, to demonstrate the superposition of K^0 and $\bar{K}^0$.

[explanation on the next slide]

Abraham Pais   Oreste Piccioni

K^0 regeneration : the idea

- Start with a pure K^0 beam in vacuum (equal amounts of K^0_S and K^0_L).
- After $t \approx 10\ \tau_S$ the K^0_S intensity down by factor $e^{(-t/\tau S)} = e^{-10} \approx 45\times10^{-6}$ (none left).
- [*For* K^0 *with 1 GeV momentum this corresponds* to ~*0.5 m.*]
- The K^0_L intensity is down by $e^{(-t/\tau L)} \approx 0.98$, i.e. all left.
- After 0.5 m, 100% K^0_L (50% K^0 + 50% $\bar{K}^0$).
- If we put another target at [say] $t = 20\ \tau_S$ [1 m downstream], we will get K^0 interactions as well as $\bar{K}^0$.
- K^0 and $\bar{K}^0$ interact (strongly) differently in the target :

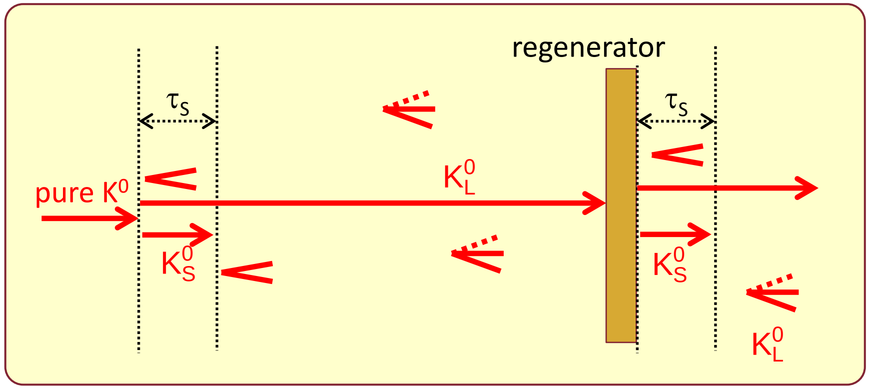
  $K^0 p \to K^0 p,\ K^+ n;$

  $K^0 n \to K^0 n;$

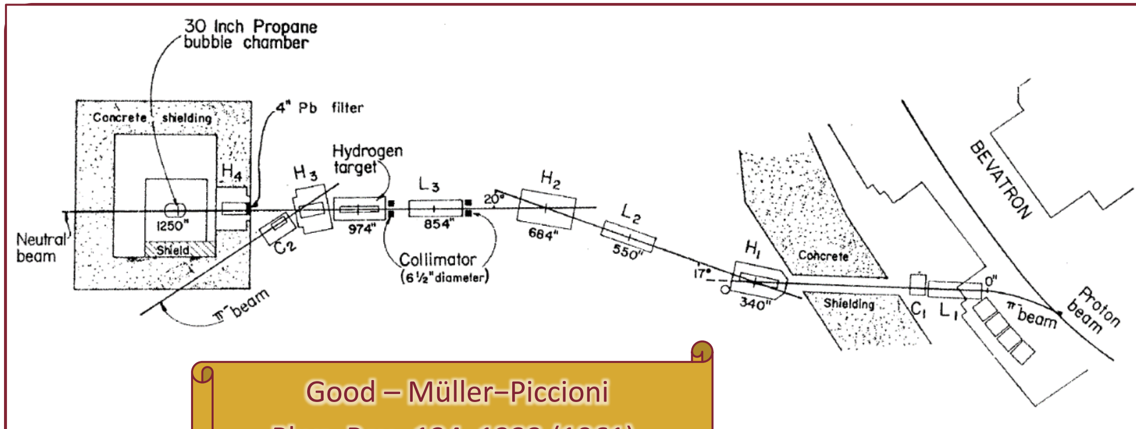
  $\bar{K}^0 p \to \bar{K}^0 p,\ \Lambda \pi^+; \to \Sigma^0 \pi^+, \Sigma^+ \pi^0;$

  $\bar{K}^0 n \to \bar{K}^0 n, \Lambda \pi^0; \to \Sigma^+ \pi^-, \Sigma^0 \pi^0, \Sigma^- \pi^+;$

- The s quark from the $\bar{K}^0$ can swap with one of the quarks in the proton or neutron, but the $\bar{s}$ from the K^0 cannot [e.g. $\bar{K}^0 p \to \Lambda X$, but $\cancel{K^0 p \to \Lambda X}$] .
- Hence there are more $\bar{K}^0$ processes, so the $\bar{K}^0$ are more strongly absorbed.
- Then, no longer 50% K^0 +50% $\bar{K}^0$ (as in K^0_L), but an amount of K^0_S has "born".
- So will have some K^0_S decays again.



K^0 regeneration : experiment

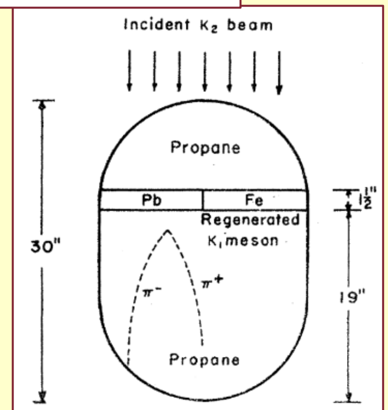


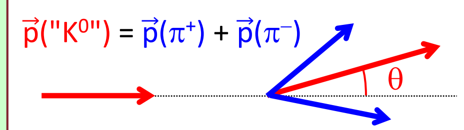
Good – Müller–Piccioni
Phys. Rev., 124, 1223 (1961).

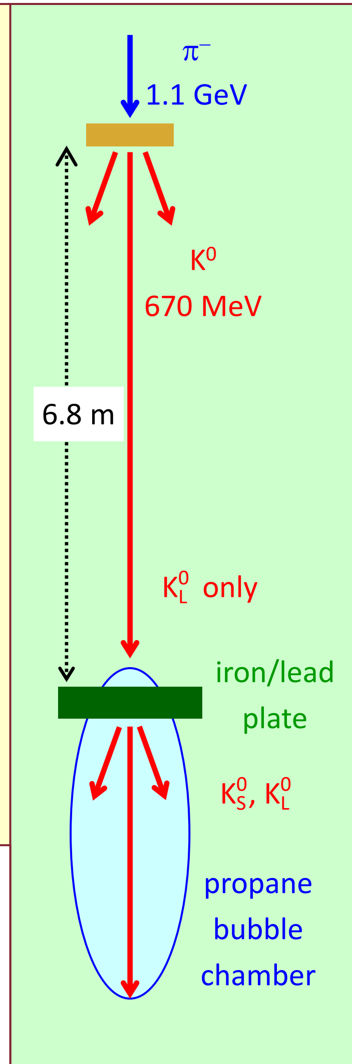
The experiment used a beam of 1.1 GeV π^- from the "Bevatron", the 6.2 GeV ("BeV“, old American) proton synchrotron at LNL, Berkeley.

The propane bubble chamber was able to measure the $\pi^\pm$ momenta by their curvature in magnetic field.

Therefore the angle θ (shown in the fig) is measured.







K^0 regeneration : results

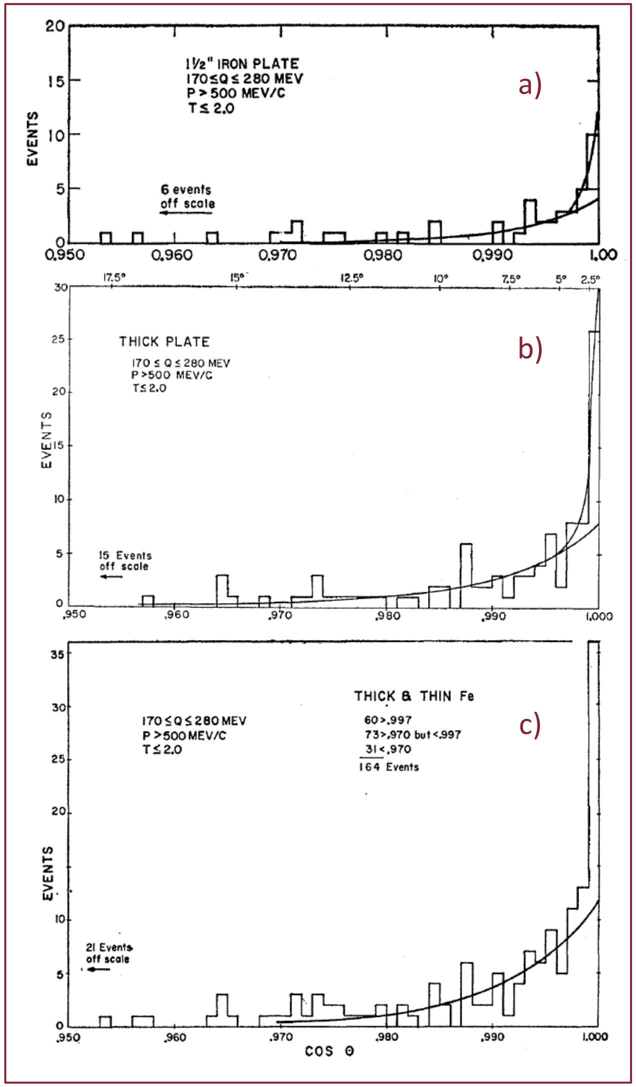
A study of the phenomenon by M. Good (1957) considered three types of regeneration, with different distributions of the angle θ between the incoming and the regenerated particle :

1. Regeneration for transmission ("forward") : θ = 0. No momentum transfer to the nucleus : coherent.
2. Regeneration for diffraction : elastic scattering, θ distribution as in diffraction.
3. Inelastic regeneration : interaction with individual nucleons, θ distribution as in scattering.

- The relative amount of the three depends on the small mass difference $\Delta m_K = m(K^0_L) - m(K^0_S)$;
- 200 observed 2π decays;
- they were able to confirm oscillations and regeneration;
- … and to measure the mass difference (units $\hbar/\tau_s$) :

$$\Delta m_K = 0.84^{+0.89}_{-0.22};$$

[*very clever result, despite present best value is 2 σ smaller*]



$\mathbb{CP}$ violation

Redefine the K^0 mesons system :

- K^0 and $\bar{K}^0$ as the particle produced in strong interactions (i.e. s.i. eigenstates) :
  - $|K^0\rangle = |d\bar{s}\rangle$, S = +1; $|\bar{K}^0\rangle = |s\bar{d}\rangle$, S = -1;
  - $\mathbb{C}\ |K^0\rangle = -|\bar{K}^0\rangle$; $\mathbb{C}\ |\bar{K}^0\rangle = -|K^0\rangle$;
- K_1^0 and K_2^0 as the $\mathbb{CP}$ eigenstates :
  - $|K_1^0\rangle = 1/\sqrt{2}\ [\ |K^0\rangle + |\bar{K}^0\rangle\ ]$;
  - $|K_2^0\rangle = 1/\sqrt{2}\ [\ |K^0\rangle - |\bar{K}^0\rangle\ ]$;
  - $\mathbb{CP}\ |K_1^0\rangle = +\ |K_1^0\rangle$;
  - $\mathbb{CP}\ |K_2^0\rangle = -\ |K_2^0\rangle$;
- K_S^0 and K_L^0 as the states with lifetimes τ_S, τ_L [*NOT necessarily* $\mathbb{CP}$ *eigenstates*] :
  - $\tau_S = 0.90 \times 10^{-10}$ s; $\tau_L = 0.51 \times 10^{-7}$ s;
- the $(\pi^+\pi^-)$, $(\pi^0\pi^0)$, $(\pi^+\pi^-\pi^0)$ systems are $\mathbb{CP}$ eigenstates:
  - $\mathbb{CP}\ |2\pi\rangle = +\ |2\pi\rangle$; $\mathbb{CP}\ |3\pi\rangle = -|3\pi\rangle$;

- Clearly, if $K_1^0 = K_S^0$, $K_2^0 = K_L^0$, then $\mathbb{CP}$ is conserved in the K^0 decays; i.e. $\mathbb{CP}$ conservation implies

$$K_S^0 \to 2\pi,\ K_L^0 \to 3\pi;$$

???

- On the contrary, decays

$$K_L^0 \to 2\pi,\ K_S^0 \to 3\pi$$

with small, but non-0 BR, would be an experimental evidence of **the NON-CONSERVATION of $\mathbb{CP}$**.

In the following slides we do NOT assume $\mathbb{CP}$ conservation in K^0 decays.

The little "$\mathbb{CP}$" in the upper right corner has disappeared.

$\mathbb{CP}$ violation: test of the theory

A textbook "experimentum crucis".

Consider three possible interactions:

a. $\mathbb{C}$ and $\mathbb{P}$ conserved ["strong i."] :
- $\mathbb{C}$, $\mathbb{P}$ conserved separately,
- strangeness conserved;
- eigenstates K^0, $\bar{K}^0$;

b. $\mathbb{CP}$ conserved :
- $\mathbb{C}$, $\mathbb{P}$ not conserved separately, but $\mathbb{CP}$ conserved;
- strangeness NOT conserved;
- eigenstates $K_1^0 \rightarrow 2\pi$, $K_2^0 \rightarrow 3\pi$ [because 2π and 3π states are $\mathbb{CP}$ eigenstates];

c. $\mathbb{CP}$ non conserved ["weak i."] :
- K_S^0, K_L^0 decay with lifetimes τ_S, τ_L;
- strangeness NOT conserved;
- eigenstates K_S^0, K_L^0 [K_S^0 and K_L^0 NOT $\mathbb{CP}$ eigenstates].

Strong interactions follow [a].

If weak interactions conserve $\mathbb{CP}$, then they follow [b]:

$|K_1^0\rangle = |K_S^0\rangle$, $|K_2^0\rangle = |K_L^0\rangle$,
$K_S^0 \rightarrow 2\pi$, $K_L^0 \rightarrow 3\pi$.

Instead, if $\mathbb{CP}$ is violated in w.i., then [b] is only a first approx. of [c].

The discriminant is the existence (at least with a small BR) of the decays:
$K_S^0 \rightarrow 3\pi$, $K_L^0 \rightarrow 2\pi$.

Conclusion :

since a small amount of ($K_S^0 \rightarrow 3\pi$) is not observable, due to the background ($K_L^0 \rightarrow 3\pi$), the key observation is $\boxed{(K_L^0 \rightarrow 2\pi)}$.

$\mathbb{CP}$ violation: summary

a) Flavor eigenstates :

$|K^0\rangle = d\bar{s};\ S = +1; \quad \mathbb{CP}\ |K^0\rangle = +|\bar{K}^0\rangle;$

$|\bar{K}^0\rangle = s\bar{d};\ S = -1; \quad \mathbb{CP}\ |\bar{K}^0\rangle = +|K^0\rangle.$

(strong interactions)

b) CP eigenstates :

$|K^0_1\rangle = 1/\sqrt{2}\,[\,|K^0\rangle + |\bar{K}^0\rangle\,];\ \ CP = +1;$

$|K^0_2\rangle = 1/\sqrt{2}\,[\,|K^0\rangle - |\bar{K}^0\rangle\,];\ \ CP = -1;$

$|K^0\rangle = 1/\sqrt{2}\,[\,|K^0_1\rangle + |K^0_2\rangle\,];$

$|\bar{K}^0\rangle = 1/\sqrt{2}\,[\,|K^0_1\rangle - |K^0_2\rangle\,].$

(K^0 oscillations+decay, regeneration)

c) Mass eigenstates in vacuum :

$|K^0_S\rangle = (|K^0_1\rangle + \varepsilon|K^0_2\rangle) / \sqrt{1+|\varepsilon|^2};$

$|K^0_L\rangle = (\varepsilon|K^0_1\rangle + |K^0_2\rangle) / \sqrt{1+|\varepsilon|^2}$

($\mathbb{CP}$ violation in vacuum)

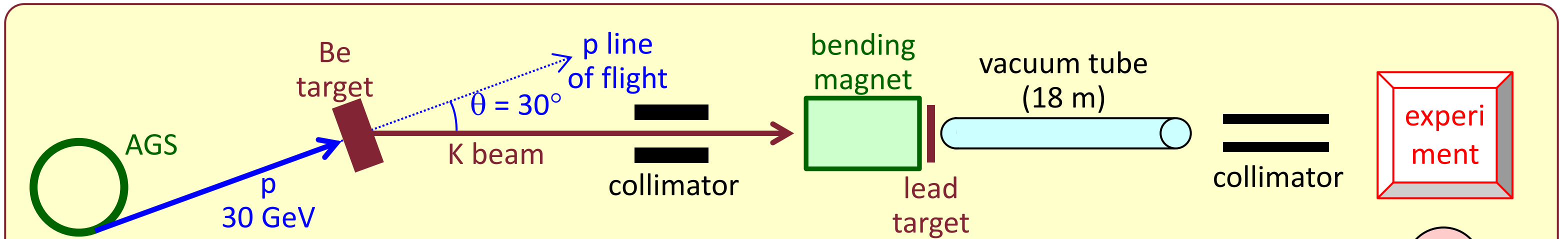
d) Mass eigenstates in matter :

$|K^0_{S,M}\rangle = (|K^0_1\rangle + \varepsilon^M|K^0_2\rangle)/\sqrt{1+|\varepsilon^M|^2};$

$|K^0_{L,M}\rangle = (\varepsilon^M|K^0_1\rangle + |K^0_2\rangle)/\sqrt{1+|\varepsilon^M|^2}.$

($\mathbb{CP}$ violation in matter)

ℂℙ violation: experimental layout



CP violation experiment (1964)
schematic layout

In 1964 an experiment was built to search for ℂℙ violation at the Brookhaven AGS (Alternating Gradient Synchrotron).

The schematic layout is shown in the fig.:

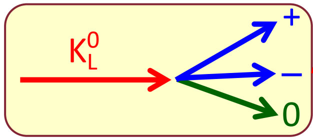
- the primary proton beam (30 GeV) hits a beryllium target;
- secondaries at θ = 30° are selected;
- if charged, collimated and bent away;
- if neutral, collimated and let decay;
- the resultant K^0_L (long lifetime) hit a second lead target, regenerate and are let decay again in a long decay tube;

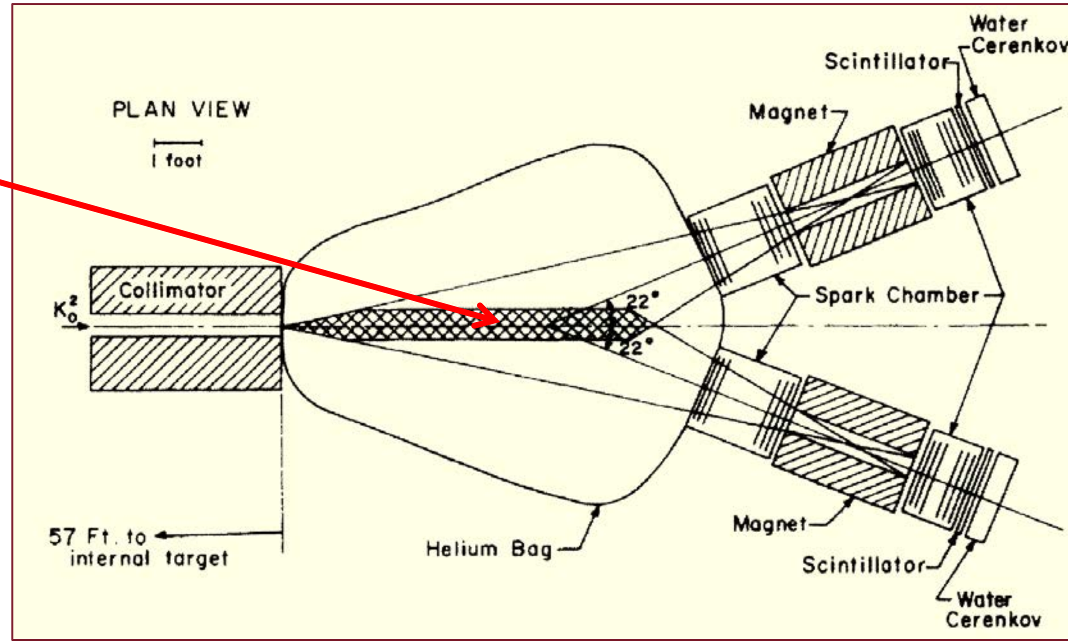
- no K^0_S left → if ℂℙ is conserved, only long lifetime K^0_L [= K^0_2] should remain and decay → 3π;
- if (2π) observed → $\boxed{\text{ℂℙ is violated}}$!!!

- *16 years after, in Stockolm*

James Cronin   Val Fitch

$\mathbb{CP}$ violation: the experiment





Helium bag for K^0_L decays + two-arm-spectrometer.

Each of the two arms :

- spark chambers ($\to$ position);
- magnetic field ($\to$ momentum measurement);
- scintillators ($\to$ trigger + tof);
- water Cerenkov ($\to$ particle id);

main background : n ($\to$ tof rejects).

Other selection criteria :

- two opposite charged particles, one for each arm;
- measure $\vec{p}_+$ and $\vec{p}_-$ (direction and module);
- assume $m_+ = m_- = m_\pi \to m_{+-} \approx m_K \to$ test;
- angle θ between $\vec{p}_{sum}$ (= $\vec{p}_+ + \vec{p}_-$) and $\overrightarrow{dir}_{collimator} \approx 0 \to$ test.

The three-body decays (e.g. $K^0_L \to \pi^+\pi^-\pi^0$) do NOT satisfy those conditions :

- ($\vec{p}_+ + \vec{p}_- = \vec{p}_K - \vec{p}_0$) not collinear with $\overrightarrow{dir}_{collimator}$;
- $m_{+-} \leq (m_K - m_\pi) < m_K$.

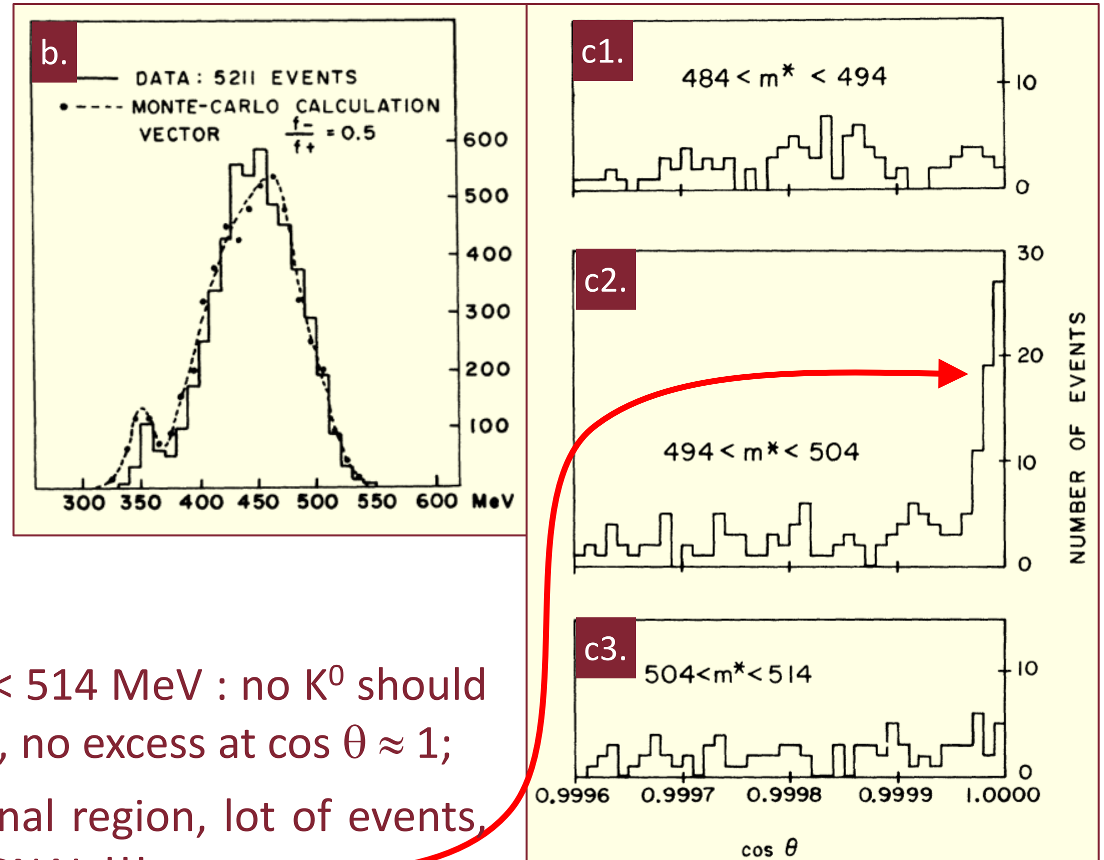
$\mathbb{CP}$ violation: results

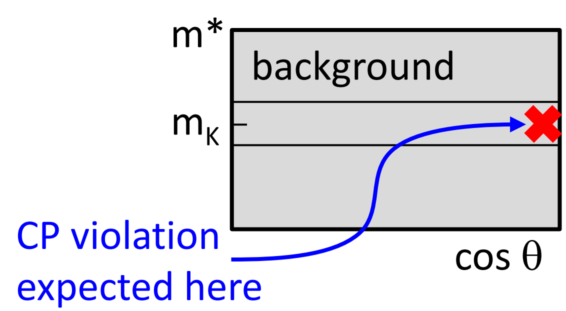
a. (not in figs.) just for calibration, a tungsten plate was put in front of the spectrometer for K^0 regeneration: $\pi^\pm$ identification and mass reconstruction [OK !];

b. distribution of m* [=mass($\pi^+\pi^-$)] for real events and MC simulation [OK!];

c. distribution of cos θ for 3 mass bins, with improved resolution :

- 484 < m* < 494 and 504 < m* < 514 MeV : no K^0 should be there : therefore few events, no excess at cos $\theta \approx 1$;
- 494 < m* < 504 MeV : the signal region, lot of events, clear peak at cos $\theta \approx 1$: THE SIGNAL !!!





d. final result (similar result for the neutral decay $\to\pi^0\pi^0$) :

$$R = BR(K^0_L \to \pi^+\pi^-) \,/\, BR(K^0_L \to \text{charged}) = (2.0 \pm 0.4) \times 10^{-3}$$

$\Rightarrow$ $\mathbb{CP}$ is violated !!!

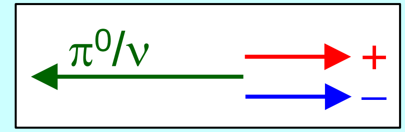
CP violation: $K^0_L \to \pi^+\pi^-, \pi^+\pi^-\pi^0, \pi^\pm e^\mp \nu/\bar\nu$

Q.: study the mass m*

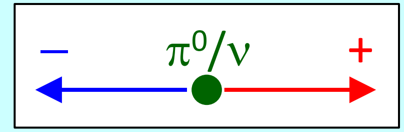
[a typical kin. problem with ambiguities + mass hypoteses]

- work in the K^0_L ref. system;
- define $m^* = \text{mass}(+ve, -ve)$;
- approx. : $m_\nu \approx 0$, $m_e^2 << m_\pi^2$;
- look at the box

min(m*) happens when + and – are at rest wrt each other:
$m^*|_{min} = m_+ + m_-$.



max(m*) happens when the neutral is at rest:
$m^*|_{max} = m_K - m_0$.



a) $K^0_L \to \pi^+\pi^-$
$m^* = m_K$ [easy, no problem];

b) $K^0_L \to \pi^+\pi^-\pi^0$
$m^*|_{min} = 2\, m_\pi \approx 270$ MeV;
$m^*|_{max} = m_K - m_\pi \approx 360$ MeV;

c) $K^0_L \to \pi^\pm e^\mp \nu$
$m^*|_{min} = m_\pi + m_e \approx m_\pi$;
$m^*|_{max} = m_K - m_\nu \approx m_K$;
[*apparently easy, but ...*]

d) $K^0_L \to \pi^\pm e^\mp \nu/\bar\nu$, "$e^\mp$" interpreted as $\pi^\mp$:

"m*"$_{min}$ = m_π + "m_e" = $2m_\pi \approx 270$ MeV;

for "m*"$_{max}$ compute $|\vec p_{\pi/e}|$ and $E_{\pi/e}$ when $|\vec p_\nu| \approx 0$:

$$p_\pi = p_e = \frac{m_K^2 - m_\pi^2}{2m_K} \text{ [see e.g. § 4];}$$

$$E_\pi = \text{"}E_e\text{"} = \sqrt{m_\pi^2 + p_p^2} = \sqrt{m_\pi^2 + \frac{m_K^4 + m_\pi^4 - 2m_K^2 m_\pi^2}{4m_K^2}} =$$

$$= \sqrt{\frac{m_K^4 + m_\pi^4 + 2m_K^2 m_\pi^2}{4m_K^2}} = \frac{m_K^2 + m_\pi^2}{2m_K};$$

$$\text{"}m^*\text{"}_{max} = E_\pi + \text{"}E_e\text{"} = 2E_\pi \approx m_K\left(1 + m_\pi^2/m_K^2\right).$$

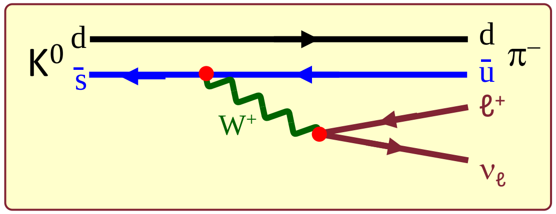
$m^*_{max} \approx 534$ MeV $> m_K$!!!

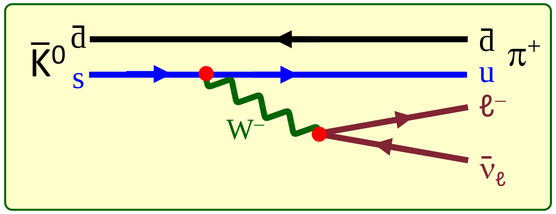
$\mathbb{CP}$ violation: semileptonic decays

- The $(K^0_L \to \pi^+\pi^-)$ is NOT the only channel, which shows $\mathbb{CP}$ violation;
- another important process is the semileptonic decay $(K^0_L \to \pi^\pm \ell^\mp \nu_\ell)$;
- it is an important channel, since :

  $BR(K^0_L \to \pi^\pm e^\mp \nu_e) \approx 40.6\ \%$;

  $BR(K^0_L \to \pi^\pm \mu^\mp \nu_\mu) \approx 27.0\ \%$;
- if $\mathbb{CP}$ were conserved, the rate with the +ve and the –ve charge would be the same, since they are connected by a $\mathbb{CP}$ transformation;





- instead, they are different; it is customary to express the difference as :

$$\delta_L = \frac{\Gamma\left(K^0_L \to \ell^+\nu_\ell\pi^-\right) - \Gamma\left(K^0_L \to \ell^-\bar{\nu}_\ell\pi^+\right)}{\Gamma\left(K^0_L \to \ell^+\nu_\ell\pi^-\right) + \Gamma\left(K^0_L \to \ell^-\bar{\nu}_\ell\pi^+\right)};$$

it is measured $\delta_L = (3.32 \pm 0.06) \times 10^{-3}$.

- NOT "just another boring number".
- First evidence for difference matter-antimatter :

  "*the real matter contains the electron with smaller BR in the* $K^0_L \to \pi^\pm e^\mp \nu_e$ *decay*".
- In fact, some mechanism MUST have generated the asymmetry matter-antimatter of the Universe [*if primordial universe was symmetric*].
- However $\delta \sim 10^{-3}$ is too small to account for the large asymmetry of our world.
- In addition, if the K^0_L decay is the only source, at the big bang time who provided all these K^0_L's ?

$\mathbb{CP}$ violation: the Sandro's view

From [Bettini] :

[… *A*]t late times, when only K_L's survive, they decay through $K_L \to \pi^- \ell^+ \nu_\ell$ a little more frequently than through the $\mathbb{CP}$ conjugate channel $K_L \to \pi^+ \ell^- \bar{\nu}_\ell$. […] This shows, again and independently, that matter and antimatter are somewhat different.

Let us suppose that we wish to tell an extraterrestrial being what we mean by matter and by antimatter. We do not know whether his/her world is made of the former or the latter.

We can tell him/her : "prepare a neutral K meson beam and go far enough from the production point to be sure to have been left only with the long-lifetime component." At this point s/he is left with K_L mesons, independently of the matter or antimatter constitution of her/his world.

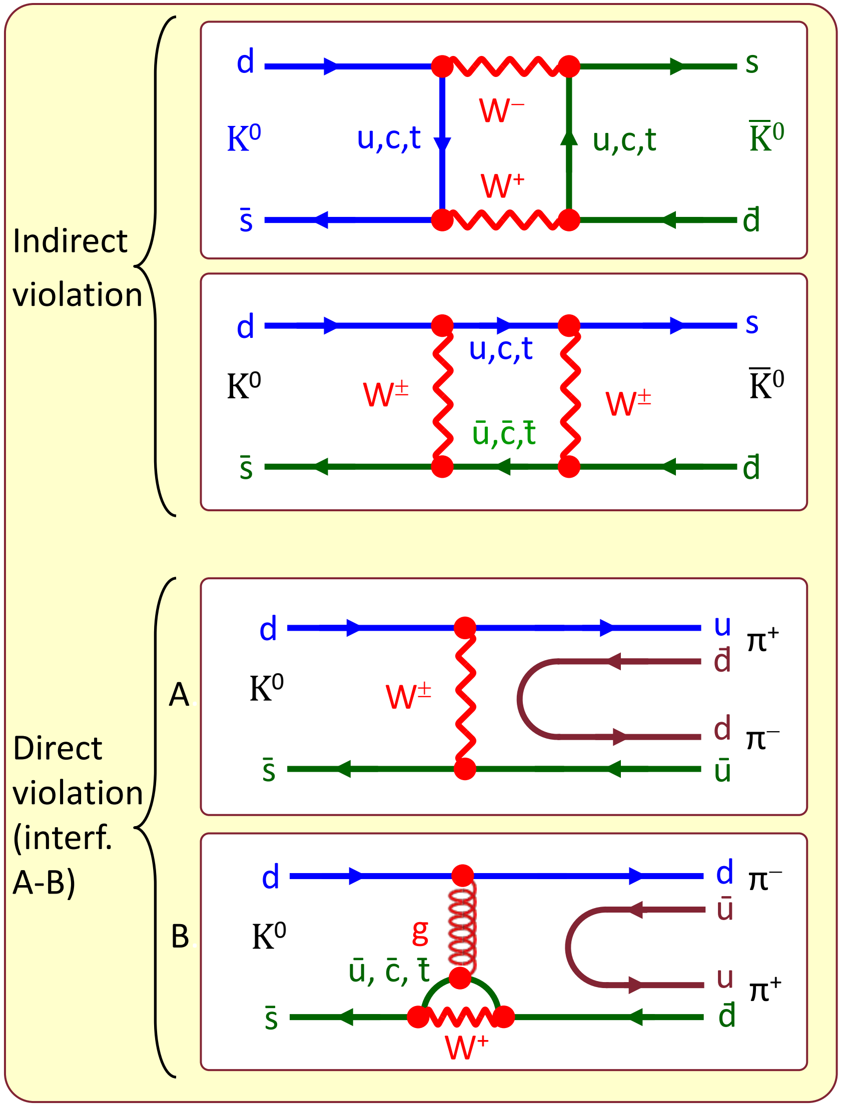
We continue: "count the decays with a lepton of one or the other charge and call positive the charge of the sample that is about three per thousand larger. Humans call matter the one that has positive nuclei."

If, after a while, our correspondent answers that his nuclei have the opposite charge, and comes to meet you, be careful, apologize, but do not shake his/her hand.

Sandro's version of the famous Feynman joke [see § 4].

Direct/indirect $\mathbb{CP}$ violation

- The previous examples/experiments show $\mathbb{CP}$ violations in the decay of neutral flavored mesons (K^0, in the following B^0).
- In fact, three different types of $\mathbb{CP}$ violation have been identified and measured:
  a. in the mixing of neutral mesons ($M \leftrightarrow \bar{M}$) (**indirect violation**);
  b. difference in the decay of a particle: $\Gamma(M \to X) \neq \Gamma(\bar{M} \to \bar{X})$ (**direct violation**);
  c. **interference** between direct and indirect violation : $\Gamma(M \to X) \neq \Gamma(M \to \bar{M} \to X)$.
- in the K^0 system (a) is important, while in the B^0 system b/c dominate; the relative importance of the effect is determined by the values of the V_{CKM} matrix [*see later*];
- (a) and (b) are usually parametrized by the coefficients ε and ε'.



[the indirect violation has been discussed before, e.g. for the 1964 experiment; the couplings qqW are regulated by the V_{CKM} matrix, see later]

Direct/indirect $\mathbb{CP}$ violation: ε and ε'

- The complex parameter ε is associated with the indirect $\mathbb{CP}$ violation;
- this parameter decouples the states with definite lifetimes from the $\mathbb{CP}$ eigenstates :

$$|K^0_S\rangle = \frac{|K^0_1\rangle + \varepsilon|K^0_2\rangle}{\sqrt{1+|\varepsilon|^2}} = \frac{(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K}^0\rangle}{\sqrt{2\left(1+|\varepsilon|^2\right)}};$$

$$|K^0_L\rangle = \frac{|K^0_2\rangle + \varepsilon|K^0_1\rangle}{\sqrt{1+|\varepsilon|^2}} = \frac{(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\overline{K}^0\rangle}{\sqrt{2\left(1+|\varepsilon|^2\right)}};$$

- no $\mathbb{CP}$ violation → ε = 0 → → ($|K^0_S\rangle = |K^0_1\rangle$, $|K^0_L\rangle = |K^0_2\rangle$);
- other commonly used parameters are :

$$\eta_{00} \equiv |\eta_{00}|\exp(i\phi_{00}) \equiv \frac{\langle\pi^0\pi^0|\mathbb{H}|K^0_L\rangle}{\langle\pi^0\pi^0|\mathbb{H}|K^0_S\rangle};$$

$$\eta_{+-} \equiv |\eta_{+-}|\exp(i\phi_{+-}) \equiv \frac{\langle\pi^+\pi^-|\mathbb{H}|K^0_L\rangle}{\langle\pi^+\pi^-|\mathbb{H}|K^0_S\rangle};$$

- the direct violation is parametrized by a complex parameter ε' :

$$\eta_{+-} = \varepsilon + \varepsilon'; \quad \eta_{00} = \varepsilon - 2\varepsilon';$$

- no direct $\mathbb{CP}$ violation → ε' = 0 and $|\eta_{00}| \approx |\eta_{+-}| \approx \varepsilon$;
- ε' is an important parameter for our understanding of Nature;
- as of today, the best measurement, assuming $\mathbb{CPT}$ invariance, are :

$$|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3};$$
$$|\eta_{00}| = (2.221 \pm 0.011) \times 10^{-3};$$
$$|\phi_{+-}| = (43.51 \pm 0.05)^\circ;$$
$$|\phi_{00}| = (43.7 \pm 0.8)^\circ;$$
$$|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3};$$
$$\Re e(\varepsilon'/\varepsilon) = (1.65 \pm 0.26) \times 10^{-3};$$

which are obtained in a long series of dedicated experiments on $\mathbb{CP}$ violation.

Direct/indirect ℂℙ violation: summary_1

D. Kirkby (UC Irvine), Y. Nir (Weizmann Inst.) [PDG 2012] :

- The ℂℙ transformation combines charge conjugation ℂ with parity ℙ.
- Under ℂ, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, e.g., $Q \to -Q$ for electromagnetic charge.
- Under ℙ, the handedness of space is reversed, $\vec{x} \to -\vec{x}$. [... A] left-handed electron e^-_L is transformed under ℂℙ into a right-handed positron e^+_R.
- If ℂℙ were an exact symmetry, the laws of Nature would be the same for matter and for antimatter. We observe that most phenomena are ℂ- and ℙ-symmetric, and therefore, also ℂℙ-symmetric.
- In particular, these symmetries are respected by the gravitational, electromagnetic, and strong interactions.
- The weak interactions, on the other hand, violate ℂ and ℙ in the strongest possible way. For example, the charged W bosons couple to left-handed electrons, e^-_L, and to their ℂℙ-conjugate right-handed positrons, e^+_R, but to neither their ℂ-conjugate left-handed positrons, e^+_L, nor their ℙ-conjugate right-handed electrons, e^-_R.

(*... continue ...*)

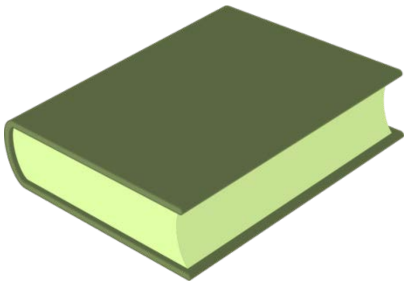
Direct/indirect ℂℙ violation: summary$_2$

D. Kirkby (UC Irvine), Y. Nir (Weizmann Inst.) [PDG 2012] :

(... continued ...)

- While weak interactions violate ℂ and ℙ separately, ℂℙ is still preserved in most weak interaction processes.
- The ℂℙ symmetry is, however, violated in certain rare processes, as discovered in neutral K decays in 1964 [...], and observed in recent years in B decays. A K_L meson decays more often to $\pi^- e^+ \nu_e$ than to $\pi^+ e^- \bar{\nu}_e$, thus allowing electrons and positrons to be unambiguously distinguished, but the decay-rate asymmetry is only at the 0.003 level.
- The ℂℙ-violating effects observed in B decays are larger: the ℂℙ asymmetry in $B^0/\bar{B}^0$ meson decays to ℂℙ eigenstates like $J/\psi K_S$ is about 0.7 [...].
- These effects are related to $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing, but ℂℙ violation arising solely from decay amplitudes has also been observed, first in $K \to \pi\pi$ decays [...], and more recently in various neutral [...] and charged B [...] decays.
- Evidence for ℂℙ violation in the decay amplitude at a level higher than 3σ (but still lower than 5σ) has also been achieved in neutral D [...] and B_s [...] decays.
- ℂℙ violation has not yet been observed in the lepton sector.

LHCb observed ℂℙ violation in D decays in 2019 at 5.3σ.



CKM matrix

Reinterpret the $\mathbb{CP}$ violation using the CKM matrix [§ 4]:

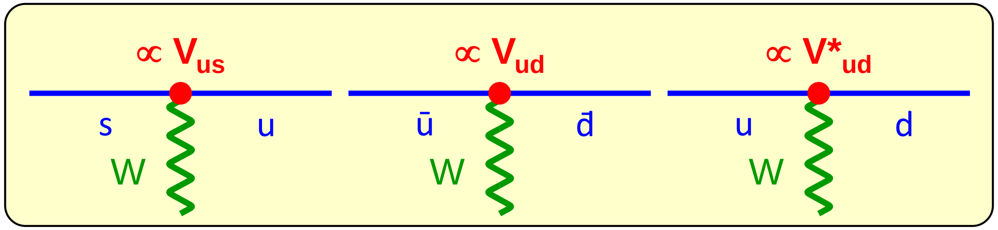
NB

V_{CKM} is a fundamental ingredient of the SM; the actual values V_{ij} are observable (→ measurable, see later), but not predictable inside the SM (like fermion masses, number of families, …)

- the weak charged current for quarks
  [d,s,b are down-quark spinors and ū,c̄,t̄ are the adjoint spinors for up-quarks]

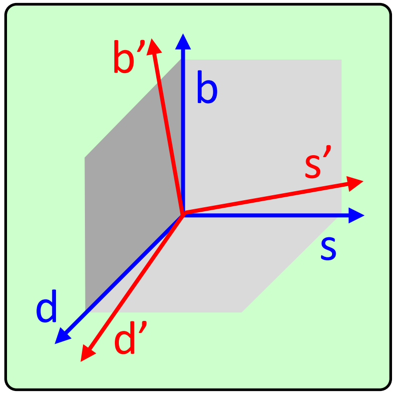
$$j^{\mu}_{qq} = -i\frac{g}{\sqrt{2}}(\bar{u},\ \bar{c},\ \bar{t})\gamma^{\mu}\frac{1}{2}\left(1-\gamma^{5}\right)\boxed{V_{CKM}}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- therefore, e.g. [notice the "*"; the definition is "V_{ij} when bds is a spinor and ūc̄t̄ the adjoint spinor" and "V^*_{ij} when uct is a spinor and b̄d̄s̄ the adjoint spinor".]



- the V_{CKM} matrix represents the rotation, i.e. the amount of mixing among rotated quarks.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



CKM matrix: α_{ij}, δ

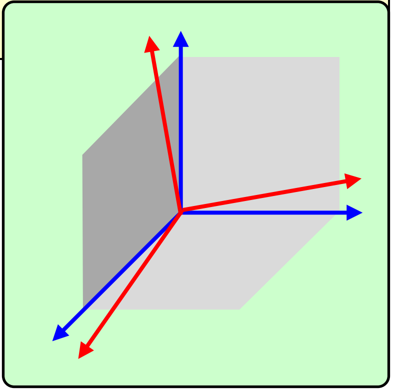
- in a N-family scheme with N=3, V_{CKM} requires n_{rot}=3 real rotations α_{ij} and n_{ph}=1 imaginary phase δ (see box);
- the rotations α_{ij} are "Euler angles" in the quark space ("Cabibbo angles in 3-dim");
- $\delta \neq 0$ $\to$ some V_{ij} complex
  $\to$ $\mathbb{CP}$ violation [next slides];
- many representations, give the most common [PDG] ($c_{ij} \equiv \cos\alpha_{ij}$, $s_{ij} \equiv \sin\alpha_{ij}$):

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{+i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

the K-M approach [IE, §9]:

$$\begin{pmatrix} n_{rot} = N(N-1)/2 \\ n_{ph} = (N-1)(N-2)/2 \\ \mathbb{CP} \text{ violation} \end{pmatrix} \to (n_{ph} \geq 1) \to (N \geq 3).$$



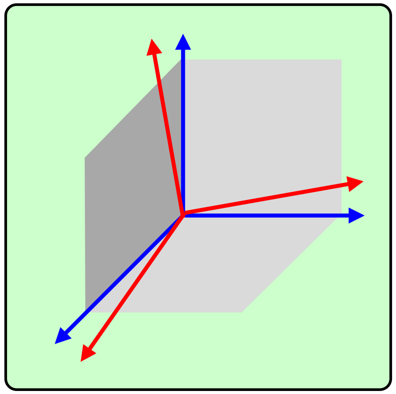
CKM matrix: phenomenology

The representation is chosen to highlight the agreement with experimental data:

- α_{ij} small $\to$ cos α_{ij} >> sin α_{ij}
  $\to V_{CKM} = \mathbb{1}$ + "small rotations"
  $\to$ q'-dynamics = q-dynamics
  + small effects;
- α_{13} small $\to \alpha_{12} \cong \theta_c$;
- Cabibbo theory works well, when considering N=2 (udsc only);
- s_{12} and s_{13} small $\to$ matrix almost real
  $\to$ ℂℙ violation small.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

$$\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \cong \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}.$$



CKM matrix: Wolfenstein parameters

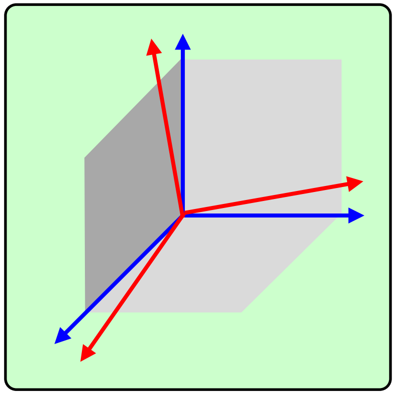
The violations associated with V_{CKM} are usually studied with the Wolfenstein parameterization V^W_{CKM}, which singles out the "small" terms and their physical meaning:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cong V^W_{CKM} + \mathcal{O}\left(\lambda^4\right);$$

$$V^W_{CKM} \equiv \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & A\lambda^2 & 1 \end{pmatrix}.$$

As the "Euler" parameterization, V^W_{CKM} has 4 independent real parameters (**λ A ρ η**):

- $\lambda \cong s_{12}$ ($\to \sin\theta_c$, mixing 1st/2nd);
- $A\lambda^2 \cong s_{23}$ ($\to$ mixing 2nd/3rd);
- $A\lambda^3(\rho + i\eta) \cong s_{13}e^{i\delta}$ ($\to \delta \cong \tan^{-1} \eta/\rho$);
- i.e. $\eta=0 \to \delta=0 \to V_{CKM}$ real $\to$ no $\mathbb{CP}$ violation.



CKM matrix: ℂℙ violation in K^0

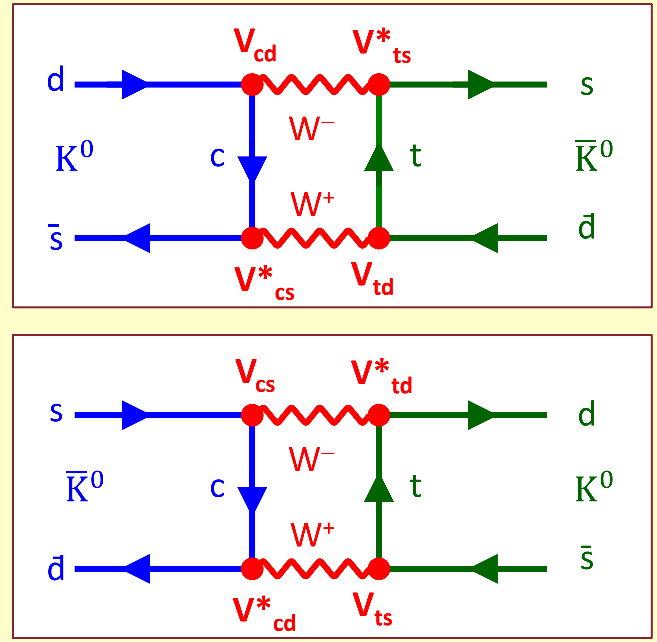
The indirect ℂℙ violation in the K^0 system can be explained with the CKM formalism [Thoms, 393]:

- for each of the $K^0 \leftrightarrow \bar{K}^0$ diagrams
  - *look the t-channel exchange: 9 couples of diagrams (uu, uc, ut, cu, cc, ct, …);*
  - *here discuss only (ct) case, others similar;*
- $\mathcal{M}(K^0 \to \bar{K}^0) \propto V_{cd}\, V^*_{ts}\, V^*_{cs}\, V_{td}$;
- $\mathcal{M}(\bar{K}^0 \to K^0) \propto V^*_{cd}\, V_{ts}\, V_{cs}\, V^*_{td}$;
- V_{ij} real $\to \mathcal{M}(K^0 \to \bar{K}^0) = \mathcal{M}(\bar{K}^0 \to K^0)$
  $\to$ no ~~ℂℙ violation~~;
- V_{ij} complex $\to \mathcal{M}(K^0 \to \bar{K}^0) \neq \mathcal{M}(\bar{K}^0 \to K^0)$
  $\to$ ℂℙ violation.
- in this case $\mathcal{M}(K^0 \to \bar{K}^0) \neq \mathcal{M}(\bar{K}^0 \to K^0)$:

  $$\mathcal{M}(K^0 \to \bar{K}^0) - \mathcal{M}(\bar{K}^0 \to K^0) \propto i\Im(V_{td}) = i\eta A\lambda^3;$$

  [$\Delta\mathcal{M}$ imaginary, small, $\propto \eta$]

  $\to$ CP violation $\propto \eta A^2\lambda^6$ [Jarlskog invariant]

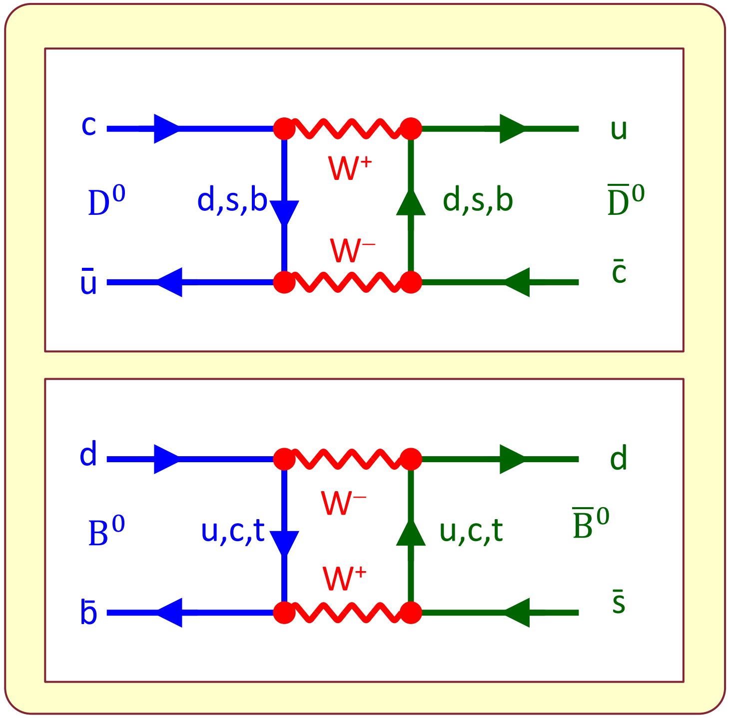


It can be shown [Thoms 403] that the ε parameter of the ℂℙ violation can be written as:

$$|\varepsilon| \propto \eta\,(1 - \rho + \text{const.})$$

CKM matrix: ℂℙ violation in D^0 / B^0

- The ℂℙ violation is expected to occur in the SM also in the D^0–$\bar{D}^0$ and B^0–$\bar{B}^0$ systems through the same dynamical mechanism [*see box*].
- However the importance of the phenomenon depends on the value of the CKM matrix elements, i.e. by the quark mixing.
- In the D^0–$\bar{D}^0$ case:
  - main contribution from b quark exchange;
  - but product $V_{cb}V_{ub}$ very small;
  - therefore predicted D^0–$\bar{D}$ mixing minute;
  - only been observed in 2019 by LHCb.
- Instead B^0–$\bar{B}^0$ mixing:
  - dominated by t quark exchange;
  - expected substantial level of mixing;
  - [*see next slides for some results*].



it could be a golden opportunity: since the SM prediction is small (and computable), a bSM effect would not be obscured.

CKM matrix: measure $|V_{ij}|$

How to measure (the real part of) V_{ij} ?

- from decays ([YN2, §6], [PDG]):
  - $|V_{ud}|$: $p \to ne\bar{\nu}$ and other β decays;
  - $|V_{cs}|$: c-mesons C(abibbo)-allowed;
  - $|V_{us}|$: s-mesons (e.g. $K^\pm$);
  - $|V_{cd}|$: c-mesons C-suppressed,
    : dileptons in ν scattering;
  - $|V_{ub}|$: b-mesons $\to$ non_c-mesons;
  - $|V_{cb}|$: b-mesons $\to$ c-mesons;
  - $|V_{td}|, |V_{ts}|$: $(B^0 \leftrightarrow \bar{B}^0)$ oscillations;
  - $|V_{tb}|$: $t \to W^\pm b$ [*not accurate*];
- conceptually simple, the problem is to disentangle the clean weak decay from the dirty hadron corrections;
- semi-leptonic decays cleaner;
- a technically difficult job (hundreds of papers, theses, conferences…);

- nice final result [PDG 2016]:
  - V_{CKM} quasi-diagonal, as expected;
  - well consistent with SM (unitary, 3 families).

$$|V_{CKM}| \equiv \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} =$$

$$= \begin{pmatrix} .97417 & .2248 & .0409 \\ .220 & .995 & .0405 \\ .0082 & .0400 & 1.009 \end{pmatrix} \pm$$

$$\pm \begin{pmatrix} .00021 & .0006 & .0039 \\ .005 & .016 & .0015 \\ .0006 & .0027 & .0031 \end{pmatrix}.$$

CKM matrix: $|V_{ij}|$ and SM

How to interpret V_{CKM} ?

$$|V_{CKM}| \equiv \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} =$$

$$= \begin{pmatrix} .97417 & .2248 & .0409 \\ .220 & .995 & .0405 \\ .0082 & .0400 & 1.009 \end{pmatrix} \pm$$

$$\pm \begin{pmatrix} .00021 & .0006 & .0039 \\ .005 & .016 & .0015 \\ .0006 & .0027 & .0031 \end{pmatrix}.$$

- tests of SM from $V^\dagger V = \mathbb{1}$:

  $\sum_i V_{ij} V_{ik}^* = \delta_{jk}; \quad \sum_j V_{ij} V_{kj}^* = \delta_{ik}.$

  (e.g. $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$;
- if (a) test(s) fail(s)
  - more generations (missing pieces) ?
  - general breakdown of the model ?
- if all tests succeed
  - general fit imposing unitarity;
  - improved accuracy;
  - stricter tests;
  - more accuracy;
  - and so on, forever [see §LEP].

Unitarity triangle

- from one of the unitarity relations:

$$\sum_i V_{i1} V_{i3}^* = V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = \delta_{13} = 0;$$

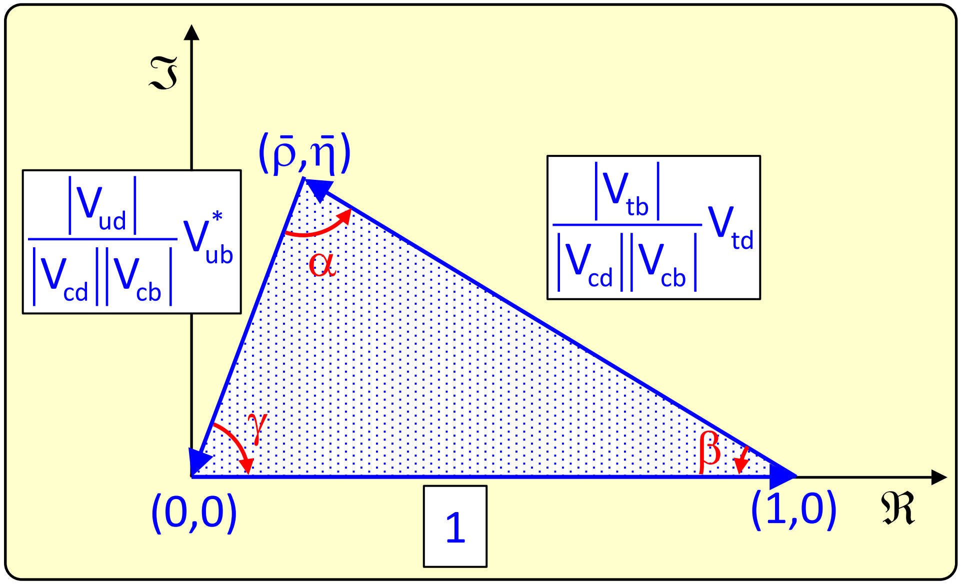
- add some simple math:

V_{ud}, V_{cb}, V_{tb} real > 0;

V_{cd} real < 0 (see V_{cd}^W);

$$\rightarrow |V_{ud}| V_{ub}^* - |V_{cd}||V_{cb}| + V_{td} |V_{tb}| = 0;$$

$$\rightarrow \boxed{1 - \frac{|V_{tb}|}{|V_{cd}||V_{cb}|} V_{td} - \frac{|V_{ud}|}{|V_{cd}||V_{cb}|} V_{ub}^* = 0;}$$



- put the relation in complex plane $\Re\Im$;
- interpreted it as a triangle (unitarity triangle, u.t.);
- define angles (α, β, γ) (see fig.);
- relate $V_{ij} \rightarrow$ Wolfenstein param. ρ^W, η^W;
- the vertex is at ($\bar\rho \cong \rho^W$, $\bar\eta \cong \eta^W$)

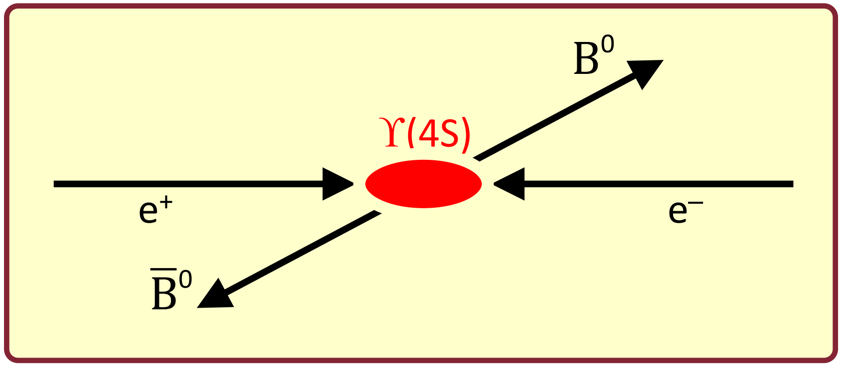
The exact relation is [*check it !*] :

$$\bar\rho + i\bar\eta = (\rho + i\eta)\left(1 - \frac{\lambda^2}{2}\right) + \mathcal{O}(\lambda^4).$$

Note:

- u.t. defined by using V_{ij} only;
- nice adimensional parameters (ratios);
- experiments measure triangle "geometry" (sides, angles);
- lot of relations (e.g. $\alpha+\beta+\gamma=180°$):
  - consistency tests of SM,
  - global fits to parameters assuming SM.

Unitarity triangle: meas β at BaBar, Belle

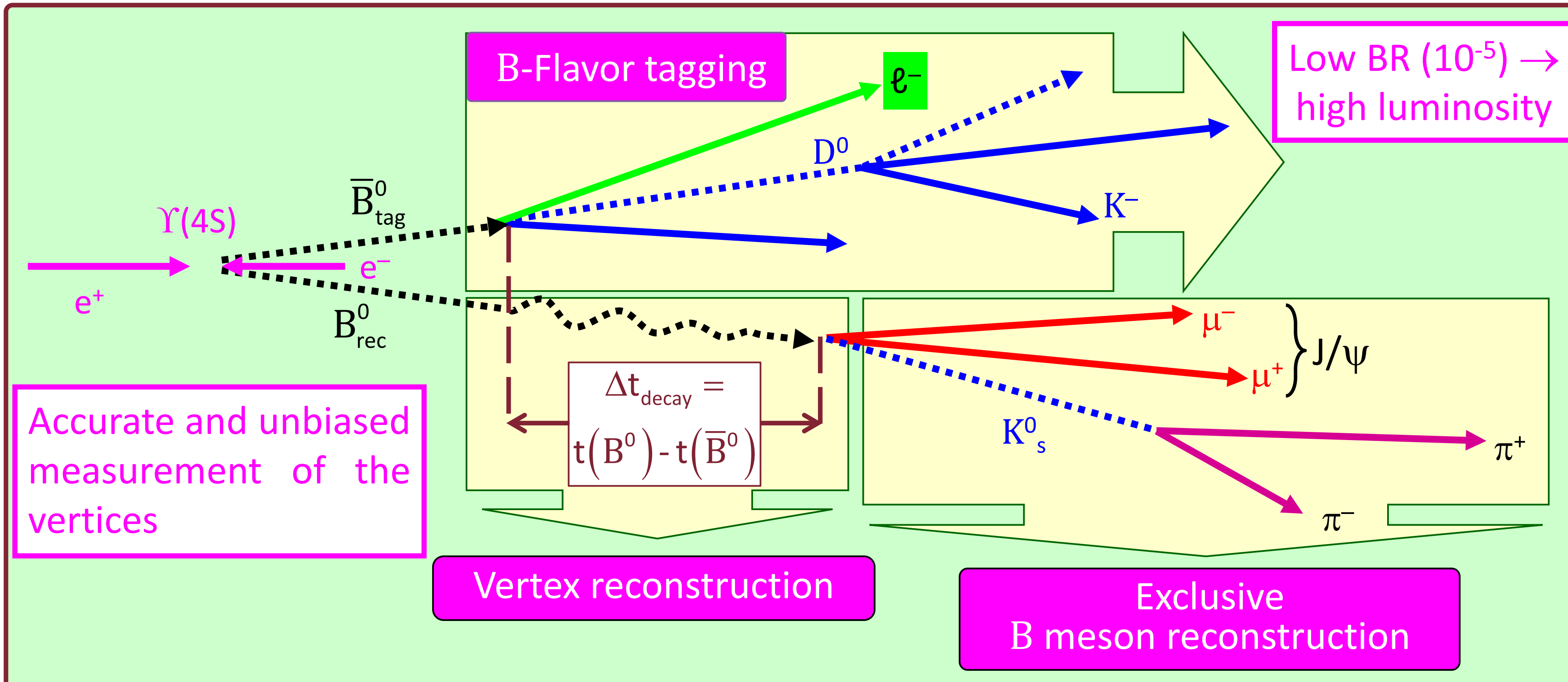


A typical event used for $\mathbb{CP}$ violation in asymmetric e^+e^- at $\sqrt{s} = m(\Upsilon_{4S}) \approx 10.579$ GeV :

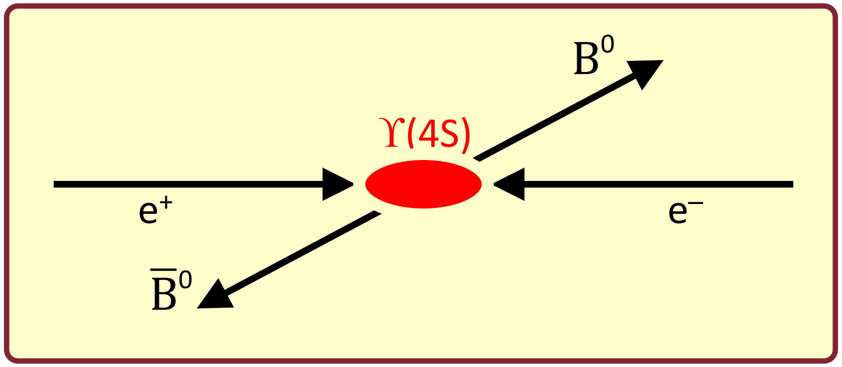
$$e^+e^- \to \Upsilon(4S) \to \overline{B}^0 B^0;$$

$$\overline{B}^0 \to \ell^- D^0 X^+; \quad D^0 \to K^- X^+;$$

$$B^0 \to J/\psi K^0_S; \quad J/\psi \to \mu^+\mu^-; \quad K^0_S \to \pi^+\pi^-.$$

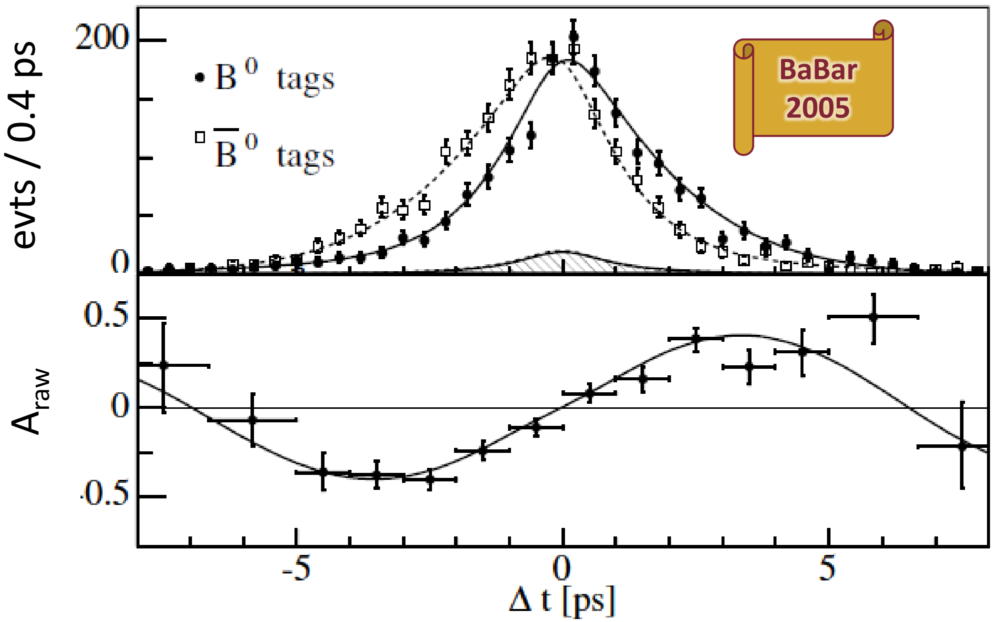


Unitarity triangle: results for β at BaBar



$$A_{raw} = \frac{n\left[\overline{B}(\Delta t) \to J/\psi K_s^0\right] - n\left[B^0(\Delta t) \to J/\psi K_s^0\right]}{n\left[\overline{B}(\Delta t) \to J/\psi K_s^0\right] + n\left[B^0(\Delta t) \to J/\psi K_s^0\right]} \propto$$

$$\propto \sin(2\beta)\sin(\Delta m \Delta t).$$



sin2β = 0.722±0.040

±0.023

[*now improved*]

NB: sinβ > 0

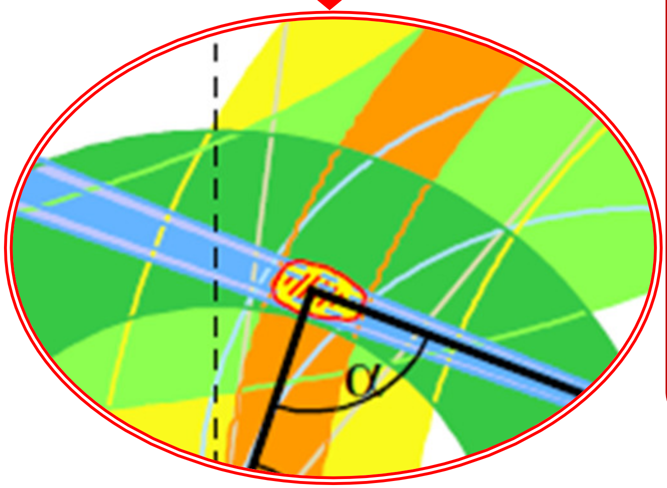
→ η > 0

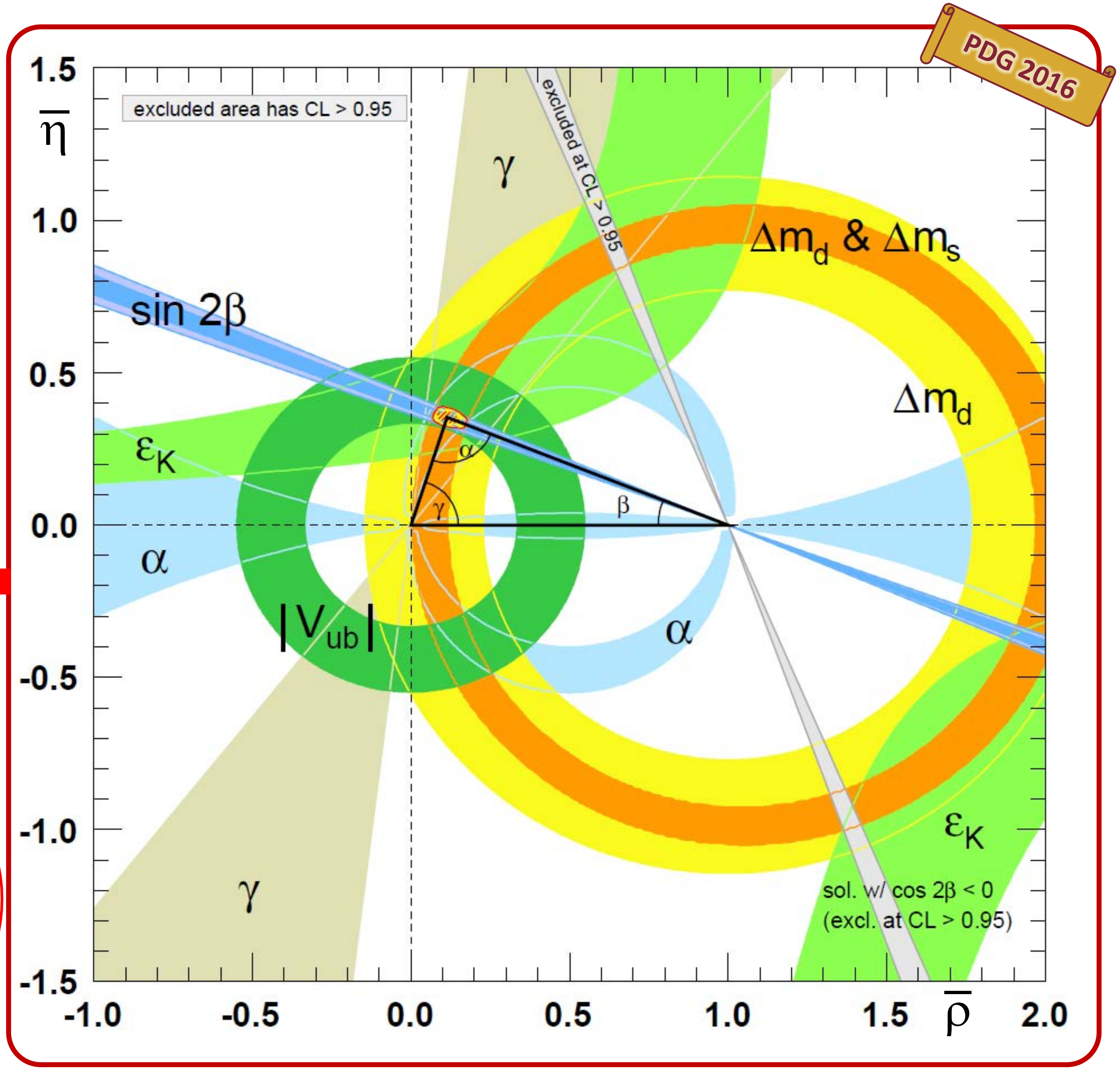
→ ℂℙ violated !!!

Unitarity triangle: measure ρ,η

As of today [PDG 2016]:

- converging measurements (mainly asymmetric e^+e^- factories BaBar, Belle);
- no deviation from 3_f-SM, e.g. $[\alpha+\beta+\gamma]_{fit}$ = $(183\pm8)°$;
- try harder, one of the most promising frontiers !!!



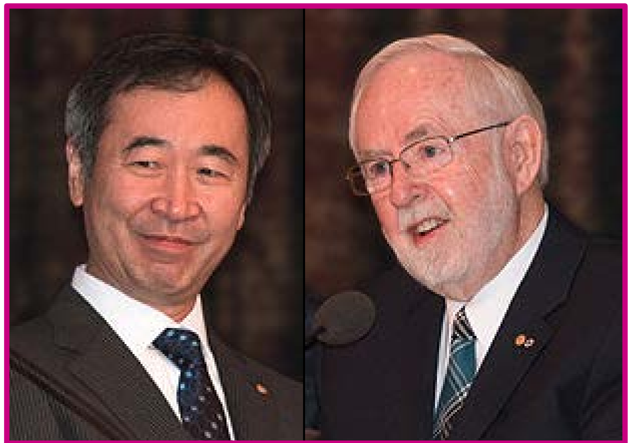


ν oscillations

Quarks of same charge and different flavor mix together → composite hadrons "oscillate" (e.g. $K^0 \leftrightarrow \bar{K}^0$).

The CKM matrix parameterizes the process in the context of the SM.

And the lepton sector ? Do the ν's oscillate ?



T. Kajita – A. McDonald

The answer to the previous question is **YES**.

The results are **important** (Nobel Prize 2015):

- $m_\nu > 0$ (at least for two of them);
- there is mixing in the lepton sector;
- and possibly $\mathbb{CP}$ violation (not easy to see);
- the first discovery bSM (even though, if ν's are Dirac fermions, they can be easily incorporated in the SM).

In the following the ν's will be considered as massive neutral Dirac fermions (sort of neutral electrons), sometimes called "Weyl ν's":

- *this hypothesis is simple, but not the favorite of most physicists;*
- *(as of today) it is NOT falsified by the exp.;*
- *other comments on § Standard Model.*

The ν's are very complicated objects! many (most ?) of the important discoveries in particle physics of the last 80 years came from them !!!

ν oscillations: toy model

Assume mixing in the ν sector and look for possible observables.

notice: $\nu_{1,2}$ = mass eigenstates (= $K^0_{S,L}$) with $m_{1,2}$, $\nu_{e,\mu}$ = lepton eigenstates (= K^0, $\bar{K}^0$) with $n_{e,\mu}$.

Simple toy model, inspired to Cabibbo angle:

- 2 families ($\nu_1, \nu_2 \to \nu_e, \nu_\mu$);

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_\nu & \sin\theta_\nu \\ -\sin\theta_\nu & \cos\theta_\nu \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix};$$

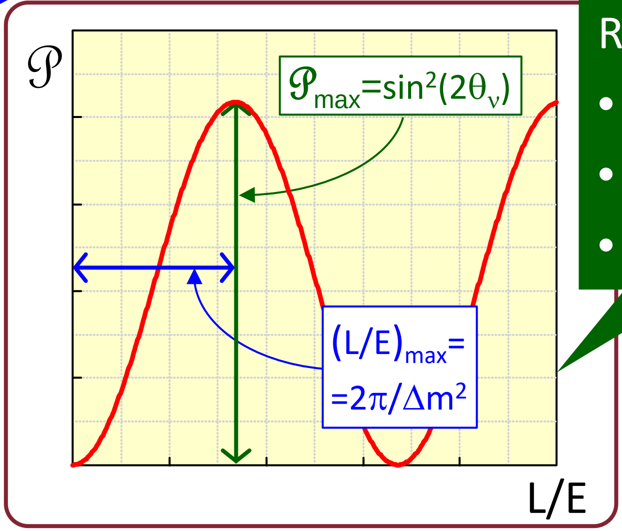
- free parameters: masses, mixing angle θ_ν;
- same formalism as in the ($K^0_1 \leftrightarrow K^0_2$) case;
- time evolution of a pure $\nu_{e,\mu}$ at t=0:

$$|\nu_e(t)\rangle = \cos\theta_\nu e^{-iE_1 t}|\nu_1\rangle + \sin\theta_\nu e^{-iE_2 t}|\nu_2\rangle$$
$$|\nu_\mu(t)\rangle = -\sin\theta_\nu e^{-iE_1 t}|\nu_1\rangle + \cos\theta_\nu e^{-iE_2 t}|\nu_2\rangle$$

- the oscillation probability $\mathcal{P}$ is [*next slide*]:

$$\mathcal{P}_L(\nu_e \to \nu_\mu) = \sin^2[2\theta_\nu]\sin^2\left[\frac{\Delta m^2 L}{4E}\right];$$

$$\frac{\Delta m^2 L}{4E} \approx \frac{1.27 \times (m_2^2 - m_1^2)[eV^2] \times L[km]}{E[GeV]}.$$



Required:

- large θ_ν;
- $m_\nu > 0$;
- large L/E;

→ since θ_ν and $m_{1,2}$ are not up to us, the relevant exper. parameter is **L/E**; with present technologies, the observation is:

- difficult (= impossible) with accelerators;
- needs astrophysical exp.

[actual experiments are NOT discussed here: they belong to the astroparticle course]

ν oscillations: the math

$$\left|\langle\nu_e(t)|\nu_e(0)\rangle\right|^2=\left|\left(\cos\theta_\nu e^{-iE_1t}\langle\nu_1|+\sin\theta_\nu e^{-iE_2t}\langle\nu_2|\right)\left(\cos\theta_\nu|\nu_1\rangle+\sin\theta_\nu|\nu_2\rangle\right)\right|^2=$$

$$=\left|\cos^2\theta_\nu e^{-iE_1t}+\sin^2\theta_\nu e^{-iE_2t}\right|^2=$$

$$=\left|\cos^2\theta_\nu\cos(E_1t)-\imath\cos^2\theta_\nu\sin(E_1t)+\sin^2\theta_\nu\cos(E_2t)-\imath\sin^2\theta_\nu\sin(E_1t)\right|^2=$$

$$=\cos^4\theta_\nu\cos^2(E_1t)+\sin^4\theta_\nu\cos^2(E_2t)+2\sin^2\theta_\nu\cos^2\theta_\nu\cos(E_1t)\cos(E_2t)+$$

$$+\cos^4\theta_\nu\sin^2(E_1t)+\sin^4\theta_\nu\sin^2(E_2t)+2\sin^2\theta_\nu\cos^2\theta_\nu\sin(E_1t)\sin(E_2t)=$$

$$=\cos^4\theta_\nu+\sin^4\theta_\nu+2\sin^2\theta_\nu\cos^2\theta_\nu\cos\left[(E_2-E_1)t\right]\boxed{+1-(\cos^2\theta_\nu+\sin^2\theta_\nu)^2}=$$

$$=1-2\sin^2\theta_\nu\cos^2\theta_\nu\left\{1-\cos\left[(E_2-E_1)t\right]\right\}=1-4\sin^2\theta_\nu\cos^2\theta_\nu\sin^2\left[(E_2-E_1)t/2\right]=$$

$$=1-\sin^2(2\theta_\nu)\sin^2\left(\frac{\Delta m^2L}{4E}\right).$$

$$\mathcal{P}_L(\nu_e\to\nu_\mu)=1-\left|\langle\nu_e(t)|\nu_e(0)\rangle\right|^2=$$
$$=\sin^2(2\theta_\nu)\sin^2\left(\frac{\Delta m^2L}{4E}\right).$$

$\mathcal{P}_L$ is the oscillation probability after a distance L.

$$(E_2-E_1)t=\left(\sqrt{p^2+m_2^2}-\sqrt{p^2+m_1^2}\right)t\approx$$
$$\approx p\left[\left(1+\frac{m_2^2}{2p^2}\right)-\left(1+\frac{m_1^2}{2p^2}\right)\right]\frac{L}{c}\approx$$
$$\approx\frac{m_2^2-m_1^2}{2p}L\approx\frac{\Delta m^2L}{2E}.$$

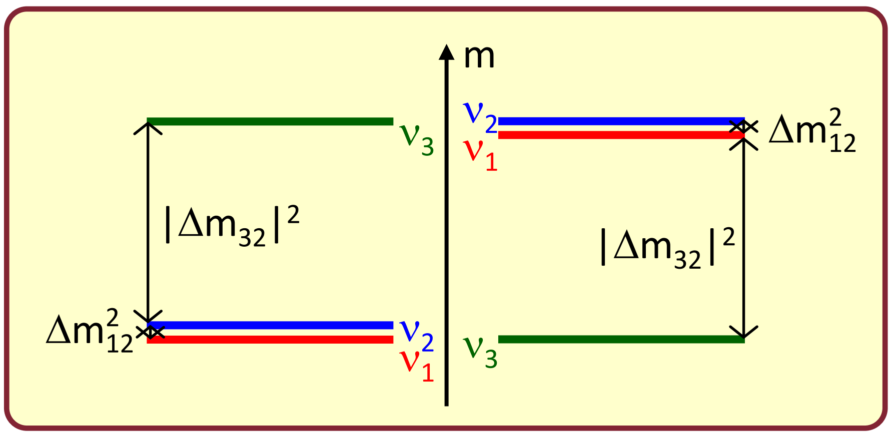
ν oscillations: results

Current ν oscillation experiments measure:

$\Delta m^2_{12} = m^2_2 - m^2_1 \approx 7.37 \times 10^{-5}\ eV^2;$

$|\Delta m_{32}|^2 = |m^2_3 - m^2_2| \approx 2.56 \times 10^{-3}\ eV^2;$

compatible with the two "hierarchies" shown in the box (ambiguity still not solved).



Q. why ν's from the sky and not from an accelerator ? compute the value of L/E for the oscillation maxima using these values.

In the SM there are three families → the ν mixing matrix is 3 × 3, with the same math properties of the CKM one (three angles + a CP-violating phase).

It is called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = V_{PKMS} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix};$$

the present best measurements are [PDG]:

$$|V_{PKMS}| = \begin{pmatrix} 0.826 & 0.544 & 0.151 \\ 0.427 & 0.642 & 0.635 \\ 0.368 & 0.540 & 0.757 \end{pmatrix}.$$

The CP-violating phase (δ_ν) is $\approx 3\pi/2$.

$\mathbb{CPT}$ theorem

If (Quantum field theory) and (Special relativity) and ($\mathbb{H}$ invariant under Lorentz transformation),

then

the physical states are $\mathbb{CPT}$ invariant, i.e. invariant under the consecutive application of the operators $\mathbb{C}$harge-conjugation, $\mathbb{P}$arity and $\mathbb{T}$ime-reversal.

Nota bene :

- The states may be invariant for the application of any of the three, like in strong interaction processes.
- In this case, *a fortiori*, they will be invariant under the three together.
- But even processes which violate one (left-handed neutrinos, K^0 oscillations) or even two (K^0 semileptonic decays), must be invariant under the combined application of the three together.

Consequences of the $\mathbb{CPT}$ theorem :

- mass, charge and lifetime of a particle and its antiparticle are exactly equal :
  - $|m(K^0) - m(\bar{K}^0)|$ / aver. $< 6 \times 10^{-19}$;
  - $|m(e^+) - m(e^-)|$ / aver. $< 8 \times 10^{-9}$;
  - $|q(p) - q(\bar{p})|$ / $q(e^-) < 2 \times 10^{-9}$;
  - $[\tau(\mu^+) - \tau(\mu^-)]$ / aver. $= (2 \pm 8) \times 10^{-5}$;
- any violation in an individual or pair of symmetries must be compensated by an asymmetry in the other operation(s), so to save exact symmetry under $\mathbb{CPT}$.
- (e.g.) The weak interactions violate $\mathbb{C}$ and $\mathbb{P}$ separately but in general they are invariant under the combined operation of $\mathbb{C}$ and $\mathbb{P}$ (and $\mathbb{T}$ alone).
- (e.g.) The weak decays of the K^0 mesons violate $\mathbb{CP}$, but this is accompanied by a corresponding violation of $\mathbb{T}$, so that [$\mathbb{CP}$ $\mathbb{T}$] is respected.

References

1. [BJ, 11.13]], [YN1, 16];
2. the CPT theorem is discussed in [MQR, 12];
3. the $\mathbb{CP}$ violation and the FCNC are discussed in [IE, 12-13]

Gian Lorenzo Bernini – Apollo and Daphne – 1622-25 – Galleria Borghese

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End of chapter 5

Particle Physics - Chapter 6
The Standard Model

Paolo Bagnaia
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AA 18-19

last mod.
2-May-19

6 – The Standard Model

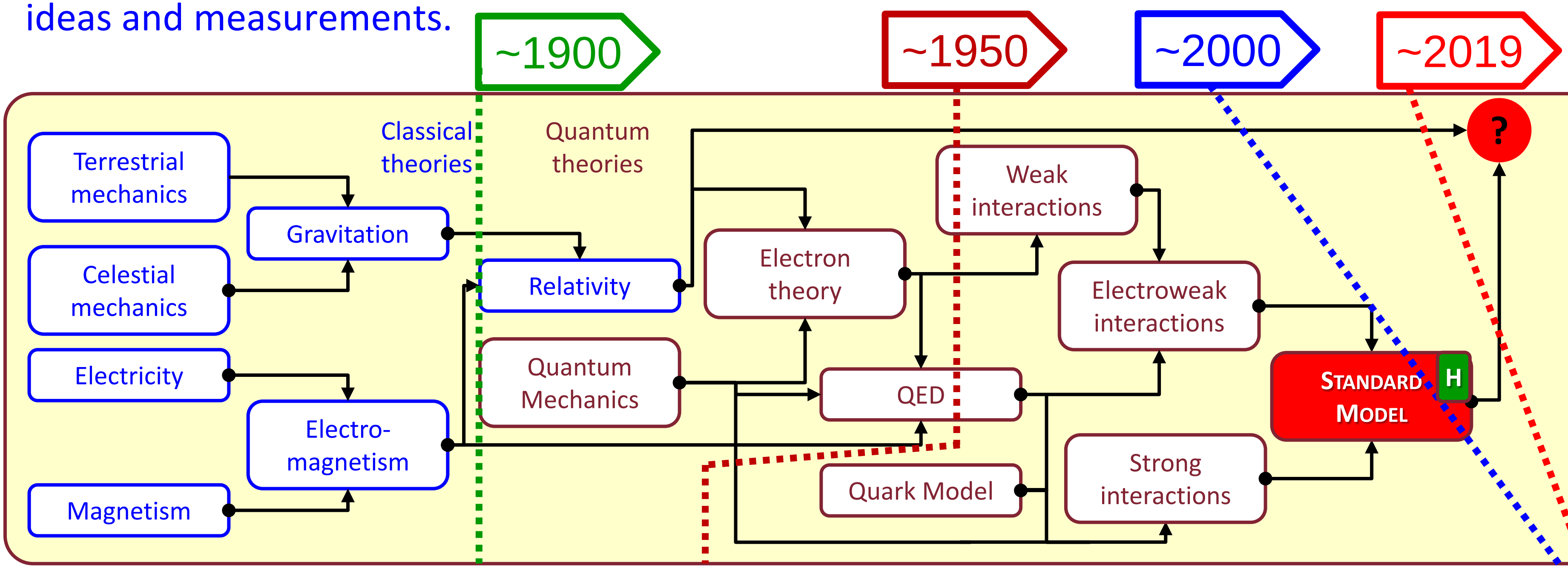
u	c	t	γ
d	s	b	g
ν_e	ν_μ	ν_τ	Z
e	μ	τ	$W^\pm$
			H

The style of this chapter depends on our CdL:

- the electroweak part is an overlap with previous semester → just for reference;
- the QCD part is different: still amateurish, but I'm the first to tell you;
- the Higgs part is in § LEP and § LHC.

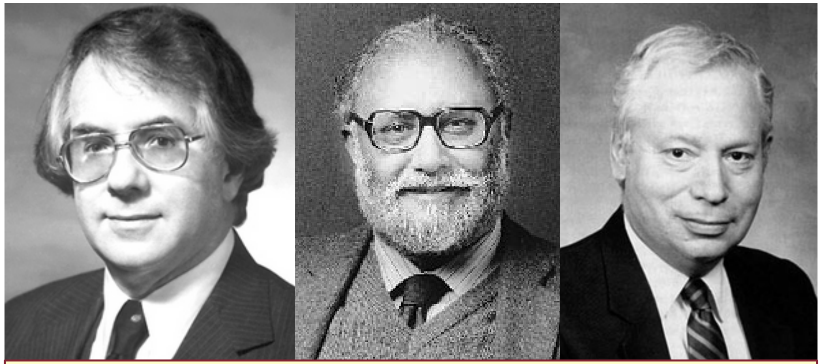
The Standard Model

- The name **SM** (not a nice name) designates the theory of the Electromagnetic, Weak and Strong interactions.
- The theory has grown in time, the name went together.
- The development of the SM is a complicated interplay between new ideas and measurements.
- Many theoreticians have contributed : since the G-S-W model is at the core of the SM, it is common to quote them as the main authors.
- The little scheme [BJ] of its time evolution may help (missing connections, approximations, …).



the electroweak theory

- Glashow (1961), Salam (1964), Weinberg (1967) provided the main ingredients for the unification of weak and electromagnetic interactions.
- The fundamental interactions are described by field theories, invariant under local gauge transformations.
- Technically, by a Lagrangian $\mathscr{L}$, invariant under the appropriate symmetries.
- The symmetries correspond, via the Noether theorem, to the conservation laws of the Theory.
- The conservation laws are local [*i.e. in a given space-time point*]: electric charge is the usual example of such a quantity.
- In the Standard Model, the electro-magnetic and weak interactions (both CC and NC) are related to the symmetry group SU(2) $\otimes$ U(1).
- The parameters of the theory controls all the phenomena: "few" independent masses and couplings for the full theory.
- The dynamics is fully regulated : ($e^{\pm}$, $\mu^{\pm}$, ν) DIS, e^+e^- processes (LEP), IVB and Higgs production and decay (Sp$\bar{p}$S, LHC) are fully described by the e.w. Theory.
- Among the successful predictions neutral currents, $W^{\pm}$, Z, Higgs.
- [in the '60s/'70s no strong interactions theory, but now QCD occupies the role.]
- [as of today, no quantum gravity theory.]



Sheldon Lee Glashow | Abdus Salam (عبد السلام) | Steven Weinberg

the e.w. theory: properties

- Any theory (including the e.w.) has to be free from logical and mathematical inconsistencies.
- In mathematical terms, it MUST be renormalizable, i.e. it must exist a mathematically correct procedure, that eliminates the infinities that arise in calculations of physical observables, such as cross sections and decay rates.
- To achieve this objective, the e.w. $\mathscr{L}$ must not contain explicitly mass terms; i.e., at the $\mathscr{L}$ level, both the Gauge bosons (the "fields") and Fermions (the "matter") must be massless.
- The proof of the renormalizability of the theory was provided by 't Hooft and Veltman (in 1971, Nobel Prize in 1999).
- The masses are then generated in the theory, without destroying the renormalizability, with the mechanism of spontaneous symmetry breaking, usually called *Higgs mechanism*, proposed by Englert & Brout (1964), Higgs (1964) and Gularnik, Hagen & Kibble (1964).
- The mechanism predicts the existence of (at least) one scalar, the Higgs boson H.
- The values of the fermion masses are left as free parameters; however, once they are fixed, all the couplings of the H boson to the other bosons and fermions are predicted by the theory.

G.t'Hooft – M.Veltman

the e.w. theory: I_W, Y

The properties of the **gauge bosons** **$W^{\pm}$, Z** and **γ** come out from the theory.

- The fundamental representation of SU(2) ⊗ U(1) is given by three [SU(2)] and one [U(1)] Gauge fields.
- The quantity called "weak isospin" **I_W** [*here called simply "isospin"*(*)] belongs to the SU(2) sector.
- For U(1), there is the "weak hypercharge" **Y_W** [*here "hypercharge"*].
- All the members of the same isospin multiplet have the same hypercharge.
- Similarly to the flavor case, the hypercharge is defined as twice the difference between the electric charge and the third component of the isospin :

  $Y_W \equiv 2\,(Q - I_{Wz})$.
- The triplet of fields corresponding to SU(2) is called W = (W_1, W_2, W_3). The fields W have I_W = 1 and Y_W = 0. They interact with the weak isospin of the particles.
- The field corresponding to U(1) is called **B**. Its isospin, electric charge and hypercharge are zero. It interacts with the weak hypercharge of the particles.
- These four fields (W_i, B) are NOT the physical fields which mediate the interactions.
- The CC weak interactions are mediated by $W^{\pm}$, which are linear combinations of W_1 and W_2.
- The photon and the Z, mediators of the electromagnetic and NC weak interactions, are linear combinations of W_3 and B.

(*) *Notice that the weak isospin and hypercharge do NOT have any dynamical relation with those defined in precedence for the hadrons, although their mathematical properties are the same.*

the e.w. theory: fermions

The value of I_W and Y_W of the particles depends on the fact that the $W^\pm$, the mediators of the CC, are coupled only to states with negative chirality.

The leptons. In each family there are two left-handed leptons in a I_W = ½ doublet :

$$I_W = ½,\ I_{Wz} = +½ \ :\ \nu_{eL},\ \nu_{\mu L},\ \nu_{\tau L};$$

$$I_W = ½,\ I_{Wz} = -½ \ :\ e^-_L,\ \mu^-_L,\ \tau^-_L.$$

- the ν's have a (small but non-zero) mass and mix together (mixing matrix 3×3);
- unlike the charged currents, the neutral currents also interact with the charged right-handed fermions, but NOT with right-handed neutrinos;
- the right-handed charged lepton of each family is an isospin singlet (I_W = 0) :

$$I_W = 0,\ I_{Wz} = 0 : e^-_R,\ \mu^-_R,\ \tau^-_R.$$

- right-handed ν's DO NOT EXIST [*more precisely, if existing, they have* ($I_W = Y_W$ = 0) *and do NOT interact with anything except possibly through gravity*].

The quarks. Their structure is similar, apart from a different mixing (the CKM matrix) and the color :

- The $W^\pm$ is universally coupled with the CKM-rotated states d', s' and b'.
- [three isospin doublets, one for each family] × [three colors] = nine doublets :

$$I_W = ½,\ I_{Wz} = +½ \ : u_L,\ c_L,\ t_L;$$

$$I_W = ½,\ I_{Wz} = -½ \ : d'_L,\ s'_L,\ b'_L;$$

- the singlets (18 in total) are :

$$I_W = 0 \ : d_R,\ u_R,\ s_R,\ c_R,\ b_R,\ t_R;$$

- for NC, the quark mixing is irrelevant; therefore we can study the interactions of the "non-rotated" states.

the e.w. theory: a remark on ν's

Some alternative hypotheses on **ν's** :

a) Dirac particles, charge=0, spin=½, mass=0, helicity = –1, partners of charged leptons:
   → ν's do not mix;

b) as (a), but $m_{\nu's} > 0$, although very small:
   → ν's can mix, define the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix;
   → helicity NOT intrinsic, depends on L-system [*but see comment*];

   b_1) PMNS diagonal → ν's do not mix;

   b_2) PMNS NOT diagonal → ν's do mix;

c) any hypothesis bSM (e.g. Majorana ν's).

in [MQR] and [IE], (a) and (b) are called "Weyl ν's", while "Majorana ν's " are in (c).

Results:

(a) believed to be correct for most of the XX century; **falsified** in 1998 by discovery of ν-oscillations [ν-o.];

(b_1) [*ugly*] **falsified** by ν-o.;

(b_2) **current working hypothesis** [*because minimal extension of the SM*]; however it looks unlikely to most [*???*] physicists;

(c) **much appreciated**; however, as of today, no data supports it (many new ongoing experiments: ***good luck !!!***).

Comment:

- ν oscillations (→ mixing) appear only in astro-physical or long-baseline experiments;
- in all other experiments, data (**until today**) consistent with (a);
- no contradiction: ($m_{\nu's} << E_{\nu's}$) → (ν's ultra-relativistic) → (a) good approx. of (b).

Conclusion (as of today):

- (at least two) ν's have mass > 0;
- ν's can and do oscillate (PMNS ≠ $\mathbb{1}$);
- for most exp., approx. $m_{\nu's}$=0, no-oscillation, fixed helicity (–1 for ν, +1 for $\bar{\nu}$);
- ***hope for new exp., or more precise data***.

the e.w. theory: antiparticles

- **The antiparticles.** For each particle, there exists an antiparticle, with opposite quantum numbers.
- In the lepton sector, for CC there are the following three doublets of antileptons :

  $I_W = ½, I_{Wz} = + ½$: e^+_R, μ^+_R, τ^+_R;

  $I_W = ½, I_{Wz} = – ½$: $\bar{\nu}_{eR}$, $\bar{\nu}_{\mu R}$, $\bar{\nu}_{\tau R}$;
- Plus the following singlets :

  $I_W = 0, I_{Wz} = 0$: e^+_L, μ^+_L, τ^+_L.
- For the $\bar{\nu}$'s, the same rules apply as for ν's.
- In the antiquark sector, three doublets of isospin and six singlets for each family (9 plus 18 in total) :

  $I_W = ½, I_{Wz} = + ½$: $\bar{d}'_R, \bar{s}'_R, \bar{b}'_R$;

  $I_W = ½, I_{Wz} = – ½$: $\bar{u}_R$, $\bar{c}_R$, $\bar{t}_R$;

  $I_W = 0$: $\bar{d}_L$, $\bar{u}_L$, $\bar{s}_L$, $\bar{c}_L$, $\bar{b}_L$, $\bar{t}_L$.

P.A.M. Dirac, 1933 Nobel Lecture:

"If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regard it rather as an accident that the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons.

It is quite possible that for some of the stars it is the other way about, these stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods."

Great !!! But presently we know much more (more precisely, **we ignore** much more).

Find where we "improved" in the last 85 years.

the e.w. theory: summary of q.n.

	Spin	I_W	I_{Wz}	Y_W	Q [(*)]
$\nu_{\ell L}$	½	½	+½	−1	0
ℓ^-_L	½	½	−½	−1	−1
ℓ^-_R	½	0	0	−2	−1
u_L	½	½	+½	+⅓	⅔
d'_L	½	½	−½	+⅓	−⅓
u_R	½	0	0	+4/3	⅔
d_R	½	0	0	−⅔	−⅓
W^+	1	1	+1	0	+1
W^-	1	1	−1	0	−1
Z	1	1,0	0	0	0
γ	1	1,0	0	0	0
H	0	0	0	0	0

(*) $Q = I_{wz} + ½\, Y_W$.

- Weak isospin (I_W) and weak hypercharge (Y_W) are conserved in all known interactions.
- I_W and Y_W have nothing to do with those of hadrons.
- I_W is the source of the weak charged fields $W^\pm$.
- Y_W and I_{Wz} are the sources of the weak neutral field Z and of the e.m. field γ.
- The L(eft) components of the spinors have $I_W \neq 0$; they emit and absorb $W^\pm$.
- The R(ight) components have $I_W = 0$; they do not emit or absorb $W^\pm$.
- Both components have $Y_W \neq 0$; they emit and absorb Z.
- the ν_R have $I_W = 0$ and $Y_W = 0$; they do not exist or are not observable (in the m=0 limit).

the e.w. theory: the IVBs W/Z

The field W_μ = ($W_{\mu 1}$, $W_{\mu 2}$, $W_{\mu 3}$) is a 4-vector in the space-time$^{(*)}$, and a vector in the space of the weak isospin I_W of SU(2) (index $_{123}$), because it has $I_W = 1$:

- The fields of the physical charged bosons:
  $W^\pm = (W_1 \mp iW_2) / \sqrt{2}$;
- For each doublet of fermions there is a 4-vector, which is at the same time a 3-vector in the I_W space, which represents the weak current :
  $j_\mu \equiv (j_{\mu 1}, j_{\mu 2}, j_{\mu 3})$;
- The field W_μ is coupled to j_μ as ($g\ W_\mu\ j_\mu$) through the dimensionless coupling constant g.
- The charged currents are linear combinations of two current components
  $j^\pm = (j_1 \pm j_2)$.
- E.g., consider the doublet (ν_{eL}, e^-_L); the corresponding charged currents are
  $j^+_{e\mu} = \bar{\nu}_{eL} \gamma_\mu e^-_L$; $j^-_{e\mu} = \bar{e}^-_L\ \gamma_\mu \nu_{eL}$.

The field B_μ is a 4-vector in space-time and a scalar in isospin ($I_W = 0$). It interacts with the neutral current of the leptons j_μ (4-vector - isoscalar) through the coupling constant g'.

- The current generated by the hypercharge is twice the difference between the electric current j^{EM}_μ and the neutral component of the NC :
  $Y_W = 2\ (Q - I_{Wz}) \rightarrow j^Y_\mu = 2 j^{EM}_\mu - 2 j_{3\mu}$.
- The first term is the electromagnetic current, which for charged fermions is
  $j^{EM}_{f\mu} = \bar{f} \gamma_\mu f$.
- The chirality is not specified because the electro-magnetic interactions do not depend on it.

$^{(*)}$ *warning : here the μ index refers to the space-time dimensions, NOT to the $\mu^\pm$ lepton.*

the e.w. theory: mixing angle θ_w

Call **A** and **Z** respectively the physical fields that mediate the electromagnetic and neutral currents.

- They are two mutually orthogonal linear overlap of W_3 and B, which can be determined by requiring that the photon does not couple to the neutral particles, contrary to the Z.
- The transformation is given as a function of two couplings g and g', i.e. as a rotation of an angle θ_w, the mixing angle of the weak interactions [a.k.a. the *Weinberg angle*] :

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix} = $$

$$= \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix};$$

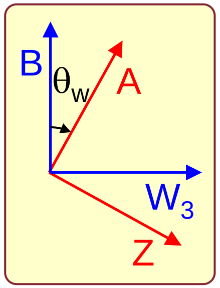
$$\tan\theta_w = g'/g.$$

- The mixing angle is quite large, $\theta_w \approx 29°$.
- The interaction Lagrangian, being symmetric under the gauge group, is an isoscalar :

$$\mathscr{L} = g\,(j_{\mu 1}W_{\mu 1} + j_{\mu 2}W_{\mu 2} + j_{\mu 3}W_{\mu 3}) +$$
$$+ \tfrac{1}{2}g' j_\mu^Y B_\mu,$$

which can also be written as:

$$\mathcal{L} = g/\sqrt{2}\,(j_\mu^- W_\mu^+ + j_\mu^+ W_\mu^-) +$$
$$+ j_\mu^3\,(gW_{\mu 3} + g'B_\mu) +$$
$$+ g'\, j_\mu^{EM} B_\mu.$$



the e.w. theory: interactions

- Then, introducing the neutral physical fields:

$$\mathscr{L} = \frac{g}{\sqrt{2}}\left(j_\mu^- W_\mu^+ + j_\mu^+ W_\mu^-\right) + \qquad \underline{\textbf{CC}}$$

$$+\frac{g}{\cos\theta_w}\left(j_{\mu 3} - \sin\theta_w j_\mu^{EM}\right) Z_\mu + \qquad \underline{\textbf{NC}}$$

$$+ g \sin\theta_w j_\mu^{EM} A_\mu . \qquad \underline{\textbf{EM}}$$

- The equation contains three terms :

  CC : the charged current interactions;

  NC : the neutral current interactions;

  EM : the electromagnetic interactions.

- The constant which multiplies the last term has to be proportional to the electrical charge, to ensure that the photon is NOT coupled to neutral particles ($\hbar = c = 1$) :

  $g \sin\theta_w = q_e = \sqrt{4\pi\alpha}$.

- All the interactions, mediated by the four vector bosons, are expressed in terms of two constants, the electric charge q and the weak angle θ_w.

- However, the model does not predict the values of the two fundamental constants, which must be determined experimentally.

- The numerical relations between the fundamental constants, obtained from low-energy value of α ($\approx 1/137$) and the best measurement of θ_w) ($\sin^2\theta_w \approx 0.232$), are :

$$\frac{1}{\alpha} = \frac{4\pi}{g^2} + \frac{4\pi}{g'^2}; \quad \frac{4\pi}{g^2} = 31.8; \quad \frac{4\pi}{g'^2} = 105.2.$$

- The second term in the equation gives the coupling between the Z and the fermions.

the e.w. theory: summary of formulæ

- The Z coupling is "universal“: it only depends on the electric charge and weak isospin:

$$g_z \equiv \frac{g}{\cos\theta_w}\left[I_{w_z} - Q\sin^2\theta_w\right] =$$
$$= \frac{\sqrt{4\pi\alpha}}{\sin\theta_w \cos\theta_w}\left[I_{w_z} - Q\sin^2\theta_w\right].$$

warning: these formulæ (and all those of this chapter) are simple and elegant, but only valid at 1st order; *the devil lies in the details* (= higher order corrections).

Summary :

$$\boxed{m_w^2 = \frac{\sqrt{2}g^2}{8G_F} = \frac{\pi\alpha}{\sqrt{2}G_F\sin^2\theta_w} = \left(\frac{37.3}{\sin\theta_w}\ \text{GeV}\right)^2}$$

$$\boxed{G_F = \frac{\sqrt{2}g^2}{8m_w^2}}$$

$$\boxed{\tan\theta_w = \frac{g'}{g}}$$

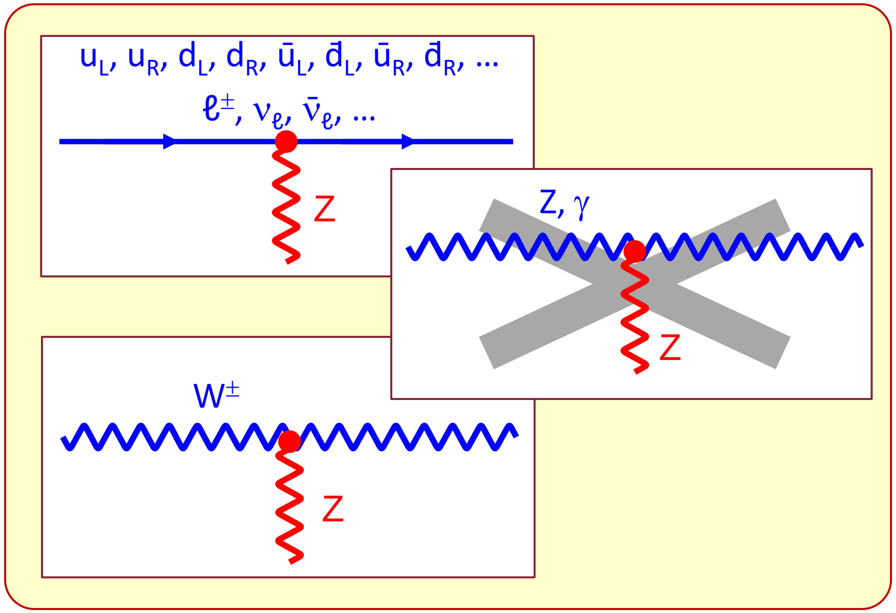
$$\boxed{g\sin\theta_w = q_e = \sqrt{4\pi\alpha}}$$

$$\boxed{\frac{1}{\alpha} = \frac{4\pi}{g^2} + \frac{4\pi}{g'^2}}$$

$$\boxed{m_w = m_z\cos\theta_w}$$

the e.w. theory: NC

- **The Neutral Currents (NC)** have important differences compared to CC.
- NO FCNC, i.e. fermions are only coupled with themselves (e.g. $e^- \leftrightarrow e^-$, $u_{red} \leftrightarrow u_{red}$, NOT $u_{red} \leftrightarrow u_{blue}$, NOT $u \leftrightarrow c$, etc).
- They do not have the simple coupling [$\gamma_\mu(1-\gamma_5)$, i.e. "V – A"], but are a mixture of both left and right couplings.
- The currents of the 1st family (the other families are similar) are :



$$J(\nu) = \frac{1}{2} g_L^{\nu_e} \overline{\nu}_e \gamma_\mu (1-\gamma_5) \nu_e = g_L^{\nu_e} \overline{\nu}_{eL} \gamma_\mu \nu_{eL};$$

$$J(e) = \frac{1}{2} g_L^e \overline{e} \gamma_\mu (1-\gamma_5) e + \frac{1}{2} g_R^e \overline{e} \gamma_\mu (1+\gamma_5) e = g_L^e \overline{e}_L \gamma_\mu e_L + g_R^e \overline{e}_R \gamma_\mu e_R;$$

$$J(u) = \frac{1}{2} g_L^u \overline{u} \gamma_\mu (1-\gamma_5) u + \frac{1}{2} g_R^u \overline{u} \gamma_\mu (1+\gamma_5) u = g_L^u \overline{u}_L \gamma_\mu u_L + g_R^u \overline{u}_R \gamma_\mu u_R;$$

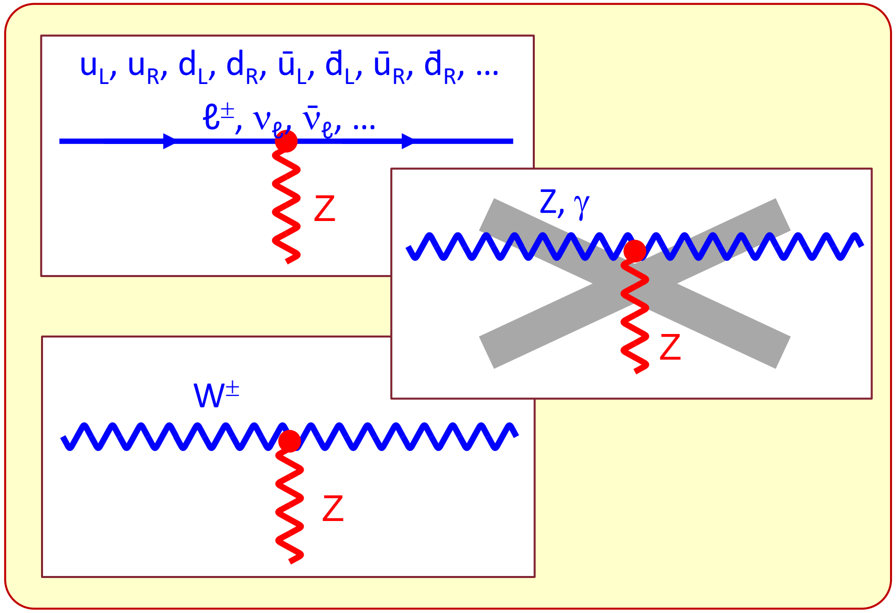
$$J(d) = \frac{1}{2} g_L^d \overline{d} \gamma_\mu (1-\gamma_5) d + \frac{1}{2} g_R^d \overline{d} \gamma_\mu (1+\gamma_5) d = g_L^d \overline{d}_L \gamma_\mu d_L + g_R^d \overline{d}_R \gamma_\mu d_R.$$

i.e. 7 parameters [given in § 7]

the e.w. theory: NC couplings

- In the SM the 7 couplings are equal for the 3 families and functions of only two parameters (α_{em} and θ_w).
- The Z couples with quarks/leptons:
  - ➢ charged fermions, both L and R;
  - ➢ ν's and $\bar{\nu}$'s, even if they have no charge, because they have $I_{Wz} \neq 0$;
  - ➢ $W^\pm$.
- The Z is NOT coupled (<u>in lowest order</u>) to particles with both Q = 0 and I_{Wz} = 0, i.e. the γ and the Z itself (and the gluons).
- In NC processes, the unification of the weak and electromagnetic interactions is particularly evident.
- The following tests have been performed [*those with "➢" will be discussed in these lectures*]:

- parity violation in atoms (scale = eV);
- ➢ DIS ν_μ on electron (scale = MeV);
- scattering of polarized electrons on D_2 (GeV);
- ➢ asymmetries in $e^+e^- \to \mu^+\mu^-$ (from 10 GeV to 200 GeV);
- ➢ DIS ν_μ and $\bar{\nu}_\mu$ on nuclei (several GeV);
- ➢ the measurement of Z parameters themselves.



electroweak results: theory ↔ exp

- All the couplings can be expressed in terms of the values of g and g' (usually α and θ_w).
- The experiments measure observables (cross sections, decay rates, …) and compare calculated ↔ measured quantities.
- The calculation is based on a "perturbative series", up to a certain order : lowest order (= "tree level"), subsequent orders (= "radiative corrections").
- The table shows an incomplete set of e.w. results since '70s: **hundreds** of measurements, **no inconsistency** found, **no disagreement**.
- In the following chapters a small part of the measurements will be examined, in the context of their experimental environment (e.g. LEP).
- The overall picture is impressive.

Score

✓ CC processes at low energy : well described by Fermi theory.

✓ NC processes : direct test of unification.

✓ Gauge boson ($W^{\pm}$, Z) existence.

✓ Gauge boson ($W^{\pm}$, Z, γ) coupling.

✓ **Fermion mass generation (Higgs boson existence).**

? **Higgs boson couplings** $^{(*)}$.

✓ Quark mixing and CP violation $^{(*)}$.

? **Neutrino masses.**

? **Neutrino mixing.**

____

$^{(*)}$ *Looks OK, with some possibility of surprises.*

QCD

- The **color** quantum number had been introduced [*see § 1*] to explain the **presence of the Δ^{++}**, in apparent violation of the Pauli principle;
- color is also necessary to explain the value of **R** [$=\sigma_{hadrons}/\sigma_{\mu+\mu-}$] in e^+e^- interactions, which shows an excess, exactly of a factor 3 [*see § 3*];
- in a similar way, it is necessary to explain the decay rate **$\pi^0 \to \gamma\gamma$** [*next slide*].
- all these observations have convinced the physicists that the color is at the core of the dynamics of the hadrons.
- the theory must also explain **confinement** (= no free quarks) and **asymptotic freedom** (= scaling at high Q^2).
- the modern **QCD** is a gauge theory based on the symmetry $SU(3)_{color}$, mathematically equivalent to $SU(3)_{flavor}$, but based on a completely different dynamics.
- the carriers of the force are 8 colored massless vector (= spin 1) bosons, called **gluons** [*"glue" as an example of a strong force with short range*].
- compared to QED, the differences are in the behavior of the fields :

	"matter" = fermions			"fields" = bosons				
	name	spin	"charge"	name	spin	"charge"	mass	self-coupling
QED	leptons+quarks	½	yes	γ	1	no	no	no
QCD	quarks	½	yes	**8 gluons**	1	**yes**	no	**yes**

QCD : π^0 decay

[the π^0 decay is an e.m. process, NOT a strong one; we discuss it here because it critically depends on the number of colors, i.e. on a QCD parameter.]

- An independent test of the color charge of the quarks comes from a completely different measurement, the **π^0 decay**;
- compute the decay amplitude, by introducing an (a-priori unknown) arbitrary normalization factor "N_c" :

$$\langle \gamma\gamma | \mathbb{H}_{em} | \pi^0 \rangle = \left\langle \gamma\gamma \left| \mathbb{H}_{em} \right| \frac{(u\bar{u} - d\bar{d})}{\sqrt{2}} \right\rangle \propto f_\pi \frac{N_c}{\sqrt{2}} \left(\frac{4}{9} e^2 - \frac{1}{9} e^2 \right);$$

where f_π is the decay constant of the π^0, which is related to the wave-function overlap of the quark and antiquark;

- the full computation gives(*) :

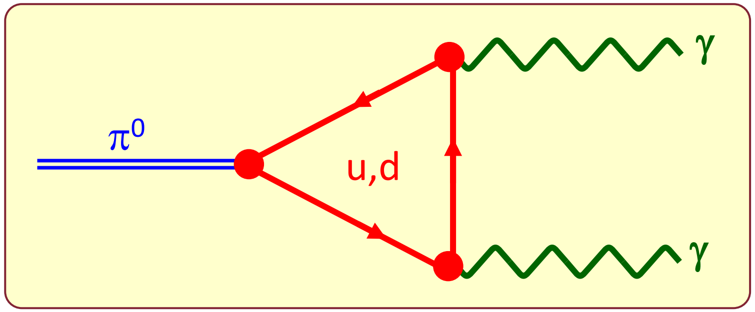
$$\Gamma(\pi^0 \to \gamma\gamma)\Big|_{theo.} = \left(\frac{q_u^2 - q_d^2}{e^2} \right)^2 \frac{N_c^2 \alpha^2 m_\pi^3}{32\pi^3 f_\pi^2} = 7.64 \left(\frac{N_c}{3} \right)^2 \text{ eV};$$

$$\Gamma(\pi^0 \to \gamma\gamma)\Big|_{exp.} = (7.84 \pm 0.6) \text{ eV};$$

$$\to N_c = 2.96 \pm 0.11,$$

not compatible with N_c=1, but with the QCD prediction N_c = 3.

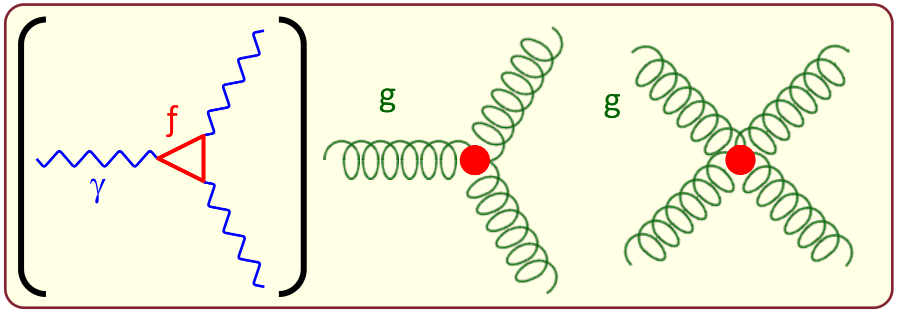
(*) warning : "$32\pi^3 f_\pi^2$" depends on the definition of f_π; also 16 or 64 in the literature.



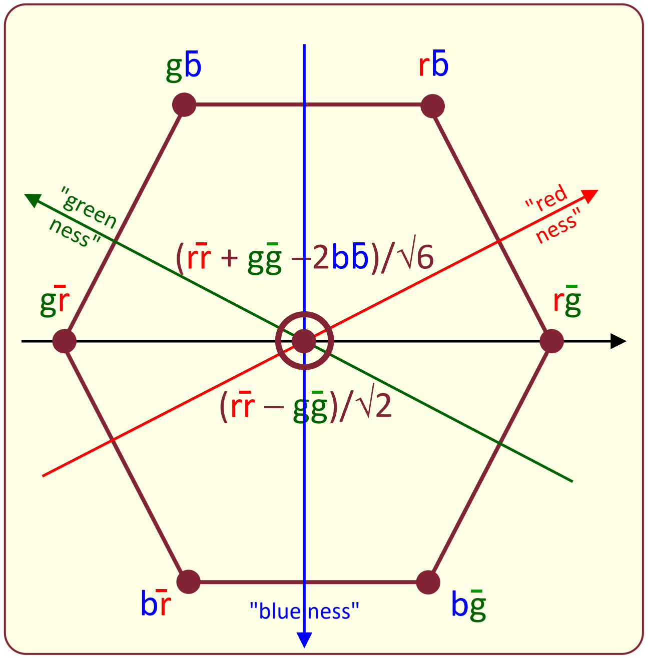
QCD : color

The **color** is a charge, equivalent to the electric charge :

- it is exactly conserved in the processes;
- it obeys the "video-display" rgb rules (e.g. red + green = yellow, so yellow = anti-blue);
- a gluon carries two colors (which is equivalent to an anti-color, see above);
- gluons are colored, therefore **self-coupled**; the vertex with three bosons is allowed in QCD, while in QED it happens only on higher orders (with a triangle of fermions); also 4-gluons verteces are allowed;



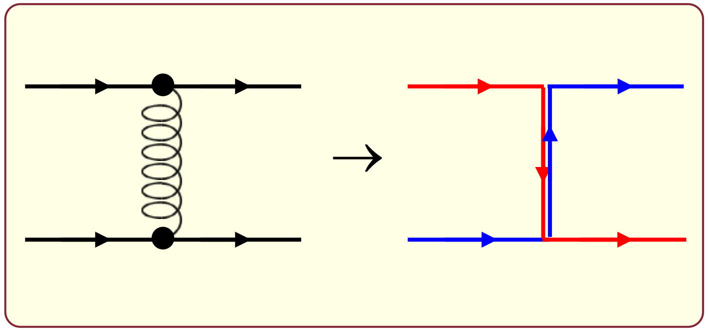
- the number of gluons (8) comes from the number of generators of SU(3) (Gell-Mann matrices [*see* § *1*]);
- similarly, it comes from the independent combinations of two rgb ($r\bar{r}$, $g\bar{g}$, $b\bar{b}$, $r\bar{b}$, $r\bar{g}$, $g\bar{b}$, $g\bar{r}$, $b\bar{r}$, $b\bar{g}$), after removing the singlet combination [$(r\bar{r} + g\bar{g} + b\bar{b}) / \sqrt{3}$];
- the gluon octet is similar to the $q\bar{q}$ one :

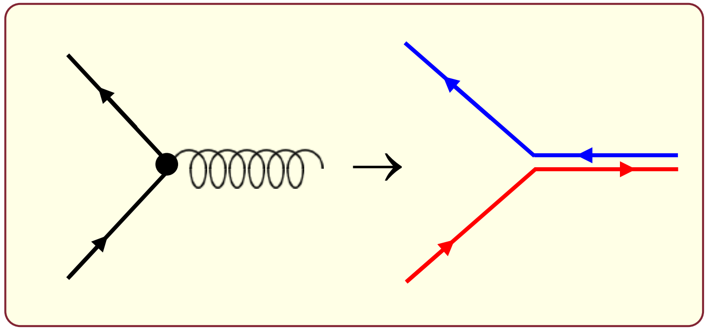


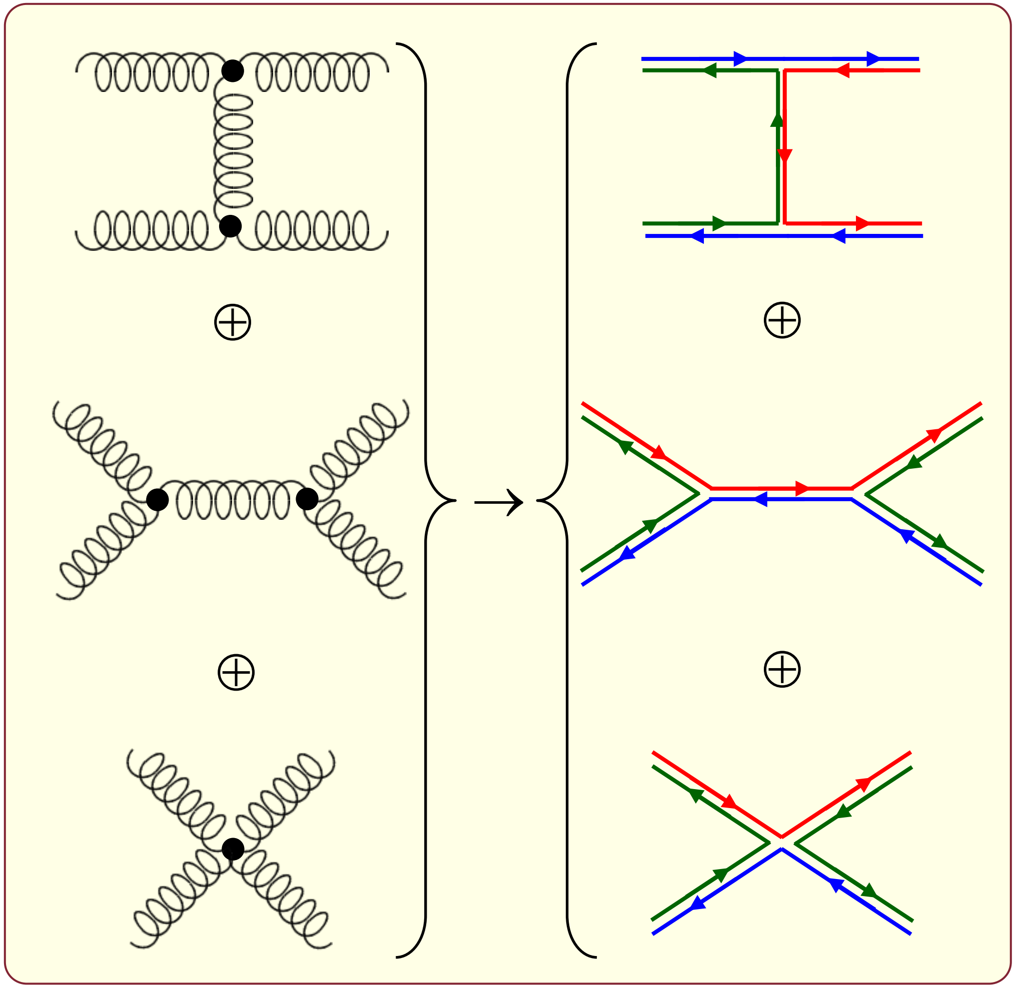
QCD : gluon color

Examples of quark-gluon diagrams with emphasis on color conservation :

- in this page, color is "QCD-color";
- only one shown of the many color permutations;

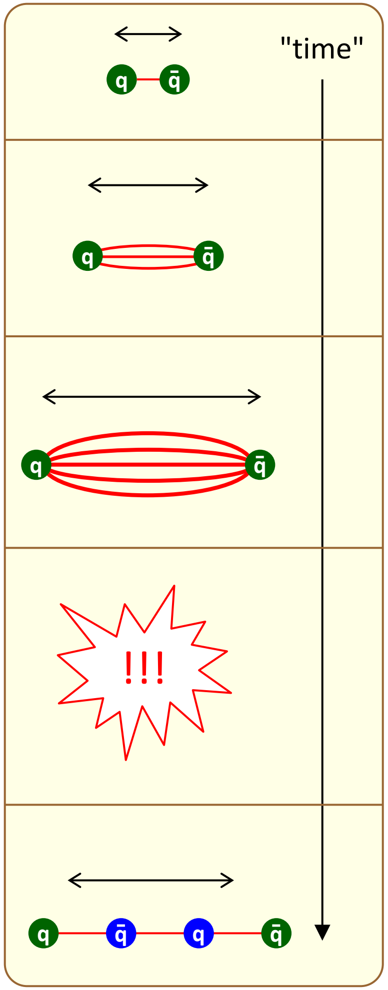






QCD : confinement

- The color does NOT manifest directly in an observable property of the particles (something like a "red" particle has never been observed).
- The standard explanation of this fact requires that only "white" ("*color singlets*") states be physically existent.
- The consequence is that quarks and gluons themselves cannot be observed as free states (***confinement***); they exist only inside "molecules" (= hadrons)
- The mathematical formalism of QCD gives an account for that.
- Some naïve classical models, with similarity to springs and magnetism (the "broken magnet") are often quoted.
- An important consequence is that partons (quarks and gluons), created in e^+e^- or hadronic scattering, must undergo a complicated mechanism which finally produces only color singlets (hadrons) in a spray of particles (***jets***).



QCD : asymptotic freedom

- From the study of the DIS, we have learned that at high Q^2, the projectile "sees" smaller scales inside the nucleon.
- At small distances the force between quarks and gluons is apparently smaller and smaller : the quarks behave as free objects; the scattering onto free quarks is the origin of the Bjorken scaling [§ 2].
- This effect is called **asymptotic freedom** [Gross, Politzer, Wilczek – Nobel Prize 2004].
- With increasing distance among the quarks, the intensity of the strong force increases, keeping the quarks "confined" in the nucleon.
- As we have already mentioned, at some distance the available energy becomes sufficient to create a new quark-antiquark pair, eventually leading to the production of new hadron(s), but PREVENTING the emission of quarks as free particles.
- Summary : among quarks there exists a "color" field. The gluons that mediate this force act as additional sources of the color field ("*gluons are non-abelian*"). The gluon-gluon interaction "pulls" the lines of force of the color in a narrow tube, a sort of a "string", similar to a spring, whose tension ("= *potential energy*") increases with length.

David Gross | David Politzer | Frank Wilczek

QCD : hadrons

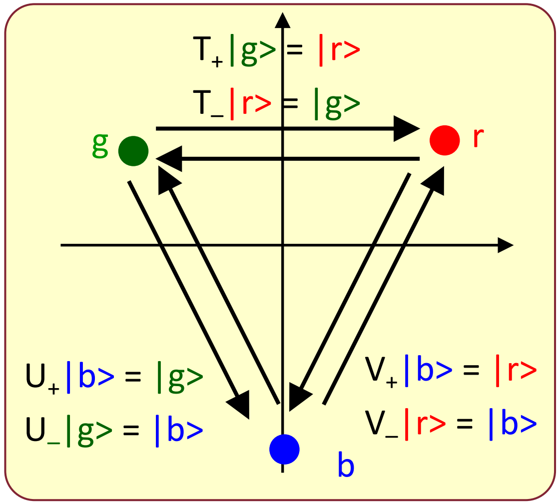
remember §1, "color"

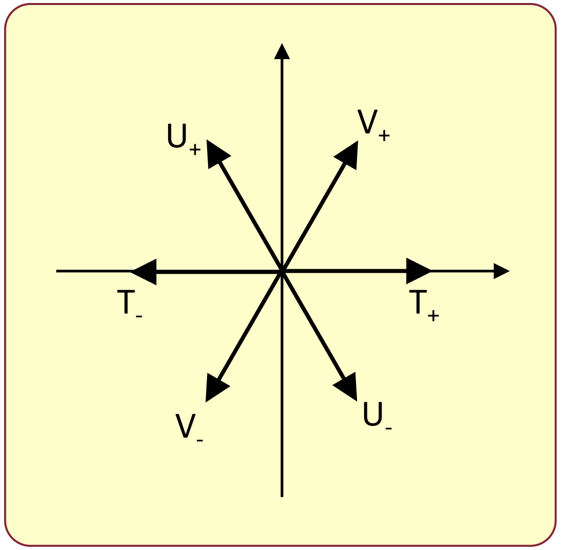
here "rgb" are quarks with the appropriate flavor.

The particles of the theory are built from the SU(3) rules.

Technically, introduce the ladder operators $T_\pm$, $U_\pm$, $V_\pm$ [§1 + **the QCD dynamics**]:

- mesons are **color singlets** [a q$\bar{q}$ pair with symmetric wave function: $(r\bar{r} + g\bar{g} + b\bar{b}) / \sqrt{3}$];
- baryons are also color singlets [qqq with antisymmetric w.f.: $(rgb + gbr + brg - grb - bgr - rbg) / \sqrt{6}$];
- [mesons are their own anti-particles]
- [anti-baryons are $\bar{q}\bar{q}\bar{q}$ states with the same rules]
- *puzzling : there are also other possible color singlets : $q\bar{q}q\bar{q}$, $qqqq\bar{q}$* [*next slide*]*, or glue-glue bound states …*
- *no (QCD-based) rule forbids their existence; in the past there have been (well founded ?) claims of discovery (tetraquarks, pentaquarks, glueballs, …).*

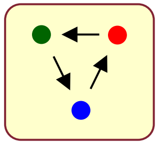




QCD : SU(3)$_C$

C	$\mathcal{B}$	I_{Cz}	Y_C	C	$\mathcal{B}$	I_{Cz}	Y_C
r	+⅓	+½	+⅓	$\bar{r}$	−⅓	−½	−⅓
g	+⅓	−½	+⅓	$\bar{g}$	−⅓	+½	−⅓
b	+⅓	0	−⅔	$\bar{b}$	−⅓	0	+⅔

- mathematically identical to SU(3)$_{flavor}$ (see);
- define a "color isospin" (I_C, I_{Cz}) and a "color hypercharge" (Y_C);
- rgb just names, no connection with ordinary "colors";
- for baryons, ψ_{color} :
  - $\mathcal{B}$ = 1 ⇒ 3-quarks (or 3 antiquarks);
  - normalized;
  - overall color = 0;
  - anti-symmetric (Pauli principle);
- therefore, only one solution :



$$\psi_{color}^{baryons} = \frac{1}{\sqrt{6}}\left(\begin{array}{l} r_1 g_2 b_3 + g_1 b_2 r_3 + b_1 r_2 g_3 + \\ -r_1 b_2 g_3 - b_1 g_2 r_3 - g_1 r_2 b_3 \end{array}\right);$$

The most general quark combination is :

$$\psi_{color} = r^\alpha g^\beta b^\gamma \bar{r}^{\bar{\alpha}} \bar{g}^{\bar{\beta}} \bar{b}^{\bar{\gamma}}; \qquad [\alpha,\beta,\ldots \text{ integer}]$$

define (whithout loss of generality) :

$$m \equiv \alpha+\beta+\gamma \geq n \equiv \bar{\alpha}+\bar{\beta}+\bar{\gamma};$$

$$I_{Cz} = (\alpha-\bar{\alpha})/2 - (\beta-\bar{\beta})/2 = 0;$$

$$Y_C = (\alpha-\bar{\alpha})/3 + (\beta-\bar{\beta})/3 - 2(\gamma-\bar{\gamma})/3 = 0;$$

$$\Rightarrow (\alpha-\bar{\alpha}) = (\beta-\bar{\beta}) = (\gamma-\bar{\gamma}) \equiv p;$$

$$\Rightarrow \alpha+\beta+\gamma-\bar{\alpha}-\bar{\beta}-\bar{\gamma} = m-n = 3p; \quad p \geq 0;$$

$$\Rightarrow \psi_{color} \equiv q^m \bar{q}^n = q^{3p+n}\bar{q}^n = (qqq)^p (q\bar{q})^n.$$

The simplest cases are :

- p = 1, n = 0 → baryons qqq (+ anti–);
- p = 0, n = 1 → mesons $q\bar{q}$;
- many other possibilities NOT forbidden, e.g. (p=n=1; p=0,n=2) → ($qqq\,q\bar{q}$; $q\bar{q}q\bar{q}$);
- searches (and claims...).

QCD: multiquarks ? glueballs ?

Modern remark on tetra- and penta-quarks

M.Karliner et al, Annu. Rev. Nucl. Part. Sci. 68:17-44 (2018) [*emphasis mine*]:

Why do we see certain types of elementary particles and not others ? This question was posed more than 50 years ago in the context of the quark model. Gell-Mann and Zweig proposed that the known mesons were $q\bar{q}$ and baryons qqq, with the quarks known at the time, u (up), d (down), and s (strange), having charges of ⅔, −⅓, and −⅓, respectively. Mesons and baryons would then have integral charges.

Mesons such as $qq\bar{q}\bar{q}$ and baryons such as $qqqq\bar{q}$ would also have integral charges. Why weren't they seen? *They have now been seen*, but only with additional heavy quarks and under conditions that tell us a lot about the strong interactions and how they manifest themselves.

(...)

A look back at the experimental developments in hadron spectroscopy in the new millennium shows that heavy quarks have done it again! After converting us into firm believers in the quark model in the 1970s, heavy quark systems have taught us a new lesson: *Not all hadronic states are minimal quark combinations*. In addition to $q\bar{q}$ mesons, four-quark $qq\bar{q}\bar{q}$ configurations become important, especially near and above the $q\bar{q}$ + $q\bar{q}$ meson thresholds. Similarly, not all baryons are qqq states; $qqqQ\bar{Q}$ configurations also play a role.

Theoretical disputes continue as to whether the observed multiquark configurations are tightly bound tetra- and penta-quarks or loosely bound meson-meson and baryon-meson *molecules*. ***In our opinion***, the case for the latter is stronger. It is also beyond dispute that baryon-baryon *molecules* exist and have been known for a long time as nuclei.

(...)

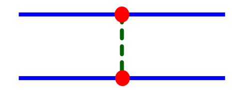
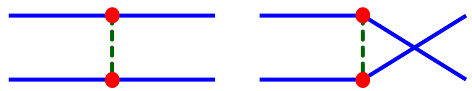
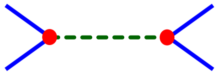
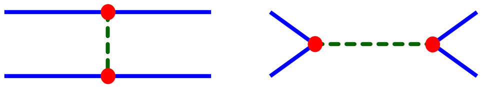
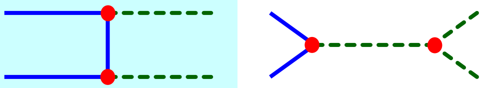
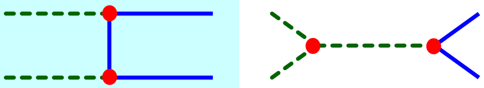
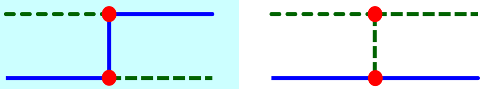
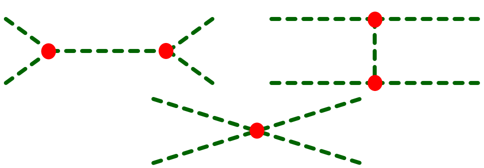
Strong interactions : the QCD game

Process **(qq̄g) (qq̄g) → (qq̄g) (qq̄g)**:

- picture only valid at high Q^2: at low Q^2 hadrons scatter coherently (see § 2-8), → rest of discussion assumes high Q^2;
- 8 cases, according to (qq̄g) [next slide];
- impossible to disentangle on an event-by-event basis (in QED e^+e^+ and e^+e^- not in the same accelerator, while in QCD qq and qq̄ possible, because of valence/sea) [an advantage, but a difficult one];
- therefore all processes mixed together, difficult (≈ impossible) to disentangle on an event-by-event basis: only statistical mixtures measurable;
- weights of stat. mixture are **couplings** at parton level (get from theory) * **PDF** (parametrize/evolve) [a difficult game];
- in hadronic initial states (h.i.s.) the energy at parton level (**ŝ**) is different from energy at hadron level (s); same for **t̂** and **û** ↔ t and u [next slide uses stu, but means ŝt̂û];
- in h.i.s. s from beam energy, but ŝ difficult [§ 8] and different for each event;
- jets, not single partons in final state: in general (q ↔ q̄ ↔ g) not distinguishable, single quarks (e.g. b ↔ u d) difficult;
- t̂−channels much more abundant than ŝ-channels; gluon channels more abundant than quark- : a disgrace for the search of $W^\pm$, Z, H, which mainly come from q-q̄ processes in the ŝ channel.

Conclusion: a rich and difficult game, which requires a lot of events, strong computing power, intelligent analysis.

Strong interactions: 2→2 processes

process	$\left[\frac{d\sigma}{d\Omega} = \alpha_s^2 f(s,t,u)/(9s)\right]$ f(s,t,u)	f(θ=90°) [t=u=−s/2]	diagram(s)	QED equivalent
$qq' \to qq'$ $\bar{q}q' \to \bar{q}q'$	$(s^2+u^2)/t^2$	5		$e^-\mu^- \to e^-\mu^-$ $e^+\mu^- \to e^+\mu^-$
$qq \to qq$ $\bar{q}\bar{q} \to \bar{q}\bar{q}$	$(s^2+t^2)/u^2+(s^2+u^2)/t^2-2s^2/(3ut)$	7+⅓= 7.3		$e^-e^- \to e^-e^-$ $e^+e^+ \to e^+e^+$
$q\bar{q} \to q'\bar{q}'$	$(t^2+u^2)/s^2$	½= 0.5		$e^+e^- \to \mu^+\mu^-$
$q\bar{q} \to q\bar{q}$	$(t^2+u^2)/s^2+(s^2+u^2)/t^2-2u^2/(3st)$	5+⅚= 5.8		$e^+e^- \to e^+e^-$
$q\bar{q} \to gg$	$8/3(t^2+u^2)[1/(tu) -9/(4s^2)]$	2+⅓= 2.3		$e^+e^- \to \gamma\gamma$
$gg \to q\bar{q}$	$3/8(t^2+u^2)\ [1/(tu) -9/(4s^2)]$	$^{21}/_{64}$= 0.3		$\gamma\gamma \to e^+e^-$
$gq \to gq$ $g\bar{q} \to g\bar{q}$	$(s^2+u^2)[9/(4t^2)-1/(su)]$	13+¾= 13.8		$\gamma e^- \to \gamma e^-$ $\gamma e^+ \to \gamma e^+$
$gg \to gg$	$81/8[3-ut/s^2-su/t^2-st/u^2]$	68+$^{11}/_{32}$= 68.3		$[\gamma\gamma \to \gamma\gamma]$

The **lowest order processes of the strong interactions in QCD:**

- s,t,u,θ at parton level ($\hat{s}$, $\hat{t}$, $\hat{u}$);
- q' : q' ≠ q.

——— q or $\bar{q}$;
- - - - γ_{QED} or g_{QCD}.

Strong interactions : the coupling α_s

A semi-classical approach for the QCD potential from experimental data :

- for small distances ($r \to 0$), Coulomb shape, with a stronger coupling α_s (instead of α_{em}) :

  $V(\text{small } r) = - 4\alpha_s / (3 r)$

- at high distances ($r \to \infty$), a linearly increasing function, responsible for confinement :

  $V(\text{large } r) = k\, r$

- all together (see fig) :

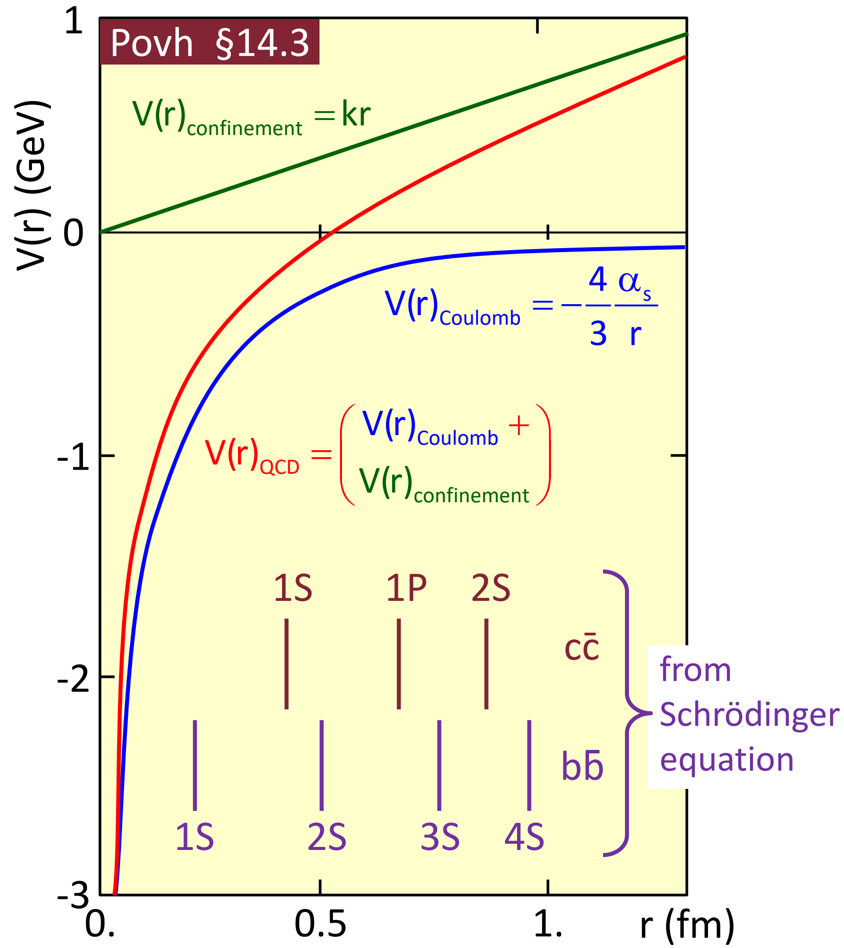
  $V(r) = - 4\alpha_s / (3r) + k\, r$

- parameters α_s and k adjusted to fit data : $\alpha_s \approx 0.15 \div 0.25$, $k \approx 1$ GeV fm^{-1};

- then (numerically) solve the Schrödinger equation and derive (e.g.) the properties of bound states;

- approximation supposed to work better in non-relativistic case, V << m;

- (fair) agreement with reality, especially in the heavy quark sector.



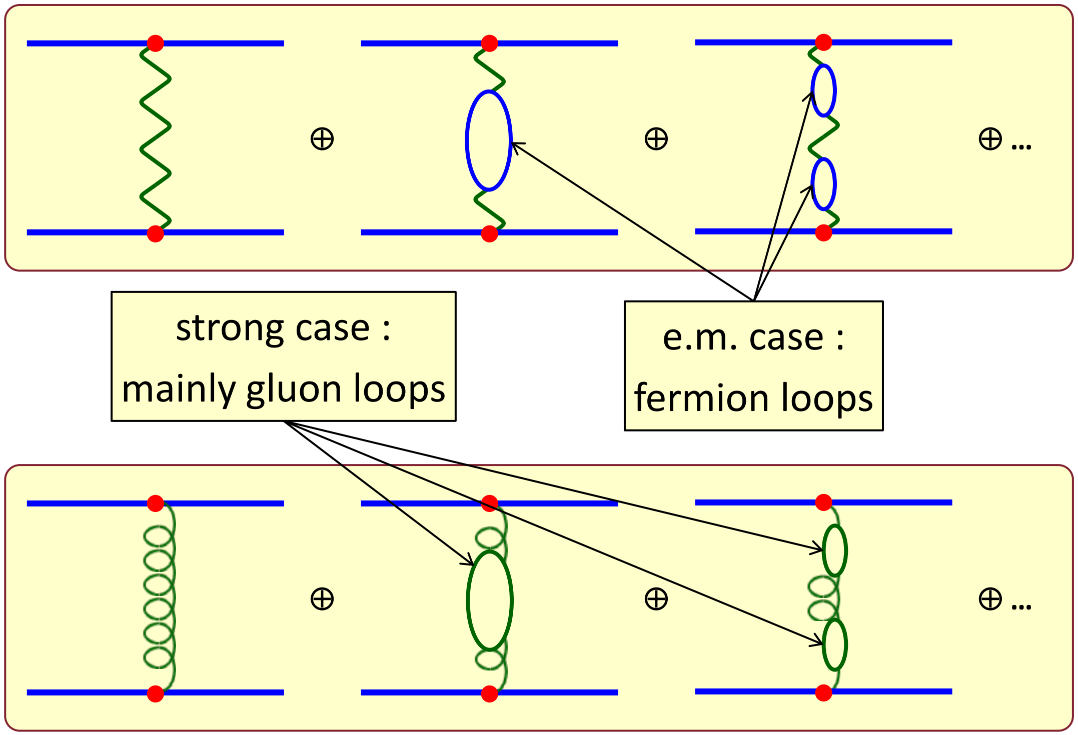
see also [MS, 6.4.3] for V(r) = A ln (r/B).

Strong interactions : $\alpha_s = \alpha_s(Q^2)$

More effective approach for scattering processes : *reabsorb higher orders* into an effective α_s :

→ loops increases for higher Q^2 ;

→ evolution of the coupling α_s from its low-Q^2 value, with standard Feynman techniques;



Important difference $\alpha_s \leftrightarrow \alpha_{em}$:

- higher order loops in α_{em} only due to fermions → *increase* of α_{em} as a function of Q^2;
- instead, since the gluons are self-coupled, loops in α_s mainly due to bosons → *decrease* of α_s with Q^2;
- the formulæ show the "*running*" of α_{em} and α_s with Q^2 :
- (confinement and asymptotic freedom automatically produced).

$$\alpha_{em}\left(Q^2\right) = \alpha_{em}\left(Q_0^2\right) \Bigg/ \left[1 - \frac{\alpha_{em}\left(Q_0^2\right)}{3\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right];$$

$$\alpha_s\left(Q^2\right) = \alpha_s\left(Q_0^2\right) \Bigg/ \left[1 + \alpha_s\left(Q_0^2\right) \frac{11N_c - 2N_f}{12\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right].$$

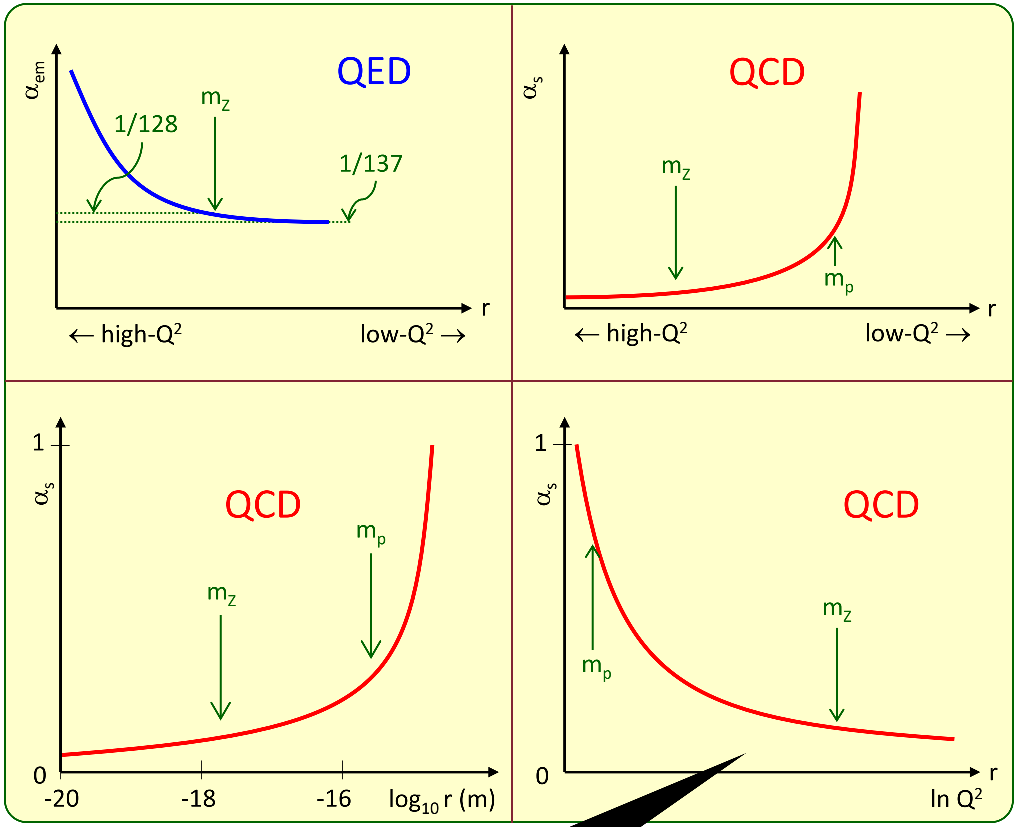
Strong interactions : $\alpha_s \leftrightarrow \alpha_{e.m.}$

$$\alpha_s(Q^2) = \alpha_s(Q_0^2) \Big/ \left[1 + \alpha_s(Q_0^2)\frac{11N_c - 2N_f}{12\pi}\ln\left(\frac{Q^2}{Q_0^2}\right)\right]; \quad N_c = 3; \quad N_f = 3 \to 4 \to 5 \to 6$$

The effective value of α_s decreases as a function of Q^2, thus explaining the nucleons (strongly bound partons) and the DIS (quasi-free partons).

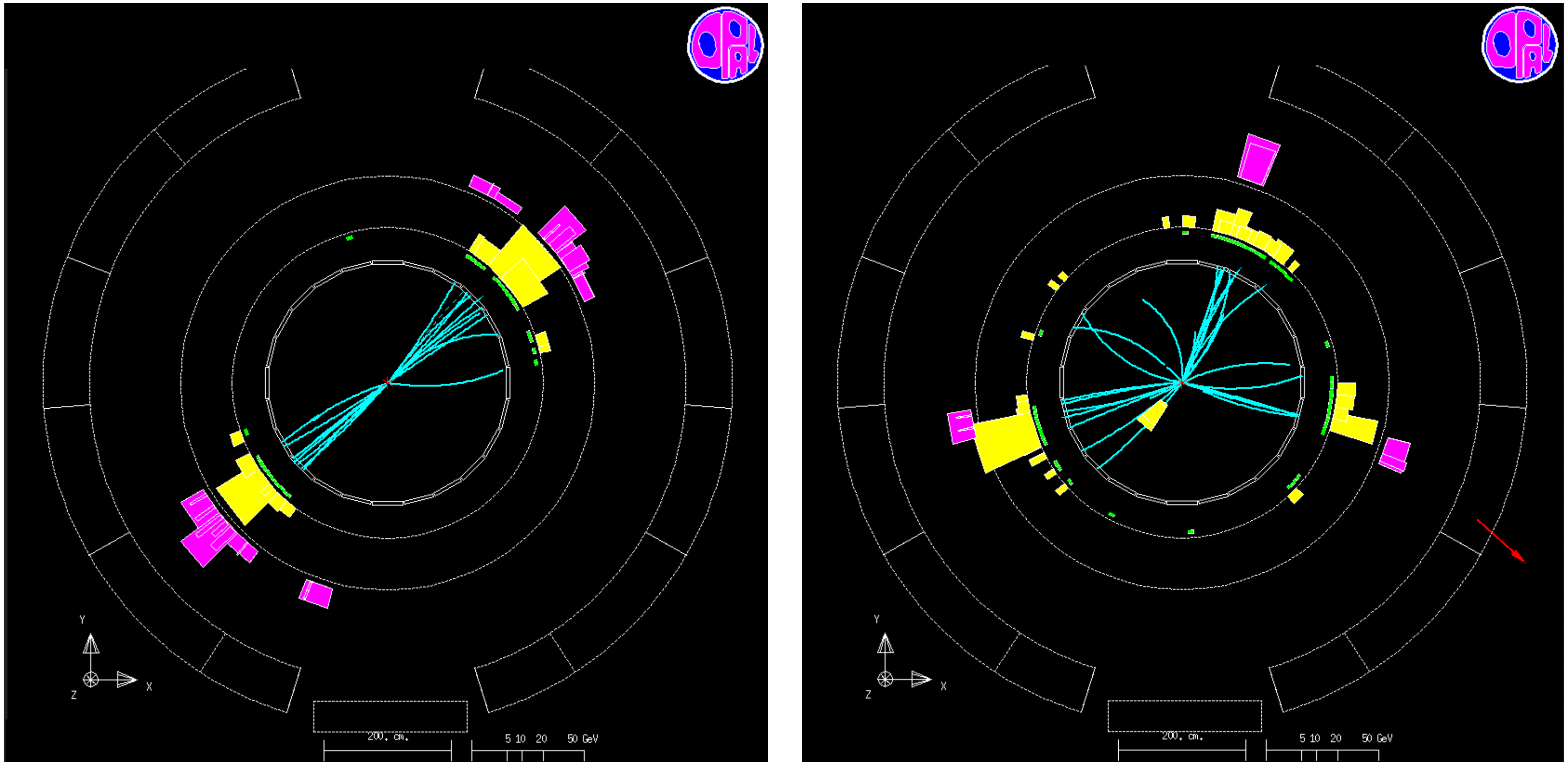
Some care for the coefficients :

- "N_c" = number of colors = 3;
- "N_f" = number of flavors = 6;
- but $N_f = N_f(Q^2)$, i.e. the number of flavors which participate in the interactions at a given Q^2;
- to be simple (but not entirely correct), at a given value of Q^2, only flavors with $(2m_f)^2 < Q^2$ enter in the computation of α_s.



for the exp. data, see later

Hadrons in the final state



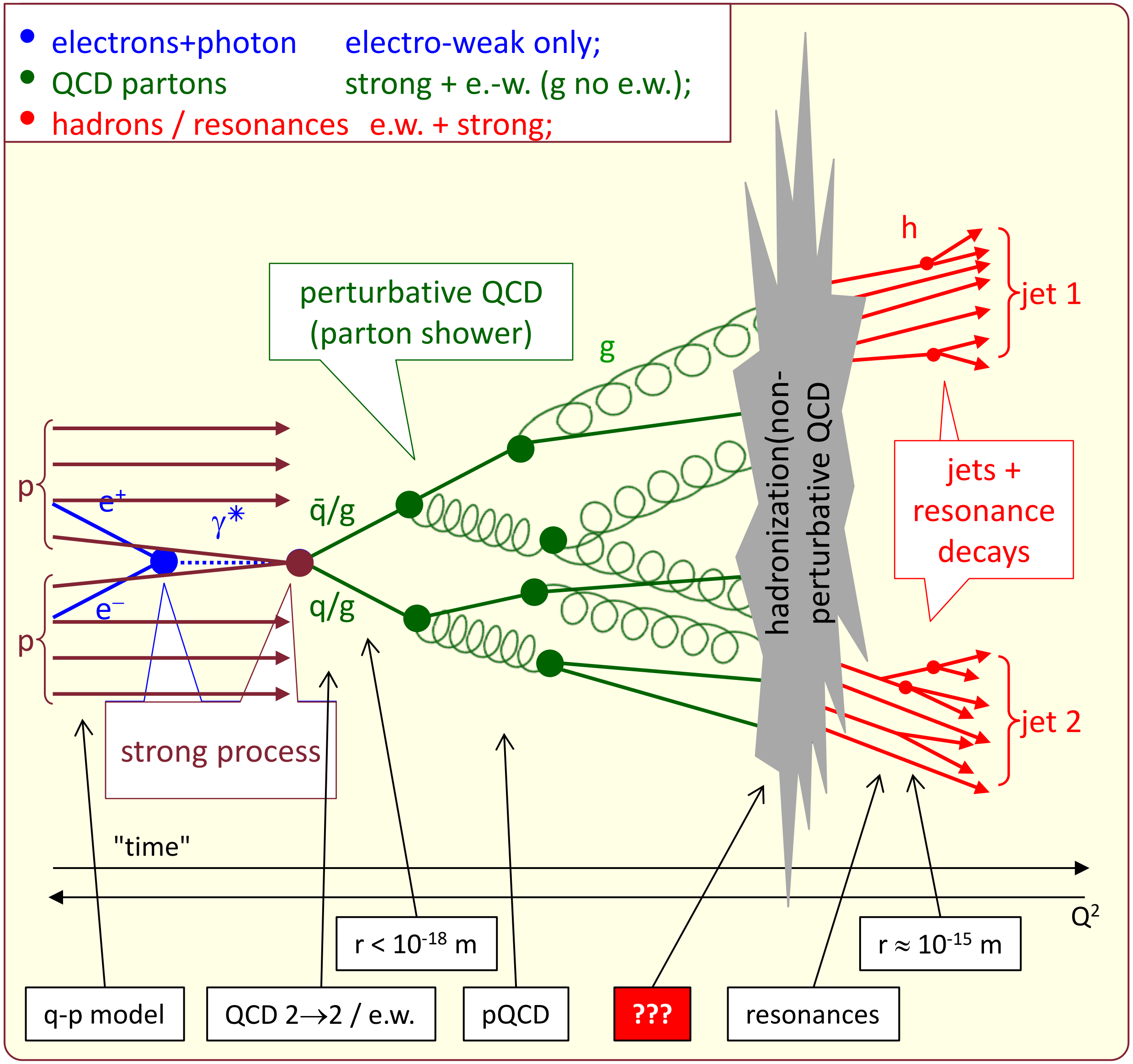
A two-jet and a three-jet event in OPAL.

Hadrons in the final state

Three phases after elementary process :

- **parton shower** : perturbative cascade of (q q̄ g); notice the gluon self-coupling (non-abelian);
- **hadronization** : low-Q^2 parton processes, <u>no well-funded calculation</u>;
- **hadrons** : decays of resonances and emergence of jets.

NB Lot of work in parameterizations, fitting, algorithms, speculations …

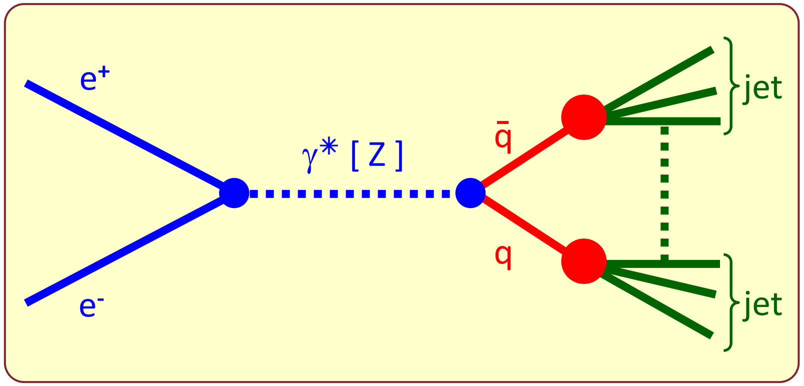


Hadrons in the final state

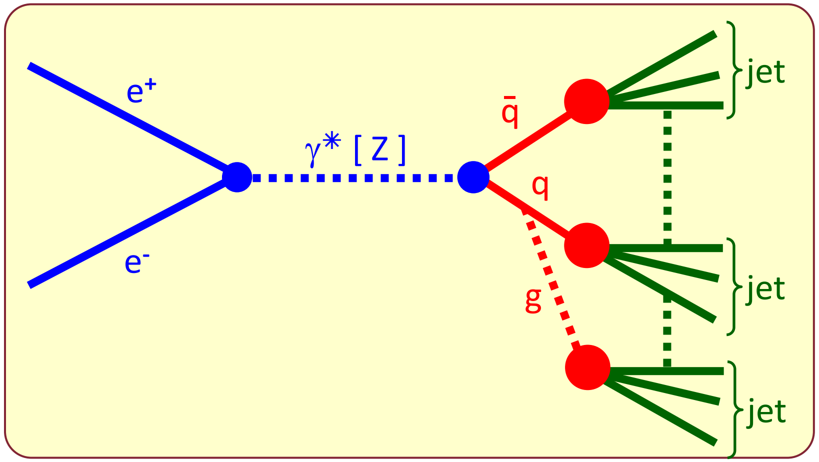
The jets can be identified with the partons of the final state;

- problems :
  - to preserve the color, the two jets in the final state must "talk" each other (e.g. by exchange of gluons);
  - so it is impossible, *strictly speaking*, to assign in a given event a hadron (and hence a jet) to a "father" parton;
- however, as soon as $Q^2 >$ (few GeV)^2, the majority of the events presents two (rarely three) well identified jets, with essentially no ambiguity;
- from the experimental point of view, the situation is relatively simple:
  - *(in practice all)* the events $e^+e^- \to$ hadrons for $\sqrt{s} \geq$ (few GeV) [SPEAR 1975] have two collimated jets of particles, opposite in θ and φ.

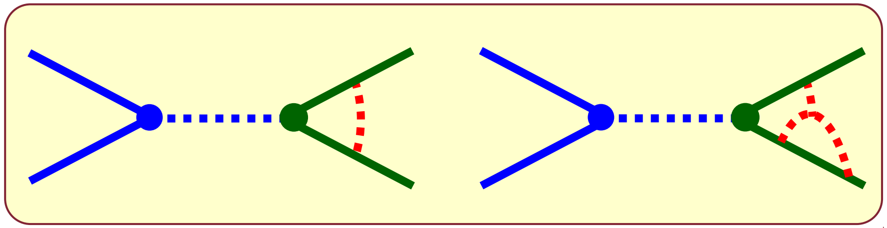
- the direction and momentum of the partons can be reconstructed from the vector sum of the 4-momenta of the hadrons (many subtleties, but the essence is simple);
- it is also possible to measure the "fragmentation function" of the partons : $f(z)$, $z = E_{hadron} / E_{parton}$;
- more discussions in (§ Sp$\bar{p}$S) and (§ LEP).



Hadrons in the final state : three-jet events



- Sometimes, a parton emits a gluon of bremsstrahlung, at an angle and with an energy such as to produce a third jet, well separated from the other two;
- usual litany : "*the fraction of three-jet events* $\propto \alpha_s$"; however :
  - ➢ jets are "ill-defined" quantities : the number and 4-mom. of jets in an event depends on the analysis (the so-called jet-finding algorithm, *JFA*);
  - ➢ the real meaning is that one has to compute (e.g. via montecarlo) the yield of multi-jet events with a given JFA and a given value of α_s; then the comparison with the data, <u>analysed with the same JFA</u>, is a "meas." of α_s (*e.g. too few three-jets in MC wrt data* $\to \alpha_s^{MC} < \alpha_s^{true}$);
- similarly, 4-jet, 5-jet, …;
- with the previous caveats :
  - ➢ σ(2-jet) $\propto \alpha_{em}^2$; σ(3-jet) $\propto \alpha_{em}^2 \times \alpha_s$;
  - ➢ σ(3-jet) / σ(2-jet) $\propto \alpha_s$;
  - ➢ α_s can be measured by the ratio 3-jet/2-jet [also many other ways];
- high value of α_s [> 0.10] $\to$ importance of <u>higher orders</u> of the strong interactions, particularly true for multi-jets final states.



QCD results : quark spin

- If quark-spin = lepton-spin = ½, in $e^+e^- \rightarrow$ jets[1], $d\sigma/d\Omega \propto d\sigma/d\Omega|_{\mu\mu} \propto (1+\cos^2\theta)$;
- however, the heavy quarks have a non-negligible mass; their θ dependence is :

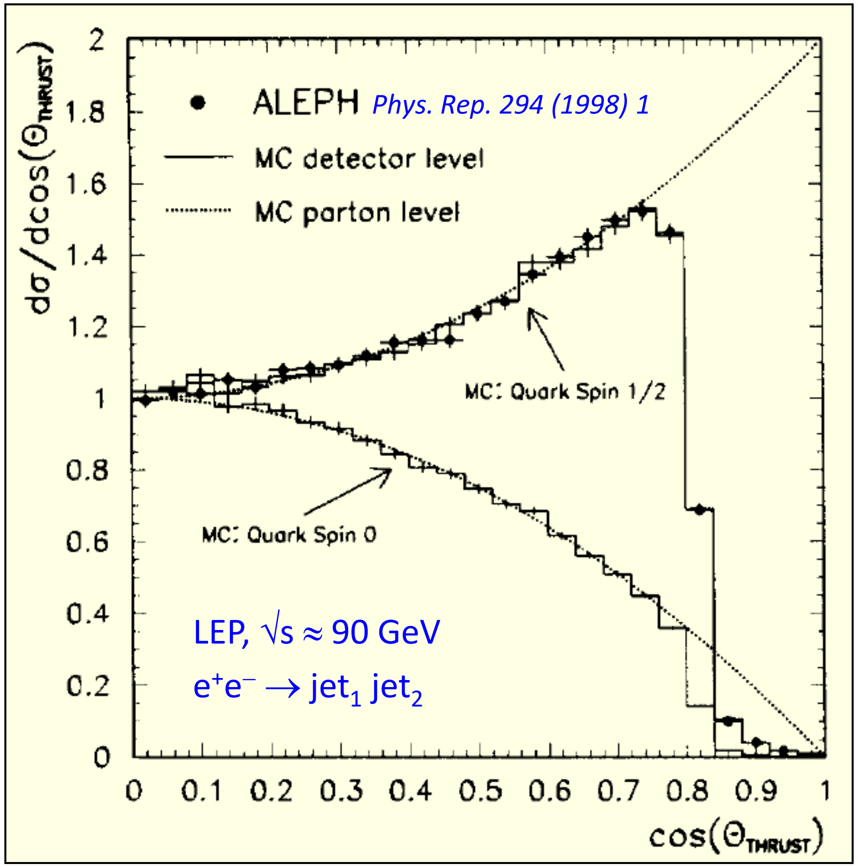
$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 e_q^2 N_c \beta_q^*}{2s}\left(2-\beta_q^{*2}+\beta_q^{*2}\cos^2\theta\right);$$

$$\beta_q^* = \sqrt{1-4m_q^2/s}.$$

- in reality, cannot distinguish jets from q and $\bar{q}$ → exp. ambiguity ($\theta \leftrightarrow 180°-\theta$), ($\cos\theta \leftrightarrow -\cos\theta$) → plot $|\cos\theta|$;
- the value of θ, i.e. the quark direction, is given by the jet axis [see previous pages], usually identified with the "thrust" axis[2];
- after all that, the comparison is possible, and definitive (e.g. ALEPH, 1998).

(1) True for e.m. and not for NC; but at the Z pole, the $q\bar{q}$ asymmetry is small [see § LEP].

(2) The thrust axis is the direction which minimizes the sum of the transverse momenta of the final state particles respect to it.



QCD results : the gluon

Naïvely, the existence (both σ and $d\sigma/d\Omega$) of three-jet events (apart from pedantic caveats on the JFA) is a convincing test of the existence of the gluon.

Other "proofs" include :

- the integral of the structure function $F_2(x)$, which demonstrates that ~50% of the nucleon momentum is NOT carried by charged partons;
- the overall agreement between QCD and measurements, e.g. for hadron colliders;

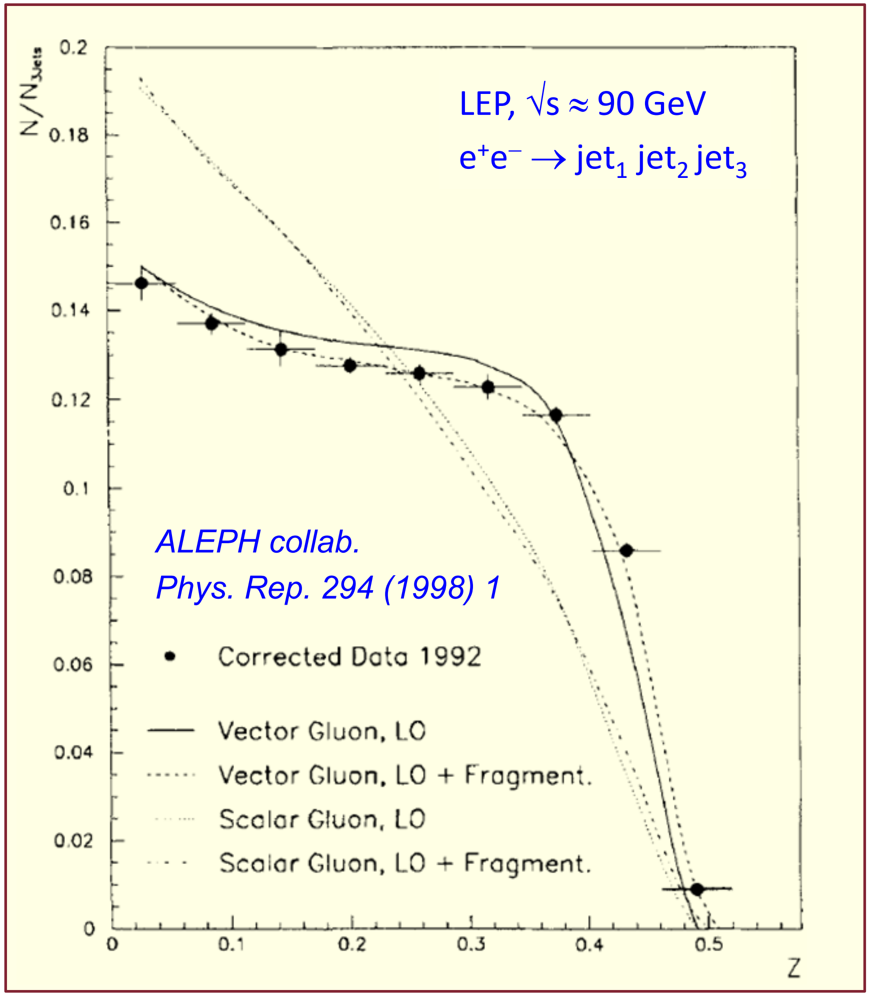
The spin of the gluon is measured :

- in e^+e^-, the third jet in three-jet events comes from gluon brem (theory : 75%);
- after ordering the jets according to energy, the variable

$$Z = 2\,(E_2 - E_3)\,/\,\sqrt{3s}$$

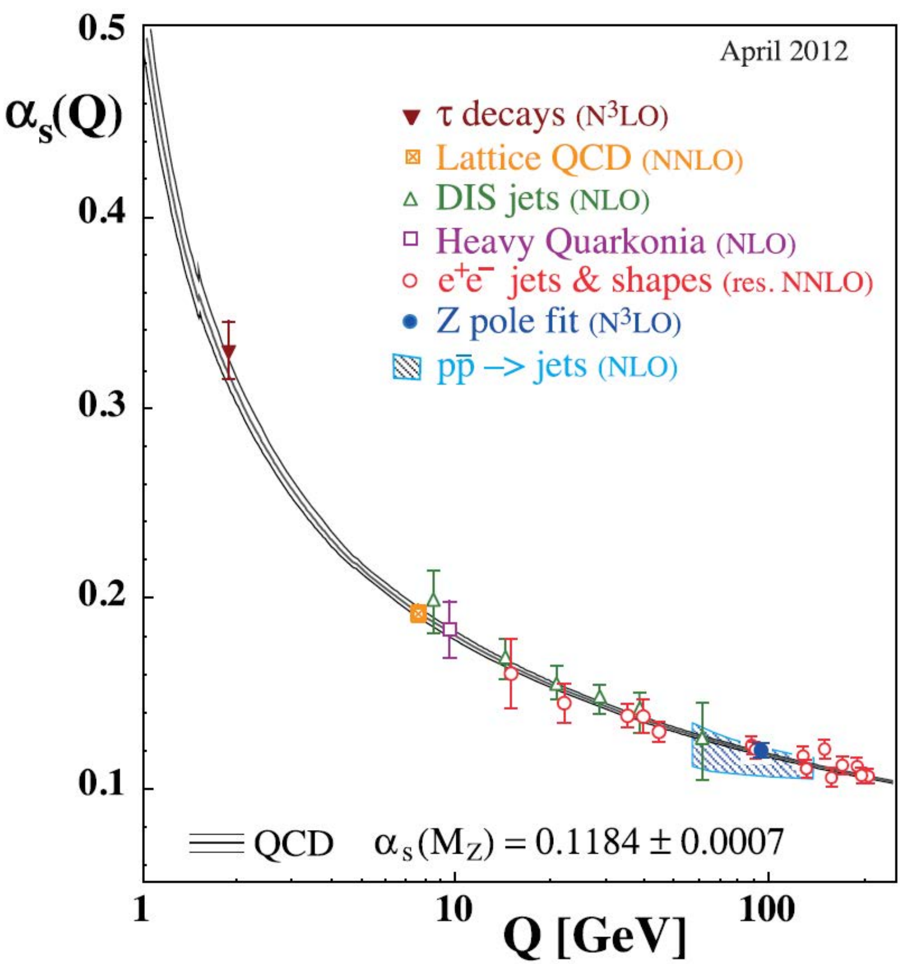
is sensitive to the gluon spin value.

- OK !!! (e.g. ALEPH, notice the quality of the result, insensitive to fragmentation) ["vector"/"scalar" : spin 1/0].



QCD results : the running of α_s

- Actually the running of $\alpha_s = \alpha_s(Q^2)$ has been shown, by measuring the strength of the coupling at different Q^2.
- The data of the figure show a variation of 100 in $\sqrt{Q^2}$, which ranges from τ decay to jets at LHC energies.
- The measurements are compared with predictions, normalized to the value with smallest error, i.e. at $Q^2 = m^2(Z)$ [only the "running" can be computed in QCD, not the abs. value].
- The funny acronyms (N^3LO, NNLO) refer to the computations : they are performed at a given order of Feynman diagrams : NLO = "next to leading order", NNLO = "next to next ..."...
- *What does "Lattice QCD" mean ?*

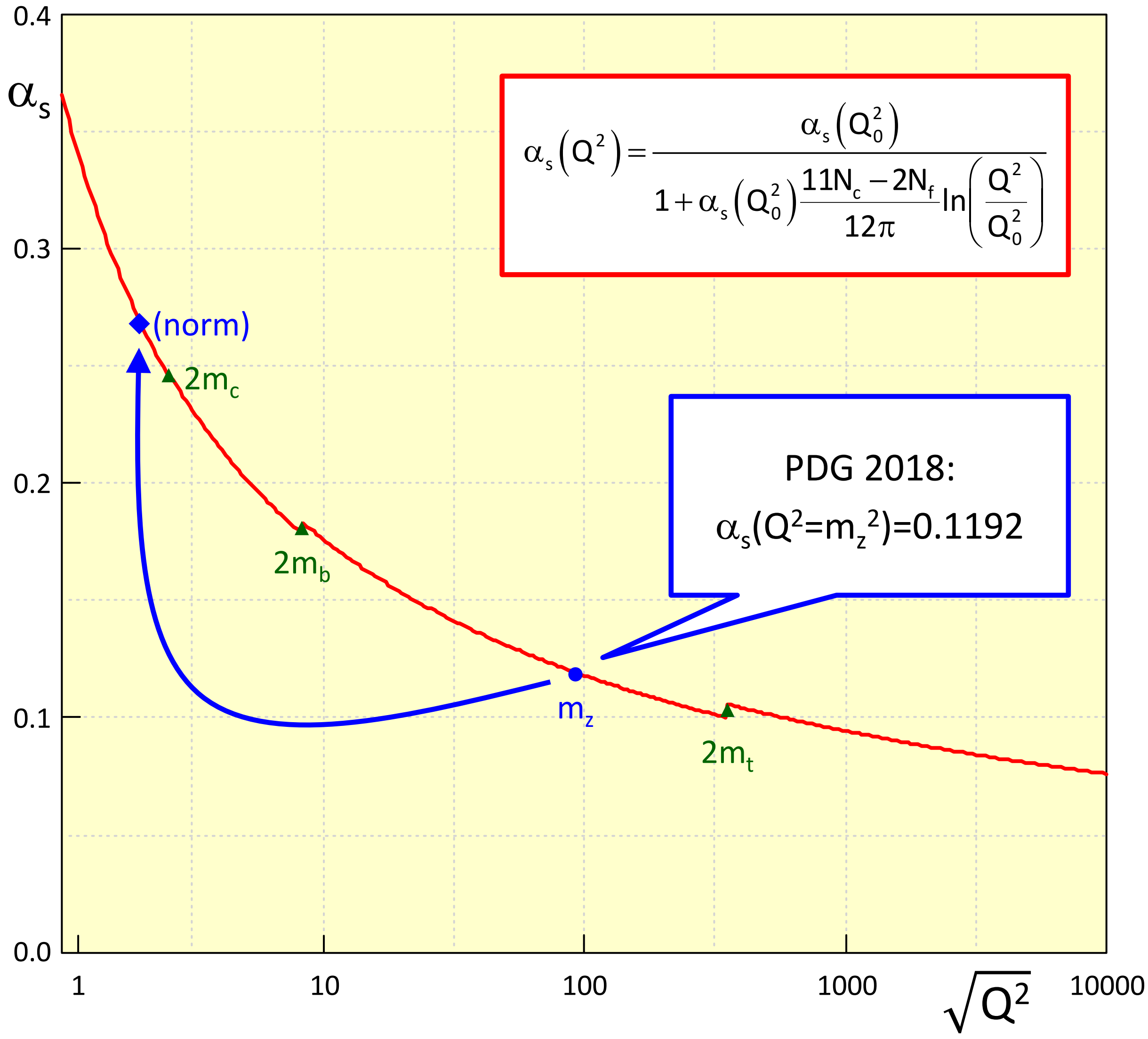


$$\alpha_s\left(Q^2\right) = \alpha_s\left(Q_0^2\right) \Big/ \left[1 + \alpha_s\left(Q_0^2\right)\frac{11N_c - 2N_f}{12\pi}\ln\left(\frac{Q^2}{Q_0^2}\right)\right].$$

QCD results : $\alpha_s = \alpha_s(Q^2)$

***do NOT use** for real computations, only useful as an exercise !!!*

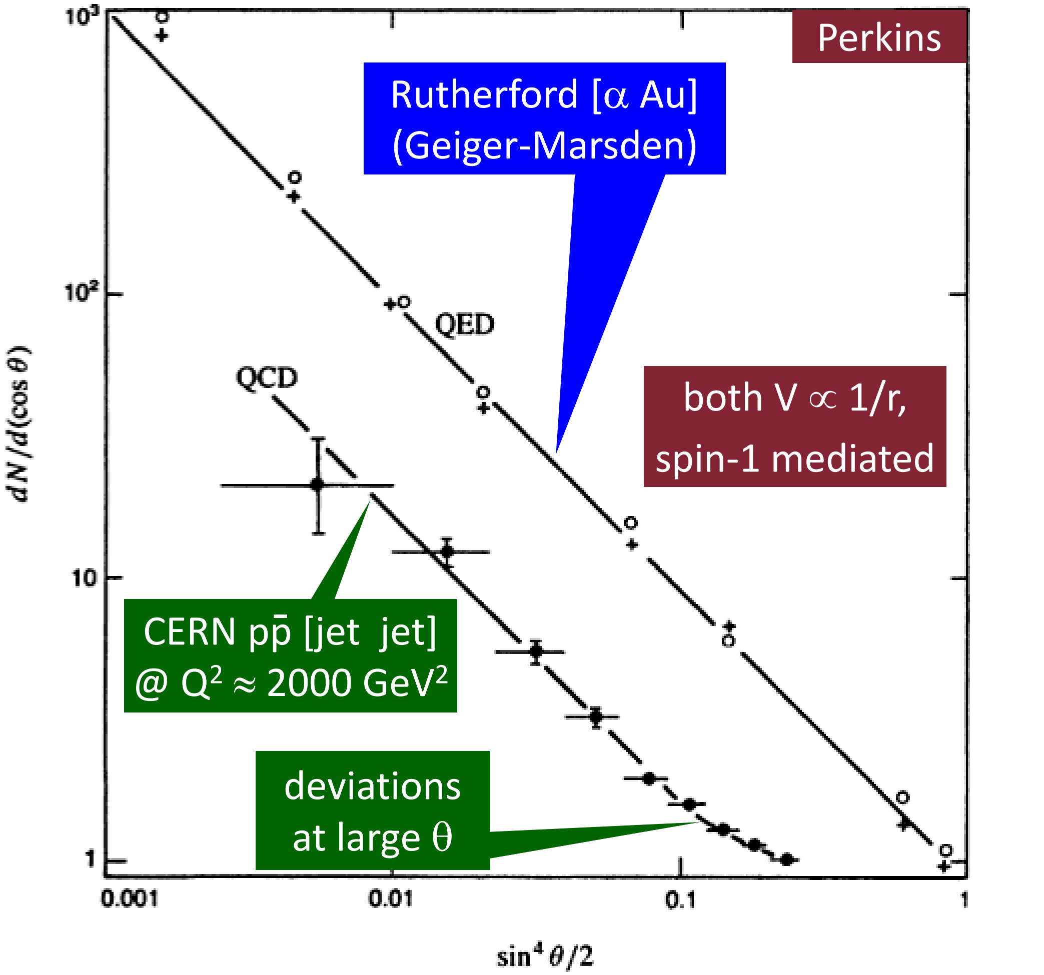
- A "home-made" plot, just for understanding;
- notice the steps at $\sqrt{Q^2} = 2m_q$, clearly an artefact of the approx.



QCD results : angular distribution

Finally, an angular comparison is shown between QED and QCD :

- upper : Rutherford scattering (QED) in the famous Geiger-Marsden plot; the dependence $\propto 1 / \sin^4(\theta/2)$ is evident;
- lower (arbitrary normalization): the same angular plot for $p\bar{p}$ QCD jets at $Q^2 \approx 2000$ GeV^2;
- [notice that, one century ago, it was not customary to show errors on the plots; *maybe in good ole time, they did not make errors*].



"Grand unification bSM" ?

Two curiosities on α_s.

- Since $\alpha_s > 0$ [*any objection ?*], it follows

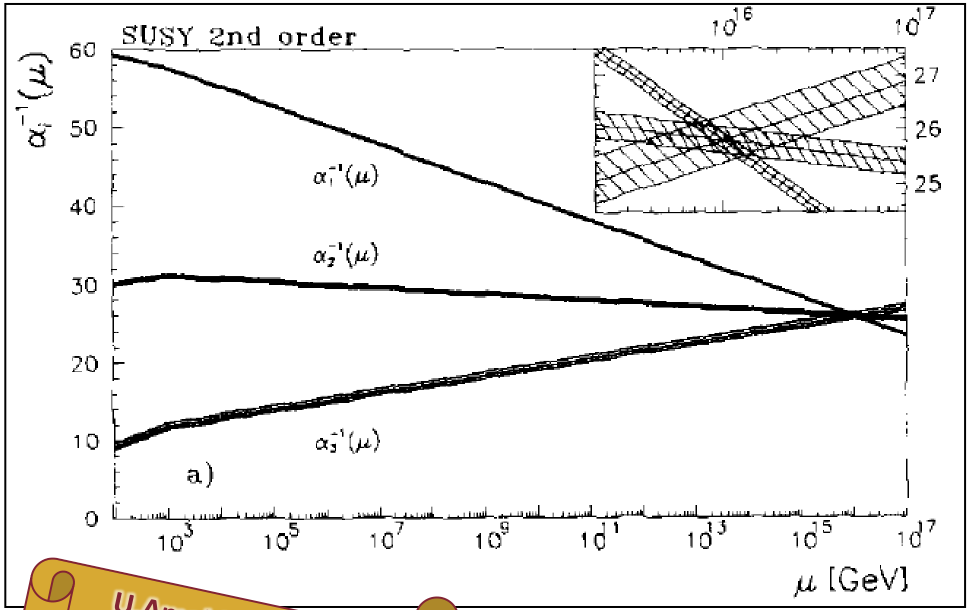
  $11\ N_c - 2\ N_f > 0 \rightarrow$

  $N_f < 11\ N_c / 2 = 16.5;$

  i.e. an upper limit on the number of flavors; after the LEP measurement of Γ_Z, the argument has lost importance, even though there is a logical possibility (lots of heavy flavors with heavy neutrinos …);

- Since $\alpha_s = \alpha_s(Q^2)$ is decreasing, while α_{em} is increasing, do they cross each other ? and, at this value of Q^2, what happens to gravity ?

  It turns out that a model "beyond the Standard Model" (SUSY) predicts that, at $\sqrt{Q^2} \approx 10^{15}$ GeV (μ in the fig.), the three couplings [$SU(3)_c \otimes SU(2)_L \otimes U(1)$] have the same value, therefore suggesting "grand unification".



U.Amaldi et al., Phys.Lett. B260, 447.

Notice that in the SM the three constants all run, but badly miss each other (!!!).

References

1. e.g. [BJ, 13], [YN2, 1], [YN2, 7];
2. the e-w theory is fully discussed in [IE];
3. QCD experimental tests in [BJ, 14].

End of chapter 6

Particle Physics - Chapter 7
High energy ν interactions

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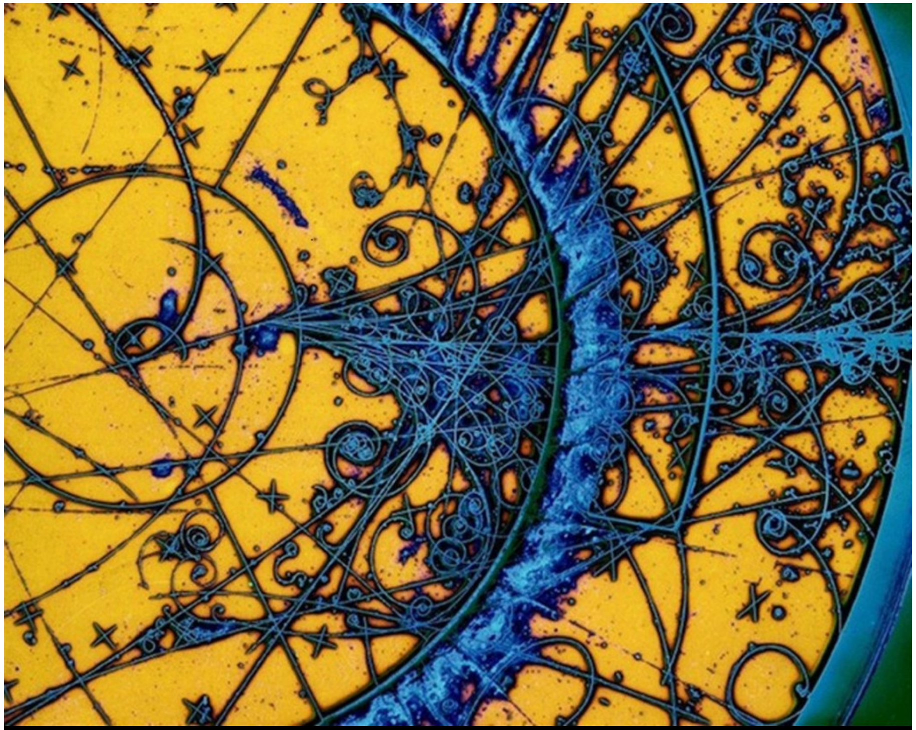
AA 18-19

last mod.
8-Apr-19

7 – High energy ν interactions

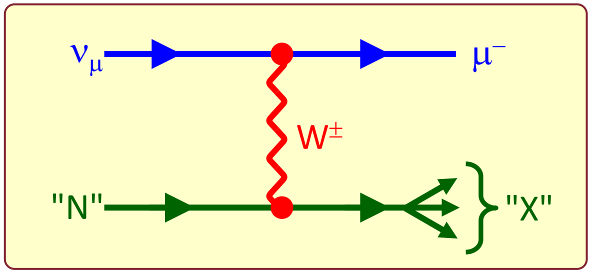


A ν interaction in BEBC

[*original in bw, the colors are an artistic invention*]

High energy ν interactions

After 1960, the <u>accelerator</u> production of ν-beams of high intensity and high energy has led to a dramatic development of our understanding of weak interactions.

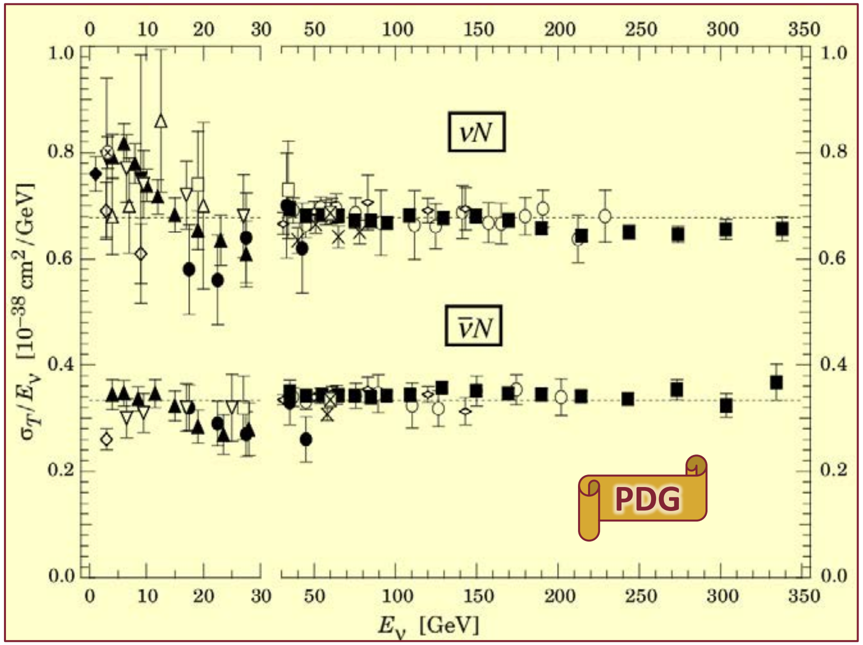


"N" and "X" are all the relevant hadrons/quarks /systems [many different cases]

It is important to explain, albeit in a schematic way, what are the key points to realize a scattering experiment ν-hadrons :

$\sigma(\nu N) = kE_\nu$; $k \approx 0.67 \times 10^{-38}$ cm^2/GeV;
$\sigma(\bar{\nu} N) = k'E_\nu$; $k' \approx 0.34 \times 10^{-38}$ cm^2/GeV.

- The neutrino cross-sections are very small (for E_ν = 1 GeV, $\sigma(\nu N) \sim 10^{-38}$ cm^2, while for the same energy $\sigma(pp) \sim 10^{-26}$ cm^2.
- Beams, detectors, experimental setups have to compensate (bulky, intense, expensive …)



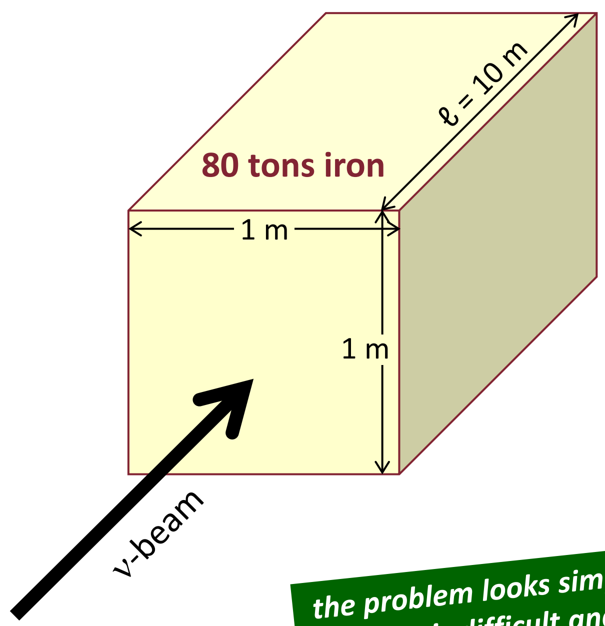
Q. : from the plot, it seems that ($\sigma_{cc} \propto E_\nu$); why ? it looks ugly (actually impossible, because of high energy divergences ("unitarity violations"). [*Wait and see …*]

High energy ν interactions: problem

Problem. How many 1-GeV ν's are necessary to produce 100 interactions in a detector of "reasonable" size and material (e.g. iron, $1 \times 1 \times 10$ m^3) ?

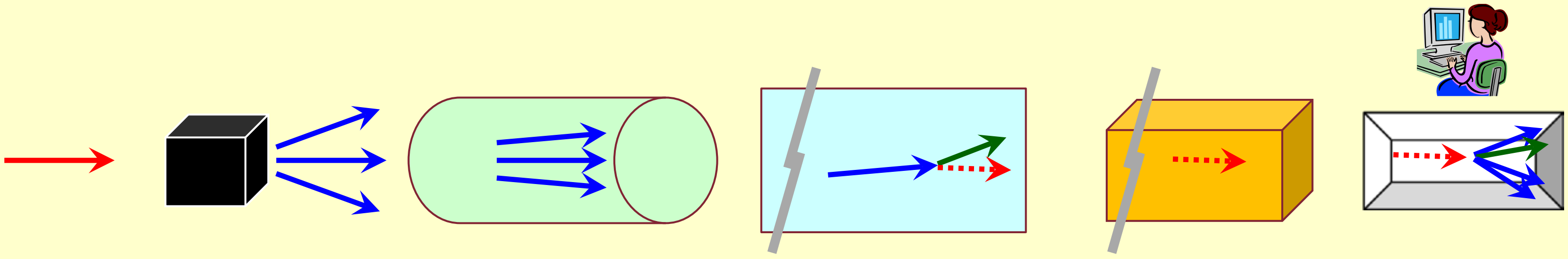
- Interaction probability $\mathcal{P}$ for 1 ν :
  - σ = cross section @ 1 GeV,
  - ℓ = length of traversed material,
  - M = nucleon mass,
  - n = [$N_{nucleons}$ per unit volume] = $= m_{detector} / (M\, V_{detector}) = \rho_{Fe} / M$;
  - $\mathcal{P} = \sigma\, n\, \ell = \sigma\, \rho_{Fe}\, \ell / M =$ [MKS] $\approx (0.7\times10^{-42}) \times (7.9\times10^{3}) \times (10) / (1.7\times10^{-27}) =$ $\approx 4\times10^{-13} \times (\rho_{Fe}/\rho_{H2O}) \times (\ell/1\text{ m}) =$ $\approx 3.2\times10^{-11}$.

- i.e. we need 30 billions ν's, in order to get one interaction in 10 meters of iron !
- Other used quantities : $\lambda_{int} = M / (\rho\sigma)$ = interaction length, the length of material to be traversed by a beam, to have a reduction 1/e of its intensity [*compute it in our case*].



the problem looks simple, but the solution is difficult and expensive

The ν beam : schema



p 450 GeV | target (Cu, Be) | π,K,… | collection + magnetic selection | decay tunnel | absorber | detector(s)

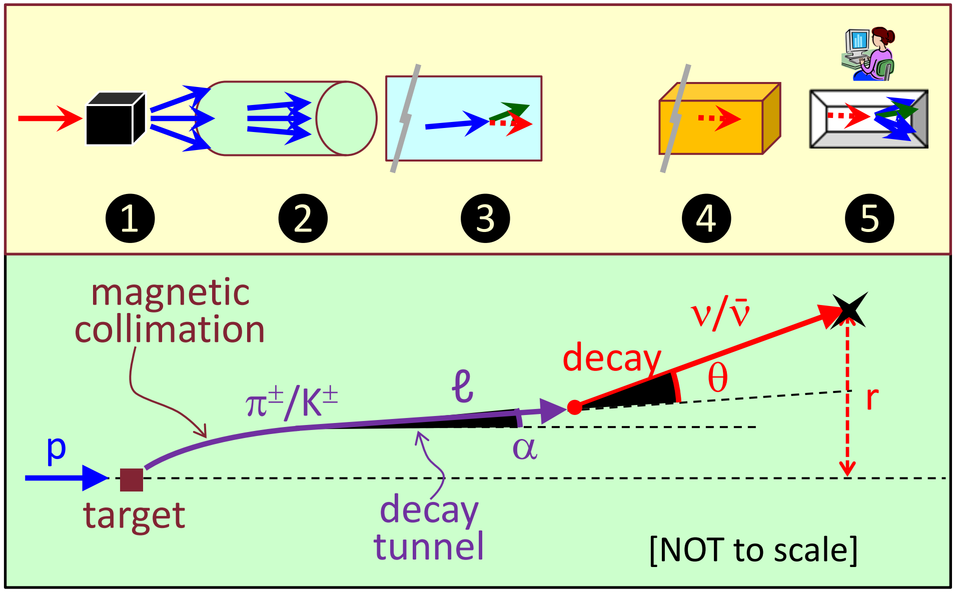
~10^{13} /cycle | ~10 /p | (WB / NB) | $\pi^{\pm}/K^{\pm}\rightarrow\mu^{\pm}\nu_{\mu}$ | no $\mu^{\pm}$ | 10^{11} $\nu_{\mu}/\bar{\nu}_{\mu}$ (WB) /cycle

<u>Schematic view of a ν beam (especially CERN WBB/NBB).</u>

[*not to scale*]

[NB a) in all the beam discussion, *mutatis mutandis* "ν" means both "ν" and "$\bar{\nu}$";
b) in this presentation the focus is on beams from CERN SPS: similar beams from PS, Fermilab, Serpukhov]

The ν beam: computation method



The relevant observable is the cross-section σ (or dσ/dΩ). In order to measure it, the experiments need **the flux of incoming $\nu/\bar{\nu}$**.

A $\nu/\bar{\nu}$ cannot be observed before its interaction ❺. Therefore the flux can only be computed statistically, together with its stat. and syst. uncertainties. The ingredients are:

❶ the inclusive differential cross sections of the $\pi^\pm$ and $K^\pm$ production in the target;

❷ the collection and collimation of $\pi^\pm/K^\pm$;

❸ the distribution of the decay length $f(\ell)$;

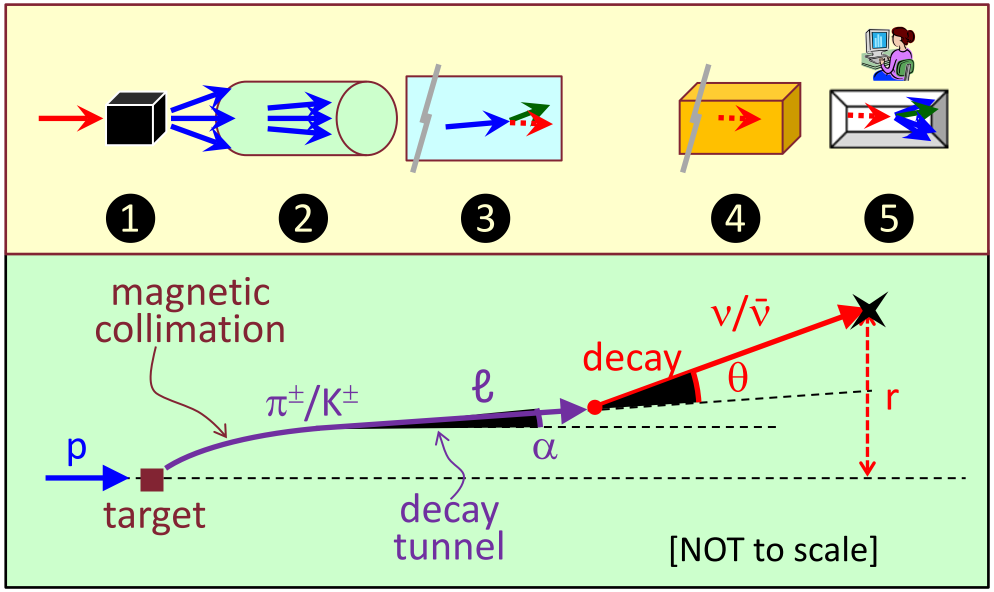
❹ the distribution of the $\nu/\bar{\nu}$ decay angle $f(\theta^*)$ [boost $\pi^\pm/K^\pm$ CM system → lab];

Using all these distributions, the flux, as a function of the $\nu/\bar{\nu}$ angles, energy and positions, is numerically computed, usually with a MC, and used in the analysis.

In the next slides some of these features will be examined.

despite all the efforts, in ν data analysis the beam is "the" problem. (Almost) all the systematics, mistakes, discussions, fights, come from the wrong control of the beam.

The ν beam : details of the method



Some details:

- the statistical distribution of ❶ and ❷ can be directly measured;
- the momentum distribution of $\mu^\pm$ from $\pi^\pm/K^\pm$ decay can be computed and checked using their measurement in the decay and absorber tunnels; the $\nu/\bar{\nu}$ flux is then inferred;
- the collection and collimation system ❷ may use different stategies (an option for the user):
  - "**wide band beam**" (WBB): more intense beam, but not "monochromatic" (π/K collection with high acceptance, e.g. van der Meer horn);
  - "**narrow band beam**" (NBB): more monochromatic and higher energy, but much less intense (standard $\pi^\pm/K^\pm$ selection);

  in practice, both beams are optimized for different physics measurements;
- $f(\ell)$ and $f(\theta^*)$ can be analytically calculated and boosted to the LAB system, using β,γ [$\beta=|p_{\pi/K}|/E_{\pi/K}$, $\gamma=E_{\pi/K}/m_{\pi/K}$] and the lifetimes $\tau_{\pi/K}$;
- many more subtleties, e.g. rare $\pi^\pm/K^\pm$ decays, punch-throughs, … are included in the computations.

The ν beam : $\pi^\pm/K^\pm$ decays

- Only beams of ν_μ (or $\bar\nu_\mu$) can be created: ν_e (or $\bar\nu_e$) are small contaminations (e.g. from K^+_{e3} decays);
- the ν's are not directly measurable → some info about their 4-momentum comes from the kinematics of the decay of the $\pi^\pm$'s and $K^\pm$'s ($\pi^\pm$ / $K^\pm \to \mu^\pm\nu_\mu$);
- the $\pi^\pm$ ($K^\pm$) has spin 0 → in its CM-frame isotropic decay (φ*, cos θ* flat);
- boost it (β_π, γ_π) to get the longitudinal momentum $p^{//}_\nu$ and its distribution;
- no boost for the transverse momentum $p^\perp_\nu$ distribution.

Results [*see next slides*] :

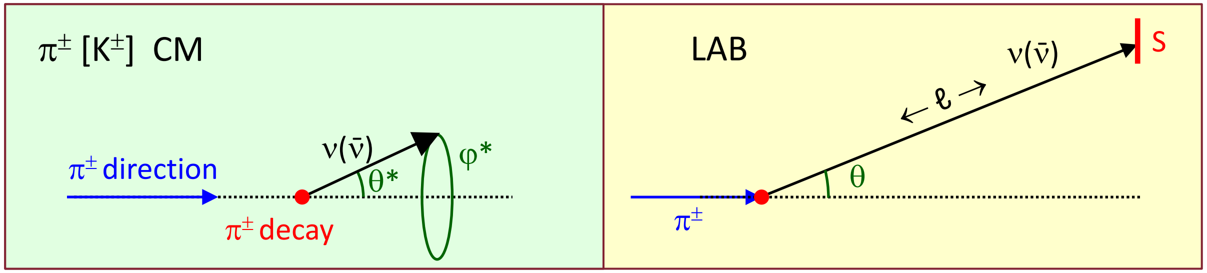
- the angular distribution for a ν, respect to a $\pi^\pm$ of energy $E_\pi = m_\pi\gamma$, is

$$\frac{dn}{d\Omega} \approx \frac{1}{4\pi}\frac{4\gamma^2\left[1+\tan^2\theta\right]^{3/2}}{\left(1+\gamma^2\tan^2\theta\right)^2};$$

[Kopp, Phys. Rep. 439, 101]

- therefore, a detector of surface S, positioned at distance ℓ and angle θ, sees a flux ϕ of ν's :

$$\phi \approx \frac{S}{4\pi\ell^2}\left(\frac{2\gamma}{1+\gamma^2\theta^2}\right)^2.$$



The ν beam : decay kinematics

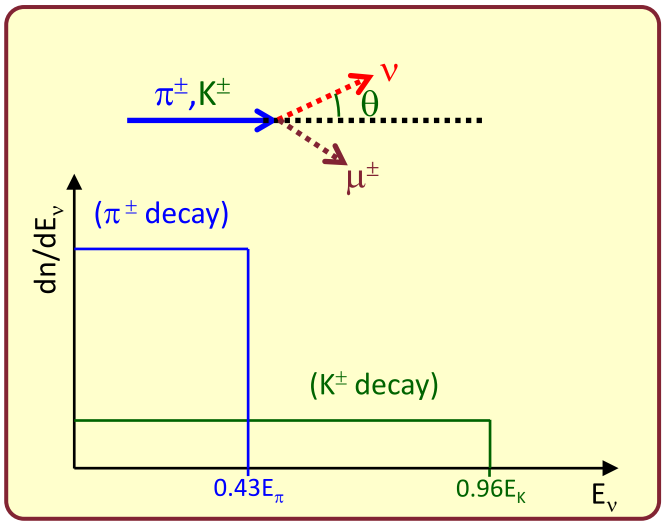
Kinematics is simple :

- since the $\pi^\pm$ have spin 0, the $(\nu\mu)$ distribution in the CM system is flat;

$\rightarrow$ the momentum of the ν's in the LAB has a (roughly) flat distribution;

$\rightarrow$ the distribution ranges between $E_\nu^{min} \approx 0$ and $E_\nu^{max} = 0.43\ E_\pi$.

- [for $K^\pm$ decay, the same formula gives a higher maximum : $E_\nu^{max} = 0.96\ E_K$]



$$\text{CM:} \begin{cases} \pi: & (\ m_\pi, & 0, & 0\ ) \\ \nu: & (\ p^*, & p^*\cos\theta^*, & p^*\sin\theta^*) \\ \mu: & (m_\pi - p^*, & -p^*\cos\theta^*, & -p^*\sin\theta^*) \end{cases}$$

$$m_\mu^2 = m_\pi^2 + p^{*2} - 2m_\pi p^* - p^{*2} = m_\pi^2 - 2m_\pi p^* \quad \rightarrow$$

$$p^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}; \qquad E^*_\mu = m_\pi - p^* = \frac{m_\pi^2 + m_\mu^2}{2m_\pi};$$

$$p_\nu^{//}\Big|_{LAB} = p\cos\theta = \gamma p^*\cos\theta^* + \beta\gamma p^*;$$

$$\frac{dn}{dp_\nu^{//}\big|_{LAB}} = \left|\frac{dn}{d\cos\theta^*}\right|\left|\frac{d\cos\theta^*}{dp_\nu^{//}\big|_{LAB}}\right| = \frac{\text{const}}{\gamma p^*} = \text{const};$$

$$p_\nu^{//}\Big|_{LAB}^{max} = p_\nu^{//}\Big|_{LAB}(\cos\theta^* = 1) = \gamma p^*(1+\beta) \approx 2\gamma p^* =$$

$$= 2\frac{E_\pi}{m_\pi}\frac{m_\pi^2 - m_\mu^2}{2m_\pi} = E_\pi\frac{m_\pi^2 - m_\mu^2}{m_\pi^2} = E_\pi\left(1 - \frac{m_\mu^2}{m_\pi^2}\right).$$

$$p_\nu^{//}\Big|_{LAB}^{min} = p_\nu^{//}\Big|_{LAB}(\cos\theta^* = -1) = \gamma p^*(\beta - 1) \approx 0.$$

$$p_\nu^\perp = p^*\sin\theta^* = \mathcal{O}(m_\pi) \ll p_\nu^{//}\Big|_{LAB}^{max} \approx E_\nu\Big|_{LAB}^{max} = \mathcal{O}(E_\pi).$$

The ν beam : dn/dΩ

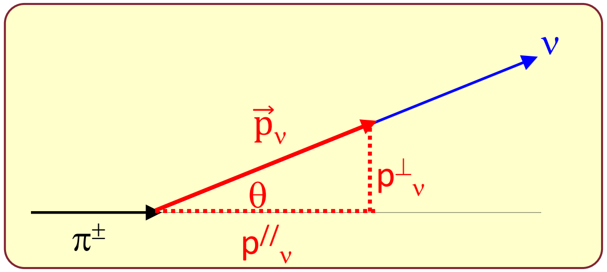
Moreover :

- 2-body decay;
- in the CM (p^*, Ω^*, θ^*), the angular distribution is flat (=$1/4\pi$);
- in the LAB (p, Ω, θ), boost β,γ;
- long, but simple (see box) :

$$\frac{dn}{d\Omega} = \frac{dn}{d\Omega^*}\left|\frac{d\Omega^*}{d\Omega}\right| = \frac{dn}{d\Omega^*}\left|\frac{d\cos\theta^*}{d\cos\theta}\right| =$$

$$= \frac{dn}{d\Omega^*}\left|\frac{d\cos\theta^*}{d\tan^2\theta}\right|\left|\frac{d\tan^2\theta}{d\cos\theta}\right| =$$

$$\approx \boxed{\frac{1}{4\pi}\,\frac{4\gamma^2\left[1+\tan^2\theta\right]^{3/2}}{\left(1+\gamma^2\tan^2\theta\right)^2}}.$$



$$p_\nu^\perp = p_\nu \sin\theta = p^* \sin\theta^*;$$

$$p_\nu^{//} = p_\nu \cos\theta = \gamma\left(p^*\cos\theta^* + \beta E^*\right) \approx \gamma p^*\left(\cos\theta^* + 1\right);$$

$$p_\nu^\perp / p_\nu^{//} = \tan\theta = \sin\theta^* / \left[\gamma\left(1+\cos\theta^*\right)\right];$$

$$\gamma^2\tan^2\theta = \left(\frac{\sin\theta^*}{1+\cos\theta^*}\right)^2 = \frac{1-\cos^2\theta^*}{\left(1+\cos\theta^*\right)^2} = \frac{1-\cos\theta^*}{1+\cos\theta^*};$$

$$\boxed{b = \frac{1-a}{1+a} \rightarrow b + ab = 1 - a \rightarrow a = \frac{1-b}{1+b}} \rightarrow \cos\theta^* = \frac{1-\gamma^2\tan^2\theta}{1+\gamma^2\tan^2\theta};$$

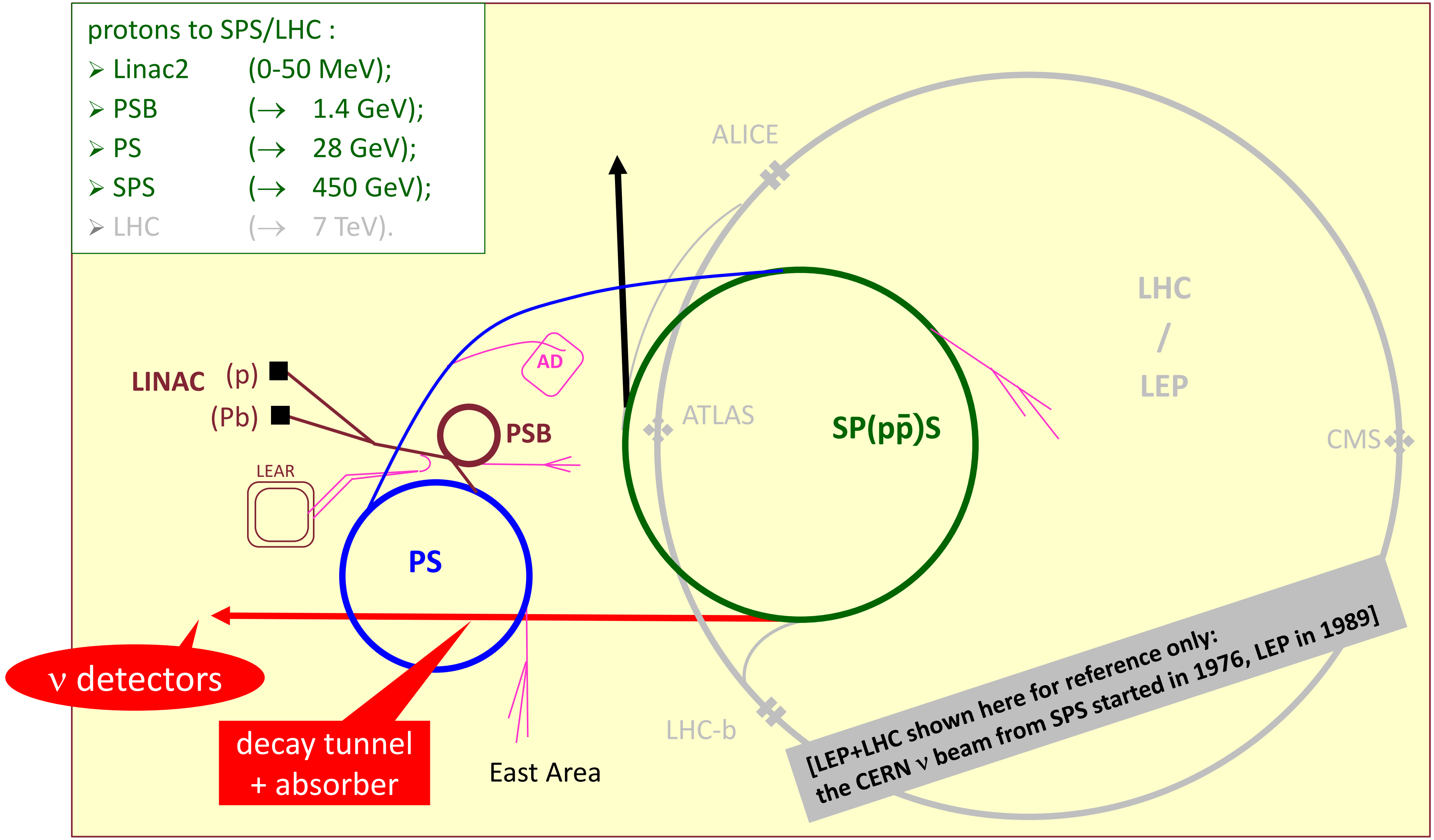
$$\frac{d\cos\theta^*}{d\tan^2\theta} = \frac{-\gamma^2}{1+\gamma^2\tan^2\theta} - \frac{\gamma^2\left(1-\gamma^2\tan^2\theta\right)}{\left(1+\gamma^2\tan^2\theta\right)^2} =$$

$$= \boxed{\frac{-2\gamma^2}{\left(1+\gamma^2\tan^2\theta\right)^2}}; \qquad \boxed{\frac{1}{\cos^2} = \frac{\cos^2+\sin^2}{\cos^2} = 1+\tan^2}$$

$$\frac{d\tan^2\theta}{d\cos\theta} = \frac{d}{d\cos\theta}\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right) = \frac{d}{d\cos\theta}\left(\frac{1}{\cos^2\theta} - 1\right) =$$

$$= -2/\cos^3\theta = \boxed{-2\left(1+\tan^2\theta\right)^{3/2}};$$

The ν beam : CERN accelerators



The ν beam : CERN SPS

The accelerator : as an example, the Super Proton Synchrotron (SPS) at CERN, which (today) accelerates ~5 × 10^{13} protons per cycle to an energy E_p = 450 GeV.

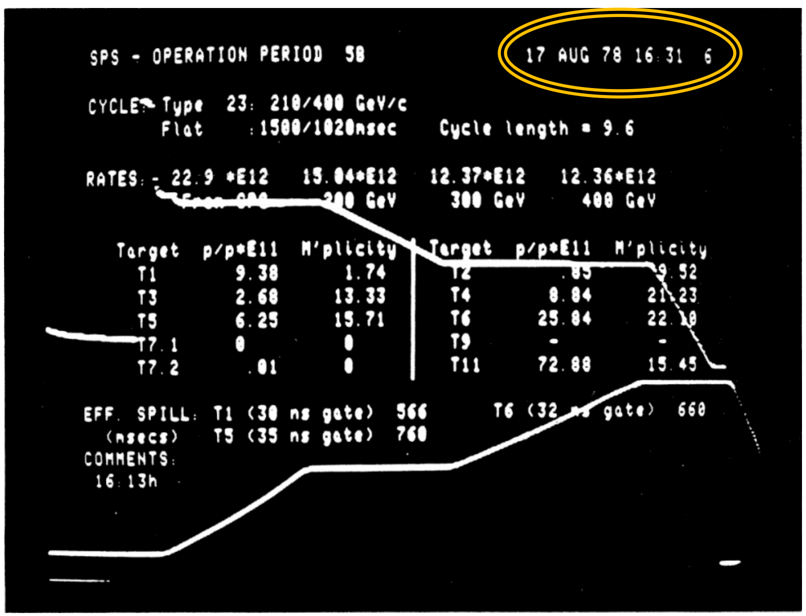
The proton beam is extracted and sent to a target (copper, beryllium, graphite). The average secondary multiplicity is ~10 charged, with energies from 10 to 100 GeV.

The secondaries ($\pi^\pm$, $K^\pm$) are focused and let decay.

The focusing is a compromise: resolution [ideally a monochromatic ν beam] vs intensity [as many ν's as possible].

A good solution is the WBB beam, where a "Van der Meer horn" selects with good acceptance $\pi^\pm$ and $K^\pm$, with given sign :

- +ve for a ν beam from $\pi^+/K^+ \to \mu^+\nu_\mu$,
- −ve for a $\bar{\nu}$ beam.



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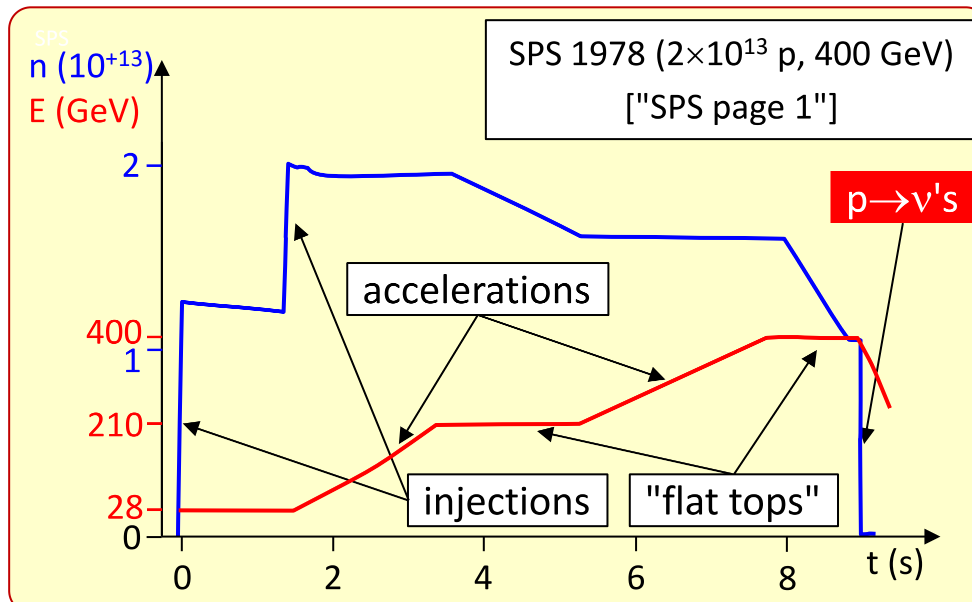
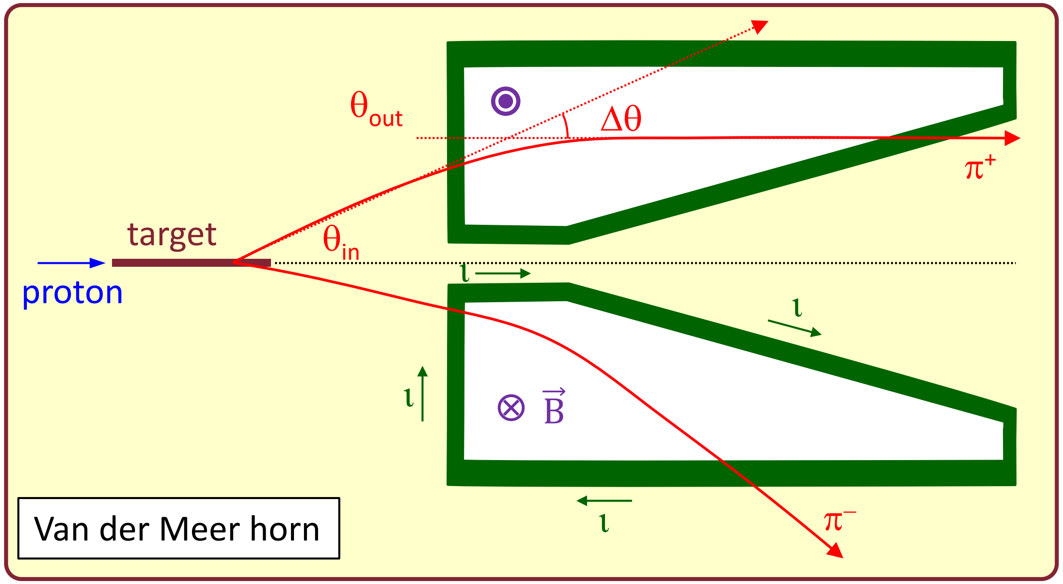


The ν beam : the horn in the WBB

- The Van der Meer horn consists in a magnet, pulsed with currents (up to 100 kA), positioned just after the target.
- It collimates particles of a given sign (π^+,K^+ in the scheme) and sweeps away the opposite charge (π^-, K^-). Multi-horn setups have also been built.
- The direction of the current in the horn(s) **selects a beam of $\nu_\mu \leftrightarrow \bar{\nu}_\mu$** : $(\pi^+ \to \mu^+ \nu)$ vs $(\pi^- \to \mu^- \bar{\nu})$.



Van der Meer horn

Imho, one of the two great contributions of SVdM to particle physics (he got the Nobel prize for the other).

The ν beam : decay tunnel

In the decay tunnel $\pi^{\pm}$'s and $K^{\pm}$'s decay.

The length of the tunnel is a compromise between cost and intensity : it should be about the average decay length.

→ In the laboratory frame :

$$\ell = \beta\gamma c\tau = p\, c\, \tau \,/\, m.$$

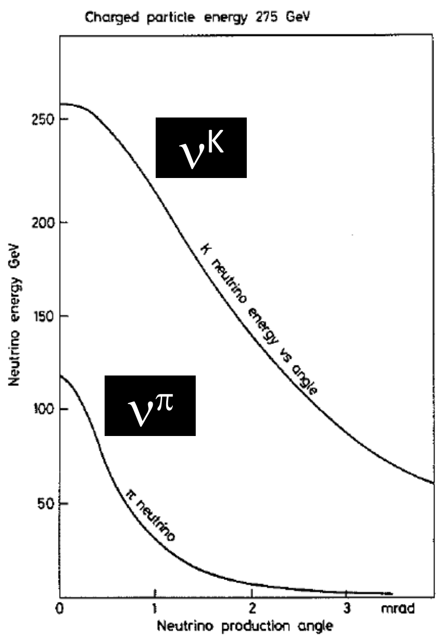
E.g. for 50 GeV π^+, [$c\tau(\pi^+)$ = 7.8 m] :

$$\ell = 50 \times 7.8 \,/\, .140 = 2800 \text{ m}.$$

(in reality the tunnels are only few×100 m).

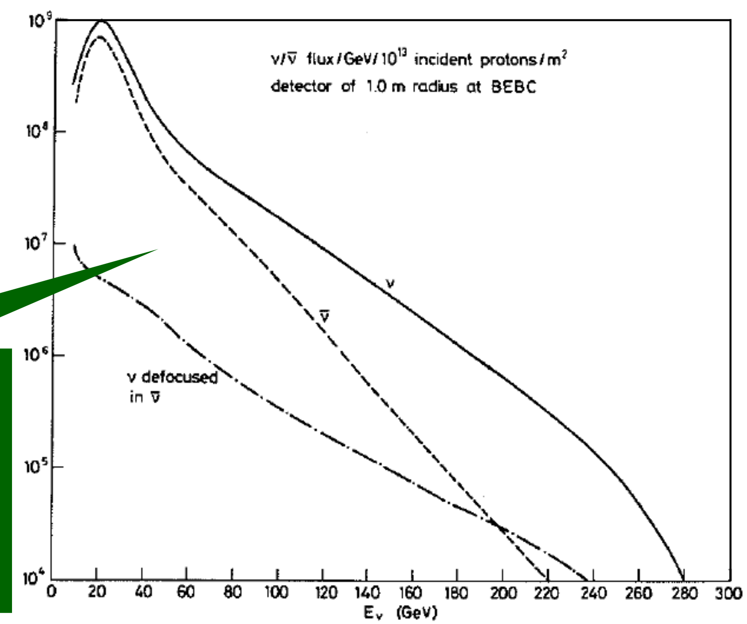
The figures show :

- the angle between the ν and its parent (i.e. the additional angular spread of the beam due to the decay), for ν originating from K or π (ν^K and ν^π);
- the energy distribution of the ν and $\bar\nu$ beams for 10^{13} protons on target.



Q. Why more ν than $\bar\nu$?

A. No exotic motivation, but the initial state pN is +ve, so π^+ are more abundant than π^-.



The ν beam : the μ's absorber

The Absorber : the detectors must obviously get ONLY ν's and NOT the μ's (initially as many as ν's), π's and K's (few, but not zero).

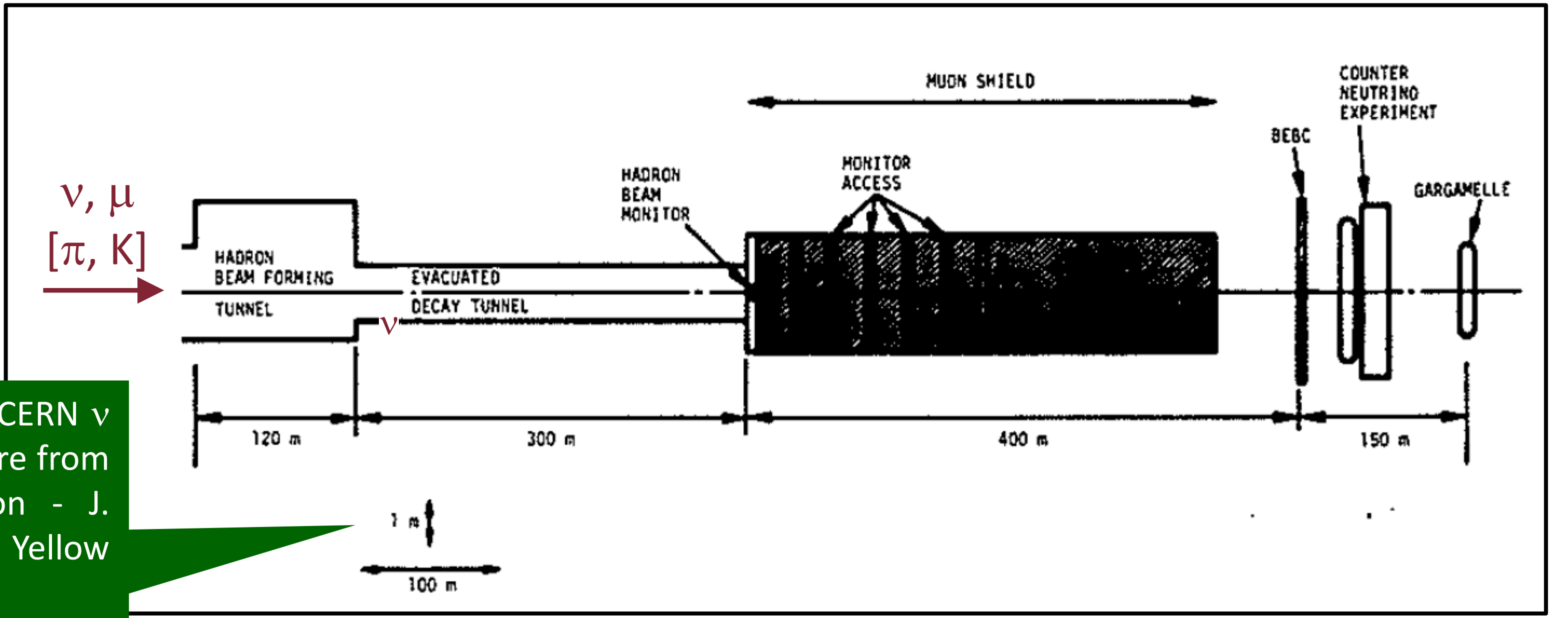
Therefore a thick absorber is positioned at the end of the decay tunnel.

At the CERN SPS it was made with 185 m iron + 220 m rock.

As an exercise, compute the range in iron for a high energy μ. From the numerical integration of the function

$$E = \int_0^{\text{range}} (dE/dx)dx :$$

E_μ (GeV)	range(Fe)	range(rock)
100 GeV	56 m	156 m
500 GeV	180 m	583 m



The setup of the CERN ν beam [a dark figure from a clear discussion - J. Steinberger, CERN Yellow Report 76-20].

The ν beam : conclusions

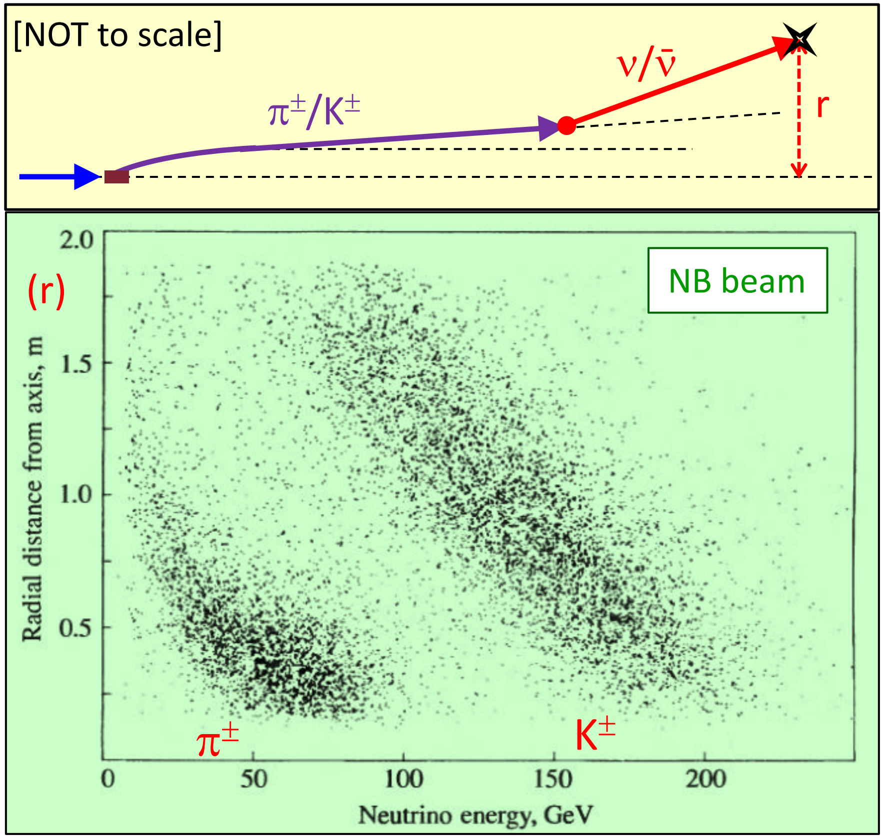
The table and the plot summarize the main performances of the two CERN beams :

- for WBB the relative contaminations:

	WBB beam	
	ν_μ	$\bar{\nu}_\mu$
ν_μ	91%	15%
$\bar{\nu}_\mu$	8%	84%
ν_e	1%	0.4%
$\bar{\nu}_e$	0.1%	0.7%

E.g., it means : you think you have built a WBB $\bar{\nu}_\mu$ beam, but actually you have only 84% $\bar{\nu}_\mu$, plus 15% ν_μ, 0.40% ν_e, 0.70% $\bar{\nu}_e$.

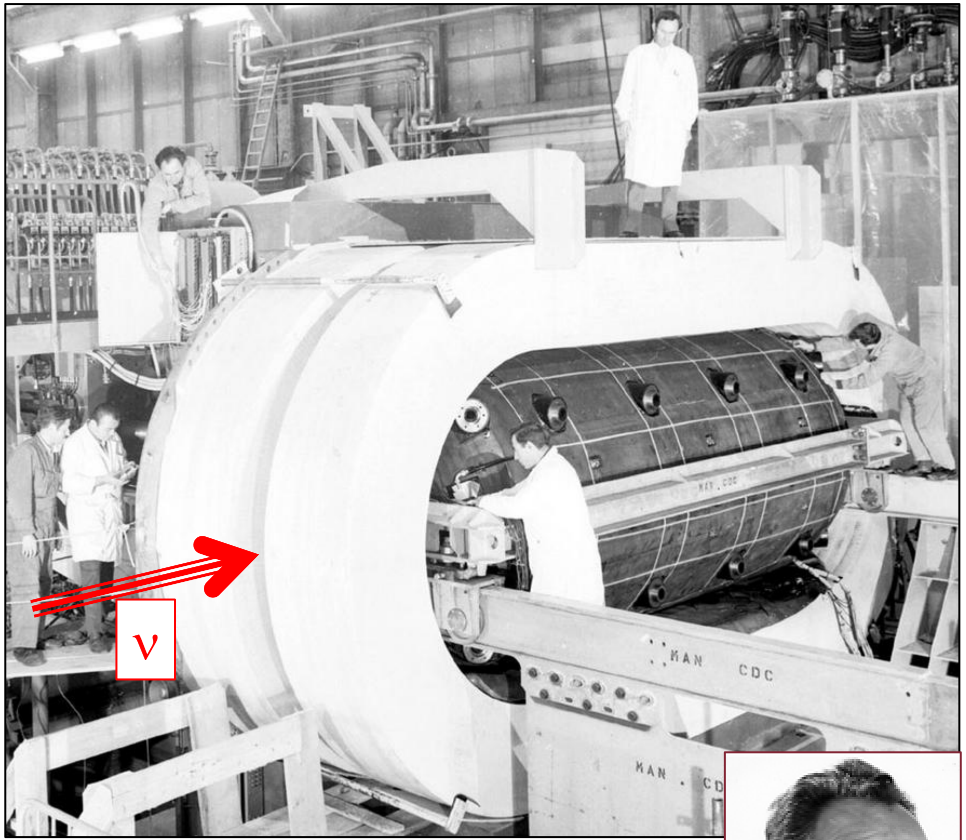
- for NBB the relation between the radial distance (r) of the impact point in the detector (P) and the ν energy allows for a determination of the ν energy with a certain resolution, and little π/K ambiguity.



The ν detectors: Gargamelle

The ν detectors are of different types, but have to share common characteristics :

- big size (detect small cross sections);
- good lepton identification (CC vs NC);
- meas. of the hadronic shower (NC);
- rate capability is NOT a bonus, due to the small number of events.
- traditionally, the best ν detectors were heavy liquid bubble chamber, filled with (**freon CF_3BR**, Ne, propane), and embedded in a strong magnetic field.
- Gargamelle is one of the first and most glorious of them : "she" discovered the neutral currents [*many thanks to her "father" A. Lagarrigue*].

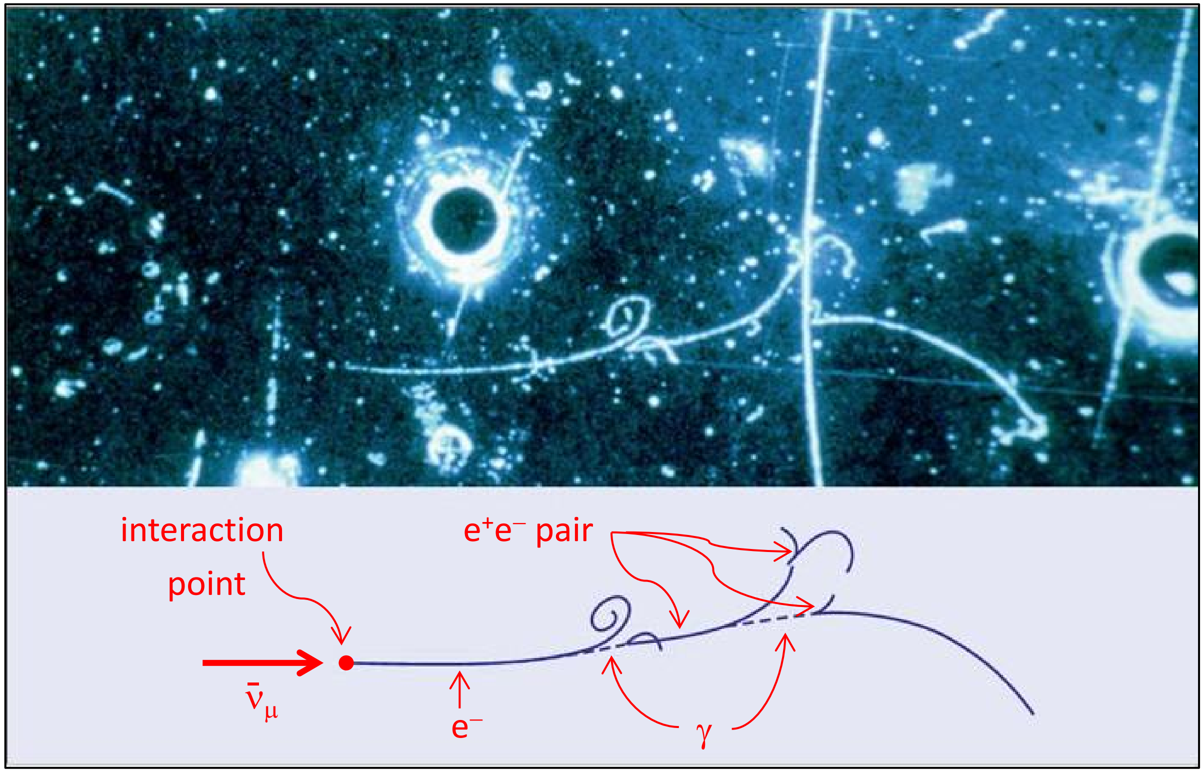


André Lagarrigue (1924-1975)

Notice :

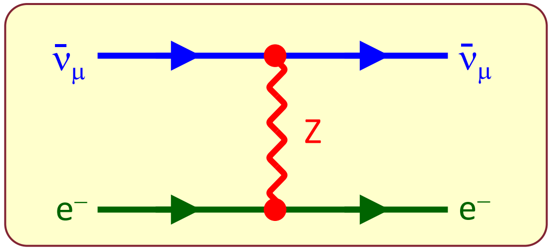
- coils for mag. field generation;
- holes for the cameras;
- big size (for the 70's);
- absence of cryostat;
- ν's enter from the left.

The ν detectors: Gargamelle



Gargamelle discovery of NC [1973] - the famous event:

- the key point is the e⁻ identification, via its brem(s);
- … and the <u>absence</u> of anything else (especially a $\mu^\pm$ candidate);
- the event was interpreted as a purely leptonic NC process [$\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$].



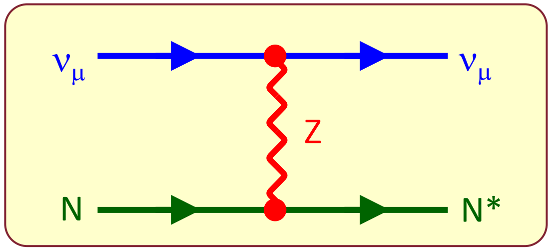
The ν detectors: Gargamelle

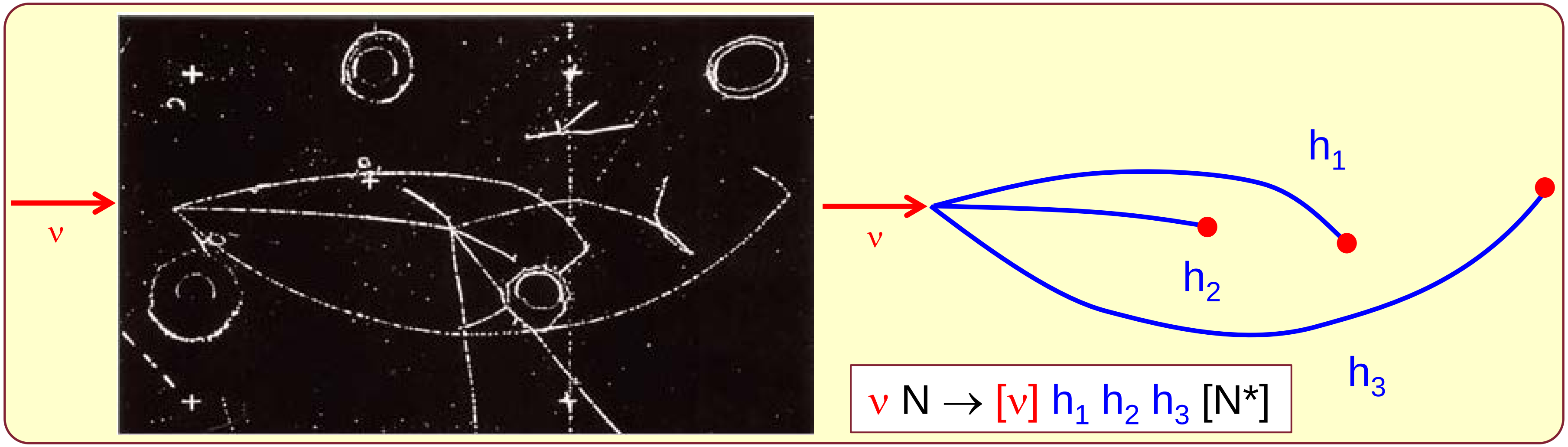
Gargamelle discovery of NC.

A beautiful hadronic neutral current event, where the interaction of the neutrino coming from the left produces three secondary particles, all clearly identifiable as hadrons, as they interact with other nuclei in the liquid. There is no charged lepton (muon or electron).

(D.Cundy, CERN Courier)

this is the key point





$\nu\, N \to [\nu]\; h_1\, h_2\, h_3\, [N^*]$

The ν detectors: BEBC

In ≥ 1976 the CERN SPS was operational : new ν beam, higher energy, new detectors.

BEBC (Big European Bubble Chamber) :

- cryostatic (H_2, D_2, Ne, mixtures) [*cryo not shown*];
- giant solenoid around (not shown); at the time the largest superconducting coil in the world;
- millions of frames : extensive studies of exclusive processes (see next slide)

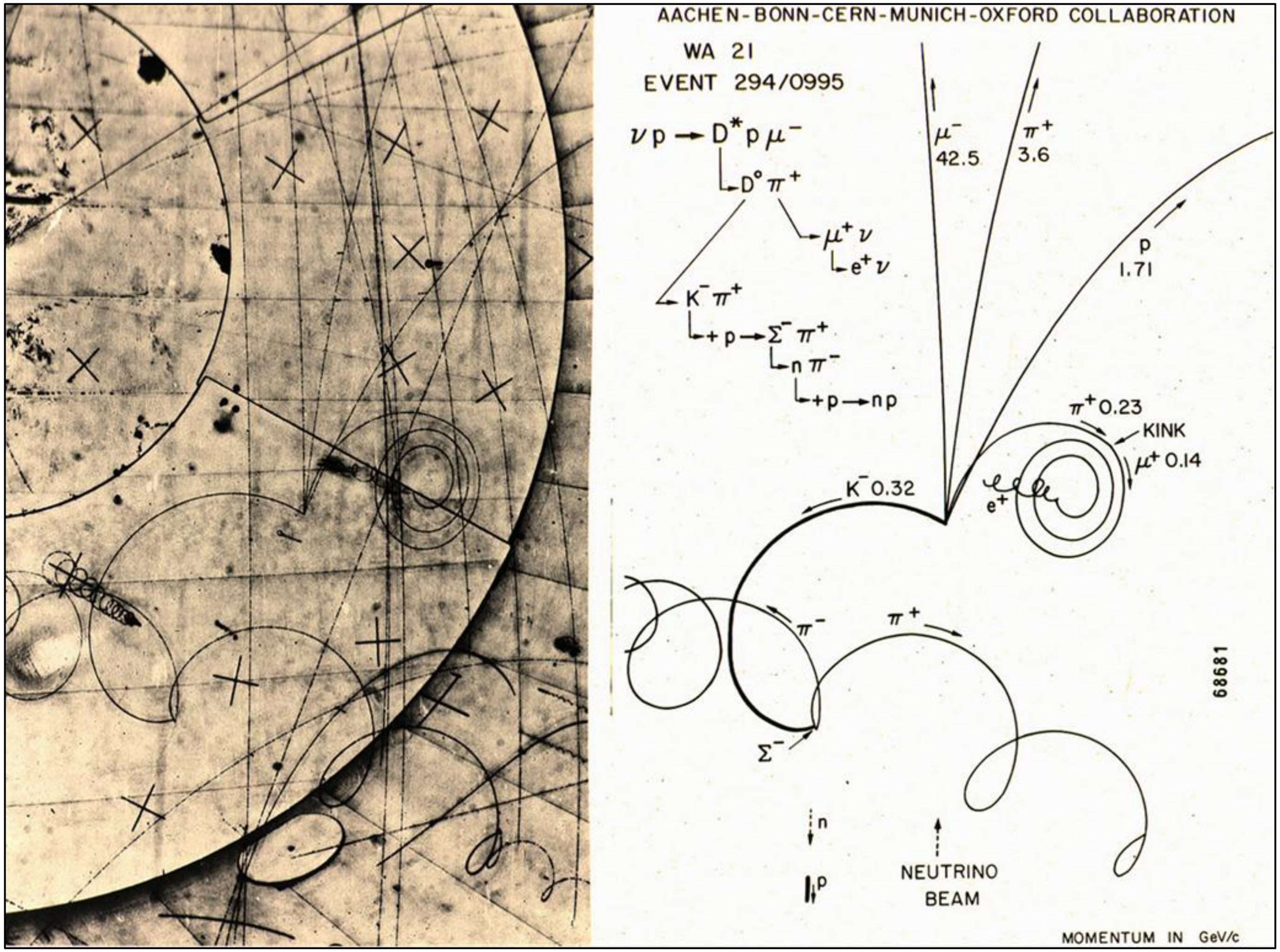
Curiosity : in 1977, an emulsion stack in front, to act as a target; aim : select and measure charm production in ν interactions, and subsequent decays, by identifying the decay vertex;

- first direct identification of charmed mesons and baryons; first measurement of their lifetime;
- Spokesman : Marcello Conversi [*believe me, it was a lot of fun*].

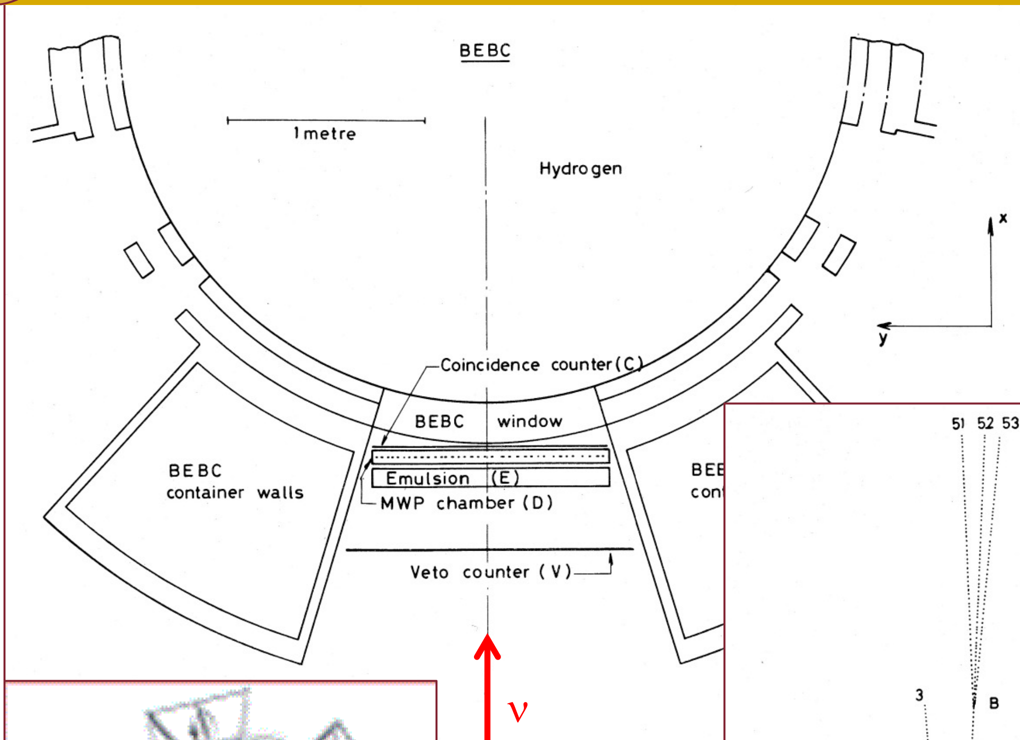
The ν detectors: BEBC

A beautiful charm event inside BEBC :

- very clear;
- 4 photo / event (at different angles → 3D reconstruction);
- momenta / charges measured by the mag. deflections;
- $e^{\pm}$ via energy loss;
- $\mu^{\pm}$ by external device (EMI);
- then, combined masses, kinematical fits, … fun.



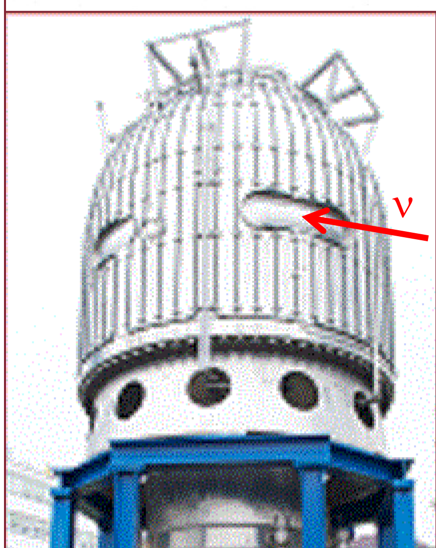
The ν detectors: BEBC + emulsions

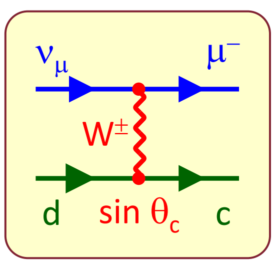


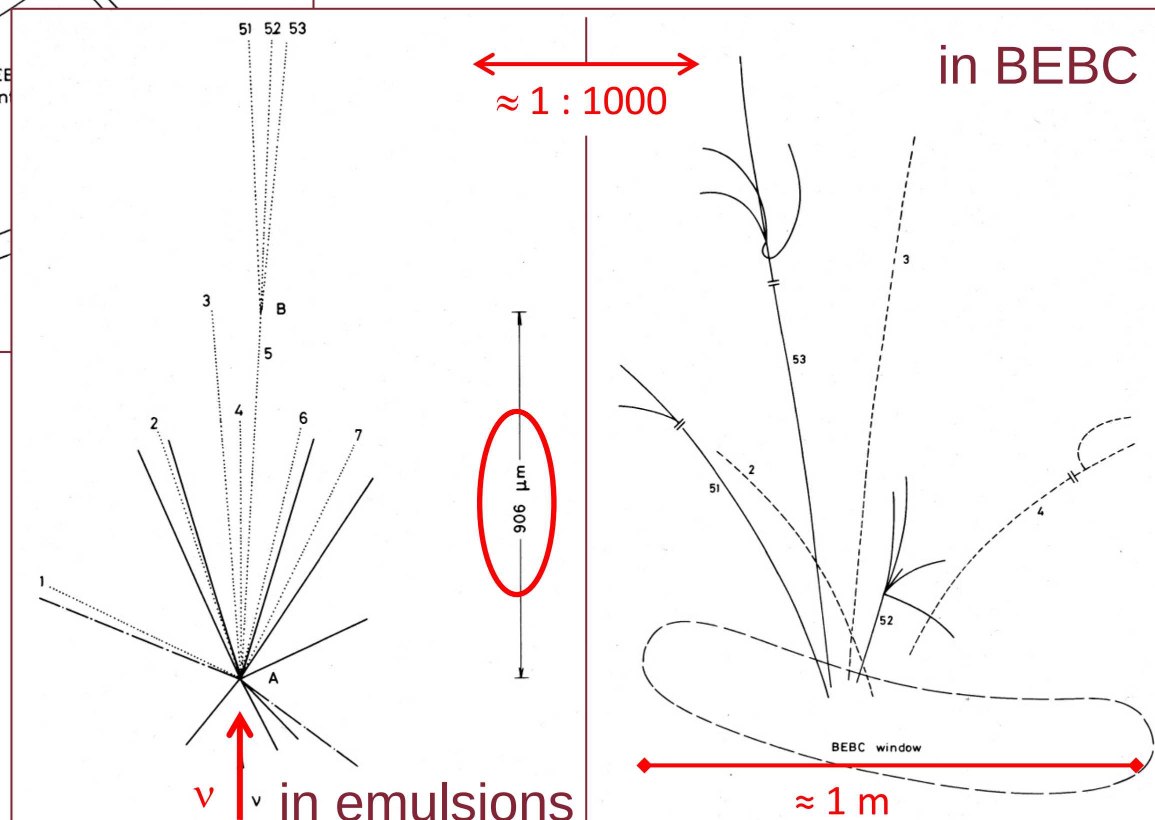
An event in the Conversi experiment;

interpreted as $D^+ \to \pi^+\pi^+\pi^-\overline{K}^0$;

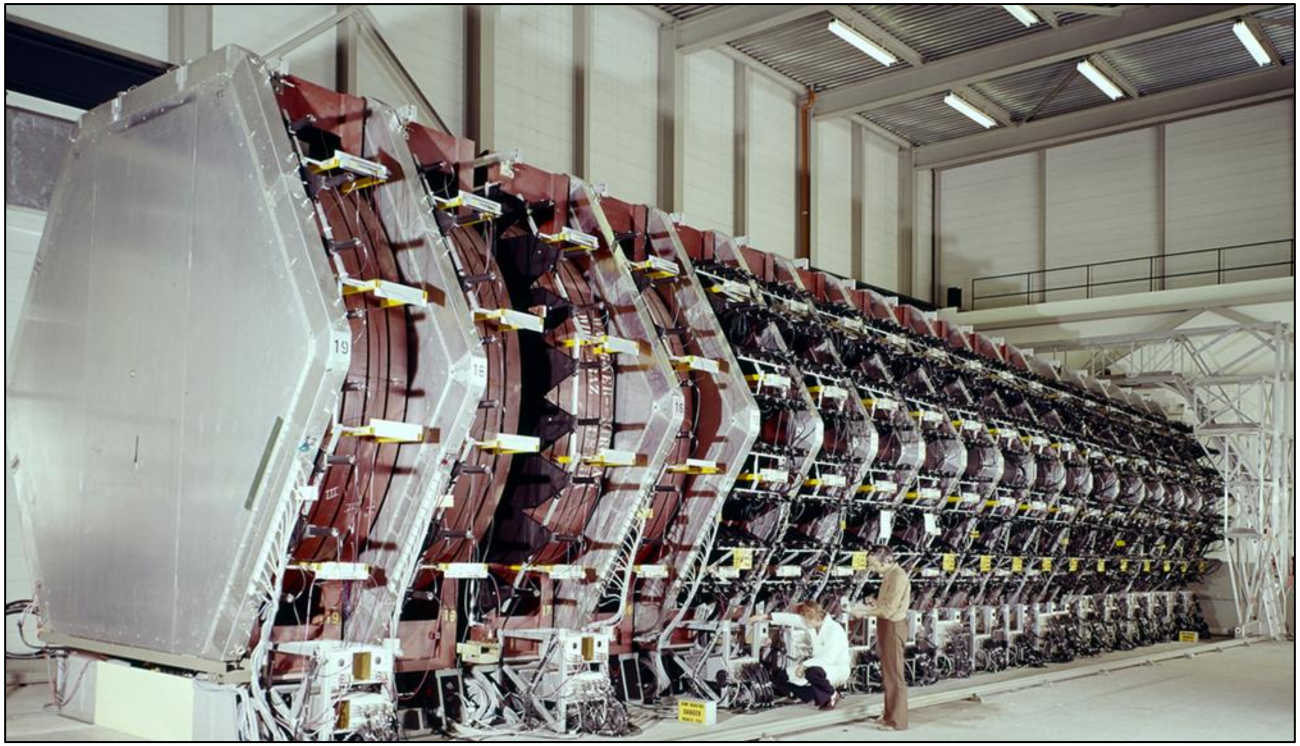
$t(D^+) = 2$ (or 4) $\times 10^{-13}$ s [two kin. solutions].







The ν detectors: CDHS



The lion share went to two electronic calorimeters :

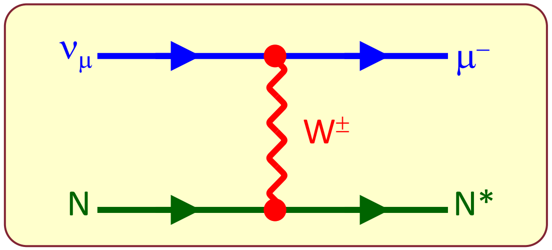
- **CDHS** (J. Steinberger et al.), a sandwich of magnetized iron disks and scintillator planes;

- [ν's from the left];
- huge mass, great $\mu^{\pm}$ identification via the iron absorbers;
- almost all the measurement which we will discuss in the next slides are from it.

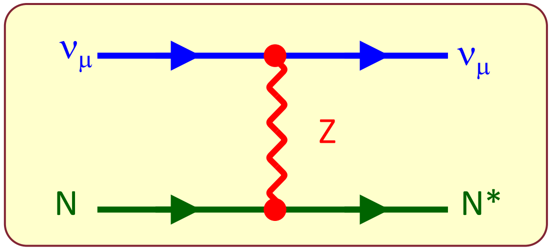
The ν detectors: CDHS events

Display of two events in CDHS :

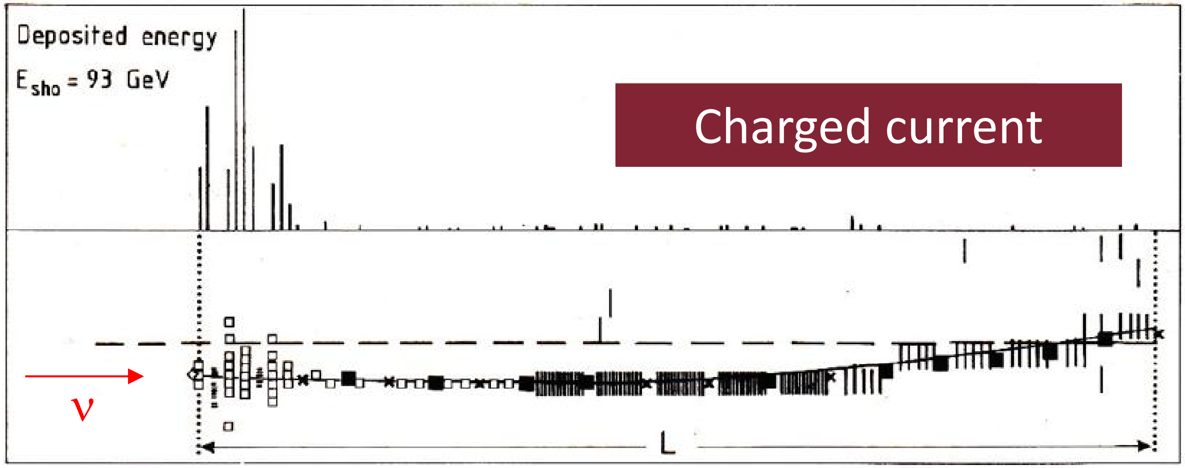
- ν ($\bar{\nu}$) from the left;
- upper event, interpreted as CC (early hadronic shower + penetrating μ^-);

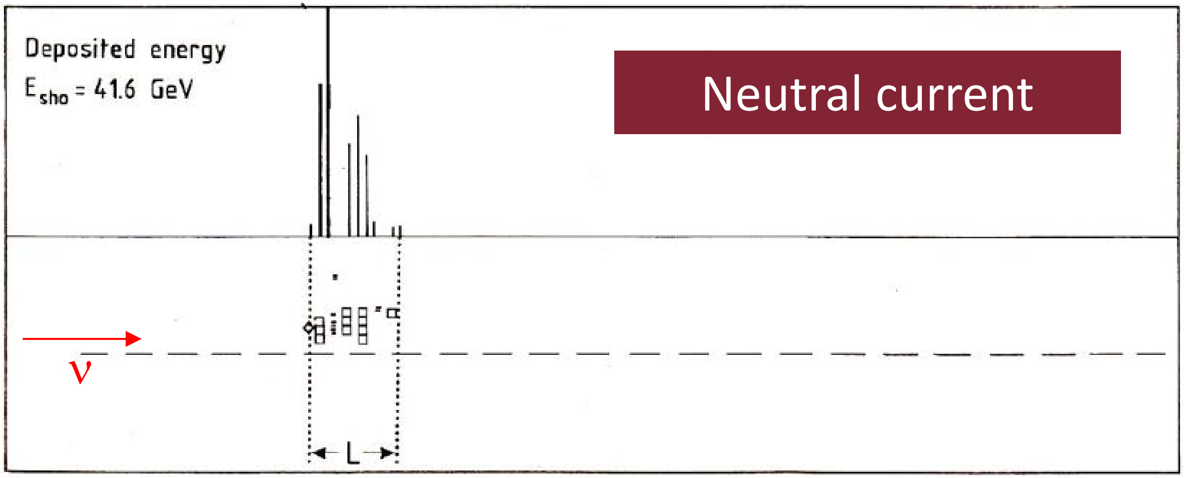


- lower event is a NC (no μ);



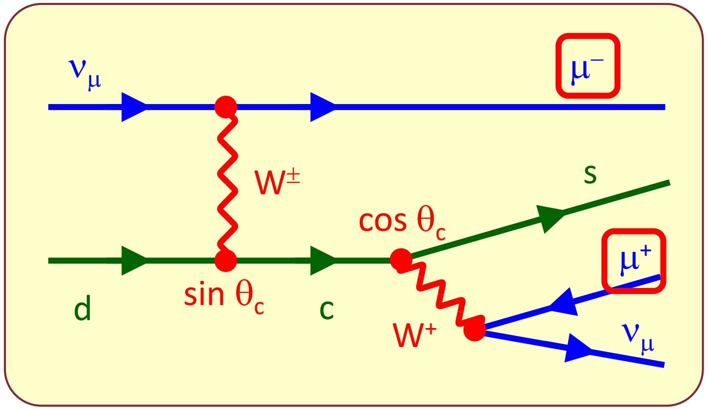
- notice the $E_{sho[wer]}$ measurement.



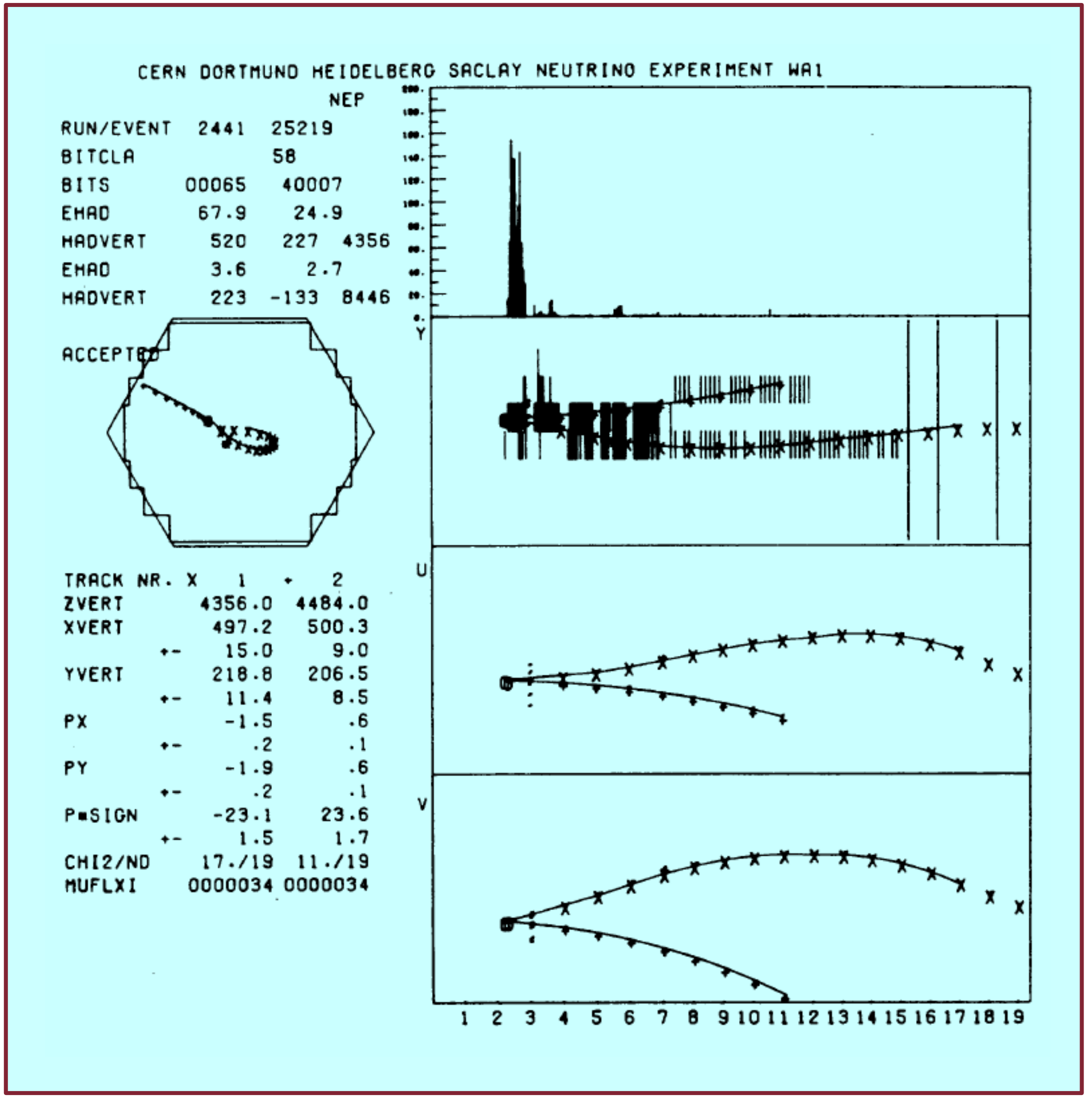


The ν detectors: CDHS 2μ

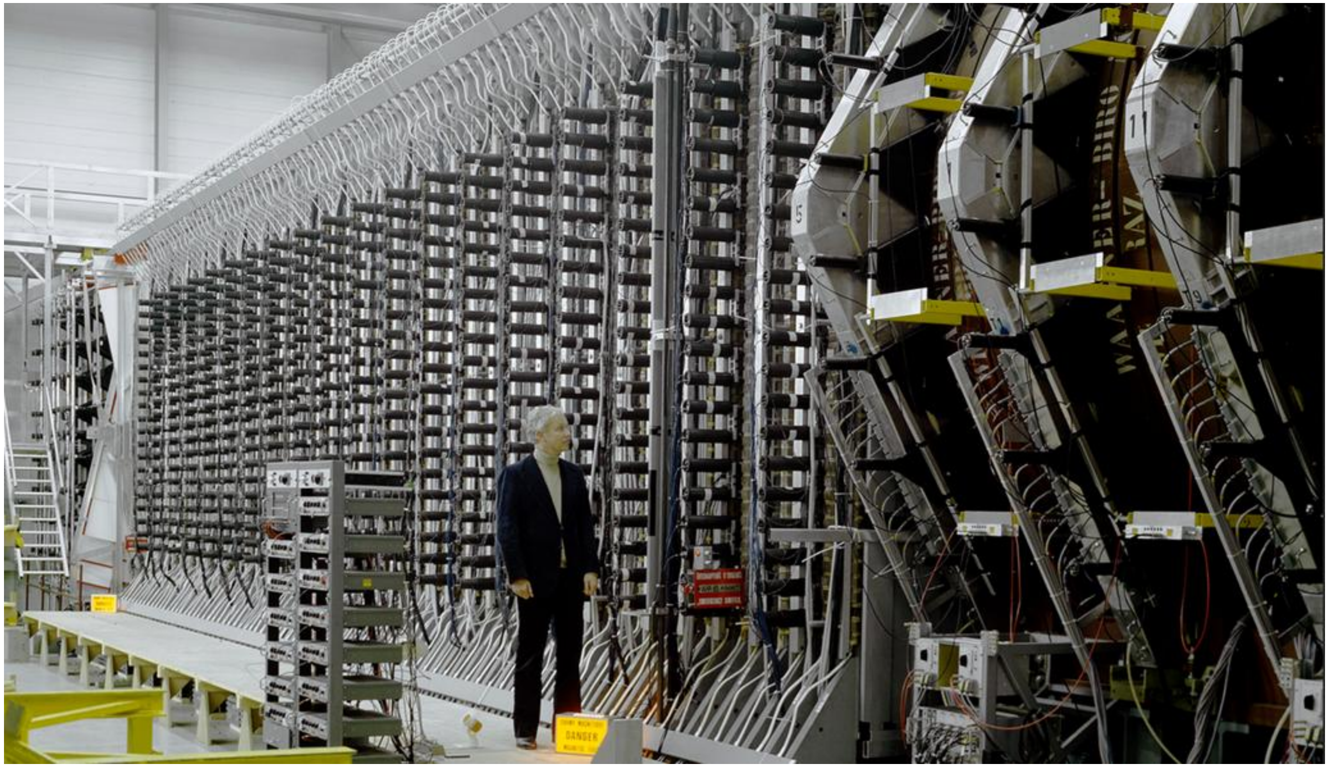
An "**opposite sign dimuon**" event in CDHS:



- *today this explanation looks almost trivial;*
- *but many years ago the origin of the "dimuons" was hardly understood, because of the lack of knowledge / confidence in the quark model and Cabibbo theory;*
- *they had an important role in convincing the physics community.*



The ν detectors: CHARM

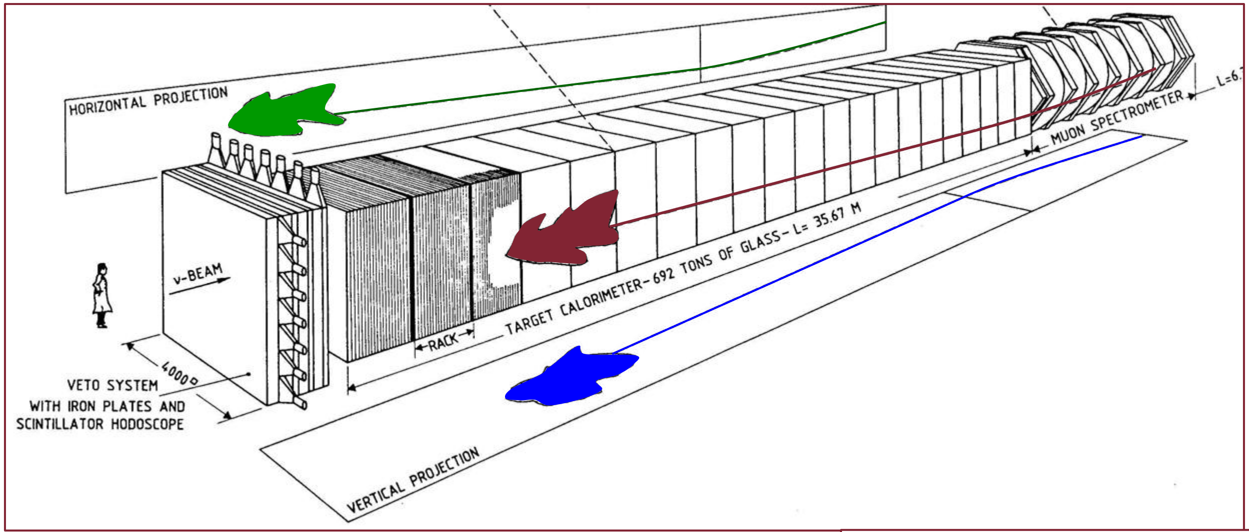


… and this is **CHARM** (CERN-Hamburg-Amsterdam-**Roma**-Moscow) :

- less massive, more granular;
- sandwich of 78 marble planes (1 X_0) + scintillators, drift and streamer tubes;

- almost 100 tonnes in total;
- designed to measure Energy and direction of the hadronic shower;
- ideal for NC.

The ν detectors: CHARM detector



Data taking : 1987-1991 :

2.5×10^{19} p on target →

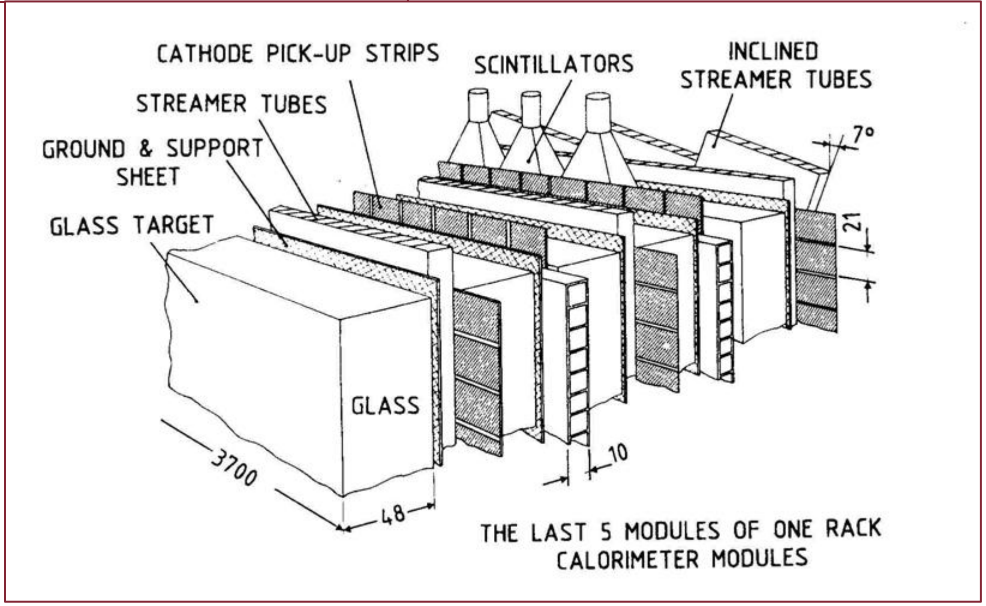
~ 10^8 ν and $\bar{\nu}$ interactions.

$\langle E(\nu)\rangle = 23.8$ GeV;

$\langle E(\bar{\nu})\rangle = 19.3$ GeV.

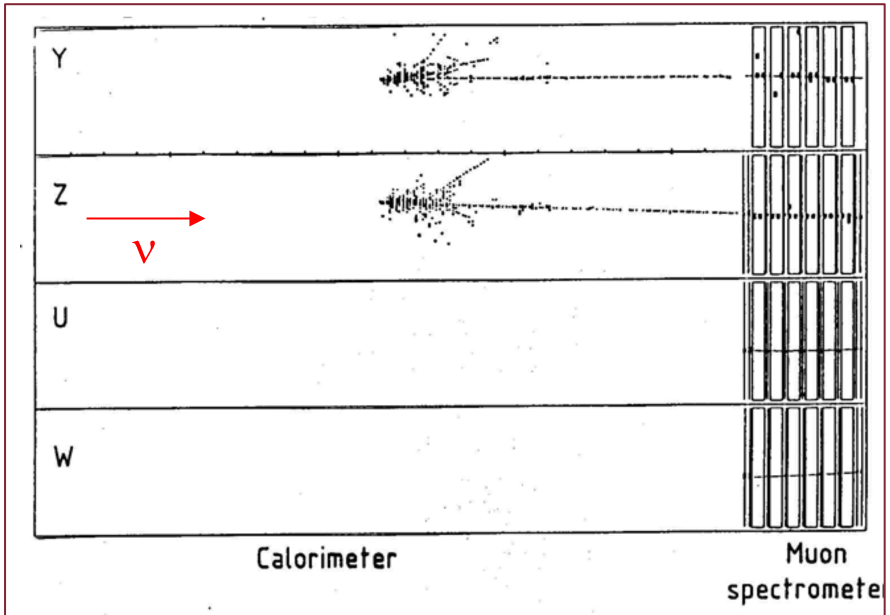
1. large mass: 692 t;
2. good angular resolution, because of low-Z absorber (glass) :

   $\sigma(\theta) / \theta \propto Z \sqrt{E}$
3. granularity for vertex definition (e/π^0 separation) : fine-grained trackers, Iarocci tubes with cells of 1 cm.

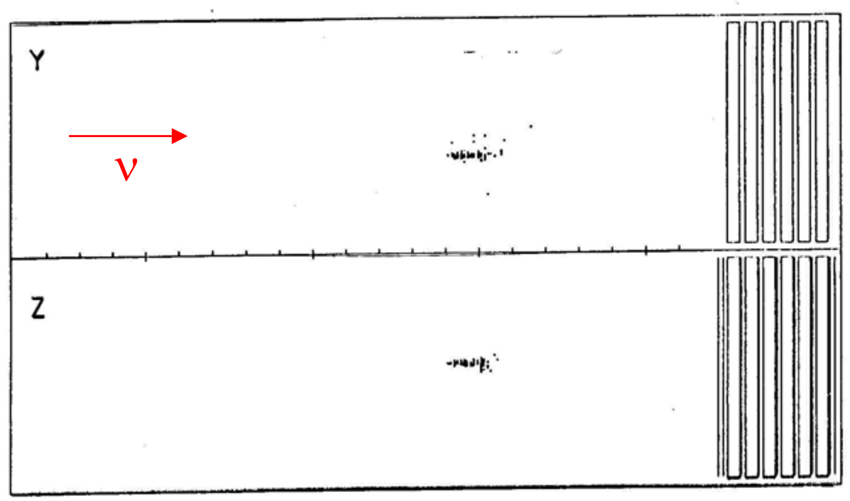


[*tech. detail: in previous page CHARM-1 (marble, ca 1978), while in this page CHARM-2 (glass, ca 1987)*]

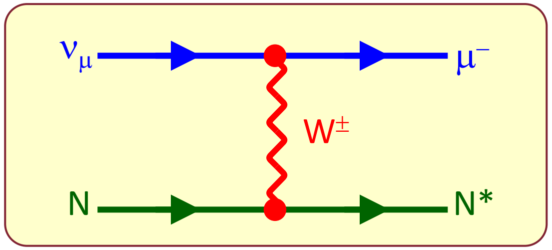
The ν detectors: CHARM event

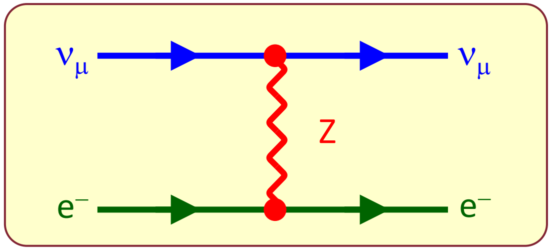


Charged current : $\nu_\mu N \to \mu^- N^*$

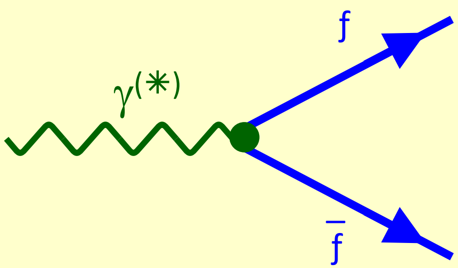


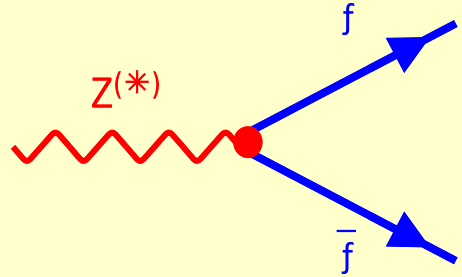
Neutral leptonic current : $\nu_\mu e^- \to \nu_\mu e^-$

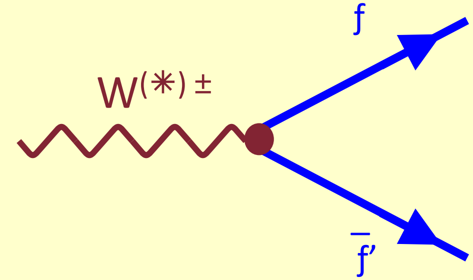




[*remember* : summary : e.m., NC, CC]







photon (γ)
(electromagnetism)

$$\mathscr{L}_F = -eJ^{\mu}_{e.m.}A_{\mu};$$
$$J^{\mu}_{e.m.} = Q_f\overline{\Psi}_f\gamma^{\mu}\Psi.$$

[V]

neutral IVB (Z)
(neutral current)

$$\mathscr{L}_F = \frac{-e}{\sin\theta_W\cos\theta_W}J^{\mu}_{nc}Z_{\mu};$$
$$J^{\mu}_{nc} = \overline{\Psi}_f\gamma^{\mu}\frac{g^f_V - g^f_A\gamma^5}{2}\Psi_f.$$

[combination g^f_V V + g^f_A A]

charged IVB (W±)
(charged current)

$$\mathscr{L}_F = \frac{-e}{\sqrt{2}\sin\theta_W}J^{\mu}_{cc}\tau^{\pm}W^{\pm}_{\mu};$$
$$J^{\mu}_{cc} = \overline{\Psi}_f\gamma^{\mu}\frac{1-\gamma^5}{2}\Psi_f.$$

[V - A]

from § 4

ν interactions : the landscape

How many types of $\nu/\bar{\nu}$ processes exist ?

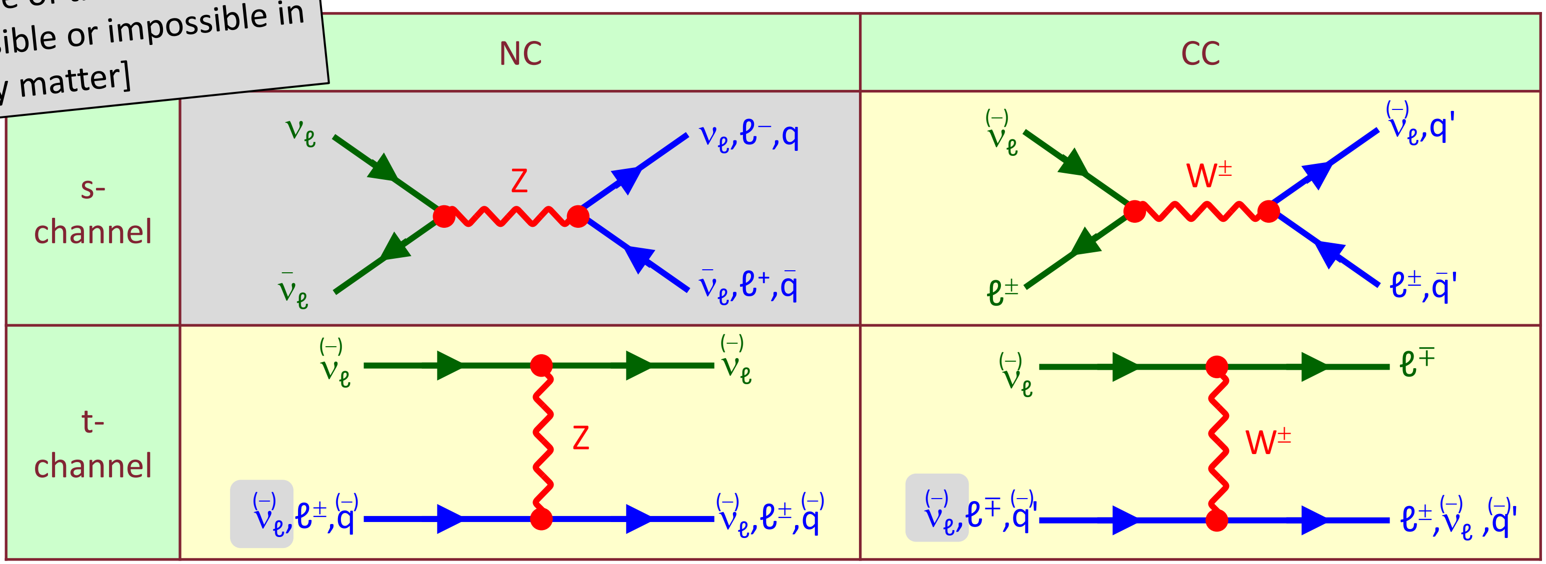
A lot, even in lowest order :

- (NC + CC) $\times$ (s-, t-channel);
- for each of them, many lepton replica ($\ell^\pm = e^\pm, \mu^\pm, \tau^\pm$);
- the semi-leptonic case : change only one fermion pair to quarks, i.e. $q\bar{q}$ for NC and $q'\bar{q}'$ for CC (q' is a CKM-rotated quark);
- each q' line counts for three (e.g. a d' is a mixture of dsb, with coefficients given by the CKM matrix).

The key feature of the SM is that all these hundreds of processes reduce to a handful number of coupling constants and charges, which allow to quantify all of them.

E.g.: $\nu_e e^+ \to \nu_\mu \mu^+$ is CC-s;
$\nu_\mu e^\pm \to \nu_\mu e^\pm$ and $\nu_e e^- \to \nu_e e^-$ are NC-t;
$\nu_e e^+ \to \nu_e e^+$ is NC-t $\oplus$ CC-s.

[NB some of these processes are invisible or impossible in ordinary matter]

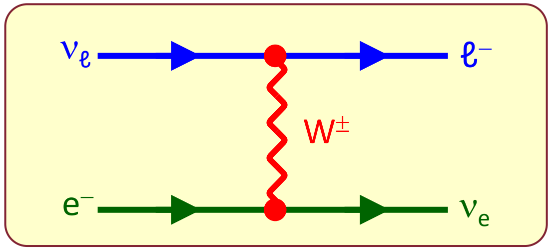


ν interactions : kinematics

An important kinematical constraint.

The threshold energy computation (§ K^0), applied to this case, puts limits on two CC processes :

- the creation of a $\mu^\pm$ requires high energy ν_μ's;
- with present accelerators, no τ's are created, even if the beam contains a ν_τ contamination.



In a process ($ab \to cd$), with b at rest :

$$E_a^{min} = \frac{(m_c + m_d)^2 - m_a^2 - m_b^2}{2m_b}.$$

For $\nu_\mu e^- \to \nu_e \mu^-$:

$m_a \approx m_c \approx 0;\quad m_b = m_e;\quad m_d = m_\mu \quad \to$

$$E_\nu^{min} = \frac{m_\mu^2 - m_e^2}{2m_e} \approx \frac{m_\mu^2}{2m_e} \approx 11 \text{ GeV}.$$

For $\nu_\tau e^- \to \nu_e \tau^-$:

$$E_\nu^{min} \approx \frac{m_\tau^2}{2m_e} \approx 3 \text{ TeV (!!!)}.$$

So, $\nu_\tau e^- \to \nu_e \tau^-$ is NOT possible with present accelerators, even if there is a small number of ν_τ's in a ν_μ beam (from D_s decays) .

CC ν processes

A very simple (possibly the simplest) CC process is the pure lepton scattering ($\nu_\mu e^- \to \mu^- \nu_e$); no hadron garbage, only CC, only one Feynman diagram in l.o. ($\hbar$ = c = 1) :

- in Fermi theory (see), when the energy $E_\nu \gg m_{e,\mu}$, since $\sqrt{s}$ is the only energy scale, for dimensional considerations :

  $\sigma \propto G_F^2 s \approx G_F^2 (2m_e E_\nu) \propto G_F^2 E_\nu$;

  or, with a more refined computation:

  $d\sigma/d\Omega = G_F^2 s/(4\pi^2) = G_F^2 m_e E_\nu / (2\pi^2)$;

  $\sigma = G_F^2 s/\pi = 2\, G_F^2\, m_e E_\nu / \pi$;

  the space isotropy of the cross section is explained by the conservation of the total angular momentum (= 0 both in initial and final state).

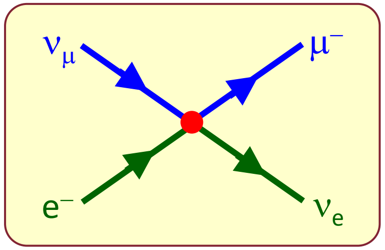
- the above equation reproduces well the data ($\sigma \propto E_\nu$), but becomes "impossible" at high energy, because σ would diverge ("violate unitarity").

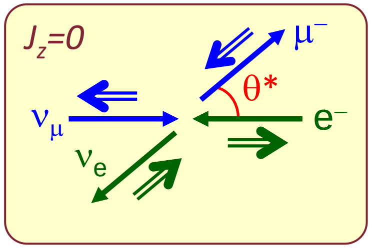
- In the SM, the process is mediated by a $W^\pm \to$ use the W propagator :

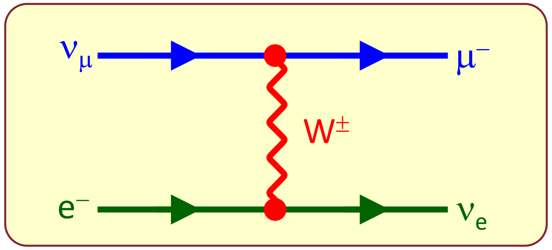
$$\frac{d\sigma}{d\Omega} = \frac{g^4 \alpha^2 m_e E_\nu}{2\pi^2 (m_w^2 + Q^2)^2} \xrightarrow{Q^2 << m_w^2} \frac{g^4 \alpha^2 m_e E_\nu}{2\pi^2 m_w^4};$$

$$\sigma_{Q^2 << m_w^2} = \frac{g^4 \alpha^2}{m_w^4} \frac{2 m_e E_\nu}{\pi} = G_F^2 \frac{2 m_e E_\nu}{\pi} = \sigma_{\mathrm{Fermi}}.$$

- instead, for $Q^2 >> m_W^2$, the cross-section has the (well-understood) 1/s behavior.







CC ν processes: quasi-elastic

However, the purely lepton process is so rare, that it is hard to compare it with data.

A more common process is $\nu_\mu\, n \to \mu^- p$, "the *quasi-elastic* scattering", where nucleons interacts coherently :

- in Fermi theory :

  $d\sigma/d\Omega = G_F^2 s / (4\pi^2) = G_F^2\, m_N E_\nu / (2\pi^2);$

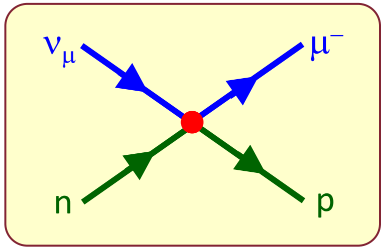
  $\sigma = G_F^2 s / \pi = 2\, G_F^2\, m_N E_\nu / \pi;$

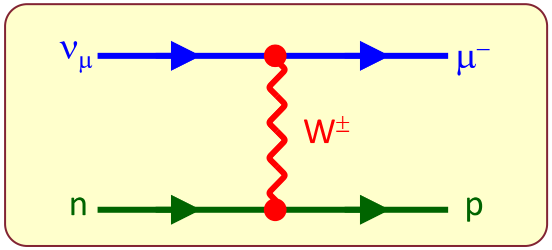
  actually the results agree pretty well with the prediction, as shown in the fig.

- In the SM, the same considerations :

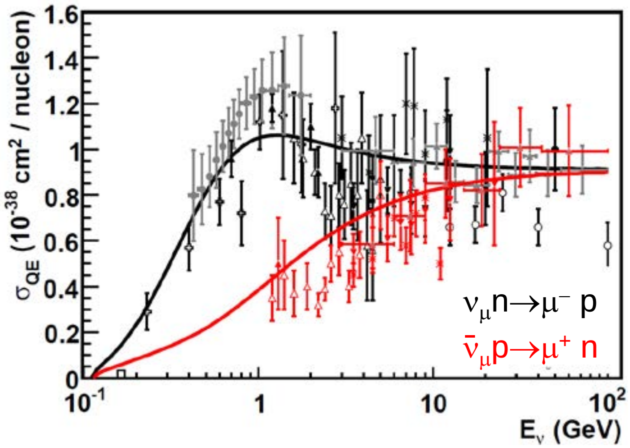
$$d\sigma/d\Omega = g^4\alpha^2\, m_N E_\nu / [2\pi^2\, m_W{}^4] = d\sigma/d\Omega|_{\text{Fermi}};$$

$$\sigma = 2\, g^4\alpha^2\, m_N E_\nu / [\pi\, m_W{}^4] = \sigma_{\text{Fermi}}.$$





- Advantage of the nucleon process over the purely lepton one : the factor m_N/m_e, [$\approx$ 2,000] $\to$ yield measurable with the present experiments.
- …, but paid by the theoretical approximation (the demand of "coherence") and the less clean experimental condition.
- Also valid for $\bar{\nu}_\mu\, p \to \mu^+ n$, which has a similar cross section [*Problem : discuss the spin structure for angular momentum conservation*].



CC ν processes: parton level

- Individual hadronic or semileptonic processes happen at parton level (*at high Q^2 "coherence" becomes meaningless*).
- Partons (=quarks) are :
  - elementary;
  - spin ½;
  - (almost) massless.
- Consider the process :

$$\nu_\mu\, d \to \mu^-\, u.$$

- Do some simple kinematics at parton level, using the DIS variables.
- *The variables y ("inelasticity") and θ* will be used a lot:*

$$\text{LAB sys}\begin{cases}\nu_\mu & (E,\quad E,\quad 0\quad)\\ d & (m_d,\; 0,\quad 0\quad)\\ \mu^- & (E',\; E'\cos\theta,\; E'\sin\theta)\\ u & (\,\ldots,\;\ldots,\quad\ldots\quad)\end{cases}$$

$$\text{CM sys}\begin{cases}\nu_\mu & (E^*,\, E^*,\quad 0\quad)\\ d & (E^*,\, -E^*,\quad 0\quad)\\ \mu^- & (E^*, E^*\cos\theta^*,\; E^*\sin\theta^*)\\ u & (\,\ldots,\;\ldots,\quad\ldots\quad)\end{cases}$$

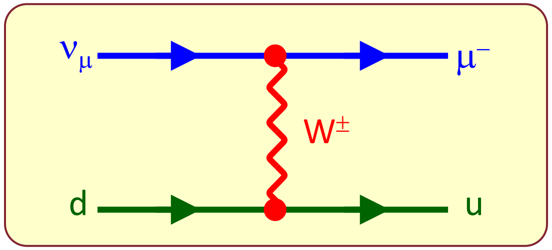
$$p_\mu \cdot p_d\big|_{LAB} = E' m_d = p_\mu \cdot p_d\big|_{CM} = E^{*2}\left(1+\cos\theta^*\right);$$

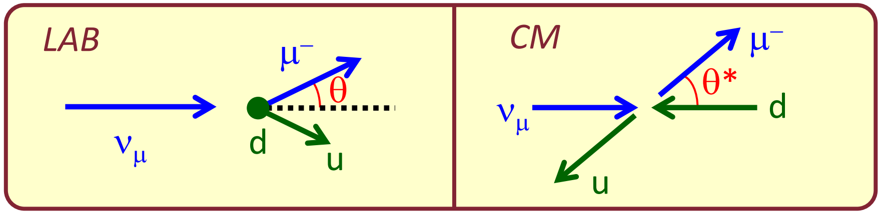
$$p_\nu \cdot p_d\big|_{LAB} = E m_d = p_\nu \cdot p_d\big|_{CM} = 2E^{*2};$$

$$y = \frac{q\cdot P}{k\cdot P} = \frac{\nu}{E} = \frac{E-E'}{E} = 1-\frac{E'}{E} = \frac{1-\cos\theta^*}{2}.$$

$\cos\theta^* = 1 - 2y$

$d\cos\theta^* = -2\, dy$

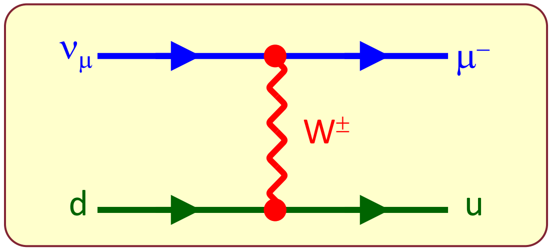


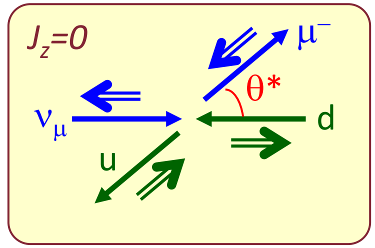


CC ν processes: helicity

Using a "quasi-Fermi" approximation, it is possible to compute angular cross sections for the CC semileptonic processes.

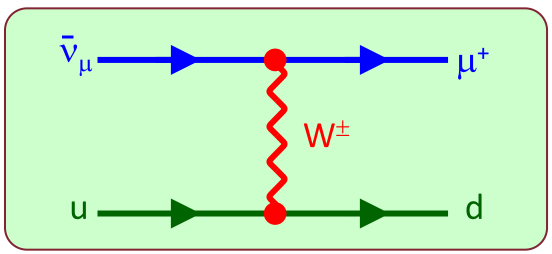
"Quasi-Fermi" means "Fermi-style" total cross-section $\times$ angular dependence from V–A, i.e. CC current $\propto (1-\gamma_5)$.

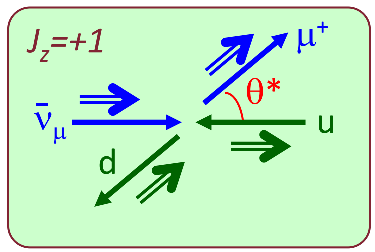




$\nu_\mu d \to \mu^- u:$

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2}; \quad \frac{d\sigma}{dy} = \frac{G_F^2 \hat{s}}{\pi}.$$





$\bar{\nu}_\mu u \to \mu^+ d:$

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2} \times \left(\frac{1+\cos\theta^*}{2}\right)^2;$$

$$\frac{d\sigma}{dy} = \frac{G_F^2 \hat{s}}{\pi} \times (1-y)^2.$$

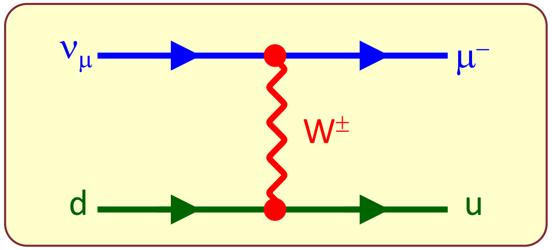
In the $(\bar{\nu}_\mu\ u)$ case, $\theta^*=180°$ clearly violates angular momentum conservation, while $\theta^*=0°$ is allowed : hence the $(1\text{-}y)^2$ factor [*next slide*].

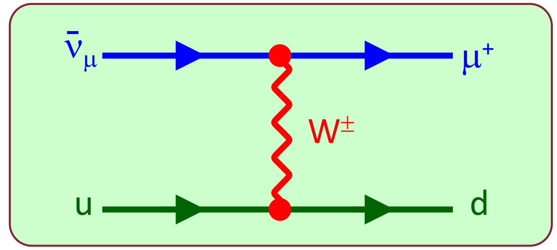
[*notice : θ^* and $\hat{s}$ are the CM variables at parton level, very useful for understanding, but y=(E-E')/E is the experimental variable, which is really measured; in fact, it is independent from the "hadronic garbage"*].

CC ν processes: dσ/dy

all $ff \to ff$

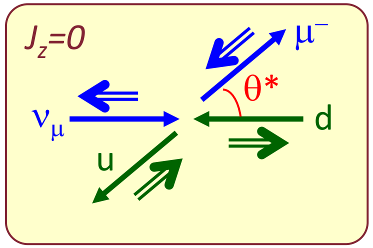
$\bar{f}\bar{f} \to \bar{f}\bar{f}$

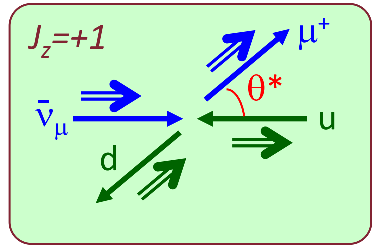




all $f\bar{f} \to f\bar{f}$

constant, NOT cosθ*-dependent





suppressed for θ*=180°

Some simple kinematics :

$$y = 1 - \frac{E'}{E} = \frac{1-\cos\theta^*}{2};$$

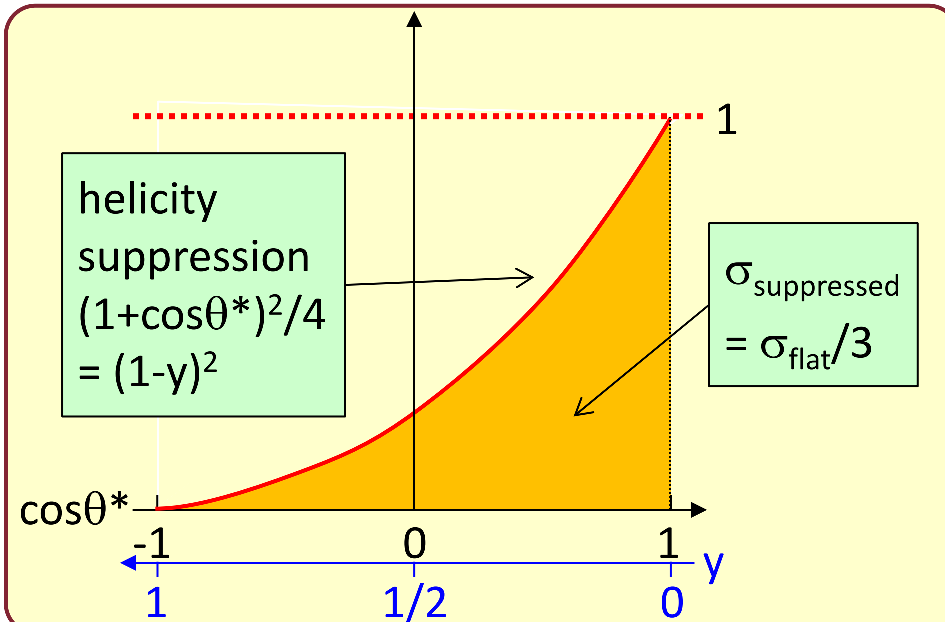
$$\cos\theta^* = 1 - 2y;$$

$$(1+\cos\theta^*)/2 = 1 - y;$$

$$(1+\cos\theta^*)^2/4 = (1-y)^2;$$

$$|d\cos\theta^*| = 2dy;$$

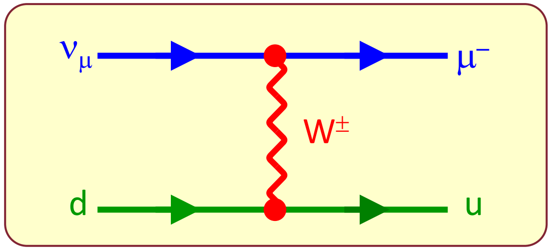
$$d\Omega = d\phi\, d\cos\theta^* = 4\pi dy.$$

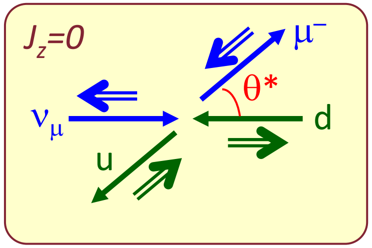


CC ν processes: score

$\nu_\mu d \to \mu^- u$:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2}; \quad \frac{d\sigma}{dy} = \frac{G_F^2 \hat{s}}{\pi}.$$

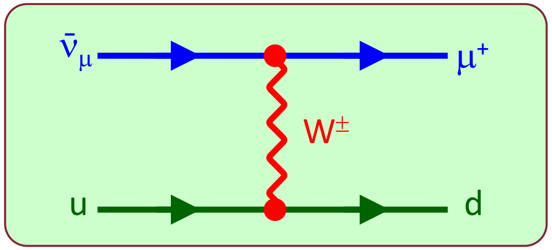


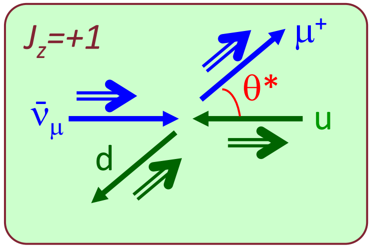


$\bar{\nu}_\mu u \to \mu^+ d$:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2} \times \left(\frac{1+\cos\theta^*}{2}\right)^2;$$

$$\frac{d\sigma}{dy} = \frac{G_F^2 \hat{s}}{\pi} \times (1-y)^2.$$





	process	**J_z**	**$d\sigma/d\cos\theta^*$**	**$d\sigma/dy$**	**σ**
score	$\nu_\mu u \to \mu^-$?, $\bar{\nu}_\mu \bar{u} \to \mu^+$?	**impossible**			
	$\boxed{\nu_\mu d \to \mu^- u}$, $\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}$	0	flat	flat	~1
	$\nu_\mu \bar{u} \to \mu^- \bar{d}$, $\boxed{\bar{\nu}_\mu u \to \mu^+ d}$	1	~$(1+\cos\theta^*)^2/4$	~$(1-y)^2$	~1/3
	$\nu_\mu \bar{d} \to \mu^-$?, $\bar{\nu}_\mu d \to \mu^+$?	**impossible**			

→ isoscalar target
$\sigma(\nu N) > \sigma(\bar{\nu} N)$!!!

Structure functions

Goal : describe the νN ($\bar{\nu}N$) scattering.

All the building blocks have been studied; put everything together :

- the elementary cross section $d\sigma/d\Omega$ (better, $d\sigma/dy$) for individual ν-parton scattering;
- the parton distribution in the nucleon [$f(x)$; x is the fraction of the nucleon momentum, carried by a single parton];
- the "factorization" hypothesis of DIS [*i.e. the interaction regards only one single parton; the other partons do NOT participate*].

For both ν and $\bar{\nu}$, and each final state F:

$$\left.\frac{d^2\sigma(\nu N \to F)}{dxdy}\right|_{s=2E_\nu M} = \sum_j f_j(x) \left.\frac{d\sigma(\nu p_j \to F)}{dy}\right|_{\hat{s}=sx} \quad ;$$

$\hat{s} = sx = 2E_\nu Mx$ = energy^2 at parton level; the sum runs on all interacting partons p_j (q_j, $\bar{q}_j$, both valence and sea).

Connect this picture with the studies of the nucleon structure in eN DIS :

- the quark distributions (**pdf**) have already been defined; [*e.g. u(x)dx is the number of u quarks in the proton with fractional momentum between x and x+dx* ($0 \le x \le 1$)];
- the same for $d(x)$, $s(x)$, $\bar{u}(x)$, $\bar{d}(x)$, $\bar{s}(x)$ …;
- a general formula for ($d^2\sigma$ / $d\Omega dE'$) has been developed, which includes two structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$;
- the transformation $(\Omega, E') \to (x,y)$ is pure (trivial) kinematics [see §2];
- a third function $W_3(Q^2, \nu)$ [$\to F_3(x, Q^2)$] has to be defined, because of terms, like the interference between V and A, which were absent in the ep case;
- if Bjorken scaling holds, the functions F_1 F_2 F_3 are functions of x and not of Q^2.
- the next slides contain the math.

Structure functions : $d^2\sigma/dxdy$

$$\left.\frac{d^2\sigma}{dxdy}\right|_{\substack{ep\\DIS}} = \frac{4\pi\alpha^2\left(s-M^2\right)}{Q^4}\left[xy^2F_1(x,Q^2)+\left(1-y-\frac{M^2xy}{s-M^2}\right)F_2(x,Q^2)\right] =$$

$$\xrightarrow{s>>M^2}\frac{4\pi\alpha^2 s}{Q^4}\left[xy^2F_1(x,Q^2)+(1-y)F_2(x,Q^2)\right];$$

$$\left.\frac{d^2\sigma}{dxdy}\right|_{\substack{\nu p\\DIS}} = \frac{G_F^2 s}{2\pi}\left[xy^2F_1^{\nu p}(x,Q^2)+(1-y)F_2^{\nu p}(x,Q^2)+xy\left(1-\frac{y}{2}\right)F_3^{\nu p}(x,Q^2)\right];$$

$$\left.\frac{d^2\sigma}{dxdy}\right|_{\substack{\bar{\nu} p\\DIS}} = \frac{G_F^2 s}{2\pi}\left[xy^2F_1^{\bar{\nu} p}(x,Q^2)+(1-y)F_2^{\bar{\nu} p}(x,Q^2)-xy\left(1-\frac{y}{2}\right)F_3^{\bar{\nu} p}(x,Q^2)\right].$$

For the νn scattering, $\left(F_1^{\nu p},F_2^{\nu p},F_3^{\nu p}\right)\rightarrow\left(F_1^{\nu n},F_2^{\nu n},F_3^{\nu n}\right)$, and so on.

Structure functions : $d^2\sigma/dxdy$

- Define u(x), d(x), ū(x), đ(x) the x-distribution of quarks u, d, ū, đ in the proton;
- then, some simple consistency relations between p and n follows :
- [*first* ❶ *the algebra on the right, then* ❷ *the case* νp *fully computed in the next slide, finally* ❸ *the results, equating the coefficients with same power of y*];
- notice that the Callan-Gross equation (see next slide) comes out again, together with other "rules".

❶

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[d(x) + (1-y)^2\,\overline{u}(x)\right];$$

$$\frac{d^2\sigma(\overline{\nu} p)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[\overline{d}(x) + (1-y)^2\,u(x)\right];$$

$$\frac{d^2\sigma(\nu n)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[d^n(x) + (1-y)^2\,\overline{u}^n(x)\right];$$

$$\frac{d^2\sigma(\overline{\nu} n)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[\overline{d}^n(x) + (1-y)^2\,u^n(x)\right];$$

$$u^n(x) = d(x); \quad \overline{u}^n(x) = \overline{d}(x);$$

$$d^n(x) = u(x); \quad \overline{d}^n(x) = \overline{u}(x);$$

$$\frac{d^2\sigma(\nu n)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[u(x) + (1-y)^2\,\overline{d}(x)\right];$$

$$\frac{d^2\sigma(\overline{\nu} n)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[\overline{u}(x) + (1-y)^2\,d(x)\right];$$

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G_F^2 s}{2\pi}\left[\begin{array}{l} xy^2F_1^{\nu p}(x) + (1-y)F_2^{\nu p}(x) + \\ +xy(1-y/2)F_3^{\nu p}(x) \end{array}\right].$$

Structure functions: $d^2\sigma/dxdy$

- math for the νp case shown in ❷;
- *neglect heavy quarks, i.e.* $s(x) = \bar{s}(x) = 0$;
- νn, $\bar{\nu} p$, $\bar{\nu} n$ left as an exercise; results for νn shown in ❸ together with νp.

❷

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G_F^2 sx}{\pi}\frac{2}{2}\left[d(x) + (1-y)^2\,\bar{u}(x)\right];$$

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G_F^2 s}{2\pi}\left[\begin{array}{l} xy^2 F_1^{\nu p}(x) + (1-y)F_2^{\nu p}(x) + \\ + xy(1-y/2)F_3^{\nu p}(x)\end{array}\right];$$

a) $y^0 \rightarrow \boxed{2x(d+\bar{u}) = F_2}$;

b) $y^1 \rightarrow 2x(-2)\bar{u} = -4x\bar{u} = -F_2 + xF_3 =$

$= -[2xd + 2x\bar{u}] + xF_3$;

$xF_3 = 2xd - 2x\bar{u} = 2x(d-\bar{u})$;

$\boxed{F_3 = 2(d-\bar{u})}$;

c) $y^2 \rightarrow 2x\bar{u} = xF_1 - xF_3/2$;

$xF_1 = 2x\bar{u} + x[2(d-\bar{u})]/2 = x(d+\bar{u})$;

$\boxed{F_1 = d+\bar{u}}$;

$\boxed{2xF_1 = F_2}$ (Callan − Gross).

❸

$$F_2^{\nu p}(x) = 2xF_1^{\nu p}(x) = 2x[d(x) + \bar{u}(x)];$$
$$xF_3^{\nu p}(x) = 2x[d(x) - \bar{u}(x)];$$
$$F_2^{\nu n}(x) = 2xF_1^{\nu n}(x) = 2x[u(x) + \bar{d}(x)];$$
$$xF_3^{\nu n}(x) = 2x[u(x) - \bar{d}(x)].$$

Structure functions: results

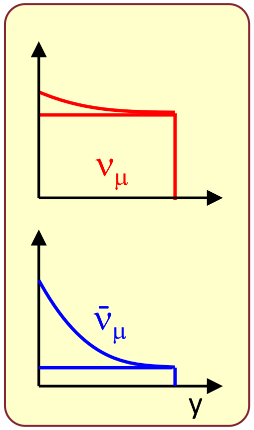
For CC process (ν_μ N) and ($\bar{\nu}_\mu$ N), expect [target "isoscalar", i.e. composed by same number of p / n (all heavy materials] :

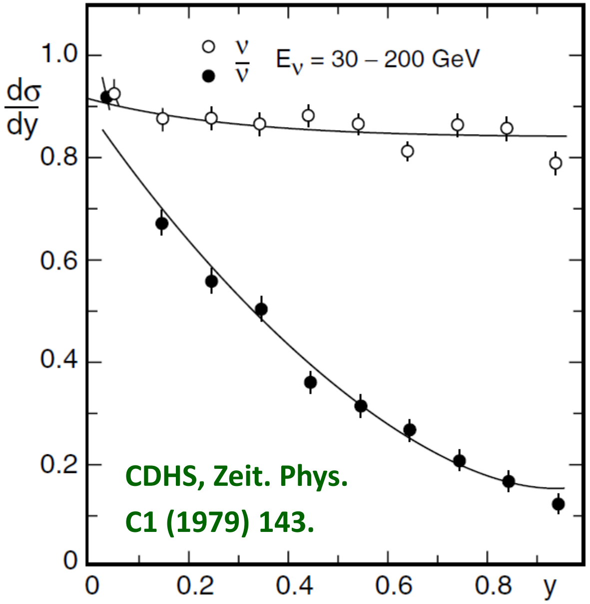
- same number of u and d (valence), and much smaller amount of ū đ (sea); s and s̄ are negligible;
- for ν_μ a mixture of (ν_μ d) and (ν_μ ū), because (ν_μ u) and (ν_μ đ) do NOT interact in CC;
- for $\bar{\nu}_\mu$ a mixture of ($\bar{\nu}_\mu$ u) and ($\bar{\nu}_\mu$ đ);
- (ν_μ d), ($\bar{\nu}_\mu$ đ) have flat y distributions;
- (ν_μ ū), ($\bar{\nu}_\mu$ u) proportional to $(1-y)^2$;

➢ for ν_μ, expectation is large constant + some minor parabolic contribution;

➢ for $\bar{\nu}_\mu$, it is the opposite: a dominant parabola + a small constant;

- plot dσ/dy for ν and $\bar{\nu}$ after integrating over x and E_ν: ***great success !!!***





$$\frac{d^2\sigma(\nu N)}{dxdy} = \frac{1}{2}\left[\frac{d^2\sigma(\nu p)}{dxdy} + \frac{d^2\sigma(\nu n)}{dxdy}\right] = \frac{G_F^2 sx}{2\pi}\left\{\left[u(x)+d(x)\right] + (1-y)^2\left[\bar{u}(x)+\bar{d}(x)\right]\right\} = \frac{G_F^2 sx}{2\pi}\left[q(x) + (1-y)^2\bar{q}(x)\right];$$

$$\frac{d^2\sigma(\bar{\nu} N)}{dxdy} = \frac{1}{2}\left[\frac{d^2\sigma(\bar{\nu} p)}{dxdy} + \frac{d^2\sigma(\bar{\nu} n)}{dxdy}\right] = \frac{G_F^2 sx}{2\pi}\left\{\left[\bar{u}(x)+\bar{d}(x)\right] + (1-y)^2\left[u(x)+d(x)\right]\right\} = \frac{G_F^2 sx}{2\pi}\left[\bar{q}(x) + (1-y)^2 q(x)\right].$$

Structure functions: $\nu N \leftrightarrow eN$

- For an isoscalar target, we get

$$F_2^{\nu N} = (F_2^{\nu p}+F_2^{\nu n}) \ / \ 2 = \\ = x\,[\,u(x) + d(x) + \bar{u}(x) + \bar{d}(x)];$$

$$F_2^{eN} = (F_2^{ep}+F_2^{en}) \ / \ 2 = \\ = 5x/18\,[\,u(x) + d(x) + \bar{u}(x) + \bar{d}(x)];$$

therefore :

$$F_2^{eN}(x) = 5/18\ F_2^{\nu N}(x).$$

[the value 5/18 is just the average of the quark charges squared : $[(\frac{1}{3})^2 + (\frac{2}{3})^2\,]/2$.]

[*in other words, in e.m. processes the interactions are proportional to e*^{2}*, while in CC scattering they are normalized to 1; there is no relative normalization between e.m. e CC in the rule*].

→ eN

- For F_3, we get

$$F_3^{\nu N} = (F_3^{\nu p}+F_3^{\nu n}) \ / \ 2 = \\ = [u(x) + d(x) - \bar{u}(x) - \bar{d}(x)];$$

the structure functions have contributions from valence and sea :

- $u(x) = u_v(x) + u_s(x) = u_v(x) + Sea(x);$
- $\bar{u}(x) = \bar{u}_s(x) = Sea(x);$
- $\int_0^1 u_v(x)\,dx = 2; \quad \int_0^1 d_v(x)\,dx = 1,$

then

$$F_3^{\nu N} = [\,u(x) + d(x) - \bar{u}(x) - \bar{d}(x)] = \\ = u_v(x) + d_v(x);$$

$$\int_0^1 F_3^{\nu N}(x)\,dx = \int_0^1 [u_v(x) + d_v(x)]\,dx = 3;$$

known as the Gross – Llewellyn-Smith sum rule.

- Experimentally, the G.-L.S. rule is well verified = 3.0 ± 0.2.

Structure functions: $\nu N \leftrightarrow eN$

- In the same Q^2 range, F_2^ν from CDHS data shows a nice agreement with 18/5 × e.m. (μ^- from EMC, e^- from MIT).
- The figure shows also the contribution of F_3^ν and the antiquarks alone.
- Since $\int (1-y)^2\, dy = 1/3$, if there were no $\bar{q}$ in the nucleon, we would expect :

  $\sigma^{\nu N} / \sigma^{\bar{\nu} N} \approx 3.$
- If instead the cross-sections are written in terms of quarks and antiquarks :

  $\sigma^{\nu N} = G_F^2\, s / (2\pi)\ \ [f_q + ⅓\, f_{\bar{q}}];$

  $\sigma^{\bar{\nu} N} = G_F^2\, s / (2\pi)\ \ [⅓\, f_q + f_{\bar{q}}];$

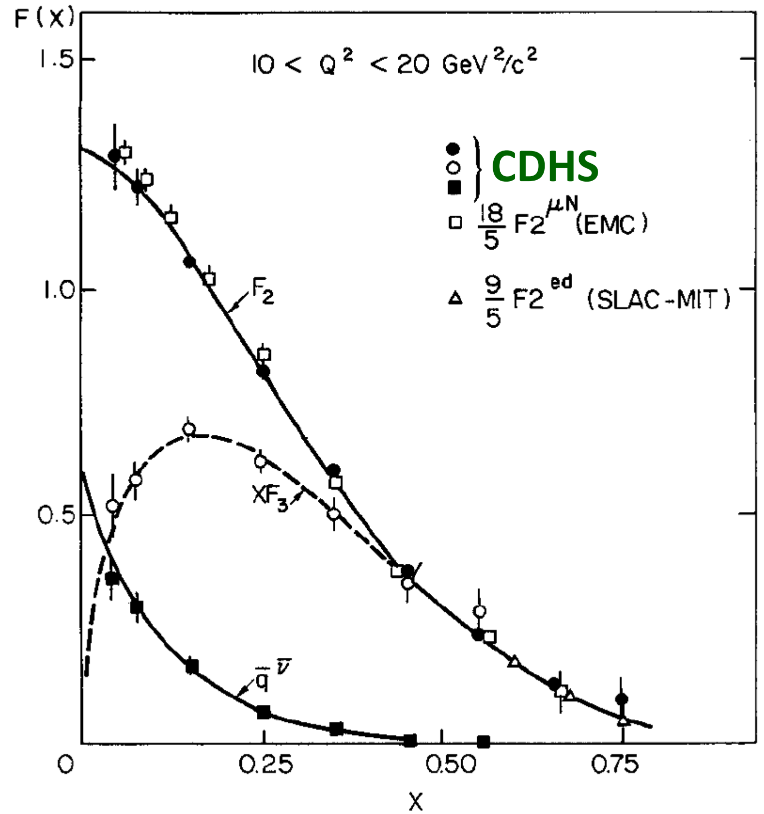
  then, the value of f_q and $f_{\bar{q}}$ can be measured :

  $f_q \approx 0.41;\ f_{\bar{q}} \approx 0.08 \ \rightarrow\ f_g \approx 0.50;$

- taking into account the $\bar{q}$ fraction, we expect

  $\sigma^{\nu N} / \sigma^{\bar{\nu} N} \approx [f_q + ⅓\, f_{\bar{q}}] / [⅓\, f_q + f_{\bar{q}}] \approx 2;$

  in reasonable agreement with the measurement [*see page 1 !!!*].



The discovery of neutral currents

- The search for NC events began in the early 1960s, when the e.w. theory of Glashow – Weinberg – Salam was still thought not to be "renormalizable".
- The searches were limited to FCNC: possible NC "non-FC" processes were thought to be obscured by e.m. currents [*in analogy with weak CC, which is visible only when flavor is violated*].
- Decays like $K^+ \to \pi^+ e^+ e^-$ and $K^0 \to \mu^+ \mu^-$ were searched and NOT found.
- The only escape from this difficulty is to make use of neutral particles, which do NOT sense e.m. interactions : the ν's.
- The signature for this process is given by the absence in the final state of a charged lepton, which is unavoidable in the CC coupling $\nu \ell^\pm W^\mp$.
- Motivated by the recent discovery of the renormalizability of the SM ('t Hooft and Veltman, 1971), the experimentalists from both sides of the Atlantic began a new "hunt" for neutral currents.

Historical Note: In 1960, experiments at CERN, by using a heavy liquid chamber and a ν beam, looked for NC. Unfortunately, they found that the ratio NC/CC is < 3%, a value much smaller than the correct one. This error was eventually corrected, but the new limit (12%) was published only in 1970.

The discovery of neutral currents

- The events [*see before*] were of the type
  - (a) $\nu_\mu + N \rightarrow \nu_\mu + X$;
  - (b) $\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + X$;
  - (c) $\nu_\mu + e^- \rightarrow e^- + \nu_\mu$;
  - (d) $\bar{\nu}_\mu + e^- \rightarrow e^- + \bar{\nu}_\mu$;

  ["X" = hadronic system, without leptons].
- In 1973, the newly built Gargamelle was filled with 15 tons of Freon ($C\,F_3\,Br$).
- The first event interpreted as a pure leptonic NC.
- They had the following criteria :
  - fiducial volume 3 m^3;
  - events were defined as NC if :
    - i. no visible $\mu^\pm$ is present;
    - ii. no charged track escapes the confidence volume;
  - Instead, events were CC if :
    - i. a clearly visible $\mu^\pm$ is present;
    - ii. the $\mu^\pm$ has to exit out of the chamber.
- Results:
  - ν beam : 102 NC, 428 CC, 15 n$^{(*)}$;
  - $\bar{\nu}$ beam : 64 NC, 148 CC, 12 n$^{(*)}$.
- The result is then :
  - NC/CC (ν) = 0.21 $\pm$ 0.03;
  - NC/CC ($\bar{\nu}$) = 0.45 $\pm$ 0:09;
  - **inconsistent with the absence** of NC.

$^{(*)}$ *The main background was due to neutrons produced by ν's in the chamber structure.*

There was also an American team, looking for NC. After an exciting race, they were unable to publish conclusive results before the Europeans.
Actually, the discovery of NC marks a clear turning point in high energy physics : after that, Europe was not anymore the expected looser in the game.

NC ν processes: couplings

The NC couplings do depend on the fermion type f :

symbol	formula	definition (physical meaning)
g		SU(2) coupling constant
g'		U(1) coupling constant
$\tan\theta_W$	$\equiv g' / g$	tangent (Weinberg angle)
e	$\equiv g \sin\theta_W$	e^+ charge (= – e^- charge)
g^f_V	$= I^f_{Wz} - 2 Q^f \sin^2\theta_W$	NC vector coupling (also v_f, c_v)
g^f_A	$= I^f_{Wz}$	NC axial coupling (a_f, c_a)
g^f_L	$= ½ (g^f_V + g^f_A) = I^f_{Wz} - Q^f \sin^2\theta_W$	"left-handed" NC coupling
g^f_R	$= ½ (g^f_V - g^f_A) = - Q^f \sin^2\theta_W$	"right-handed" NC coupling
m^2_W	$\equiv \pi\alpha / (\sqrt{2}\, G_F \sin^2\theta_W)$	$[W^\pm \text{ mass}]^2$ *[careful : m^2_W !!!]*
m_Z	$= m_W / \cos\theta_W$	Z mass

f	Q_f	g^f_V ($\sin^2\theta_W$=0.231)		$I^f_{Wz} = g^f_A$	g^f_L	g^f_R
$\nu_e\ \nu_\mu\ \nu_\tau$	0	+½+0	= +0.500	+½	+½	0
$e^-\ \mu^-\ \tau^-$	–1	$-½ + 2 \sin^2\theta_W$	$\boxed{= -0.038}$	–½	$-½ + \sin^2\theta_W$	$+ \sin^2\theta_W$
u c t	⅔	$+½ - {}^4/_3 \sin^2\theta_W$	= +0.192	+½	$+½ - ⅔ \sin^2\theta_W$	$- ⅔ \sin^2\theta_W$
d s b	–⅓	$-½ + ⅔ \sin^2\theta_W$	= –0.346	-½	$-½ + ⅓ \sin^2\theta_W$	$+ ⅓ \sin^2\theta_W$

remember: $g^e_V \approx 0$

NC ν processes: σ

Some algebra, not really difficult, but quite tedious, produces for NC the analogous formulas already derived for CC :

ƒ : point-like fermions (ℓ⁻, ν, q);

ƒ̄ : point-like anti-fermions (ℓ⁺, ν̄, q̄);

N : "isoscalar" nucleon (p+n)/2;

N' : final state hadronic system.

$$\frac{d\sigma(\nu_\mu f \to \nu_\mu f)}{dy} = \frac{G_F^2 \hat{s}}{\pi}\left[\left\{g_L^f\right\}^2 + (1-y)^2\left\{g_R^f\right\}^2\right];$$

$$\frac{d\sigma(\overline{\nu}_\mu f \to \overline{\nu}_\mu f)}{dy} = \frac{G_F^2 \hat{s}}{\pi}\left[\left\{g_R^f\right\}^2 + (1-y)^2\left\{g_L^f\right\}^2\right];$$

e.g. Rev. Mod.Phys. 70, 1341 (1998)

$$\frac{d^2\sigma(\nu_\mu N \to \nu_\mu N')}{dxdy} = \frac{G_F^2 sx}{2\pi}\left\{\begin{array}{l}\left[\left(\left\{g_L^u\right\}^2+\left\{g_L^d\right\}^2\right)+(1-y)^2\left(\left\{g_R^u\right\}^2+\left\{g_R^d\right\}^2\right)\right]q(x)+ \\ +\left[\left(\left\{g_R^u\right\}^2+\left\{g_R^d\right\}^2\right)+(1-y)^2\left(\left\{g_L^u\right\}^2+\left\{g_L^d\right\}^2\right)\right]\overline{q}(x)\end{array}\right\};$$

$$\frac{d^2\sigma(\overline{\nu}_\mu N \to \overline{\nu}_\mu N')}{dxdy} = \frac{G_F^2 sx}{2\pi}\left\{\begin{array}{l}\left[\left(\left\{g_R^u\right\}^2+\left\{g_R^d\right\}^2\right)+(1-y)^2\left(\left\{g_L^u\right\}^2+\left\{g_L^d\right\}^2\right)\right]q(x)+ \\ +\left[\left(\left\{g_L^u\right\}^2+\left\{g_R^d\right\}^2\right)+(1-y)^2\left(\left\{g_R^u\right\}^2+\left\{g_R^d\right\}^2\right)\right]\overline{q}(x)\end{array}\right\}.$$

NC ν processes: R_ν and $R_{\bar\nu}$

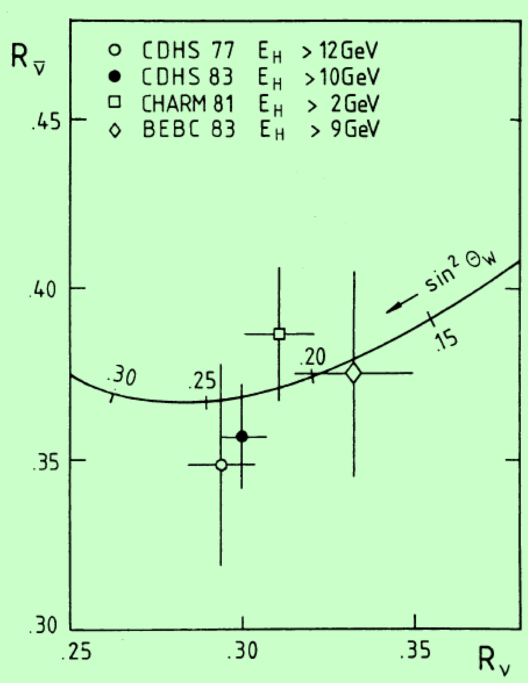
To measure $\sin^2\theta_w$:

- produce some algebra [*next slide, not for the exam*]:
  1. start with the CC and NC cross sections for isoscalar targets;
  2. neglect the sea contributions ū(x), đ(x);
  3. integrate over x and y (∫(1-y)²dy = ⅓);
  4. divide the cross sections, to cancel the dependence of all the other parameters;
  5. use g_L and g_R for each f(ermion) :

$$R_\nu \equiv \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)} \approx \frac{1}{2} - \sin^2\theta_w + \frac{20}{27}\sin^4\theta_w;$$

$$R_{\bar\nu} \equiv \frac{\sigma_{NC}(\bar\nu N)}{\sigma_{CC}(\bar\nu N)} \approx \frac{1}{2} - \sin^2\theta_w + \frac{20}{9}\sin^4\theta_w.$$

- The values of R_ν and $R_{\bar\nu}$ are well defined and, <u>at least in principle</u>, easy to measure :
  - ➢ unknown or difficult-to-measure parameters cancel out;
  - ➢ exp. systematics, beam effects, detector … (see next slides).



[Dieter Haidt, CERN school '84, CERN yellow report 85-11]

old data, but useful to explain the method :

$$\begin{cases} R_\nu = R_\nu(\sin^2\theta_w) \\ R_{\bar\nu} = R_{\bar\nu}(\sin^2\theta_w) \end{cases} \rightarrow$$

a point for each value of $\sin^2\theta_w$ → a curve in the plane R_ν / $R_{\bar\nu}$ → measure $\sin^2\theta_w$.

NC ν processes: $d^2\sigma/dxdy$

1. Start with the CC and NC cross sections for isoscalar targets;

not difficult, but NOT for the exam.

2. Neglect the sea contributions $\bar{u}(x)$, $\bar{d}(x)$;

3. Integrate over y $\left[\int_0^1 (1-y)^2 dy = 1/3\right]$;

4. Use g_L^f and g_R^f from the previous tables $\left[g_R^{u2} + g_R^{d2} = \frac{5}{9}\sin\theta_w^4, \quad g_L^{u2} + g_L^{d2} = \frac{1}{2} - \sin\theta_w^2 + \frac{5}{9}\sin\theta_w^4\right]$;

5. Divide NC/CC.

$$\text{CC}: \left[\begin{array}{l} \dfrac{d^2\sigma(\nu_\mu N \to \mu^- N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}\left[q(x) + (1-y)^2\,\bar{q}(x)\right]; \\ \dfrac{d^2\sigma(\bar{\nu}_\mu N \to \mu^+ N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}\left[\bar{q}(x) + (1-y)^2\,q(x)\right]; \end{array}\right. \to \text{CC}: \left[\begin{array}{l} \dfrac{d^2\sigma(\nu_\mu N \to \mu^- N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi} q(x); \\ \dfrac{d^2\sigma(\bar{\nu}_\mu N \to \mu^+ N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}(1-y)^2\,q(x); \end{array}\right.$$

$$\text{NC}: \left[\begin{array}{l} \dfrac{d^2\sigma(\nu_\mu N \to \nu_\mu N')}{dxdy} = [\text{prev.slide}]; \\ \dfrac{d^2\sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu N')}{dxdy} = [\text{prev.slide}]; \end{array}\right. \to \text{NC}: \left[\begin{array}{l} \dfrac{d^2\sigma(\nu_\mu N \to \nu_\mu N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}\left[\begin{array}{l}\left(g_L^{u2} + g_L^{d2}\right) + \\ +(1-y)^2\left(g_R^{u2} + g_R^{d2}\right)\end{array}\right] q(x); \\ \dfrac{d^2\sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}\left[\begin{array}{l}\left(g_R^{u2} + g_R^{d2}\right) + \\ +(1-y)^2\left(g_L^{u2} + g_L^{d2}\right)\end{array}\right] q(x). \end{array}\right.$$

NC ν processes: $\sin^2\theta_w$

Most recent results :

- $\sin^2\theta_w$ = 0.2356 ± .0050 CHARM
- = 0.2250 ± .0050 CDHS
- = 0.2332 ± .0015 (a)
- = 0.2251 ± .0039 (b).

Notes :

- (a) and (b) are "today's best" [PDG], for ν's on isoscalar target:
- they differ because of two different "definitions" of higher order parameters (see the radiative corrections in § LEP).

The quantities REALLY measured are R_ν ($R_{\bar\nu}$) :

$$R_\nu \equiv \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)} = \frac{\varepsilon_{NC}\left[n_{NC}^{tot} - n_{NC}^{bckg}\right]}{\int \Phi(\nu)dE} \frac{\int \Phi(\nu)dE}{\varepsilon_{CC}\left[n_{CC}^{tot} - n_{CC}^{bckg}\right]} = \frac{\varepsilon_{NC}\left[n_{NC}^{tot} - n_{NC}^{bckg}\right]}{\varepsilon_{CC}\left[n_{CC}^{tot} - n_{CC}^{bckg}\right]}.$$

The flux cancels out; this is not a good news, because ε_{NC} and ε_{CC} DO depend on E_ν, and are very different for CC and NC, so better know the E_ν dependence on σ.

In fact :

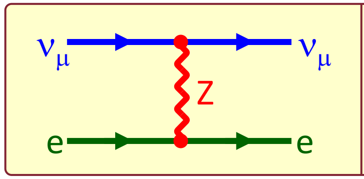
- CC, due to the presence of a charged $\mu^\pm$, are "easy" to detect, and relatively background free (n^{bckg} small);
- NC, however, are hardly distinguishable from cosmics and CC-low-energy;
- at low y, $\mu^\pm$ id. is difficult → the selection algorithm gets confused : CC → NC .

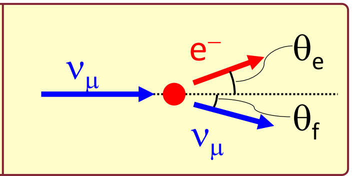
Therefore :

- ➢ accurate computation of the flux as a function of E_ν;
- ➢ accurate understanding of the systematics;
- ➢ reproduction via montecarlo, to study algorithms and systematics.

Pure leptonic ν processes : kinematics

- The cleanest NC process are
  $(\nu_\mu e^- \to \nu_\mu e^-)$ and $(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-)$.
- In fact, no hypothesis on "isoscalarity", no dependence on structure functions, on sea-content of the nucleon, …
- Only one problem : cross section ($\propto s = 2m_eE_\nu$) VERY small :
  $s(\nu_\mu e^-) = 2\,m_eE_\nu \approx s(\nu_\mu N)$ / 2,000.
- However, the process has been extensively studied.
- The problem : select the tiny number of signal events from the overwhelming NC (hadronic) events.
- The key is the very particular kinematics (see box).





Lab sys. ($i = \nu_{initial}$, $f = \nu_{final}$, $p_i \approx E_i$, $p_f \approx E_f$, $p_e \approx E_e$) :

E) $E_i + m_e = E_e + E_f$;

x) $E_i = E_e \cos\theta_e + E_f \cos\theta_f$;

y) $0 = E_e \sin\theta_e + E_f \sin\theta_f$.

Subtract (x) from (E) and $\times$ 2 :

$2m_e = 2E_e\,(1 - \cos\theta_e) + 2E_f\,(1 - \cos\theta_f)$;

$0 \leq 2\,E_e\,(1 - \cos\theta_e) \approx E_e\,\theta_e^2 \leq 2\,m_e$;

i.e.

1. the value of E_e is (almost always) many GeV (think to the y distribution);
2. The angle θ_e must be very small : $\theta_e^2 \leq 2\,m_e/E_e$;
3. the ν variables (E_i, E_f, θ_f) are not measured;
4. it is therefore compulsory to measure the <u>e.m. shower</u> (= E_e) very well;
5. … and (even more important) its <u>direction</u> θ_e;
6. and SELECT on ($E_e\,\theta_e^2$).

Pure leptonic ν processes : data selection

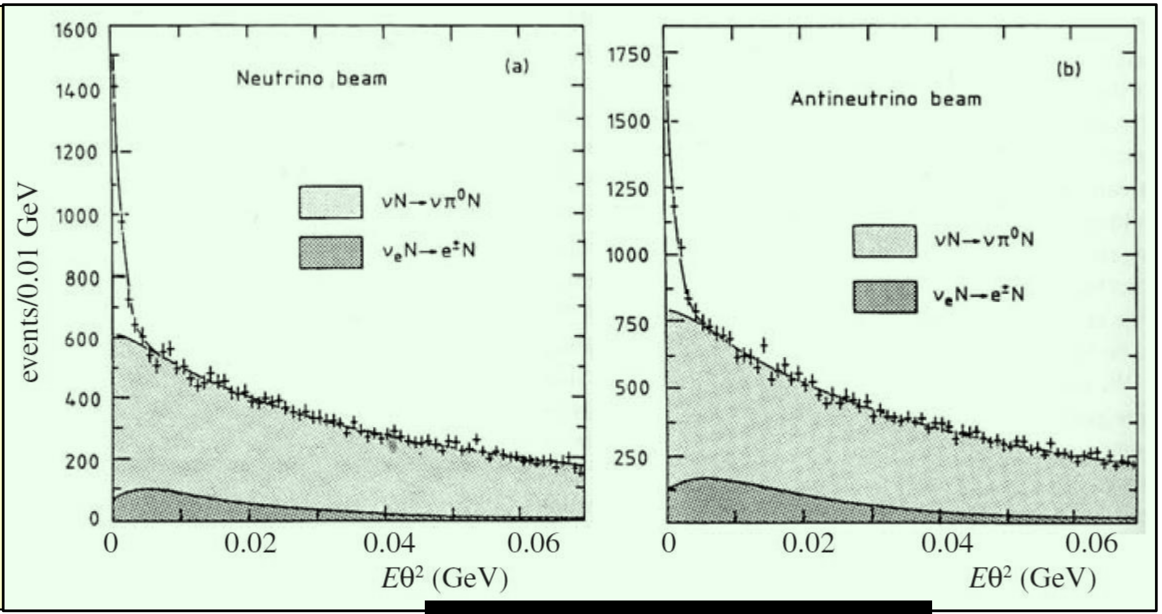
- The extraction of the signal requires the rejection of the background.
- The main one is due to NC hadronic interactions, without $\mu^\pm$ in the final state, with one or more π^0's; the photons due to π^0 decays mimic the electron shower.
- To reject those events, the deposit of energy in the early scintillators is used.
- Since $\pi^0 \rightarrow 2\gamma \rightarrow 4e^\pm$, a scintillator, if crossed at a very early stage of the shower development, sees 4 minimum ionizing particles, instead of only one.
- In this way, by using only the part of the detector immediately upstream of the scintillator, a much better isolation of the signal is obtained, at the price of a reduced statistics.

Three "populations" :

- the signal;
- hadronic NC;
- CC due to ν_e beam background;

The selection is statistical, NOT on an event-by-event basis.

[NOT because of quantum mechanics, but selection method]



CHARM, Phys. Lett. B 335, 246 (1994)

Pure leptonic ν processes : analysis

- The pure leptonic process is the cleanest and most systematic-free NC process.
- It has been used to measure θ_w.
- The price is a reduction ~2,000 in statistics and a difficult selection procedure.

$$\frac{d\sigma_{NC}(\nu_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi}\left[\left(g_L^e\right)^2 + (1-y)^2\left(g_R^e\right)^2\right];$$

$$\frac{d\sigma_{NC}(\bar{\nu}_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi}\left[\left(g_R^e\right)^2 + (1-y)^2\left(g_L^e\right)^2\right];$$

$$\sigma_{NC}(\nu_\mu e^-) = \frac{G_F^2 s}{4\pi}\left[1 - 4\sin^2\theta_w + \frac{16}{3}\sin^4\theta_w\right];$$

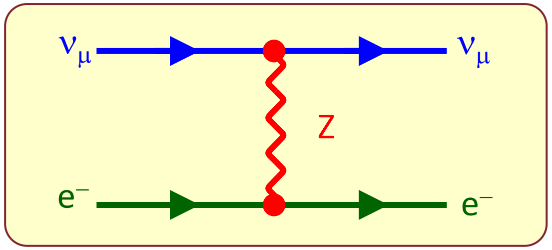
$$\sigma_{NC}(\bar{\nu}_\mu e^-) = \frac{G_F^2 s}{12\pi}\left[1 - 4\sin^2\theta_w + 16\sin^4\theta_w\right];$$

$$R_{NC}^{\nu_\mu e} \equiv \frac{\sigma_{NC}(\nu_\mu e^-)}{\sigma_{NC}(\bar{\nu}_\mu e^-)} = 3\frac{\left[1 - 4\sin^2\theta_w + \frac{16}{3}\sin^4\theta_w\right]}{\left[1 - 4\sin^2\theta_w + 16\sin^4\theta_w\right]}.$$

- The ratio being really measured is

$$R_{NC}^{\nu_\mu e} \equiv \frac{\sigma(\nu_\mu e^- \to \nu_\mu e^-)}{\sigma(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-)} = \frac{\varepsilon_\nu\left[n_\nu^{tot} - n_\nu^{bckg}\right]}{\int \Phi(\nu)dE}\frac{\int \Phi(\bar{\nu})dE}{\varepsilon_{CC}\left[n_{\bar{\nu}}^{tot} - n_{\bar{\nu}}^{bckg}\right]}.$$

- A key point is the ratio of the fluxes, which is computed in many ways (as simulations of the primary interactions + measurements in the decay tunnel, cross-checks with other known processes).
- Final result in the fluxes ratio : ± 2% (syst), $\to \Delta\sin^2\theta_w = \pm 0.005$.

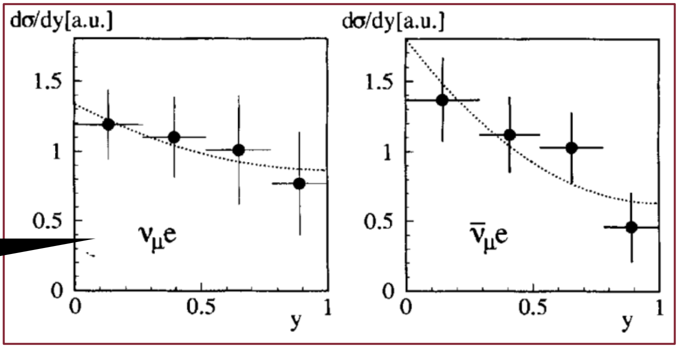


Pure leptonic ν processes : results

Results (from $\nu_\mu e$) :

- $\sin^2\theta_W$ = 0.2324 ± .0058 ± .0059 CHARM
- = 0.2311 ± .0077 (a)
- = 0.2230 ± .0077 (b).

____



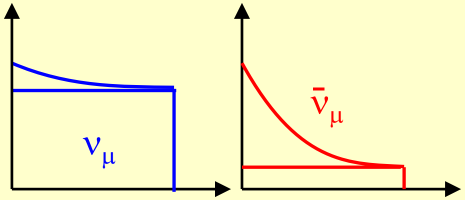
CHARM, Phys. Lett. B 320, 203 (1994).

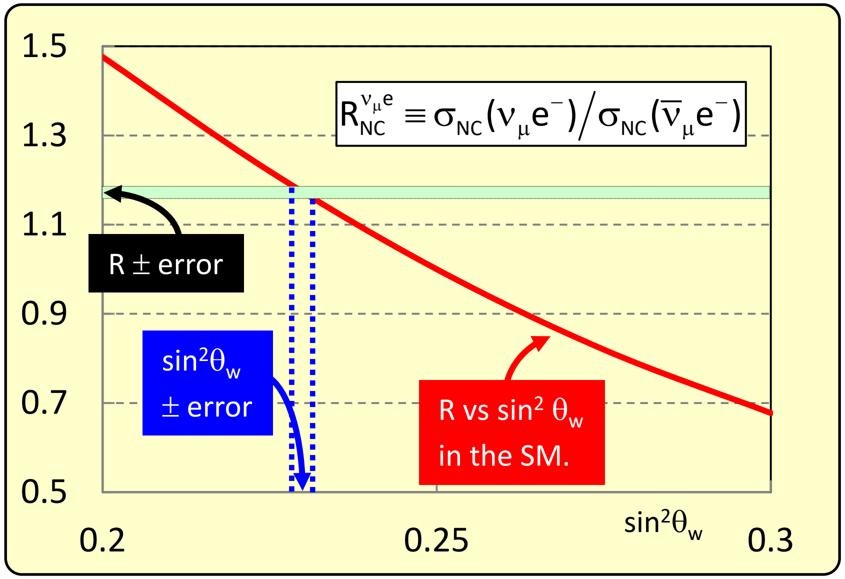
(a) and (b) are from current PDG, for ν's on isoscalar target:

- ➢ different because of definition of higher order parameters ("scheme", see the radiative corrections in § LEP).
- ➢ the y-distributions contain information on g_L and g_R (i.e. a new determination of the couplings) + a cross-check.

$$\frac{d\sigma_{NC}(\nu_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi}\left[g_L^{e2} + (1-y)^2 g_R^{e2}\right];$$

$$\frac{d\sigma_{NC}(\bar\nu_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi}\left[g_R^{e2} + (1-y)^2 g_L^{e2}\right].$$





sketch of the method $R \to \sin^2\theta_W$ [see previous page]

sketch of the method $d\sigma(\nu,\bar\nu)/dy \to g_L, g_R$.

References

1. e.g. [BJ, 14.3], [YN1, 17.7-8], [YN2, 2.1-3];
2. old review : Steinberger, CERN 76-20 (Yellow report);
3. more modern review : Rev.Mod.Phys. 70 (1998) 1341;
4. ν beams : Kopp, Phys.Rep. 439 (2007) 101.

Found on the web – Courtesy of an unknown author.

Gustave Doré (1832–1883) - Pantagruel with his father Gargantua and mother Gargamelle - watercolor

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End of chapter 7

Particle Physics - Chapter 8
Colliders : $\bar{p}p$ – e^+e^- – pp

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AA 18-19

last mod.
15-Apr-19

8 – Colliders : p̄p – LEP – pp

i. Accelerators

1) Colliders

2) Synchrotron

3) Luminosity

ii. Physics

4) Scattering

5) Rapidity and pseudo-rapidity

6) Log s physics

7) The quark parton model

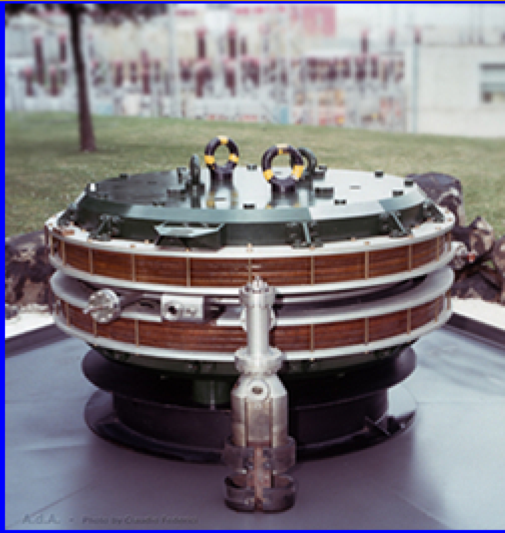
8) High-p_T processes

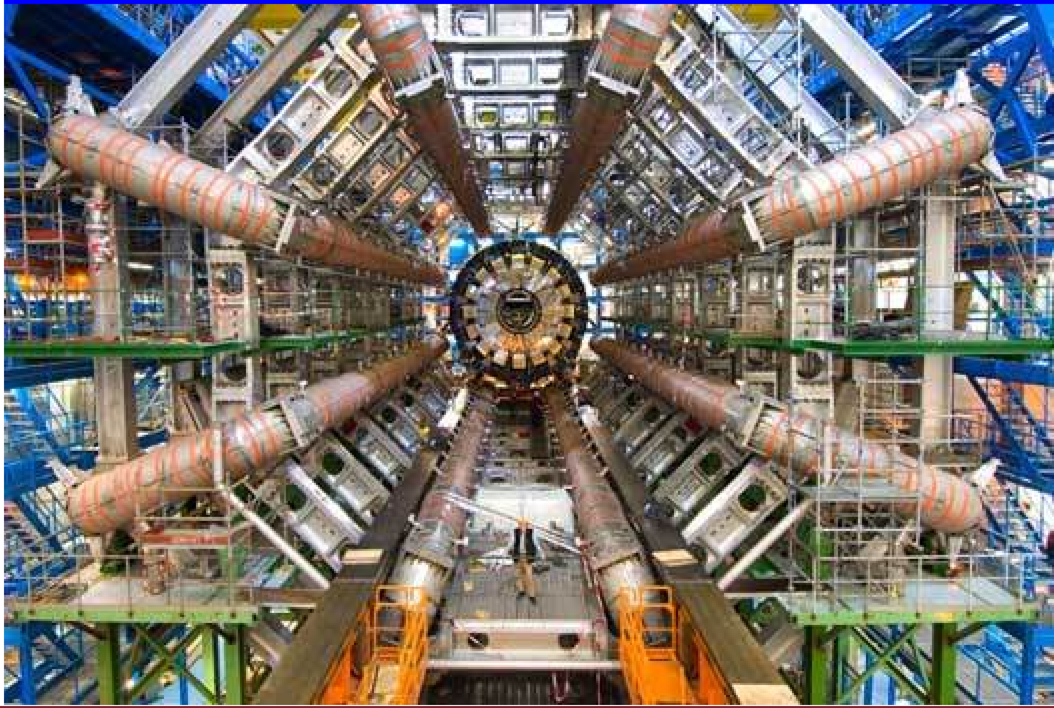
iii. comparisons

9) $e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$.

An introduction to collider physics: specific items + a discussion of $\ell^+\ell^-$ vs had colliders.

The full machine ADA (e^+e^-, R=65 cm) and a single detector like ATLAS (pp, R=12 m) at LHC (R = 4.2 km).





i. Accelerators

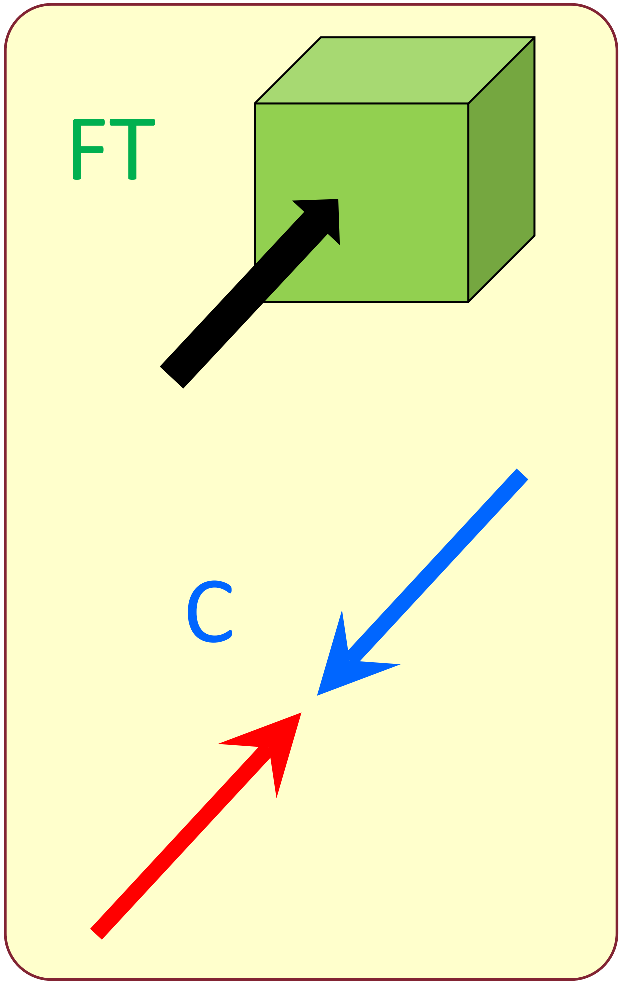
In modern particle physics, a good machine is the essential part of the game. E.g. the $W^{\pm}$, Z, H discoveries are mainly due to two new extraordinary colliders (the CERN $\bar{p}p$ and LHC respectively).

Colliders : introduction

- Hadronic collisions ($Sp\bar{p}S$ + LHC at CERN, TeVatron at Fermilab) share common dynamical and kinematical features, different from e^+e^- (Spear, LEP, …).
- Hadrons are composite, as explained by the QCD-quark-parton model :
  - coherent pp ($\bar{p}p$) scattering at low p_T;
  - $qq/\bar{q}\bar{q}/q\bar{q}/qg/\bar{q}g/gg$ scattering at high p_T, dominated by t-channel gg.
- Instead in e^+e^- Colliders only point-like interactions, dominated by s-channel.
- The historical order $Sp\bar{p}S$ – LEP – LHC is unnatural (hadrons, leptons, hadrons), but we will follow it, at the price of some repetitions and logical leaps.
- In the $Sp\bar{p}S$ and LHC chapters, the order will be the traditional one, increasing p_T and decreasing cross-section :
  1. [*total cross-section*],
  2. low-p_T interactions,
  3. high-p_T hadronic processes,
  4. high-p_T electro-weak;
  5. [*searches for new physics, if any*].
- For LEP, the order will be the history, i.e. the increasing beam energy :
  1. Z-pole electroweak physics,
  2. W^+W^- pair creation,
  3. [a digression on the method of searches and the analysis of negative results, the "limits"],
  4. Higgs searches;
  5. [*searches for new physics, if any*].
- In this first chapter, there are some definitions and discussions, useful for all the following parts, especially for hadron colliders.

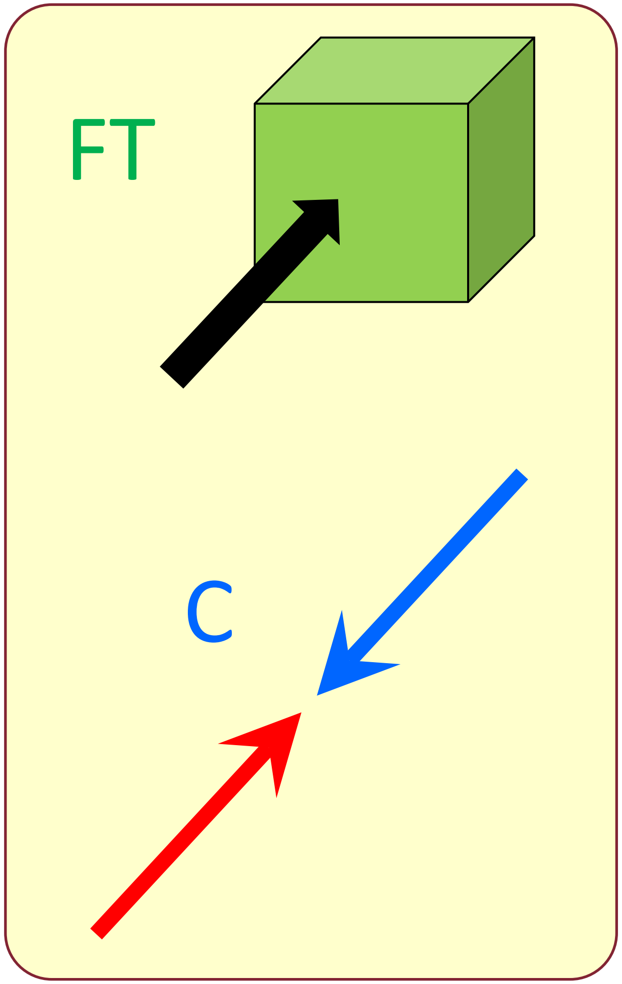
Colliders : vs fixed target

- Dynamics is invariant under a Lorentz boost; the processes depend on the relative motion of particles only : fixed target experiments (**FT**) and colliders (**C**) are dynamically equivalent;
- however, the explored kinematical region (and the experiments) are very different;
- a general (simplified) discussion of the relative merits of FT vs C in the next slides;
- for general purpose experiments, the quest for higher energy gives C a definitive advantage over FT [*imho, but widely shared*];
- the [*obvious*] reason is the CM energy $\sqrt{s}$:
  - **FT** : $s \approx 2\, m_N E_{beam} \rightarrow \sqrt{s} \propto \sqrt{E_{beam}}$;
  - **C** : $s = (2E_{beam})^2 \rightarrow \sqrt{s} \propto E_{beam}$;
- future alternatives : e^+e^- linear C, $\mu^+\mu^-$ circular C.



Colliders : types

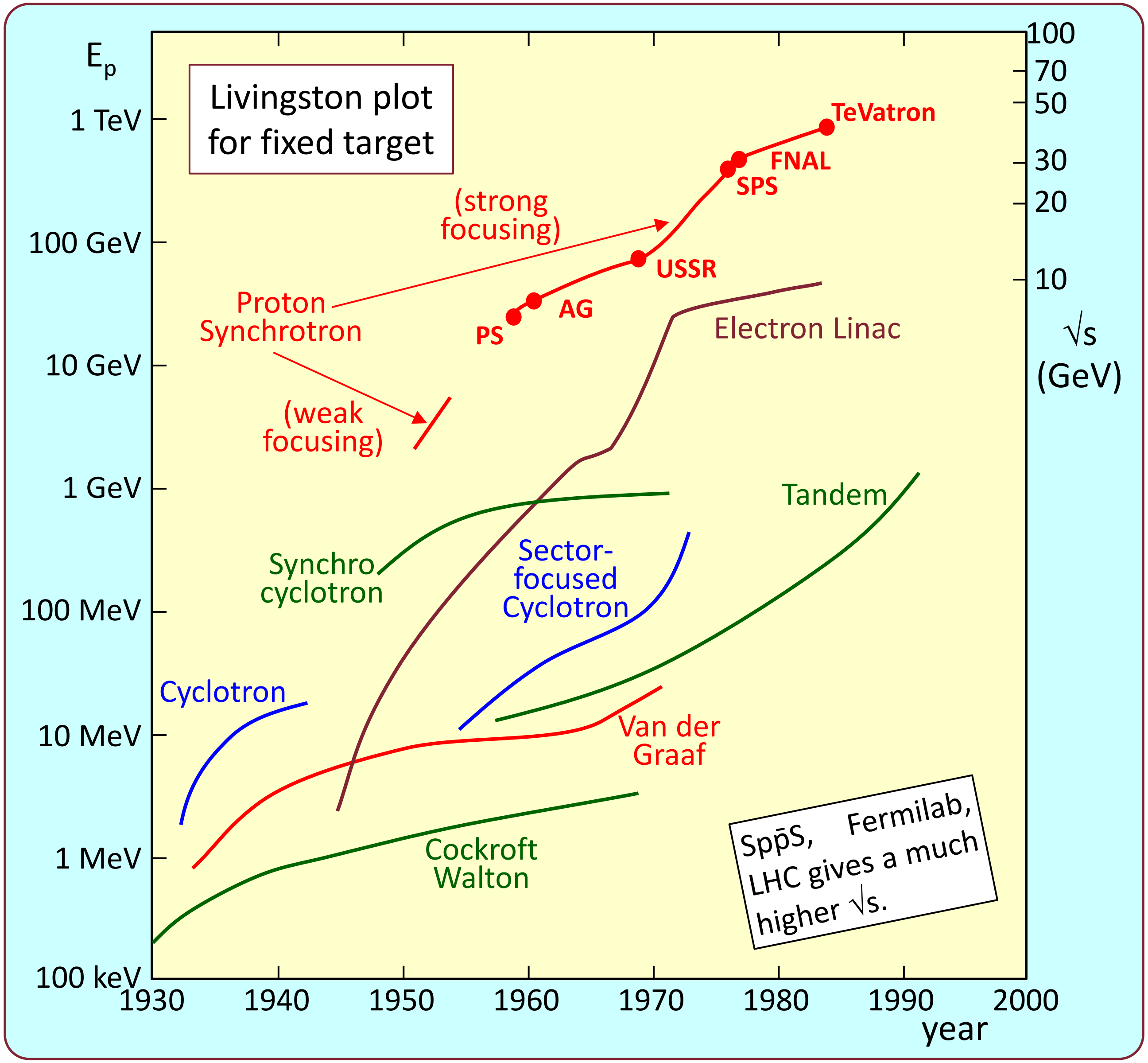
- FT's offers a plethora of initial states (nucleons, mesons, charged and neutral leptons, …), while C's have been realized with only few initial states:
  - e^+e^- AdA, ADONE, SPEAR, DESY, LEP, DAΦNE, …;
  - $\bar{p}p$ CERN and Fermilab Colliders;
  - pp ISR, LHC;
  - $e^{\pm}p$ Hera;
  - (+ heavy ions and specialized machines);
- projects for $\mu^+\mu^-$ Colliders; $\mu^{\pm}$ are dynamically equal to $e^{\pm}$, but produce (much) smaller brem; so they can be accelerated to higher energy;
- colliders e^+e^- have been realized since 50 years; they have discovered new leptons (τ), new hadrons (J/ψ, charm), new dynamics …
- The successes of pp ($\bar{p}p$) are $W^{\pm}$, Z, top, H.
- The swan songs of FT have been J/ψ and b quark (+ ν physics, which is a special case).



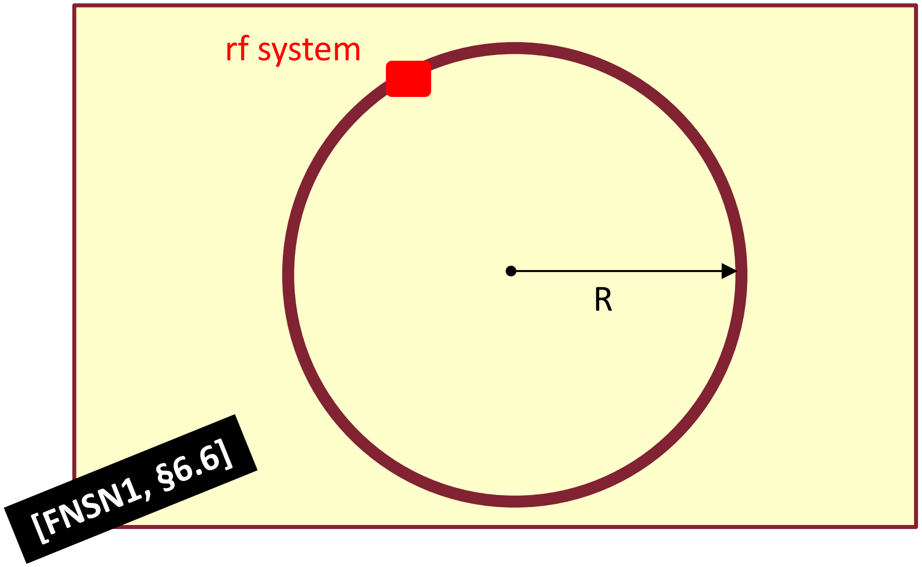
Colliders: Livingston plot

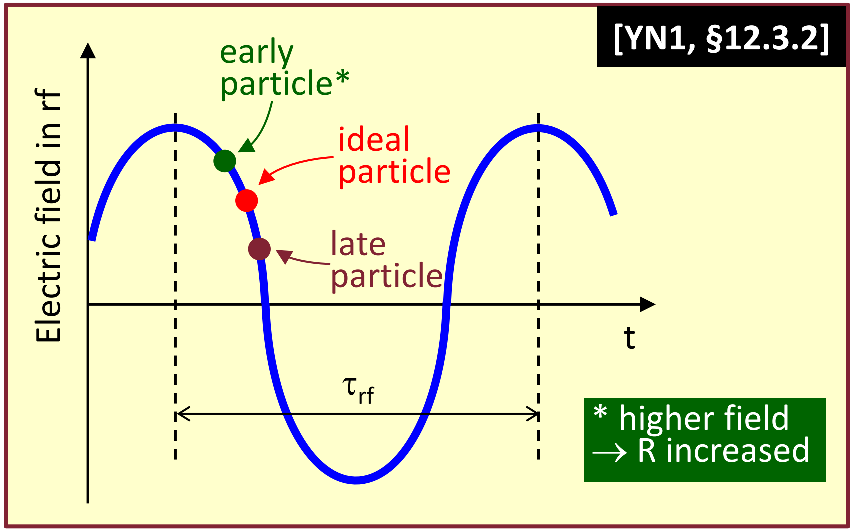
In addition, FT has plenty of applications out of the "energy frontier".

[our department, together with INFN and the SBAI department, hosts a PhD programme in accelerator physics ("dottorato in Fisica degli acceleratori")]



Synchrotron





Build a machine with a circular tube of small size and large radius, instrumented with dipoles and radiofrequencies of small-aperture and big power (+ auxiliaries) :

- from Lorentz force:

  p (GeV) = m$\beta\gamma$ = 0.3BR (T,m);

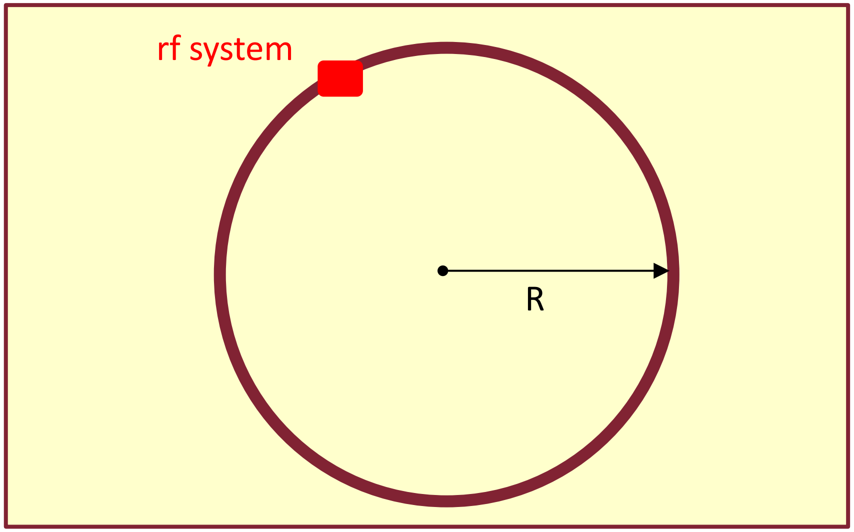
→ the mag. field $|\vec{B}|$ must be continuosly **synchro**nized to keep the beam on the same R, by varying the current ι in the magnet coils ($|\vec{B}| = \mu_0 n\iota$).

- the revolution period must be an integer multiple n_R of the radio-frequency period τ_{rf} [Povh, § A.1] :

$$t_R = \frac{2\pi R}{p/E} = n_R \tau_{rf} = \frac{2\pi n_R}{\omega_{rf}} \to \omega_{rf} = \frac{n_R p}{RE};$$

→ ω_{rf} must be continuosly re-adjusted (i.e. **synchro**nized) to follow the beam velocity (β=p/E), in order to always get the beam in the correct phase;

Synchrotron: parameters



Present limitations for parameters :

- mag. field B < 1.4 T (warm, iron core) or B < 10 T (superconductivity, but requires cryo magnets);
- R limited by civil engineering (costs, availability) to few (max tens) Km;
- radiofrequency limited by energy costs;
- brem problem for electrons [§ LEP].

Results:

- beam(s) bunched : $n_{bunch} < n_{bucket}$ (= n_R);
- $\sqrt{s}_{collider}$ (TeV) $\approx$ 2p $\approx$ 0.6 B(T) R(Km);
- $\sqrt{s}_{fixed}$ (GeV) $\approx \sqrt{2M_pE} \approx \sqrt{0.6BR}$ (T,m).

Problems:

- beam manipulation is complicated (next);
- interaction rate [see *Luminosity* in the following] is smaller wrt continuous accelerators;
- however, in practice this is the only known method to achieve high energy/high intensity;

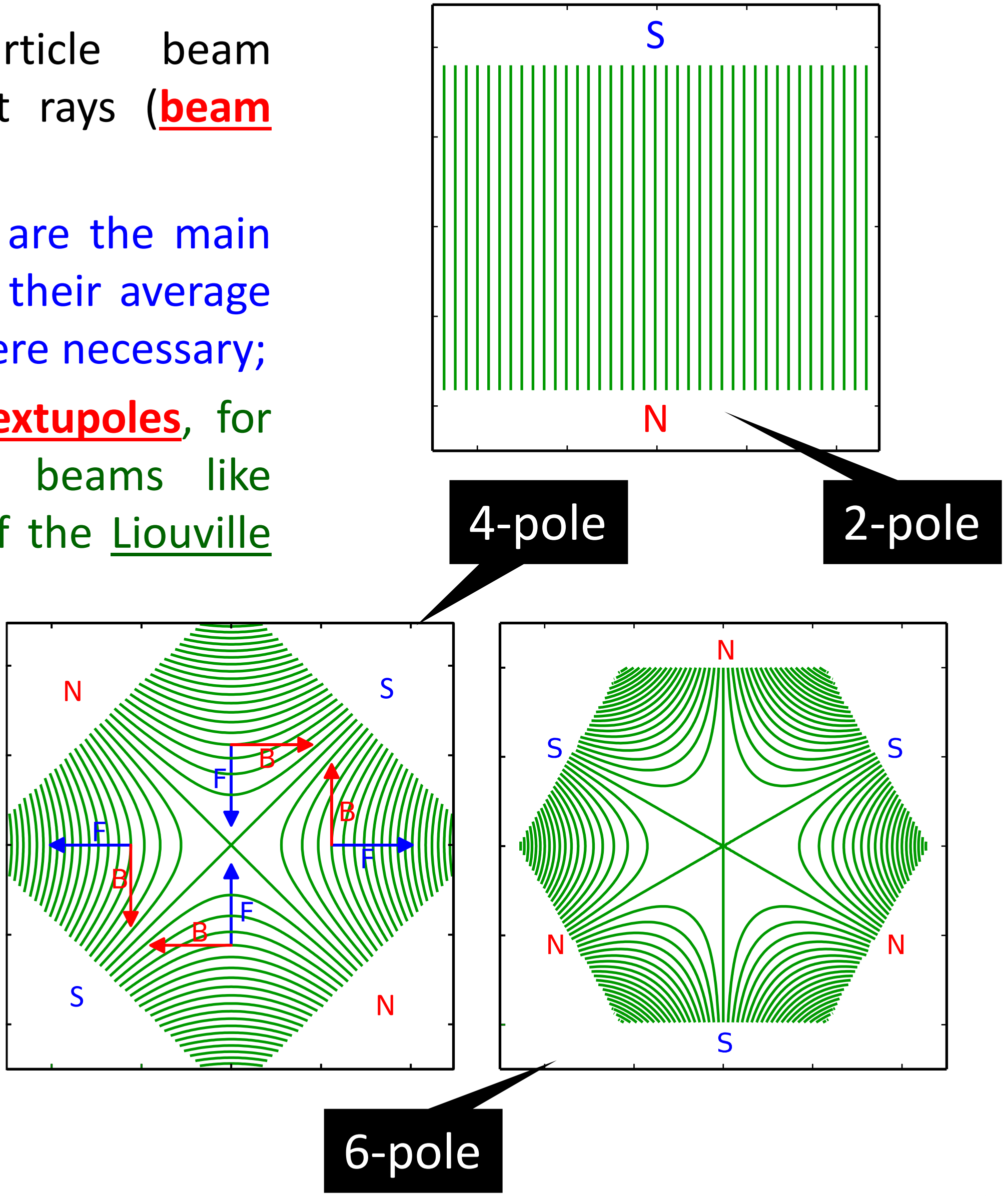
→ all modern accelerators are based on the principle of synchrotrons.

Synchrotron: magnets

The conventional approach to particle beam manipulations is to treat them as light rays (**beam optics**). The "lenses" are magnets :

- **dipoles** for beam bending; the dipoles are the main elements; if all the particles behave as their average ("ideal trajectory") no other elements were necessary;
- higher multipoles, like **quadrupoles**, **sextupoles**, for (de)focalization; they (de-)focus the beams like (di/con)vergent lenses (but be aware of the Liouville theorem !!!);
- the overall control is in the hands of very smart physicists/engineers, fast and big computers, under the goddess Fortuna.

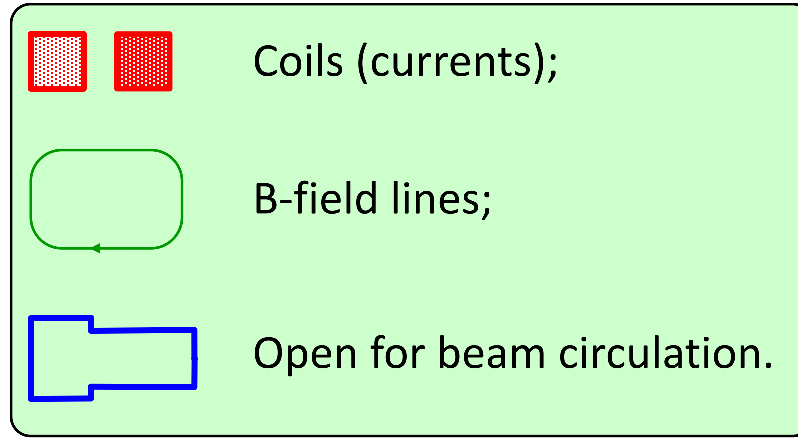
Liouville's theorem: if the particles obey the canonical equations of motion, then every element of a volume phase space is constant with respect to time.
[in this case: every gain in space density has to be compensated by a loss in momentum density]

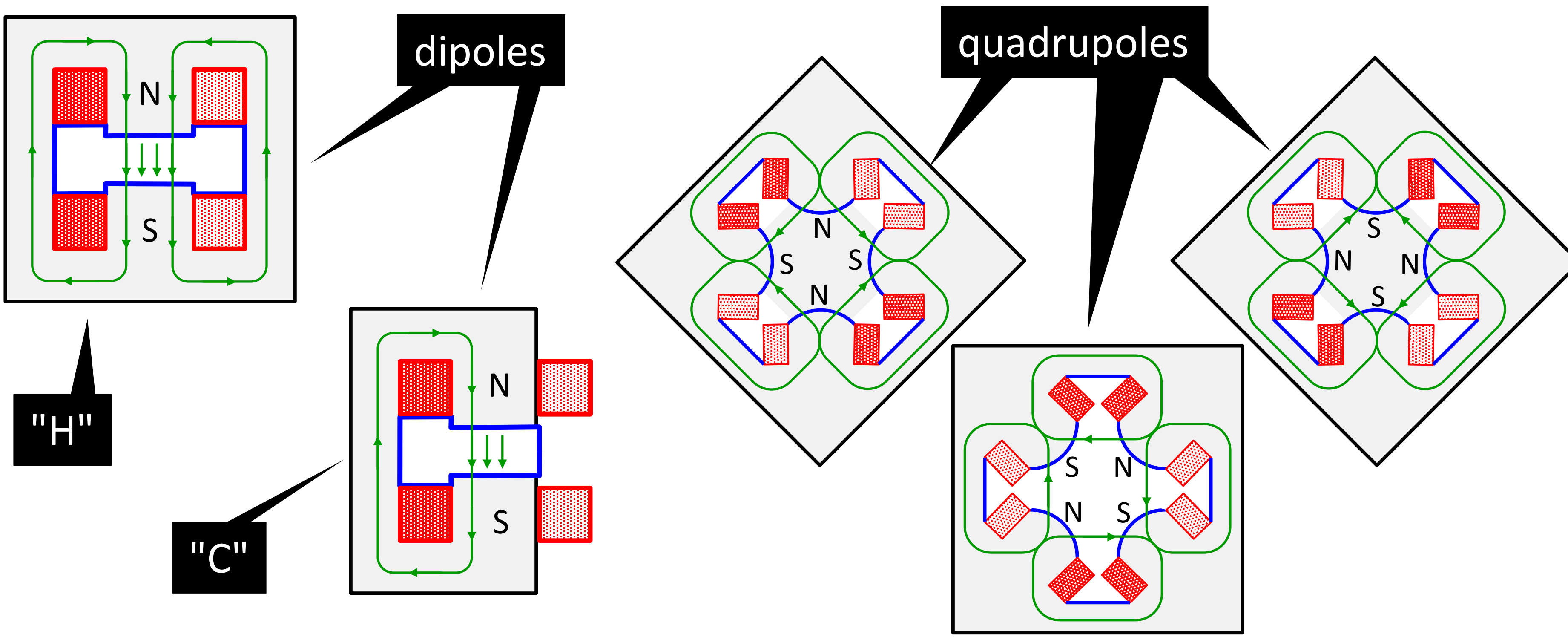


Synchrotron: magnet coils

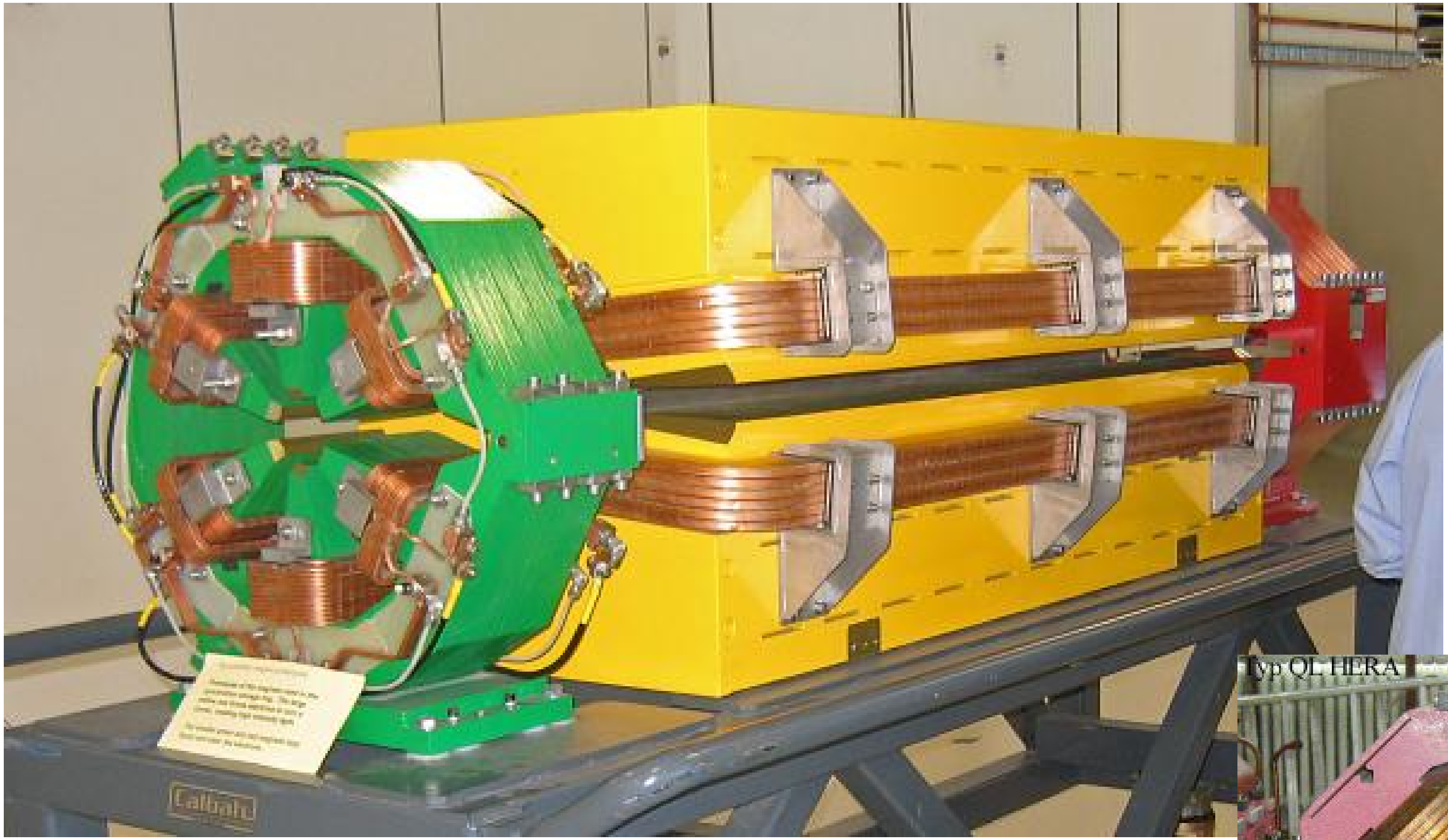
The magnets are built with two different techniques :

- warm : coils with high continuous currents + iron yoke;
- cold : superconducting coils at cryo temperature and (almost) no iron.





Synchrotron: examples of magnets



a sextupole in front of a "C" dipole

a quadrupole of the HERA accelerator

Synchrotron: the brem effect

[FNSN1, §6.6]

$$W_{Larmor} = \frac{1}{6\pi\varepsilon_0}\frac{e^2a^2\gamma^4}{c^3};$$

$$a(\text{circle}, \beta \approx 1) = \frac{v^2}{R} \approx \frac{c^2}{R};$$

$$W_{Larmor} = \frac{1}{6\pi\varepsilon_0}\frac{e^2c\gamma^4}{R^2};$$

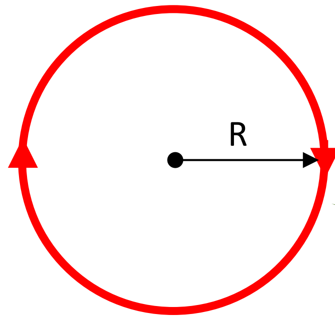
$$T_{1orbit} = \frac{2\pi R}{v} \approx \frac{2\pi R}{c};$$

$$\Delta E_{1orbit} = W_{Larmor} T_{1orbit} = \frac{1}{3\varepsilon_0}\frac{e^2\gamma^4}{R} \propto \frac{E^4}{M^4 R};$$

$$\Delta E^{electron}_{1orbit} = 8.85\times10^{-5}\left(\frac{E_e}{1\text{GeV}}\right)^4 \Big/ \left(\frac{R}{1\text{Km}}\right) \text{MeV};$$

$$\Delta E^{proton}_{1orbit} = 7.8\times10^{-3}\left(\frac{E_p}{1\text{TeV}}\right)^4 \Big/ \left(\frac{R}{1\text{Km}}\right) \text{KeV}.$$

$(m_p/m_e)^4 \approx 1.1\times10^{13}$



in practice $R_{tunnel} > R_{magnet}$.

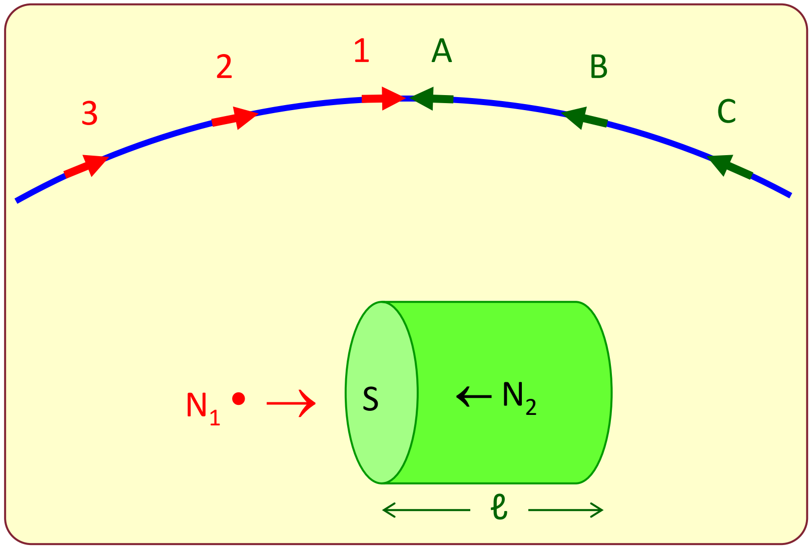
$\Delta E \propto M^{-4}$		$\sqrt{s}$ (GeV)	ΔE
LEP 1	e^+e^-	90	**121 MeV**
LEP 2	e^+e^-	200	**2,500 MeV**
LHC	pp	14,000	**6.9 KeV**

in circular e^+e^- colliders $R_{best} \propto s$ (severe limitation, see § LEP).

therefore, in future try :

- **$\mu^+\mu^-$ colliders;**
- **linear e^+e^- colliders.**

Luminosity: toy model



The fundamental figure to quantify collider performances is the **Luminosity** $\mathscr{L}$. Define it with a toy model:

- N_1 particles/bunch turning "clockwise";
- N_2 … "anti-clockwise";
- cylindrical bunches $S\times\ell$, ρ = const. [*this is the toy assumption*];
- for each of N_1, while traveling inside the cylinder N_2 for a small step x, the probability of interaction is:

$$\mathscr{P}_1(x) = 1 - e^{-\rho\sigma_T x} \cong \rho\,\sigma_T\, x = N_2\sigma_T x/(S\,\ell);$$

- the average number of interactions / crossing is :

$$\langle n_I\rangle = N_1\,\mathscr{P}_1(\ell) = N_1 N_2\, \sigma_T / S;$$

[$\langle n_I\rangle$ independent from ℓ]

- the crossings rate is

$$n_c = k \times f$$

[k = bunch number, f = revolution frequency]

therefore, the interaction rate is :

$$R \equiv \mathscr{L}\,\sigma_T = \langle n_I\rangle \times n_c = N_1\, N_2\, k\, f\, \sigma_T / S,$$

where $\mathscr{L}$, the "**luminosity**", contains the parameters of the machine, while σ_T reflects the particle dynamics:

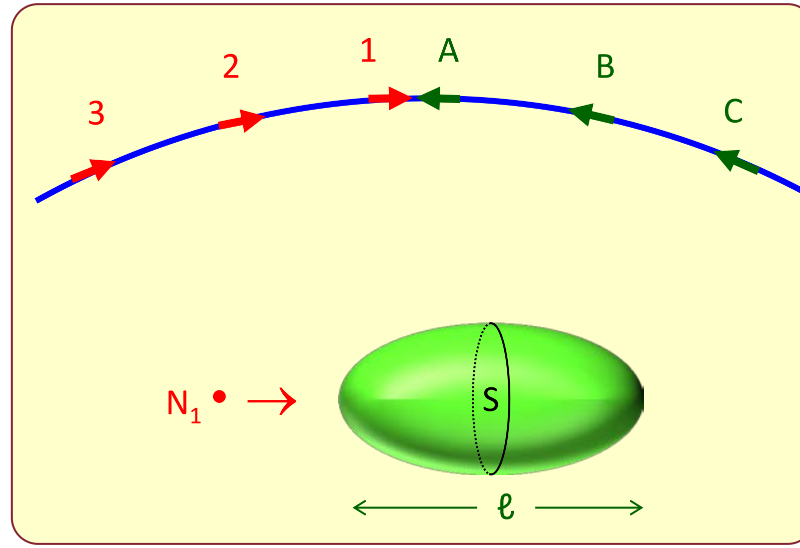
$$\boxed{\mathscr{L}^{toy} = \frac{N_1 N_2 k f}{S}.}$$

Luminosity: comments

The toy model is too naïve, however some of the conclusions are correct.

The luminosity is defined as $\mathscr{L} \equiv R/\sigma_T$, the ratio between the interaction rate and the total cross section(*). $\mathscr{L}$ is:

- **NOT** dependent (for head-on collisions) on the **bunch length ℓ**;
- proportional to the **inverse of the bunch section** (use an effective bunch section $S = 4\pi\sigma_x\sigma_y$);
- proportional to the **number of particles / bunch** of both beams ($N_1 N_2$);
- proportional to the **number of bunch crossings / second** (kf);
- [not in formula] dependent on **centroids displacement** and **beam lifetime**.



$$\mathscr{L} = \frac{N_1 N_2 k f}{4\pi\sigma_x\sigma_y}.$$

NB the total number of interactions seems to grow $\propto k^2$; however, in a given interaction point, it grows $\propto k$. *Is it clear ?* from this consideration, many clever machine developments, e.g. the *pretzel scheme*.

____

(*) for a process x : $R_x/R_T = \sigma_x/\sigma_T \rightarrow R_x = \mathscr{L}\,\sigma_x$.

Luminosity: collisions at angle α

- In case of an angle α between the beams (LHC), the formula becomes

$$\mathscr{L} = \frac{k f N_1 N_2}{4\pi\sigma_x\sigma_y} f(\alpha) \equiv \mathscr{L}_0 f(\alpha);$$

$$f(\alpha = 0) = 1; \quad f(\alpha \neq 0) < 1.$$

- It turns out(*) :

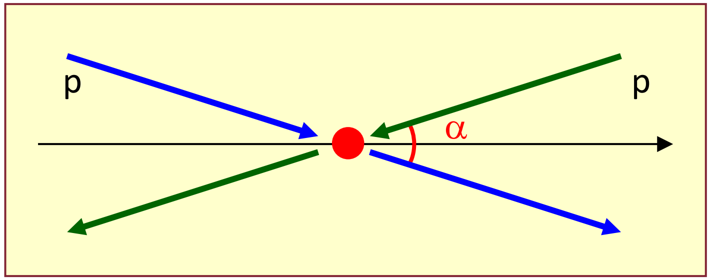
$$f(\alpha) = 1 \Big/ \sqrt{1 + \left(\alpha\sigma_\ell / 2\sigma_T\right)^2}$$

where $\sigma_\ell(\sigma_T)$ is the longitudinal (transv.) effective dimension of a bunch.

- Notice the dependence on σ_ℓ/σ_T; short bunches have other pros (better definition of the interaction point) and cons (e.g. in case of many overlapping events in the same bunch-crossing).

- At LHC, $\alpha \approx 300$ μrad $\rightarrow f(\alpha) = 0.83$.

(*) e.g. CERN CAS 2003, YR 2006-002, page 361.



- *Problem : the effect of α on $\sqrt{s}$ and p_T :*

  in LAB sys (≠ CM !!!) :

  [2E, 0, -2p sin(α/2),0] ≈ [2E, 0, - Eα, 0];

  $\rightarrow \sqrt{s} = 2E\sqrt{1-\alpha^2/4} \approx 2E\,(1 - \alpha^2/8);$

  $\rightarrow \Delta\sqrt{s} \approx -E\,\alpha^2/4$ *(negligible at LHC);*

  $\rightarrow |p_T| \approx E\alpha \approx 2$ *GeV at LHC (also negligible).*

  $\rightarrow$ *CONCLUSION : at LHC, in practice,* **LAB. sys. = CM sys., $\sqrt{s}$ = 2E, only $\mathscr{L}$ affected by α.**

Luminosity: $<n_{int}>$

Problem. How many interactions / bunch-crossing [b.c.] ? [n_{int}, also "μ", *a bad choice for an overused symbol*].

Solution [τ_{bc} = time between b.c.] :

$$\langle\mu\rangle = \frac{N_1 N_2 \sigma_T}{4\pi\sigma_x\sigma_y} = \frac{\mathcal{L}\sigma_T}{kf} \approx \mathcal{L}\tau_{bc}\sigma_T = \mathcal{L}_{bc}\sigma_T \;^{(*)};$$

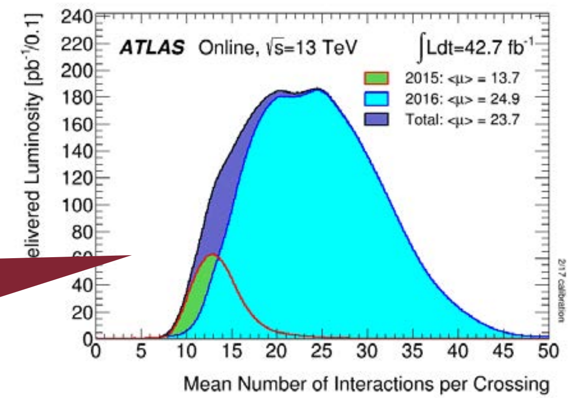
The effects of μ depend on its value:

- <u>$<\mu> \ll 1$</u> (Sp$\bar{p}$S, LEP): the probability of an interaction in a given b.c.; then "μ^2" is the probability of two events in the same b.c. (a known and not-very-important bckgd for Sp$\bar{p}$S and LEP);
- <u>$<\mu> > 1$</u> (LHC): the <u>average</u> number of overlapped events in a b.c.; the actual number is Poisson-distributed, with average $<\mu>$.

(*) some buckets are empty → larger $\mathcal{L}_{bc}$ and μ.

Comments:

- for hadronic colliders, it is better to consider $\mu_{inelastic}$ [$\sigma_T \to \sigma_{inel}$], which decreases μ by ~20%, because elastic collisions do not produce secondaries in the detectors;
- some old machines (e.g. CERN ISR) had "debunched" beams, i.e. particle uniformly spread over the whole ring; in this case the very definition of $<n_{int}>$ is meaningless; however, for LHC this setup is simply impossible [*why ? try to answer*].



μ @ LHC, see § 12b

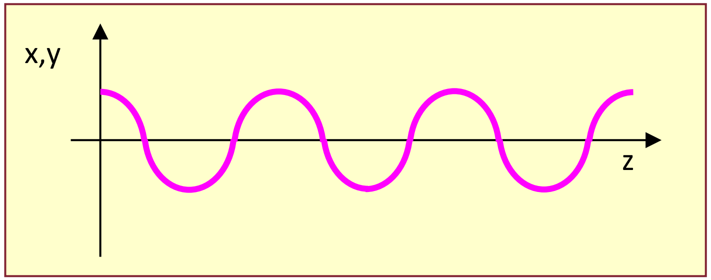
Luminosity : ε, β, β*

The dynamics of a real beam :

- real particles oscillate around the ideal trajectory (betatron oscillations);
- Reference system and definitions :
  - z : line of flight of the ideal particle;
  - x,y : deflections from ideal orbit;
  - $x' \equiv p_x / p_z$; $y' \equiv p_y / p_z$;
  - $\sigma_x \equiv$ rms beam size in x (also σ_y, $\sigma_{x'}$, $\sigma_{y'}$);
  - $\varepsilon_x = \pi \cdot \sigma_x \cdot \sigma_{x'}$ = "transverse emittance";
  - $\beta_x = \sigma_x / \sigma_{x'}$ = "amplitude function";
  - $\varepsilon_y = \pi \cdot \sigma_y \cdot \sigma_{y'}$; $\beta_y = \sigma_y / \sigma_{y'}$.
- Therefore (for the *, see on this page):

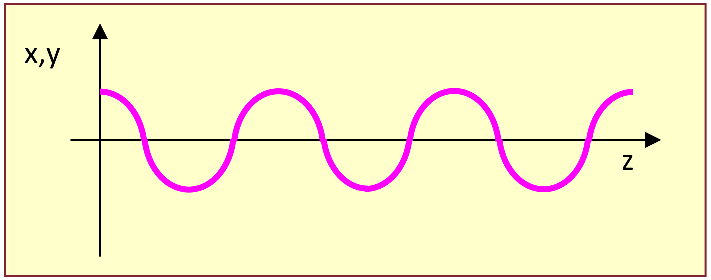
$$\mathcal{L} = \frac{k f N_1 N_2}{4\pi\sigma_x\sigma_y} f(\alpha) = \frac{k f N_1 N_2}{4\sqrt{\varepsilon_x\beta_x^*\varepsilon_y\beta_y^*}} f(\alpha);$$

- From Liouville's theorem :
  - V(6-dim) $= \sigma_x \cdot \sigma_y \cdot \sigma_z \cdot \sigma_{px} \cdot \sigma_{py} \cdot \sigma_{pz} =$ = constant;
  - $\varepsilon_{x,y}$ = const. (modulo stochastic effects, which increase it with time);
  - $\beta_{x,y}$ can be modified by accelerator devices (e.g. quadrupoles) : it MUST be SMALL in the interaction regions ("low-beta", β*), and large far from them ("high-beta", β) [next slide].

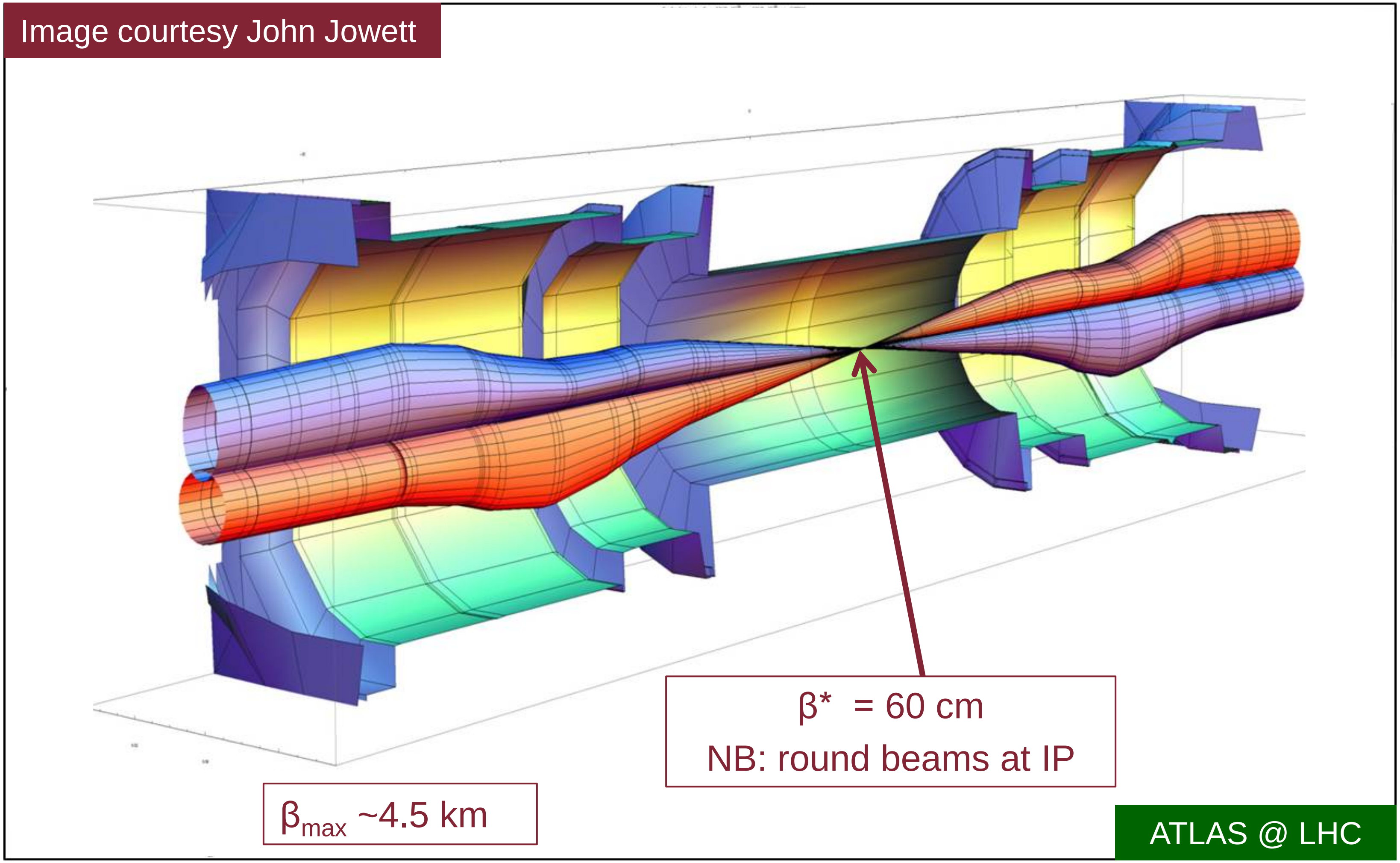


Luminosity: values of ε, β, β*

- At the **CERN Sp$\bar{p}$S** :
  - $\varepsilon_p \approx 9 \times 10^{-9}\ \pi$ rad m; $\varepsilon_{\bar{p}} \approx 5 \times 10^{-9}\ \pi$ rad m;
  - $\beta^*_H \approx 0.60$ m; $\beta^*_V \approx 0.15$ m.
- At **LEP** (remember the electron brem) :
  - $\varepsilon_H \approx (20 \div 45) \times 10^{-9}\ \pi$ rad m;
  - $\varepsilon_V \approx (0.25 \div 1.0) \times 10^{-9}\ \pi$ rad m;
  - $\beta^*_H \approx 1.50$ m; $\beta^*_V \approx 0.05$ m.
- At **LHC** ($\geq$ 2012) :
  - $\varepsilon_x \approx \varepsilon_y \approx 0.5 \times 10^{-9}\ \pi$ rad m;
  - $\beta^*_x \approx \beta^*_y \approx 0.55$ m;
  - [*see next page, from a beautiful CERN Academic training by Mike Lamont*].



Luminosity : β squeeze



Luminosity: better toy model

A mechanical analogy [Ed Wilson, 28] :

- a little ball on a falling guide [*see*];
- two forces :
  1. gravity toward z (= "acceleration");
  2. a force orthogonal to z, which depends on the <u>local</u> shape of the guide (e.g. elastic $\propto |x|$);
- choose two parameters ε, β:

$$x = \sqrt{}$$

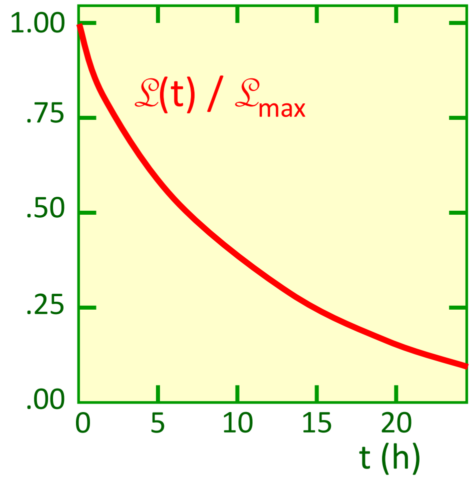
Luminosity: Liouville's theorem

- Because of the Liouville's theorem, for an "ideal fluid of balls", the [iper-] volume of the ellips[oid] keeps constant during the motion :

$$V = \pi\sqrt{}$$

Luminosity: evolution with time

- Many effects deteriorate the luminosity during a long data-taking. [following figures from LHC, but the effects are similar for all colliders].
- Parameterize as $d\mathscr{L} = -\mathscr{L}\, dt/\tau_i$; at LHC :
  - collisions $\tau_{coll} \cong$ 29 h;
  - increase of emittance $\tau_{IBS} \cong$ 80 h;
  - residual gas $\tau_{gas} \cong$ 100 h;
  - (many other minor effects ...)
- Global effect on luminosity :



$\mathscr{L}(t)=\mathscr{L}_{max}e^{(-t/\tau)}$; $\frac{1}{\tau} = \sum \frac{1}{\tau_j} \approx 1\,/\,(15\text{ h})$.

Integrated luminosity after a time T :

$$\mathscr{L}_{INT}(T) = \int_0^T \mathscr{L}(t)dt \approx \mathscr{L}_{MAX}\tau\left[1 - e^{-(T/\tau)}\right];$$

$$N(T) = \int_0^T \mathscr{L}(t)\cdot\sigma_{TOT}dt = \mathscr{L}_{INT}(T)\cdot\sigma_{TOT}.$$

- After few hours, new injection and acceleration [*see* § *LHC*].
- I.e. $\mathscr{L}_{max,effective} \approx ½\, \mathscr{L}_{max}$.
- The decision to dump the beam and restart the cycle (inject – accelerate – squeeze – data-taking) is crucial :
  - At the Sp$\bar{p}$S was dramatic (high level officials), due the scarcity of $\bar{p}$.
  - Even at LHC (plenty of protons everywhere) is a major concern.

Luminosity: $\mathscr{L}$ vs $\sqrt{s}$

Summary :

$$R = \frac{dN}{dt} \equiv \mathscr{L}\sigma_{TOT}; \quad R_x = \mathscr{L}\sigma_x;$$

$$\mathscr{L} = \mathscr{L}(\alpha = 0) = \frac{N_1 N_2 k f}{4\pi\sigma_x\sigma_y} =$$

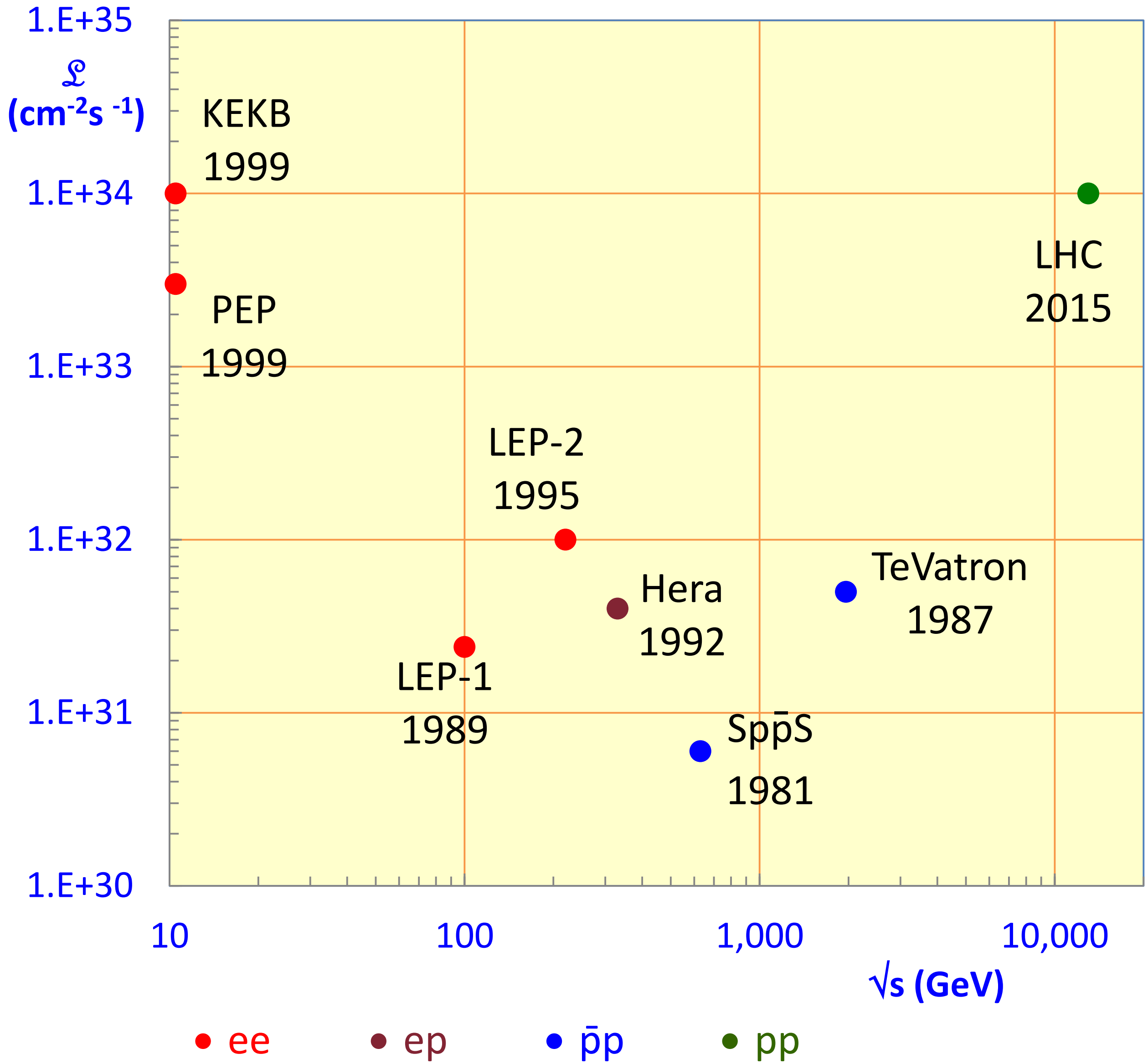
$$= \frac{k f N_1 N_2}{4\sqrt{\varepsilon_x\beta_x^*\varepsilon_y\beta_y^*}};$$

$$\mathscr{L}(t,\alpha) = \frac{\mathscr{L}(t,\alpha=0)}{\sqrt{1+\left[\alpha\sigma_\ell/(2\sigma_T)\right]^2}};$$

$$\langle n_{int}\rangle = \mathscr{L}\tau_{bc}\sigma_{TOT} \quad [\text{or } \sigma_{inel}];$$

$$\mathscr{L}_{INT} = \int \mathscr{L}(t)dt;$$

$$N_{TOT} = \mathscr{L}_{INT}\sigma_{TOT}; \quad N_x = \mathscr{L}_{INT}\sigma_x.$$



ii. Physics

Five parts:

a. Scattering: collisions in non-relativistic q.m., mainly the optical theorem and its consequences [*a memo*].

b. (Pseudo-)rapidity: kinematical variables used both at low- and high-Q^2 [*the math looks crazy, but it is very useful*].

c. Log s physics: a synonym of "low-Q^2 physics", i.e. when hadrons behave as coherent non-point-like particles [*an old subject, difficult, no clean results, but unavoidable, because it is the main source of events in hadronic physics*].

d. The quark parton model: the QCD theory and its approx., applied to the data [*the real subject of the discussion*].

e. High-p_T processes: the kinematical analysis of high-Q^2 events [*Mandelstam variables, x, $\sqrt{s}$ & c., both at parton and hadron level*].

NB. The sequence is dictated by understanding; (a-c-d-e-b) would have been more logical, but also more difficult.

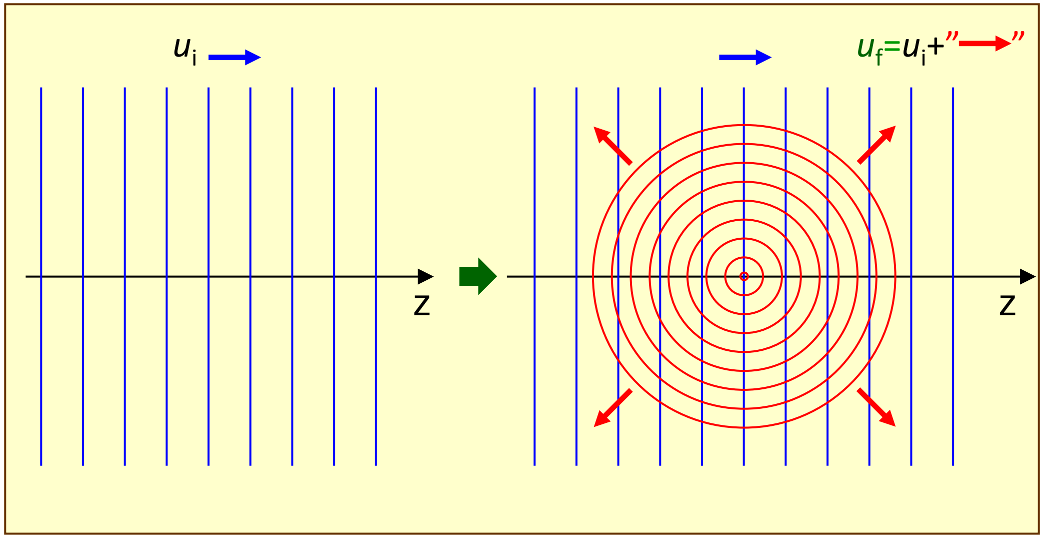
scattering

- The electromagnetic processes, treated in § 2, are a special privileged case :
  - the potential is derived from a well-known and tested theory;
  - the model is based on symmetries;
  - the dimensionless coupling constant $\alpha_{em} << 1$.
- The treatment of **nuclear interactions** is much more complex :
  - there is no classical analogue;
  - the analytic form of the interaction is [was] unknown;
  - the coupling is much larger than in electromagnetism : the perturbative approach does not give results at small Q^2 (= large distances).
- Much experimental information comes from nuclear reactions and **scattering** processes. This study is therefore crucial.

- Examine the simplest case :
  - two particles;
  - spinless;
  - non-relativistic approximation;
  - potential only dependent from relative position.

scattering: partial waves

References (many, but e.g.) :
- Sakurai, Modern q.m., 397;
- Weinberg, Lectures on q.m., 211;
- Burcham – Jobes, 286;
- Messiah, vol 2, 866;
- Perkins (ed. 1971), 265.



- Two particles, mass m_1 and m_2, both spin 0, collide with a potential $V(x,y,z)$.
- The particles are observed far from the collision region, i.e. where $V \approx 0$.
- Define :

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}; \quad \vec{r} = \vec{r}_1 - \vec{r}_2;$$

$$M = m_1 + m_2; \qquad \mu = \frac{m_1 m_2}{m_1+m_2}.$$

- If $V(\vec{r})$ depends only on $\vec{r}$, i.e. on the relative positions of $m_{1,2}$, the Schrödinger equation splits in two parts :
  - a function $\psi_{CM}(\vec{R})$, for the free motion of the CM, which behaves as a free particle, with mass M and energy E_R;
  - a function $\psi(\vec{r})$, for the motion of a particle with reduced mass μ and energy E_r, subject to $V(\vec{r})$.

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2M}\nabla_R^2 - \frac{\hbar^2}{2\mu}\nabla_r^2 + V(\vec{r})\right]\Psi;$$

$$-\frac{\hbar^2}{2M}\nabla_R^2\psi_{CM}(\vec{R}) = E_R\psi_{CM}(\vec{R});$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla_r^2 + V(\vec{r})\right]\psi(\vec{r}) = E_R\psi(\vec{r}).$$

scattering: partial waves

- The initial state is described by a plain wave along z :

$$\psi_i = e^{ikz} = \sum_\ell i^\ell \sqrt{4\pi(2\ell+1)}\; j_\ell(kr) Y_\ell^0(\theta) \xrightarrow{kr\to\infty}$$

$$\approx \frac{1}{kr}\sum_\ell i^\ell (2\ell+1) P_\ell(\cos\theta) \times \left(\frac{e^{i(kr-\ell\pi/2)} - e^{-i(kr-\ell\pi/2)}}{2i} \right).$$

$j_\ell(kr)$ = spherical Bessel functions, $P_\ell(\cos\theta)$ Legendre polynomials.

exp(ikz) = mixture of different ℓ → expand [no φ → m=0 only].

- … and the final state by the superposition of a plane and a spherical wave, modulated by f(θ,φ) :

$$\psi_f = e^{ikz} + \frac{e^{ikr}}{r} f(\theta,\varphi);\quad \psi_f^{scatt} = \psi_f - \psi_i = \frac{e^{ikr}}{r} f(\theta,\varphi);$$

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = \frac{\text{outgoing flux}@(\theta,\varphi)}{\text{ingoing flux}} = |f(\theta,\varphi)|^2;$$

definition of σ and flux.

$$\psi_f = \frac{1}{kr}\sum_\ell i^\ell \eta_\ell e^{2i\delta_\ell} (2\ell+1) P_\ell(\cos\theta) \times$$

$$\times \left(\frac{e^{i(kr-\ell\pi/2)} - e^{-i(kr-\ell\pi/2)}}{2i} \right);$$

complex factor, for each ℓ :
- $\Im(\ldots_\ell)$ = change in phase;
- $\Re(\ldots_\ell)$ = … in amplitude.

$$f(\theta,\varphi) = f(\theta) =$$

spin-0 particles + central potential = no dependence on φ.

$$= \frac{1}{2ik}\sum_\ell (\eta_\ell e^{2i\delta_\ell} - 1)(2\ell+1) P_\ell(\cos\theta);$$

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = |f(\theta)|^2 =$$

this σ refers to elastic scattering

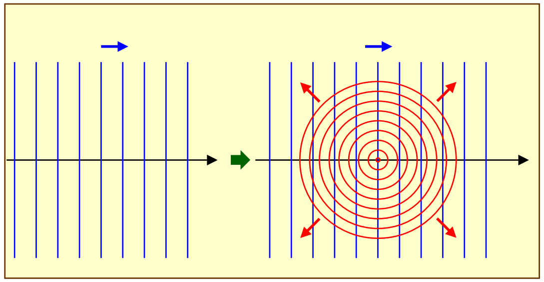
$$= \frac{1}{k^2}\left| \sum_\ell \left(\frac{\eta_\ell e^{2i\delta_\ell} - 1}{2i}\right)(2\ell+1) P_\ell(\cos\theta) \right|^2;$$

$$\sigma_{el} = \int \frac{d\sigma(\theta,\varphi)}{d\Omega} d\Omega = \int |f(\theta)|^2 d\Omega =$$

$$= \frac{\pi}{k^2}\sum_\ell (2\ell+1) \left|\eta_\ell e^{2i\delta_\ell} - 1\right|^2.$$

$$\int d\Omega P_\ell(\cos\theta) P_{\ell'}(\cos\theta) = 4\pi\delta_{\ell\ell'}/(2\ell+1)$$

[δ = Kronecker symbol].



scattering: the optical theorem

- the *phase shifts* δ_ℓ pass through a resonance when $\delta_\ell = \pi/2$:
  - $\eta_\ell \exp(2i\delta_\ell)$; $0 \le \eta_\ell \le 1$;
  - only elastic scattering $\rightarrow \eta_\ell = 1 \rightarrow$

$$\sigma_{el}^{only} = \frac{4\pi}{k^2}\sum_\ell (2\ell+1)\sin^2\delta_\ell.$$

- Finally, calculating the flux associated with ψ_f, the value of σ_{tot} is :

- [*warning : the theorem looks very smart; however, it is only a relation, based on wave mechanics, between two unknown quantities.*]

- The dynamics, carried by the potential $V(\vec{r})$, rests in **f(θ)** [**the scattering amplitude**], or, alternatively, in the **inelasticity parameters η_ℓ** and in the **phase shifts δ_ℓ**.

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} = \frac{2\pi}{k^2}\sum_\ell (2\ell+1)\left[1-\eta_\ell\cos(2\delta_\ell)\right];$$

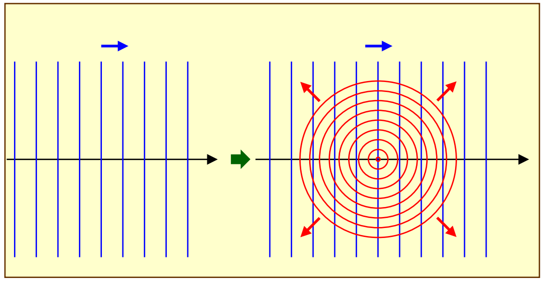
$$\sigma_{inel} = \int r^2 d\Omega\left(|\psi_i|^2 - |\psi_f^{scatt}|^2\right) = \frac{\pi}{k^2}\sum_\ell (2\ell+1)\left(1-\eta_\ell^2\right).$$

$$\Im\left[f_{el}(\theta=0)\right] = \Im\left[\frac{1}{2ik}\sum_\ell (2\ell+1)(\eta_\ell e^{2i\delta_\ell}-1)P_\ell(\cos\theta=1)\right] =$$

$$= \frac{-1}{2k}\sum_\ell (2\ell+1)\left[\eta_\ell\cos(2\delta_\ell)-1\right];$$

$$\boxed{\sigma_{tot} = \frac{4\pi}{k}\Im\left[f_{el}(\theta=0)\right].}$$

"**optical theorem**"
[Sellmeier, Rayleigh 1871; Bohr, Peierls, Placzek 1939; Bethe, de Hoffman 1955]



scattering: σ_{tot}

In hadron colliders, the standard method to measure the total cross section, e.g. at LHC **σ_{tot}(pp)**, uses the optical theorem:

a. $$\boxed{\sigma_{tot} = \frac{4\pi}{k}\Im\left[f_{el}(\theta=0)\right];}$$

$$\boxed{\theta = 0 \rightarrow \cos\theta = 1 \rightarrow t = 0}$$

$$\Im\left[f_{el}(\theta=0)\right] \equiv \Im\left[f_{el}^{t=0}\right] = \frac{k\sigma_{tot}}{4\pi} \approx \frac{\sigma_{tot}\sqrt{s}}{8\pi}.$$

b. Define the elastic cross section in terms of $f_{el}(\theta)$ and t(Mandelstam):

$$\frac{d\sigma_{el}}{d\Omega} \equiv \frac{d^2\sigma_{el}}{d\varphi d\cos\theta} = \left|f_{el}(\theta)\right|^2;$$

$$\boxed{t = -\frac{s}{2}(1-\cos\theta)} \rightarrow \cos\theta = 1 + \frac{2t}{s};$$

$$\frac{d\sigma_{el}}{dt} = \int d\varphi\left(\frac{d^2\sigma_{el}}{d\varphi d\cos\theta}\right)\left|\frac{\partial\cos\theta}{\partial t}\right| =$$

$$= 2\pi\left|f_{el}(\theta)\right|^2\frac{2}{s} = \frac{4\pi}{s}\left|f_{el}(s,t)\right|^2.$$

c. Define $\rho = \Re[f_{el}^{t=0}] / \Im[f_{el}^{t=0}]$ and put it in the equations :

$$\left|f_{el}^{t=0}\right|^2 = \left|\Re\left[f_{el}^{t=0}\right]\right|^2 + \left|\Im\left[f_{el}^{t=0}\right]\right|^2 =$$

$$= \left|\Im\left[f_{el}^{t=0}\right]\right|^2\left(1+\rho^2\right) = \frac{\sigma_{tot}^2 s}{64\pi^2}\left(1+\rho^2\right).$$

d. From the definition of the luminosity $\mathcal{L}$, for each process x, the rate is

$$\sigma_x = R_x / \mathcal{L} \rightarrow \sigma_{el} = R_{el} / \mathcal{L}; \quad \sigma_{tot} = R_{tot}/\mathcal{L};$$

$$\rightarrow (\sigma_{tot})^2 = R_{tot}\sigma_{tot} / \mathcal{L}.$$

e. Equating (b) = (c), and using (d) :

$$\left|f_{el}^{t=0}\right|^2 = \frac{\cancel{s}}{4\pi}\left.\frac{d\sigma_{el}}{dt}\right|_{t=0} = \frac{R_{tot}\sigma_{tot}\cancel{s}}{64\pi^2\mathcal{L}}\left(1+\rho^2\right).$$

f. The final equation is :

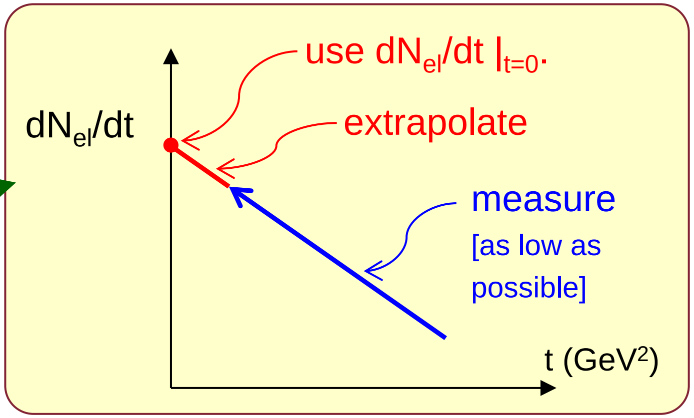
$$\boxed{\sigma_{tot} = \frac{16\pi(\hbar c)^2}{1+\rho^2}\frac{1}{R_{tot}}\left.\frac{dR_{el}}{dt}\right|_{t=0}.}$$

scattering: measure σ_{tot}

$$\sigma_{tot} = \frac{4\pi}{k}\Im\left[f_{el}(\theta=0)\right] = \frac{16\pi(\hbar c)^2}{1+\rho^2}\frac{1}{R_{tot}}\left.\frac{dR_{el}}{dt}\right|_{t=0}.$$

Since everything (but ρ) is directly measurable, σ_{tot} can be measured:

- R_{el} and R_{tot} :
  - absolute rates in arbitrary units (only the ratio counts, i.e. use N_{el} and N_{tot}, integrated over the same time interval $\rightarrow$ smaller stat. errors);
  - systematics due to dead time, faults in data-taking, … cancels in the ratio;
- the term "$dR_{el}/dt\,|_{t=0}$" :
  - produce a plot R_{el} (or N_{el}) vs $t_{Mandelstam}$;
  - N(t=0) is non-measurable $\rightarrow$ go as low as possible in t and extrapolate $\rightarrow$ t=0;
  - units do NOT count, but extrapolation errors do;



  - the histogram requires t $\rightarrow$ must know $\vec{p}_{init}$ $\rightarrow$ high-β is preferable, even if $\mathscr{L}$ (and N) are smaller;
- the ratio ρ [*a personal pessimistic view*] :
  - can be computed [*maybe "guessed"*] from first principles;
  - turns out small ($\approx$ 0.14 @ LHC) $\rightarrow$

    $$\Delta\sigma/\sigma \approx 2\rho\Delta\rho \leq 1\%;$$
  - so ρ [*is not well-understood, but it*] does not harm the result.

scattering: $\mathbb{S}$ matrix

The $\mathbb{S}$ matrix ($\mathbb{S}$ for "scattering") was introduced indipendently by J.Wheeler in 1937 and W.Heisenberg in 1940.

The following definitions and properties are discussed in [MQR § 11] in the Interaction Picture ("IP", $|\rangle_I$) :

- $\lim_{t\to\pm\infty} \mathbb{H}_I(t) = 0;$
- $\lim_{t\to\pm\infty} |\psi(t)\rangle_I \equiv |\psi(t=\pm\infty)\rangle_I = \text{const.};$
- $|\psi(t)\rangle_I = \mathbb{U}_I(t,t_0)|\psi(t_0)\rangle_I;$
- $|\, i\,\rangle \equiv |\psi(t=-\infty)\rangle_I;$
- $|\, f\,\rangle \equiv |\psi(t=+\infty)\,\rangle_I \equiv \mathbb{S}|\, i\,\rangle;$
- $\mathbb{S} \equiv \lim_{t_2\to+\infty, t_1=-\infty} \mathbb{U}_I(t_2,t_1);$
- $\mathbb{S}\,\mathbb{S}^\dagger = \mathbb{S}^\dagger\,\mathbb{S} = \mathbb{1}.$

The following properties follow :

- $\mathcal{S}_{fi} \equiv \langle\, f \,|\, \mathbb{S} \,|\, i\,\rangle;$
- $\Sigma_f |\mathcal{S}_{fi}|^2 = 1$ [conservation of probability];
- $\mathbb{S} \equiv \mathbb{1} + 2i\mathbb{T};$
- $\mathbb{T} = (\mathbb{S} - \mathbb{1}) / (2i);$
- $\langle f|\mathbb{S}|i\rangle = \delta_{fi} + i(2\pi)^4\delta^4(p_f\text{-}p_i)\langle f|\mathbb{T}|i\rangle;$
- $d\sigma = \dfrac{1}{v}\dfrac{d\vec{p}_f}{(2\pi)^3}|\mathfrak{M}_{fi}|^2\, 2\pi\delta(E_f - E_i).$

It is interesting to note that, starting from there, the optical theorem follows (almost) immediately :

- $\sigma_T = -2\,\Re[\mathfrak{M}_{ii}] / v_i = 4\pi\,\Im[f(0,\varphi)] / p_I.$

The analytical properties of the $\mathbb{S}$ matrix have been extensively studied in the '50s and '60s. After that, the success of the field theory and the SM have terminated the approach, even if some addicts are still around.

(pseudo-)rapidity

- The *rapidity* ϕ was introduced by Minkowski (NOT in particle physics):

  $\phi = \tanh^{-1}(v/c)$,

  many properties : i.e. it reduces to v/c for low speed, it is additive (unlike v), ….

- In particle physics a similar variable (y) defined by Feynman for a particle $m \neq 0$, relative to an axis z (usually the beam) :

$$\boxed{y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z};}$$

- define also :

  - $m_T^2 = m^2 + p_x^2 + p_y^2$ (transverse mass);
  - $\boxed{\eta = -\ln[\tan(\theta/2)]}$ (pseudo-rapidity);
  - $\boxed{x = 2\, p_z / \sqrt{s}}$ ("Feynman x");

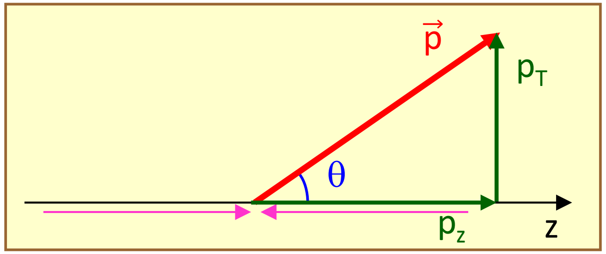
Use $p = [E, p_x, p_y, p_z; m]$; other variables will be defined. [*Unfortunately, with only 26 letters available, there is a lot of repetition, e.g. the rapidity y has nothing to do with the inelasticity y.*]

It follows (next slides) :

- $p_z \to -p_z \Rightarrow \theta \to (180° - \theta) \Rightarrow y \to -y$;
- $E = m_T \cosh(y)$; $p_z = m_T \sinh(y)$;
- $y = \ln[(E + p_z) / m_T] = \tanh^{-1}(p_z/E)$;
- $dy = dp_z / E$;
- $\boxed{\text{if } (p \gg m) \to y \approx \eta.}$
- given a Lorentz transformation $\mathbb{L}$ along z, with velocity β_z :

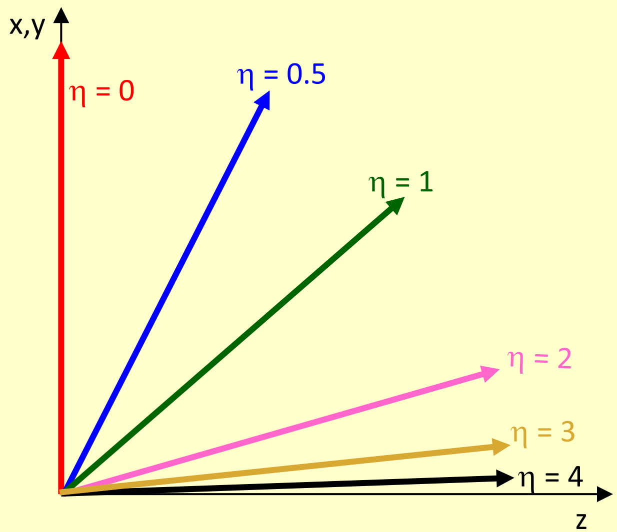
$$y' = \mathbb{L}(y) = y - \tanh^{-1} \beta_z; \ \Delta y' = \Delta y;$$

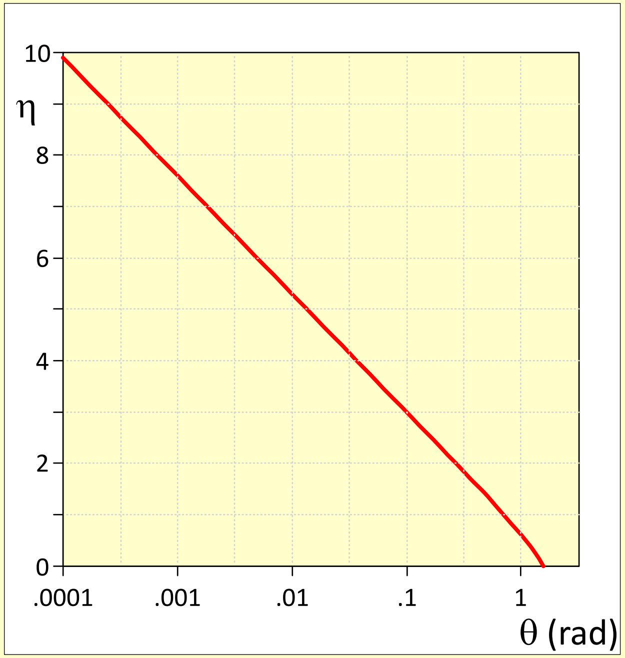
i.e. y is the variable, whose differential dy is invariant for $\mathbb{L}$-transformations along z.



(pseudo-)rapidity: plot

- The pseudorapidity η is important.
- Sometimes physicists assume to be in the extreme relativistic case, and call it "rapidity".
- Roughly, it represents the zenith θ, with a scale much expanded towards the beam axis.
- But its properties are many, and …





$\theta = 2\tan^{-1}(e^{-\eta})$

η	θ (°)
0.0	90.000
0.5	62.476
1.0	40.395
1.5	25.157
2.0	15.415
2.5	9.385
3.0	5.700
3.5	3.459
4.0	2.099
4.5	1.273
5.0	0.772
5.5	0.468

For small θ (large η) : $\eta\ [\approx y] \quad = -\ln\,[\tan\,(\theta/2)] \;\rightarrow$

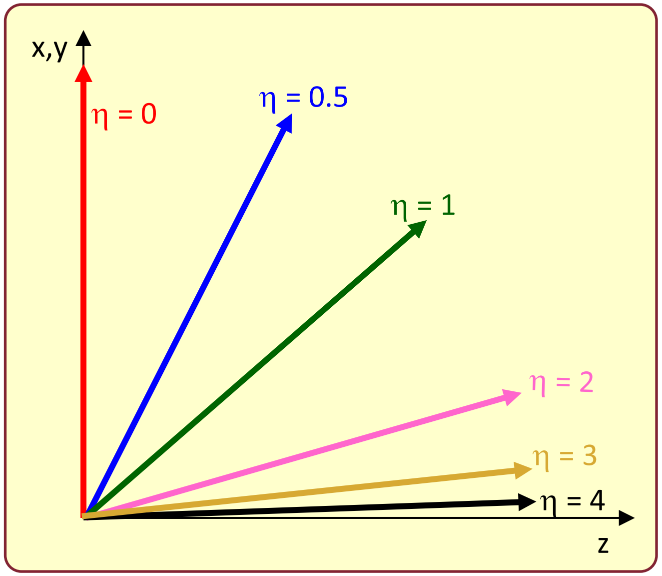
$$\approx \ln(2) - \ln[\theta(\text{rad})] = \ln(360/\pi) \; - \ln[\theta(\text{deg})] = 4.741 - \ln[\theta(\text{deg})].$$

(pseudo-)rapidity: properties (1)

Simple computations:

$$\text{a) } y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \xrightarrow{p>>m} \frac{1}{2}\ln\left(\frac{p+p_z}{p-p_z}\right) =$$

$$= \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right] = \eta;$$



$$\text{b) } y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = \frac{1}{2}\ln\left[\frac{(E+p_z)^2}{E^2-p_z^2}\right] = \ln\left(\frac{E+p_z}{m_T}\right) =$$

$$= \frac{1}{2}\ln\left[\frac{E^2-p_z^2}{(E-p_z)^2}\right] = \ln\left(\frac{m_T}{E-p_z}\right);$$

$$\text{c) } E+p_z = m_T e^y; \qquad E-p_z = m_T e^{-y};$$

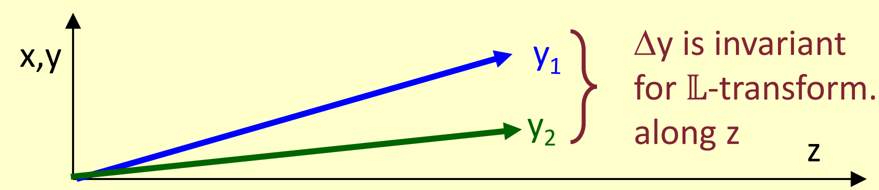
$$E = m_T\frac{e^y+e^{-y}}{2} = m_T\cosh(y);$$

$$p_z = m_T\frac{e^y-e^{-y}}{2} = m_T\sinh(y);$$

$$\rightarrow \quad y = \tanh^{-1}\left(\frac{p_z}{E}\right).$$

(pseudo-)rapidity: properties (2)

... And some others, quite long :



a) $\mathbb{L}$ transform : $p'_z = \gamma(p_z - \beta E);$

$$E' = \gamma(E - \beta p_z);$$

b) $$y' = \quad (y) = \frac{1}{2}\ln\left(\frac{E'+p'_z}{E'-p'_z}\right) =$$

$$= \frac{1}{2}\ln\left(\frac{\gamma E - \beta\gamma p_z + \gamma p_z - \beta\gamma E}{\gamma E - \beta\gamma p_z - \gamma p_z + \beta\gamma E}\right) =$$

$$= \frac{1}{2}\ln\left[\frac{E(1-\beta) + p_z(1-\beta)}{E(1+\beta) - p_z(1+\beta)}\right] =$$

$$= \frac{1}{2}\ln\left[\frac{(1-\beta)(E+p_z)}{(1+\beta)(E-p_z)}\right] = \frac{1}{2}\ln\left[\frac{(1-\beta)}{(1+\beta)}\right] + \frac{1}{2}\ln\left[\frac{(E+p_z)}{(E-p_z)}\right] =$$

$$= y + \tanh^{-1}(\beta).$$

c) $\Delta y = y_2 - y_1 = \Delta y' = y'_2 - y'_1;$

i.e. y is the variable, whose differential (even the finite Δy) is invariant for $\mathbb{L}$-transf. along z.

(pseudo-)rapidity: properties (3)

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

- Start from well-known math : $E = \sqrt{p_z^2 + p_T^2 + m^2}$;

$$dE = \frac{\partial E}{\partial p_z} dp_z = \frac{p_z dp_z}{E} \rightarrow \frac{dp_z}{E} = \frac{dE}{p_z}.$$

- Then :

$$dy = \frac{\partial y}{\partial p_z} dp_z + \frac{\partial y}{\partial E} dE = \frac{1}{2}\left(\frac{E-p_z}{E+p_z}\right)\left[\left(\frac{1}{E-p_z} + \frac{E+p_z}{(E-p_z)^2}\right)dp_z + \left(\frac{1}{E-p_z} - \frac{E+p_z}{(E-p_z)^2}\right)dE\right] =$$

$$= \frac{1}{2}\left(\frac{E-p_z}{E+p_z}\right)\left[\left(\frac{E-p_z+E+p_z}{(E-p_z)^2}\right)dp_z + \left(\frac{E-p_z-E-p_z}{(E-p_z)^2}\right)\frac{p_z dp_z}{E}\right] =$$

$$= \frac{1}{2}\left(\frac{dp_z}{E^2-p_z^2}\right)\left[2E - \frac{2p_z}{E}p_z\right] = \frac{1}{2}\left(\frac{dp_z}{E^2-p_z^2}\right)2\left(\frac{E^2-p_z^2}{E}\right) = \frac{dp_z}{E} = \frac{dE}{p_z}.$$

- i.e. the differential $dy = dp_z / E = dE / p_z$ at constant p_T.

- Definition of the **invariant cross section** ["invariant" under $\mathbb{L}$-transform. along z] :

$$\frac{Ed^3\sigma}{dp_x dp_y dp_z} = \frac{d^3\sigma}{p_T dp_T d\varphi dy}\left[= \frac{1}{\pi}\frac{d^2\sigma}{dp_T^2 dy}\right] = \frac{E' d^3\sigma}{dp'_x dp'_y dp'_z}.$$

(pseudo-)rapidity: properties (4)

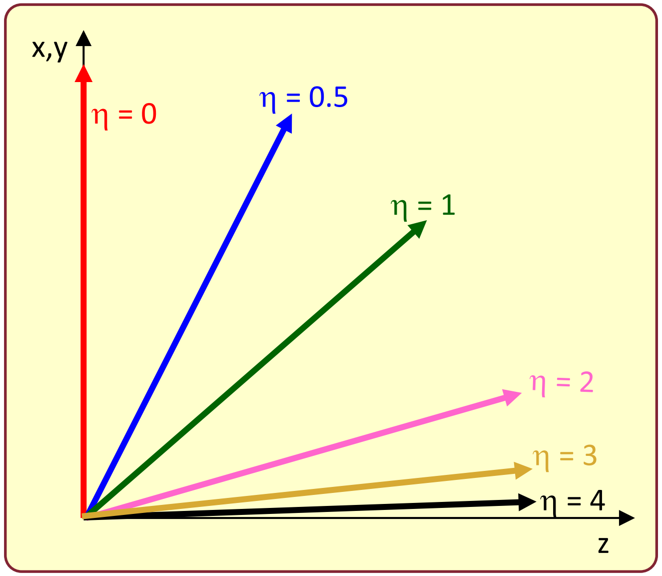
- [curiosity : an alternative way to show that y is invariant for $\mathbb{L}$-transf. along z :

$$\begin{cases} p'_z = \gamma(p_z - \beta E); \\ E' = \gamma(E - \beta p_z); \end{cases}$$

$$dp'_z = \frac{\partial p'_z}{\partial p_z} dp_z + \frac{\partial p'_z}{\partial E} dE = \gamma dp_z - \beta\gamma dE = \gamma dp_z - \beta\gamma \frac{p_z dp_z}{E} =$$

$$= \gamma dp_z \left(1 - \frac{\beta p_z}{E} \right) = \frac{\gamma dp_z}{E} (E - \beta p_z);$$

i.e. $\frac{dp'_z}{E'} = dy' = \frac{dp_z}{E} = dy$].

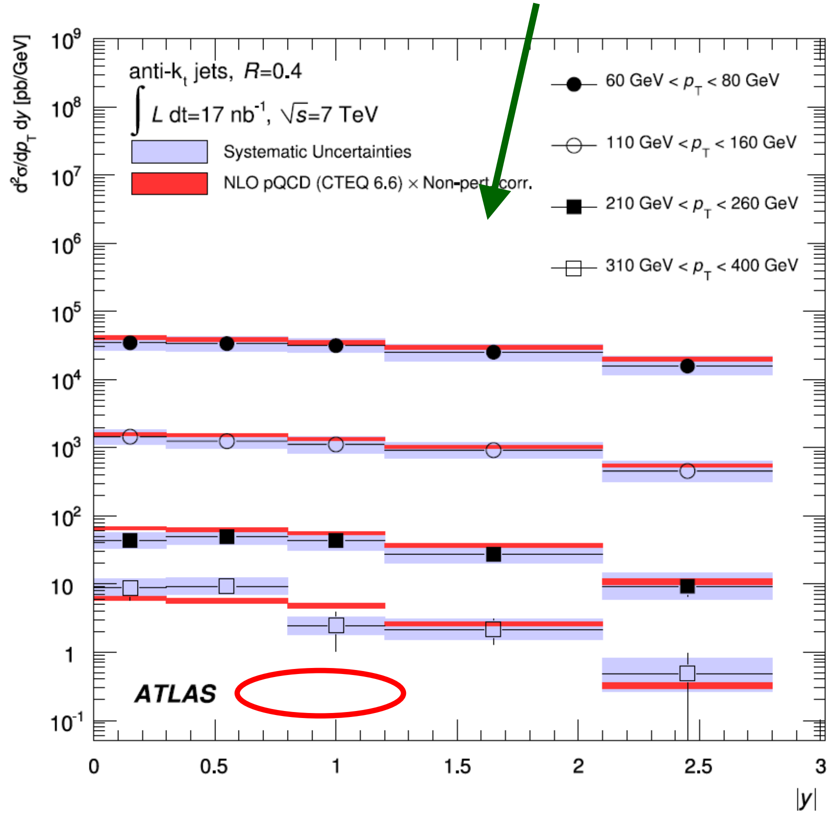


(pseudo-)rapidity: why

Why are hadronic interactions often analyzed in terms of (pseudo-)rapidity ?

Angular variables depend on each other : jacobian transformations relate all distributions; however, y looks "natural" (and produces simpler plots).

- The "Feynman argument" :
  - ➢ at high-p_T the real interaction happens at parton level;
  - ➢ the values of the parton momenta vary for each event, but they are (in 1st approx) along z;
  - ➢ therefore y is the correct variable in the lab., e.g. for jets and IVB analysis.
- The "Rutherford argument" :
  - ➢ in the parton CM, the scattering is dominated by t-channel processes;
  - ➢ the dominant processes are NOT flat in y, but $\propto t^{-2}$;
  - ➢ σ is a mixture of processes, with many t-dependences, indistinguishable on an event-by-event basis;
  - ➢ the rapidity, which expands the scale at $\theta \approx 0°$ is welcome : dσ/dy is ~ flat.



(pseudo-)rapidity: how

Why are soft hadronic interactions often analyzed in terms of (pseudo-)rapidity ?

The phenomenology of low-p_T :

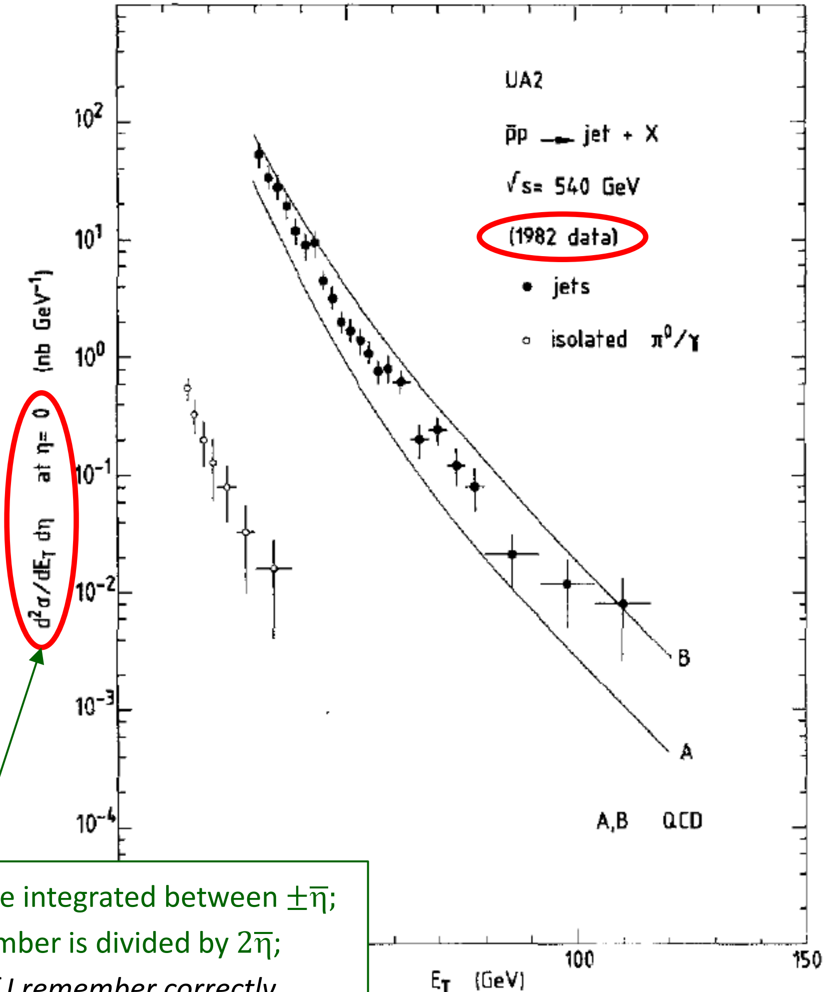
- [*maybe* reasons based on low-p_T physics, related to the invariant cross-section];
- the inclusive y distributions are ~ flat;
- so, y is very handy for fast background computations.

Why is η used often, instead of y ?

- y has important physical properties;
- y is difficult to measure, since is a small difference of two large quantities (E, p_z);
- η depends on an angle, exper. friendly;
- worst : in the literature sometimes η is given the properties of y [*but it is <u>ALMOST</u> correct*].

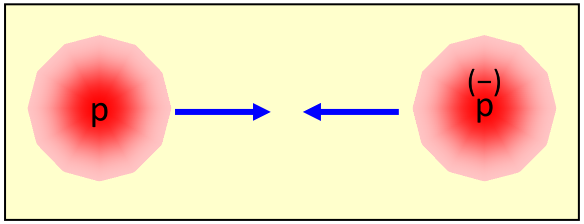
Instead, e^+e^- interactions, where partons ($=e^\pm$) interact in the LAB at x=1, are usually analyzed in terms of cos θ.

How to do it ? "typical example" : a hard interaction studied in terms of $d^2\sigma/dp_Td\eta|_{\eta=0}$.



it means : jets are integrated between $\pm\bar{\eta}$;
the resultant number is divided by $2\bar{\eta}$;
we used $\bar{\eta}$ = 1, *if I remember correctly.*

Log s physics



- An intuitive toy-model, with surprisingly good results :

  $\sigma_{tot}(pp\ or\ \bar{p}p) \approx \pi R^2 \approx \pi\ (\hbar c/m_\pi)^2 =$
  $= \pi(197\ \text{MeV}\cdot\text{fm} / 140\ \text{MeV})^2 = 62\ \text{mb}.$

- A limit ("Froissart bound") on the increase of cross-section for any pairs of particles, when $\sqrt{s}$ increases :

  for any two particles (ab) [e.g. pp, p̄p] :

  $$\lim_{s\to\infty} \sigma_{ab} \leq \text{const}\times\left(\ln^2 s\right),$$

  i.e."*at sufficiently high energies, the total cross-section for scattering on a given target* [*e.g* . $\sigma(\bar{p}p)$, $\sigma(pp)$, $\sigma(\pi^\pm p)$, $\sigma(\pi^\pm n)$] *cannot grow faster than ln² s*".

- A theorem, based on quantum field theory (NOT on dynamical assumptions, i.e. valid for any type of interaction), knows as the "Pomeranchuk theorem" :

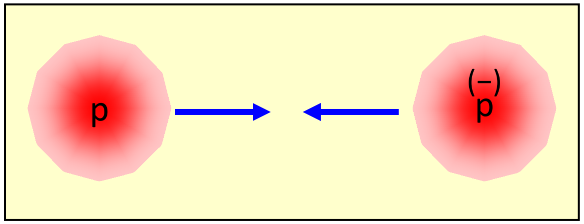
  $$\lim_{s\to\infty}\left(\frac{\sigma_{ab}}{\sigma_{\bar{a}b}}\right) = 1, \text{ for any two particles (a,b).}$$

  i.e. "*at sufficiently high energies, the total cross-section on a given target is the same for particle and antiparticle*" [*e.g.* $\sigma(\bar{p}p) \approx \sigma(pp)$, $\sigma(\pi^+n) \approx \sigma(\pi^-n)$].

- The (unexpected) experimental behavior that indeed *hadron cross-sections grow with* $\sqrt{s}$, [$\propto \ln(s)$ or maybe $\propto \ln^2(s)$], and that the "Pomeranchuk regime" is reached at accelerator energies.

↓

Log s physics: comments



- … gave rise (50 years ago) to much excitement and phenomenological models of low p_T hadronic interactions ("Regge poles", "Pomeron", "cylindrical phase space", ...).

- Then, no real breakthrough for many years …

there are books with an extensive treatment of the subject; instead we summarize everything here.

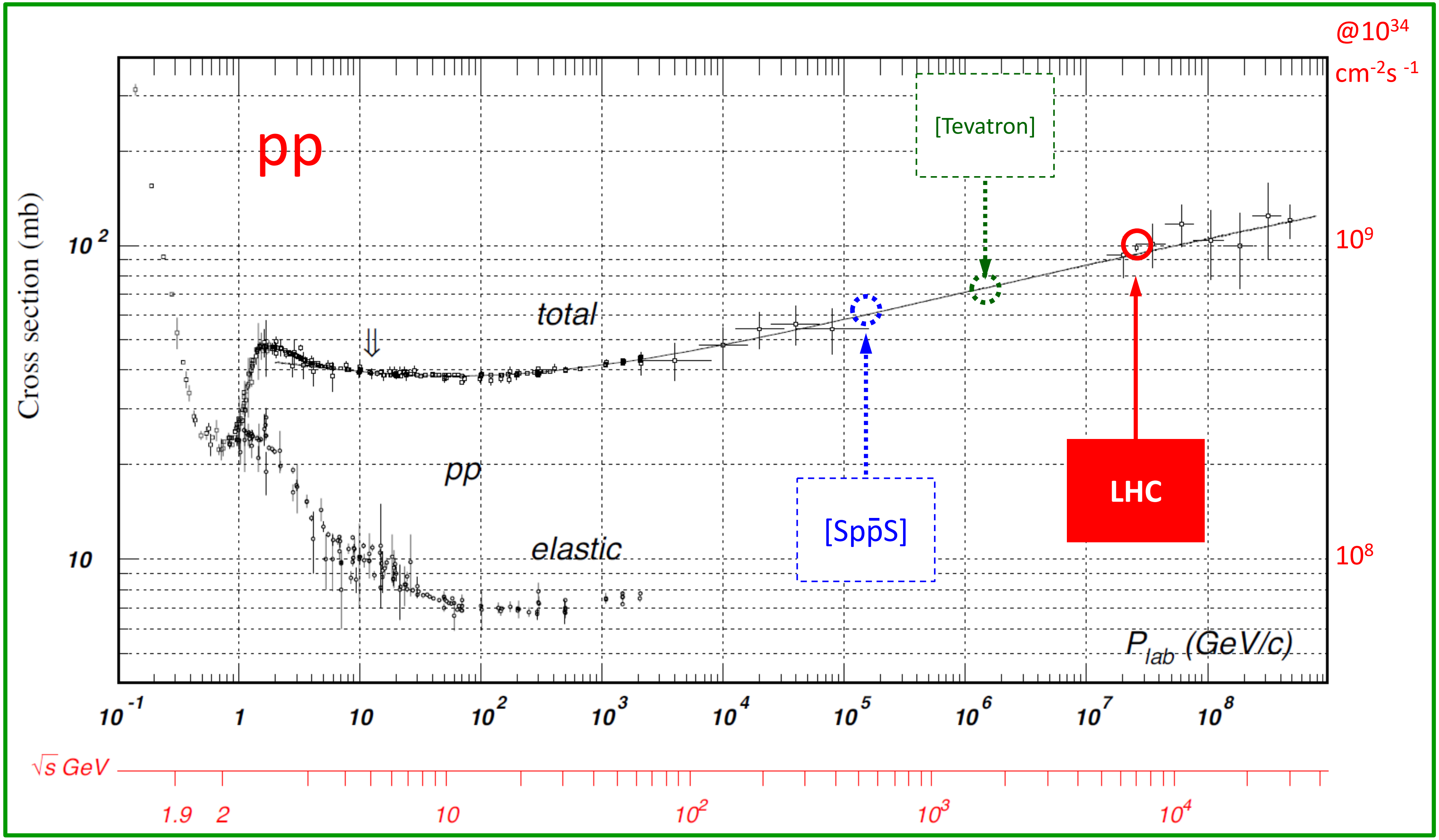
Comments (*very personal*) :

- physics born many years ago ('50s + CERN ISR), before the advent of QCD;
- poor conceptual foundations, but many phenomenological successes;
- many mysteries remain (perhaps no mystery, only complex many-body interactions, e.g. chemistry);
- today the main motivation of the study is to predict, parameterize and filter out the background.

In the following, we will assume this attitude.

The funny name "Log s physics" comes from the fact that, in low-p_T processes, the evolution with s of many quantities is logarithmic; the reasons are not really understood (Froissart ?).

Log s physics: $\sigma_{tot}(pp)$

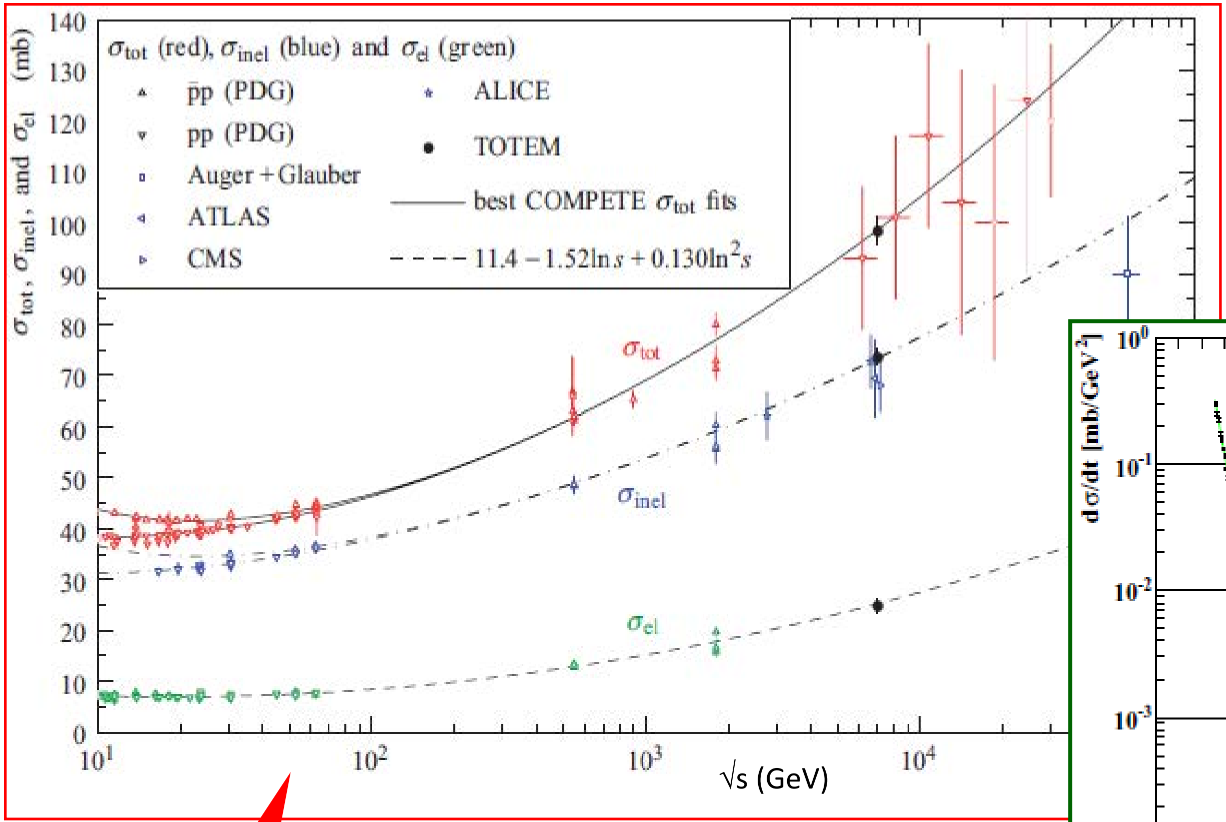


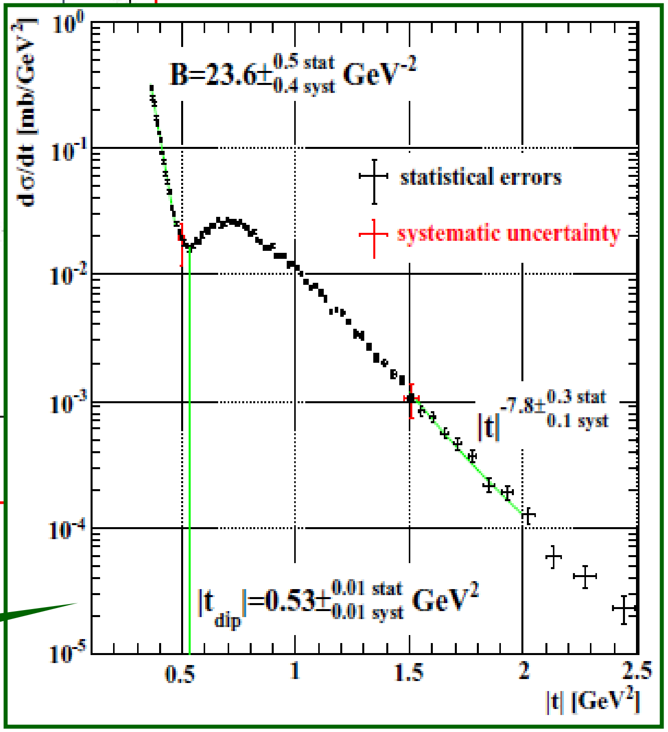
1 b = 10^{-28} m^2 = 10^{-24} cm^2
1 mb = 10^{-31} m^2 = 10^{-27} cm^2

The data of $\sigma(\bar{p}p)$, i.e. Sp$\bar{p}$S and Tevatron, are dashed, to show the similarity of the cross sections.

Log s physics: σ_{tot}, $d\sigma_{el}/dt$ @ LHC

TOTEM (LHC)
EPL, 96 (2011) 21002

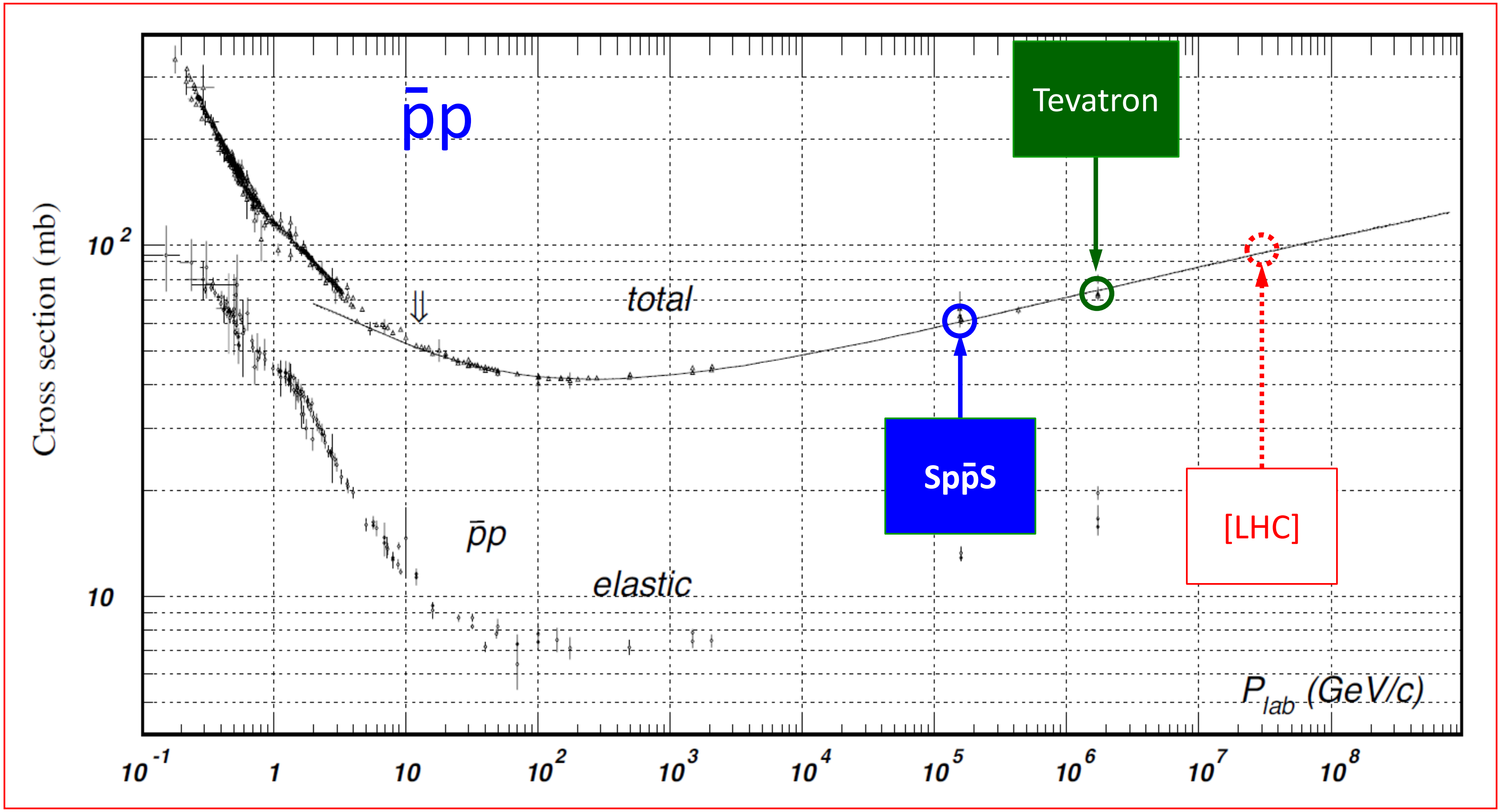




σ_T, σ_{anel}, σ_{el}

$d\sigma_{el}/dt$

Log s physics: $\sigma_{tot}(\bar{p}p)$



The data of σ(pp), i.e. LHC, do NOT belong to this plot; they are plotted dashed, to show the similarity of the cross sections ("Pomeranchuk theorem").

Log s physics: "rapidity plateau"

A heuristic computation :

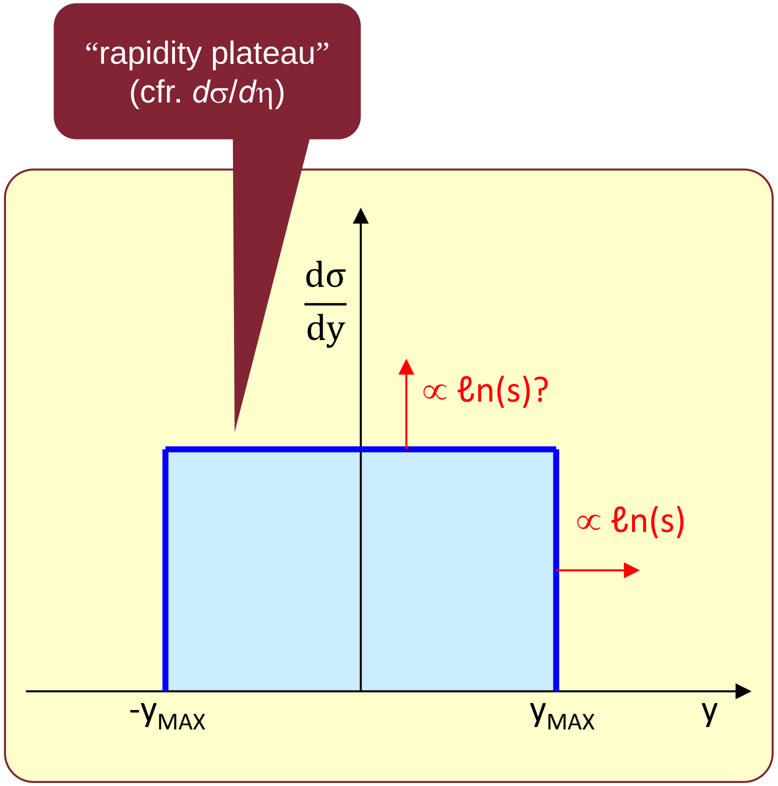
- Compute the limits on y :

$$y = \ln\left(\frac{E+p_z}{m_T}\right) \le \ln\left(\frac{\sqrt{s}}{m_T}\right) \le \frac{1}{2}\ln\left(\frac{s}{m^2}\right) \equiv y_{MAX};$$

- i.e. y_{max} increases $\propto \ln(s)$;
- if there is a "rapidity plateau", the total cross section is represented by the area of the rectangle :

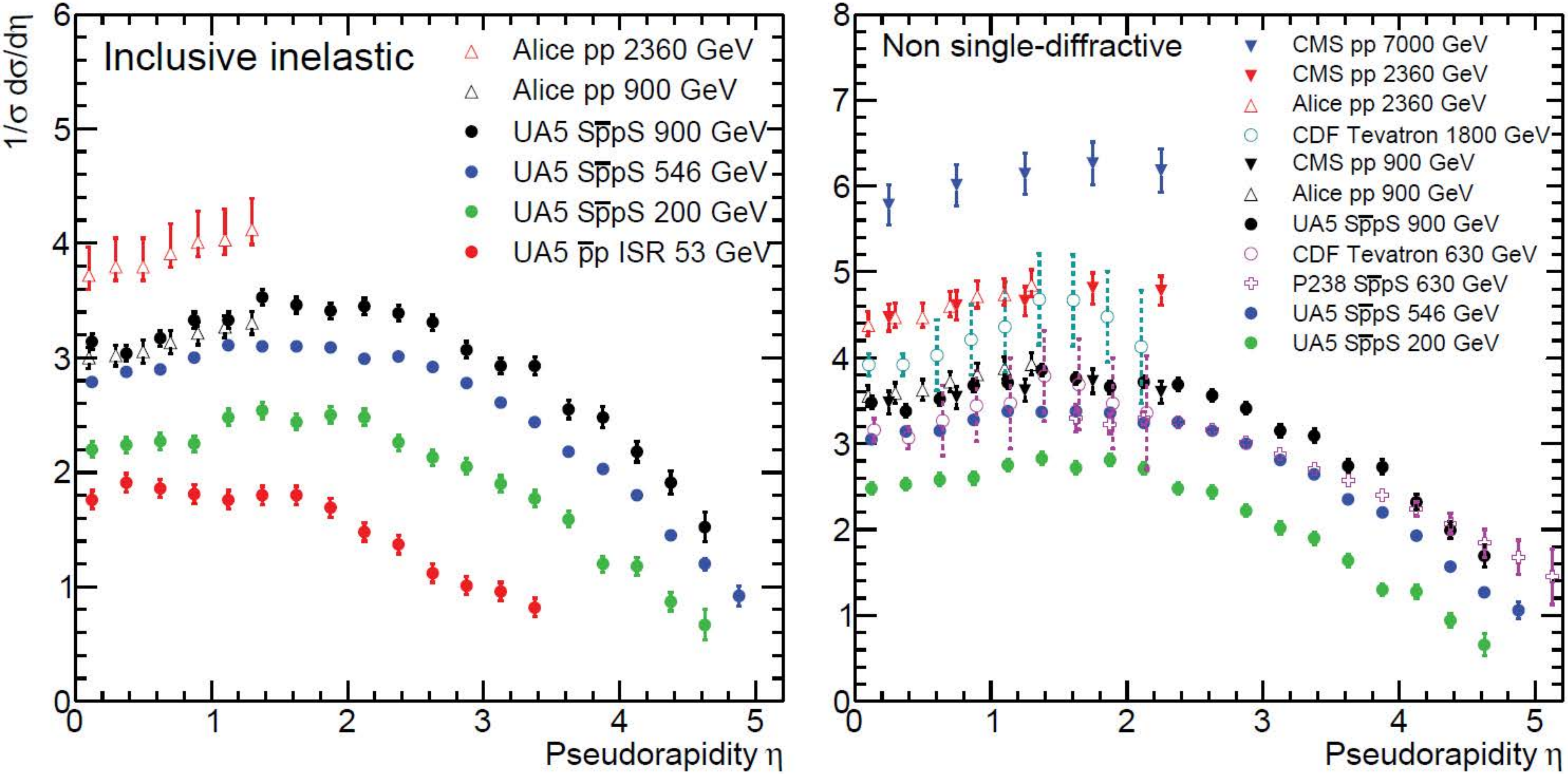
$$\sigma_{tot} = \int_{-y_{MAX}}^{-y_{MAX}} \left(\frac{d\sigma}{dy}\right) dy \approx \text{const} \times \left(\frac{d\sigma}{dy}\right) \times \ln(s);$$

- if the plateau grows $\propto$ ln s, then $\sigma_{tot} \propto \ln^2 s$, and "saturates" the Froissart bound;
- actually, this seems to be the case : both width and height of the rectangle grow $\propto$ ln s.



The real question is : why dσ/dy ∝ ln s ?

Log s physics: $d\sigma/d\eta|_{particles}$

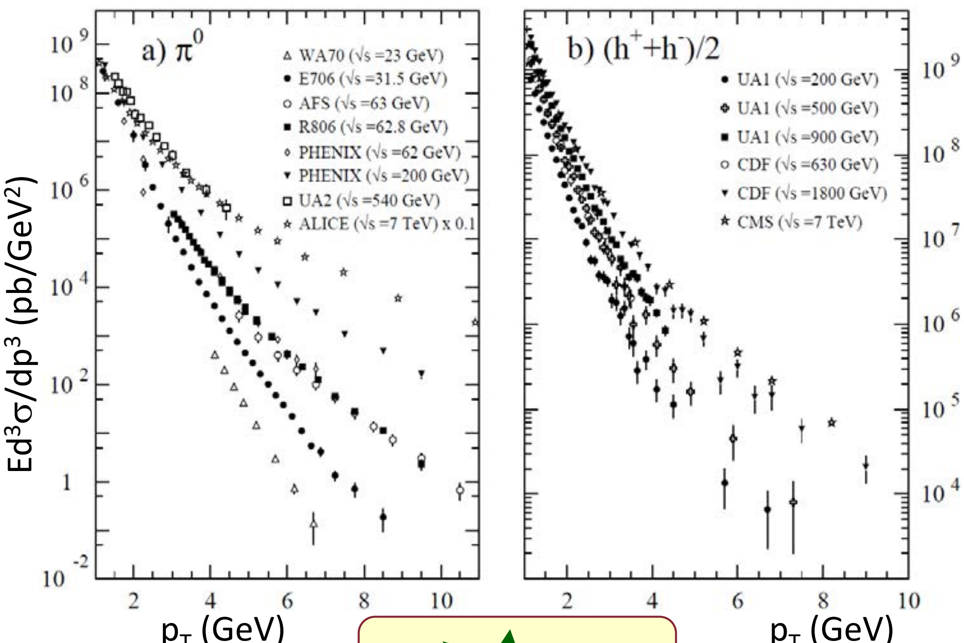


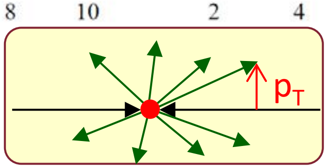
The η distributions of charged particles exhibit typical "rapidity plateaus", which increases ∝ log s.

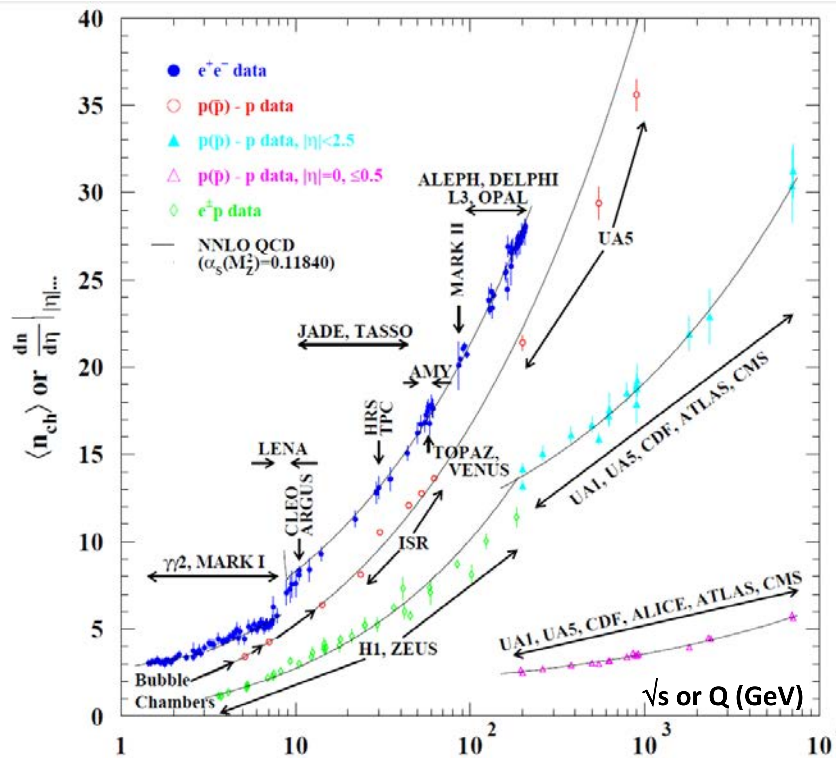
Log s physics: inclusive data

The number and p_T distribution of the charged particles of the final state exhibits interesting properties :

- they seem to follow a general law;
- the law is independent from the primary state (e^+e^-, pp, $\bar{p}$p, $e^\pm$p);
- it scales (approx) $\propto \ln s$ or $\propto \ln^2 s$.







Suggestion of a general "factorization property" of single particle production at low-p_T ["Feynman scaling"] :

$$\frac{Ed^3\sigma}{p_T dp_T dy} = f(s, p_T, y) \approx f_s(s) f_{p_T}(p_T) f_y(y);$$

In turn, the single f_i exhibits interesting properties (like the log-dependence of f_s).

The quark parton model

Hadronic collisions at high p_T (= short distance) are studied in terms of the "**quark-parton model**" (*) :

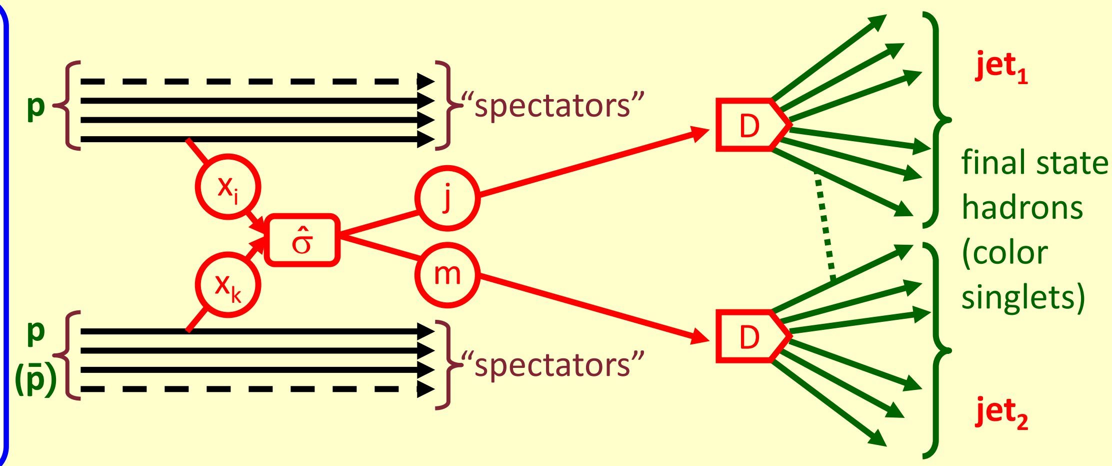
- the process take place in phases, that "**factorize**" (= take place one after the other, without mutual interference);
- the hadrons of the initial state are an **incoherent mixture** of elementary partons (= quarks and gluons of QCD);
- the partons behave as **point-like** particles **quasi- free** (like the electrons in e^+e^-);
- because of the sea contribution, the "**number**" of partons in a hadron is **not defined**; only their total momentum (= the hadron momentum) is measurable.

(… continue …)

(*) hadronic collisions at low p_T (= great distance, Q^2<[few-GeV]^2) correspond to interactions between non-point-like hadrons; they do NOT belong to this picture.

$$\sigma(\bar{p}p\,[pp] \rightarrow jet_1 jet_2) =$$

$$= \sum_{i,k} \int \left[\begin{array}{l} dx_i dx_k F_i^p\left(x_i, Q^2\right) F_k^{\bar{p}}\left(x_k, Q^2\right) \\ \hat{\sigma}\left(ik \rightarrow jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$



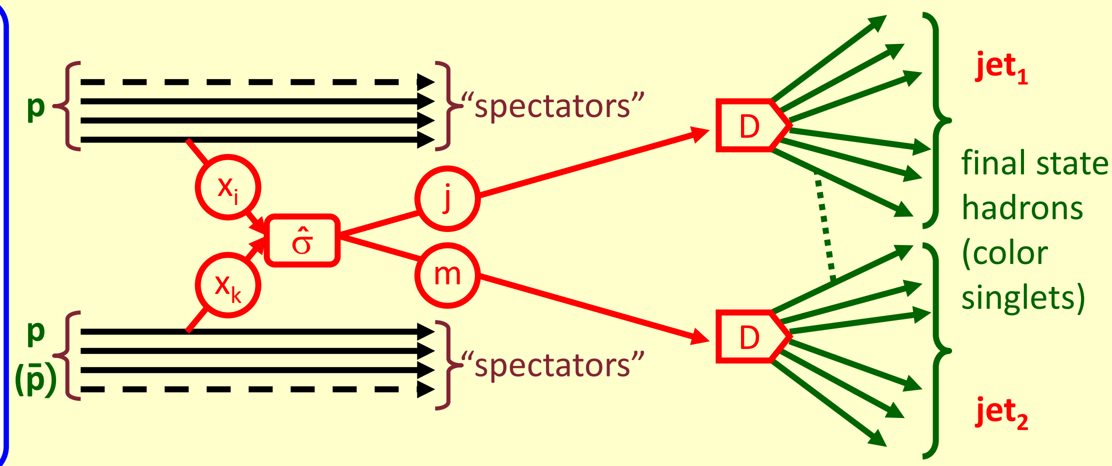
The quark parton model: initial state

- in first approximation, partons have **only longitudinal momentum** (the "Fermi motion" of partons in the hadron is small);
- each parton shares **a fraction x** of the momentum of its parent :
  $\vec{p}_{parton} = (0, 0, \pm x\, p_{hadron})$;
- the distribution function of x [$F_i^h(x,Q^2)$, for the parton i in the hadron h] are called **pdf [= parton distribution functions**, and depend both on x and Q^2 [§ 2 and 7];
- the evolution in (x, Q^2) of the pdf is regulated in non-perturbative QCD by the equation **GLAP** (Gribov – Lipatov – Altarelli – Parisi).

(… continue …)

$$\sigma(\bar{p}p\,[pp] \to jet_1 jet_2) =$$

$$= \sum_{i,k} \int \left[\begin{array}{c} dx_i dx_k F_i^p(x_i, Q^2) F_k^{\bar{p}}(x_k, Q^2) \\ \hat{\sigma}\left(ik \to jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$



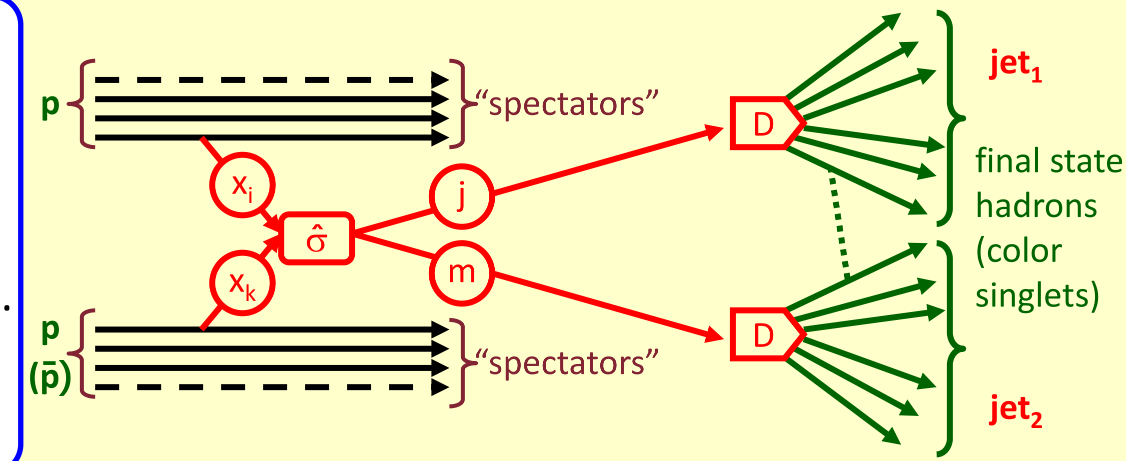
The quark parton model: collision

- collisions at high-p_T between elementary partons are two-body scatterings ("ab → cd"), to be studied in **perturbative QCD**;
- parton energy in their CM : $\hat{s} = s x_1 x_2$;
- most of the partons of the hadrons do NOT participate in the collision ("**spectator partons**"); they continue in a direction (quasi-)parallel to the hadrons of the initial state;
- after the collision, the partons of the final state "hadronize" ("**fragment**"), i.e. give rise to the hadrons of the final state;
- those particles emerge as collimated sprays ("**jets**") of particles with high p_T;
- the 4-vector sum of the momenta of the hadrons of a **jet** is identified with the 4-vector momentum of the **parton**.

(…continue…)

$$\sigma(\bar{p}p\,[pp] \to jet_1 jet_2) =$$

$$= \sum_{i,k} \int \left[\begin{array}{c} dx_i dx_k F_i^p\left(x_i, Q^2\right) F_k^{\bar{p}}\left(x_k, Q^2\right) \\ \hat{\sigma}\left(ik \to jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$

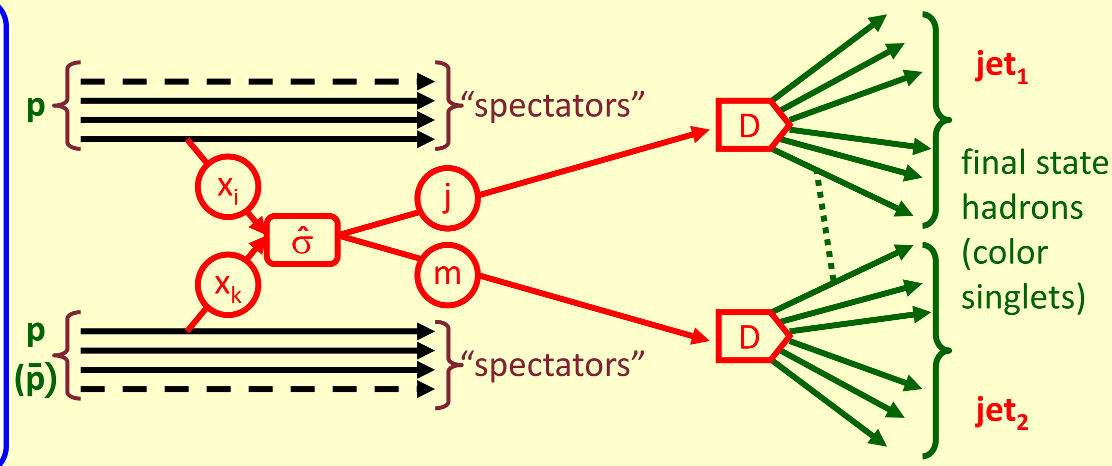


The quark parton model: fragmentation

- The distributions of the final state hadrons are called "**fragmentation functions**";
- they are functions $[D_p^h(z,Q^2)]$ of the variable z (= p_{hadron} / p_{parton}), which defines the distribution of hadron "h" in a jet from parton "p";
- they do NOT depend, to a good approximation, neither on the initial state, nor on the elementary collision, but only on the final state parton and the value of Q^2;
- however, unlike the partons of the elementary collision, the hadrons are color singlets; therefore in the process of fragmentation particles of different jets must interact.

$$\sigma(\bar{p}p\,[pp] \to jet_1 jet_2) =$$

$$= \sum_{i,k} \int \left[\begin{array}{l} dx_i dx_k F_i^p\left(x_i, Q^2\right) F_k^{\bar{p}}\left(x_k, Q^2\right) \\ \hat{\sigma}\left(ik \to jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$



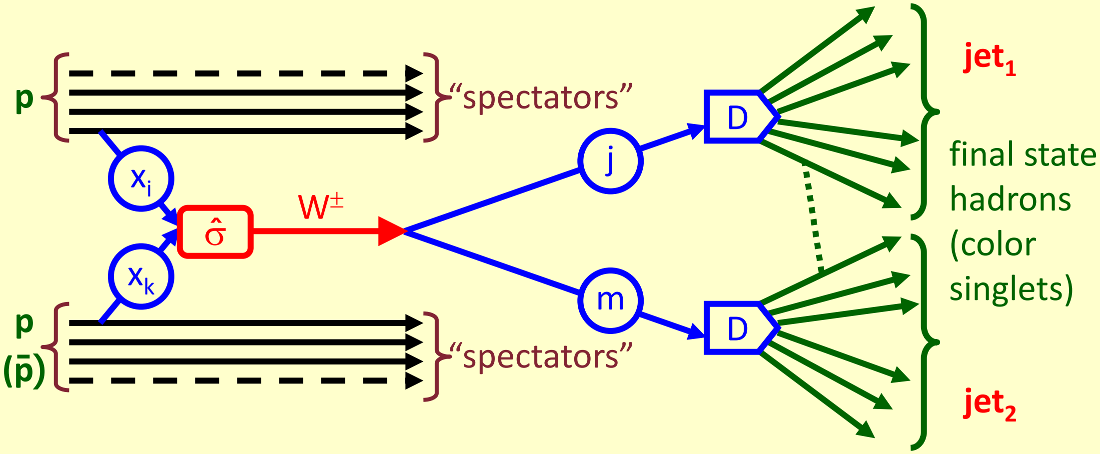
The quark parton model: electroweak

- In (few but) interesting cases, non-QCD processes happens [e.g. $\bar{u}d \rightarrow W^-$, followed by W decay into quarks];
- these processes are rare (e.g. $10^{-5} \div 10^{-6}$ of pQCD at LHC), but very valuable; they are at the origin of both the Sp$\bar{p}$S and LHC construction;
- the analysis proceeds in the same way: the two-body QCD parton scattering is replaced by the appropriate electroweak (or SUSY, or whatever) theory;

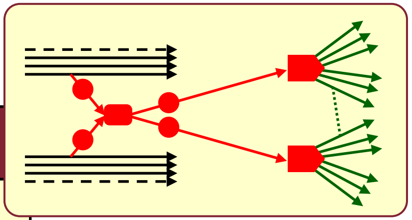
[the figure represents a Drell-Yan process (see § Sp$\bar{p}$S), with the creation of a $W^\pm$ and its successive decay into a $q\bar{q}$ pair, which fragments into two jets; other processes are treated in the same way.]

$$\sigma(\bar{p}p\,[pp] \rightarrow W^\pm) =$$

$$= \sum_{i,k} \int \left[\begin{array}{l} dx_i dx_k F_i^p\left(x_i, Q^2\right) F_k^{\bar{p}}\left(x_k, Q^2\right) \\ \hat{\sigma}\left(ik \rightarrow W^\pm @ \sqrt{s x_i x_k} = m_w\right) \end{array} \right]$$



The quark parton model: score



process	prediction ?	theory $\leftrightarrow$ exp.	why
$\sigma_{tot}(\bar{p}p \to \bar{p}p)$	no	the optical theorem is a relation, NOT a prediction.	low-p_T
$\sigma_{tot}(pp \to pp)$	no		
$\sigma_{incl}(pp/\bar{p}p \to \pi^+ X)$	no	ℓn s model ?	
$\sigma_{incl}(pp/\bar{p}p \to \text{jet } X)$	yes	fair	pQCD
$\sigma_{incl}(pp/\bar{p}p \to Z\ X)$	yes	good	electro-weak
$\sigma_{incl}(pp/\bar{p}p \to W\ X)$	yes	good	
$\sigma_{incl}(pp/\bar{p}p \to H\ X)$	yes	very good	
$\sigma_{incl}(pp/\bar{p}p \to \text{SUSY})$	if …	???	???

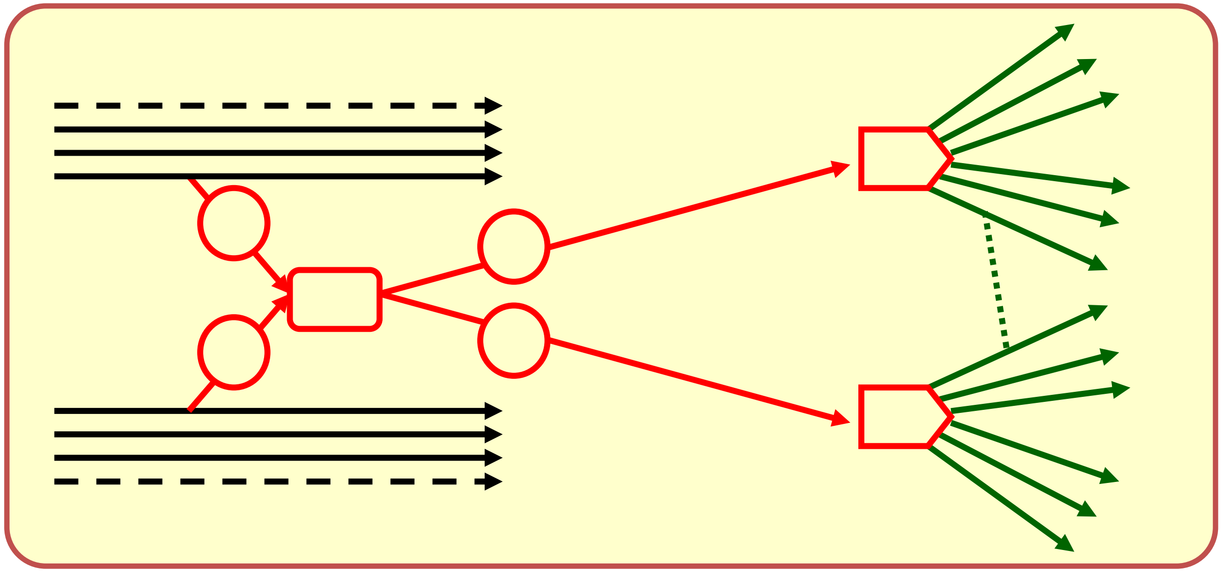
cfr. similar e.w. processes:

$\sigma_{tot}(e^+e^- \to \mu^+\mu^-)$	yes	perfect	pure e.w.
$\sigma_{tot}(e^+e^- \to Z \to f\bar{f})$	yes	perfect	pure e.w.
$\sigma_{tot}(e^+e^- \to HZ \to f\bar{f}f\bar{f})$	yes	[it will be perfect, I know]	pure e.w.

The quark parton model: method

- The scheme works for all known interactions of quarks and gluons, both e.w. and strong, if the correct definition of the elementary process ($\hat{\sigma}$) is applied.
- The present method is to reproduce the process, via Montecarlo generation of events, later analyzed as real data.
- When, according to q.m., a distribution function (e.g. $\hat{\sigma}$, pdf) appears, the random function of the computer is used.
- Many events are generated, so the a-posteriori analysis is able to predict/reproduce the statistical result.
- A single event is built in successive steps, according to the "factorization approximation":

(continue …)



The quark parton model: procedure

a. a **parton of a given type** is generated out of the first hadron; its x is also generated, according to its pdf;

b. ditto for the **second** init. state parton;

c. the elementary parton process is computed, using the appropriate **cross section at parton level**[(1)];

d. (as a part of this step) **the angular distribution** of the final state partons is generated, according to the dynamics of the elementary process;

e. each parton of the final state is fragmented, with its **fragmentation functions** (or a fragmentation model[(2)]);

f. the hadrons from **spectator** partons are added (few methods exist);

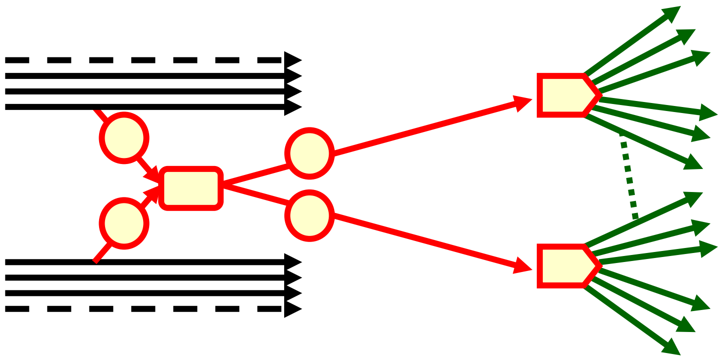
g. all the hadrons of the final state are recorded for successive analysis.

---

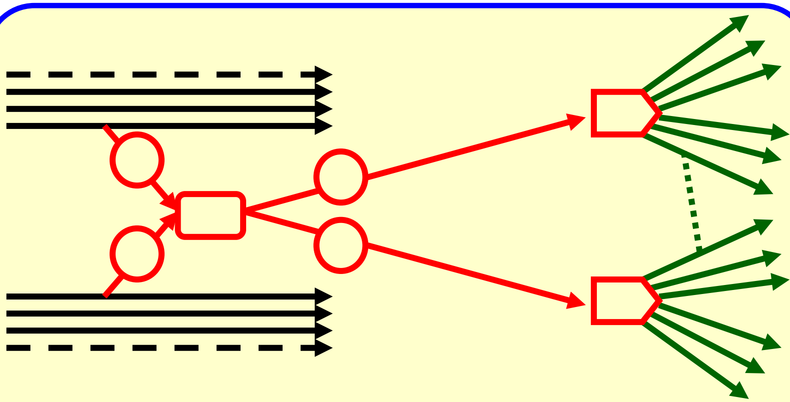
(1) In case of electroweak decays ($W^{\pm}$, Z, H), with production of leptons, the treatment of the final state has to be appropriate (in fact, it is easier, since the fragmentation step is absent or simpler).

(2) "Fragmentation models" like Lund (Pythia), Herwig, are a mixture of theory (perturbative and non-pertubative QCD), parameterization of measurements (fragmentation functions) and computing skill for easy management. They are very well-done and successful, but are NOT based on a complete reproduction of the theory.

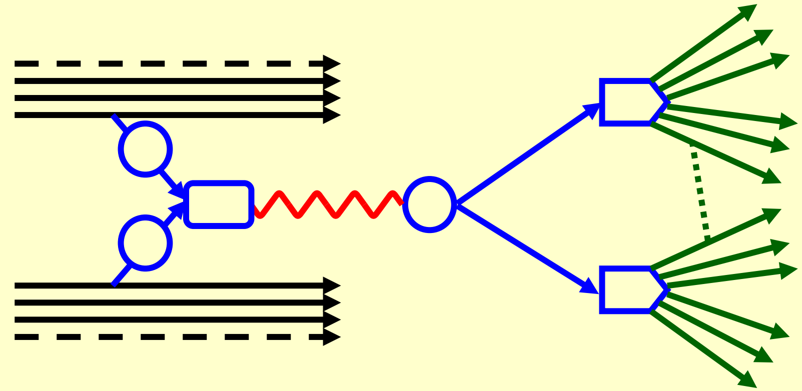
NB. The procedure just described contains some loopholes, e.g. pdf's (a-b) depend on Q^2, which is generated later (c-d); there are appropriate tricks, not described here.



The quark parton model: examples



$$\sigma(\bar{p}p\,[pp] \to jet_1 jet_2) =$$
$$= \sum_{i,k} \int \left[\begin{array}{l} dx_i dx_k F_i^p\left(x_i, Q^2\right) F_k^{\bar{p}}\left(x_k, Q^2\right) \\ \hat{\sigma}\left(ik \to jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$



$$\sigma(\bar{p}p\,[pp] \to W \to q_1 \bar{q}_2) =$$
$$= \sum_{i,k} \int \left[\begin{array}{l} dx_i dx_k F_i^p\left(x_i, Q^2\right) F_k^{\bar{p}}\left(x_k, Q^2\right) \times \\ \hat{\sigma}\left(ik \to W @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \times BR(W \to q_1 \bar{q}_2) \end{array} \right].$$

Two test-case processes for the q-p model :

a) two-jet production;

b) W (or Z) production and decay into jets.

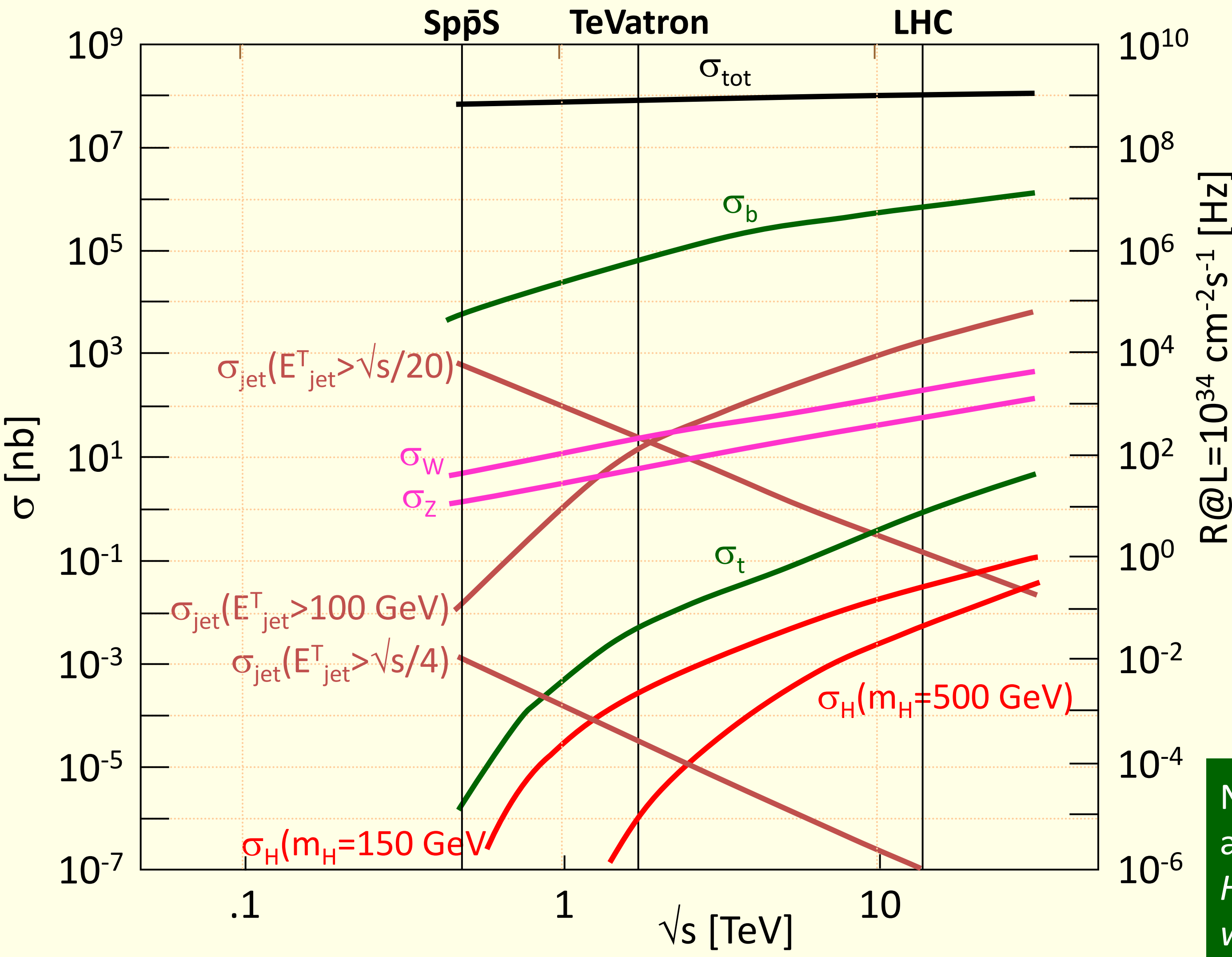
Notice the correspondence between the scheme and the corresponding formula.

The sums run over all the partons which may generate the final state, and the integrals between the kinematical limits.

The pdf's "weight" the processes, giving each parton and each x the correct share.

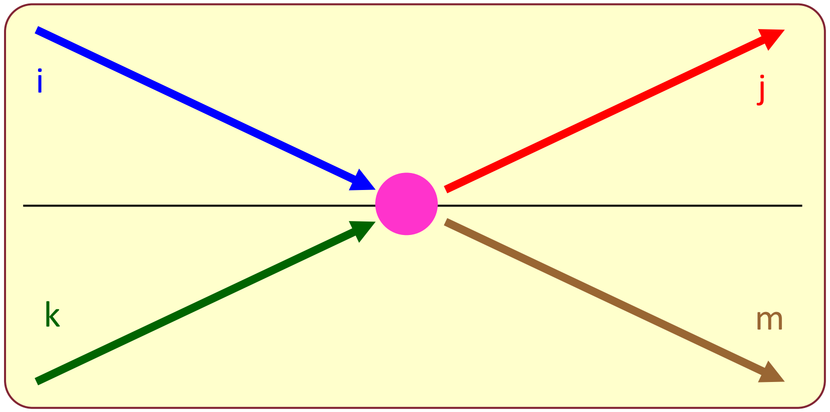
NB. a) in principle the parton type is observable $\to$ sum the σ's, NOT the amplitudes;
b) σ_W is strongly peaked for real W's $\to$ x_i, x_k are NOT kinematically independent]

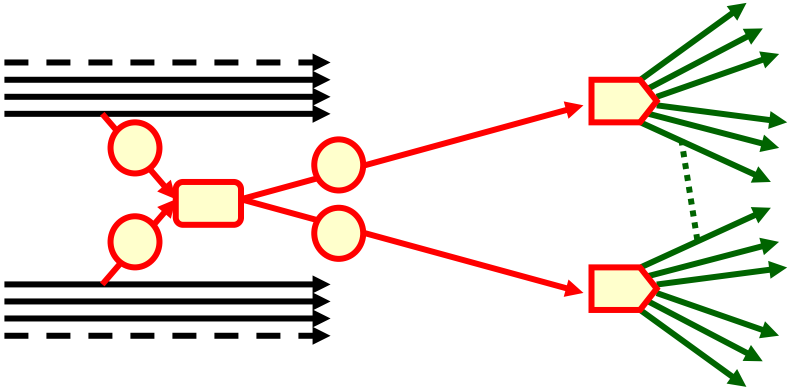
The quark parton model: Sp$\bar{p}$S → LHC



NB. Sp$\bar{p}$S and Tevatron are $\bar{p}$p, LHC is pp. *However, no difference within the accuracy of this plot.*

High-p_T: kinematics





- initial state in pp [p̄p] CM :

p_{hadron_1}= [½√s, ½√s, ~0, ~0]; p_{parton_i} = [½x_i√s, ½x_i√s, ~0, ~0];

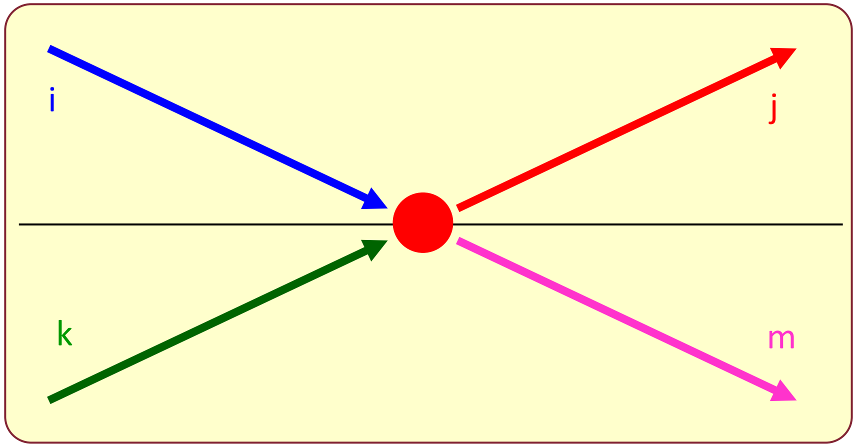
p_{hadron_2}= [½√s,-½√s, ~0, ~0]; p_{parton_k} = [½x_k√s, -½x_k√s, ~0, ~0];

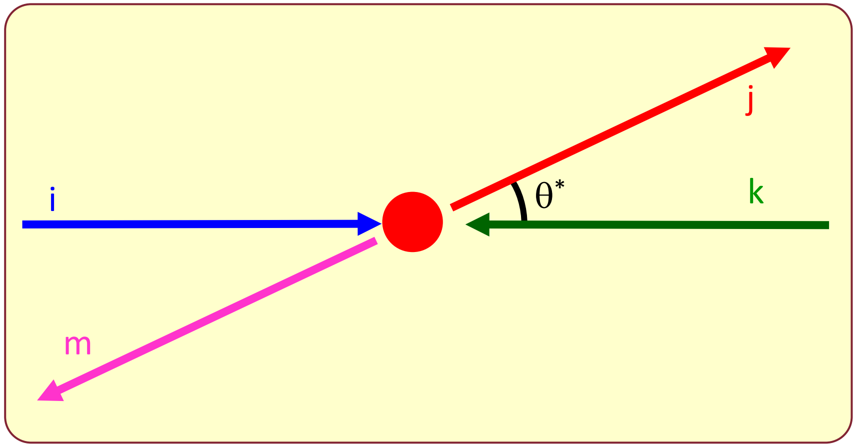
@LHC not head-on collisions + Fermi motion of partons

- sum : ik in CM_{12}: [½√s(x_i + x_k), ½√s(x_i - x_k), ~0, ~0];

ik in CM_{ik} : $[\sqrt{\hat{s}}, 0, 0, 0] \rightarrow \hat{s} = ¼s[(x_i + x_k)^2 - (x_i - x_k)^2] = s\, x_i\, x_k$.

High-p_T: parton variables

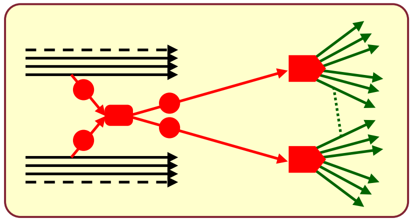




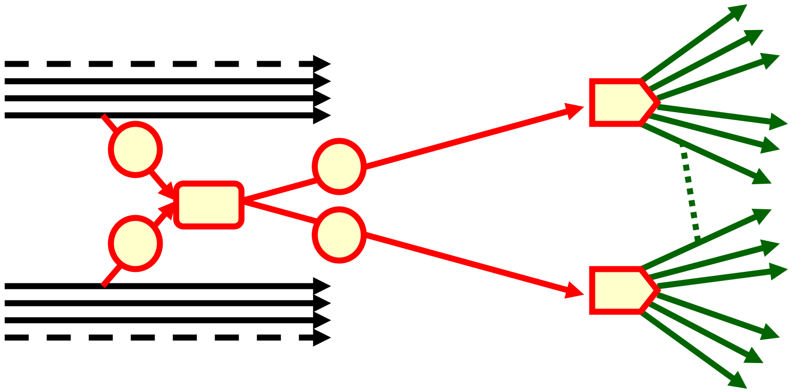
- $p_i = [\tfrac{1}{2}\sqrt{\hat{s}},\quad \tfrac{1}{2}\sqrt{\hat{s}},\quad 0,\quad 0];$
- $p_k = [\tfrac{1}{2}\sqrt{\hat{s}},\quad -\tfrac{1}{2}\sqrt{\hat{s}},\quad 0,\quad 0];$
- $p_j = [\tfrac{1}{2}\sqrt{\hat{s}},\quad \tfrac{1}{2}\sqrt{\hat{s}}\cos\theta^*,\quad \tfrac{1}{2}\sqrt{\hat{s}}\sin\theta^*,\quad 0];$
- $p_m = [\tfrac{1}{2}\sqrt{\hat{s}},\quad -\tfrac{1}{2}\sqrt{\hat{s}}\cos\theta^*,\quad -\tfrac{1}{2}\sqrt{\hat{s}}\sin\theta^*,\quad 0];$
- $\hat{s} = (p_i + p_k)^2 = (p_j + p_m)^2 = s\, x_i\, x_k;$
- $\hat{t} = (p_i - p_j)^2 = (p_m - p_k)^2 = -\tfrac{1}{2}\hat{s}\,(1 - \cos\theta^*);$
- $\hat{u} = (p_i - p_m)^2 = (p_k - p_j)^2 = -\tfrac{1}{2}\hat{s}\,(1 + \cos\theta^*);$
- $\hat{s} + \hat{t} + \hat{u} = 0$ ($\rightarrow$ in parton CM, two independent variables).

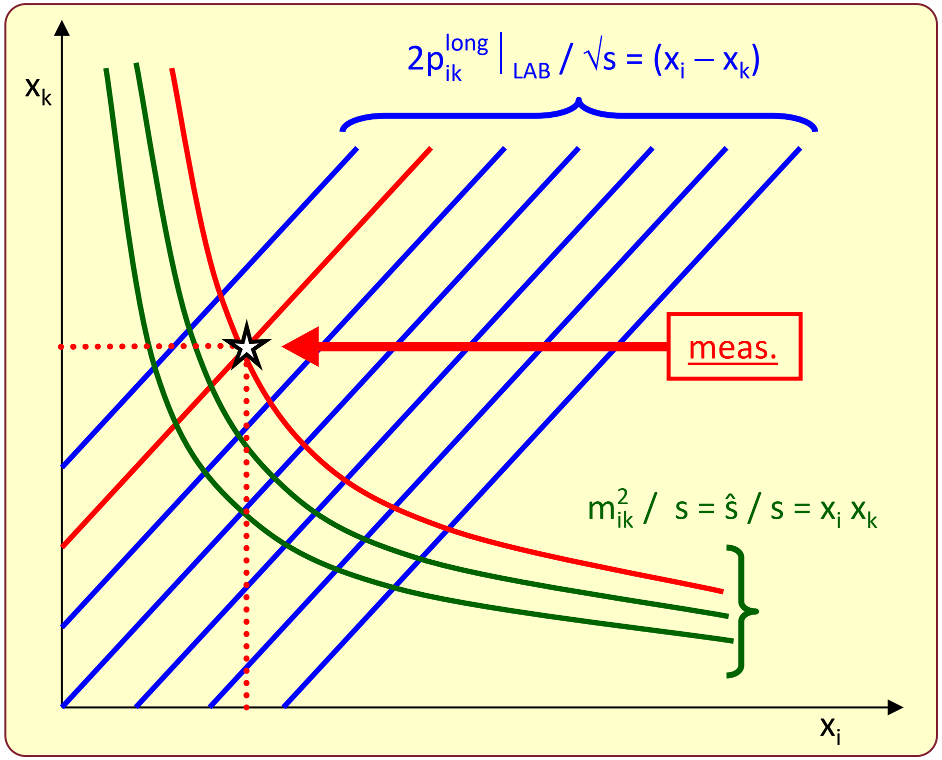
Comments:

- see § 3 for similar discussion for not-composite particles;
- zero mass approx for all partons [for $m \neq 0$, § 3 and PDG § 43.5].



High-p_T: solve the kinematics





- The overall transverse momentum MUST be balanced. A p_T imbalance is attributed to non interacting particles (ν's) or, most likely, to measurement errors.
- By measuring the 4-momenta of the final state (e.g. two jets), it is possible to compute $\hat{s}$ and p_{long}. From there, x_i and x_k and the full kinematics at parton level.

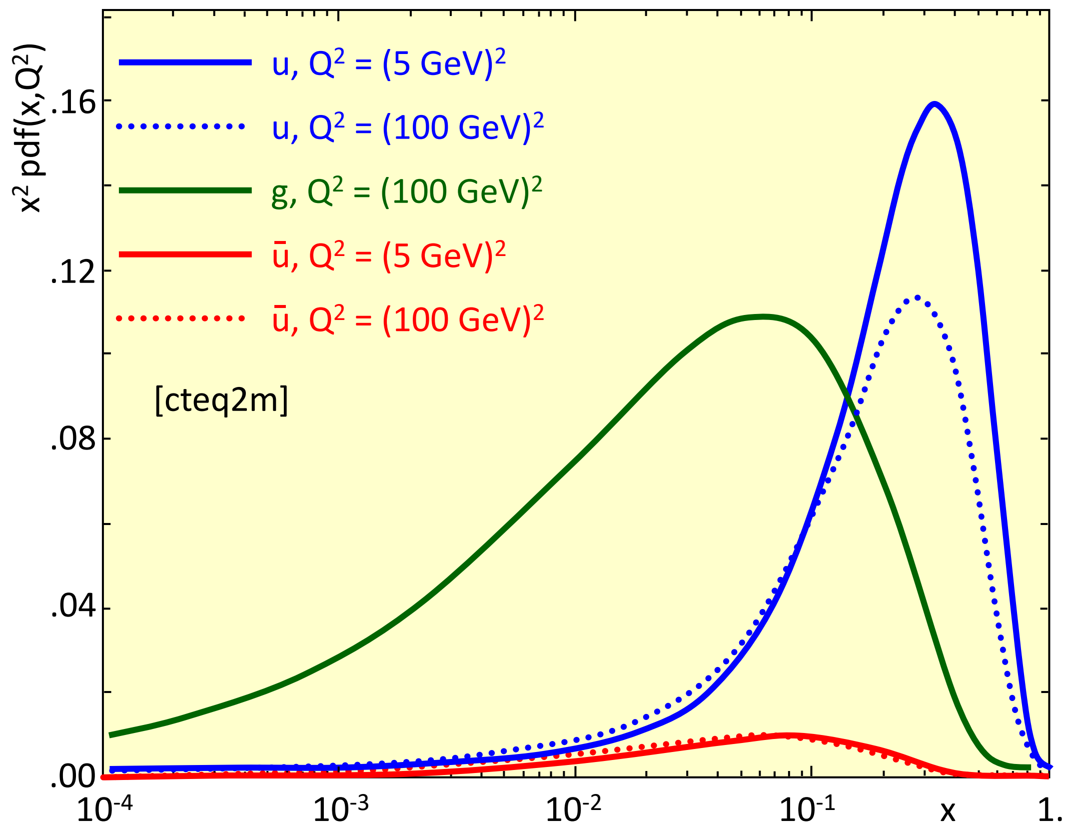
Compute $(\vec{p}_i + \vec{p}_k)$:

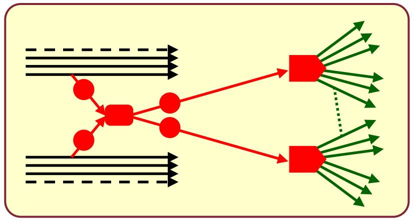
- LAB : [$\tfrac{1}{2}\sqrt{s}(x_i+x_k)$, $\tfrac{1}{2}\sqrt{s}(x_i-x_k)$, ~0, ~0];
- CM_{ik} : [$\sqrt{\hat{s}}$, 0, 0, 0];

$\rightarrow \hat{s} = \tfrac{1}{4}s[(x_i+x_k)^2 - (x_i-x_k)^2] = s\, x_i\, x_k$.

High-p_T: structure functions (pdf)

- in the quark parton model, hadrons are "wide-band beams" of elementary partons;
- in first approximation, structure functions do NOT depend on Q^2 : $\partial F_i(x, Q^2) / \partial Q^2 = 0$;
- but scaling violations do exist.





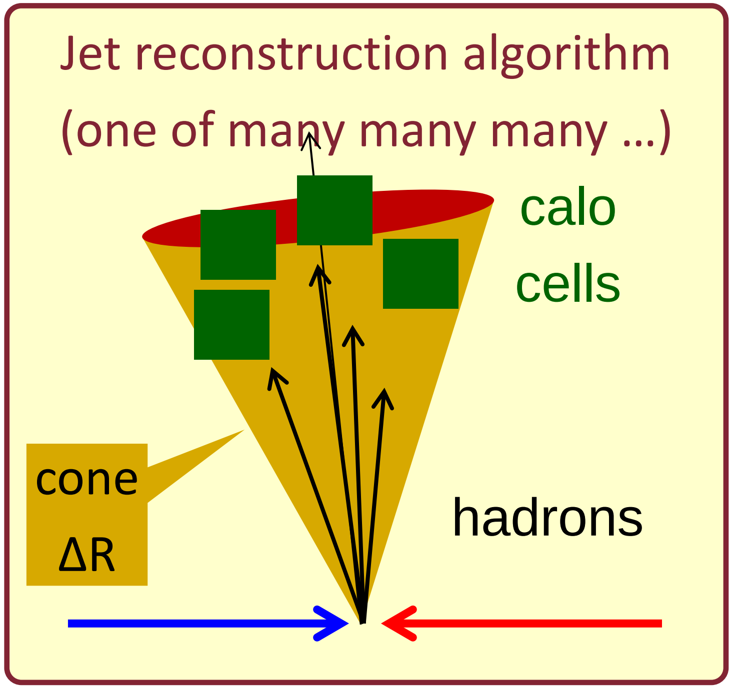
www.zebu.uoregon.edu/~parton/partongraph.html

when Q^2 increases :

- partons "get closer";
- q_{sea} and g increase at small x;
- $q_{valence}$ decreases at all x;
- at x fixed and large, rates @LHC smaller than @Sp$\bar{p}$S.

High-p_T: partons $\rightarrow$ jets

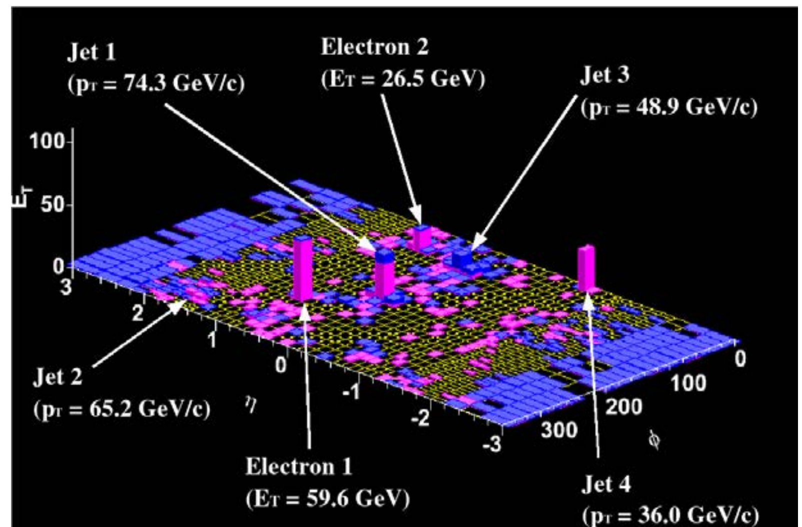
- reconstruct the jets via an algorithm :
  - simple clustering of nearby calo cells;
  - cone algo. (see fig) with fixed ΔR (very popular $\Delta R^2 = \Delta\phi^2 + \Delta\eta^2 = 1$);
  - "Durham"
  - anti-Kt
  - …



- more refined cooking (split, sum, …)
- reconstruct 4-momentum :

$$\vec{p}_{jet}=\sum\vec{p}_{hadrons}; \quad E_{jet}=\sum E_{hadrons};$$

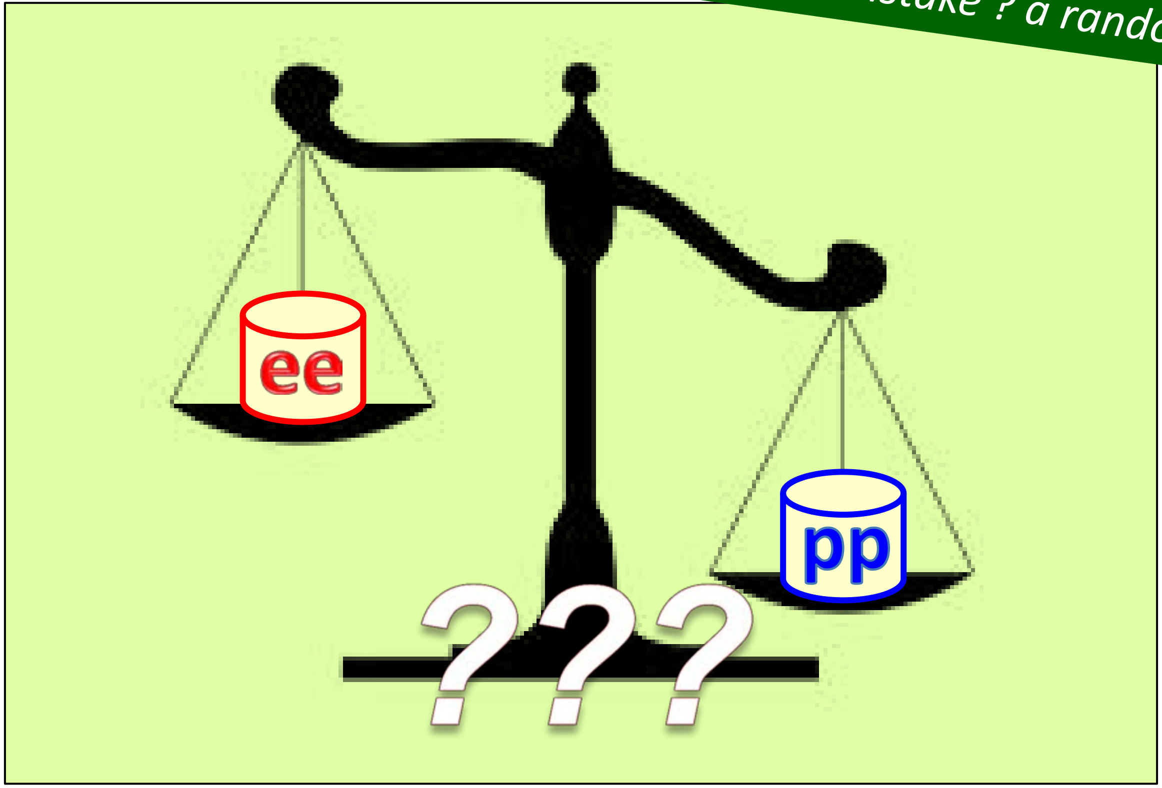
- [*notice that the above definition gives jets a mass ≠ 0, generally much larger than the tiny parton mass → more cooking …*]
- identify (jet → parton) and play with its 4-momentum;
- check the manipulations with known cases ($W^\pm$, Z → jets) and montecarlo.



CDF – Z→ e^+e^- + jets

iii. Comparisons

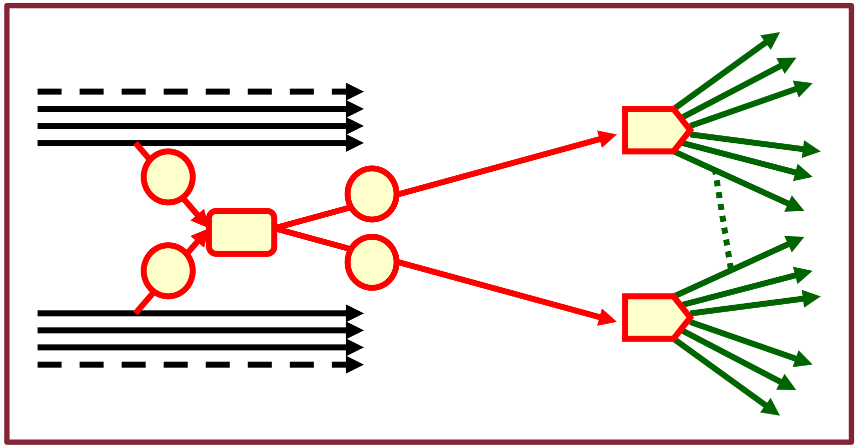
why sometimes e^+e^-, sometimes $\bar{p}p$, sometimes pp ?
and possibly $\mu^+\mu^-$? a mistake ? a random choice ?



$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$

Few general arguments : the REAL answer is in the complete set of lectures.

- ☺ a hadron is a bundle of many different partons (valence+sea quarks, sea anti-quarks, gluons);
- ☺ many initial states are simultaneously available in $pp/\bar{p}p$, i.e. hadron machines are much richer in physics;
- ☺ in $pp/\bar{p}p$, no need to scan in $\sqrt{s}$: at high Q^2, the pdf's provide a large range of $\sqrt{\hat{s}}$ simultaneously (see the J/ψ story);
- ☺ it is therefore possible to define a "differential luminosity" $d\mathcal{L}_i/d\sqrt{\hat{s}}$ for partons of type "i" (quarks, gluons) as a function of $\sqrt{\hat{s}}$ for the same $\sqrt{s}$;
- ☹ $d\mathcal{L}_i/d\sqrt{\hat{s}}$, integrated in small intervals of $\sqrt{\hat{s}}$, is small; it also decreases for $\sqrt{\hat{s}} \to \sqrt{s}$ (i.e. $x_1x_2 \to 1$), because of the pdf's;
- ☹ because of all that, the experiments and analysis are **much more difficult** in hadron machines.



$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: soft vs hard collisions

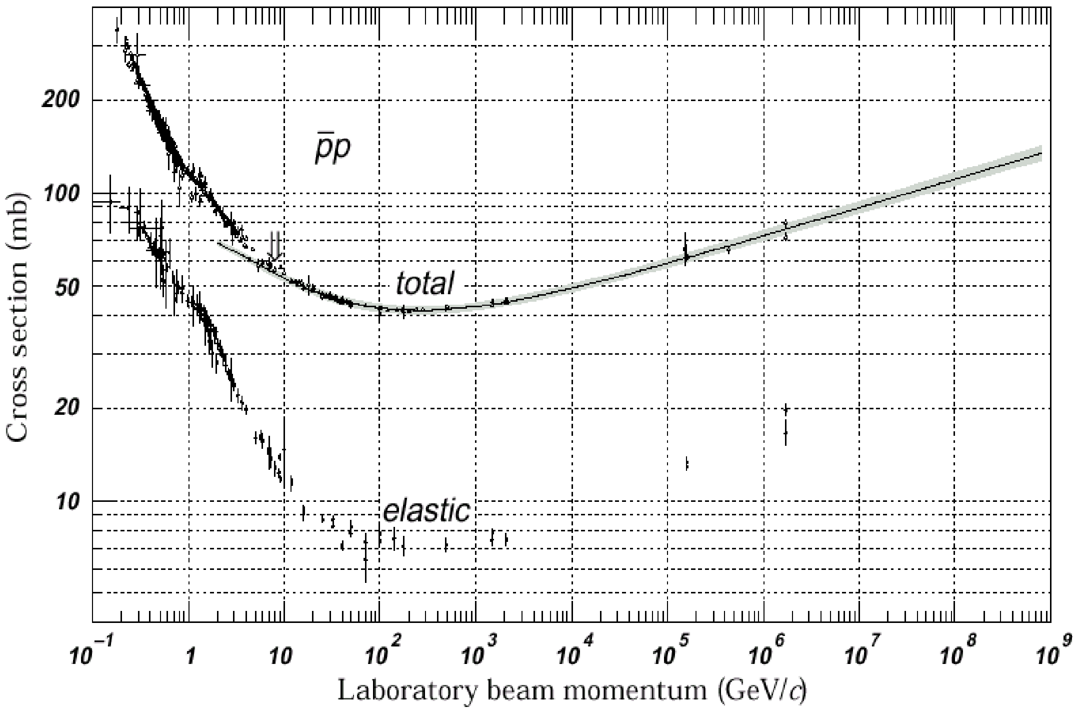
ex. : σ(LEP II, $e^+e^- \to$ hadr., $\sqrt{s}$ = 200 GeV) $\approx$ **100 pb**;

σ(LHC, pp $\to$ total, $\sqrt{s}$ = 14 TeV) $\approx$ **100 mb**;

σ(LHC, pp $\to$ jet X, E_T^{jet} > 250 GeV) $\approx$ **100 nb**.

[actual thresholds quite arbitrary, retain the order of magnitude]

~ 1 ÷ 10^9 ÷ 10^3 (!!!)

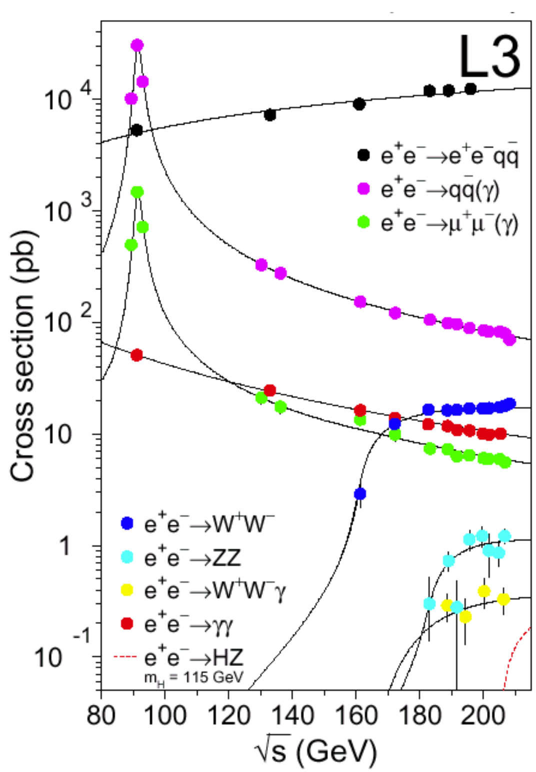


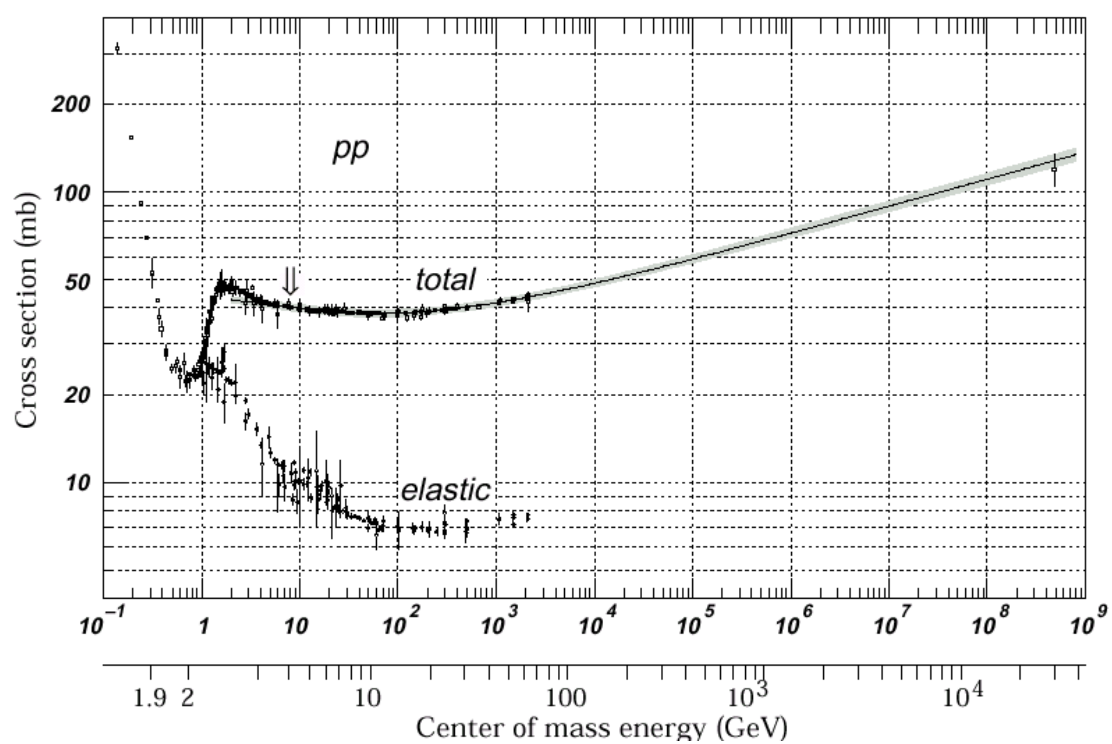
- nucleons, <u>when coherent</u>, are "one billion times" larger than electrons;
- however, when <u>individual partons</u> have to play, they are only "1,000 times" (the actual number depends on Q^2) larger;
- the factor 1,000 is due to the strength of the <u>coupling</u> ($\alpha_s \leftrightarrow \alpha_{em}$).

$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: small vs large σ

- in ee, "small" σ_{tot} (~pb, $\propto$ 1/s away from the Z pole), dominated by high-Q^2 processes mainly in the s-channel;
- therefore few events (rate ~1 Hz), all very interesting $\rightarrow$ **event trigger**;

- in pp/$\bar{p}$p, much higher σ_{tot} (~100 mb over many orders of magnitude), dominated by low-Q^2 processes (t-channel);
- therefore very high rate (~10^9 Hz), rare interesting events $\rightarrow$ **high-p_T triggers**.

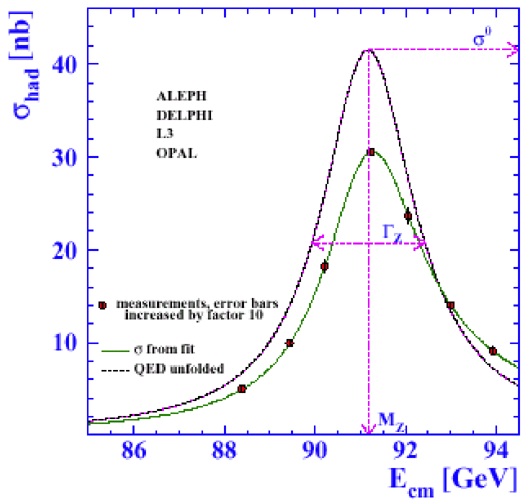




$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: data analysis

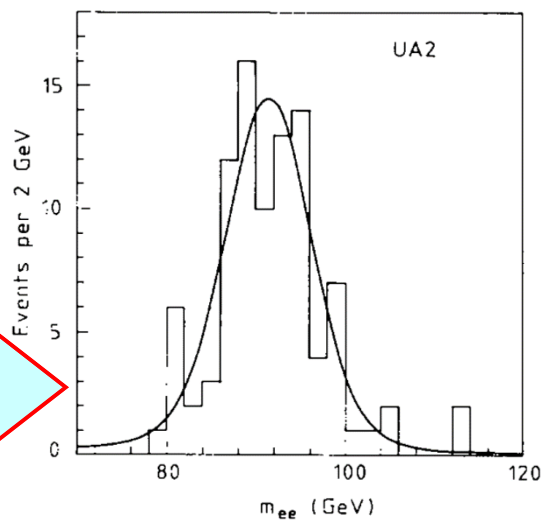
In detector and analysis many differences between e^+e^- and pp/p̄p:

- in ee "partonic" energy $\sqrt{s}$ is fixed by the machine, and known precisely;
- in pp/p̄p partonic energy $\sqrt{\hat{s}}$ changes for each event by a large factor;
- for a given $\sqrt{s}$, the average $\sqrt{\hat{s}}$ in a pp/p̄p collision is much lower;
- in ee, kinematical fits in 4D, constraints known to 10^{-5};
- in pp/p̄p, fits in 2D, (because of spectators), constraints to %;
- but $\sqrt{s}$ in ee machines is severely limited by brem.



LEP I, $e^+e^- \to Z$
m_Z from $\sqrt{s}$ (LEP)
width = Γ_Z = 2.5 GeV
$\Delta m \approx$ few MeV

UA2, $\bar{p}p \to Z$
m_Z from $m(e^+e^-)$
width = $\Gamma_Z \oplus \sigma_Z \approx$ 4.3 GeV,
i.e. Γ_Z and σ_Z comparable,
$\Delta m \approx$ few $\times$ 100 MeV



$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: a personal conclusion

In a given moment, with similar technology (and resources, *don't forget*) :

A pp/p̄p machine :

- needs a smaller ring (because of brem);
- more difficult to build (both the magnets and the detectors);
- (much) higher $\sqrt{s}$ and (fairly) higher $\sqrt{\hat{s}}$;
- analysis difficult, higher systematics;
- larger variety of both initial and final states (not only vacuum q.n.);

Therefore [*imho, but largely shared*]:

- (ee) and (pp/p̄p) are complementary, NOT competitive;
- (pp/p̄p) an exploratory machine, for first generation experiments;
- (ee) a "second generation" machine, for systematics and consolidation (and surprises in the precision meas);

This has been the CERN strategy in the last half a century :

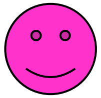
1. (pp/p̄p) (re-using an old machine);
2. civil engineering for a new ring (the long and expensive step);
3. (ee) in the new ring;
4. [back to step (1), restart the cycle].

It happens that, e.g., the value of $\sqrt{s}$ in step (3) is similar to $\hat{s}_{eff}$ in step (4/1) [e.g. both the Sp$\bar{p}$S and LEP had $W^\pm$ and Z as their main purpose.

The "luminosity frontier" (Babar, Daφne, …) is a different approach : a dedicated machine, especially optimized wrt intensity and systematics, for (a) very important (single) measurement(s).

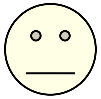
$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: matter vs antimatter

Last question : $pp \leftrightarrow \bar{p}p$?



- **pp** has major problems :
  - it needs two independent magnet rings;
  - at the same $\sqrt{s}$, the effective $\sqrt{\hat{s}}$ is smaller for $q\bar{q}$ channels (valence-sea instead of valence-valence);

- however, **$\bar{p}$p** has a larger problem:
  - antiprotons do NOT exist in nature (at least in our proximity);
  - therefore $\bar{p}$'s have to be "built", starting from pp collisions;
  - they are scarce, and have an incredible "price" (in the Sp$\bar{p}$S, one good $\bar{p}$ / 3×10^5 pp collisions);
  - they have to be cooled and stored (AA, stochastic cooling, van der Meer);
  - the resultant luminosity is small (in 1983, the golden year, $\mathcal{L}$(Sp$\bar{p}$S) < 10^{30} cm^{-2}s^{-1});



- Therefore, in spite of all the successes of the $\bar{p}$p machines, both at CERN and Fermilab, the quest for higher energies and (consequently) higher luminosities makes the pp option really superior for present and future colliders.
- The $\bar{p}$p option will probably be reserved for dedicated single-task machines at sub-TeV energy.

$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: e^+e^- linear or circular ?

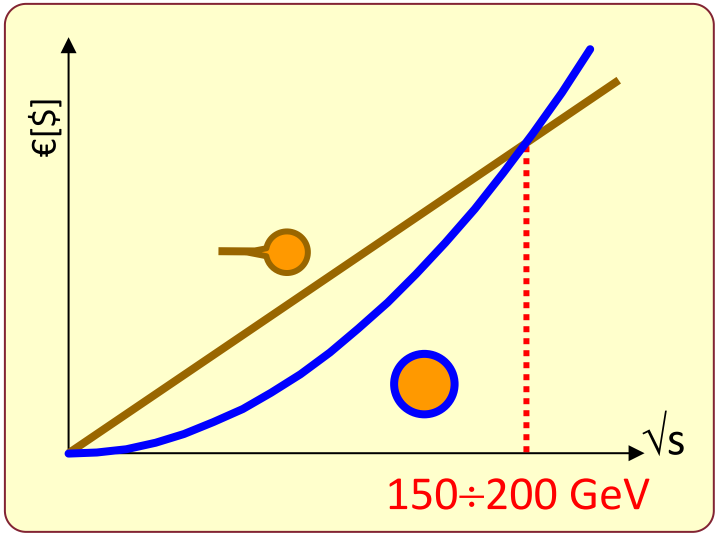
- Smart idea (SLAC '80s): build/use a powerful e^+e^- linear collider, add two arcs and produce the equivalent of a circular electron collider [*see § LEP*].
- In this way, essentially NO BREM (e^+/e^- only once in a curved path).

Pros/cons : *[thanks to Gary Feldman]*

- Circular colliders (like ADA, ADONE, SPEAR, LEP, …) :
  - cost $\propto$ radius,
  - energy to exploit $\propto$ *E^4 / R (brem)*,
  - $\$ = \alpha R + \beta E^4 / R$;
    $d\$ / dR = 0 \rightarrow \alpha = \beta E^4 / R^2 \rightarrow$
    $R_{best} = \sqrt{\beta/\alpha}\, E^2$; $\$_{min} = \sqrt{\alpha\beta}\, E^2$;
  - best choice: $R \propto E^2$; $\$ \propto E^2$.
- Linear colliders (SLC, next CERN ?) :
  - both machine and energy $\propto$ length;
  - $R \propto E$; $\$ \propto E$.

- Coefficients α,β depend on technology and market; at present the crossing is at $E_{beam} \approx 150\div200$ GeV;
- ~~possibly LEP is the highest energy e^+e^- circular collider ever built~~ [*never say never …* read the CERN strategy plan];

- $p, \bar{p}, \mu^\pm$, etc., are different (see § LHC).



References

1. e.g. [BJ, 14];
2. for the results, see next 3 chapters;
3. accelerator physics : [BJ, 2], [Povh, appendix];
4. better accelerator physics : Ed. Wilson, An introduction to particle accelerators.

God the Geometer, Frontispiece of Bible Moralisee Codex Vindobonensis 2554 (French, ca. 1250) [Österreichische Nationalbibliothek]

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End of chapter 8

Particle Physics - Chapter 9
The $Sp\bar{p}S$ – $W^{\pm}$ and Z discovery

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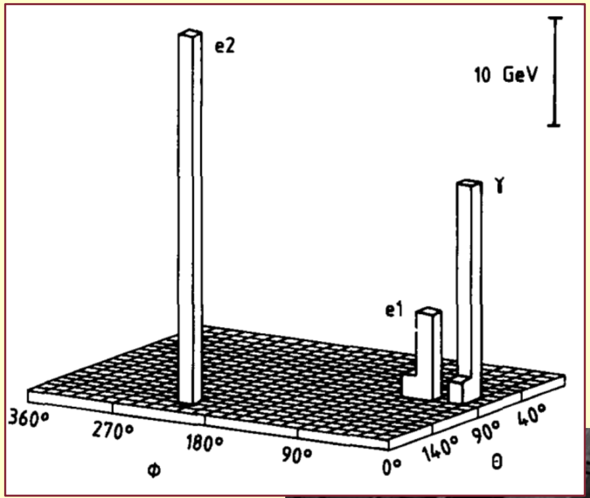
AA 18-19

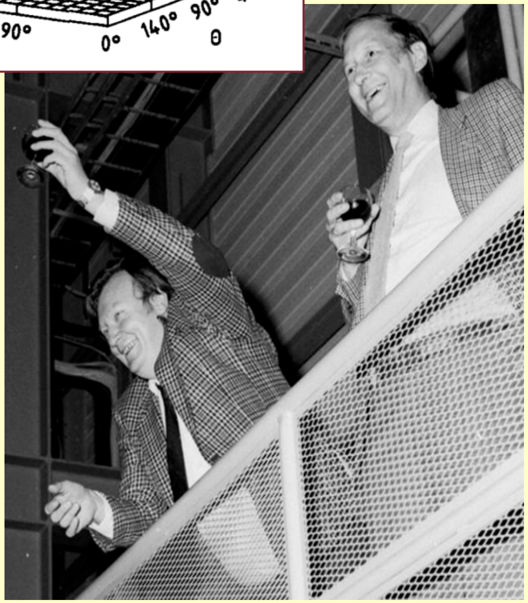
last mod.
18-Apr-19

9 – The Sp$\bar{p}$S – $W^\pm$ and Z discovery

1. [$\bar{p}$p collisions]
2. [The Sp$\bar{p}$S parameters]
3. [Detectors]
4. [Events]
5. [Hadronic interactions]
6. [The Drell-Yan process]
7. [$W^\pm$ discovery]
8. [Z discovery]
9. [$W^\pm$/Z properties]$^{(*)}$

$^{(*)}$ some of the properties of $W^\pm$ and Z are best studied in e^+e^- interactions [typical examples : Γ's and BR's] : their discussion is postponed to § LEP.





$\bar{p}p$ collisions: history

- The antiprotons ($\bar{p}$) are the antiparticles of the protons (p).
- Therefore **$\bar{p}p$** and **e^+e^-** colliders have similarities (e.g. one mag. channel with head-on collisions).
- … with the bonus of the lack of brem for $\bar{p}p$: in the same SPS tunnel, p/$\bar{p}$ were accelerated up to 273/315/450 GeV, while $e^\pm$ up to few GeV only.
- … and the disadvantage of compositeness → in high Q^2 collisions, partons$_{1,2}$ have a momentum $(x_{1,2}\sqrt{s}/2)$ and the energy of the parton collision is $\sqrt{\hat{s}} = \sqrt{s x_1 x_2}$.
- In addition $\bar{p}$'s are very scarce in our world (also e^+ are, but they are easy to produce and cheap).
- The real problem is the $\bar{p}$ "fabrication", accumulation and cooling, which has to happen before the acceleration process.
- It requires lot of clever ideas, both from Physics, Electronics, Engineering.

Once upon a time in 1976 …

Producing Massive Neutral Intermediate Vector Bosons with Existing Accelerators(*)

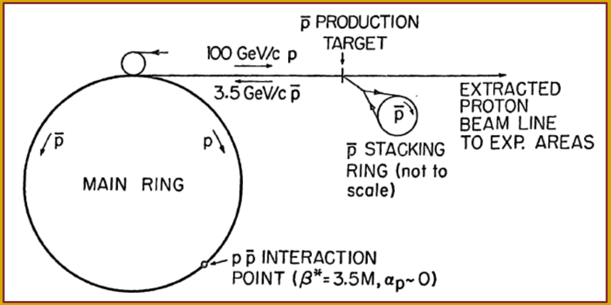
C. Rubbia and P. McIntyre
Department of Physics
Harvard University
Cambridge, Massachusetts 02138

and

D. Cline
Department of Physics
University of Wisconsin
Madison, Wisconsin 53706

March 1976

C. Rubbia, P. McIntyre and D. Cline, Proc. Int. Neutrino Conf., Aachen, 1976 (eds. H. Faissner, H. Reithler and P. Zerwas) (Vieweg, Braunschweig, 1977), p. 683.

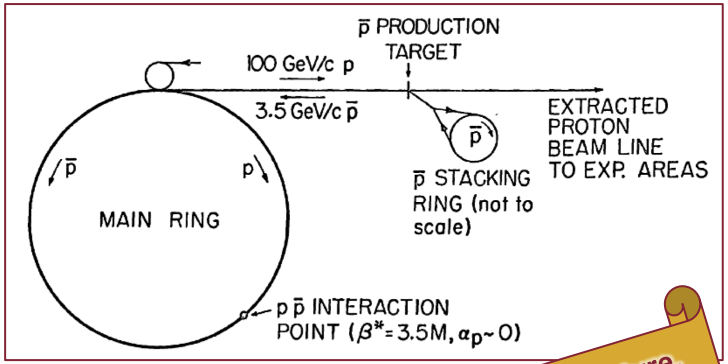


$\bar{p}p$ collisions: sequence

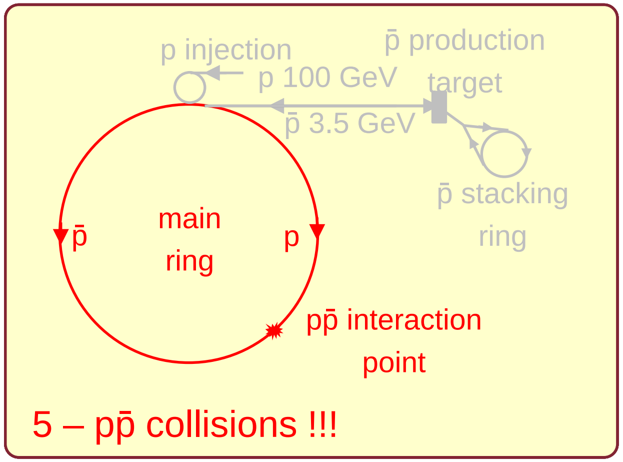
A little animation may help :

1. <u>Protons</u> are accelerated to an intermediate suitable energy [*the proposal says E_p = 100 GeV from Fermilab main ring, but it is NOT critical – at CERN E_p = 26 GeV from PS*] .
2. Then the p are extracted and sent onto a target, to produce high intensity <u>collisions</u>.
3. The <u>resultant $\bar{p}$</u> (very rare) are collected and cooled ("stacked") in a lower energy ring [*at CERN $E_{\bar{p}}$ = 3.5 GeV – can't store $\bar{p}$'s at rest, despite Dan Brown stories*$^{(*)}$].
4. After hours (days), when <u>enough $\bar{p}$ are available</u>, they are re-extracted and injected in the main ring, together with protons.
5. Both <u>$\bar{p}$ and p are accelerated</u> to the max energy, and then let collide.

Although every step requires ingenuity, step (3) and (4) are the real nightmares; have a closer look.



Rubbia, McIntyre, Cline op.cit.

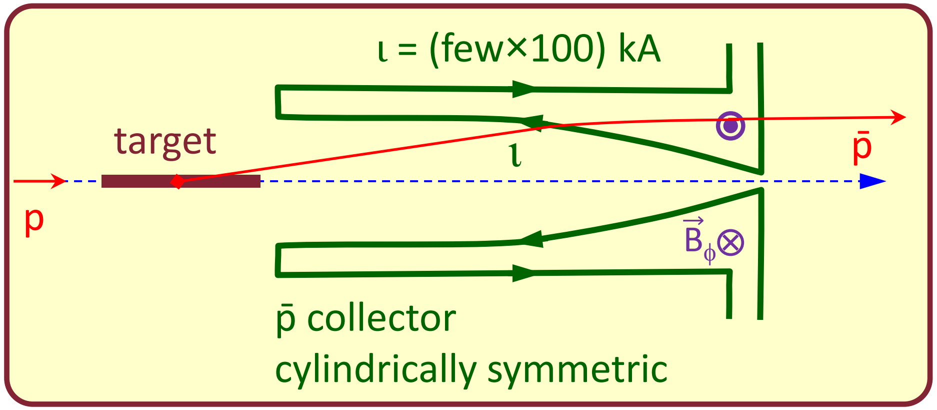


(*) *Penning traps work for few (< 10) particles.*

p̄p collisions: the making of p̄

Rubbia et al. invented an innovative scheme for p̄p collisions[(*)].

- Carlo initially offered it to Fermilab, then he built it at CERN in 1978-81, later somebody else implemented it at Fermilab [*another turning point in particle physics, people thinks that Americans are more fast and flexible*].
- The key structures were the p̄ collectors, which were a new design of the Van der Meer horn (see figs) …
- … and the AA (= Antiproton Accumulator), the ring where the p̄ were collected, cooled, accumulated and stored for up to few days (next page).



(*) *imho* the creation of the p̄p machine (and not the relatively easy W and Z discovery) was the real success of the CERN p̄p Collider.

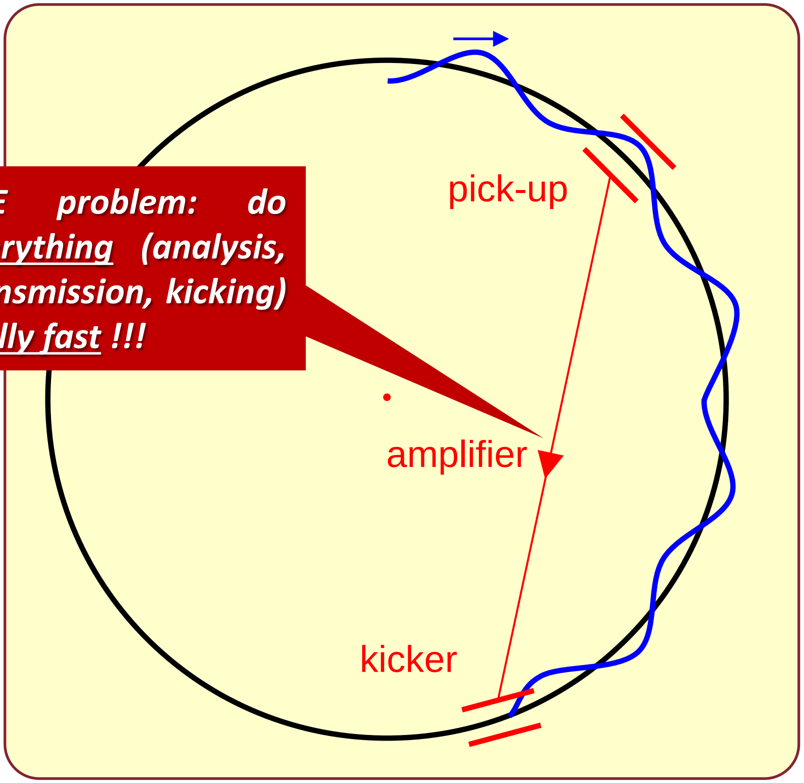
look the ν horn in §ν (same author) and comment on the difference.

p̄p collisions: pickup+kicker

The main problem : the "cooling" of p̄ :

- [*why "cooling" ? in classical physics, the* *<u>temperature</u>* *of a gas is related to its motion in the CM frame : higher temperature means higher* (<v^2> - <v>^{2}) *velocity; so "gas cooling" means reducing the* ***<u>relative</u>*** *velocity of particles;*]
- analyze a single particle (——) circulating in a ring;
- it oscillates with "<u>betatron oscillations</u>" around the ideal particle (——);
- a "**<u>pick-up</u>**" electrode detects its position respect to the nominal orbit;
- this value, appropriately amplified, is transmitted to a "**<u>kicker</u>**", displaced by (n/2 + ¼) wavelengths;
- **<u>the kicker corrects the orbit</u>**;
- notice that the **<u>space displacement</u>** produces an **<u>angle correction</u>**;
- in reality, the pick-up and kicker are traversed by a large and incoherent number of particles at the same time;
- but if their average displacement is NOT zero, they get a correction and become closer to the ideal orbit.

THE problem: do <u>everything</u> (analysis, transmission, kicking) <u>really fast</u> !!!

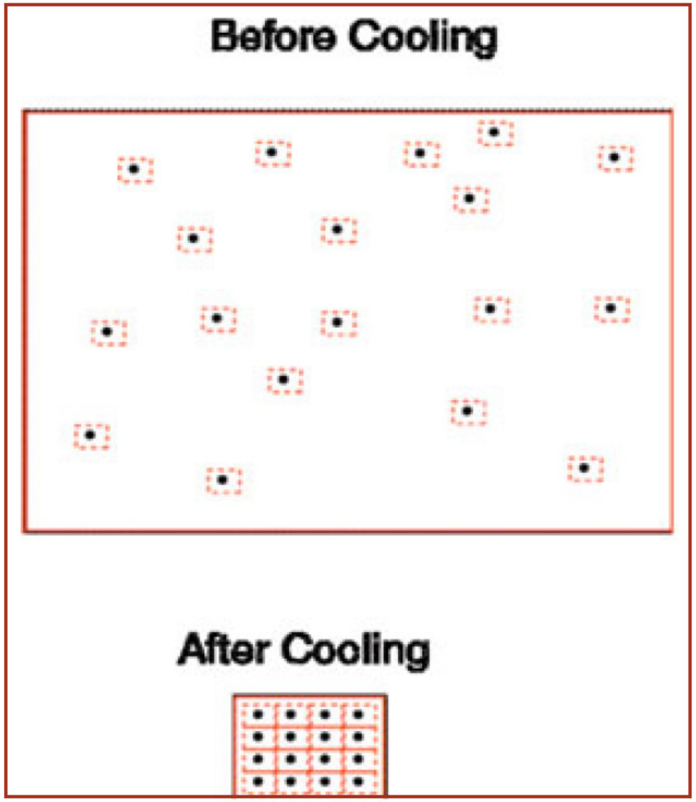


p̄p collisions: stochastic cooling

- Wikipedia : "***Liouville's theorem***, *[…] after the French mathematician Joseph Liouville, is a key theorem in classical statistical and Hamiltonian mechanics. It asserts that the phase-space distribution function is constant along the trajectories of the system.*"
- A principle well known to experts of beam optics : e.g. a quadrupole, or the principle of strong focusing.
- The cooling of p̄ in a reduced phase space region conflicts with the theorem : e.g. a squeeze in transverse momentum must result in an increase in space dimensions.
- **Stochastic cooling** : [S. van der Meer, Nobel Lecture] "*Fortunately, there is a trick - and it consists of using the fact that particles are points in phase space with empty space in between. We may push each particle towards the center of the distribution, squeezing the empty space outwards. The small-scale density is strictly conserved, but in a macroscopic sense the particle density increases. This process is called cooling because it reduces the movements of the particles with respect to each other.*"

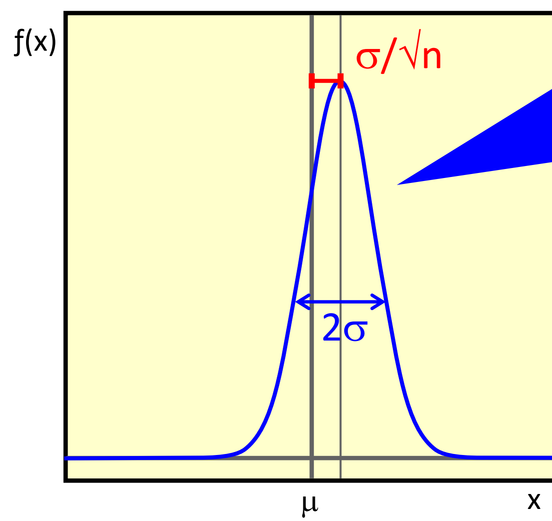
p̄p collisions: (how to avoid) Liouville theorem

Stochastic cooling



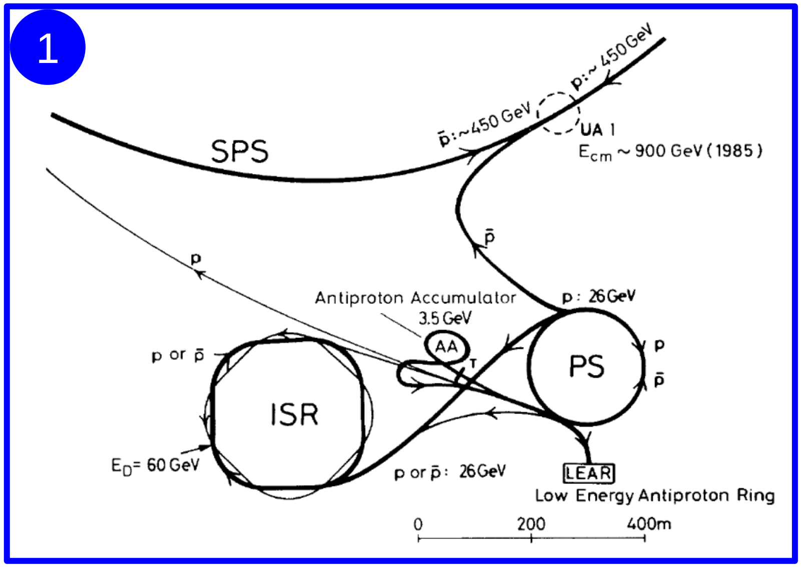
- *My understanding : cannot modify individual particle trajectories, but act on packets of n particles, small enough that their means be sensibly different from the ideal orbit ($1/\sqrt{n}$ not negligible).*
- it requires to divide the p̄'s in small packets, act on each packet, and then reassemble the beam.
- *A completely different type of cooling exists, **electron cooling**, invented by G.I. Budker. It is used in other accelerators.*

A cartoon by Carlo, to explain the previous sentence of van der Meer and the solution of the "Liouville problem".



"if a population of n elements is distributed according to a gaussian with average μ and rms σ, its mean is a random variable with average μ and rms = $\sigma/\sqrt{n}$."

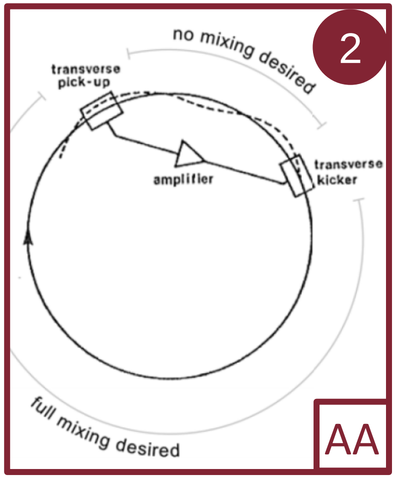
p̄p collisions: the AA

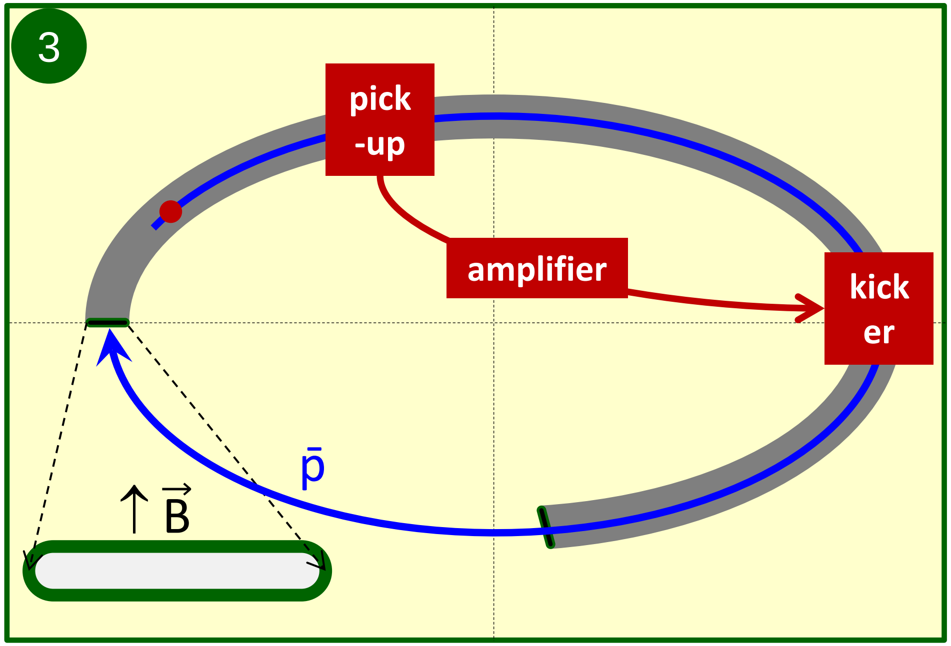


1. A view of the CERN pp̄ complex in the '80s.

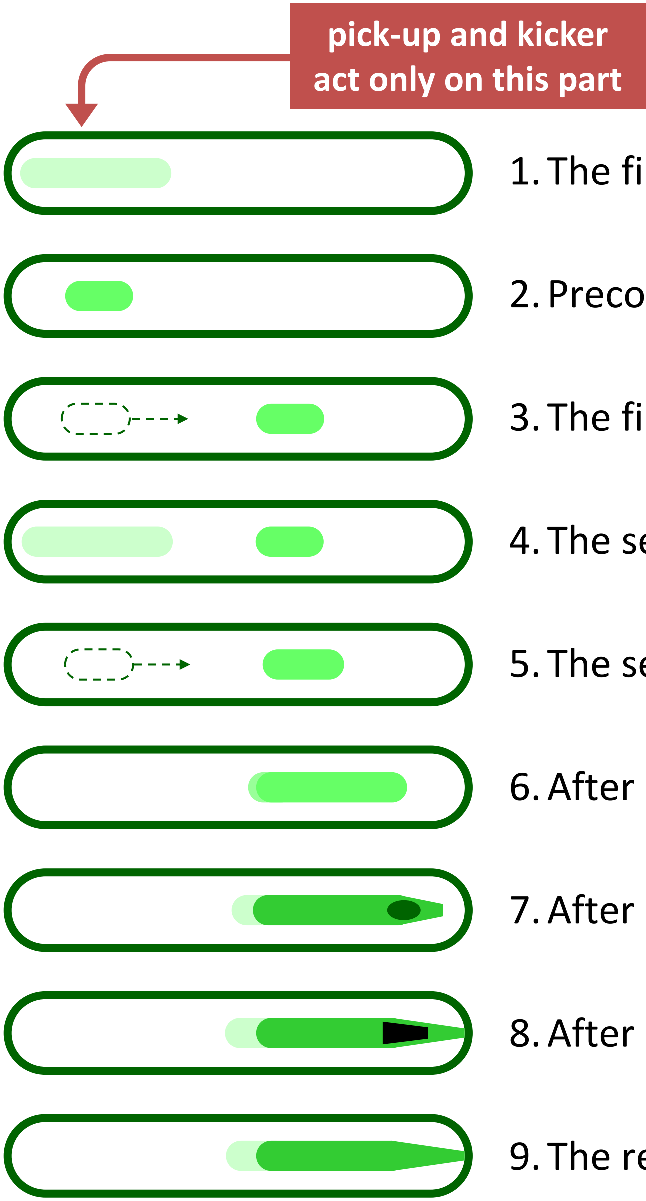
2. The AA and the its functioning principle.

3. A scheme of the AA operations.





p̄p collisions: the AA at work



Time evolution of the p̄ in the AA:

1. The first pulse of 7×10^6 p̄ has been injected into the vacuum chamber.
2. Precooling has reduced the momentum spread.
3. The first pulse has been moved to the stack-tail region.
4. The second pulse is injected, 2.4 s later.
5. The second pulse, after having been precooled, is also stacked.
6. After 15 pulses, the stack contains 10^8 p̄.
7. After 1 h, a dense core has formed inside the stack.
8. After 24 h, the core contains enough p̄ for transfer to the SPS.
9. The remaining p̄ are used to begin the next day of accumulation.

$\bar{p}p$ collisions: summary

- In hadronic interactions, partonic collisions at high Q^2 are more interesting than coherent hadron scattering at low Q^2.
- Why in some cases pp are preferred, and in other $\bar{p}p$? [*see the score card*]
- Pros and cons are balanced : the winner depends on many considerations (money, availability of the facilities …)
- However, the physics trend is clear :
  - pp machines are more expensive …
  - but pointlike cross sections decrease like 1/s; therefore as $\sqrt{s}$ increases, the luminosity is the essential requisite;
  - the level of sea quarks increases with Q^2, even at high x, therefore the argument of "valence @ high-x" loses strength;
  - probably the Sp$\bar{p}$S and the Tevatron will be the highest energy $\bar{p}p$ colliders.

Antiproton-proton $\bar{p}p$:

- [lot of $\bar{q}$ at high x → initial state with the vacuum quantum number →], more Z, $W^\pm$, Higgs with same $\sqrt{s}$ and luminosity;
- cheaper machine [only one magnetic ring];
- but lower reliability [a fault in AA, e.g. due to a storm, could block the Sp$\bar{p}$S for one week, due to the loss of $\bar{p}$]

Proton-proton pp :

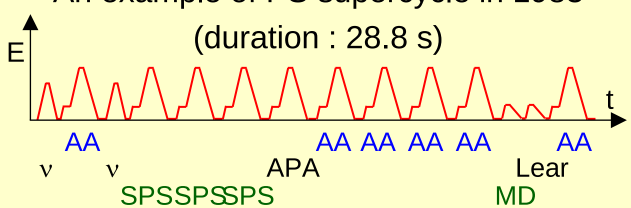
- no auxiliary machines (AA, horns, …) [no antimatter];
- higher reliability [no antimatter];
- **much higher luminosity** (~ 10^6) [no antimatter].

Sp$\bar{p}$S parameters

1983 was the "golden year" of Sp$\bar{p}$S : performances still improving, $W^\pm$ and Z discovery. Notice :

- The rate of $\bar{p}$ production : a rate ~10^6 paid to convert matter into antimatter.
- The energy for $\bar{p}$ collection (3.5 GeV) was chosen because it is optimal for production σ and acceptance.
- The cross-section of the design, from an old experiment $\sigma(p\ {}_{74}W \to \bar{p}X)$, was higher. The project had margins to (barely) survive.
- The Sp$\bar{p}$S performances were considered great, but LHC is × **10^5** in luminosity and × **20** in energy (**30** years later).

An example of PS supercycle in 1983 (duration : 28.8 s)



The Sp$\bar{p}$S in 1983		
$p\ {}_{74}W \to \bar{p}\ X$	$\lvert\vec{p}\rvert$ = 26 GeV	10^{13} / 2.4 s
$\bar{p}$	$\lvert\vec{p}\rvert$ = 3.5 GeV	$1/(10^6\ p)$ $\to$ few × 10^9/h
$\bar{p}\ p$	$\sqrt{s}$ = 546 GeV (*)	$\mathcal{L} = 1.6 \times 10^{29}$ cm^{-2}s^{-1}
$\int \mathcal{L} dt$	153 nb^{-1}	
$N_{events}(\bar{p}p)$	8 × 10^9	
$W^\pm \to e^\pm \nu$	**90**	
$W^\pm \to \mu^\pm \nu$ (UA1 only)	**14**	
$Z \to e^+e^-$	**12**	
$Z \to \mu^+\mu^-$ (UA1 only)	**4**	

Don't confuse "${}_{74}W$" (tungsten,"wolfram") with "$W^\pm$", the IVB.

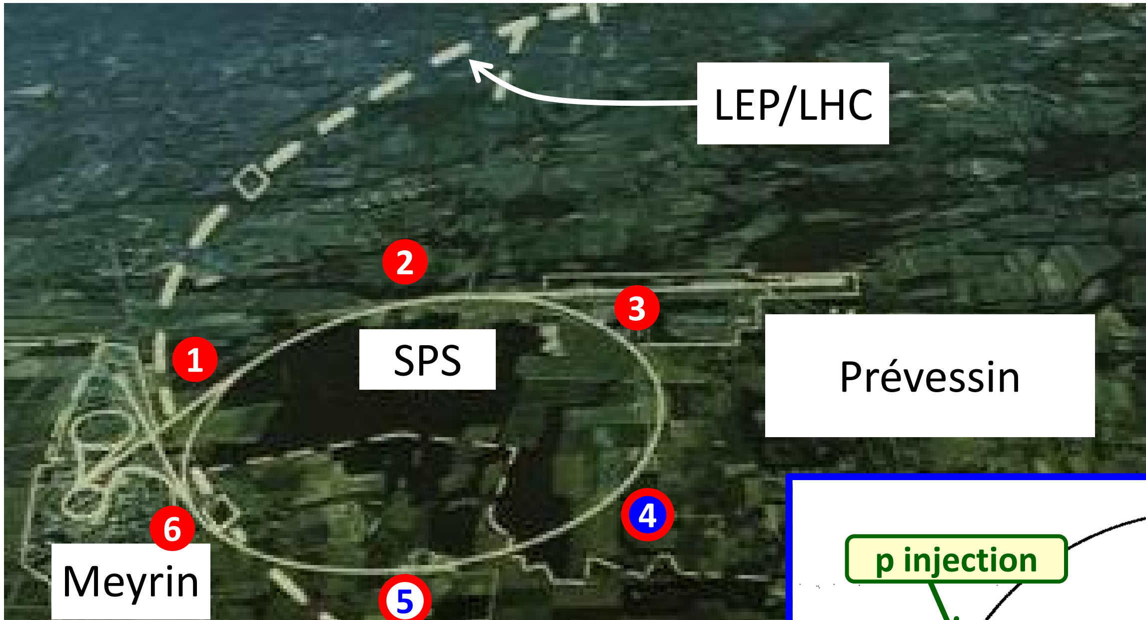
[*sorry, not my fault, only 26 letters available*]

(*) $\sqrt{s}$ = 630 GeV in ≥ 1984.

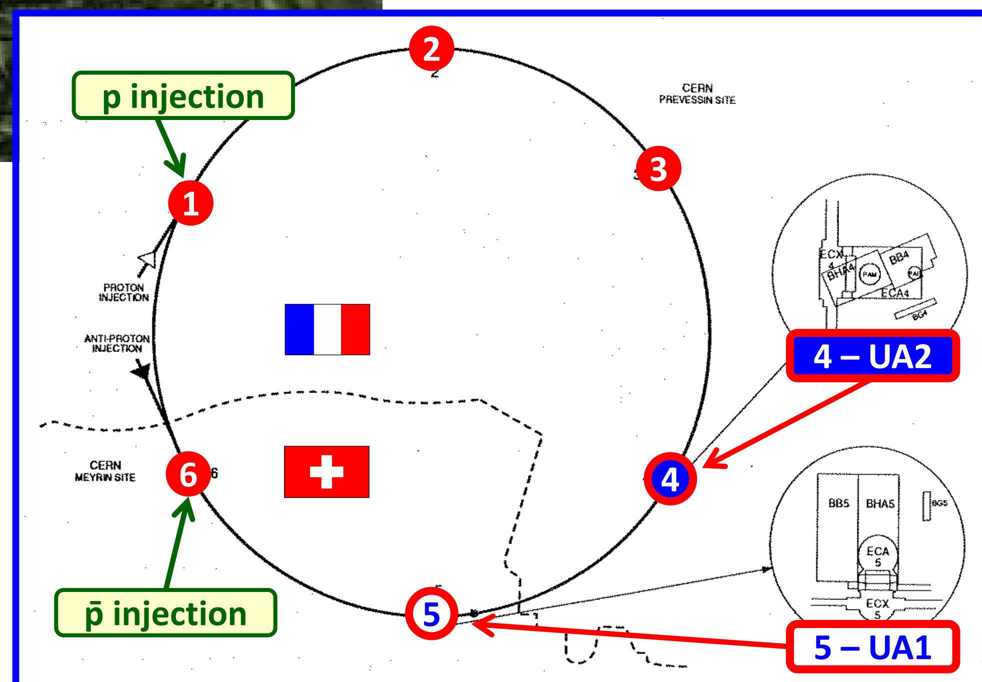
Sp$\bar{p}$S parameters: $\mathscr{L}_{int}$ / year

Year	1982	1983	1984	1985	1986	1987	1988	1989	1990
Beam energy (GeV)	273	273	315	315		315	315	315	315
β_h* (m)	1.5	1.3	1	1		1	1	1	0.6
β_v* (m)	0.75	0.65	0.5	0.5		0.5	0.5	0.5	0.15
# bunches	3+3	3+3	3+3	3+3		3+3 (6+6)	6+6	6+6	6+6
p/bunch (10^{10})	9.5	14	16	16			12	12	12
$\bar{p}$/bunch (10^{10})	1.2	1.5	2	2			4	6	7
< $\mathscr{L}_{initial}$ > (10^{30} cm^{-2}s^{-1})	0.05	0.17	0.36	0.39		0.35	1.3	1.8	3.1
< $\mathscr{L}_{int}$/coast > (nb^{-1})	0.5	2.1	5.3	8.2		2.8	31.5	40	70
# coasts/year	56	72	77	80	0	33	107	119	104
< T_{coast} > (h)	13	12	15	17			11	12	10
$\mathscr{L}_{int}$ /year (nb^{-1})	**28**	**153**	**395**	**655**	**0**	**94**	**3608**	**4759**	**7241**

The detectors



UA1 and UA2 are placed at 60° wrt each other, in the region far from the injection from PS.



The detectors: hermeticity

- Modern Collider detectors cover a solid angle as close as possible to 4π;
- there are two reasons for that :
  - detect all the particles of the final state (e.g. to reconstruct a rare multibody state with high efficiency);
  - "detect" the invisible particles (e.g. ν's), which escape without interacting with the apparatus ("hermeticity", as Carlo used to call it);
- there is a fundamental difference between e^+e^- and pp ($\bar{p}$p) :
  - in hadronic colliders (NOT in e^+e^-), most of $\sqrt{s}$ (= $1-\sqrt{x_1x_2}$) is lost in spectator fragments, which escape in the beam chamber without being detected;
  - the "visible energy" is a (small and variable) fraction of $\sqrt{s}$;
- therefore, in pp and $\bar{p}$p, the constraint of 4-mom conservation is not applicable in 4D;
- instead, a 2D constraint in the transverse plane is used;
- in the analysis, use the "missing transverse energy" $\cancel{E}_T$ (assume $\cancel{E}_T=|\vec{p}_T^{\,\nu}|$). [*"missing transverse momentum" looks more correct*].

Rules for trigger and analysis:

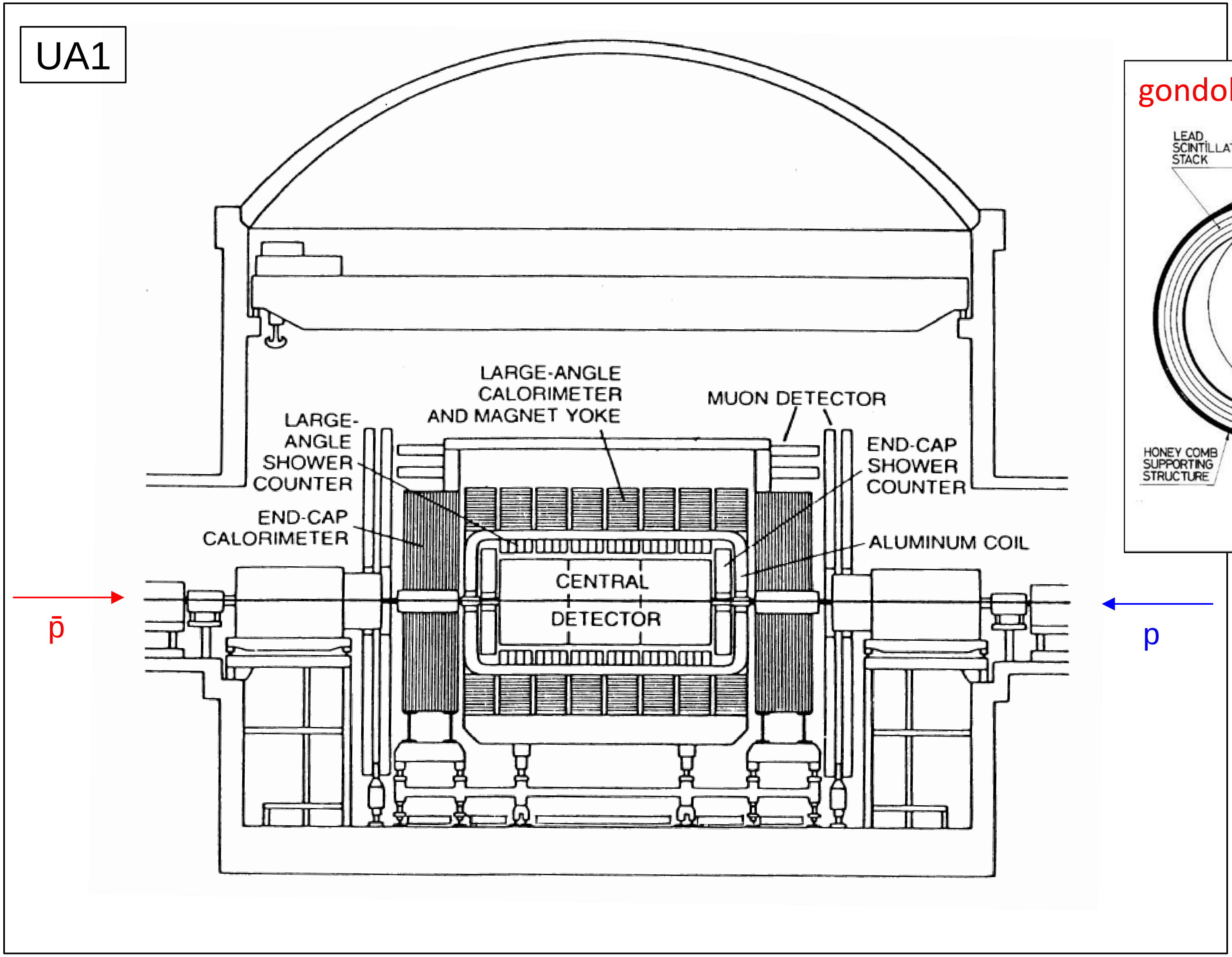
e^+e^- : "4D";

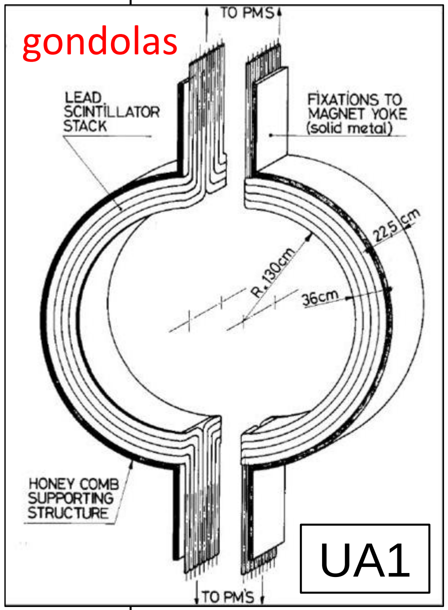
pp($\bar{p}$p) : "2D" :

ν's $\rightarrow E_T$

spectators $\rightarrow E_\ell$

The detectors: UA1 layout



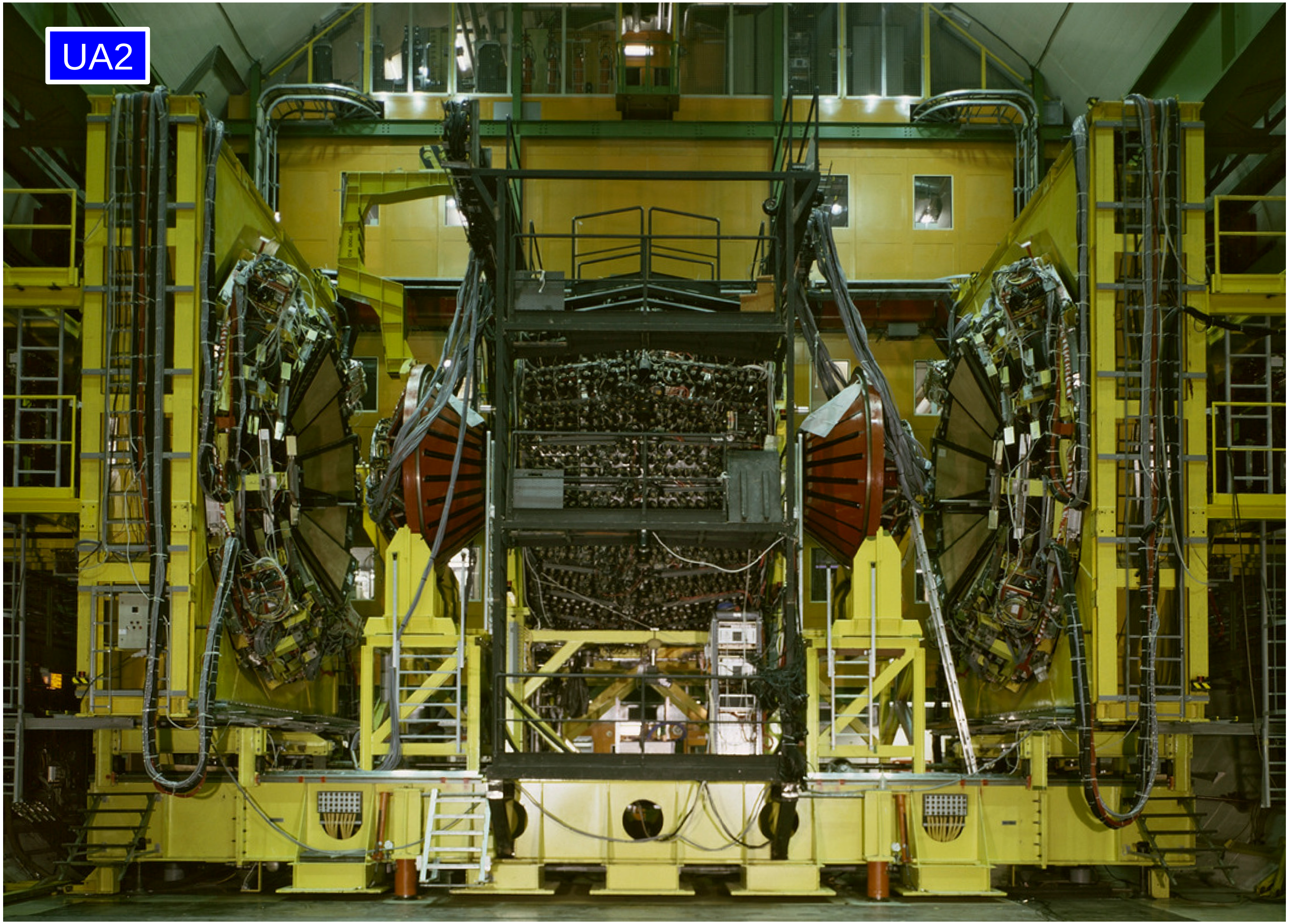


The detectors: UA1 parameters

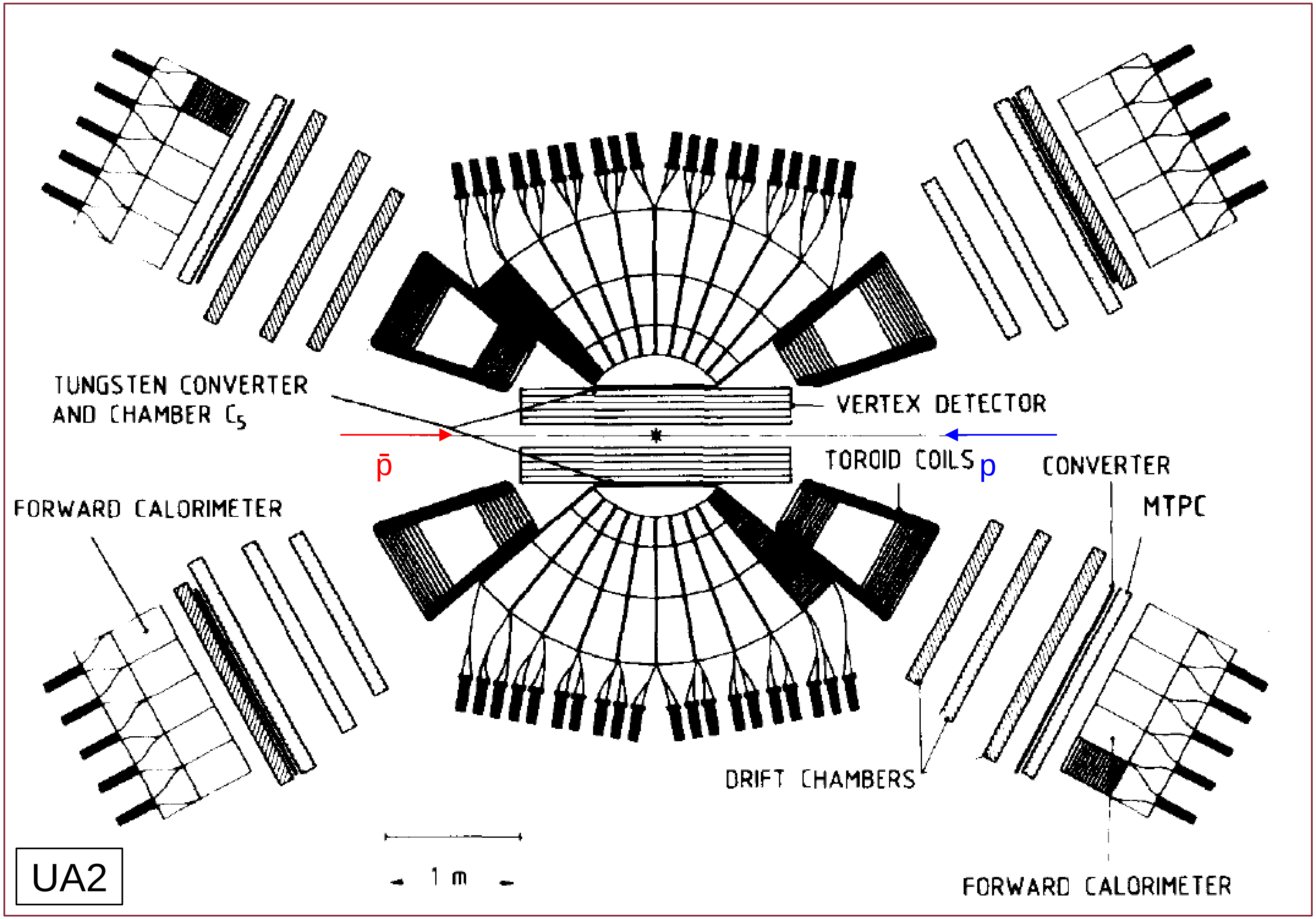
Central drift chamber	Gas	Field	v_{drift}	$\alpha_{Lorentz}$	$N_{sense\ wires}$
	Ar-ethane 40-60	1.5 kV/cm	53 μm/ns	23° @ 0.7 T	6110

UA1	Zenith θ	type	Name	e.m. rad-length	had. abs-length	Cell $\Delta\theta\times\Delta\phi$	σ_E/E
Central calorimeter	25°–155°	e.m.	gondolas	26.6/sinθ	1.1/sinθ	5°×180°	$0.15/\sqrt{E(\text{GeV})}$
		had.	C's	–	5.0/sinθ	15°×18°	$0.80/\sqrt{E(\text{GeV})}$
Endcap calorimeter	5°–25° 155°–175°	e.m.	bouchons	27/cosθ	1.1/cosθ	20°×11°	$0.12/\sqrt{E(\text{GeV})}$
		had.	I's	–	7.1/cosθ	5°×10°	$0.80/\sqrt{E(\text{GeV})}$

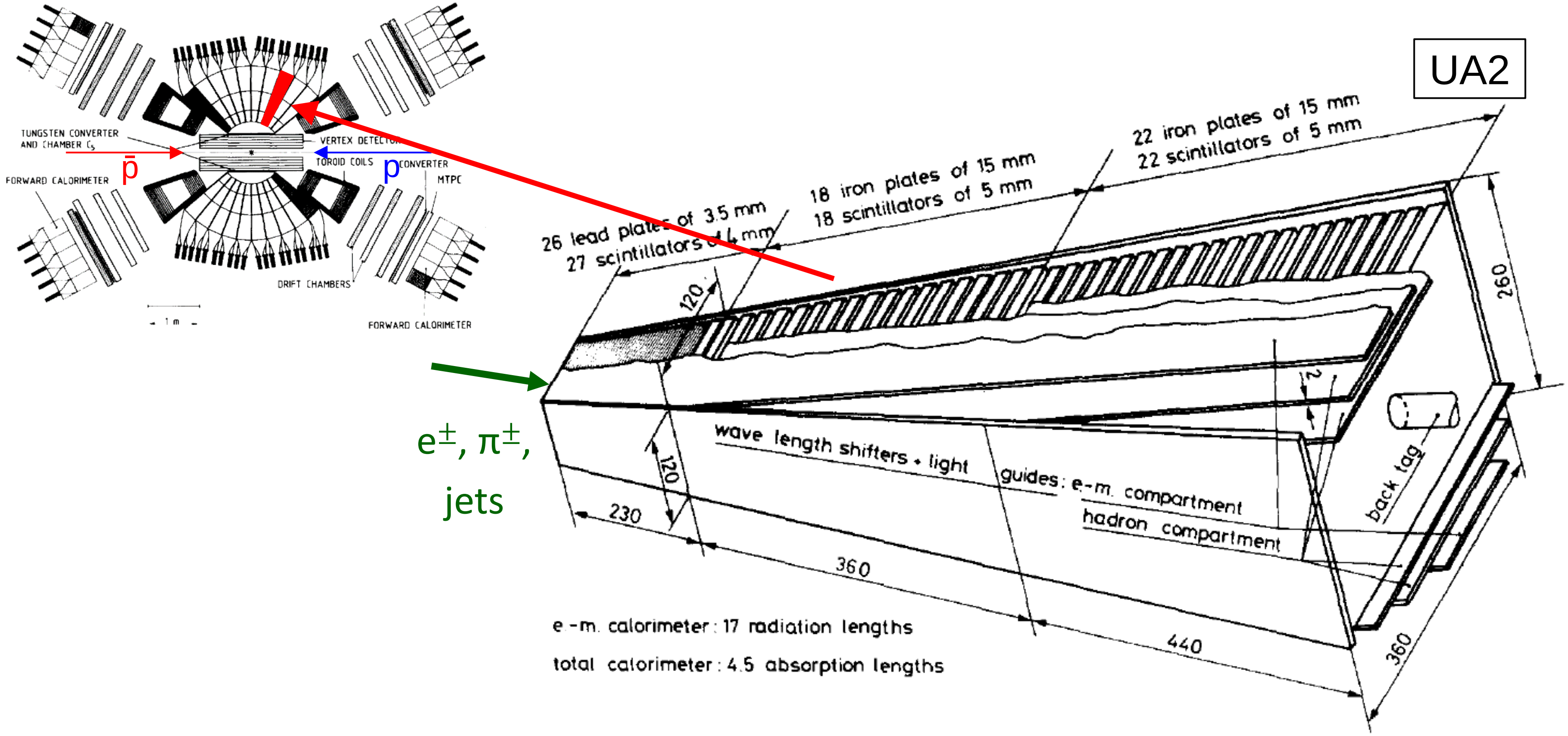
The detectors: UA2



The detectors: UA2 scheme



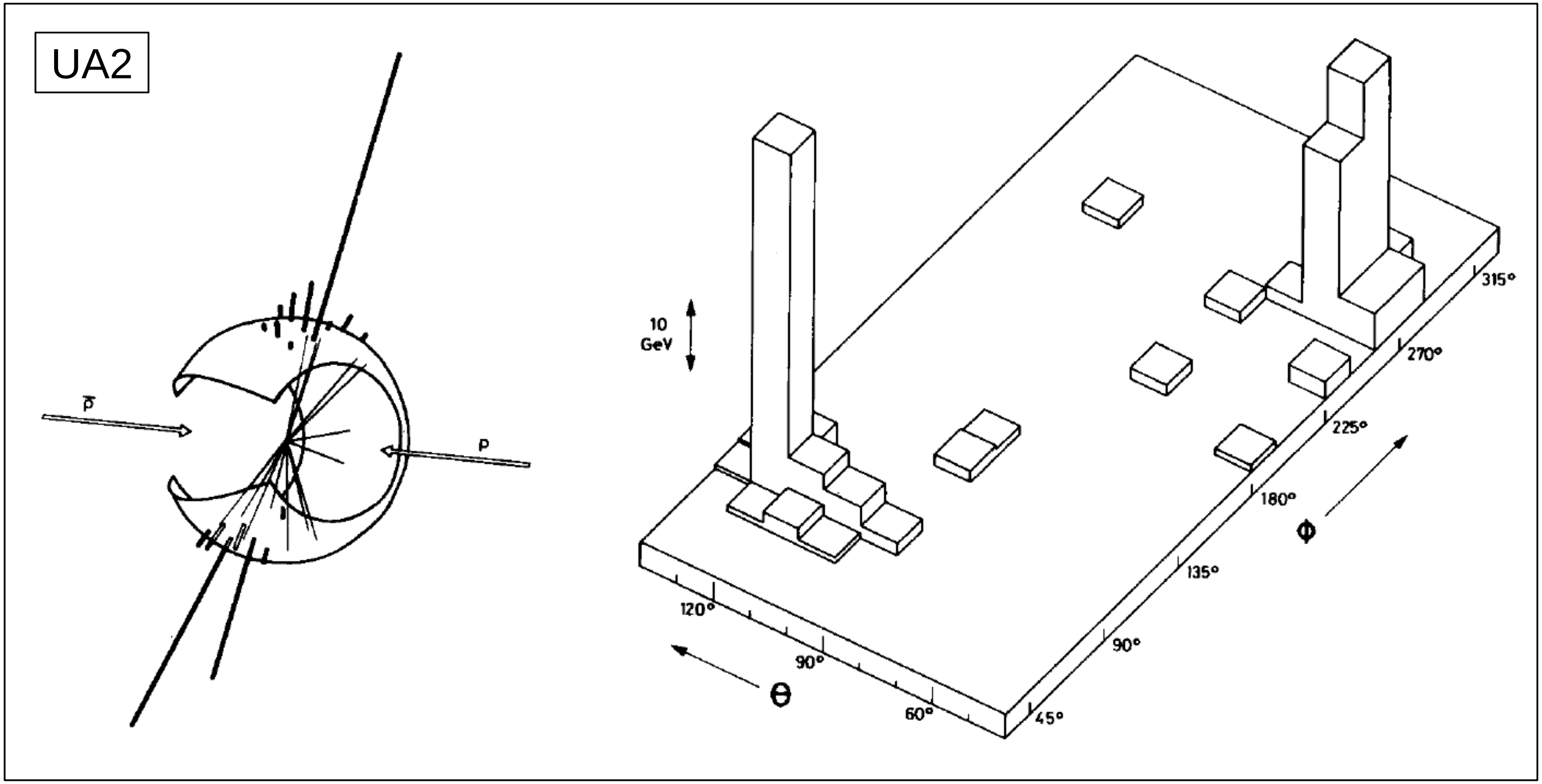
The detectors: UA2 calos



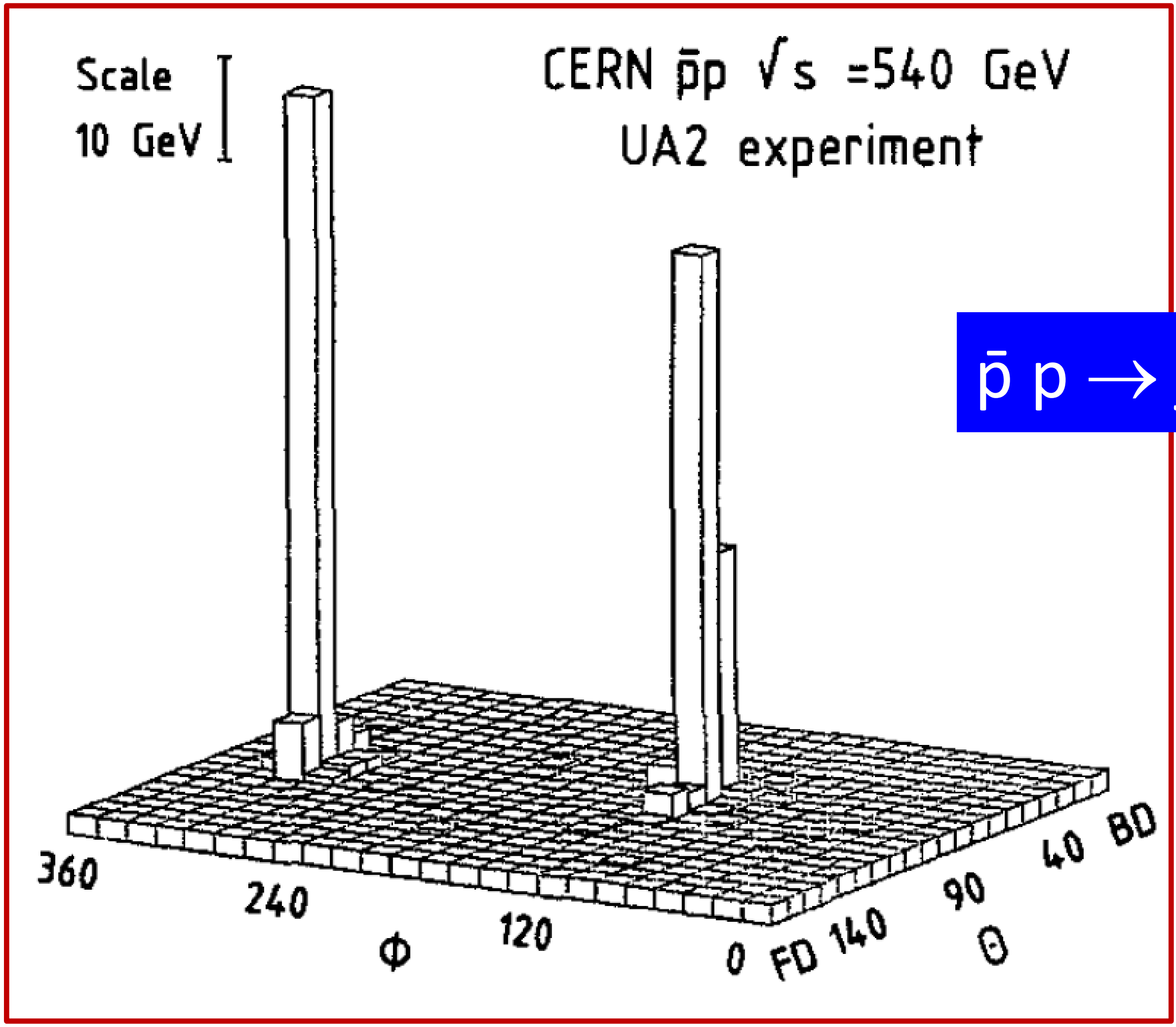
UA2 Central calorimeter	zenith θ	type	e.m. rad-length	had. abs-length	Cell Δθ×Δϕ	σ_E/E
	40°−140°	e.m.	17	~0.5	10°×15°	$0.14/\sqrt{E(\mathrm{GeV})}$
		had	–	2+2		32% – 11%

The events: jets discovery

Hadronic jets discovery :
UA2 - Paris conference, 1982



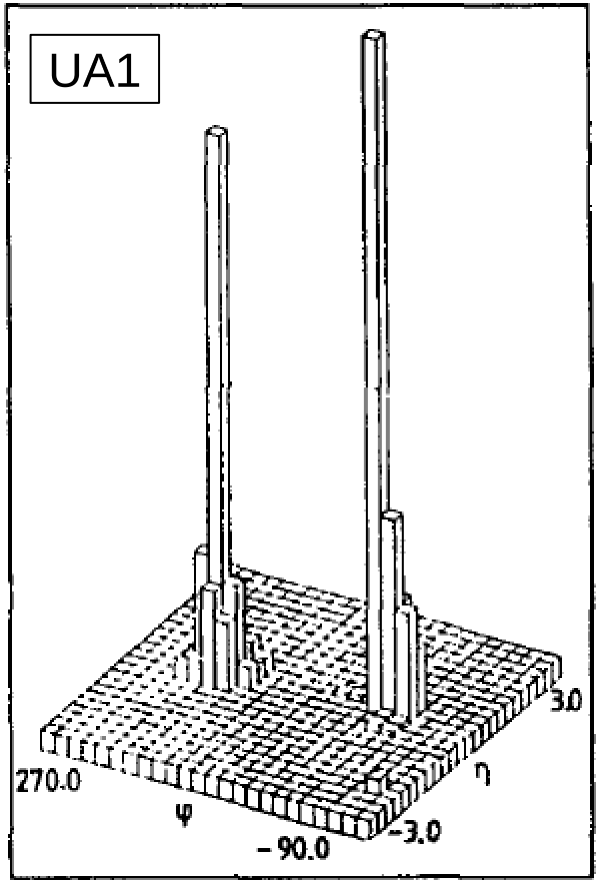
The events: UA2 jets

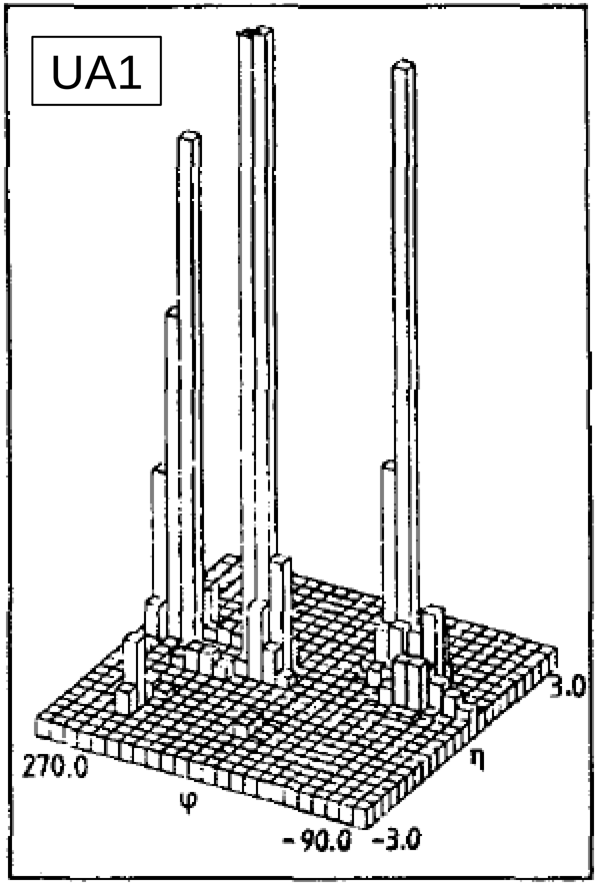


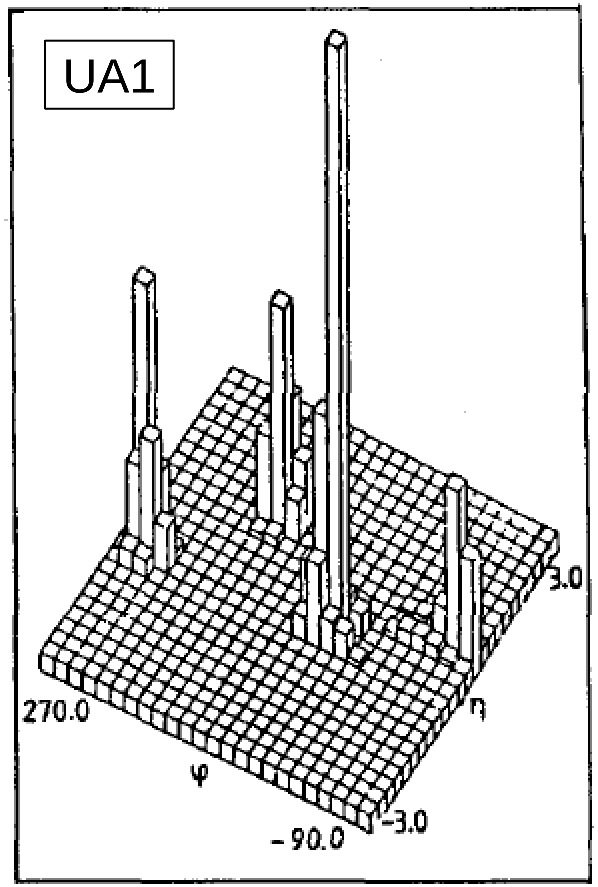
$\bar{p}\,p \to$ jet jet

The events: UA1 jets

p̄p → 2,3,4 jets

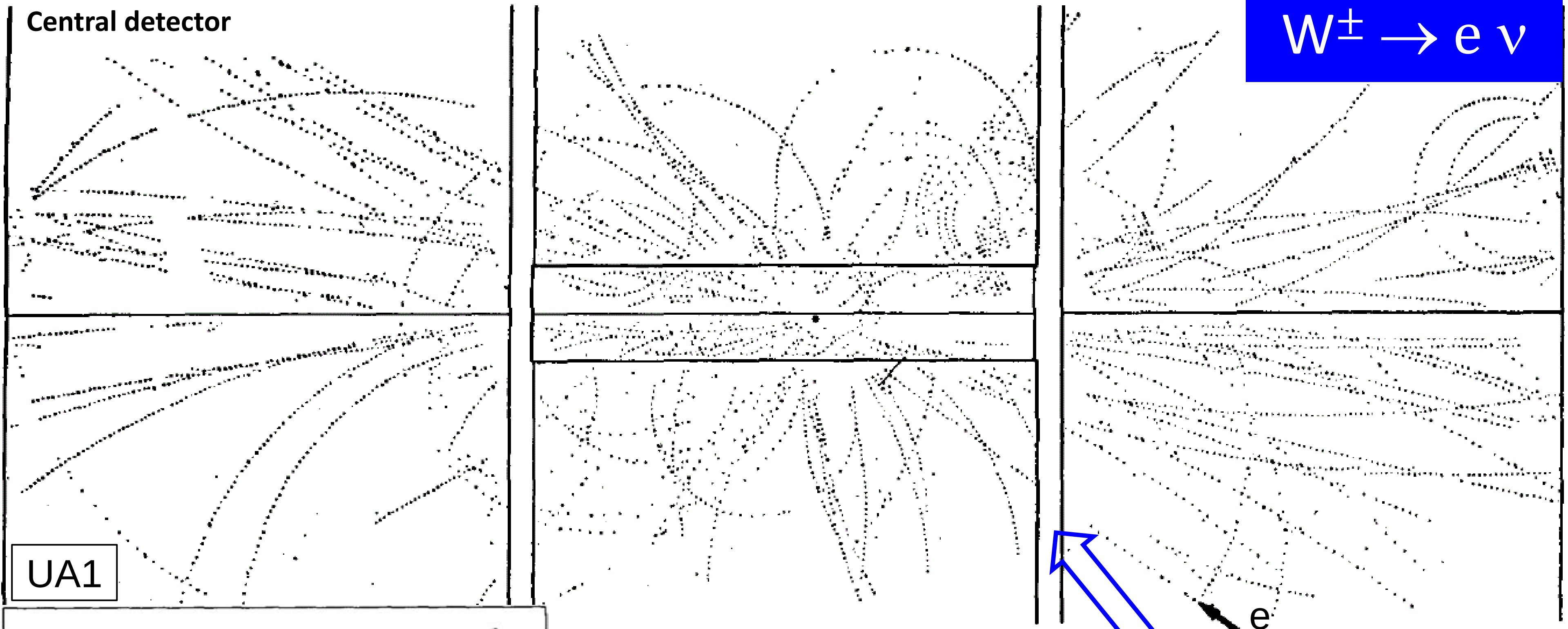


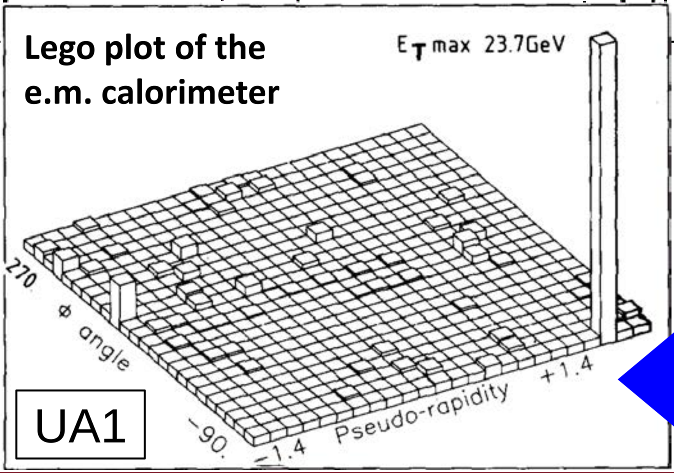




The events: UA1 $W^{\pm} \to e\nu$

$W^{\pm} \to e\,\nu$



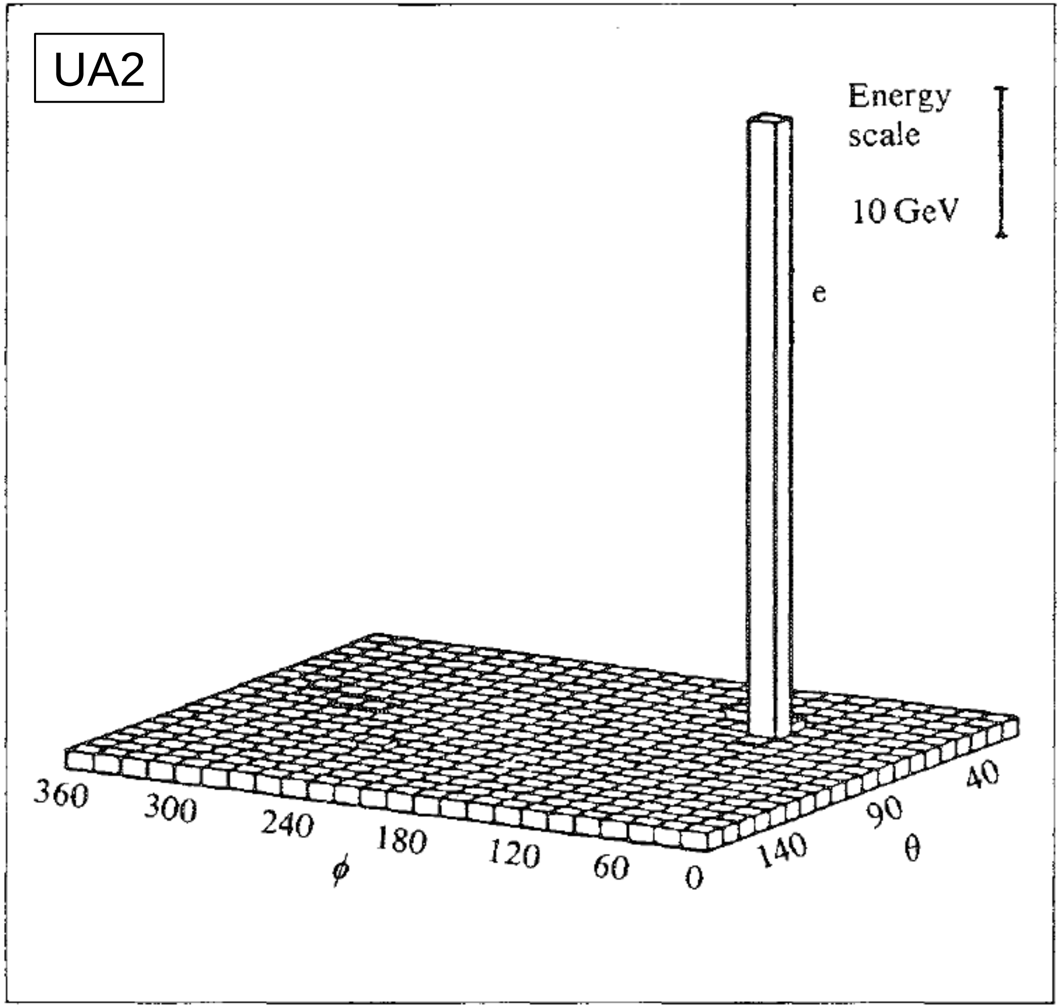


The original UA1 paper said "*Fig. 6. The digitization from the central detector for the tracks in (…) events which have an identified, isolated, well-measured high-p_T electron*".

The central detector is impressive, but the e.m. calo is definitely easier for event trigger, selection and analysis.

The events : UA2 $W^{\pm} \rightarrow e\nu$

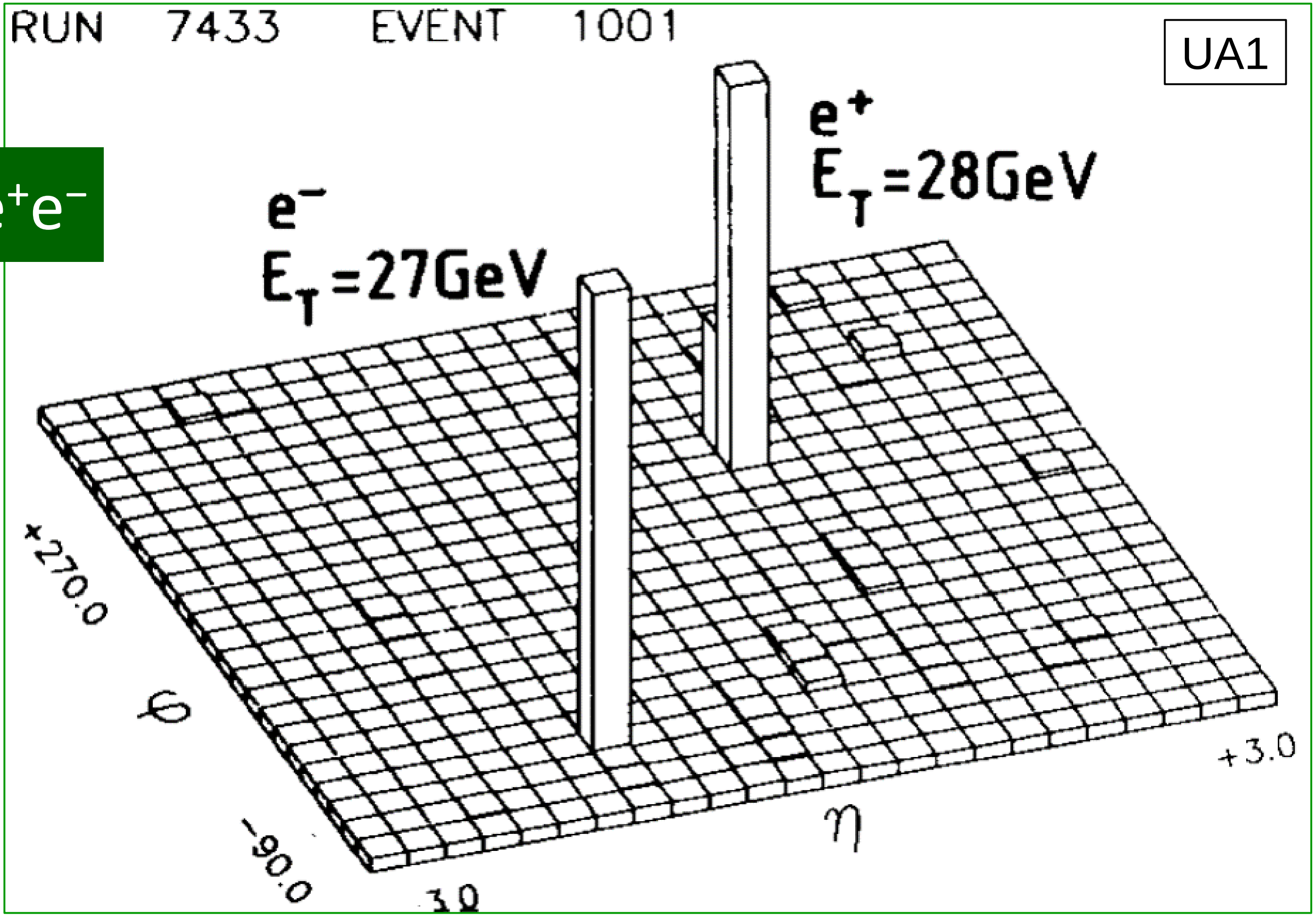
$W^{\pm} \rightarrow e\,\nu$



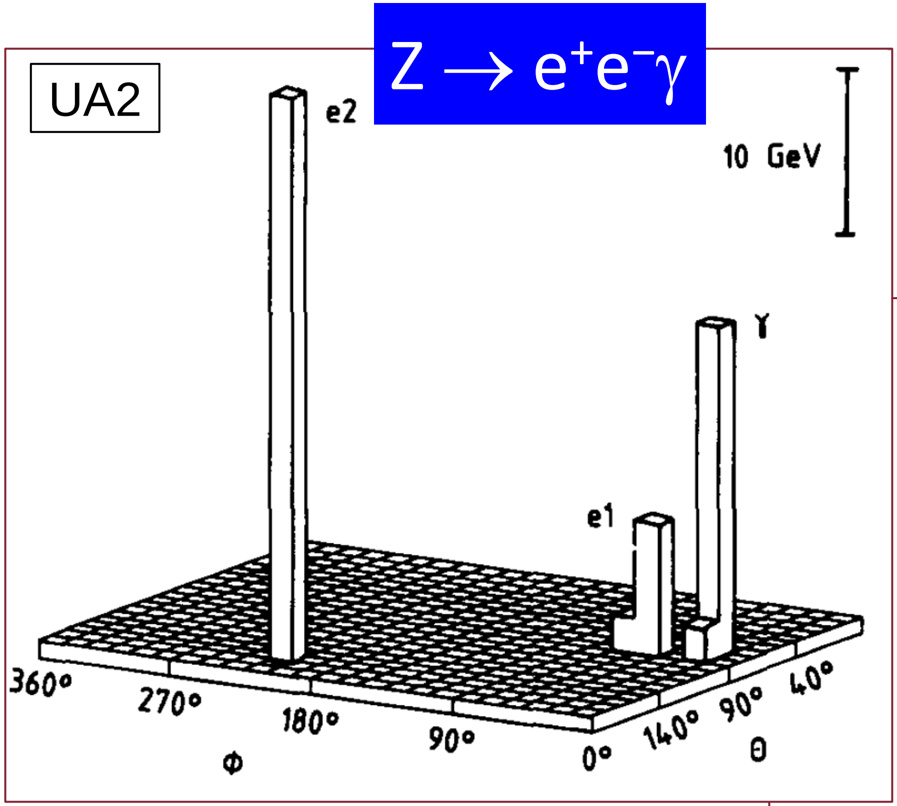
NB UA2 had a magnetic field only in the FB regions to measure $e^{\pm}$ asymmetry; in the central region, $e^{+} \leftrightarrow e^{-}$ were ambiguous.

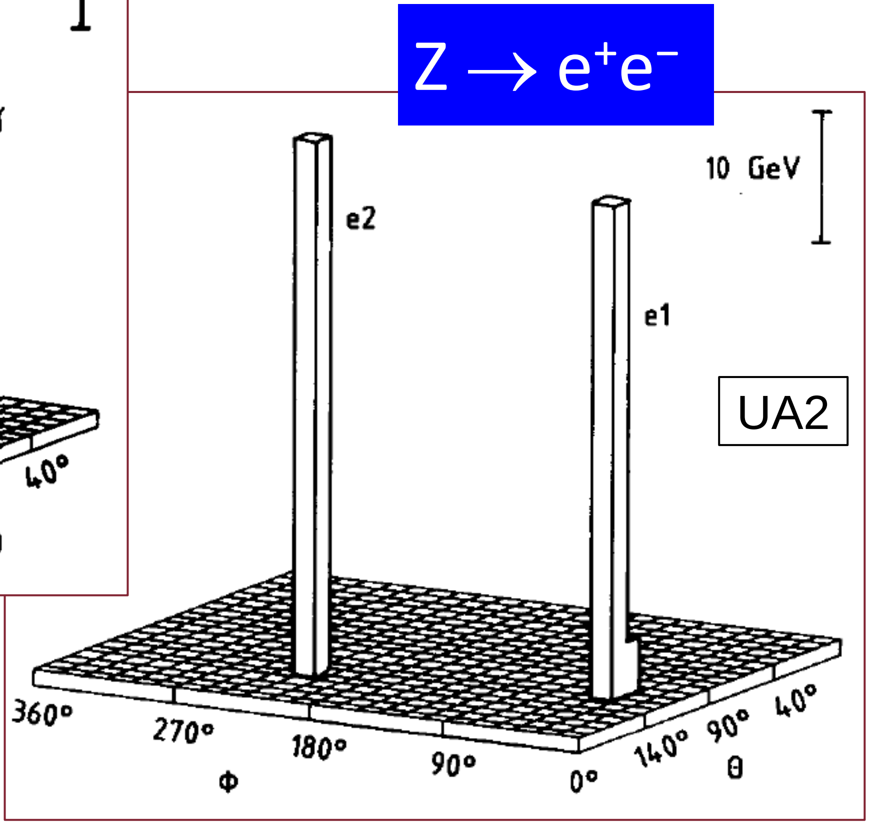
The events: UA1 $Z \rightarrow e^+e^-$

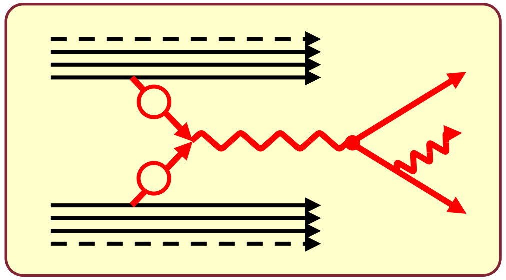
$Z \rightarrow e^+e^-$



The events: UA2 $Z\to e^+e^-$

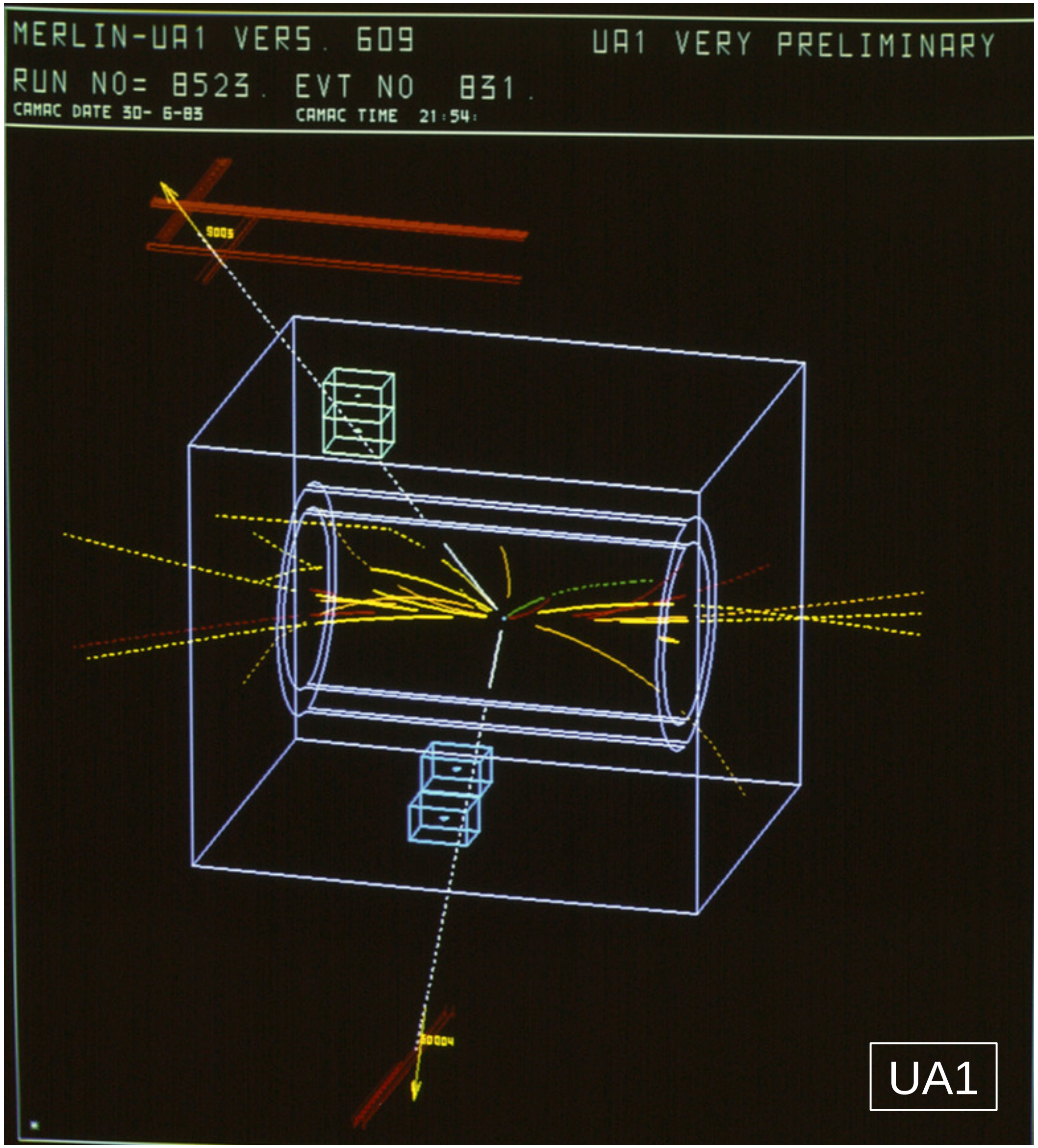






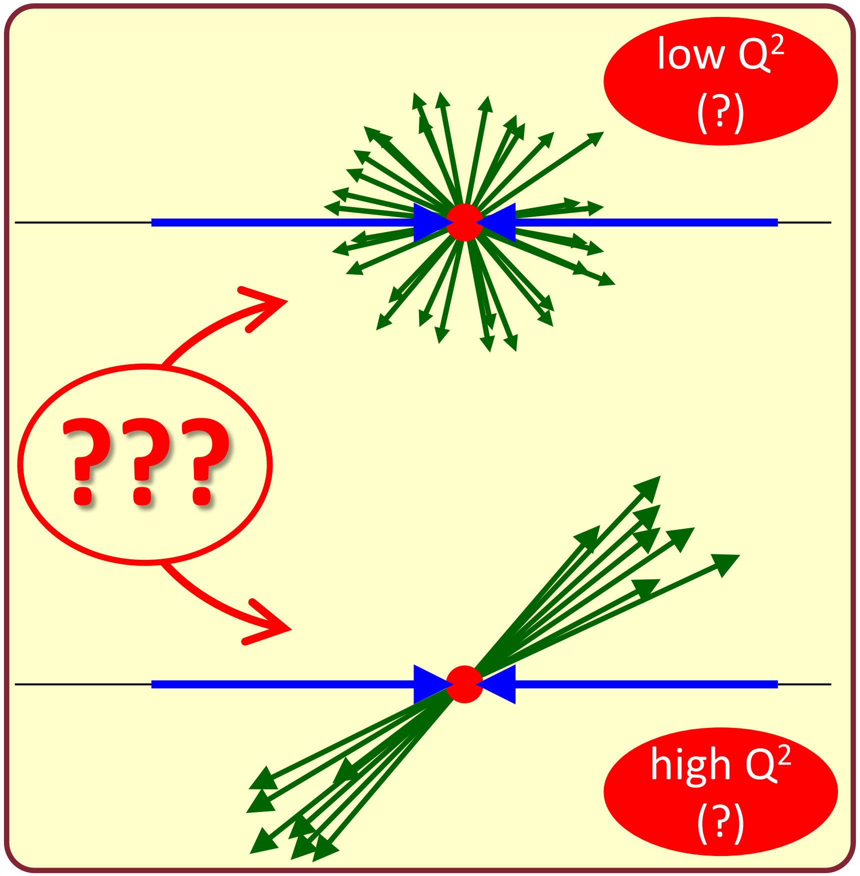
The events: UA1 $Z \to \mu^+\mu^-$

$Z \to \mu^+\mu^-$



hadronic interactions

- At the time, the scheme of the quark-parton model (qpm) was established, but not shared by everybody .
- The expected signature of qpm is the "jettyness" of the hadronic events.
- If qpm and QCD hold, the expectation is a change of regime as a function of Q^2 :
  - at low Q^2, coherent $\bar{p}p$ collisions → final state hadrons spherically distributed;
  - at high Q^2, parton-parton collisions → two thin jets.
- Otherwise, expect all types of events at any Q^2, but most should be spherical.
- A difficult experimental challenge :
  - prove jettyness without a "trigger bias" (i.e. *cherry piking the events*);
  - disentangle dynamics from kinematics (3-momentum conservation may simulate jettyness);
  - prove that the majority (?) of events at high Q^2 are "jet-like".



hadronic interactions: transition region

The solution :

- measure Q^2 independently from jets: define ΣE_T (total transverse energy, i.e. an unbiased (*) observable, in QCD $\propto \sqrt{Q^2}$) :

$$\Sigma E_T = \Sigma_k \, |E_T^{\text{hadron-k}}| = \Sigma_k \, E_k \, |\sin\theta_k|;$$

- identify the two highest jets of the events and their transverse energies E_T^1, E_T^2;
- plot, in bins of ΣE_T, the fractions :

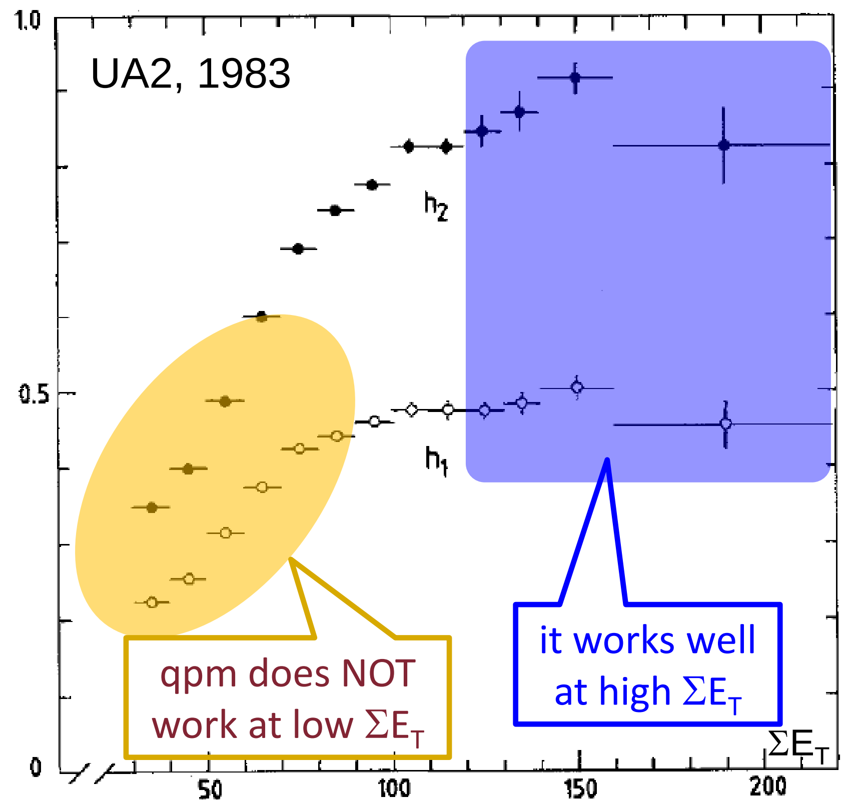
$$h_1 = \langle E_T^1/\Sigma E_T \rangle;$$

$$h_2 = \langle (E_T^1 + E_T^2)/\Sigma E_T \rangle.$$

- Ideally, in qpm+QCD :
  - ➢ $\bar{p}p$ int. @ low Q^2 : both h_1,h_2 small;
  - ➢ qpm @ high Q^2 : $h_1 \approx 0.5$, $h_2 \approx 1$.

___

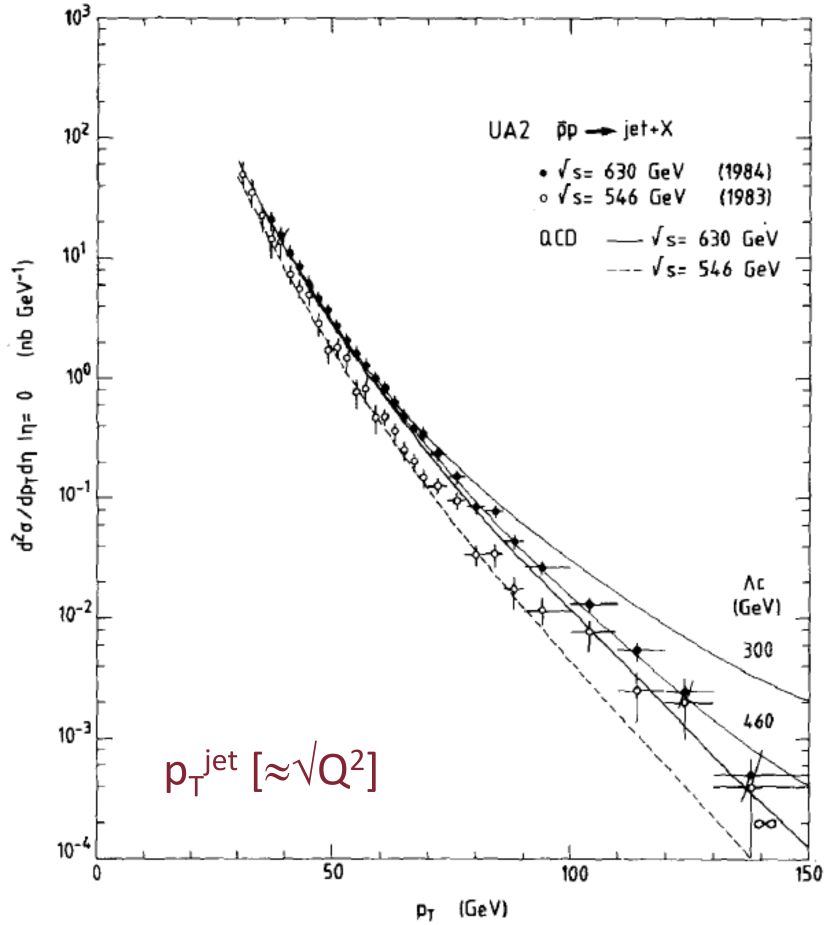
(*) events selected (triggered) by ΣE_T are unbiased respect to shape; moreover, if qpm holds, $\Sigma E_T \propto \sqrt{Q^2}$.

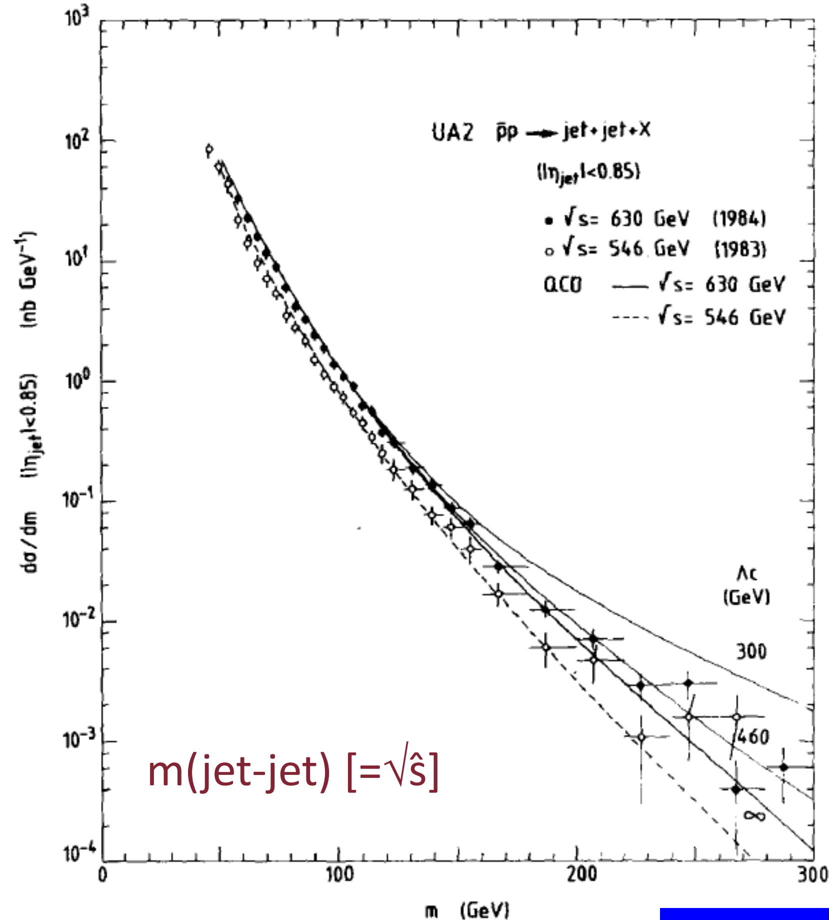


Success !!! As a function of ΣE_T , (i.e. $\sqrt{Q^2}$), the events change in the expected way; the qpm region is not precisely defined, but

$\Sigma E_T >$ ~100 GeV ($\ell <$ ~10^{-18} m).

hadronic interactions: $d^2\sigma/dp_T d\eta|_{\eta=0}$





Already discussed. Just notice :

- the increase as a function of √s;
- the comparison with pQCD;

- limit on $\Lambda \geq 370$ GeV @ 95% CL ($1/\Lambda$ hypothetical scale of a sub-structure : $(370\text{ GeV})^{-1} \approx 5\times10^{-19}$ m.

for higher limits on Λ, see § LHC.

hadronic interactions: dσ/dcosθ

The ($\mathbb{L}$-invariant) angular variable χ:

$$\chi \equiv \frac{\hat{u}}{\hat{t}} = \frac{1+\cos\theta^*}{1-\cos\theta^*};\ [\chi \text{ large} \leftrightarrow \theta \text{ small}]$$

The variable χ "flattens" the Rutherford angular cross-section, i.e. $d\sigma/d\cos\theta^* \propto \hat{t}^{-2} \propto (1 - \cos\theta^*)^{-2}$ $\to d\sigma/d\chi$ = const. [*box*].

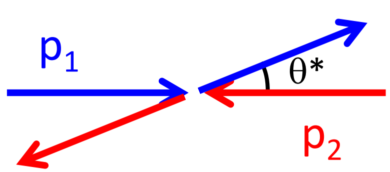
$$\frac{d\chi}{d\cos\theta^*} = \frac{1}{1-\cos\theta^*} + \frac{1+\cos\theta^*}{(1-\cos\theta^*)^2} = \frac{2}{(1-\cos\theta^*)^2};$$

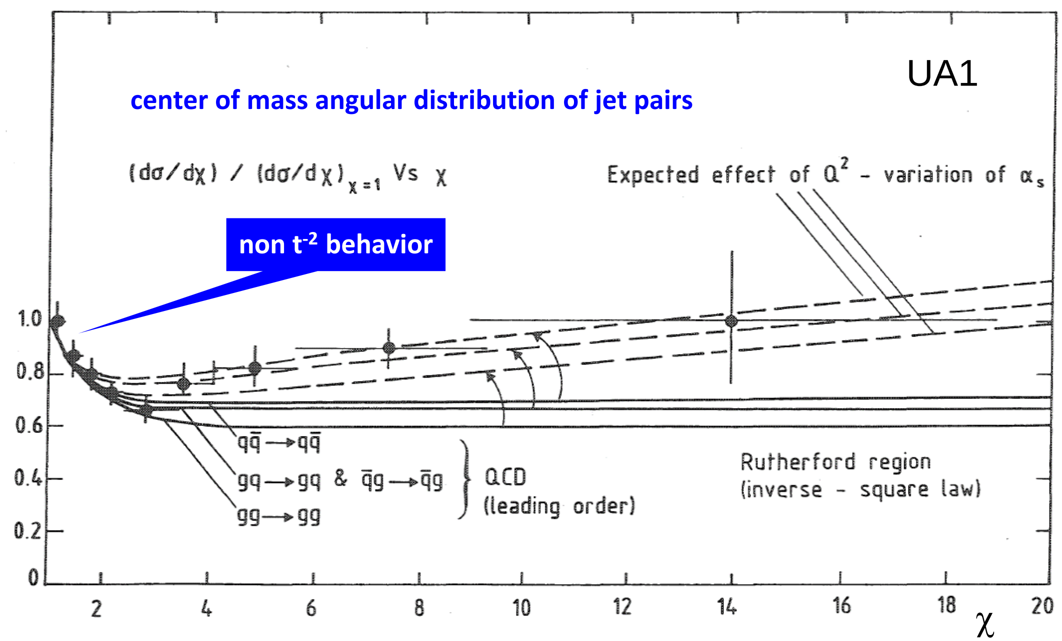
$$\frac{d\sigma_{\text{Rutherf.}}}{d\chi} = \left(\frac{d\sigma}{d\cos\theta^*}\right)\left|\frac{d\chi}{d\cos\theta^*}\right|^{-1} \propto \left(\frac{1}{\hat{t}^2}\right)\left|\frac{d\chi}{d\cos\theta^*}\right|^{-1} \propto$$

$$\propto \frac{1}{(1-\cos\theta^*)^2}(1-\cos\theta^*)^2 = \text{const.}$$

The data (UA1 1983, actually Bill Scott) show :

- $d\sigma/d\chi$ is remarkably "quasi flat";
- good agreement with pQCD: $d\sigma/d\chi$ not constant because of α_s running : χ large $\to$ θ small $\to$ Q^2 small $\to$ α_s larger $\to$ σ larger);
- in addition, non-$\hat{t}^{-2}$ processes at small χ (large θ).





The "Drell-Yan" process

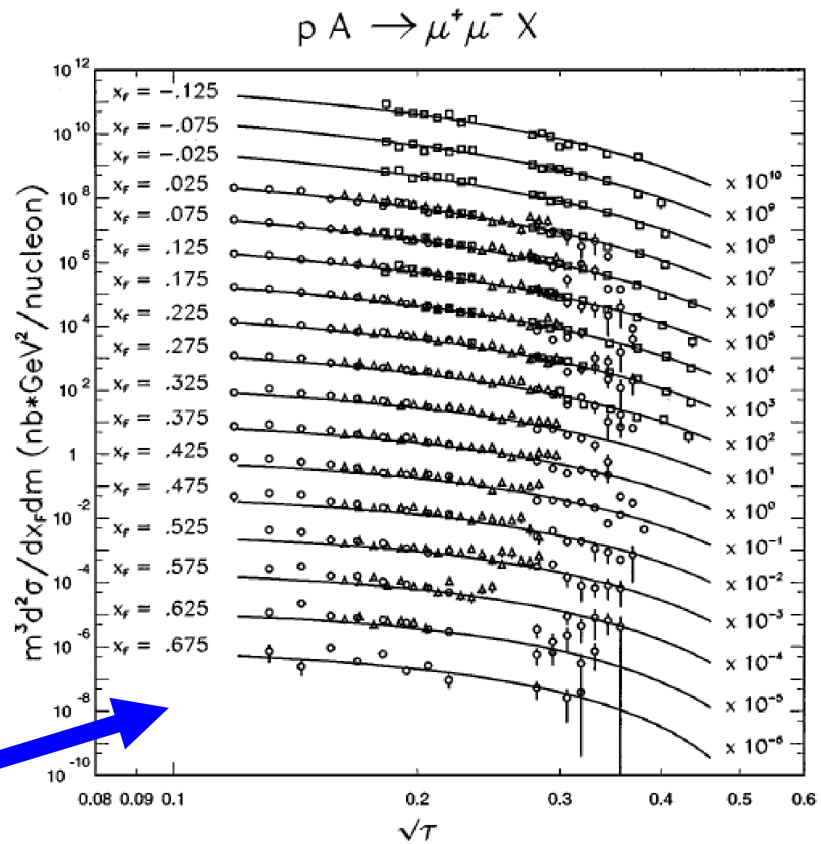
- Drell and Yan in 1971 computed in qp model:

$$q\,\bar{q} \to \gamma^* \to \ell^+\ell^-, \quad \ell = e, \mu, \tau;$$

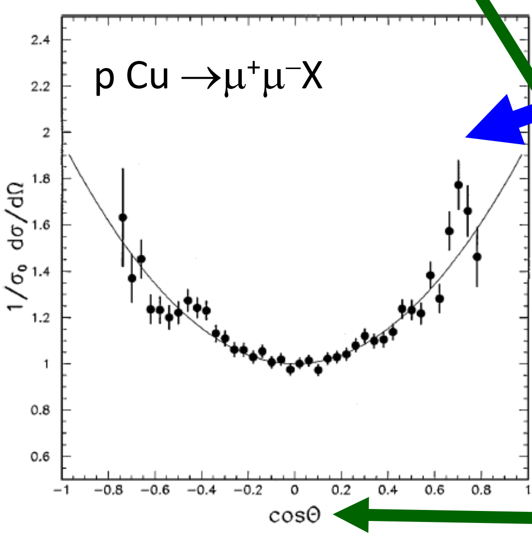
- they found :

$$\frac{d^2\sigma}{dm_{12}^2 dx_F} = \frac{4\pi\alpha^2}{9m_{12}^2 s}\frac{1}{(x_1+x_2)} \times$$
$$\times \sum_i e_i^2 \left[q_i(x_1)\bar{q}_i(x_2) + \bar{q}_i(x_1)q_i(x_2)\right];$$

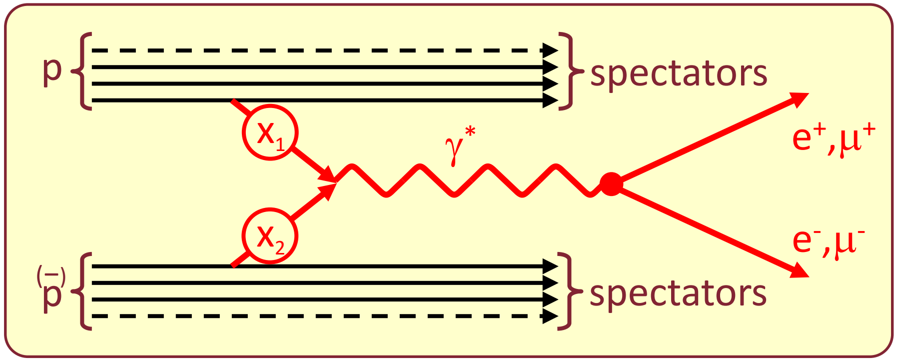
$$\frac{d\sigma}{d\Omega} = \sigma_0\left(1+\cos^2\theta\right); \quad x_F = x_1 - x_2; \quad \tau = m_{12}^2/s.$$



Ann.Rev.Nucl.Part.Sci. 49:217 (1999) [fixed-target exp.]



"θ" is the angle in the $\ell^+\ell^-$ frame, should be "θ*".

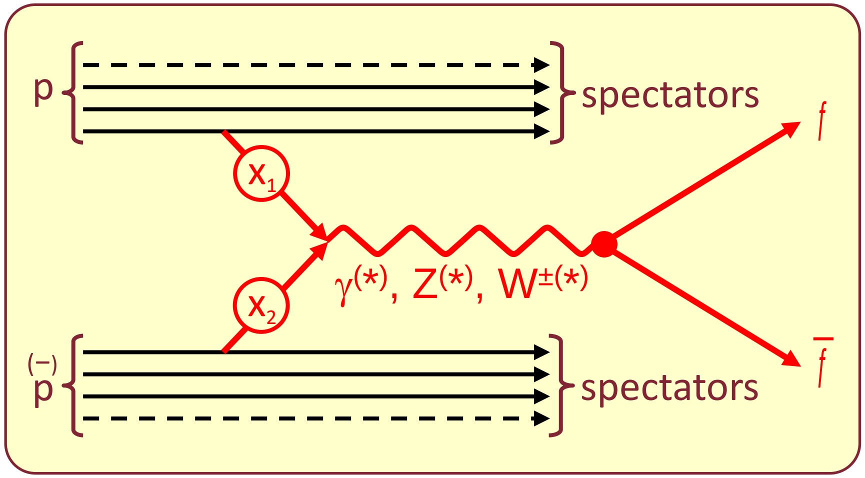


The "Drell-Yan" process: definition

- by extension, in hadronic interactions, the name "DY" was also used for processes with two leptons mediated by a (heavy) vector bosons :

  $d\bar{u} \to W^- \to \ell^-\bar{\nu}$, (+ any $q\bar{q}' \to$ leptons);

  $u\bar{u} \to Z \to \ell^-\ell^+, \nu\bar{\nu}, q\bar{q}$ (+ ...);
- by a further extension, it is also used for all processes with a fermion-antifermion pair in the final state, mediated by an electro-weak vector boson, either real or virtual ($\gamma^{(*)}$, $Z^{(*)}$, $W^{\pm(*)}$), e.g. $d\bar{u} \to W^- \to q\bar{q}'$;
- i.e. "DY" = production of a $f\bar{f}$ pair in a hadronic interaction with an electro-weak spin-1 mediator;
- when the γ^* is replaced by another IVB, at parton level the electro-magnetic process has to be replaced by the appropriate electro-weak cross-section;
- a DY process is calculable with the usual qpm scheme [*as shown in § 8*];
- computations of the DY processes were at the origin of the Sp$\bar{\text{p}}$S proposal, and the main ingredient of the comparison data-theory;
- since then, this scheme has been technically improved without basic modifications.



W$^{\pm}$ discovery

On 25 January 1983 CERN announced the discovery of the W boson. Left to right: Carlo Rubbia, Simon van der Meer, Herwig Schopper, Erwin Gabathuler, Pierre Darriulat (Image: CERN)

W± discovery: UA1

Volume 122B, number 1 PHYSICS LETTERS 24 February 1983

$W^\pm \to e^\pm \nu$

Phys. Lett. 122B (1983)

EXPERIMENTAL OBSERVATION OF ISOLATED LARGE TRANSVERSE ENERGY ELECTRONS WITH ASSOCIATED MISSING ENERGY AT $\sqrt{s}$ = 540 GeV

UA1 Collaboration, CERN, Geneva, Switzerland

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Aachen [a]–Annecy (LAPP) [b]–Birmingham [c]–CERN [d]–Helsinki [e]–Queen Mary College, London [f]–Paris (Coll. de France) [g]–Riverside [h]–Rome [i]–Rutherford Appleton Lab. [j]–Saclay (CEN) [k]–Vienna [l] Collaboration

$W^{\pm}$ discovery: UA2

$W^{\pm} \to e^{\pm}\nu$

Phys. Lett 122B (1983)

Volume 122B, number 5,6 PHYSICS LETTERS 17 March 1983

OBSERVATION OF SINGLE ISOLATED ELECTRONS OF HIGH TRANSVERSE MOMENTUM IN EVENTS WITH MISSING TRANSVERSE ENERGY AT THE CERN $\bar{p}p$ COLLIDER

The UA2 Collaboration

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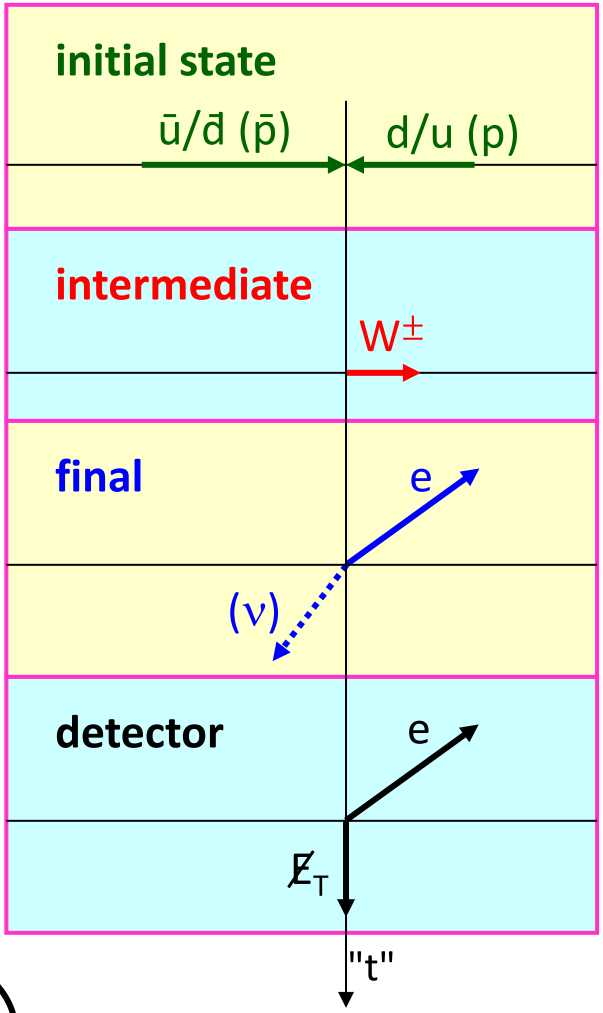
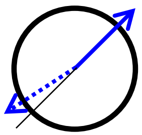
$W^{\pm}$ discovery: method

- production (assume only valence) : $\bar{u}d \to W^- \to \ell^-\bar{\nu}$
  [the case ($u\bar{d} \to W^+ \to \ell^+\nu$) is equal, *mutatis mutandis*];
- ℓ = e/μ, study the "e" case (original discovery, μ similar);
- the hadronic decay modes are dominant (see **§ LEP**), but essentially invisible at the Sp$\bar{p}$S, but an attempt by UA2;
- qpm $\to p_T(W^{\pm}) \approx 0$; $p_z(W^{\pm})$ unknown and varying;
- ν not detected (but $\not{E}_T$);
- selection :
  - trigger in E_T electromagnetic ($e^{\pm}$) : E_T > 8 GeV [UA2];
  - selection requires large $\not{E}_T$ ($\to p_T^{\nu}$);
  - ... and a true $e^{\pm}$ (from its e.m. shower);
  - reconstruct p_T^e, p_T^{ν} (= $\not{E}_T$), $\to E_T^{tot}$, p_T^{tot};
  - compute : m_T ["transverse mass"] :
    $$m_T^2 \equiv \left(E_T^{\ell} + E_T^{\nu}\right)^2 - \left(\vec{p}_T^{\,\ell} + \vec{p}_T^{\,\nu}\right)^2 \approx 2E_T^{\ell}E_T^{\nu}\left(1 - \cos\Delta\phi_{\ell\nu}\right);$$
- analysis :
  - select clean $W^{\pm}$ decays, i.e. high-p_T $e^{\pm}$ + $\not{E}_T$;
  - correlate $m_T \to m_W$, e.g. via montecarlo.



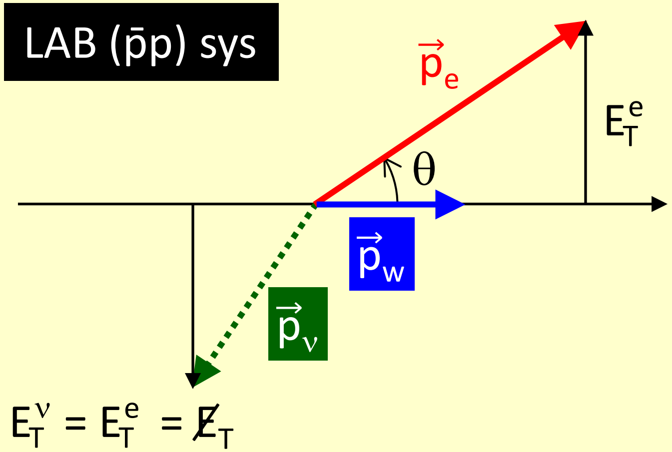
ideally $E_T^{\ell} = E_T^{\nu} \equiv E_T$,
$\Delta\phi$=180°, $m_T = 2E_T$.



$W^{\pm}$ discovery: kinematics

Problem : In a $W \to e\nu$ event, only $\vec{p}_e$ and $\not{E}_T$ are detected. Is it possible to get $\vec{p}_W$ and $\vec{p}_\nu$?

LAB ($\bar{p}p$) sys



... but:

- Γ_W neglected $\to \Delta p_W{}^{sys}$;
- better : $\vec{p}_T{}^W$ = "$\not{E}_T$(2D)" $- \vec{p}_T{}^e$ (but large error from spectators).

$$W: \left(\sqrt{m_w^2+p^2},\quad p,\quad 0\right);$$ because of q.p.m.

$$e: \left(k,\quad k\cos\theta,\quad k\sin\theta\right);$$ e almost massless

$$\nu: \left(\sqrt{m_w^2+p^2}-k,\quad p-k\cos\theta,\quad -k\sin\theta\right);$$

measured: $k,\ \theta,\ \not{E}_T$;

unknowns: $m_w,\ p = p_W$;

check: $\not{E}_T \left[\approx E_T^\nu \approx E_T^e\right] \approx k\sin\theta$ [+ planarity];

$$m_\nu^2 \approx 0 \;\to\; \left(\sqrt{m_w^2+p^2}-k\right)^2 = (p-k\cos\theta)^2 + k^2\sin^2\theta;$$

$\to$ one equation, two unknowns $\to$ no solution.

But, if m_w known : e.g. from the jacobian [next slide]

$$m_w^2+p^2+k^2-2k\sqrt{m_w^2+p^2} = p^2+k^2-2pk\cos\theta;$$

$$\left(2k\sqrt{m_w^2+p^2}\right)^2 = \left(m_w^2+2pk\cos\theta\right)^2;$$

$$4\boxed{p^2}\,k^2\left(1-\cos^2\theta\right)-4\boxed{p}\,km_w^2\cos\theta+4k^2m_w^2-m_w^4=0;$$

$\to$ two solutions for $\boxed{p_W}$ and for $\boxed{\vec{p}_\nu}$.

$W^\pm$ discovery: the jacobian peak

- the "jacobian peak" ["*" = W sys.] :

$$p_T^e{}^* = p_T^e = p^e{}^*\sin\theta^* = \tfrac{1}{2} m_W \sin\theta^*;$$

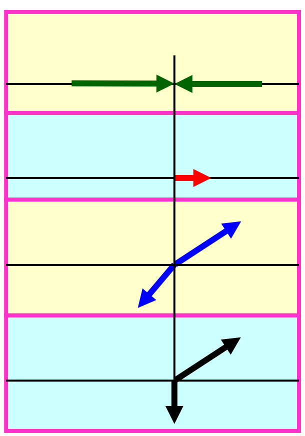
$$\cos\theta^* = \sqrt{1-\left(2p_T^e/m_W\right)^2} = \sqrt{m_W^2 - 4\left(p_T^e\right)^2}\,/m_W\,;$$

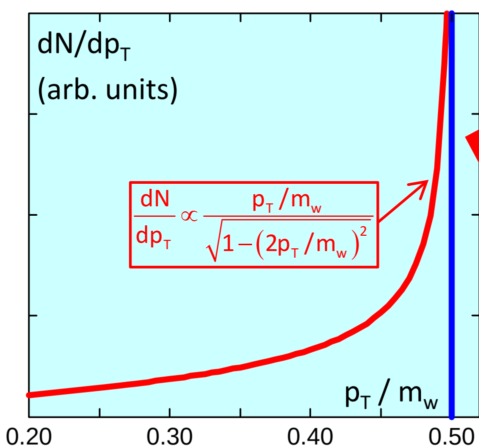
$$dN/dp_T^e = \left(dN/d\cos\theta^*\right)\times\left|d\cos\theta^*/dp_T^e\right| \propto$$

$$\propto \boxed{\left(1+\cos\theta^*\right)^2 \times \frac{4p_T^e}{m_W}} \times \boxed{\frac{1}{\sqrt{m_W^2-4\left(p_T^e\right)^2}}}.$$

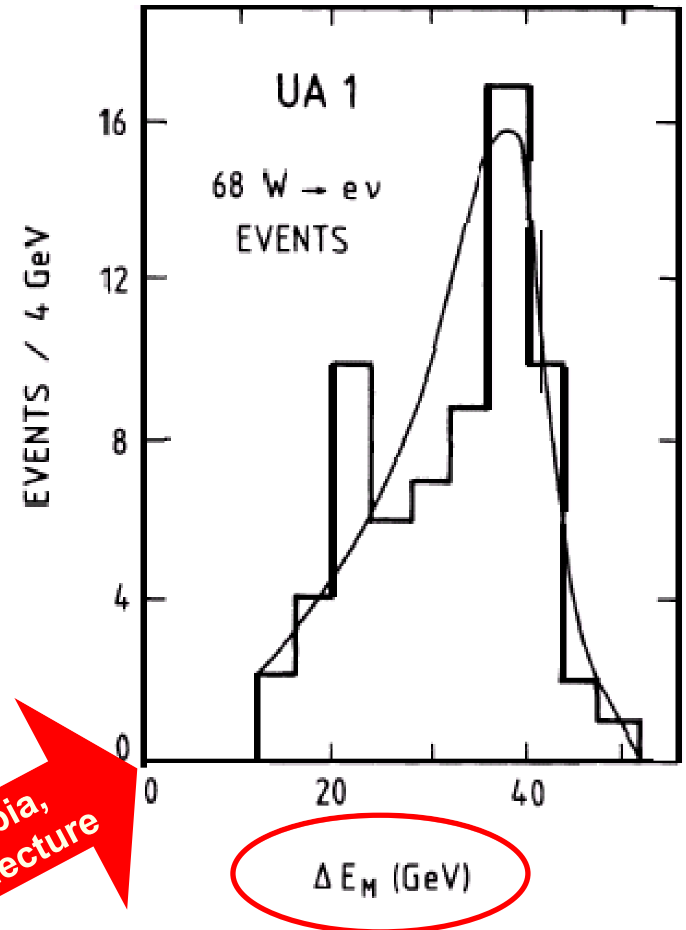
= $f(p_T{}^e, m_W)$ smooth, no-peak

- therefore the "jacobian" $|d\cos\theta^*/dp_T{}^e|$ produces a sharp peak at $p_T{}^e \approx m_W/2$, modulated by $\Gamma_W \oplus$ (detector).



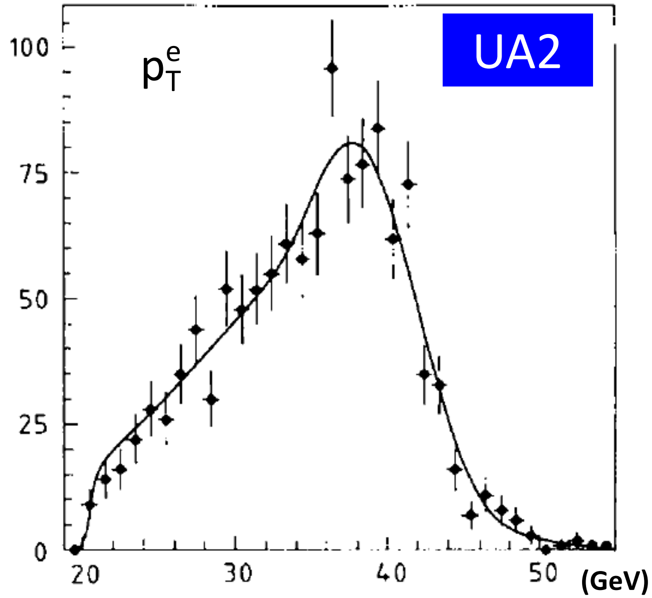


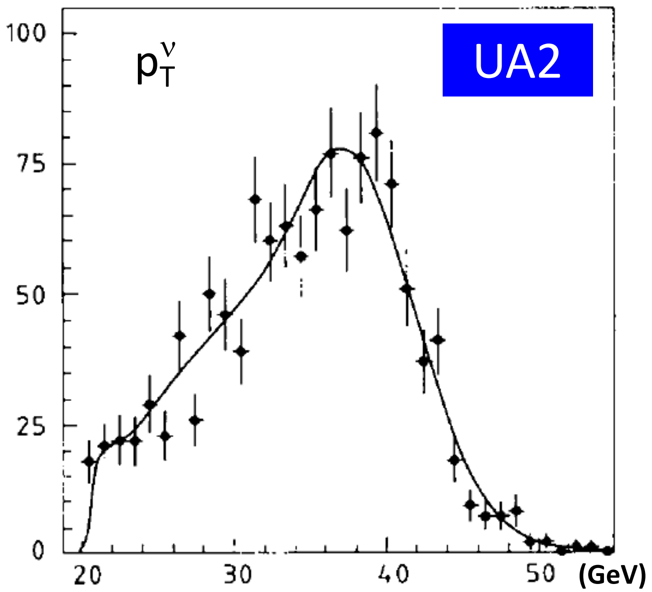
C.Rubbia, Nobel lecture



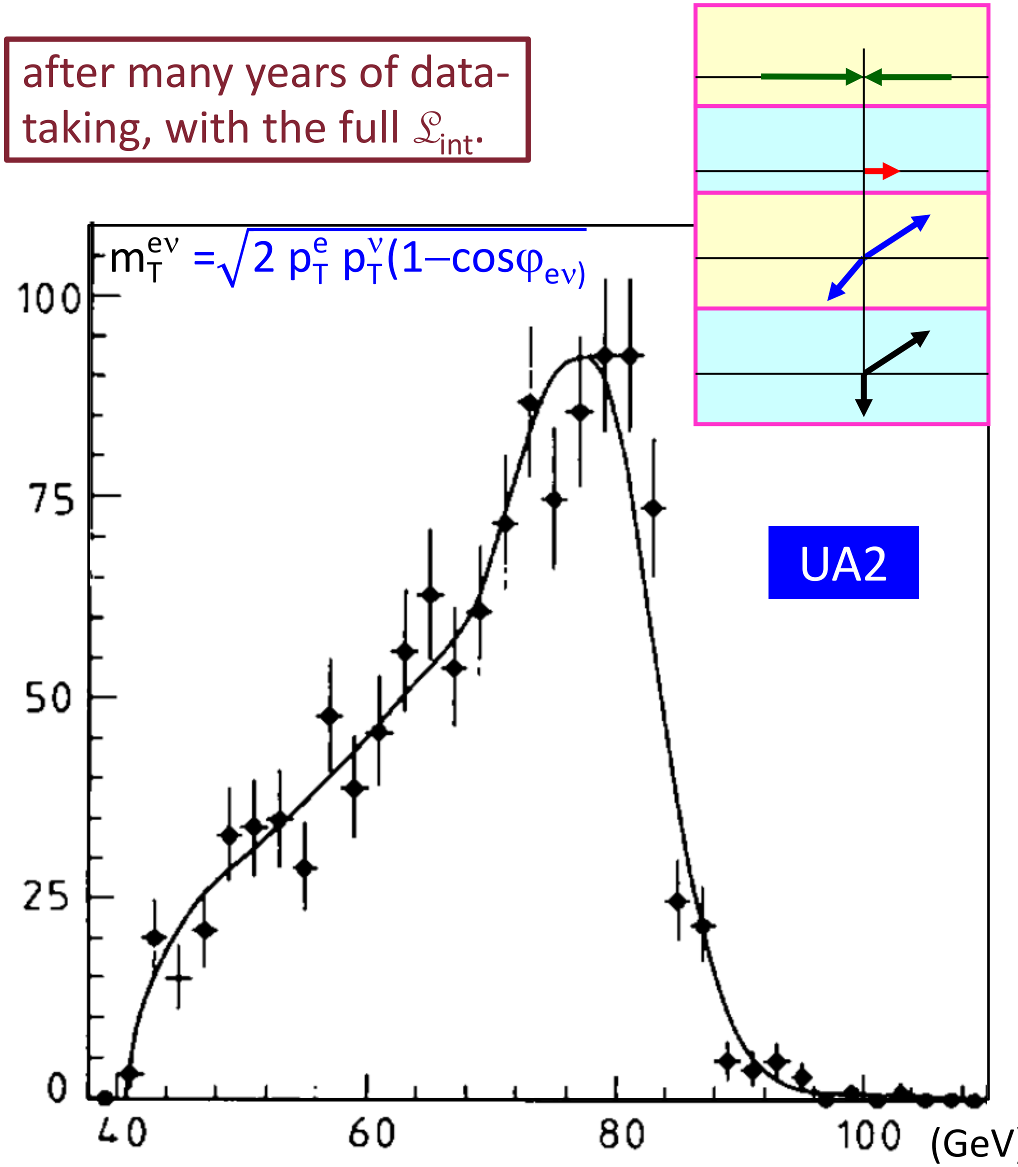
technicality : the plot shows "ΔE_M" $\equiv \not{E}_T \approx E_T{}^\nu$, which follows exactly the same kinematics as $p_T{}^e$.

$W^{\pm}$ discovery: the jacobian peak

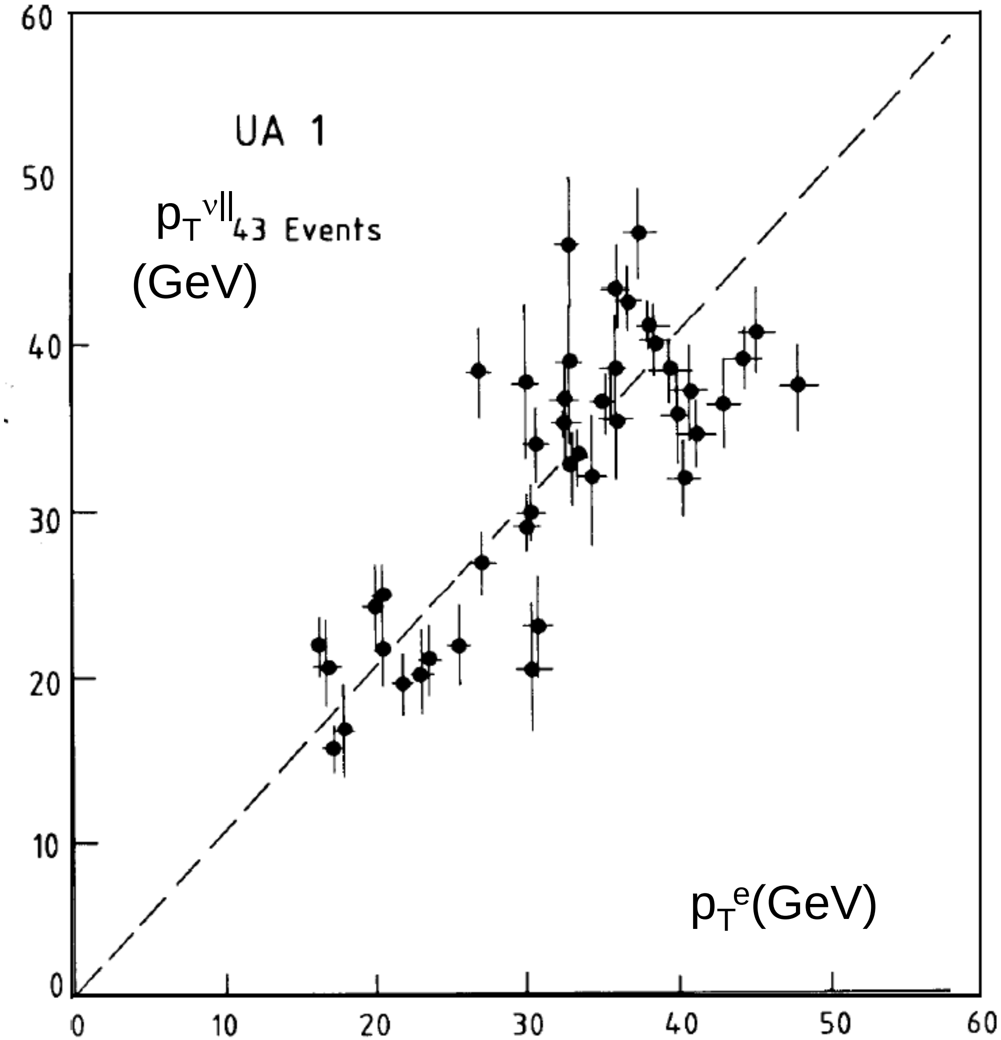




after many years of data-taking, with the full $\mathscr{L}_{int}$.



$W^{\pm}$ discovery : p_T vs $\not{E}_T$



Results (see § LEP) :

UA1 pre-LEP :

m_w = 82.7 $\pm$ 1.0 (stat) $\pm$ 2.7(syst) GeV;

Γ_W < 5.4 GeV;

UA2 pre-LEP :

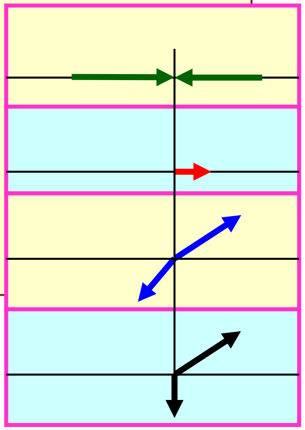
m_w = 80.2 $\pm$ 0.8 (stat) $\pm$ 1.3(syst) GeV;

Γ_W < 7 GeV;

[PDG 2016] :

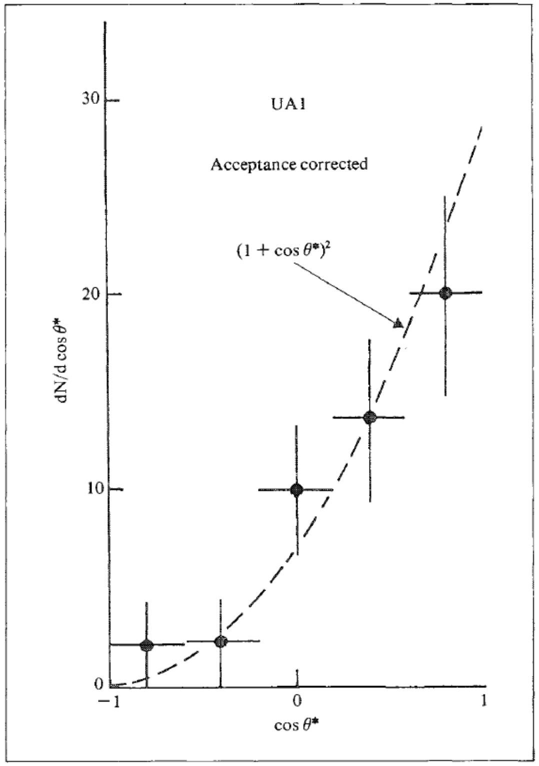
m_w = 80.385 $\pm$ .015 GeV;

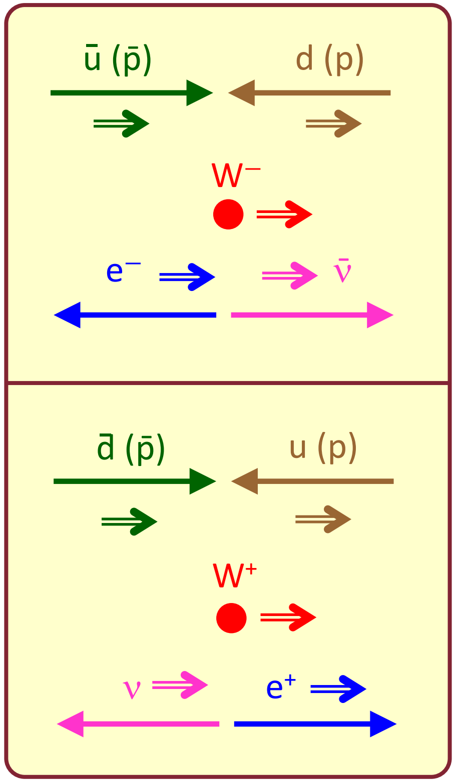
Γ_W = 2.085 $\pm$ .042 GeV;



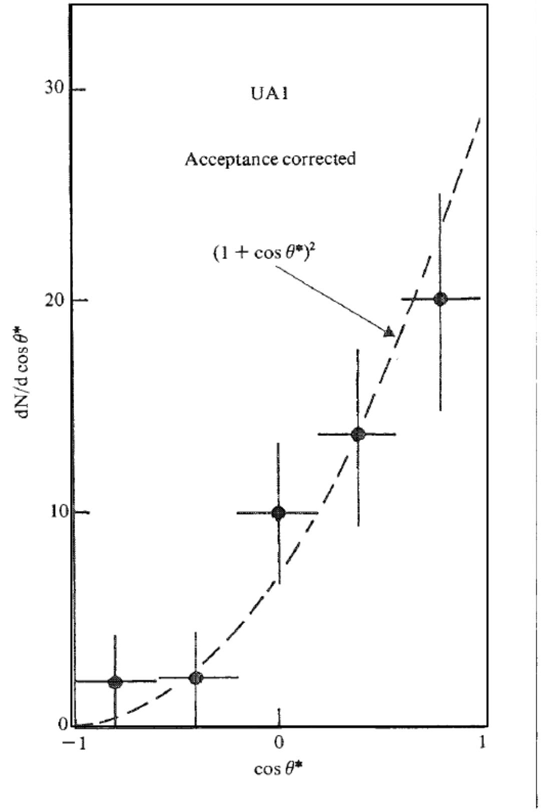
$W^{\pm}$ discovery : asymmetry

- Assume that the main process be valence-valence. The large values of the $W^{\pm}$ mass makes all the other masses negligible. Thus the particles have –ve helicity and the antiparticles +ve helicity.
- Then, the (V–A) structure of the CC favor the collinearity (e^-p), ($e^+\bar{p}$), i.e. $\cos\theta^* \approx 1$.
- As in many similar processes, $d\sigma/d\cos\theta^* \propto (1+\cos\theta^*)^2$.
- The process is a simple and powerful test of the theory …
- *… but does it discriminate between (V–A) and (V+A) ? <u>[think and answer]</u>*





$W^\pm$ discovery: asymmetry results



- As important as the pure discovery [*less media impact, of course*].
- This beautiful effect is only evident at the $Sp\bar{p}S$ [$m_w^2 = sx_1x_2$ → increasing $\sqrt{s}$, the value of $x_{1,2}$ decreases, and therefore sea-quarks become dominant].
- [*probably one of the few advantages in hadronic colliders for a low value of* $\sqrt{s}$].
- At LHC, the initial state is pp, completely symmetric, so the effect is completely absent. The W^+ yield is more abundant, especially at large x, where the valence quarks are dominant [*do not confuse difference in initial state with parity violation*].
- At LHC, cross-section larger → more precise m_w, Γ_w measurements.
- A method to increase the asymmetry at high $\sqrt{s}$ is the selection of "low-p_T" $W^\pm$ ($q\bar{q} \to W^\pm$), with respect to "high p_T" $W^\pm$ (qg, $\bar{q}g \to W^\pm$ jet).

Z discovery: UA1

Volume 126B, number 5 PHYSICS LETTERS 7 July 1983

EXPERIMENTAL OBSERVATION OF LEPTON PAIRS OF INVARIANT MASS AROUND 95 GeV/c^2 AT THE CERN SPS COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

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Aachen [a] – Annecy (LAPP) [b] – Birmingham [c] – CERN [d] – Helsinki [e] – Queen Mary College, London [f] – Paris (Coll. de France) [g] – Riverside [h] – Rome [i] – Rutherford Appleton Lab. [j] – Saclay (CEN) [k] – Vienna [h] Collaboration

$Z \to e^+e^-$

Phys. Lett 126B (1983)

Z discovery: UA2

Volume 129, number 1,2 PHYSICS LETTERS 15 September 1983

$Z \rightarrow e^+e^-$

Phys. Lett. 129B (1983)

EVIDENCE FOR $Z^0 \rightarrow e^+e^-$ AT THE CERN $\bar{p}p$ COLLIDER

The UA2 Collaboration

P. BAGNAIA [b], M. BANNER [f], R. BATTISTON [1,2], Ph. BLOCH [f], F. BONAUDI [b], K. BORER [a], M. BORGHINI [b], J.-C. CHOLLET [d], A.G. CLARK [b], C. CONTA [e], P. DARRIULAT [b], L. Di LELLA [b], J. DINES-HANSEN [c], P.-A. DORSAZ [b], L. FAYARD [d], M. FRATERNALI [e], D. FROIDEVAUX [b], G. FUMAGALLI [e], J.-M. GAILLARD [d], O. GILDEMEISTER [b], V.G. GOGGI [e], H. GROTE [b], B. HAHN [a], H. HÄNNI [a], J.R. HANSEN [b], P. HANSEN [c], T. HIMEL [b], V. HUNGERBÜHLER [b], P. JENNI [b], O. KOFOED-HANSEN [c], E. LANÇON [f], M. LIVAN [b,e], S. LOUCATOS [f], B. MADSEN [c], P. MANI [a], B. MANSOULIÉ [f], G.C. MANTOVANI [1], L. MAPELLI [b,3], B. MERKEL [d], M. MERMIKIDES [b], R. MØLLERUD [c], B. NILSSON [c], C. ONIONS [b], G. PARROUR [b,d], F. PASTORE [e], H. PLOTHOW-BESCH [b], M. POLVEREL [f], J-P. REPELLIN [d], A. RIMOLDI [e], A. ROTHENBERG [b], A. ROUSSARIE [f], G. SAUVAGE [d], J. SCHACHER [a], J.L. SIEGRIST [b], H.M. STEINER [b,4], G. STIMPFL [b], F. STOCKER [a], J. TEIGER [f], V. VERCESI [e], A.R. WEIDBERG [b], H. ZACCONE [f], J.A. ZAKRZEWSKI [b,5] and W. ZELLER [a]

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[f] *Centre d'Etudes Nucléaires de Saclay, France*

Z discovery: mass computation

- production $\bar{u}u$ ($\bar{d}d$) $\rightarrow Z \rightarrow \ell^+\ell^-$;
- both selection and analysis easier than in the $W^\pm$ case [despite smaller cross-section] :
  - ➢ require two well identified, opposite-charge, same-flavor leptons;
  - ➢ use fake eμ to study bckgd [*NOT existent, NO bckgd, the easiest analysis ever*];
- compute :

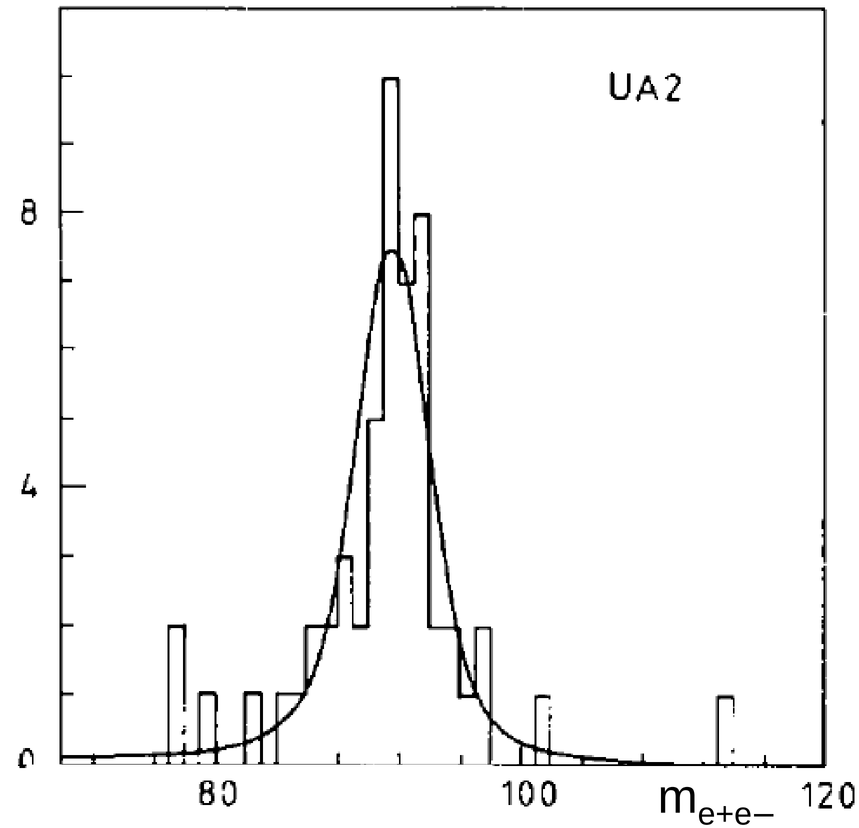
$$m(\ell^+\ell^-) \approx \sqrt{2E_+E_-(1-\cos\alpha)} = 2\sqrt{E_+E_-}\sin(\alpha/2);$$

$$\frac{\partial m}{\partial E_+} = \frac{m}{2E_+}; \qquad \frac{\partial m}{\partial E_-} = \frac{m}{2E_-}; \qquad \frac{\partial m}{\partial \alpha} = \frac{m}{2\tan(\alpha/2)};$$

$$\left(\frac{\Delta m}{m}\right)^2 = \frac{1}{4}\left[\left(\frac{\Delta E_+}{E_+}\right)^2 + \left(\frac{\Delta E_-}{E_-}\right)^2 + \left(\frac{\Delta\alpha}{\tan(\alpha/2)}\right)^2\right];$$

$$\{E_+ \approx E_-; \alpha \approx 180^\circ\} \Rightarrow \frac{\Delta m}{m} \approx \frac{1}{\sqrt{2}}\frac{\Delta E}{E}; \quad \Delta m \approx \sqrt{2}\Delta E.$$

- typically $\Delta m \approx 2$ GeV for a single event.



for Z,
$\Delta m_{exp} \approx \Gamma_{TOT}$

Z discovery: results

[interpretation and comparison with SM in § LEP]

Results :

UA1 :

m_z = 93.1 $\pm$ 1.0 (stat) $\pm$ 3.0(syst) GeV;

Γ_z = 2.7 $^{+1.2}_{-1.0}$ (stat) $\pm$ 1.3 (syst) GeV;

UA2 :

m_z = 91.74 $\pm$ .28 (stat) $\pm$ .93(syst) GeV;

Γ_z = 2.7 $\pm$2.0 (stat) $\pm$ 1.0 (sys) GeV;

[PDG > 1995, i.e. LEP] :

m_z = 91.1876 $\pm$ .0021 GeV;

Γ_z = 2.4952 $\pm$ .0023 GeV.

Comparison with SM :

- m_w/m_z;
- $\sin\theta_w$;
- SM checks;
- SM predictions (e.g. top mass);
- "bSM" physics.

the e^+e^- machine improves by >100 in m_z and >1000 in Γ_z !

... but the discovery was made in $\bar{p}p$!!!

$W^{\pm}$ / Z properties: quark decay

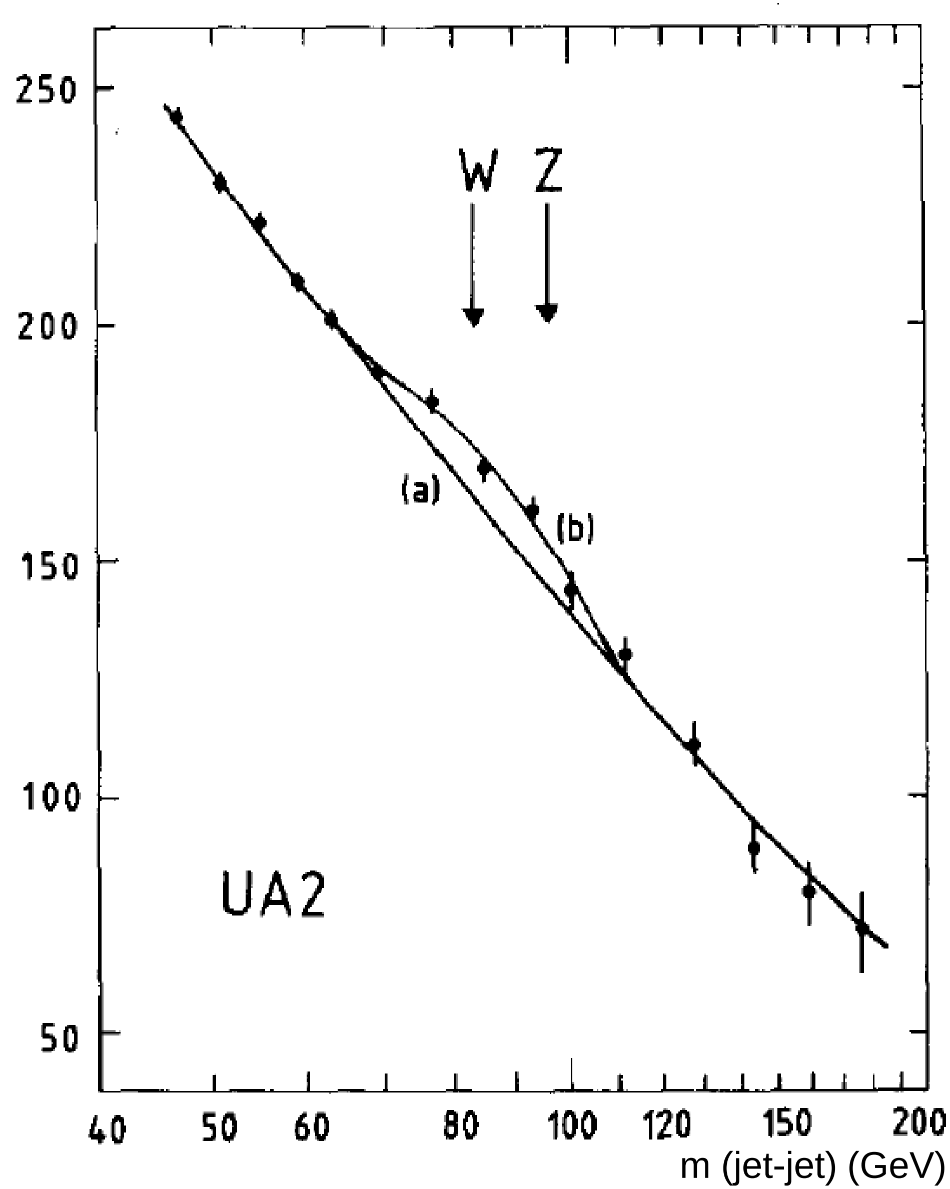
- The dominant decays of W/Z are into quark pairs :

  $W^{+} \rightarrow u\bar{d}, \rightarrow c\bar{s};$

  $W^{-} \rightarrow \bar{u}d, \rightarrow \bar{c}s;$

  $Z \rightarrow u\bar{u}, \rightarrow d\bar{d}, \rightarrow s\bar{s}, \ldots$

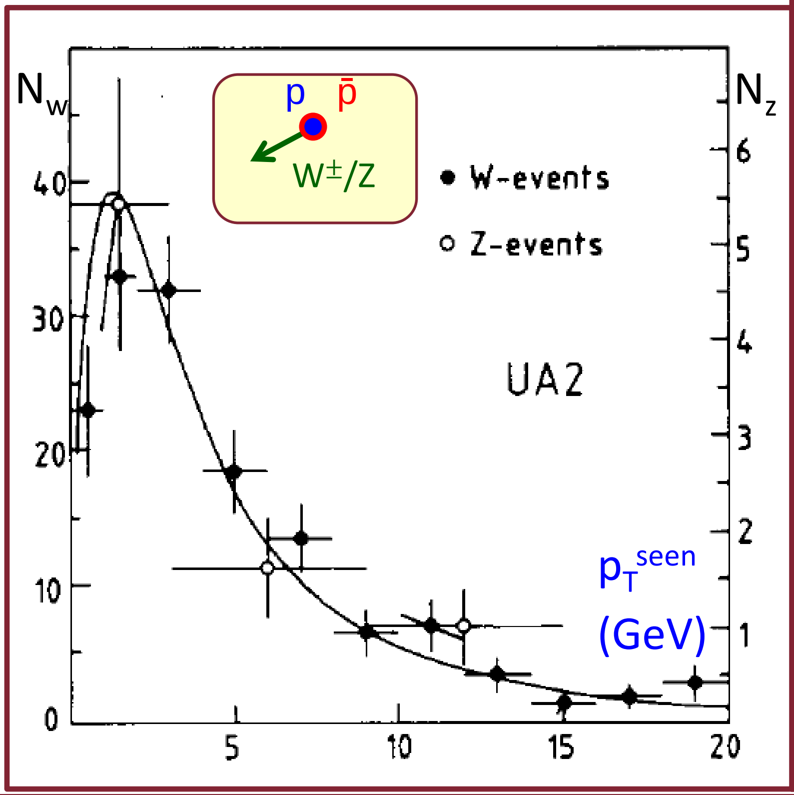
- but they are overwhelmed by the dominant QCD two-jet processes;
- the only analysis [*to my knowledge*] to select them by UA2, shown here;
- the first attempt of "jet spectroscopy", important as a method, but still quite rudimental in 1986.

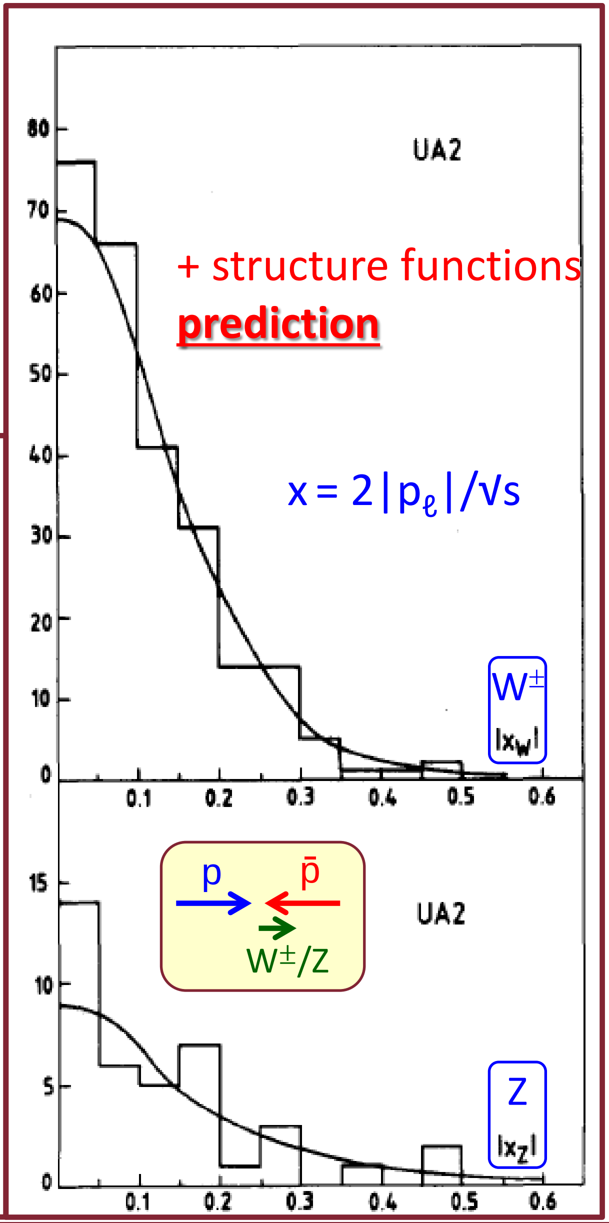


W± / Z properties: SM checks

Check the qpm with W± and Z :

- NOT a joke : if unsuccessful, serious breakdown both of the theory and the experimental method;
- x : the same variable as in structure functions and qpm;
  - the qpm **predicts** the x distribution, both for W and Z;
  - **ok**.
- p_T : the transverse momentum :
  - in qpm, NOT predicted (≈ 0);
  - expected to be "small";
  - heavily affected by detector;
  - "prediction" is a mixture of theory and exp.
  - **ok**.





References

1. all textbooks, e.g. [BJ 14], [Perkins 7-8], …
2. $Sp\bar{p}S$: Phys. Rep. 403 (2004) 91;
3. Drell-Yan : Ann. Rev. Nucl. Part. Sci. 49:217 (1999);
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5. UA1 jets : Phys. Lett. B177 (1986) 244.
6. UA2 jets : Phys. Lett. B118 (1982) 203, Phys. Lett. B160 (1985) 349.
7. UA1 W,Z : C.Rubbia, Nobel Lecture 1984.
8. UA2 W,Z : Phys. Lett. B241 (1990) 150.
9. UA2 W,Z $\rightarrow q\bar{q}$: Phys. Lett B186 (1987) 452.

NB original papers are quoted everywhere; these are reviews – usually easier to understand.

AA antiproton production target

The first version of the antiproton production target was a tungsten rod, 11 cm long (actually a row of 11 rods, each 1 cm long) and 3 mm in diameter. The rod was embedded in graphite, pressure-seated into an outer casing made of stainless steel. The casing had fins for forced-air cooling. In this picture, the 26 GeV high-intensity beam from the PS enters from the right, where a scintillator screen, with circles every 5 mm in radius, permits precise aim at the target centre.

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End of chapter 9

Particle Physics - Chapter 10

LEP — e^+e^- physics

Paolo Bagnaia

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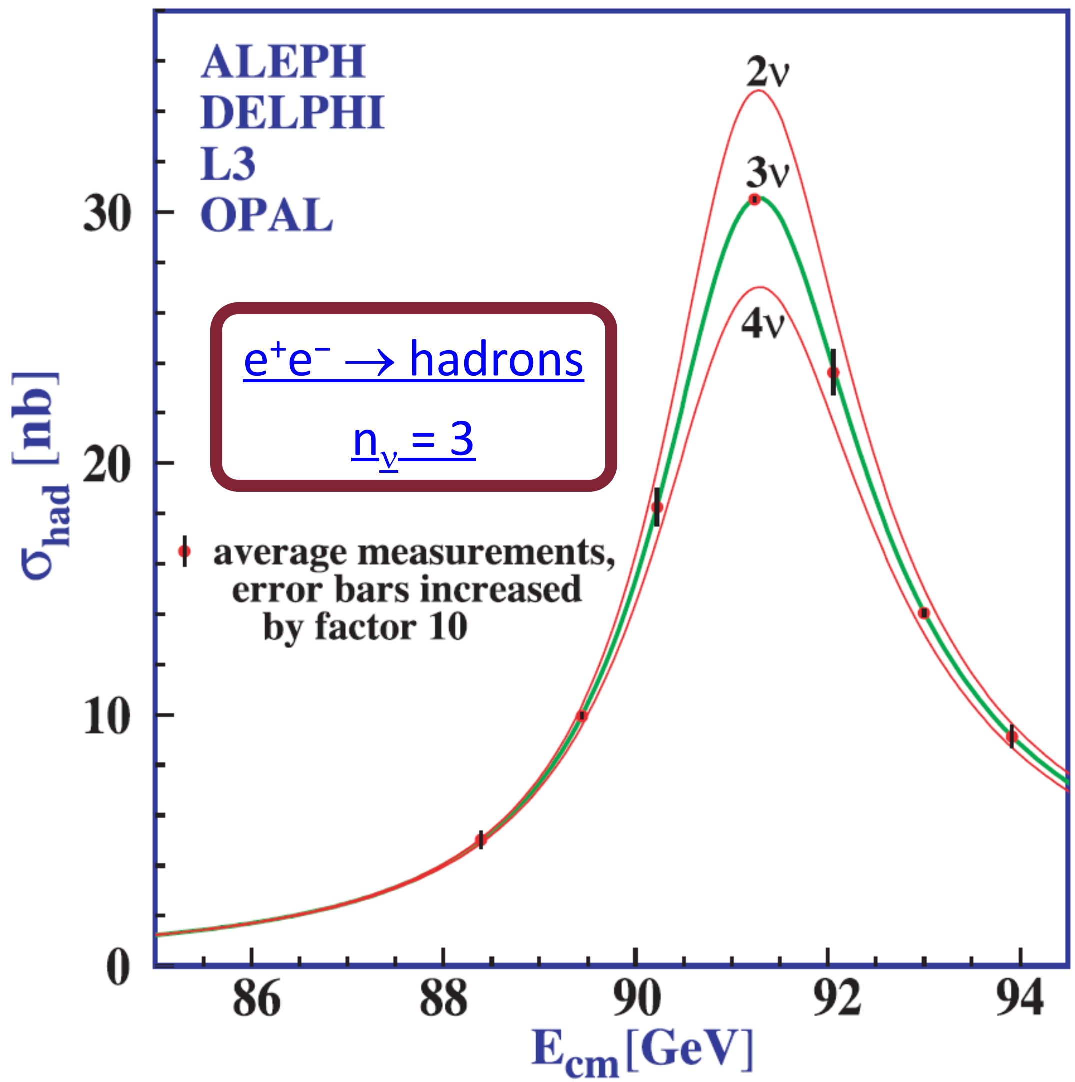
AA 18-19

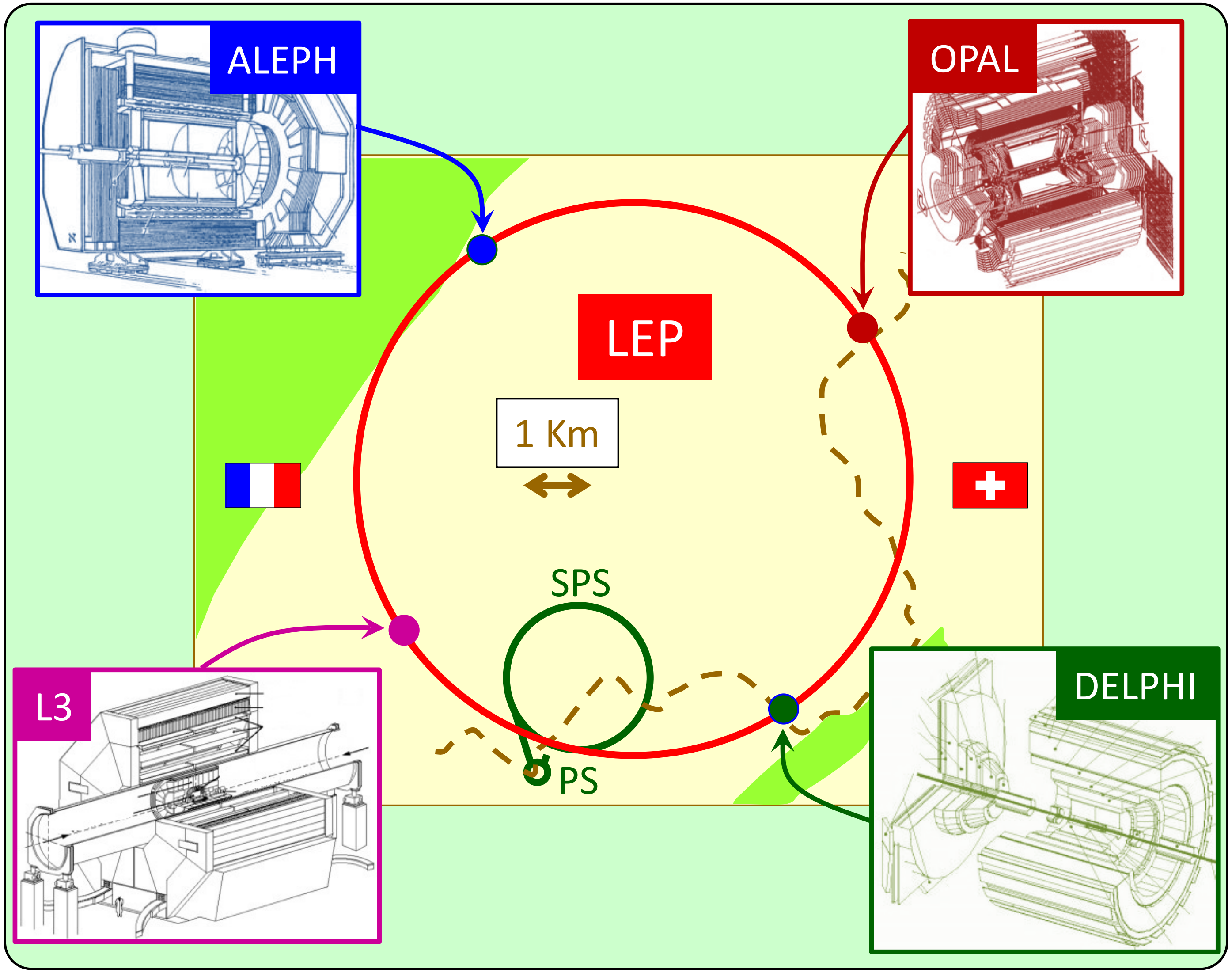
last mod.
16-May-19

10 – LEP – e^+e^- physics

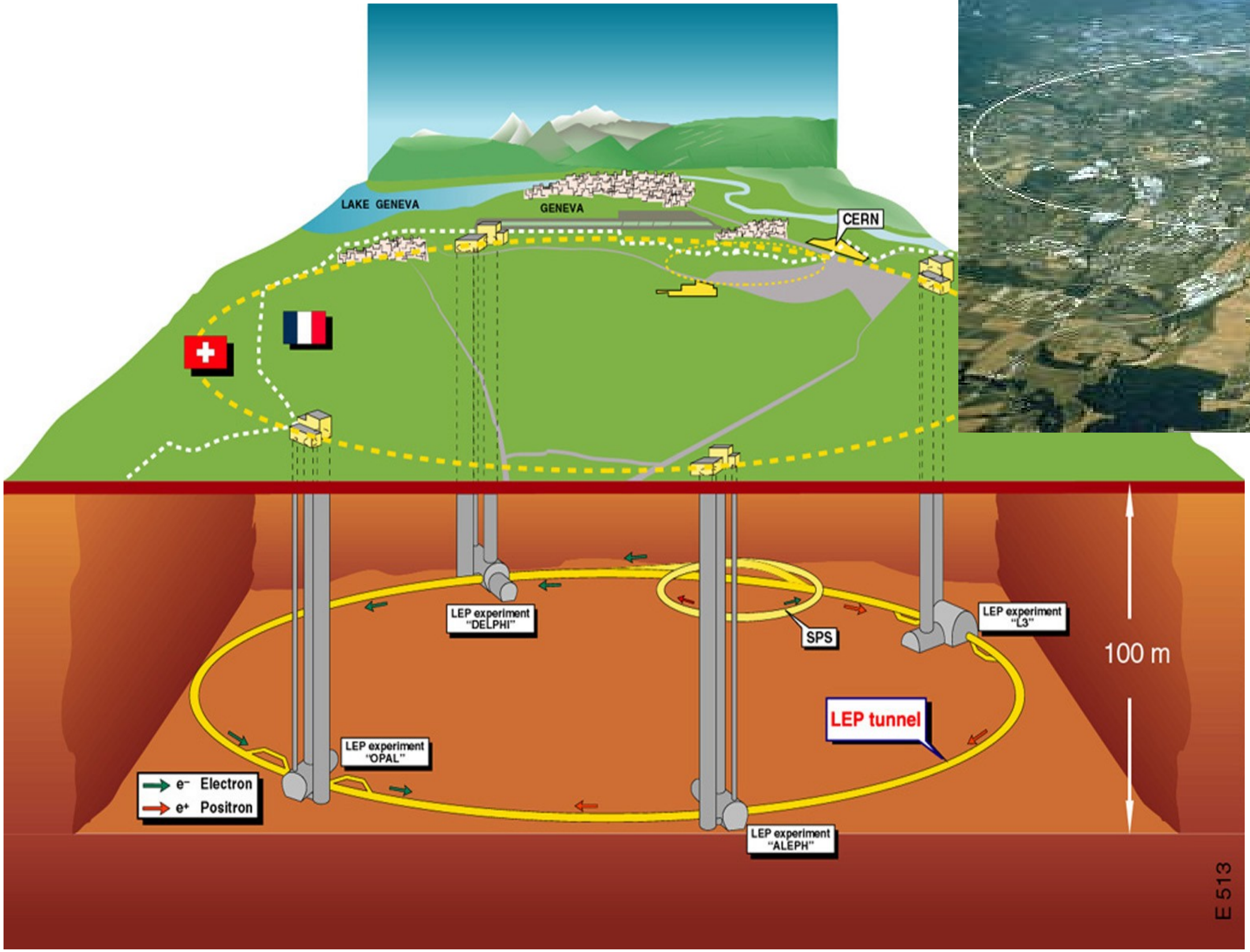


i. Machine & Detectors

1. The LEP Collider
2. Detectors
3. The L3 detector
4. LEP events
5. – 16. […]



The LEP collider



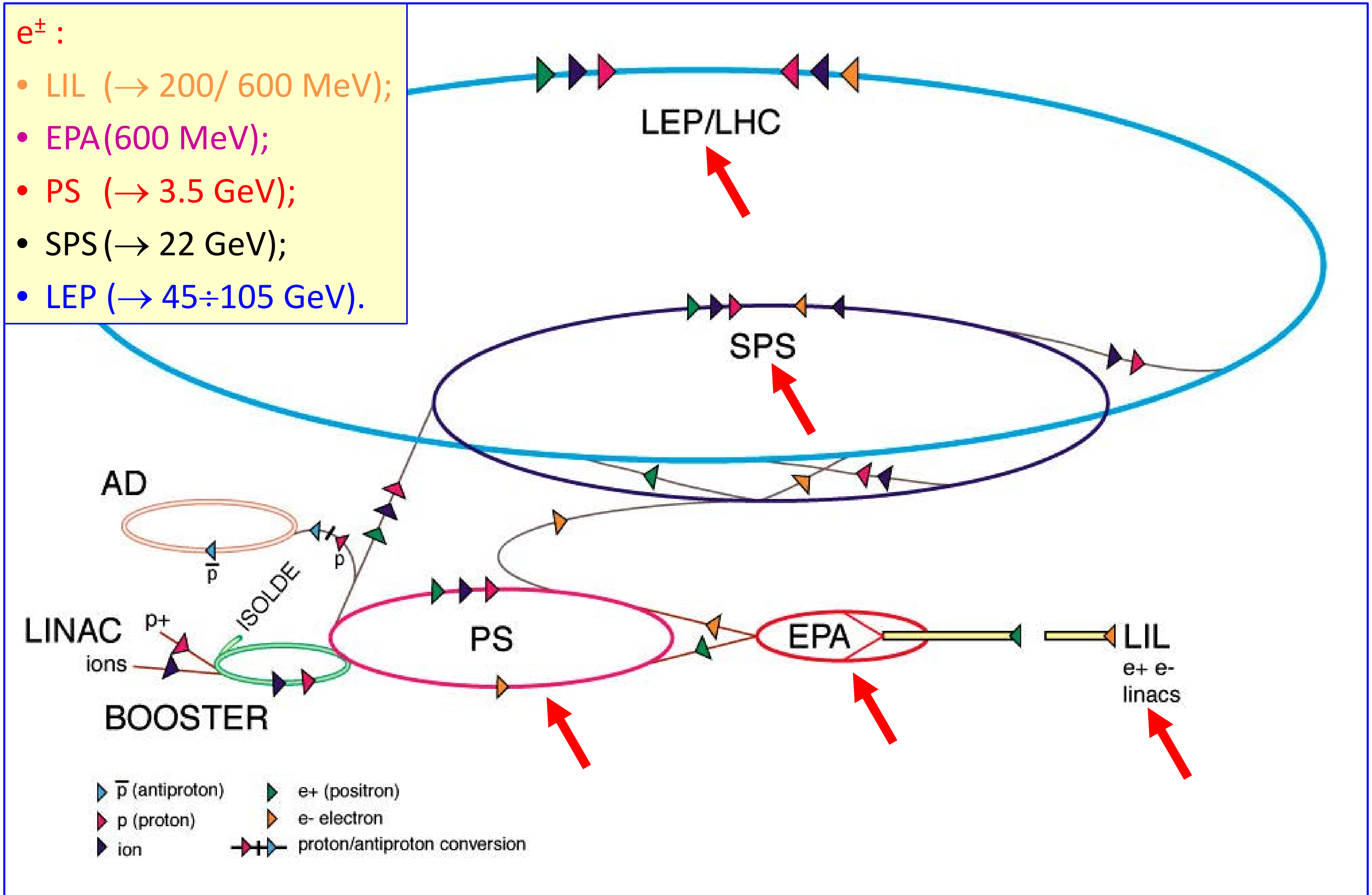
$2\pi R \approx 27$ km

~100 m underground

planar, slightly tilted wrt surface, because of geology.

The LEP collider : $e^{\pm}$ acceleration

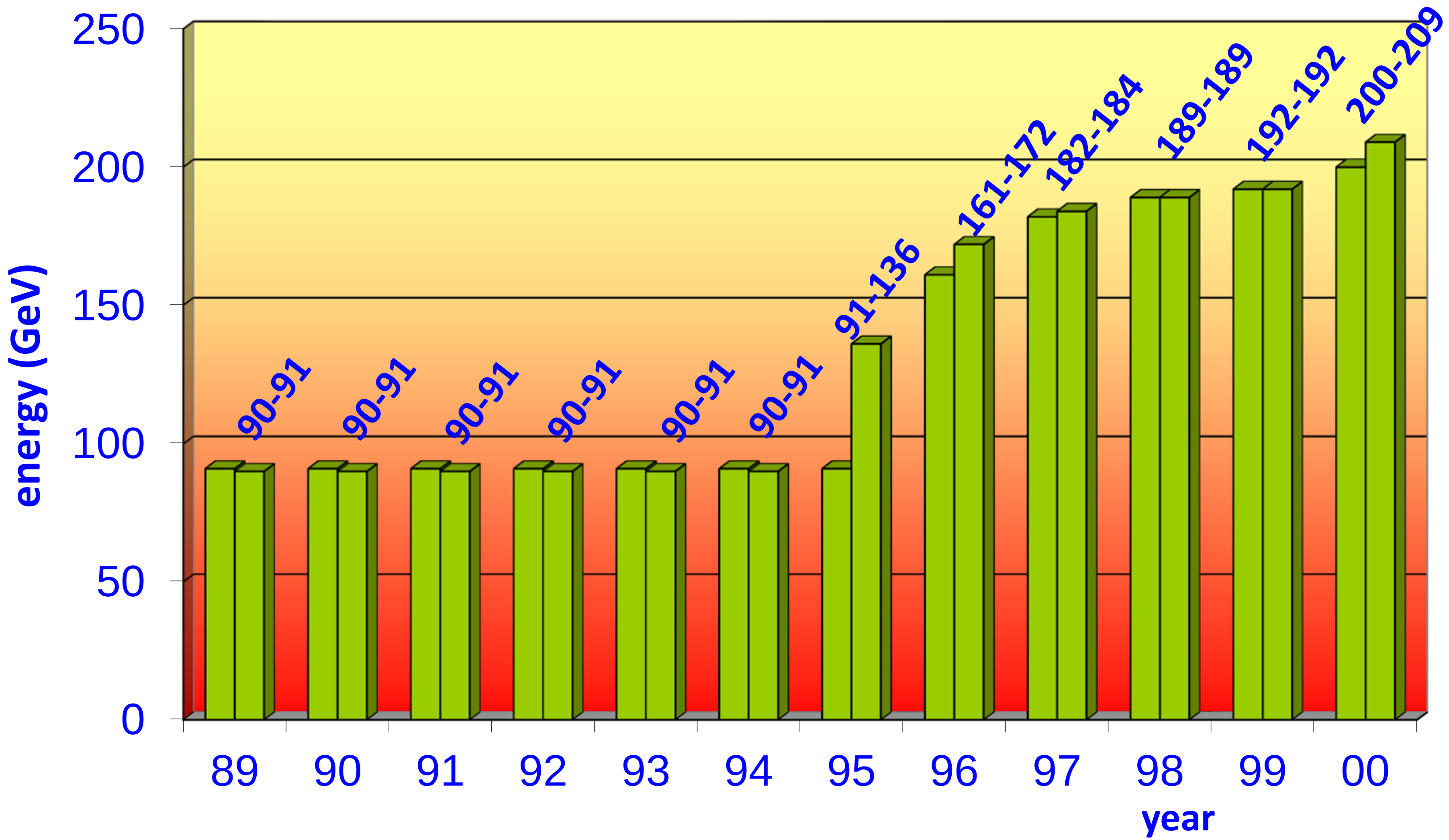
$e^{\pm}$:
- LIL (→ 200/ 600 MeV);
- EPA (600 MeV);
- PS (→ 3.5 GeV);
- SPS (→ 22 GeV);
- LEP (→ 45÷105 GeV).



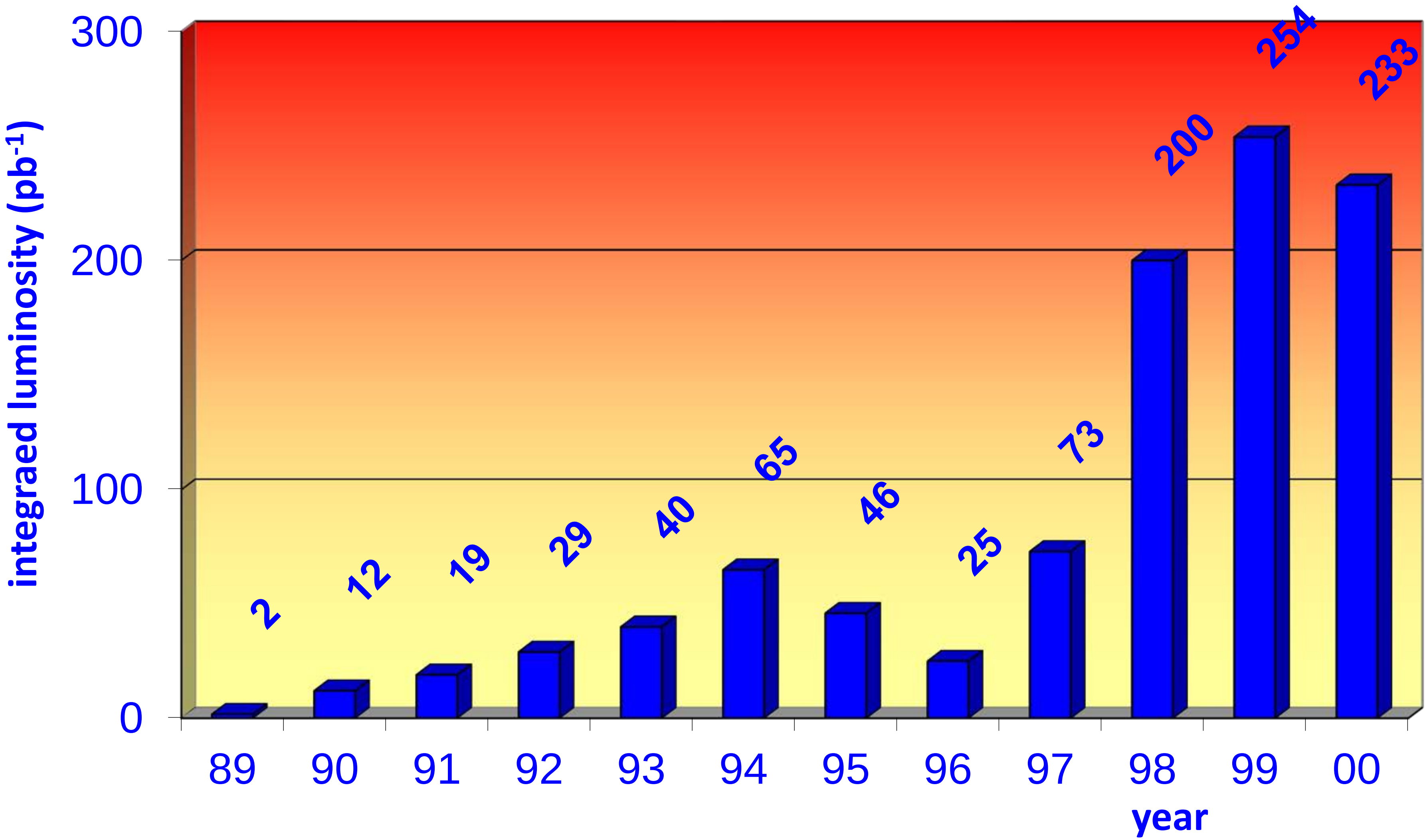
The LEP collider : parameters

	LEP 1	LEP 2
Circumference (Km)	26.66	same
E_{max} / beam (GeV)	50	105
max lumi $\mathscr{L}$ (10^{30} cm^{-2} s^{-1})	~25	~100
time between collisions (μs)	22 (11)	22
packet length (cm)	1.8	
packet radius (hori.) (μm)	200÷300	
packet radius (vert.) (μm)	2.5÷8	
injection energy (GeV)	22	same
particles/packet (10^{11})	4.5	same
packet number	4+4 (8+8)	4+4
years	1989-1995	1996-2000

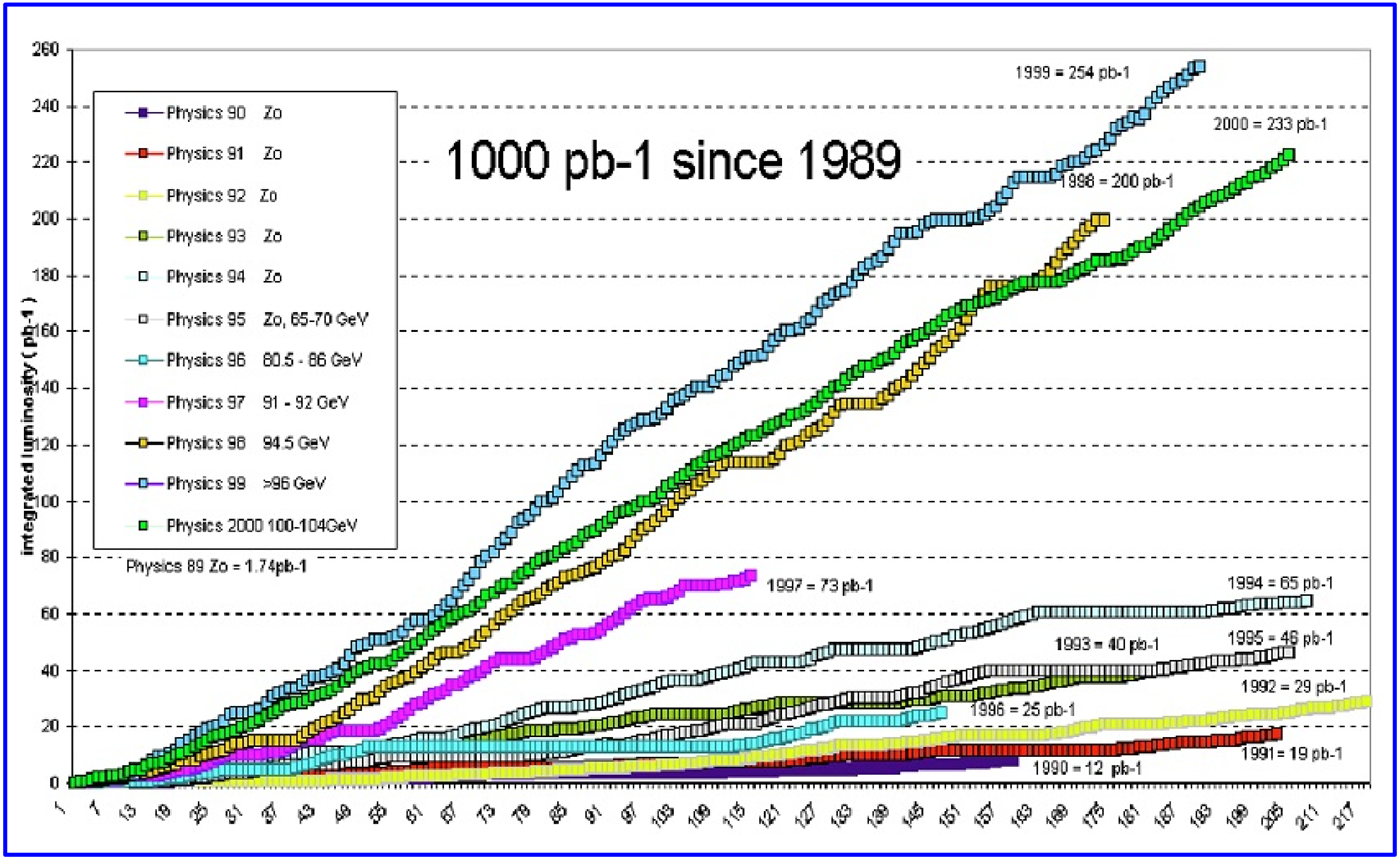
The LEP collider : √s vs year



The LEP collider : $\mathcal{L}_{integrated}$



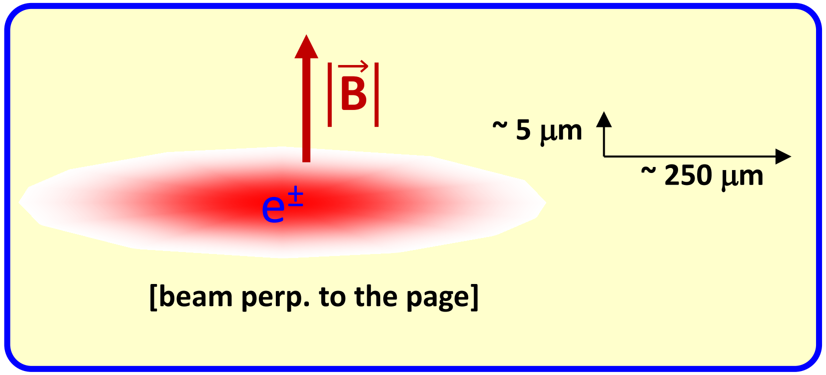
The LEP collider: $\mathcal{L}_{int}$ vs day



The LEP collider: $e^{\pm}$ brem

- $\Delta E_{orbit} \propto e^2 E^4 / (M^4 R)$; [§ 8]
- ➢ $\Delta E^{e\pm}_{orbit}$(MeV) = 8.85×10^{-5} E^4 (GeV) / R (Km);
- $\langle R_{LEP} \rangle = 4.25 \times 10^3$ m ($\rightarrow$ see table);
- in QED, the bremsstrahlung is not deterministic; the formula gives the average; a further (annoying) effect is the increase of emittance, i.e. the increase of the packets both in space and momentum; this effect is greater in the horizontal plane, as an effect of the magnetic bending:
  - ➢ σ_{hori} = 200 ÷ 300 μm;
  - ➢ σ_{vert} = 2.5 ÷ 8 μm.

E_{beam} (GeV)	$\sqrt{s}$ (GeV)	ΔE_{orbit} (GeV)
45	90	~0.1
90	180	~1.4
100	200	~2.1



The LEP collider: $\mathscr{L}$ effective

Assume $\mathscr{L}_{max} = 2\times10^{31}\text{cm}^{-2}\text{s}^{-1}$:

- $\sigma_{tot}(e^+e^- \to Z, \sqrt{s}=m_Z) \approx 40$ nb :
  - $R_{max}(e^+e^- \to Z, \sqrt{s}=m_Z) = \mathscr{L}\,\sigma_{tot} = 0.8$ Hz;
  - ~~6×10^4 events / day $\to 10^7$ events/ year;~~
  - [??? no !!!];

… because …

- the luminosity normally quoted corresponds to the "peak lumi.", i.e. the first minutes after acceleration and squeezing;

  $\mathscr{L}(t) = \mathscr{L}_{max} \exp(-t/\tau)$ (stochastic effects + optics corrections)

  $\to \langle\mathscr{L}\rangle \approx \frac{1}{2}\,\mathscr{L}_{max}$

  + techn. stops, maintenance, mistakes, …

- global efficiency ~ ¼

- also data @ $\sqrt{s} \neq m_Z$ (e.g. to measure the lineshape), where σ much smaller.

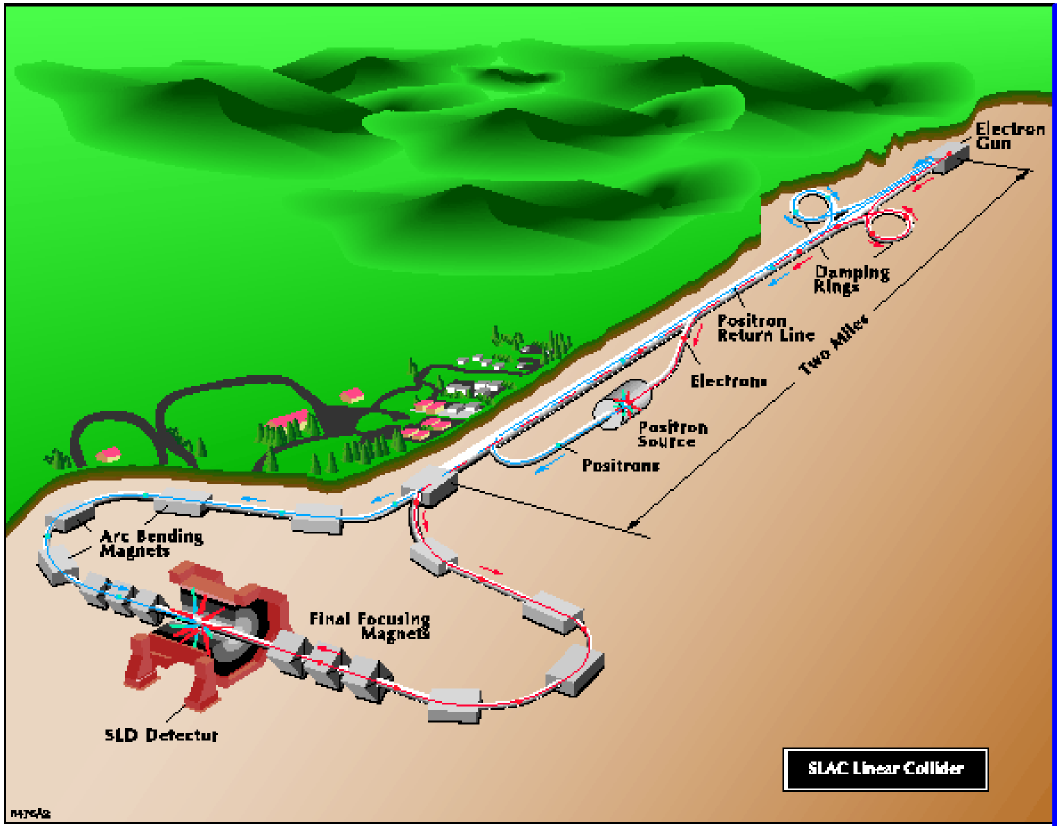
$\Rightarrow$ @ LEP 1 :

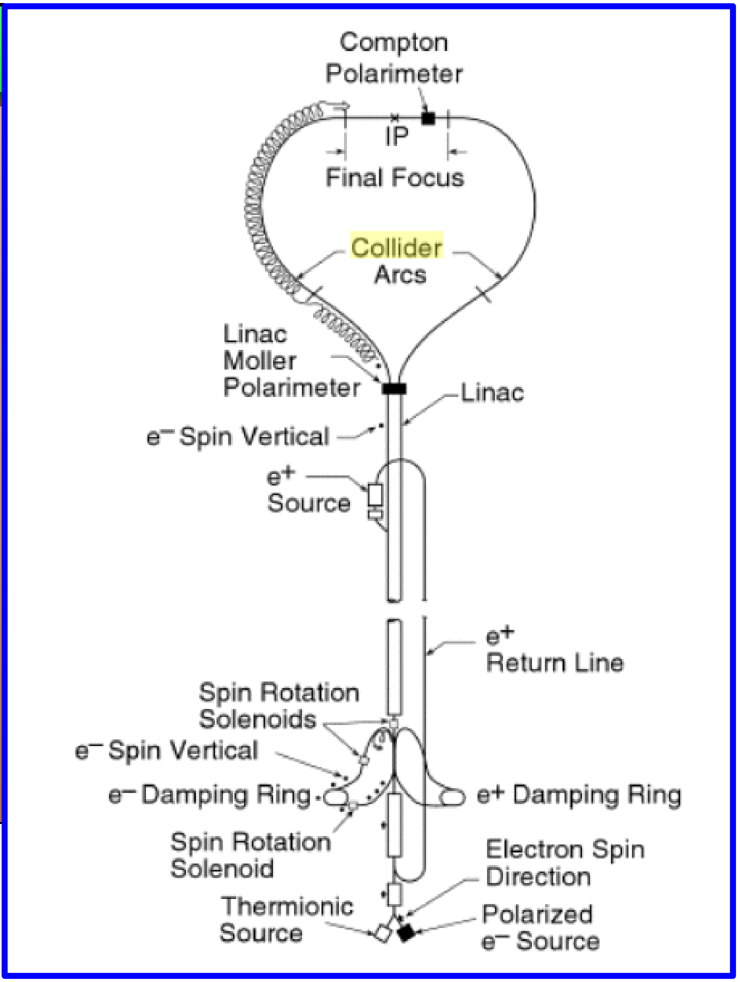
4×10^6 hadronic events $\times$ 4 exp =

= 15.5×10^6 hadronic events
+ the corresponding leptons.

Problem: use the formulæ of § 8 and the LEP parameters to compute $\mathscr{L}_{bc}$ and μ $(=\mathscr{P}_{int})$.
Comment on TDAQ requirements. Is LEP trigger/DAQ "easy" or "difficult" ?
[*please think before answering*]

The LEP collider : the competition - SLC

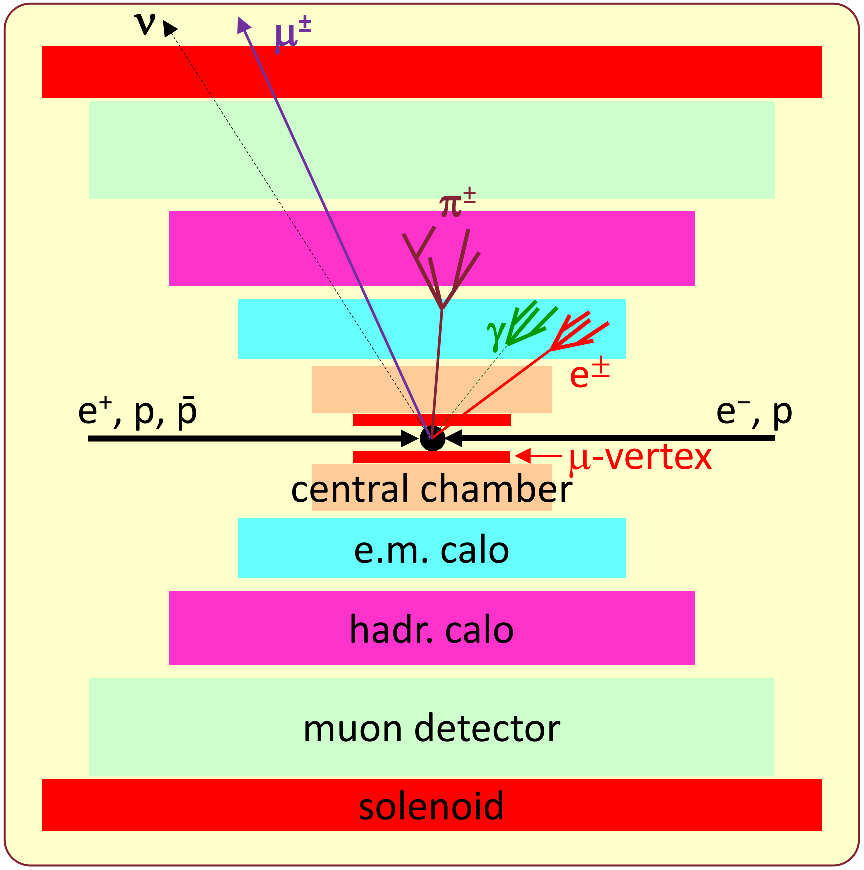


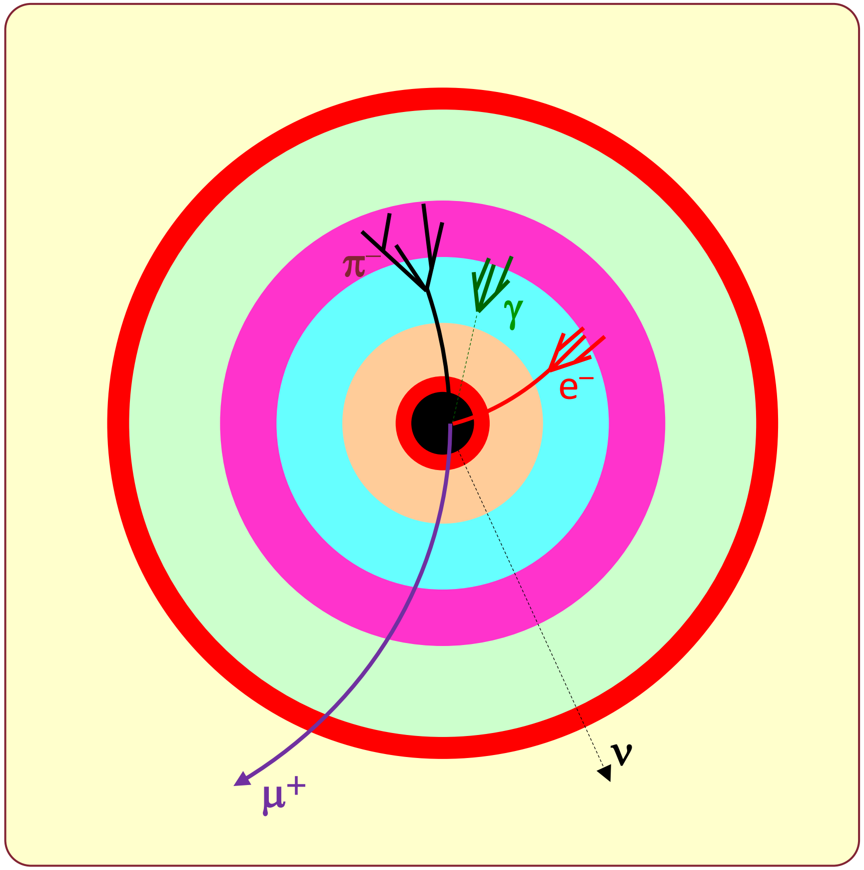


SLC : Stanford Linear Collider (1989-98):

- the first example of linear e^+e^- collider;
- lower energy (only Z pole) and less intense;
- polarized beams;
- promising new technique ($\sqrt{s}$ > 500 GeV $\rightarrow$ a circular e^+e^- requires a huge ring).

Detectors

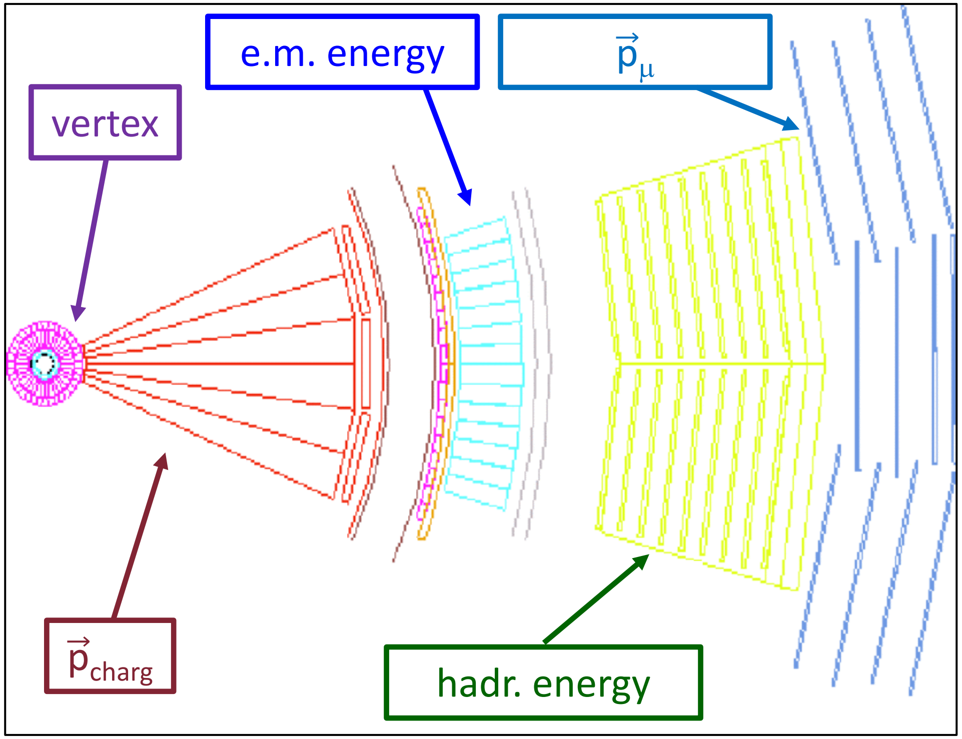




A typical detector of LEP / TeVatron / LHC (ATLAS is the only remarkable exception).

Notice both the possible measurement of E, $\vec{p}$ and the particle id. capability.

Detectors: principles



	$\vec{p}_{charg}$	E_{em}	E_h	$\vec{p}_\mu$	**sec. vtx. ?**
$e^\pm$	yes	yes	~no	no	yes
γ	no	yes	~no	no	diff.
$\pi^\pm$, $K^\pm$	yes	small	yes	no	yes
n, K^0	no	small	yes	no	diff.
$\mu^\pm$	yes	mip	mip	yes	yes
ν	no (but *hermeticity*)				

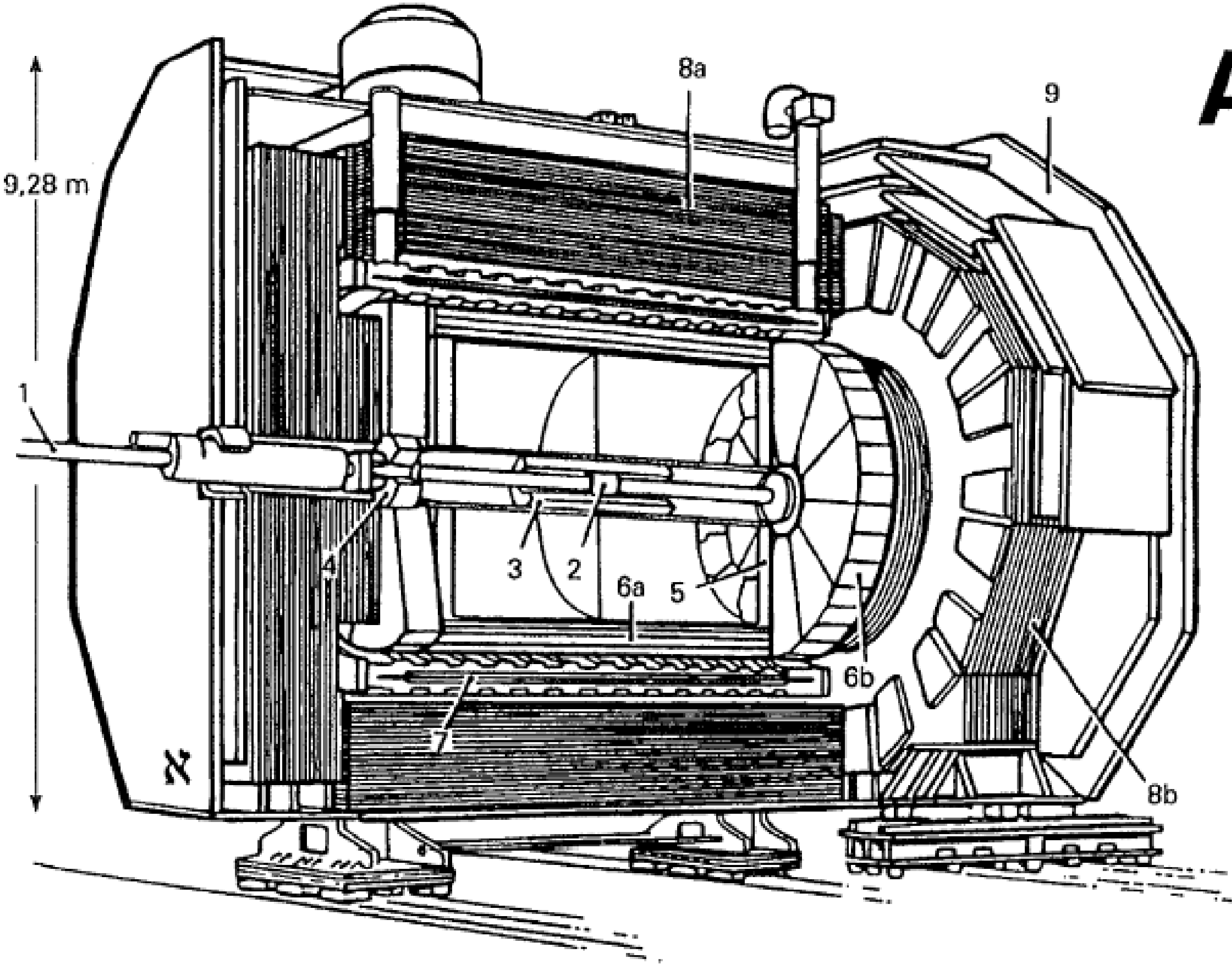
A detector fully operational allows for both the measurement of the 4-momenta of all the particles and their identification ("*part.id*"). The charge is measured by the sign of the bending.

The ν's are "detectable" from the conservation of the 4-momentum, i.e. :

$$\begin{cases} \vec{p}_\nu = -\sum_{all} \vec{p}_j; \\ E_\nu = \sqrt{s} - \sum_{all} E_j; \end{cases} \quad \left[\oplus\ m_\nu^2 = E_\nu^2 - |\vec{p}\,|_\nu^2 = 0 \right].$$

Problem : what happens if there are two ν's in the final state ?
An interesting question … and not uncommon [$Z \to \tau\tau$, $ZH \to \nu\bar{\nu}b\bar{b}$].

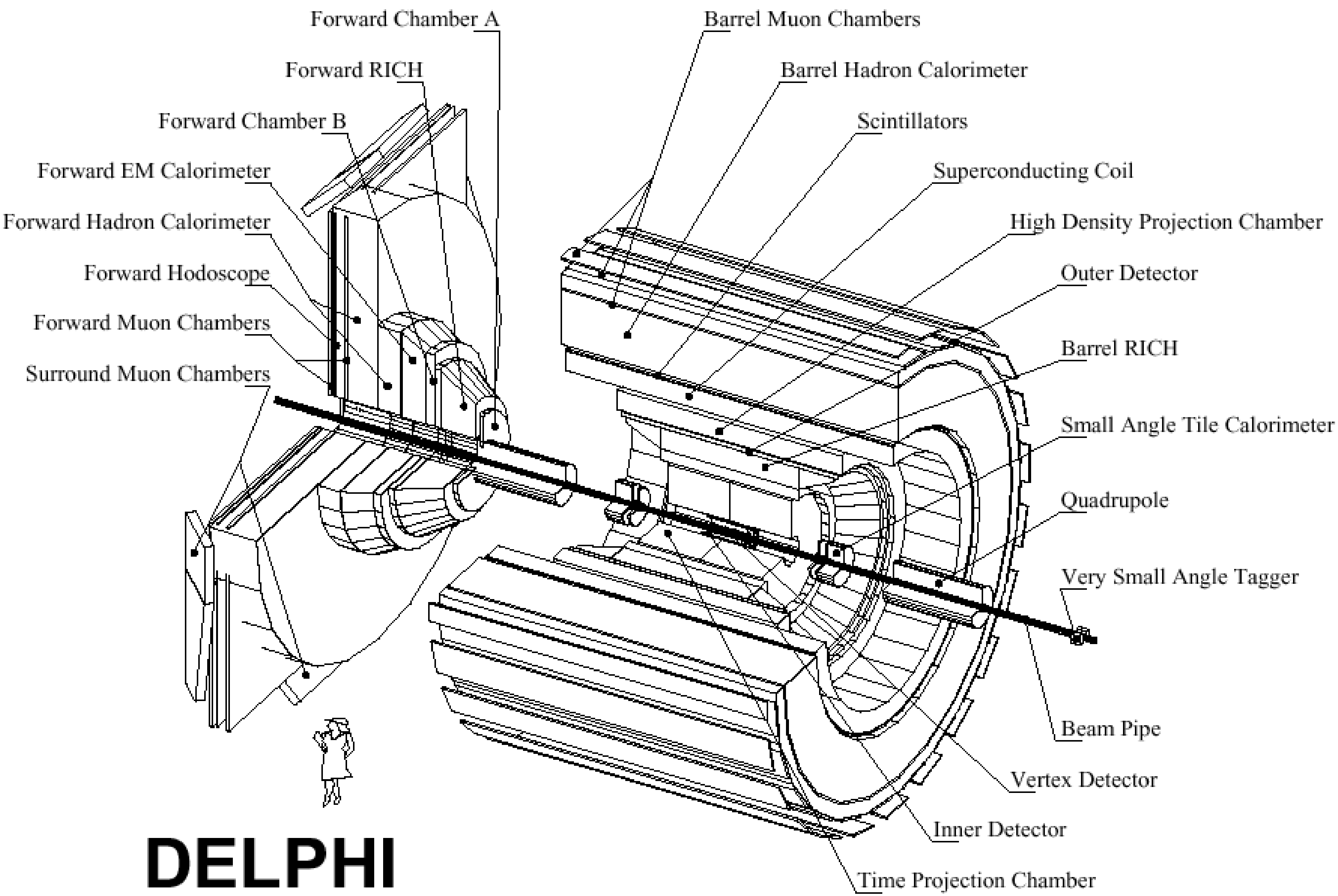
Detectors : א



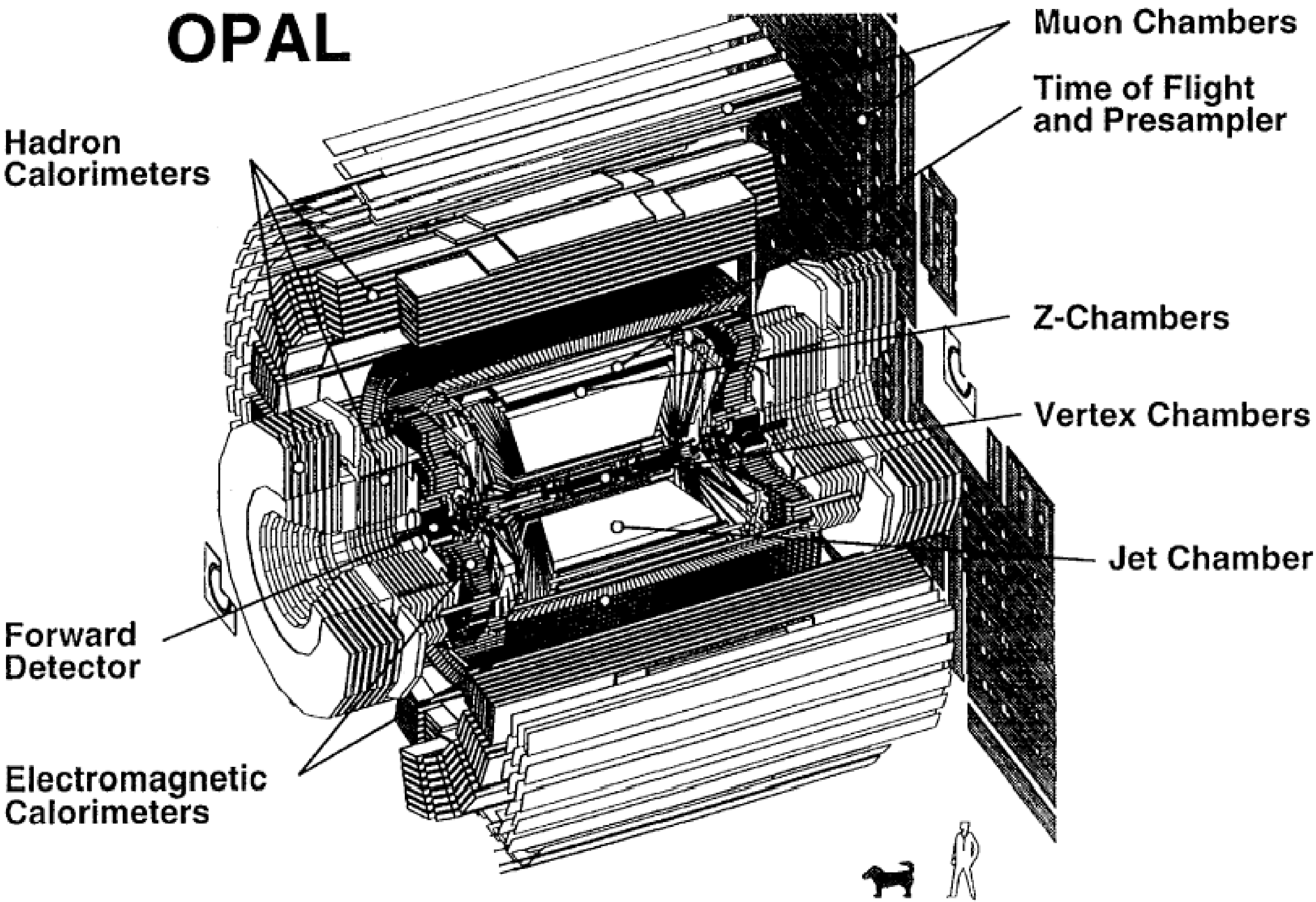
ALEPH

1. Beam Pipe
2. Silicon Vertex Detector
3. Inner Tracking Chamber
4. Luminosity Monitor
5. TPC Endplate
6. Electromagnetic Calorimeter
   - 6a Barrel
   - 6b Endcap
7. Superconducting Coil
8. Hadron Calorimeter
   - 8a Barrel
   - 8b Endcap
9. Muon Chambers

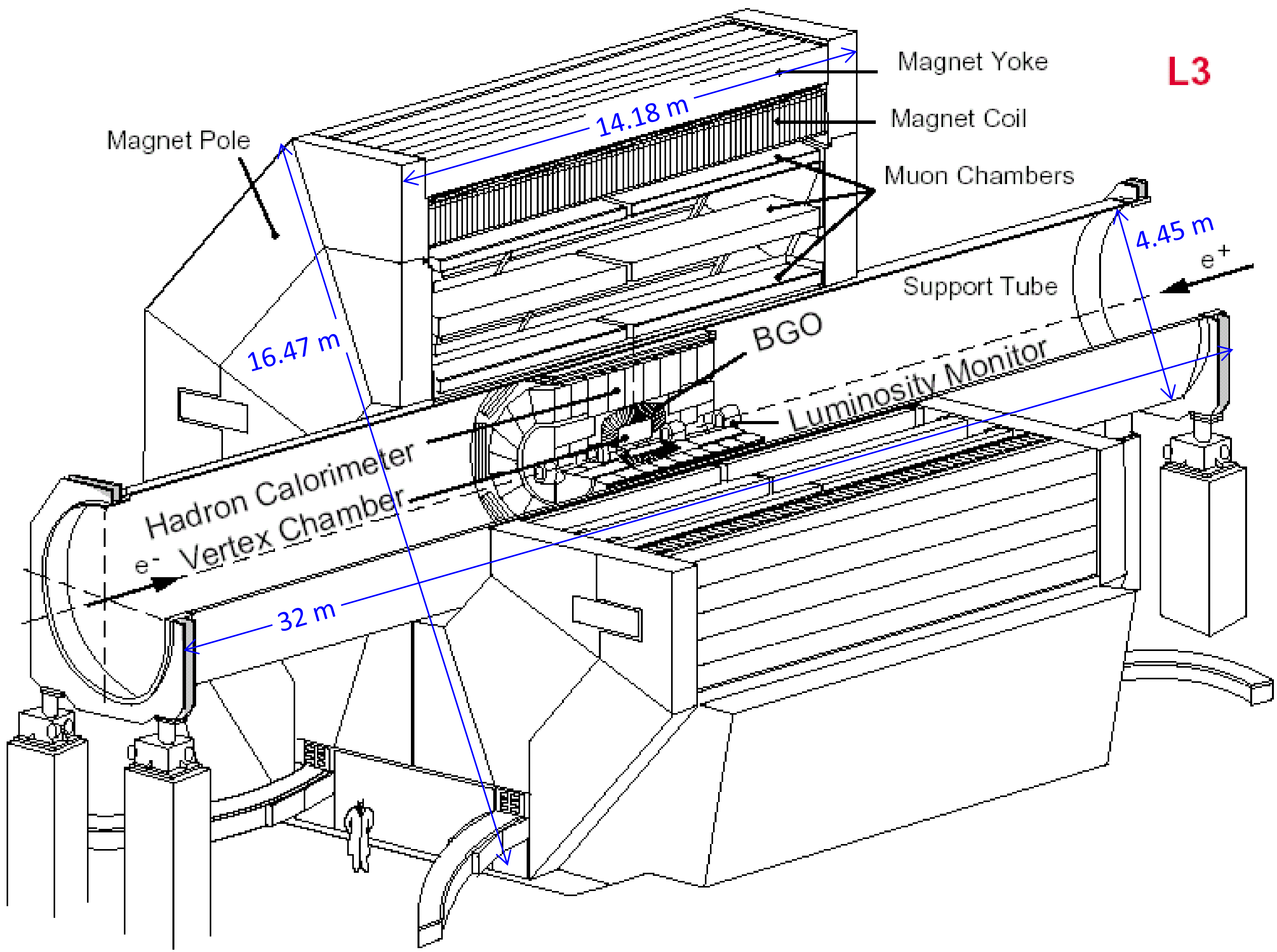
Detectors : DELPHI



Detectors : OPAL

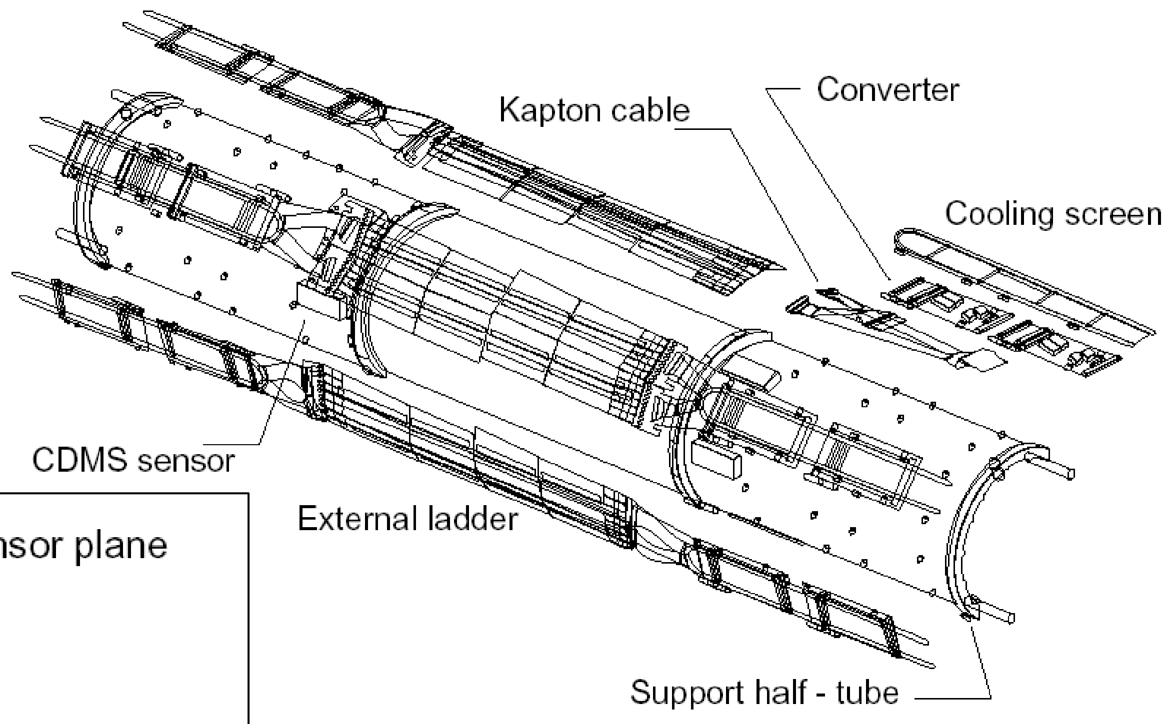


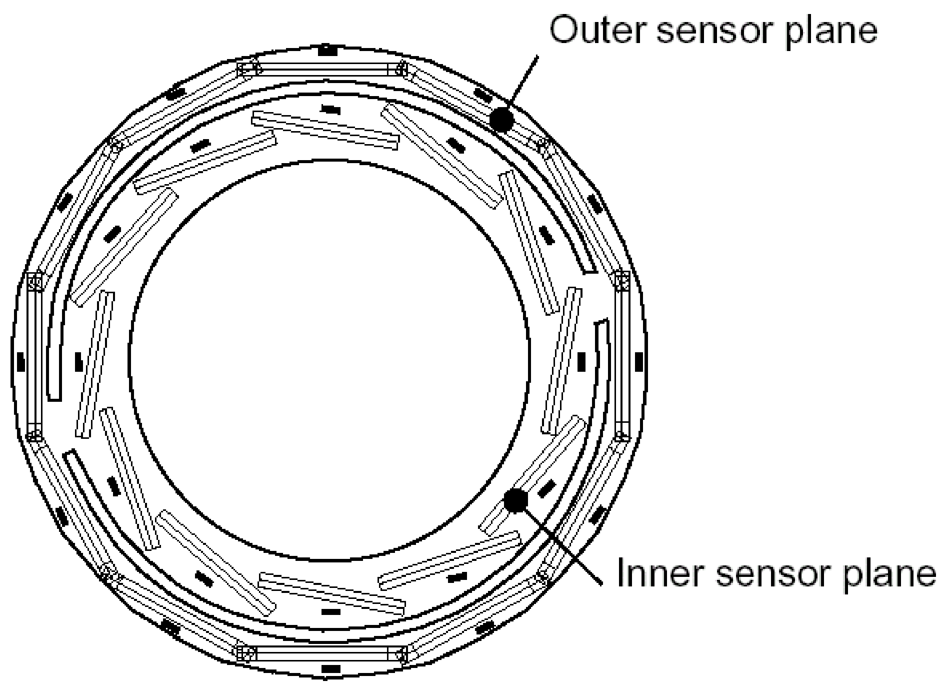
Detectors : L3



The L3 detector: SMD

- 96 silicon wafers
- 70 mm × 40 mm × 300 μm
- two layers:
  - ∅ inner layer : 120 mm
  - ∅ outer layer : 150 mm
  - zenith coverage : $|\cos\theta| < 0.93$.

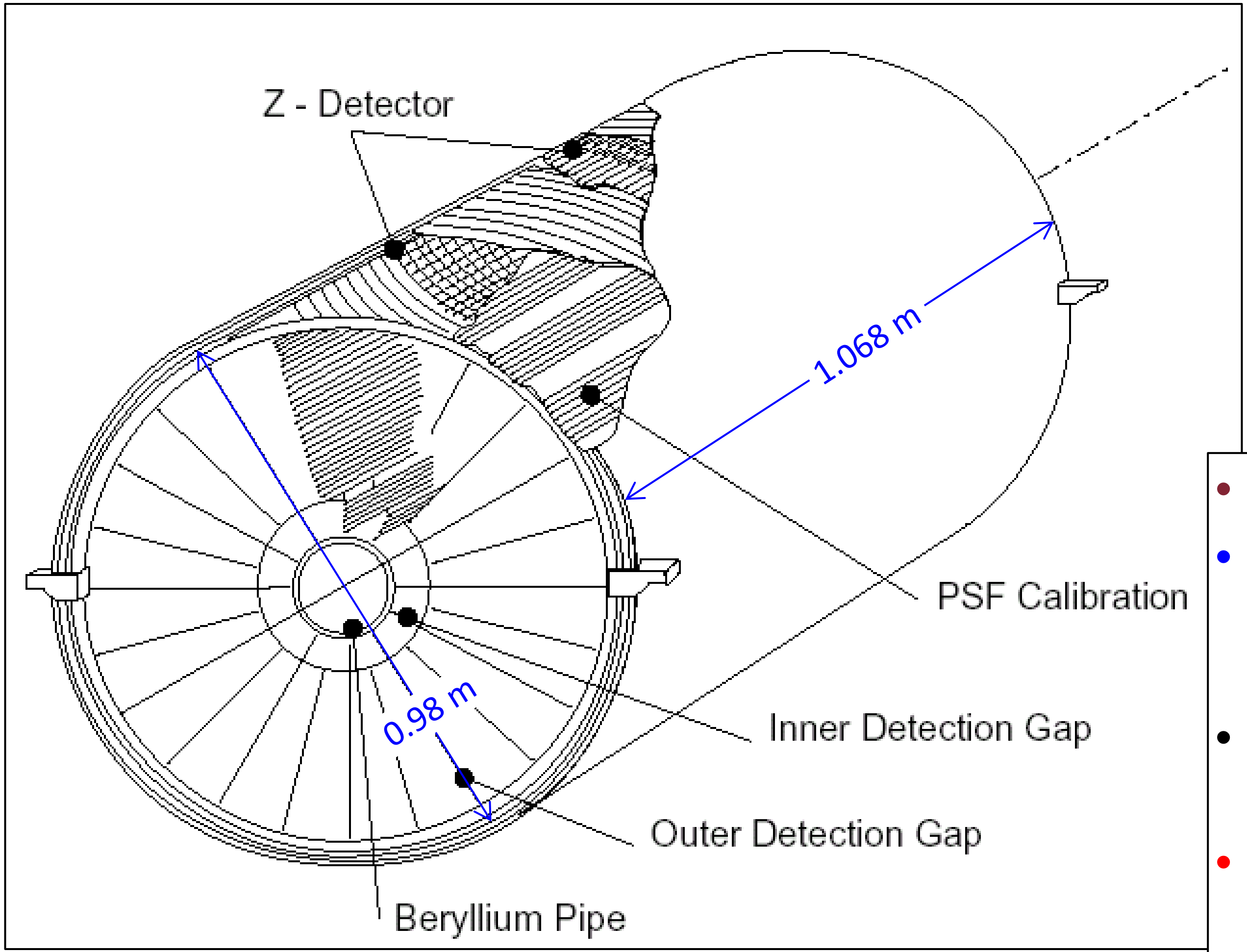




2 read outs :

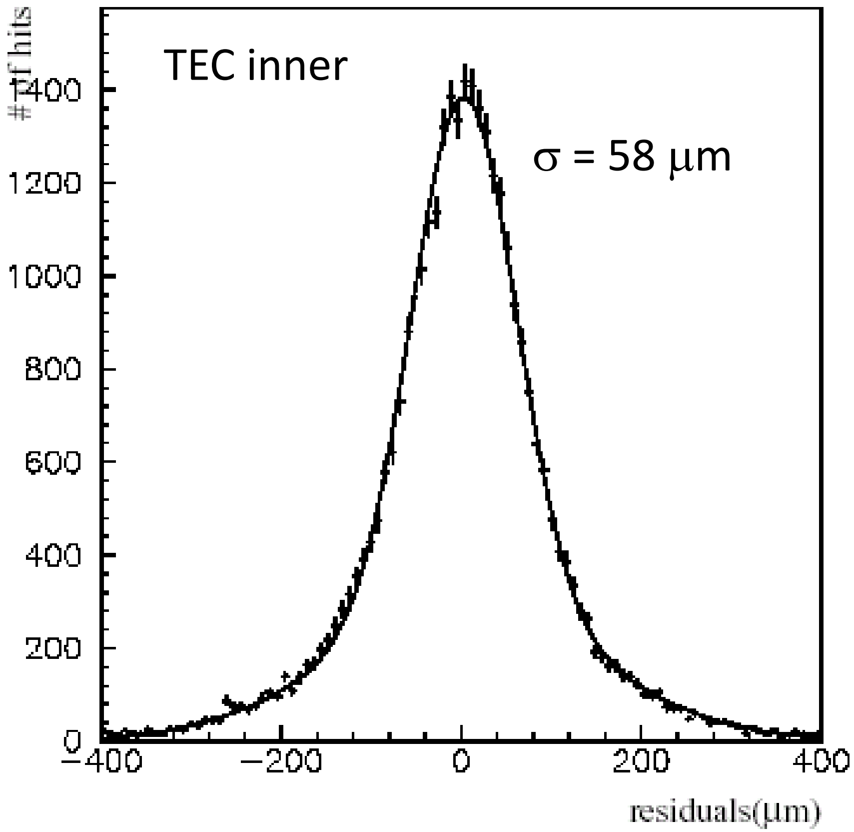
- 50 μm in $r\phi$;
- 150÷200 μm in z

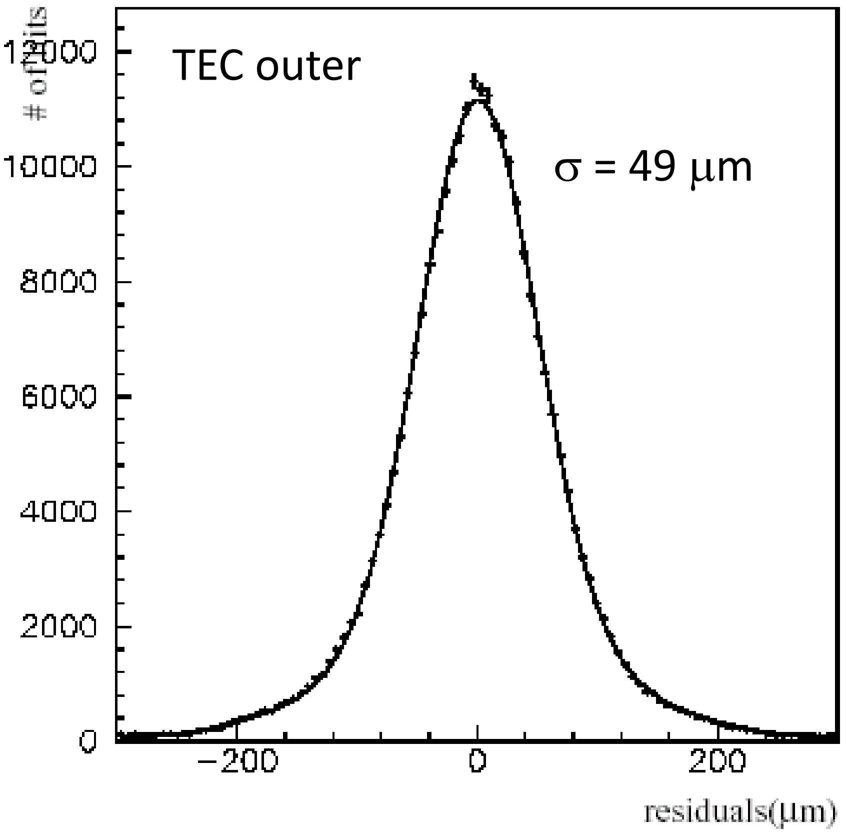
The L3 detector: TEC



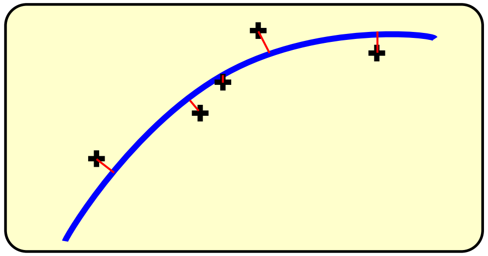
- ext. – int. radius = 317 mm;
- two separate concentrical regions : inner 8 wires + outer 54 wires;
- 80% CO_2, 20% iC_4H_{10}, 1.2 bar (abs);
- v_{drift} = 6μm / ns ("TEC" = Time *Expansion* Chamber);
- $\alpha_{Lorentz}$ = 2.3°;
- z-detector (σ= 320μm).

The L3 detector: TEC results





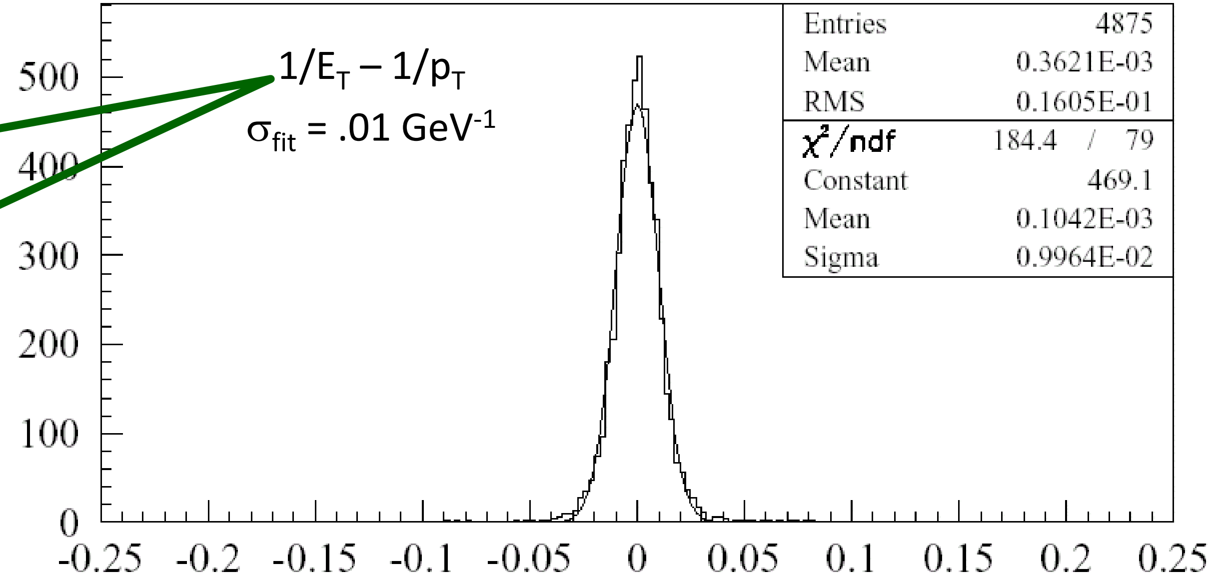
The ***residuals*** are the distances (with sign) between the measurements and the fitted trajectory. Assuming "many" measurements with the same resolution, their distribution is expected to be gaussian with mean=0 and RMS=resolution.

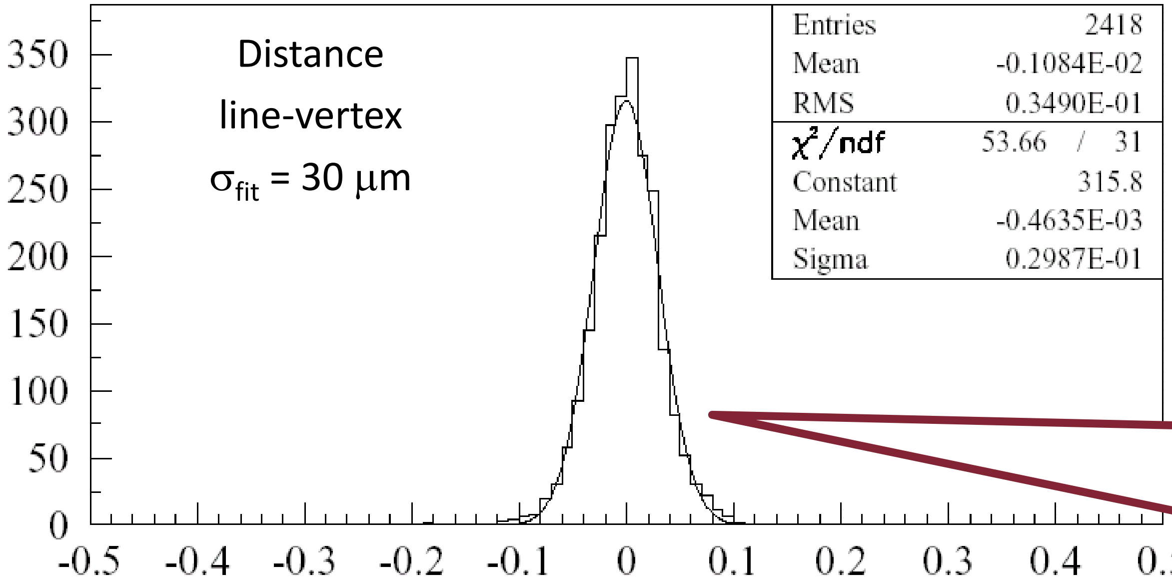


The L3 detector: SMD + TEC

Why plot (1/E – 1/p), instead of (E–p) ?

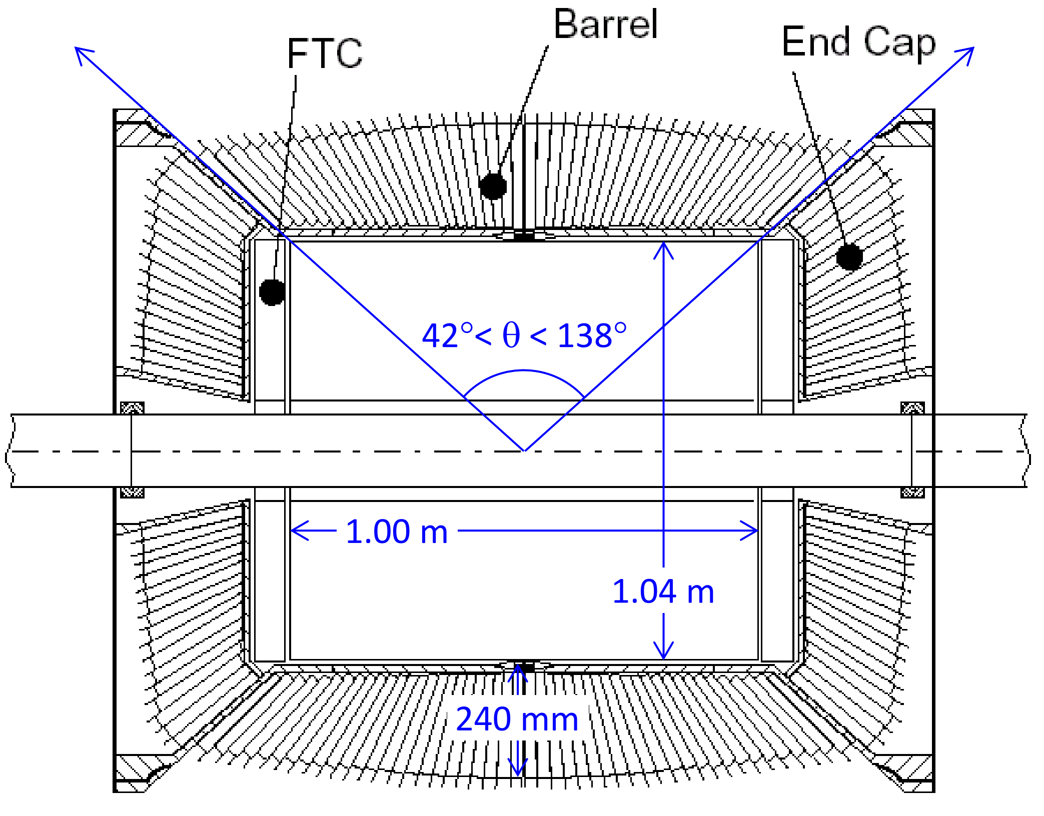
Answer in few slides, but you should be able to understand yourself.

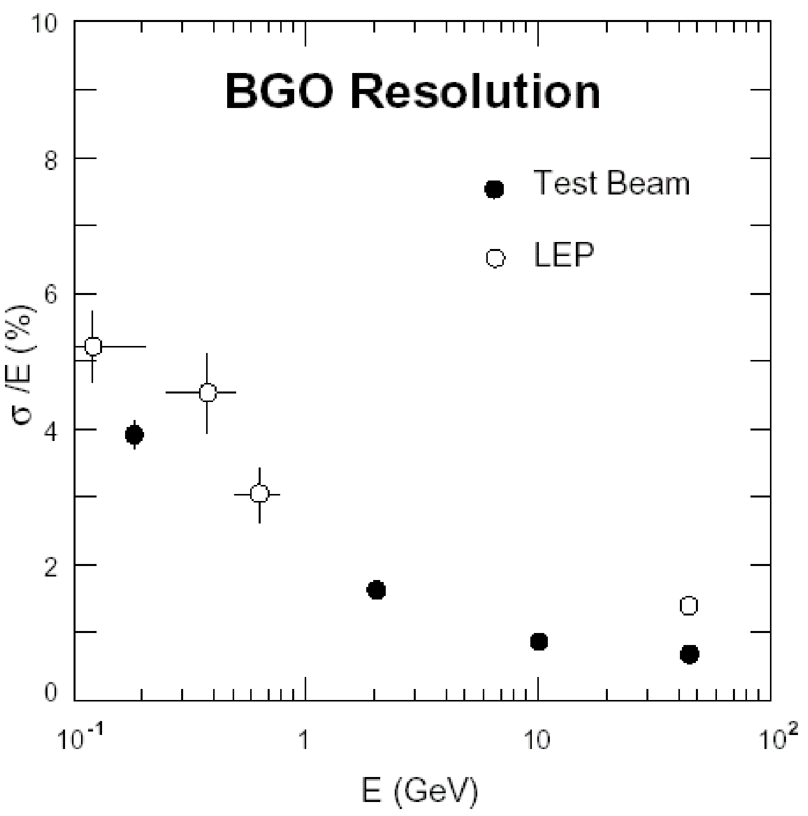




Tracks, which miss the interaction point, are a signal of secondary verteces (τ's, heavy flavors…) → the resolution on the "impact parameter" is important.

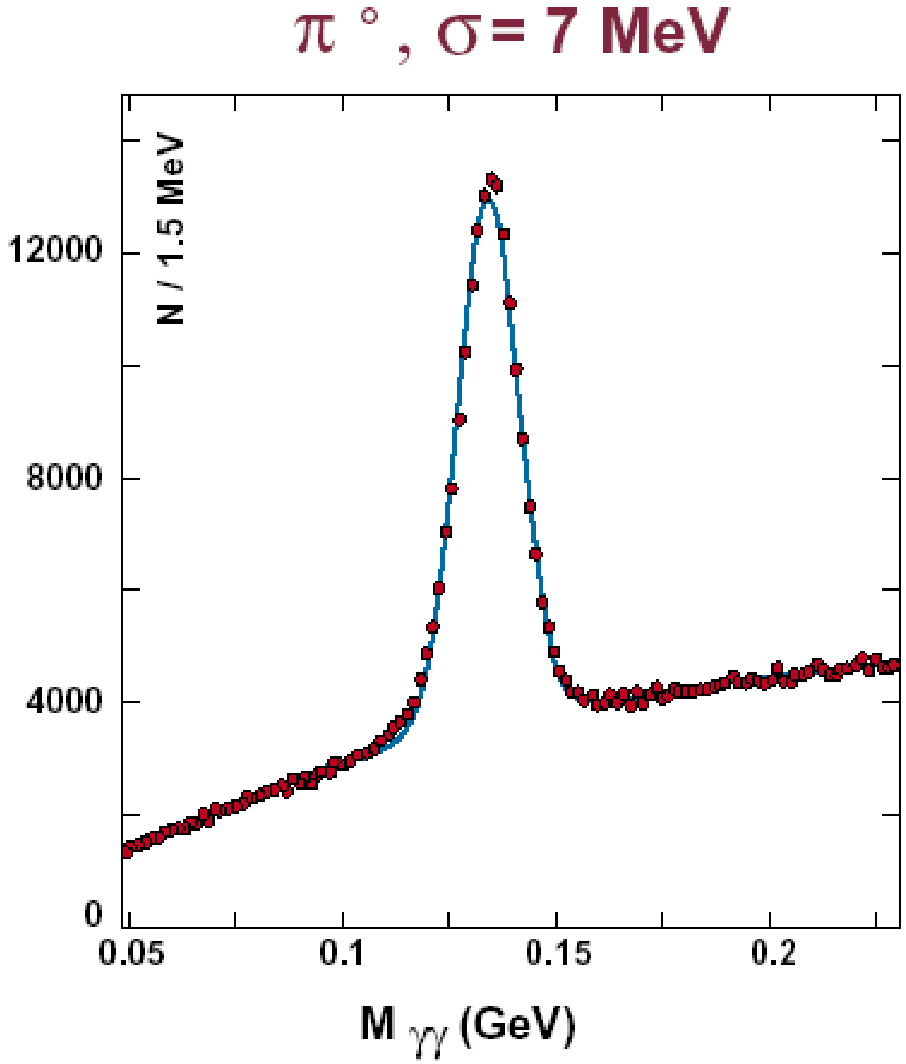
The L3 detector: BGO

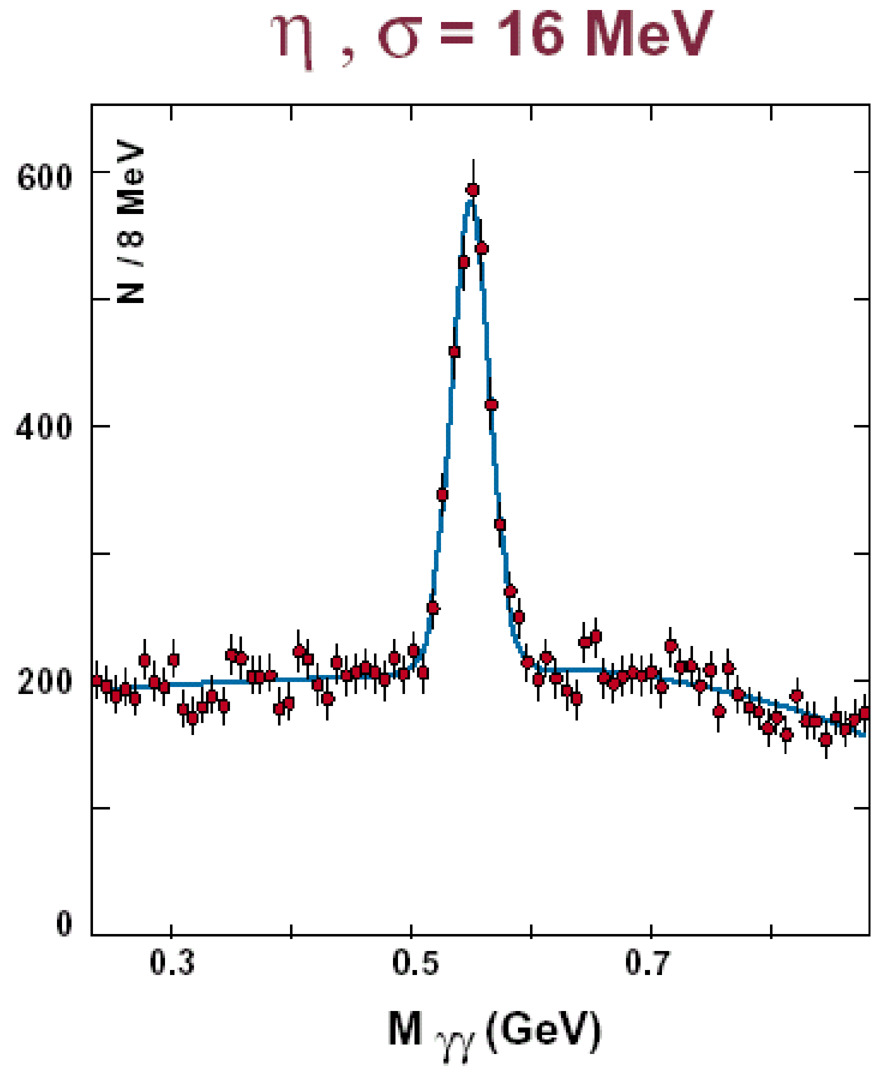




- 11,000 BGO (Bismuth germanium oxide $Bi_4 Ge_3 O_{12}$) scintillating crystals;
- pyramids 20×20 → 30×30 mm², length 240 mm;
- X_0 = 11.3 mm → 21 X_0.

The L3 detector: BGO results

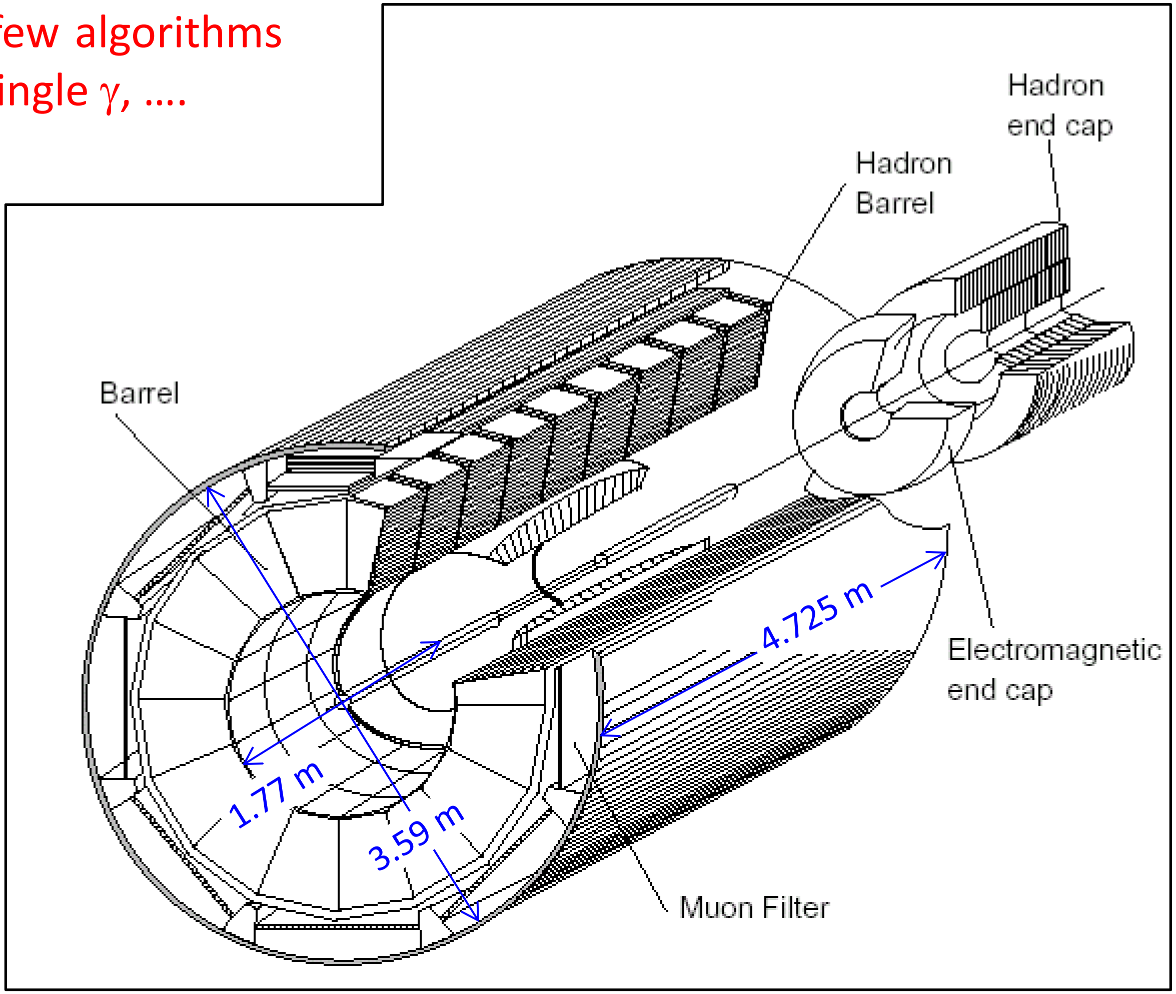




the mass resolution for particles decaying into γ's is the traditional figure of merit of the e.m. calo (true also for $H \to \gamma\gamma$ at LHC !!!).

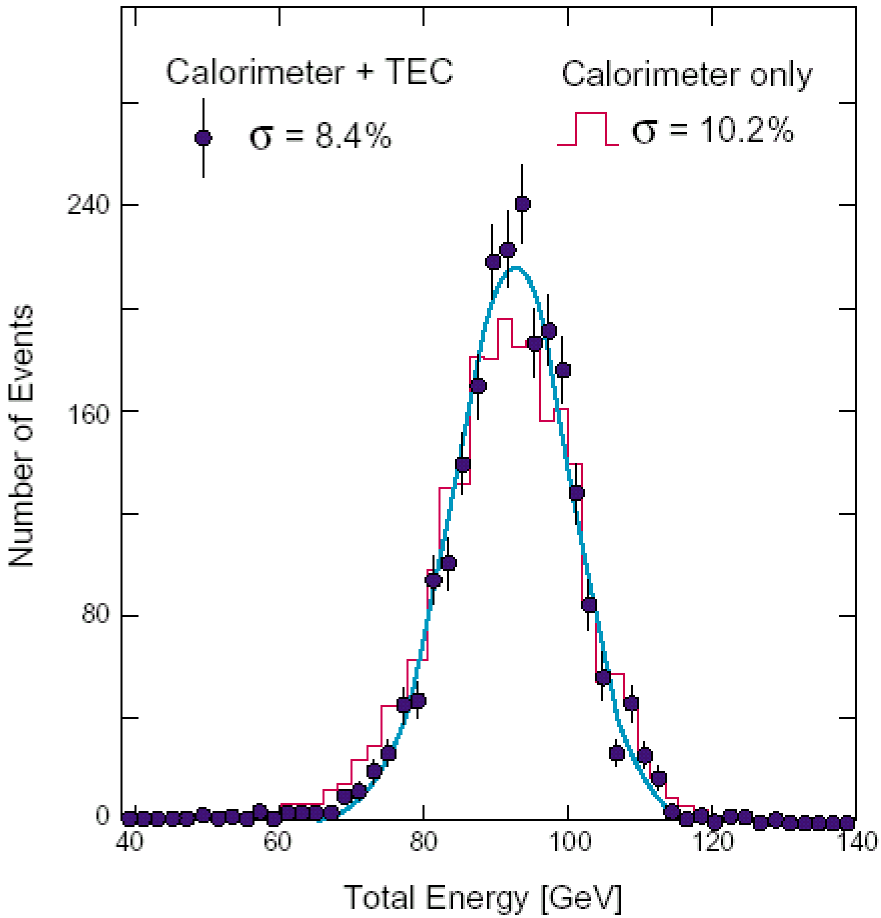
The L3 detector: HadCal

- plates of depleted U (U_{238}) + proportional wire chambers (370,000 wires);
- brass μ-filter (65%Cu, 35% Zn) + prop. tubes;
- BGO + hadcal in calo trigger (few algorithms in .OR., e.g. E_{tot}, E_{tot}^{BGO}, cluster, single γ, ….



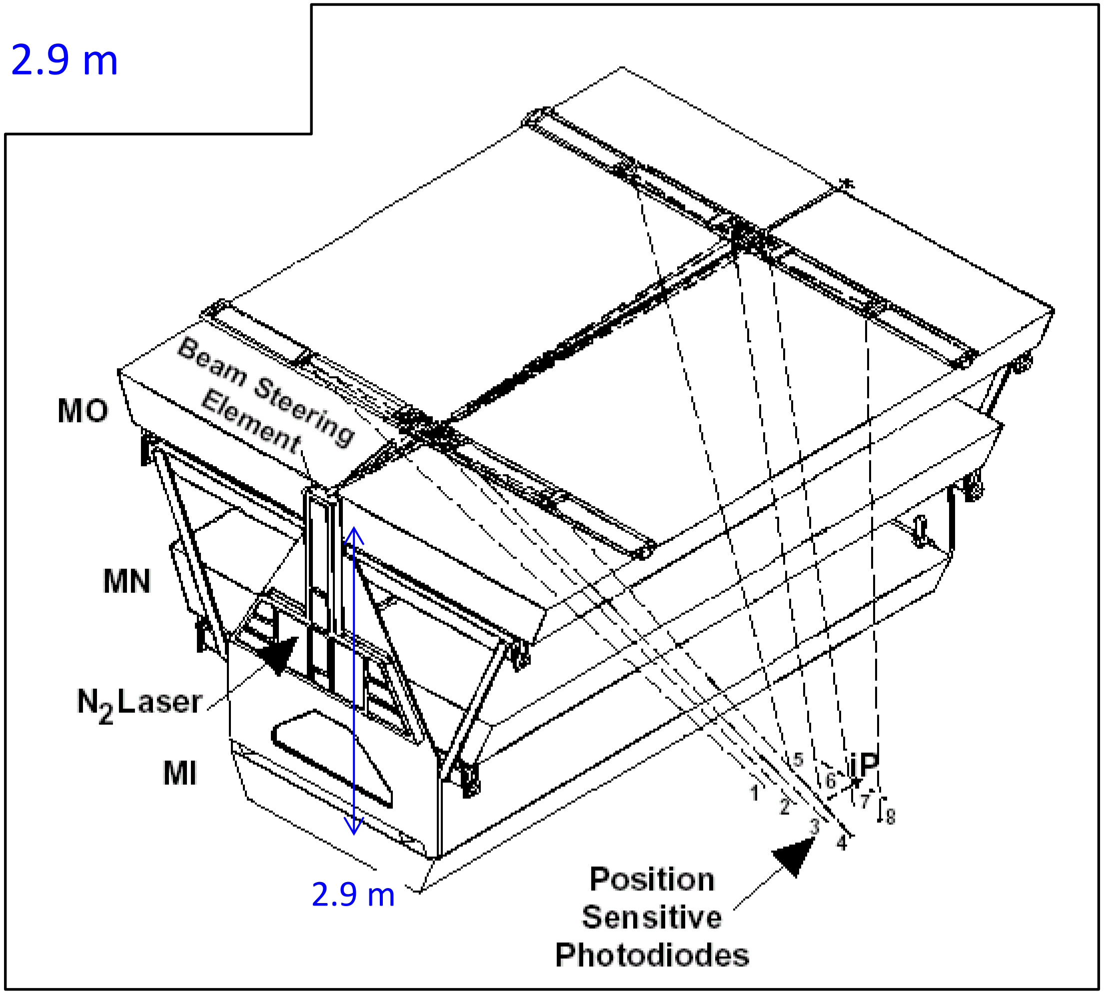
The L3 detector: HadCal results

- $Z \to q\bar{q}$ at $\sqrt{s} = m_Z$;
- E_{tot} is known and used to calibrate the detector;
- $E_{vis} / \sqrt{s} = \sum_i E_i / \sqrt{s}$ in two cases :
  - calo e.m. + had;
  - calo e.m. + had + TEC (– double-counting);
  - resolution = 10.2% with calos only;
  - resolution = 8.4% , when TEC is also used (avoiding double counting).

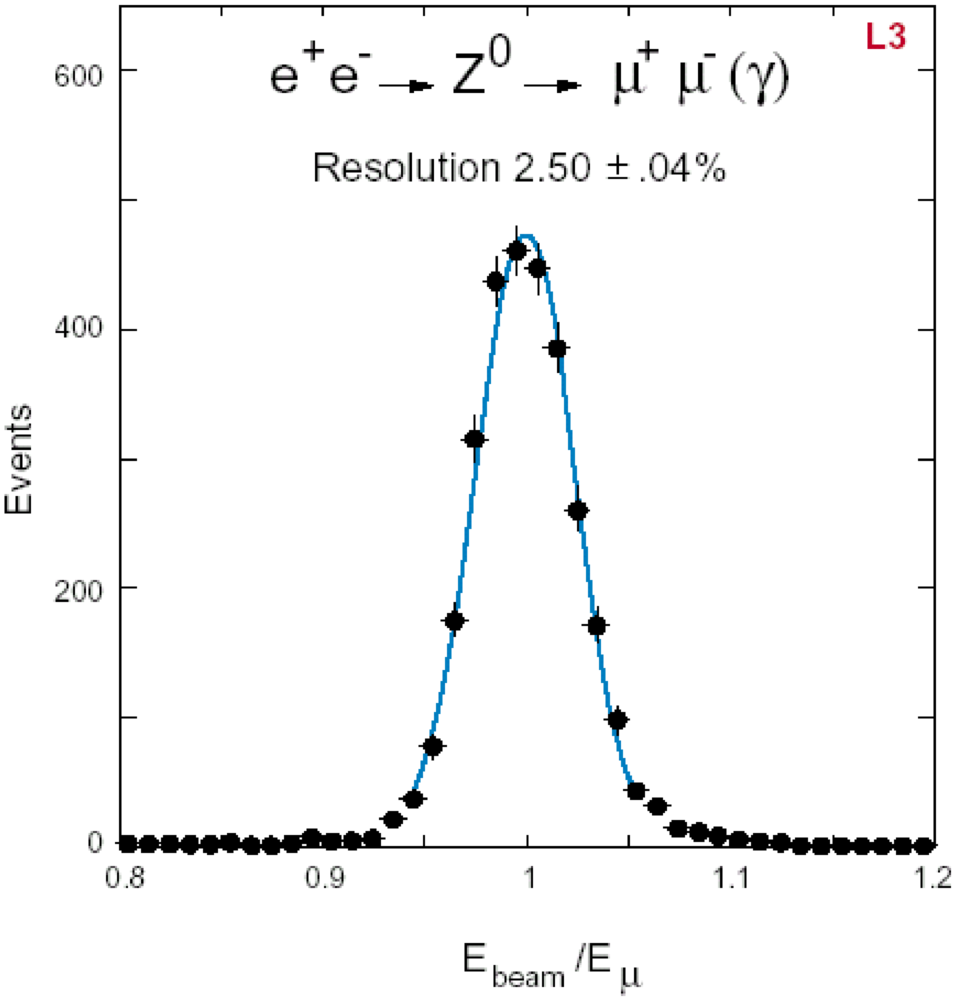


The L3 detector: μ chambers

- octants, each with three chamber types : MO + MN + MI (16 + 24 + 16 wires);
- effective length of measurement: 2.9 m
- mechanical accuracy: ~10μm;
- alignment with optical sensors.



The L3 detector: μ chambers results



Why plot E_{beam} / $E_{measured}$?

- the sagitta ($\propto$ 1/p) is the measured parameter;
- therefore 1/p expected gaussian, while p is strongly asymmetric in the tails;
- $E_{beam} / E_{\mu} = \sqrt{s} / (2\, p_{\mu})$;
- $\sigma(m_Z)/m_Z = \sigma\, [E_{beam} / E_{\mu}] / \sqrt{2}$.

For Z events, error from the machine, i.e. $\sigma(m_Z) = \sigma(\sqrt{s})$ = few MeV.

This method is used to check $\vec{p}_{\mu}$, which is used in other channels (e.g. Higgs search).

And why $(1/E - 1/p)$, or $(1/E_T - 1/p_T)$?

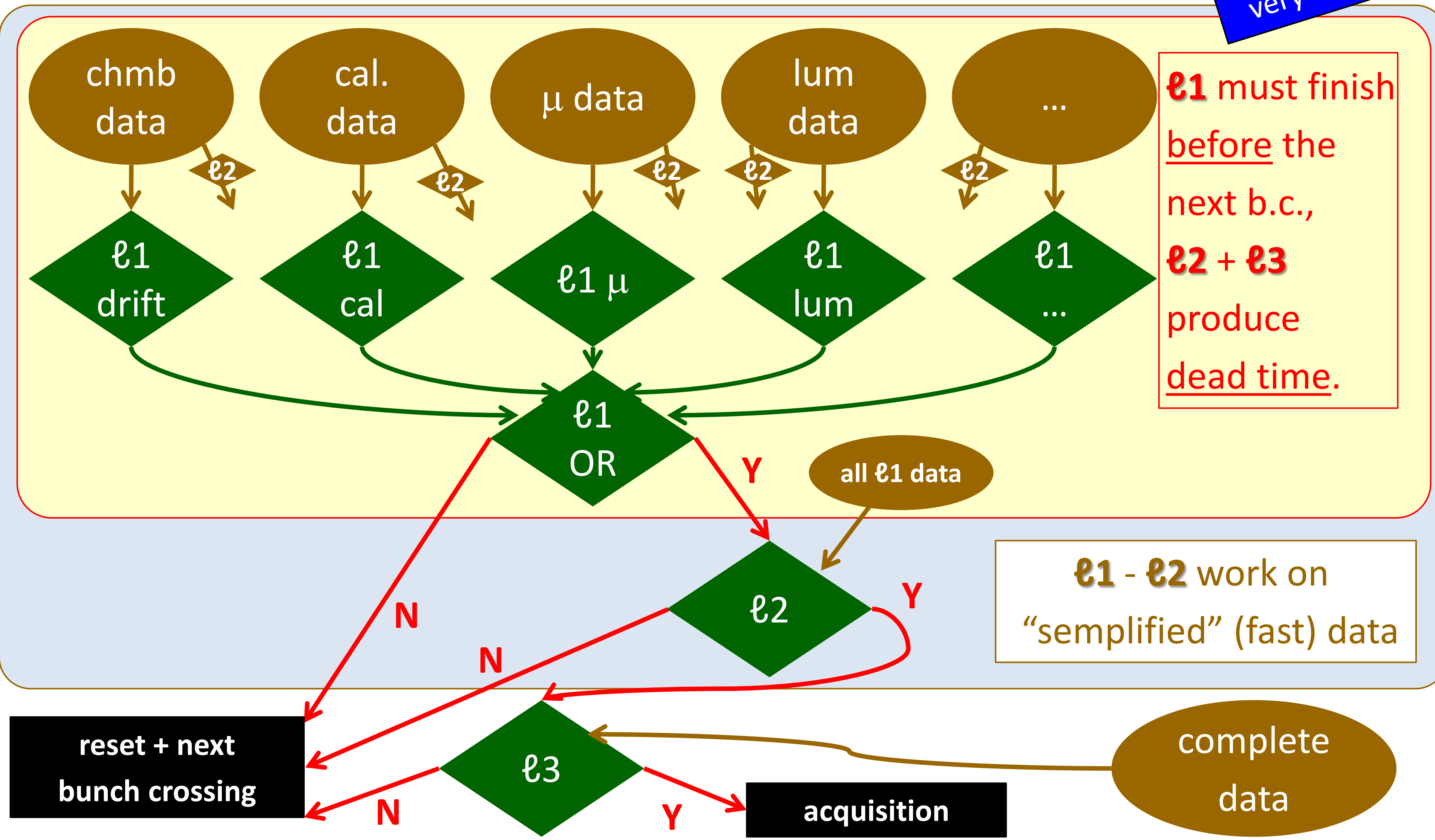
Similar, but more elaborated.

E (and E_T) comes from a calo, so it is normal, while p (and p_T) comes from a spectrometer, so it is normal in 1/p.

Plot $(E - p)$ if $\sigma(E) >> \sigma(p)$, but $(1/E - 1/p)$ if $\sigma(p) >> \sigma(E)$.

The L3 detector: trigger / DAQ

other exp.'s very similar



ℓ1 must finish before the next b.c., **ℓ2** + **ℓ3** produce dead time.

ℓ1 - **ℓ2** work on "semplified" (fast) data

The L3 detector: trigger requirements

- crossing @ 44/88 KHz $\leftrightarrow$ physics $\leq$ 1 Hz, i.e. "μ" $\approx 10^{-4} \div 10^{-5}$;
- event trigger (no selection on process type, unlike LHC);
- 3 levels of trigger;
- 1st level: simplified readout (e.g. faster ADC less precise), logical OR among:
  - TEC (e.g. 2 opposite tracks);
  - μ (at least one candidate);
  - …
  - energy (see next slides);
- 2nd level: same data as 1st lvl, but combine different detectors (e.g. a track + corresponding calo deposit);
- 3rd level: final data.

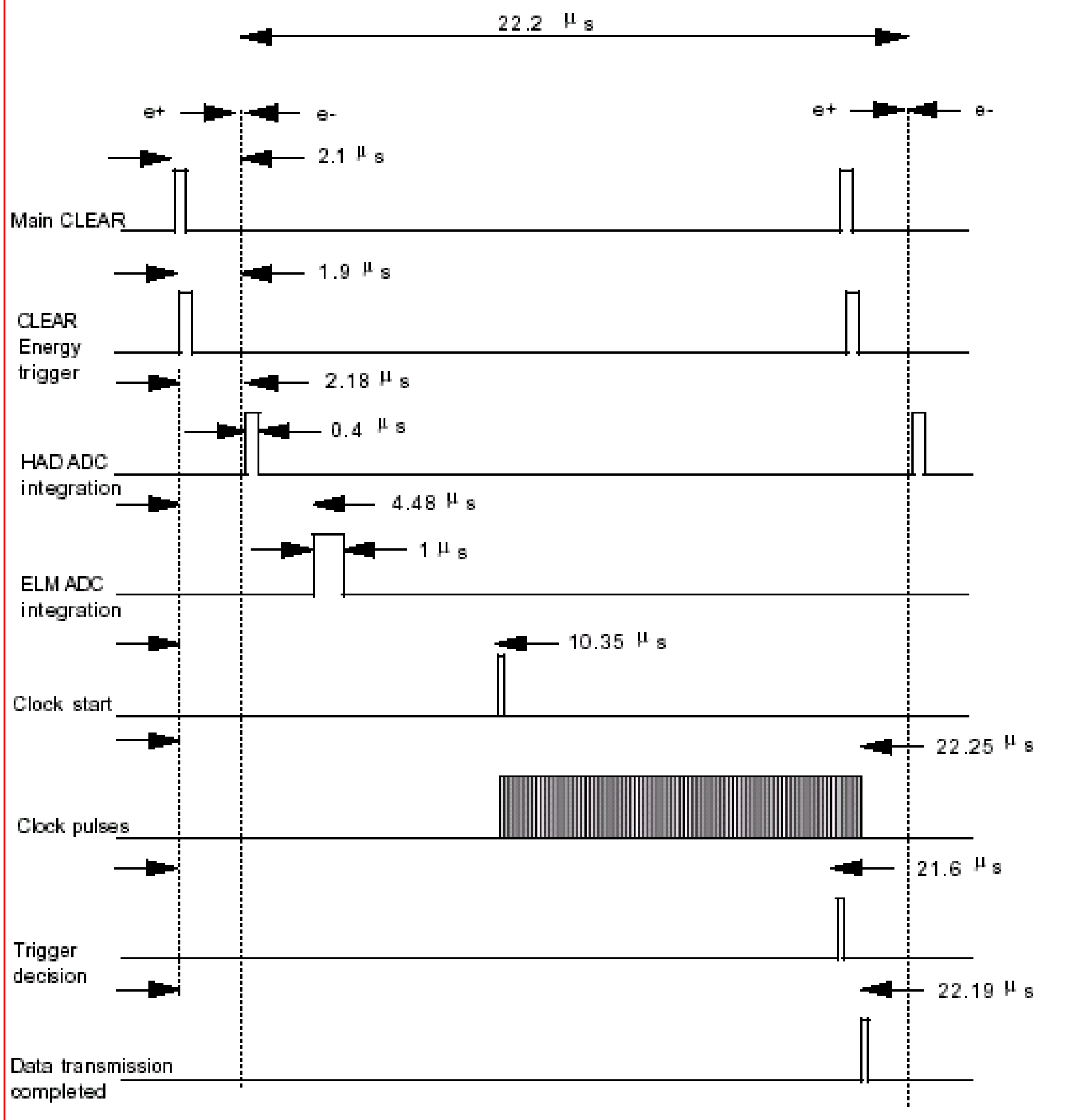
- fake triggers sources (~10÷20 Hz at 1st level) :
  - electronic noise;
  - beam halo + "beam-gas" interactions , brem photons, …;
  - cosmics, …;
- 1st level is cabled + home-made processors [*home : THIS building*];
- 2nd level: (quasi-)commercial processor;
- 3rd level: standard computer (vax-station at the time, today would use pc server + LINUX).

$\rightarrow$ inefficiency $\leq 10^{-3}$ for $Z \rightarrow e^+e^-$, $\mu^+\mu^-$, hadrons;

$\rightarrow$ dead time $\approx$ 5%.

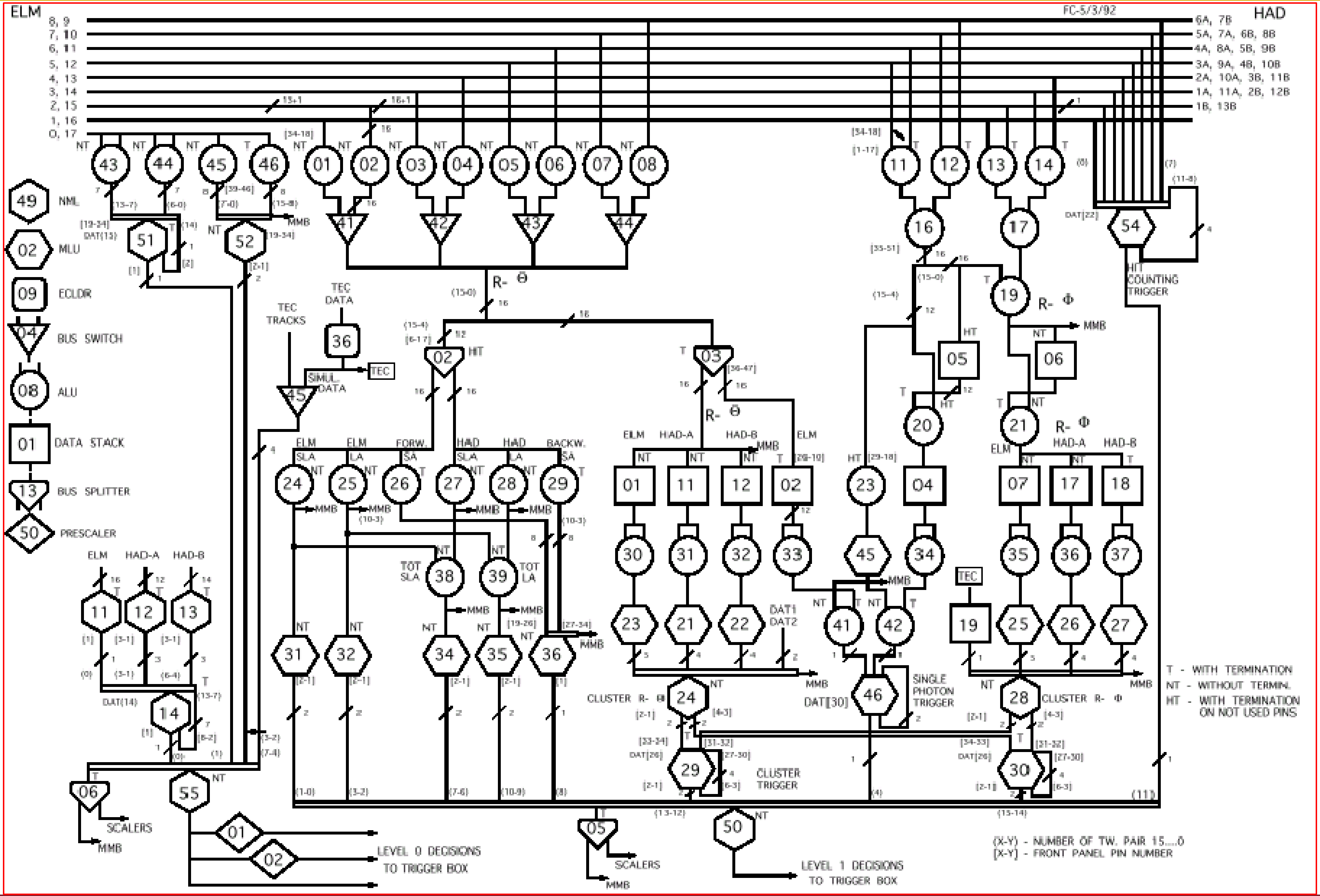
The L3 detector: energy trigger

- Roma : 1989-2000;
- CAMAC(*) processor, built by "Sezione INFN" (this building, ground floor);
- fast digitization of calo signals;
- decision algorithm based on a digital programmable processor, realized with logic and arithmetic units;
- ~200 CAMAC modules;
- decision in ~22 μs →



(*) CAMAC was an electronic standard, widely used in the '70s – 90's, now almost completely replaced by VME and other systems.

The L3 detector: energy trigger scheme



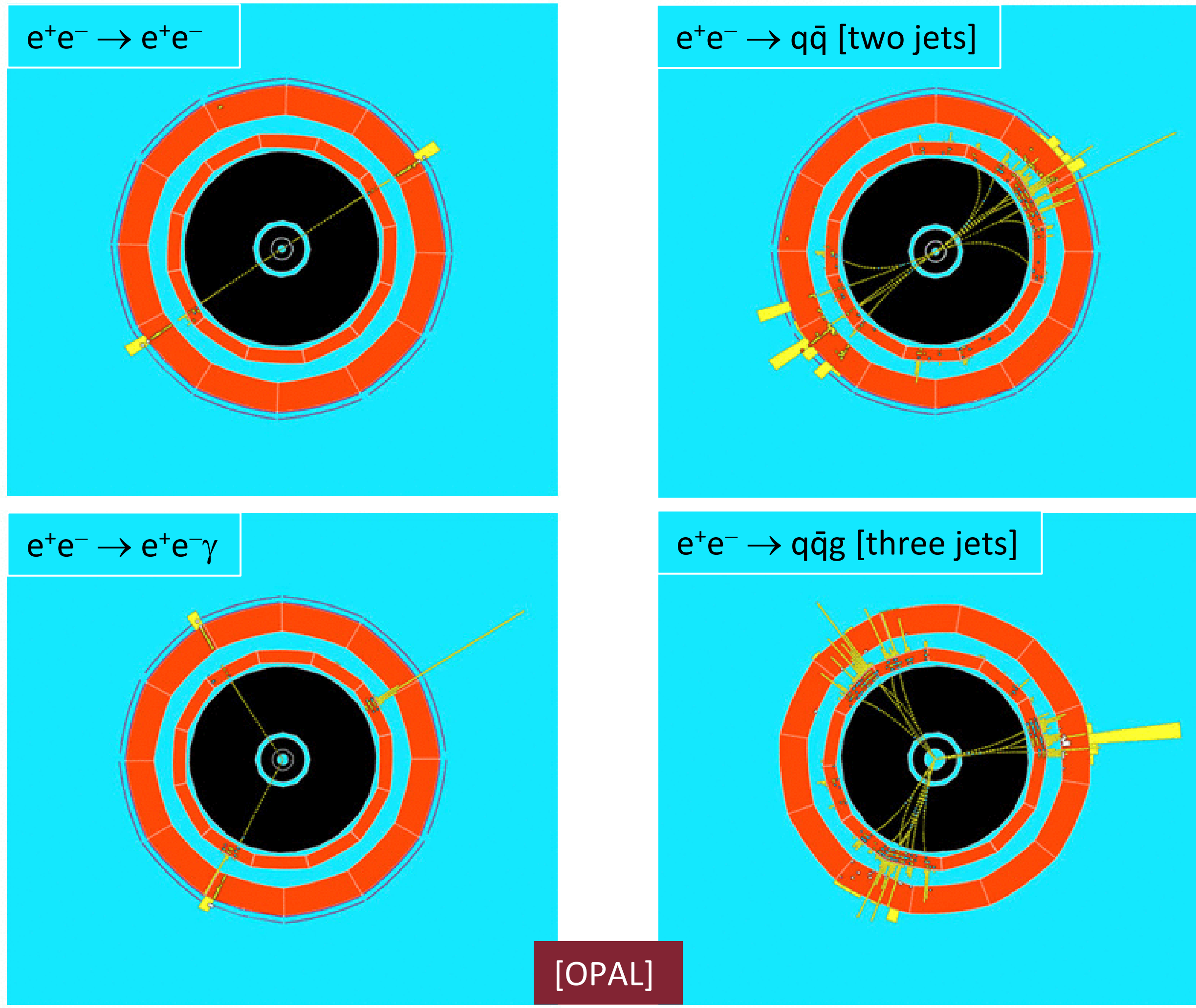
LEP events

The e^+e^- initial state produces very clean events (parton system = CM system = laboratory, no spectators).

In these four LEP events the beams are perpendicular to the page.

The recognition of the events is really simple, also for non-experts.

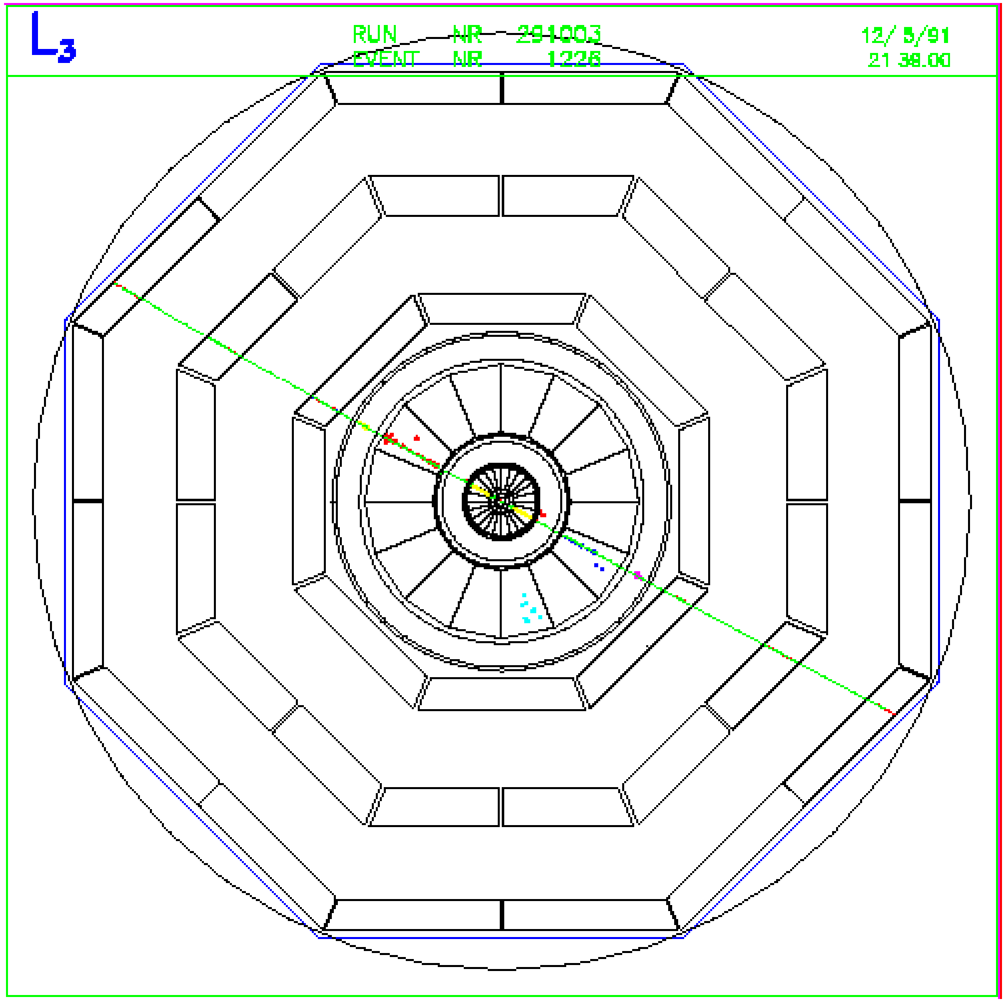
Great machines for high precision physics …



LEP events: $\mu^+\mu^-$

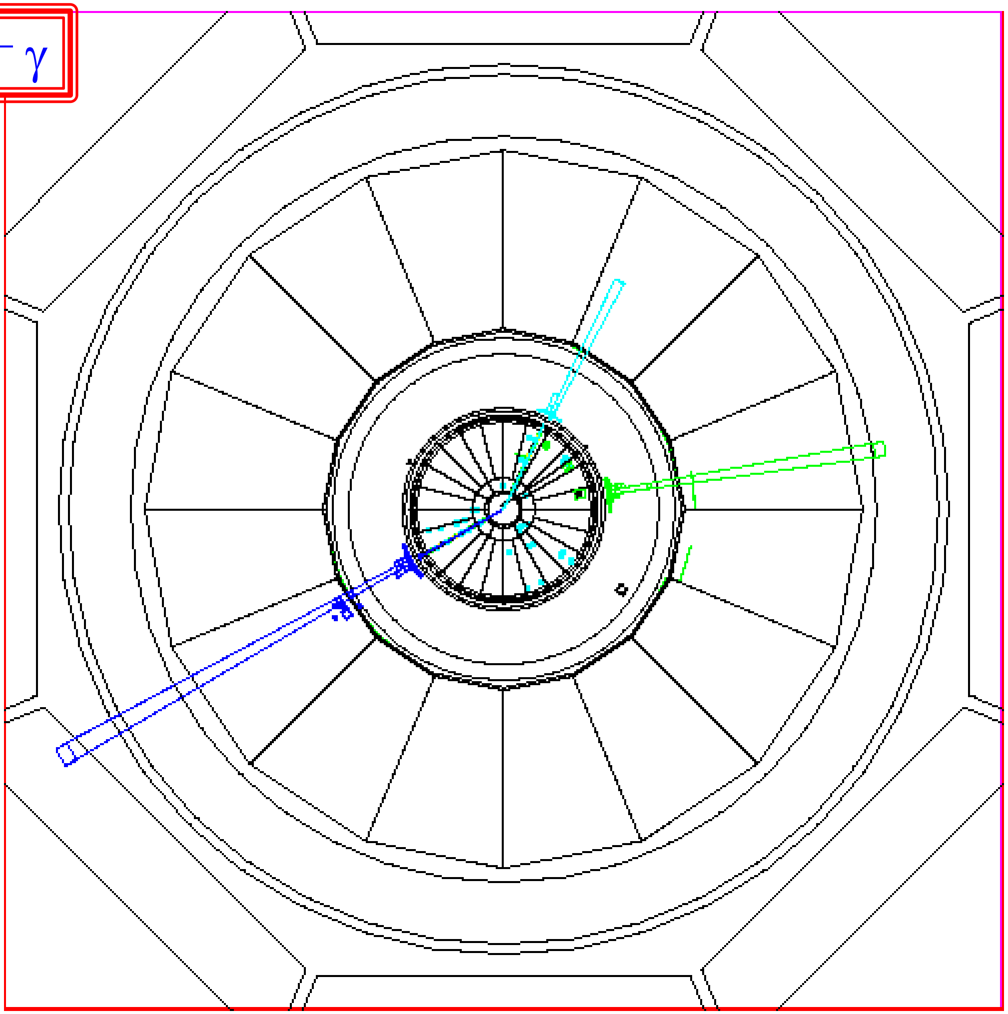
$e^+ e^- \to \mu^+ \mu^-$

+ signals in SMD
+ track in TEC ($\to$ momentum and charge)
+ mip in calos
+ signals in μ chambers ($\to$ momentum and charge)
= identified and measured $\mu^\pm$.



LEP events : $e^+e^-\gamma$

$e^+ e^- \to e^+ e^- \gamma$



+ signals in SMD
+ track in TEC ($\to$ momentum and charge)
+ e.m. shower in e.m. calo
+ (almost) nothing in had calo
+ absolutely nothing in μ chambers

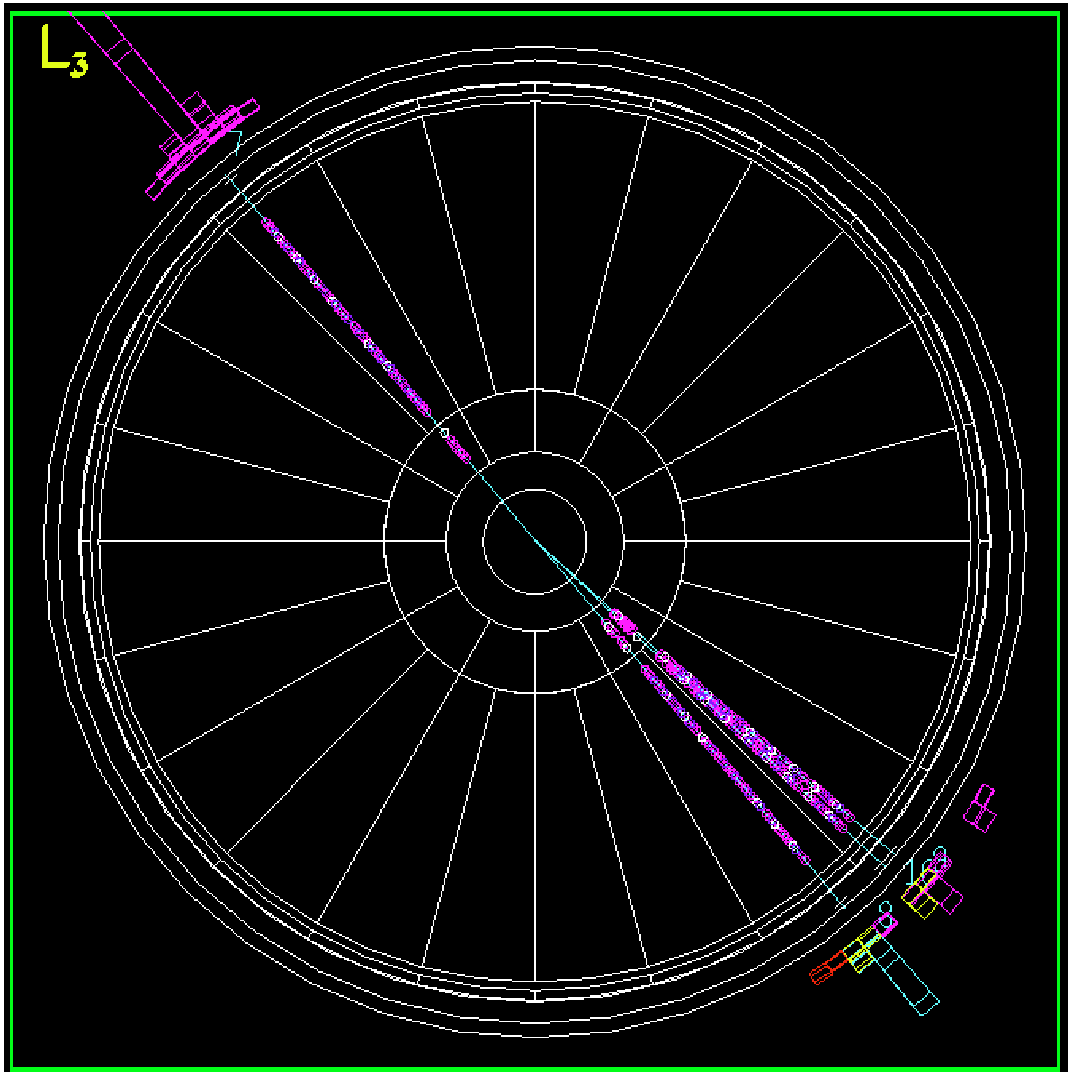
= identified and measured $e^\pm$.

+ no signal in SMD
+ no signal in TEC
+ e.m. shower in e.m. calo
+ (almost) nothing in had calo
+ absolutely nothing in μ chambers

= identified and measured γ.

LEP events : $\tau^+\tau^-$

$e^+ e^- \rightarrow \tau^+ \tau^-$



$\tau^\pm$ id. does depend on decay:

- 1/3/5 had tracks;
- [or identified single $\ell^\pm$;]

+ $\not{E}$ (i.e. a ν_τ /$\bar{\nu}_\tau$)

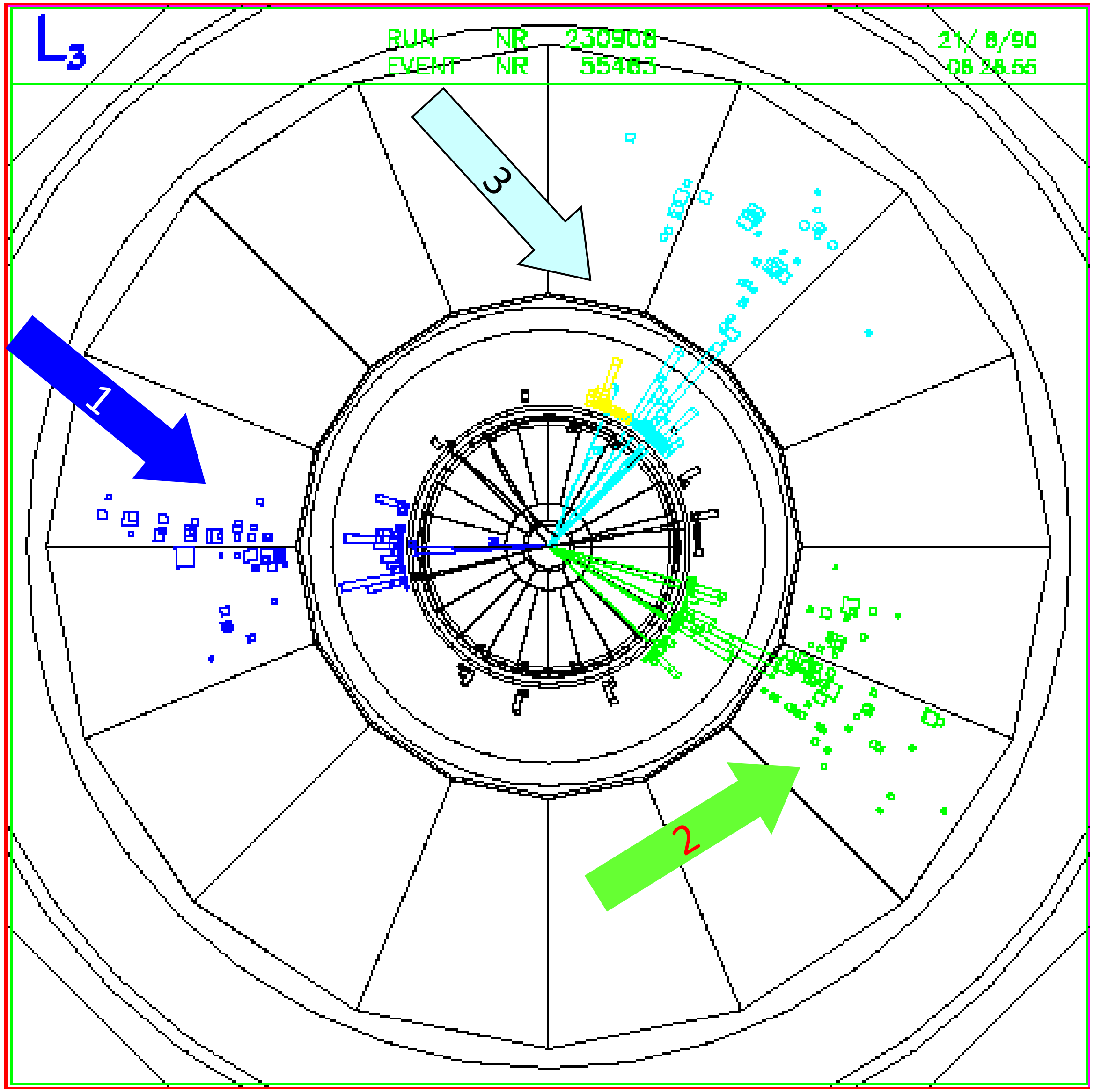
(the evidence comes from the combination of the two decays in the opposite emispheres).

LEP events : 3 jets

$e^+ e^- \to q \bar{q} g$

a (anti-)quark or a gluon gives a hadronic jet:

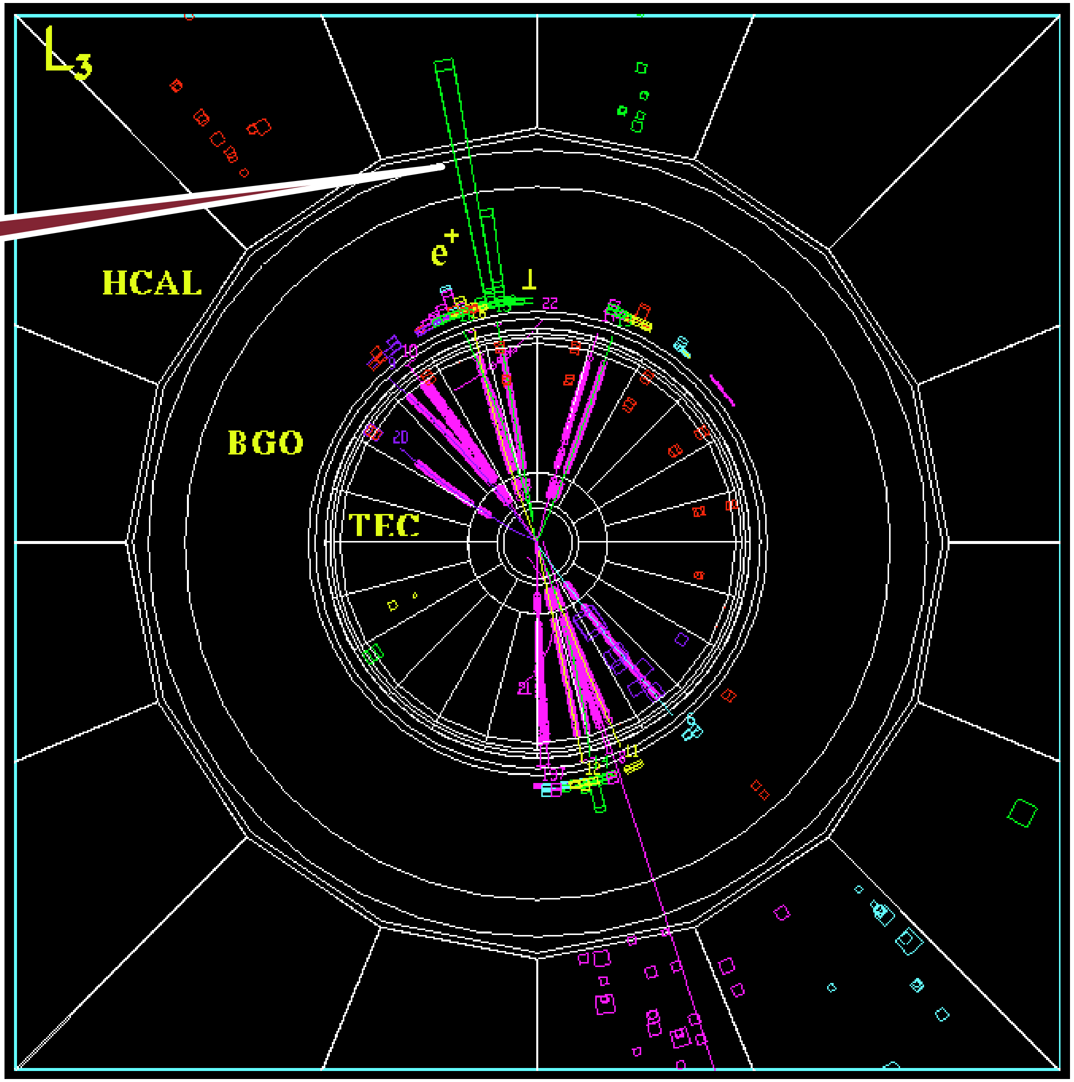
+ many collimated tracks
+ large splashes in e.m. and had calos
+ (possibly) low momentum associated $e^\pm/\mu^\pm$



LEP events : $b\bar{b}$, $b \to e^-$

$e^+ e^- \to b \bar{b}$

identified e^+



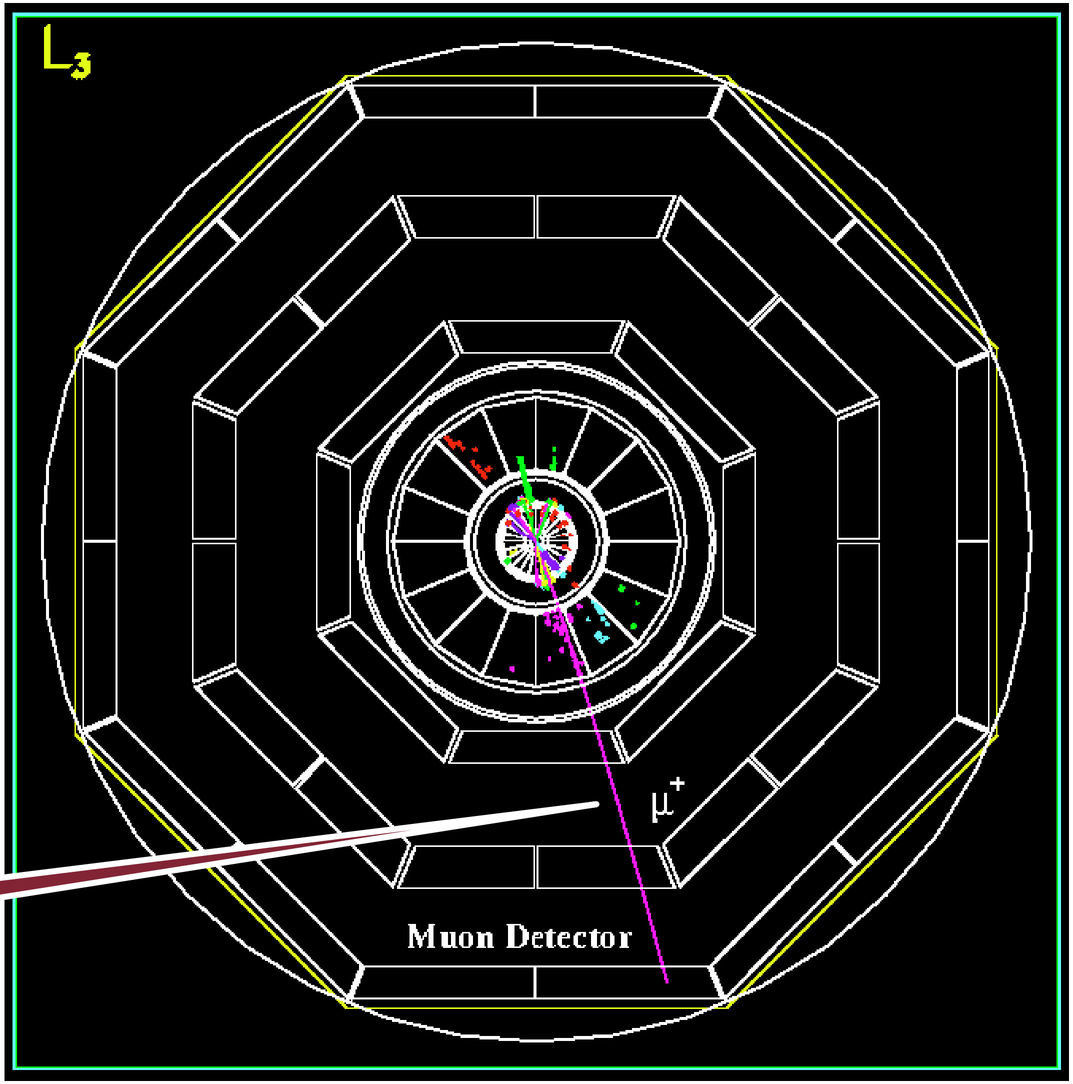
a heavy flavor quark is a quark (i.e. a jet) with:

+ displaced secondary verteces (SMD)
+ high momentum leptons from quark semileptonic decays

[not all h.f. have one or both characteristics → h.f. id. efficiency not complete (see next)]

LEP events : $b\bar{b}$, $b \rightarrow \mu^-$

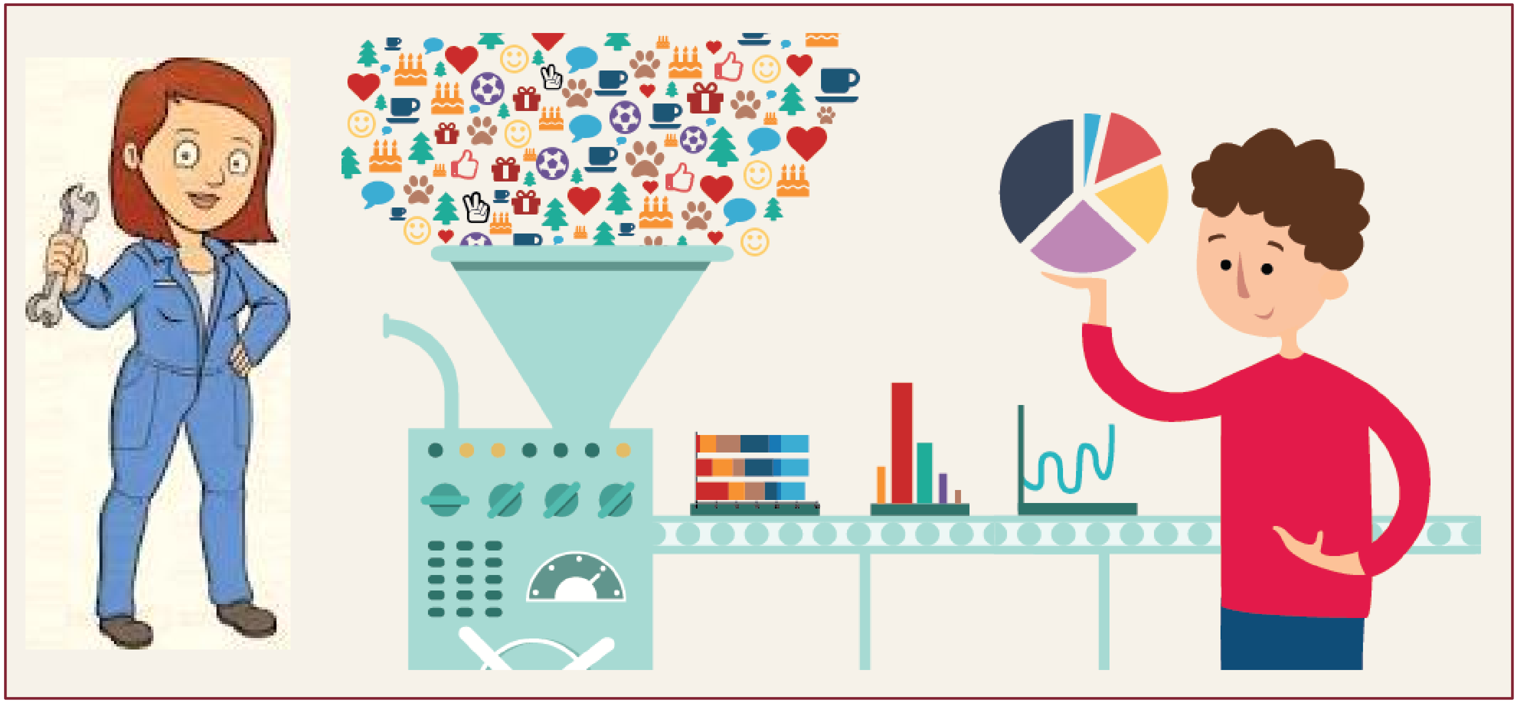
$e^+ e^- \rightarrow b\,\bar{b}$



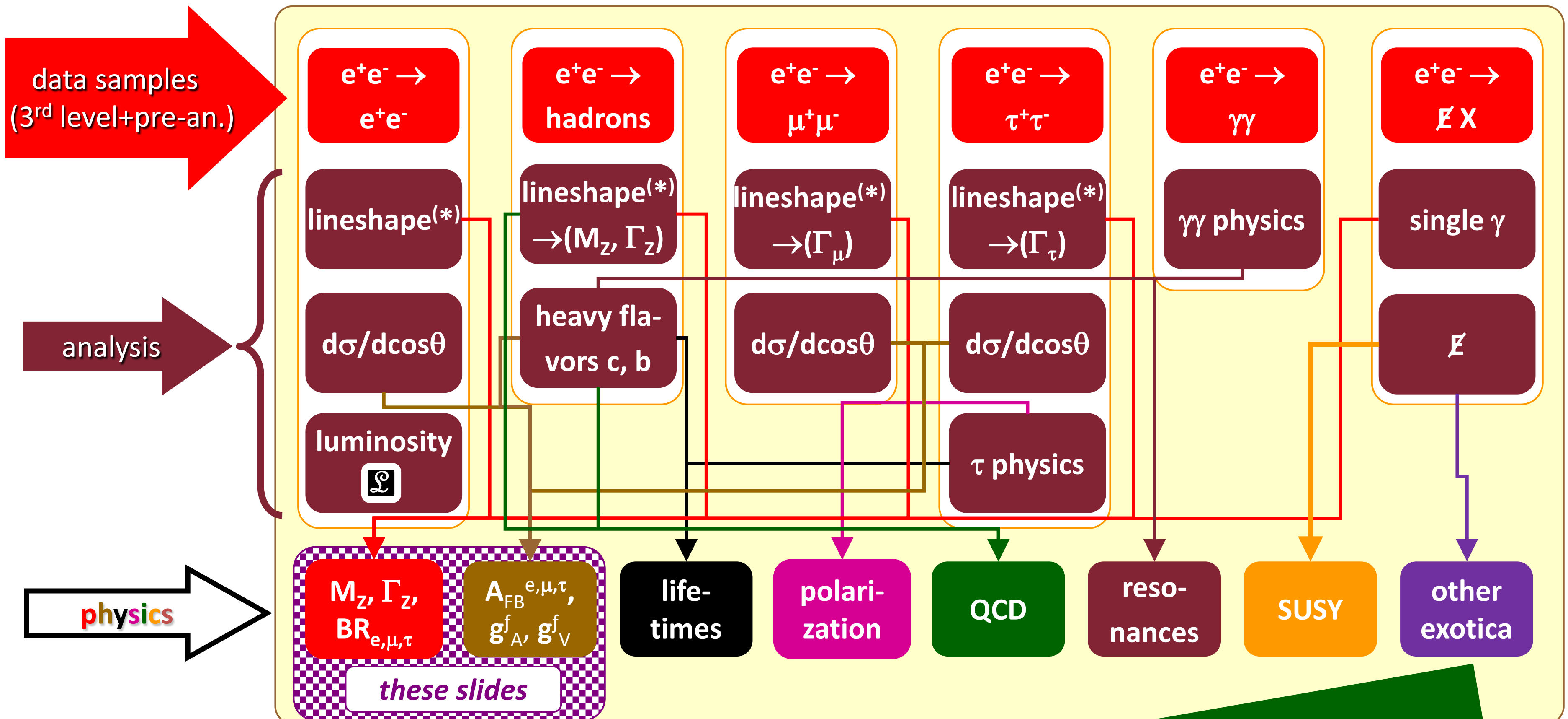
identified μ^+

ii. Exp. methods

1. – 4. […]
5. Data analysis
6. Secondary verteces
7. Efficiency and purity
8. The luminosity
9. – 16. […]



data analysis



very simplified – just for the lectures

(*) "lineshape" : $\sigma = \sigma(\sqrt{s})$

data analysis: events $\rightarrow \sigma$

- At LEP, as in any other experiment, a number of events $\mathbf{N^{exp}}$ has to be translated to a cross section $\boldsymbol{\sigma_s}$ ("signal");
- [also $\mathbf{dN^{exp}/d\Omega \rightarrow d\sigma_s/d\Omega}$;]
- straightforward : $\boldsymbol{\sigma_s = N^{exp} / \mathscr{L}_{int}}$;
- but (at least) two problems :
  - the **selection algorithm** loses true- and gains spurious-events: $N^{exp} = N_{true} - N_{lost} + N_{sp.}$;
  - the determination of $\mathscr{L}_{int}$, the **luminosity**.
- the experiment must measure/compute :
  - N^{exp} : number of selected events;
  - σ_b : cross-section of bckgd;
  - $\varepsilon_{s,b}$: efficiency (signal and bckgd);
  - $\Delta N^{exp} = \sqrt{N^{exp}}$ (statistical error);
  - $\Delta\varepsilon_{s,b}$ = "systematics";
  - $\mathscr{L}_{int}$ = int. luminosity.
- then (next slides) :
  - $N^{exp} = \mathscr{L}_{int} (\varepsilon_s \sigma_s + \varepsilon_b \sigma_b) \rightarrow$
    $\sigma_s = (N^{exp}/\mathscr{L}_{int} - \varepsilon_b\sigma_b) / \varepsilon_s$;
    $d\sigma_s/d\ldots = [\ldots]$;
- the luminosity $\mathscr{L}_{int}$ is equal for signal and bckgd and must be measured;
- LEP measures $\mathscr{L}_{int}$ from a process ("lumi process"), with a calculable cross section, triggered and acquired at the same time as other data ($\rightarrow$ so DAQ inefficiencies cancel out) :

$$\mathscr{L}_{int} = N_{lumi} / (\varepsilon_{lumi}\, \sigma_{lumi} + \varepsilon_{b\text{-}lumi}\, \sigma_{b\text{-}lumi})$$

- therefore three new errors :
  (statistics) $\Delta N_{lumi} = \sqrt{N_{lumi}}$,
  (sistematics) $\Delta\varepsilon_{lumi,b\text{-}lumi}$, $\Delta\sigma_{b\text{-}lumi}$,
  ("theory") $\Delta\sigma_{lumi}^{theory}$.

NB. In an ideal experiment, $N_{lost} = N_{sp.} = 0 \rightarrow \varepsilon_s = 1, \varepsilon_b = 0$.

data analysis: theory ↔ exp. data

An example: $e^+e^- \to \mu^+\mu^-$:

- studies for efficiency and purity with MC simulation [*see later*].
- signal: true events $e^+e^- \to \mu^+\mu^-$; the yield depends on m_Z, Γ_Z, Γ_μ (unknown);
- bckgd: events from other sources, with similar final state (because really the same or similar in the detector), e.g. :

➢ $e^+e^- \to Z \to \tau^+\tau^- \to$
$\to (\mu^+\bar{\nu}_\tau\nu_\mu)\ (\mu^-\nu_\tau\bar{\nu}_\mu)$
$\to (\mu^+\mu^-)$ (+ *not-visible*);

➢ $e^+e^- \to e^+e^-\mu^+\mu^- \to$
$\to (e^+e^-)^{\text{beam chamber}}\ (\mu^+\mu^-)^{\text{detected}}$;
$\to (\mu^+\mu^-)$ (+ *not-detected*);

result !!!

$$N_{exp} = \mathscr{L}_{int}\,[\varepsilon_s \boxed{\sigma_s} + \varepsilon_b\sigma_b].$$

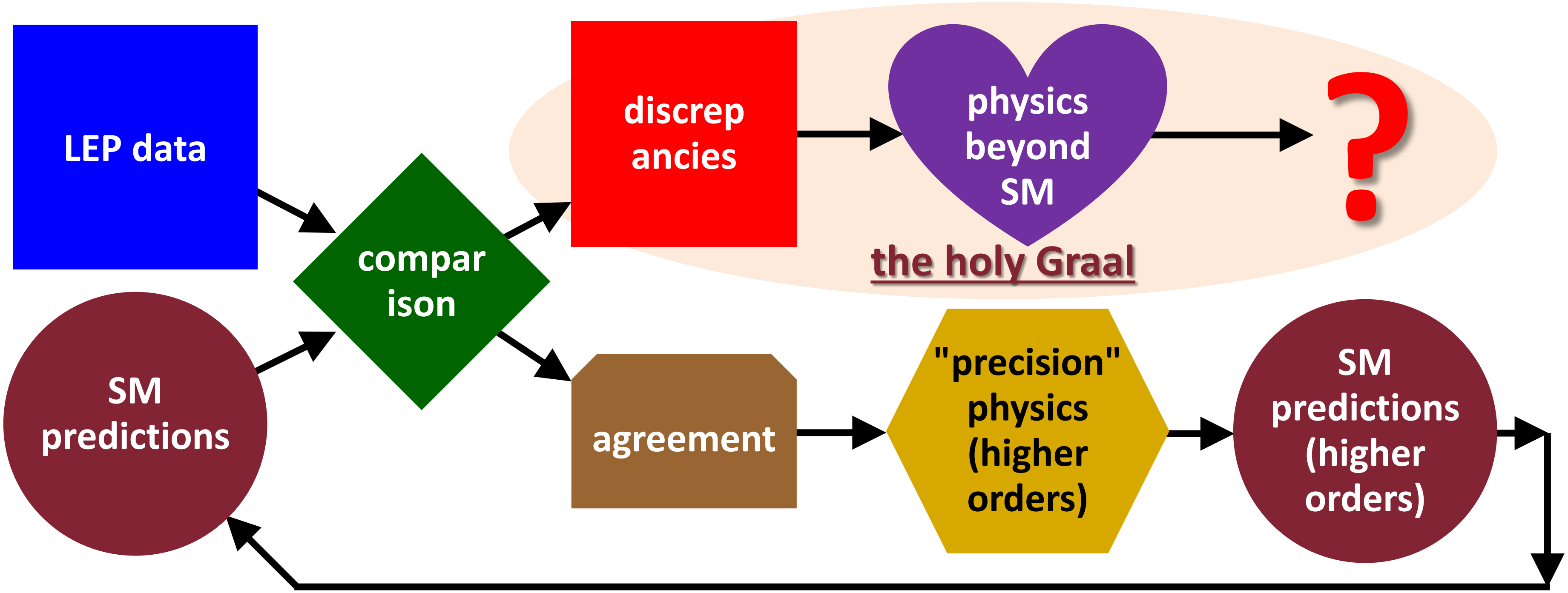
selected sample

lumi meas (see)

mc signal + bckgd

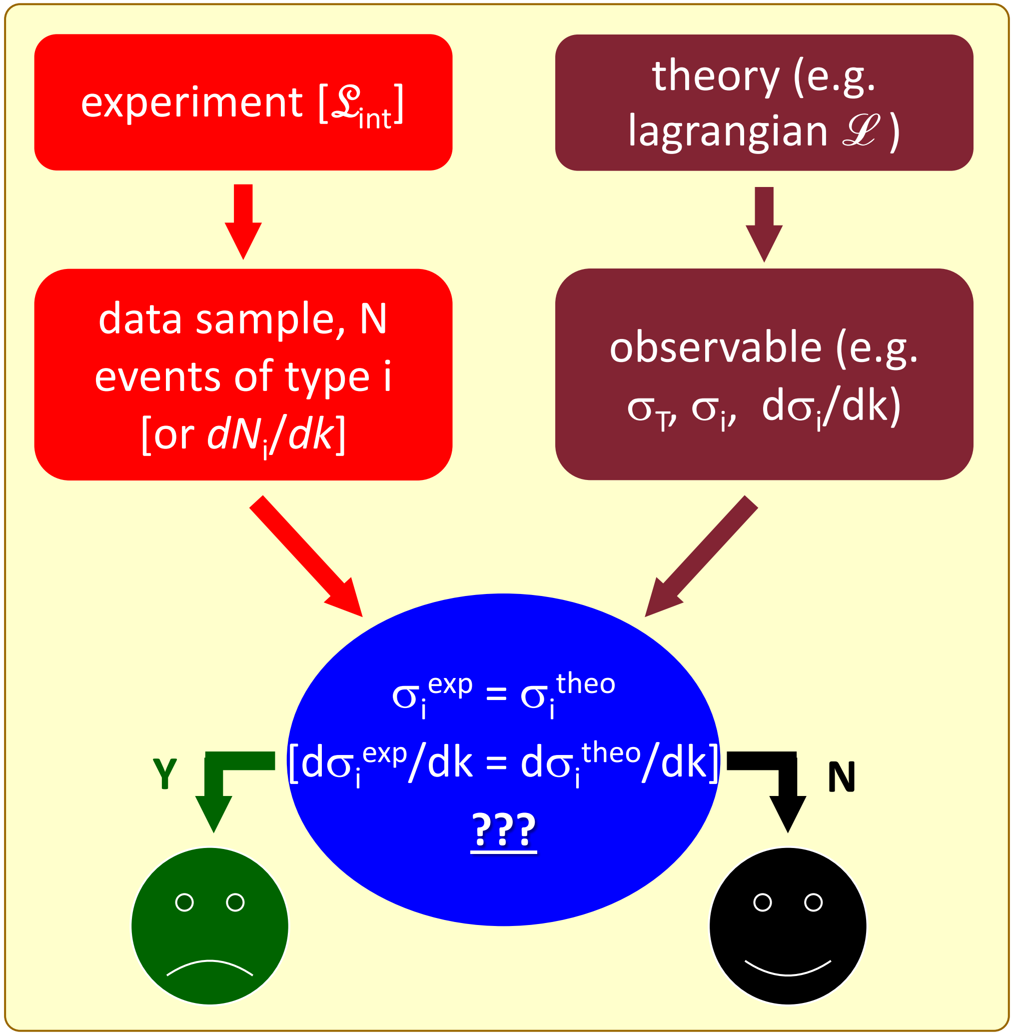
theory, other studies

data analysis: scheme



- In 1989, when LEP started, the SM was completely formulated and computed;
- the only missing pieces (at that time) were the top quark and the Higgs boson (both now discovered);
- the values of m_{top} and m_{Higgs} are such that they (in lowest order) have no role at LEP $\sqrt{s}$ [*but for H we did NOT know*];
- twelve years of LEP physics gave NO major surprise, but general agreement with SM predictions;
- tons of measurements, a superb unprecedented work of precision physics : the **number of light ν's** and the **predictions of m_{top} and m_{Higgs}** via higher orders are [*imho*] the LEP masterpieces.

data analysis: comparison theory ↔ data



Therefore, a ***measurement*** means :

- select a pure (as much as possible) sample of events N_i;
- measure the statistical significance of the experiment ($\rightarrow \mathcal{L}_{int}$);
- measure/compute the associated efficiency and purity ($\rightarrow \varepsilon$,p);
- compute $\sigma_i \equiv \sigma_i^{exp}$ = [previous slide] [or $d\sigma_i^{exp}/dk$ = (…)];

$\rightarrow$ finally **theory ↔ experiment**:

- compute σ_i^{theo} from theory;
- **compare** $\sigma_i^{theo} \leftrightarrow \sigma_i^{exp}$.

["**limits**" require a different method, see § limits].

data analysis: results

SM predictions :

- $\sigma(f\bar{f})$, $\sigma(e^+e^-)$, $d\sigma/d\cos\theta$ ... ("Born");
- radiative corrections;
- approximations;

experiment(s) (LEP, L3 as an example) :

- cross sections $\sigma(e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, \text{hadrons}, \nu\bar{\nu})$;
- differential cross sections $d\sigma(e^+e^- \to \ldots) / d\cos\theta$;
- "lineshape" (i.e. $\sigma(e^+e^- \to \ldots)$ as a function of $\sqrt{s}$ [also $d\sigma(e^+e^- \to \ldots) / d\cos\theta$ vs $\sqrt{s}$].

data analysis and interpretations : global fit (4 exp. data) $\leftrightarrow$ (SM):

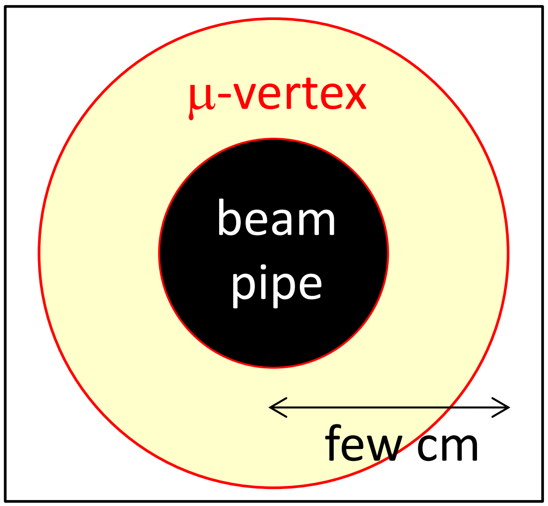
- Z mass, full and partial width (m_Z, Γ_Z, Γ_f);
- number of ν's from $\Gamma_{invisible}$ and from γ_{single};
- asymmetries $A_{forward\text{-}backward}$ for $e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$, hadrons;
- global fit data $\leftrightarrow$ SM ($\to$ consistency);
- global fit data $\leftrightarrow$ SM ($\to$ predictions of m_{top}, m_{Higgs} from radiative corrections).

secondary verteces

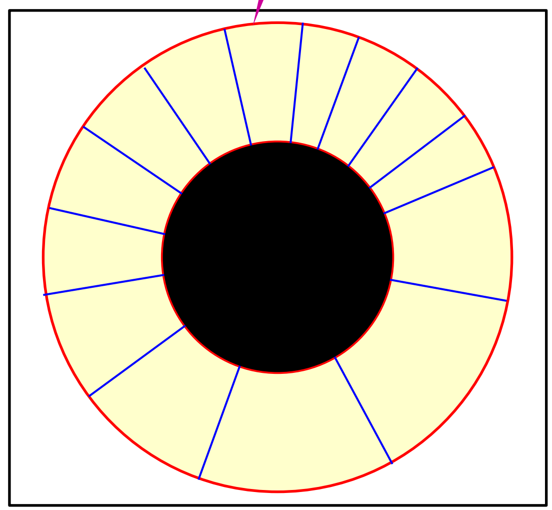
how to detect and identify c / b / τ's with a μ-vertex

do you see the difference ?

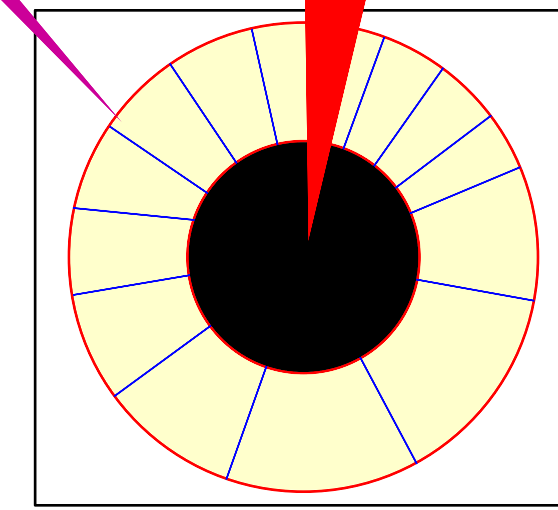
heavy quark (e.g. b) decay



the detector

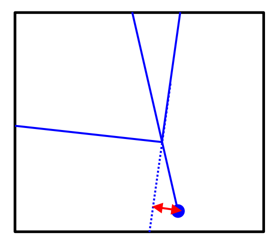


typical event: case 1



typical event: case 2

it needs a great accuracy in the "impact parameter" measurement.



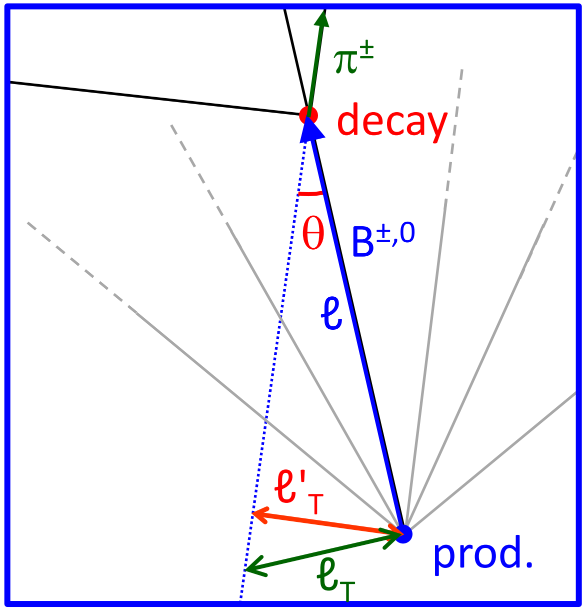
secondary verteces: kinematics

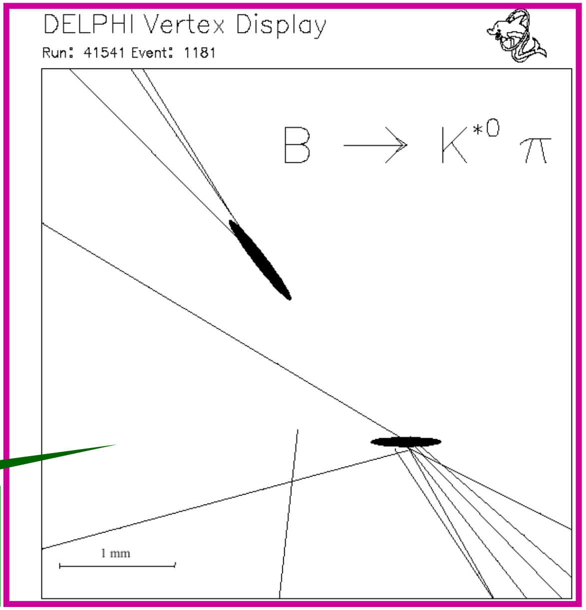
Analysis method (B as an example, similar for c-mesons/baryons, $\tau^\pm$] :

- [B conservation $\rightarrow$ 2 B in the event $\rightarrow$ 2 sec. vtxs];
- B ref. sys: $\tau(B^{\pm,0}) \approx 1.5\times10^{-12}$ s $\rightarrow \ell^* = c\,\tau_B \approx 500\ \mu$m;
- $\beta_B \approx 1 \rightarrow\ \ell\ (= \ell_B) = \ell^*\ \beta_B\gamma_B \approx c\ \tau_B\gamma_B \approx$ few mm [*see*];
  ℓ_T (= ℓ tanθ) is invariant wrt a $\mathbb{L}$-transform along β_B
  $\rightarrow \ell_T = \ell^*_T = \ell^* \sin\theta^* \approx 100 \div 500\ \mu$m
  (θ^* is the angle B/π in the B ref. sys., **NOT** small);
- ℓ_T has large errors, but ℓ'_T, the transverse distance (extrapolation of a track) $\leftrightarrow$ (primary vtx) can be meas.;
- $\theta \sim m_B/E_B \approx 1/\gamma_B$ = small $\rightarrow \sin\theta \approx \tan\theta \rightarrow \ell'_T \approx \ell_T$;
- [call both ℓ'_T and ℓ_T "impact parameter ℓ_T"];

➢ need a detector with an accuracy << 100 μm in ℓ_T (i.e. in the extrapolation of the line of flight of a charged particle after 20÷30 mm from the last meas;

➢ **i.e. a very precise microvertex detector may identify and reconstruct b, c, τ decays.**

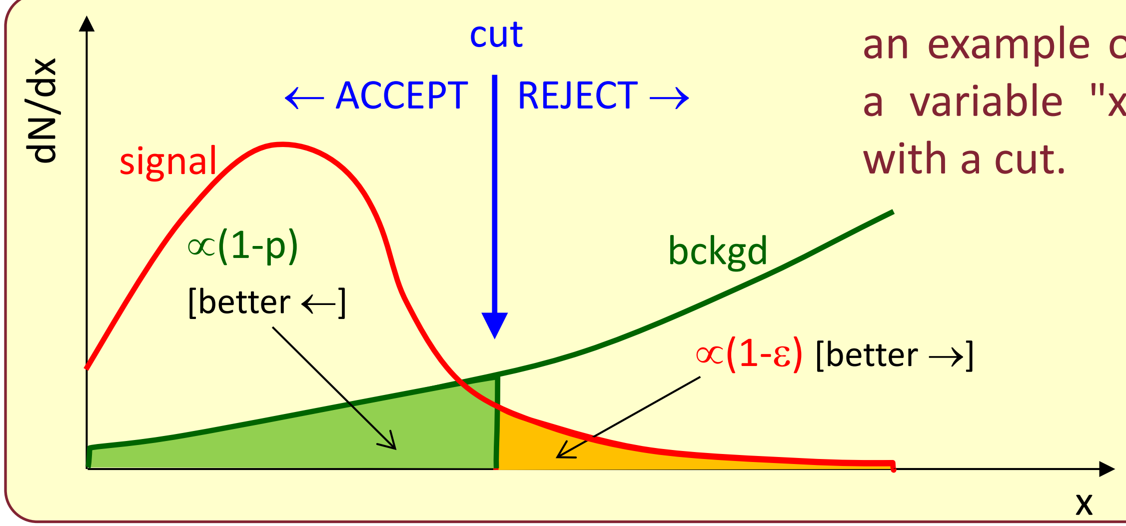




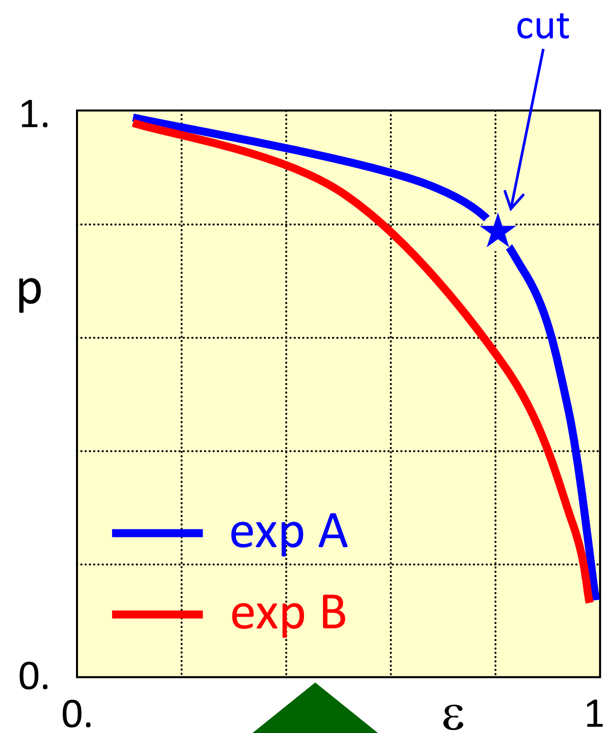
a real B^0 decay in Delphi (only one B vtx shown]

efficiency and purity

- No selection method is fully "pure" and "efficient", i.e. in a selected sample of events of type "i", there are some events "j" (j≠i), while some events "i" have been rejected;
- if N_i^{sel} is the number of events of the sample, define :
  - efficiency : $\varepsilon_i = N_i^{sel,true} / N_i^{true,all} < 1$ [ideally = 1];
  - purity : $p_i = N_i^{sel,true} / N_i^{sel,all} < 1$ [ideally = 1];
  - [contamination : $k_i = N_i^{sel,false} / N_i^{sel,all} = 1 - p_i$] ;
- in general, ε_i and p_i are anti-correlated (see below);
- an algorithm (e.g. a cut in a kin. variable) produces $\varepsilon_i + p_i$;
- the "optimal" **choice** depends on the analysis and on $\mathscr{L}_{int}$.



an example of a variable "x" with a cut.



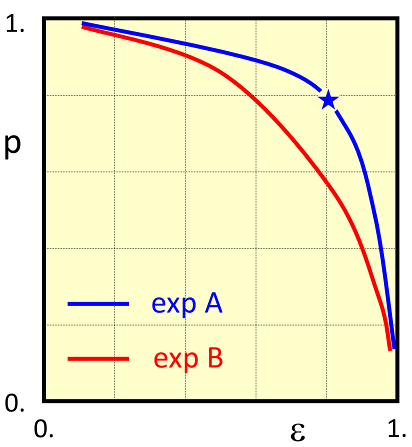
Example [no "$_i$" in the plots] :
- two cases of p_i vs ε_i, when the cut varies.
- exp. A "is better" than B.
- "★" shows a possible choice for (p_i, ε_i) in A.

efficiency and purity: methods

$N_i^{sel,true}$ and $N_i^{true,all}$ are NOT directly measurable. Few methods to determine the relation ε / p, e.g. :

- Montecarlo (commonly used) :
  - 3 steps : "physics" [→ 4-mom.] + detector [→ pseudo-meas.] + analysis [exactly the same as in real data];
  - pros : large statistics, flexible, easy;
  - cons : (some) systematics cannot be studied;
- test-beam :
  - intrinsic purity + large statistics;
  - pros : systematics;
  - cons : not flexible, difficult, expensive;
- "data themselves" [e.g. μ from $Z \to \mu\mu$ to study $b \to \mu X$] :
  - "tag and probe" [$p \approx 1$ even if ε small] to force purity;
  - ok for systematics;
  - difficult reproduction of the required case [in the example isolated μ's 45 GeV instead of low-p_T μ in a jet].

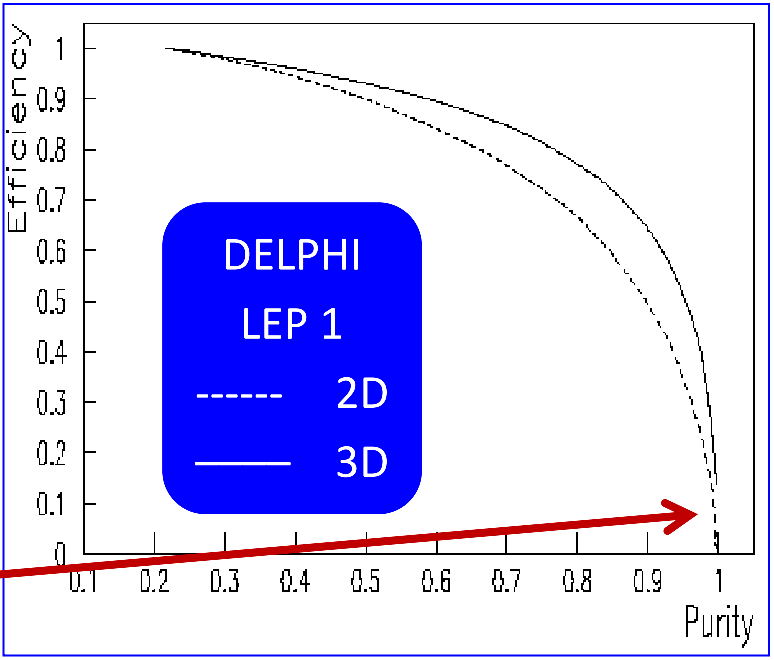
∴ Combination of the above, iterations, new ideas (i.e. **you** ☺)…

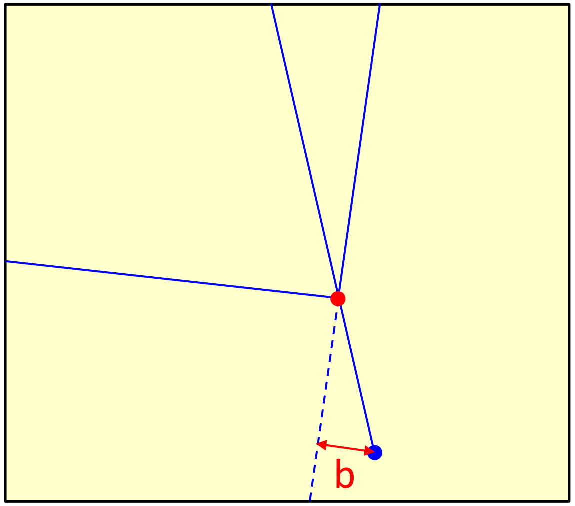


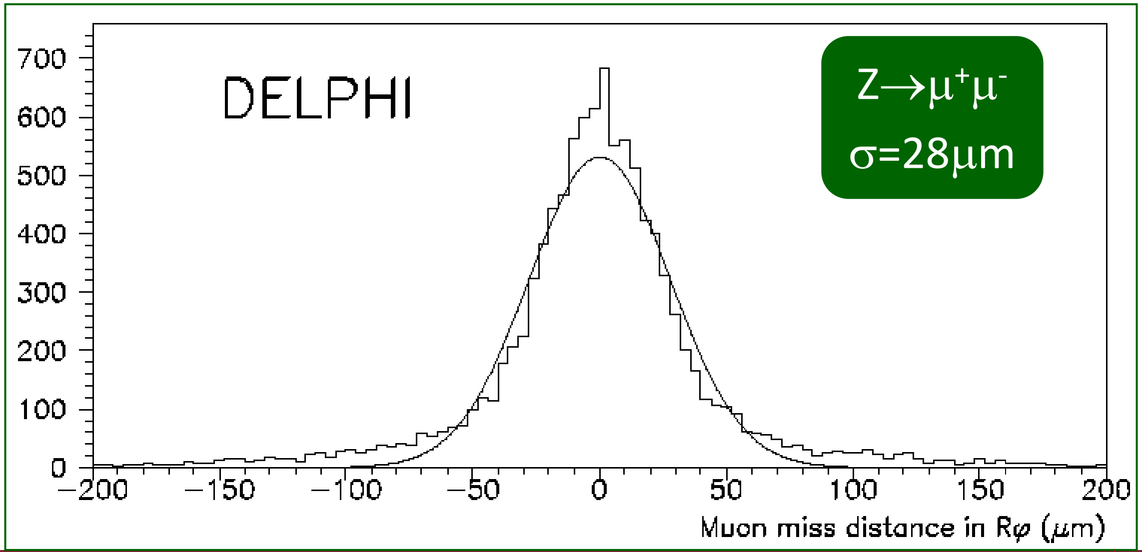
efficiency and purity: example

An example of the computation of ε vs p (secondary vtxs with impact parameter):

- use a mc (not shown) to define the distribution of impact parameter b in events with sec. vtxs;
  - a cut on b $\rightarrow \varepsilon = \varepsilon(b_{cut})$;
- use a process without secondaries ($Z \rightarrow \mu^+\mu^-$) to define the distribution of the variable b;
  - a cut on b $\rightarrow p = p(b_{cut})$;
- $\varepsilon = \varepsilon(b_{cut}) \oplus p = p(b_{cut})$ are parametric equations;
- repeat with more info → "3D" → better curve.





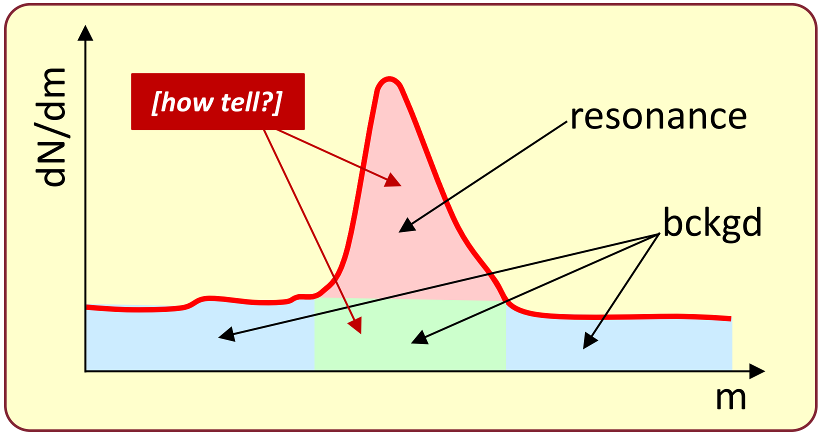


efficiency and purity: the bckgd

- The background ["bckgd"] may be conceptually divided into two categories :
  - irreducible bckgd(*): other processes with the same final state [e.g. $e^+e^- \to ZH$, $Z \to \mu^+\mu^-$, $H \to b\bar{b}$ (signal) $\leftrightarrow$ $e^+e^- \to Z_1 Z_2$, $Z_1 \to \mu^+\mu^-$, $Z_2 \to b\bar{b}$ (bckgd)];
  - reducible bckgd :
    - badly-measured events,
    - detector mistakes,
    - physics processes which appear identical (with given selection criteria) to the process under study [e.g. because part of the final state is undetected, $e^+e^- \gamma_{unseen} \leftrightarrow e^+e^-\nu$];
- the meaning of the distinction is that r.b. can be disposed with a better detector, or a more accurate selection (maybe with a loss in ε_s), while i.b. is intrinsic, and can only be subtracted statistically, by comparing [$N^{exp} \leftrightarrow$ (expected bckgd)] and [$N^{exp} \leftrightarrow$ (expected signal+bckgd)] ;

____

(*) Similar to the "resonances" of the strong interactions, where a mass distribution exhibits peaks, interpreted as short-lived particles. However, it is impossible to assign single events to the resonating peak or to the non-resonant bckgd.



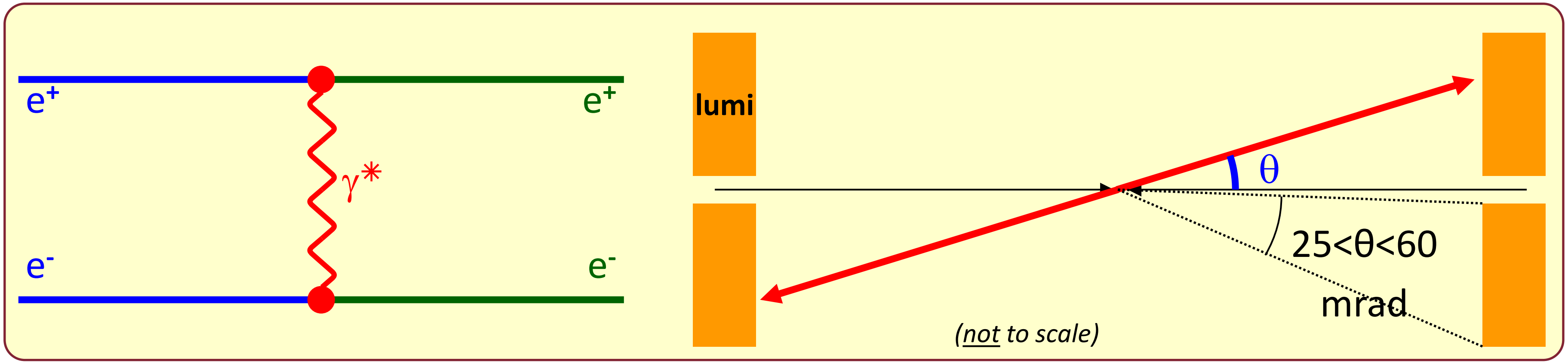
the luminosity

[few slides ago:
LEP measures $\mathscr{L}_{int}$ from a process (...):
$\mathscr{L}_{int} = N_{lumi} / (\varepsilon_{lumi}\,\sigma_{lumi} + \varepsilon_{b\text{-}lumi}\,\sigma_{b\text{-}lumi})$ *]*

- the "lumi" process (σ_{lumi}) is $e^+e^- \to e^+e^-$ (Bhabha scattering) at small θ;
- we **<u>assume</u>** that, when $\theta \to 0°$, the Bhabha scattering is dominated by the γ^* exchange in the t-channel, while both (a) the γ^*/Z exchange in the s-channel; (b) the $Z^{(*)}$ exchange in the t-channel <u>are negligible</u>;
- therefore, the LEP experiments have e.m. calorimeters at small θ, to both identify and measure $e^\pm$ ("luminometers", ring-shaped ♦);
- it is essential that the "ring" reaches very small θ, to minimize $\Delta\sigma_{stat}$ ($d\sigma_{Rutherford} / d\cos\theta \propto \theta^{-4}$);
- their position and efficiency must be known (= measured) very reliably, in order to minimize systematics;
- typically at LEP, $25 \leq \theta_{lumi} \leq 60$ mrad :

$$\sigma_{lumi} = \frac{16\pi\alpha_{em}^2}{s}\left(1/\theta_{min}^2 - 1/\theta_{max}^2\right);$$

$$\Delta\mathscr{L}/\mathscr{L} = \Delta\sigma_{lumi}/\sigma_{lumi} \approx 2\Delta\theta_{min}/\theta_{min}.$$



(<u>not</u> to scale)

the luminosity $\mathscr{L}$: results

- at the end of LEP, using sophisticated silicon calos, the final results on luminosity was :

  $\Delta\mathscr{L}_{int}/\mathscr{L}_{int}$ = [see box] (statistical);

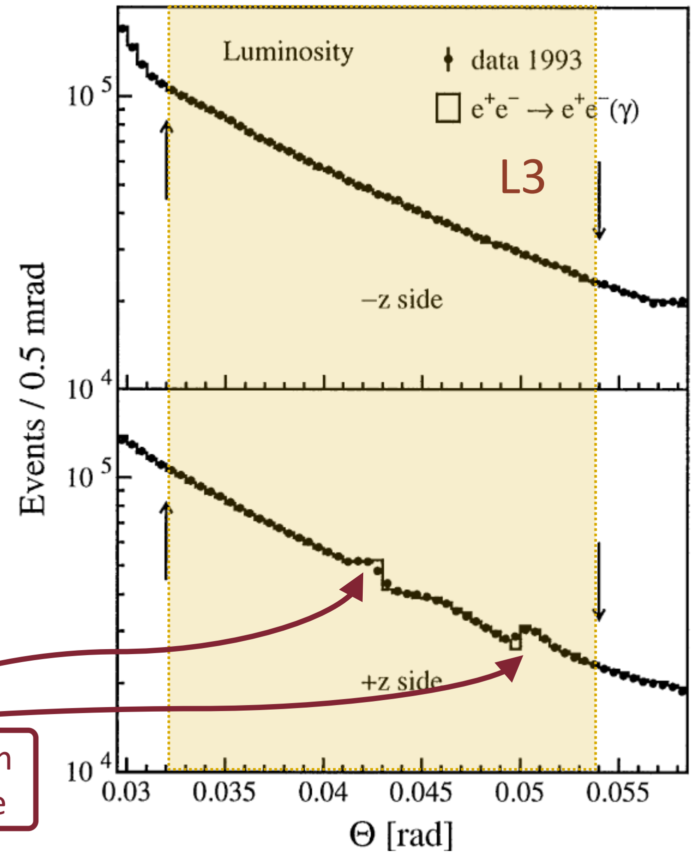
  $\oplus$ [0.03 ÷ 0.1 %] (syst. exp : $\Delta\theta$, alignment, ...);

  $\oplus$ [0.11 %] (theory, higher orders like $e^+e^- \to e^+e^-\gamma_{unseen}$);

- some of the LEP measurements, as number of ν's, asymmetries, do NOT depend on $\Delta\mathscr{L}_{int}$: because can be expressed as ratios "σ_1/σ_2 [$=N_1/N_2$]";

- [the luminosity data are an important fraction of all LEP1 data].

fake Bhaba's from beam pipe shape



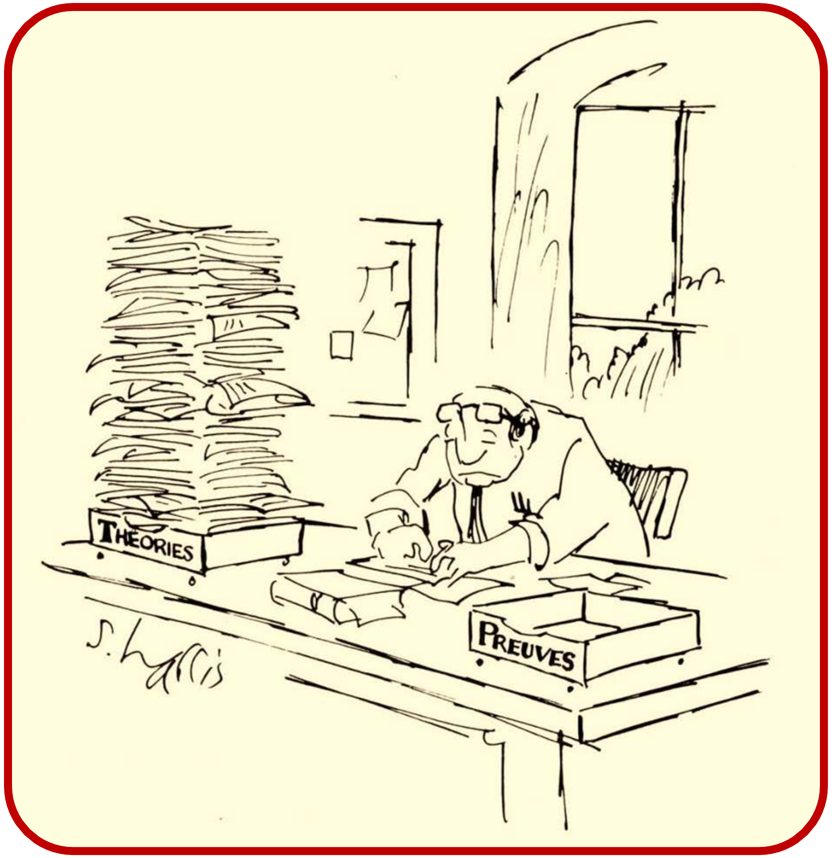
An estimate of the importance of the statistical error comes from the comparison :

- $\sigma(e^+e^- \to \text{hadrons}, \sqrt{s} = m_Z) \approx 30$ nb, the largest cross-section among all LEP processes;
- $\sigma(e^+e^- \to e^+e^-, 25 \le \theta \le 60 \text{ mrad}) \approx 100$ nb.

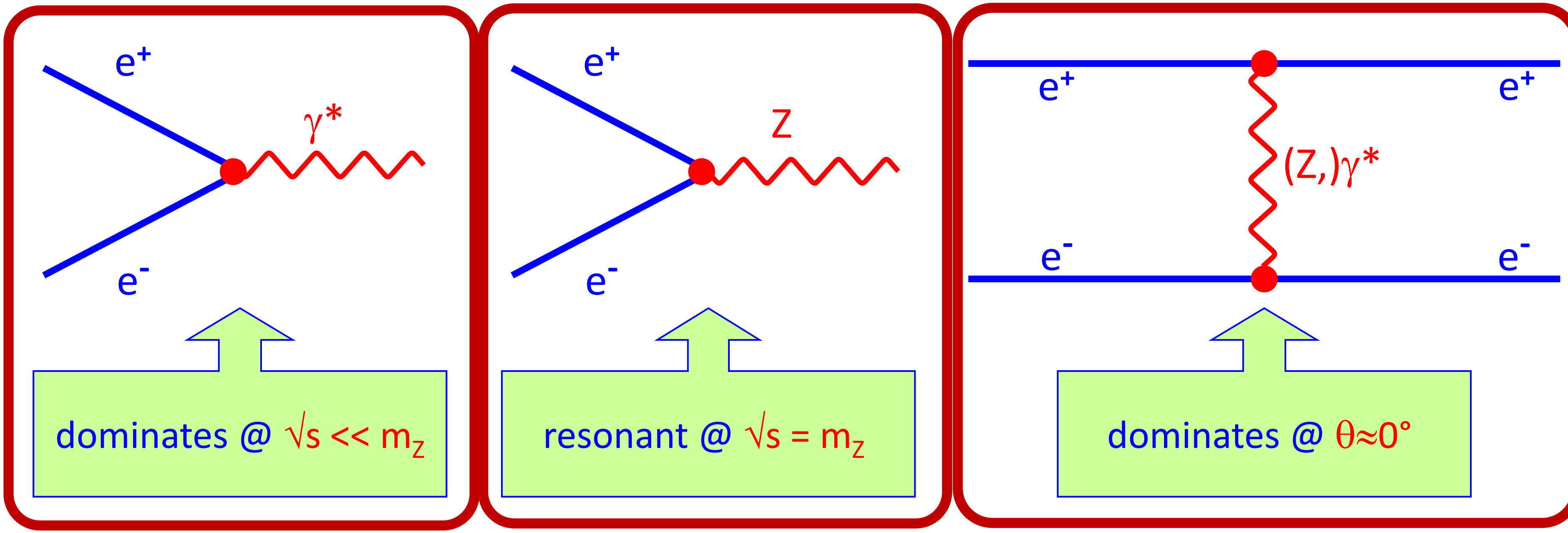
Therefore the statistical error on the luminosity is negligible, but for the hadronic cross section at $\sqrt{s} = m_Z$, where it is $\sim\sqrt{3/10}$ of the statistical error on the hadron data [but for this process the stat. error is irrelevant wrt systematics].

iii. Physics 1: Z & W

1. – 8. […]
9. $e^+e^- \to Z \to f\bar{f}$
10. $d\sigma(e^+e^- \to f\bar{f}) / d\Omega$
11. $e^+e^- \to Z \to e^+e^-$
12. Radiative corrections
13. LEP1 SM fit
14. $e^+e^- \to W^+W^-$ @ LEP2
15. Global LEP(1+2) fit
16. […]



$e^+e^- \to Z \to f\bar{f}$



- Many possibility from e+e– initial state;
- similar couplings wrt already considered processes [§3, §4, §6, §7];
- at low energy, QED only (exchange of γ^* in the s-channel);
- at $\sqrt{s} \approx m_Z$:
  - $\sigma_{res}(e^+e^- \to f\bar{f}) \propto \Gamma_f \,/\, [\,(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2\,]$;
  - for each fermion pair, two (four for e^+e^-) diagrams + interferences);
  - at higher energy, new phenomena ($W^\pm$, exchange, IVB pairs in the final state, …).

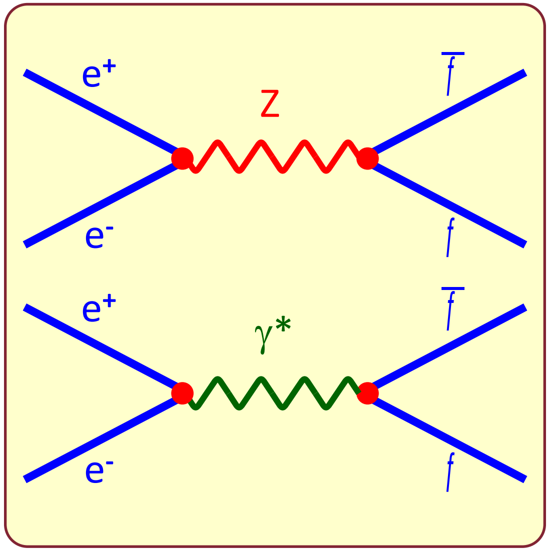
$e^+e^- \to Z \to f\bar{f}$: σ^{SM}_{Born}

In the SM, at lowest order, for $f \neq e^\pm$, $m_f << m_z$:

- $\sigma_{Born}(e^+e^- \to f\bar{f}) = \sigma_{Zs} + \sigma_{\gamma s} + J_f;$
- $\sigma_{Zs} = \dfrac{s\Gamma_Z^2}{(s-m_Z^2)^2 + s^2\Gamma_Z^2/m_Z^2} \dfrac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2};$
- $\sigma_{\gamma s} = \dfrac{4\pi\alpha^2}{3s} c_f Q_f^2;\quad \left[c_f = 1 \text{ (leptons), } 3 \text{ (quark)}\right];$
- $J_f = -\dfrac{(s-m_Z^2)m_Z^2}{(s-m_Z^2)^2 + s^2\Gamma_Z^2/m_Z^2} \dfrac{2\sqrt{2}\alpha}{3} c_f Q_f G_F g_V^e g_V^f;$
- $\Gamma_Z = \Gamma_{tot} = \sum_f \Gamma(Z \to f\bar{f});$
- $\Gamma_f \equiv \Gamma(Z \to f\bar{f}) = \dfrac{G_F m_Z^3 c_f}{6\sqrt{2}\,\pi}\left[g_V^{f\,2} + g_A^{f\,2}\right];$
- for $\sqrt{s} \approx m_z \to$ interference and γ^* negligible;
- $\sigma_{Born}(e^+e^- \to f\bar{f}, \sqrt{s} = m_Z) = \dfrac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2}.$

i.e. neglect t-channel, both Z^* and γ^*

Z (s-channel)	= bell-normalized-to-1 $\times\, \sigma(\sqrt{s}=m_Z)$
γ^* (s-channel)	[well known, see §3]
interference $Z_s \leftrightarrow \gamma^*_s$	new entry, possibly important for $\mathbb{P}$-violation



$e^+e^- \rightarrow Z \rightarrow f\bar{f}$: g_V^f and g_A^f

- the partial widths Γ_f (e.g. Γ_μ) are also easily computed in lowest order :

$$\Gamma_f = \frac{G_F m_Z^3 c_f}{6\sqrt{2}\,\pi}\left[g_V^{f\,2} + g_A^{f\,2}\right] \;\rightarrow\; (f=\mu^\pm) \;\rightarrow\; \Gamma_\mu \approx \frac{1}{4}\frac{G_F m_Z^3}{6\sqrt{2}\,\pi} \approx 83\,\text{MeV};$$

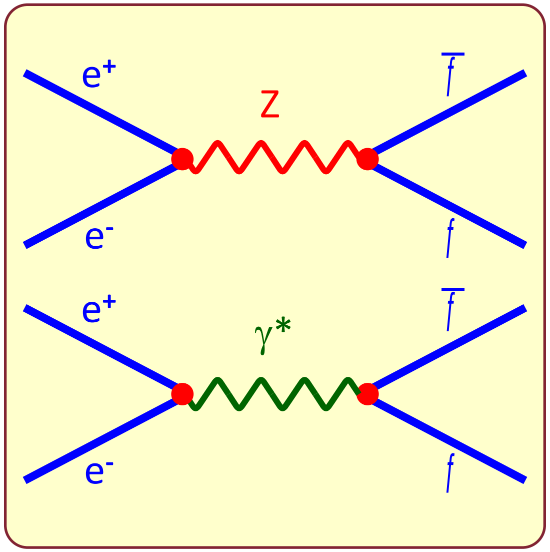
[§ν] :

$g_A^f = t_{3L}^f$

$g_V^f = t_{3L}^f - 2Q^f \sin^2\theta_W$

- for the other Γ's it is found **[lowest order values, NOT "the best"]** :

f	Q_f	g_A^f	g_V^f	Γ_f (MeV)	Γ_f / Γ_μ	R_f (%)
$\nu_e\ \nu_\mu\ \nu_\tau$	0	+½	+½	166	1.99	6.8
$e^-\ \mu^-\ \tau^-$	−1	-½	−.038	83	[1]	3.4
u c [t]	⅔	+½	+.192	286	3.42	11.8
d s b	-⅓	-½	−.346	368	4.41	15.2



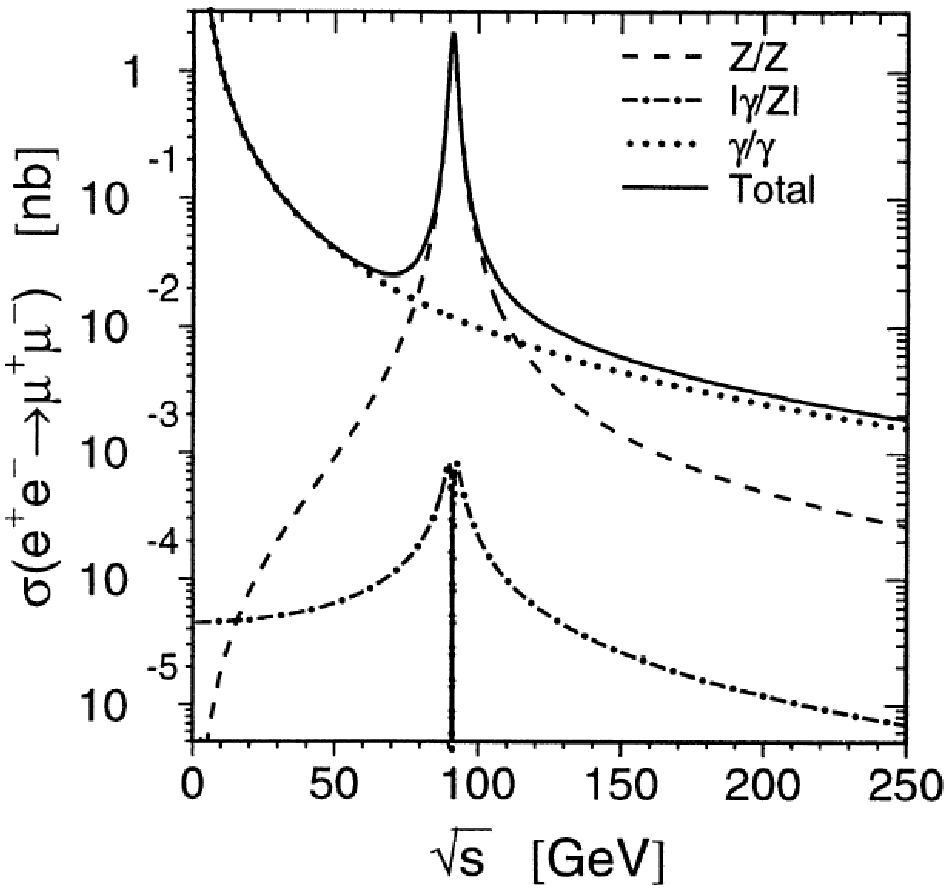
In Born approx. [B = "Born"] :

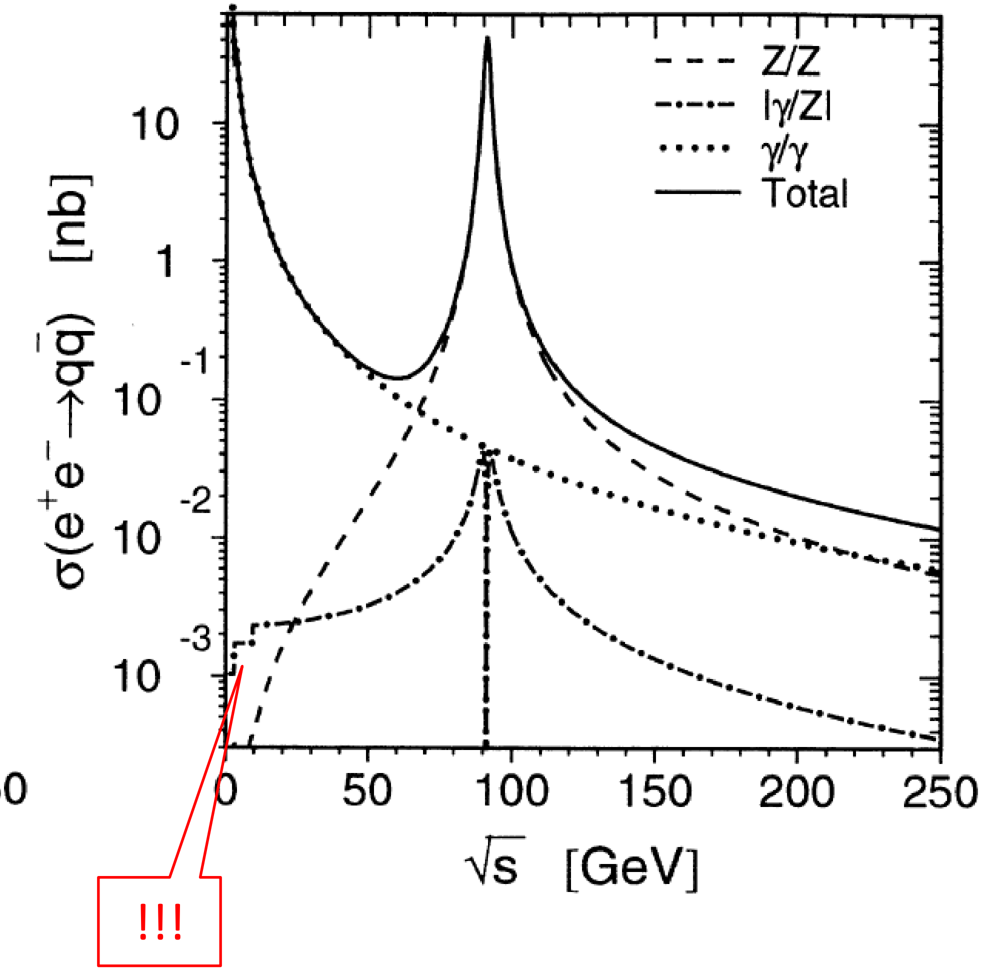
- Γ_Z^B = 2423 MeV, $\Gamma_{hadr.}^B$ = 1675 MeV, $\Gamma_{invis.}^B = \Gamma_\nu^B$ = 498 MeV;
- $R_{hadr.}^B$ = 69.1 %, $R_{lept\pm}^B$ = 10.2 %, $R_{invis.}^B$ = 20.5 %,
- $R_{hadr.}^B / R_{vis.}^B$ = 87.0 %.

remember !

- $\Gamma_Z \approx$ 2.4 GeV, $\Gamma_\nu \approx$ 0.5 GeV,
- $\nu : \ell^\pm : u : d \approx 2 : 1 : 3.4 : 4.4$, hadr : $\ell^\pm : \nu \approx 70 : 10 : 20$.

$e^+e^- \to Z \to f\bar{f}$: predictions





Z/Z and γ^*/γ^* are +ve by definition,

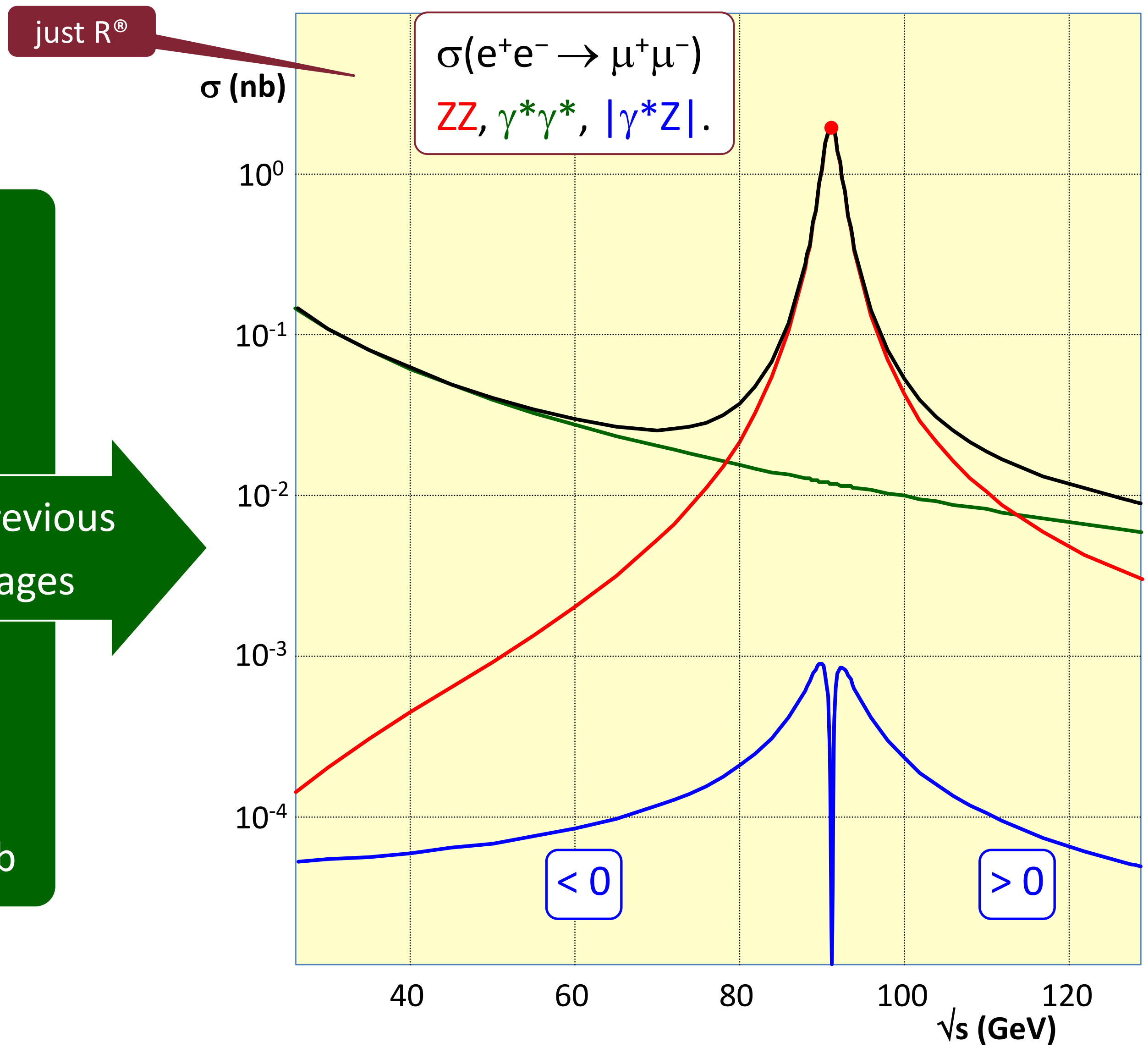
$|\gamma^*/Z|$ is plotted (**<0 @ $\sqrt{s}$<m_Z, >0 @ $\sqrt{s}$>m_Z**).

CERN 89-08

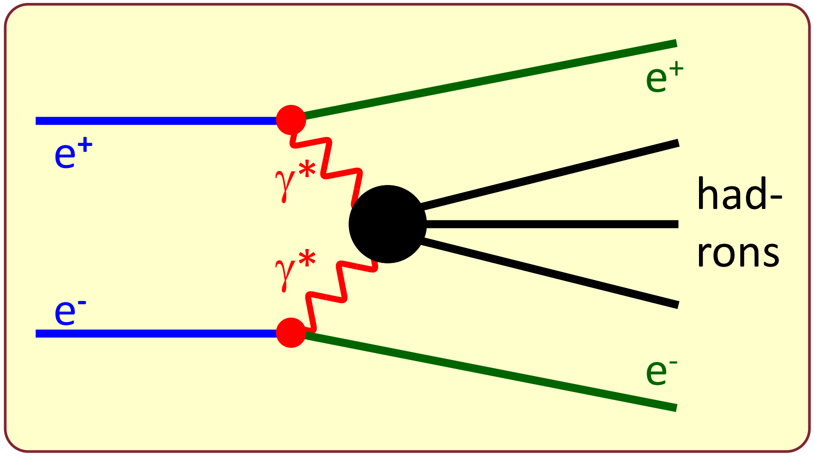
$e^+e^- \to Z \to f\bar{f}$: home-made predictions

m_Z = 91.1876 GeV
Γ_Z = 2.4952 GeV
Γ_e = 0.083984 GeV
Γ_μ = 0.083984 GeV
$1/\alpha_{em}$ = 128.877
q_μ = -1
c_μ = 1
g_V^e = -0.03783
g_V^μ = -0.03783
G_F = 1.1664×10^{-5} GeV^{-2}
$(\hbar c)^2$ = 3.8938×10^5 GeV^2 nb

+ previous pages



$e^+e^- \to Z \to f\bar{f}$: 2 γ physics



Introduce a different process: "2 γ physics":

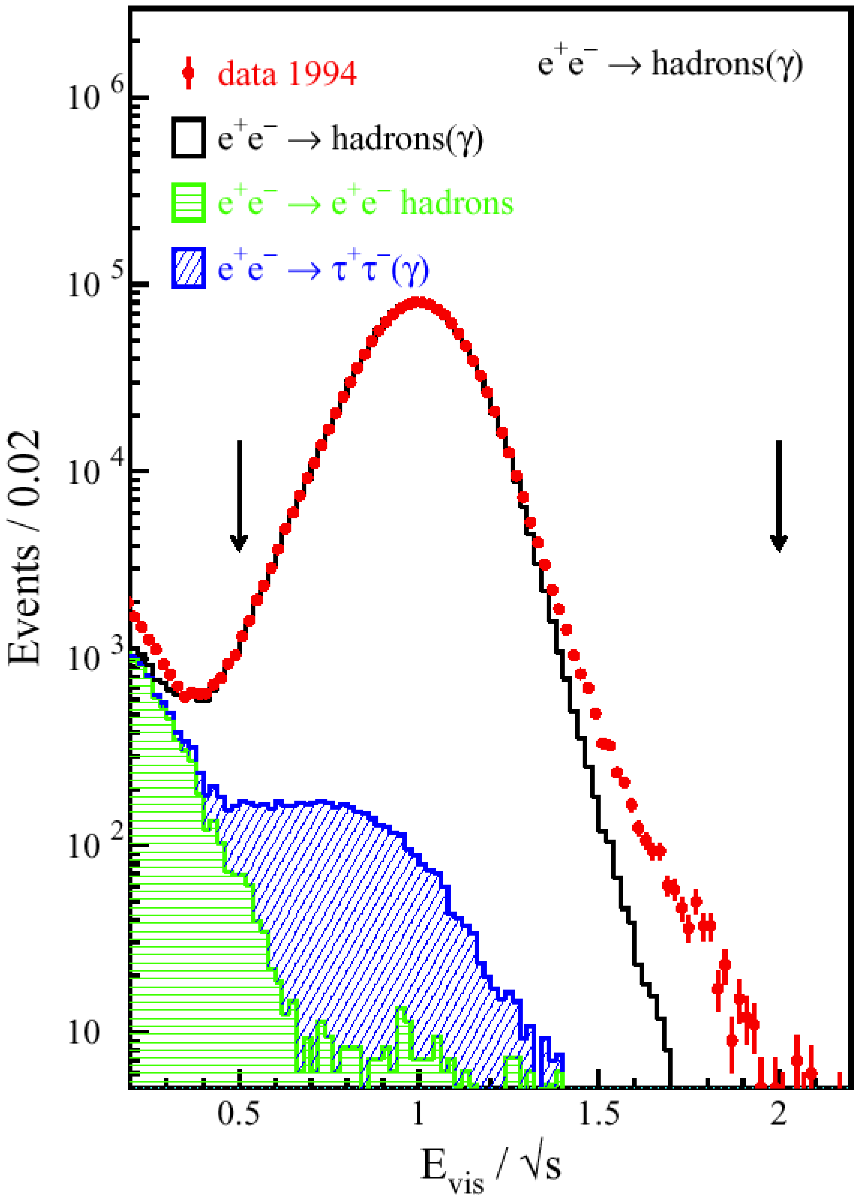
- it is so called because the initial state of the hard collision is given by two γ's;
- the two $e^{\pm}$ of the initial state retain much of the energy, and in most cases escape undetected in the beam chamber;
- classify events in "untagged", "single tag" and "double tag", depending on whether 0, 1, 2 and $e^{\pm}$ are detected;
- lot of nice kinematics [*try it*];

- events studied using two variables:
  - $\sqrt{s} = m_{ini}(e^+e^-)$;
  - $W = m(\gamma\gamma) = m(\text{hadrons})$;
- the study of $\sigma_{\gamma\gamma}$ requires a cut on W, i.e. $\sigma_{\gamma\gamma} = \sigma_{\gamma\gamma}(W > W_{cut})$, both for theory and detection:
  - $\sigma_{\gamma\gamma}$ weakly dependent on $\sqrt{s}$;
  - $\sigma_{\gamma\gamma}$ strongly dependent on W, $\sigma_{\gamma\gamma} \sim e^{-W}$.

Why study "2 γ physics" ? Two main goals:

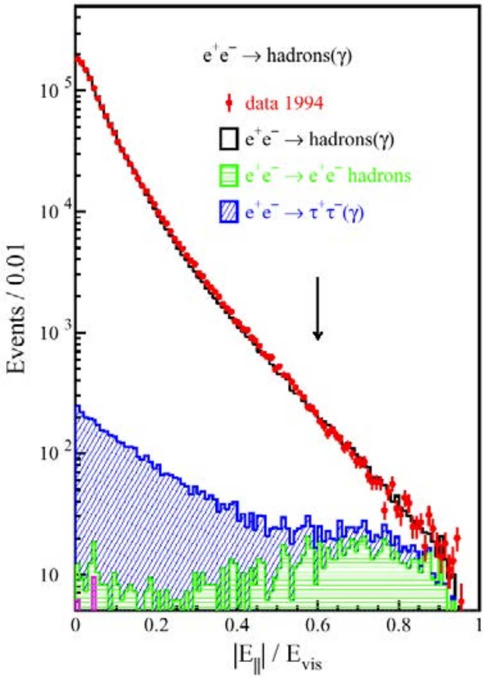
1. *intrinsic interest:*
   - *any process deserves a study;*
   - *rich "factory" of hadron resonances;*
   - *other low-energy processes;*
2. $\sigma_{\gamma\gamma}$ *is large:*
   - *LEP1: subtract from high precision meas.;*
   - *LEP2: typically tiny cross sections → an important background, especially if large* $\not{E}$ *required.*

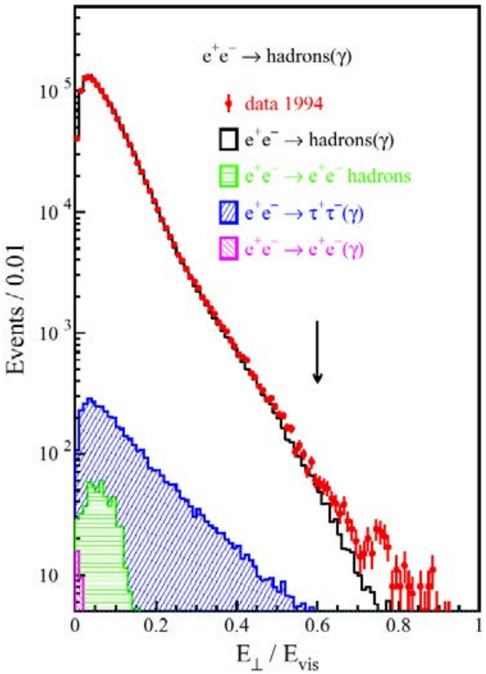
$e^+e^- \rightarrow Z \rightarrow f\bar{f}$: hadrons (1)



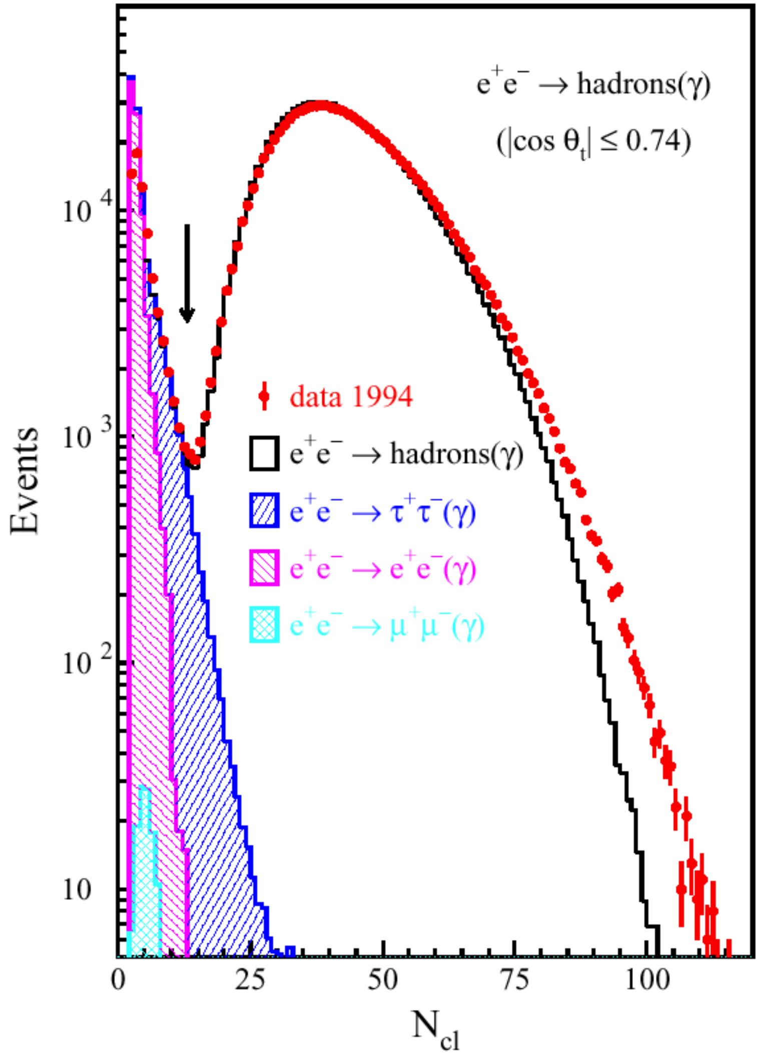
Example : $e^+e^- \rightarrow$ hadrons (i.e. $e^+e^- \rightarrow q\bar{q}$) in L3 1994 *(an old paper, chosen because well written)*. Selection :

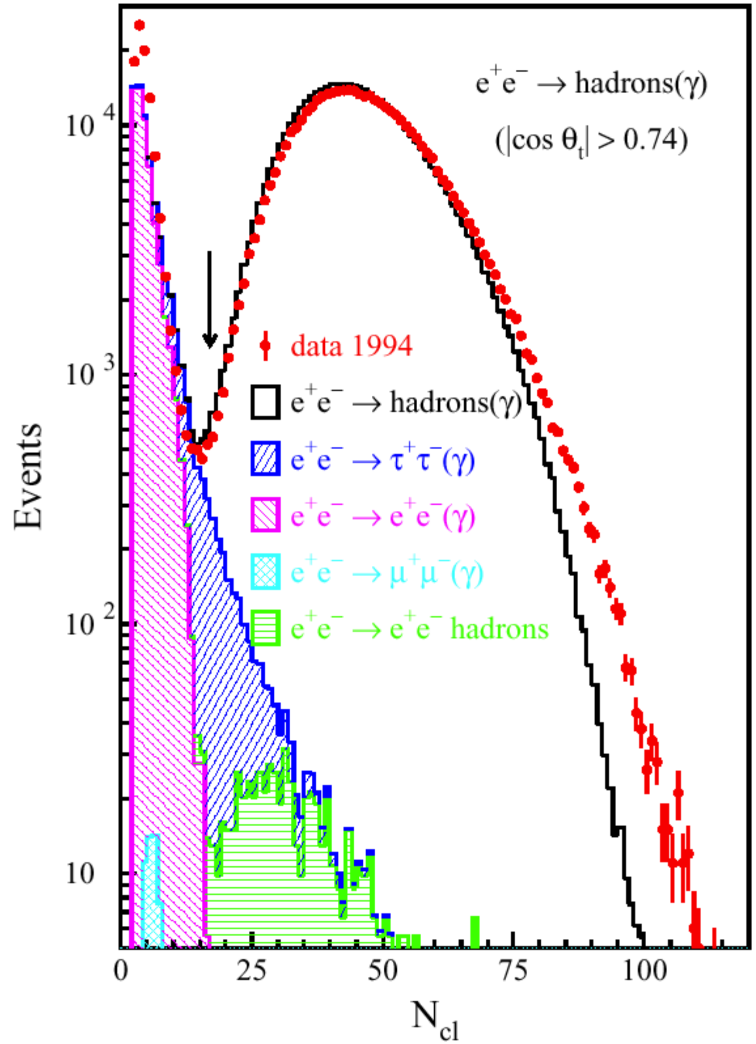
- $0.5 < E_{vis} / \sqrt{s} < 2.0$;
- $|E_{//}| / E_{vis} < 0.6$;
- $|E_{\perp}| / E_{vis} < 0.6$;
- $N_{clusters} > 13$ (barrel), > 17 (endcap) [next]





$e^+e^- \to Z \to f\bar{f}$: hadrons (2)

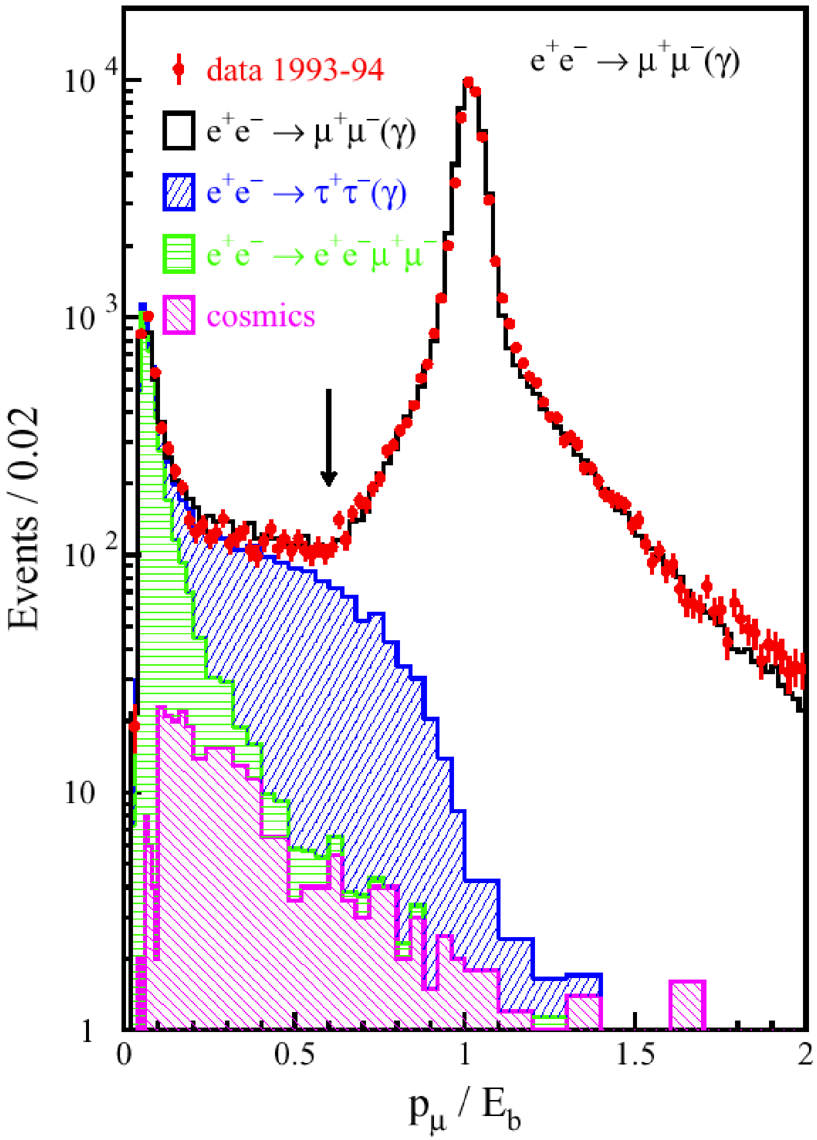


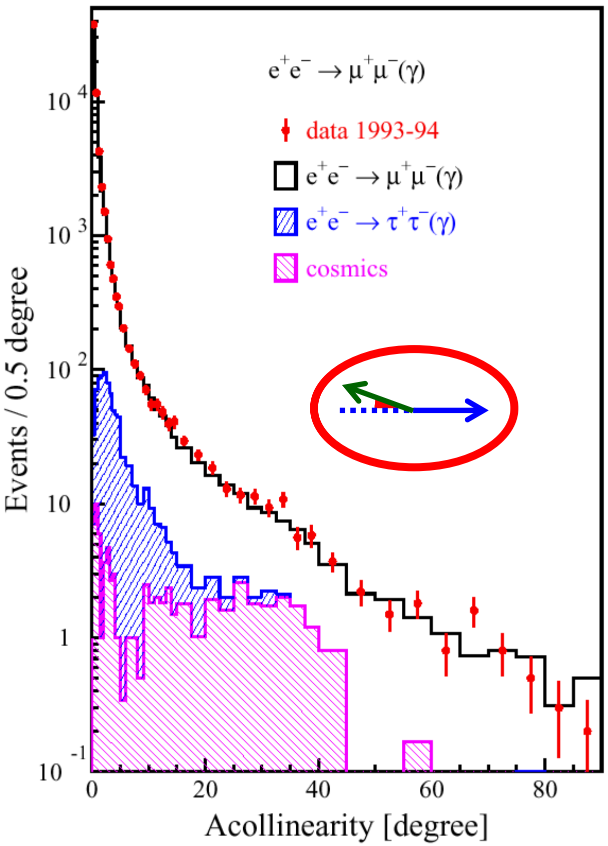


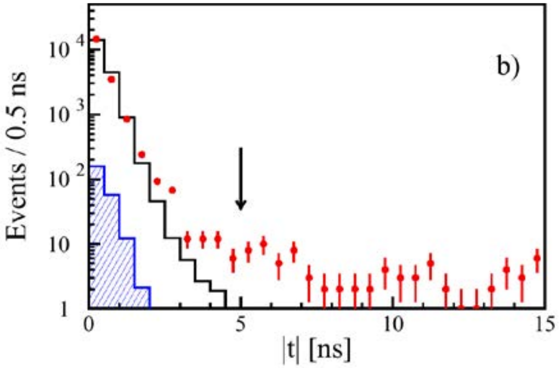
Example : $e^+e^- \to$ hadrons (i.e. $e^+e^- \to q\bar{q}$) in L3 1994 – pag. 2

[$N_{clusters}$ > 13 (barrel), > 17 (endcap)]

$e^+e^- \to Z \to f\bar{f}$: $\mu^+\mu^-$







Other example (same paper) : $e^+e^- \to \mu^+\mu^-$

Selection :

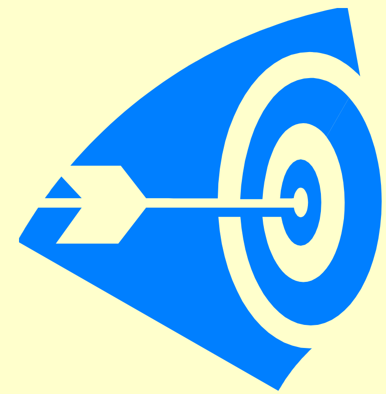
- ≥ 1 μ identified;
- $|p_\mu| > 0.6\ (\sqrt{s}/2)$;
- $\alpha(\mu\mu)$ "small";
- $N_{clusters} < 15$;
- $time_{scintillators}$.

Q. : why μ's have smaller acollinearity than τ's ?

$e^+e^- \to Z \to f\bar{f}$: from W. Tell to LEP

Problem. Two variables (x, y) are normally (=Gauss) distributed with mean (m_x, m_y) and standard deviation $\sigma_x = \sigma_y = \sigma$. Find the distribution of the distance from the center

$$r = \sqrt{(x - m_x)^2 + (y - m_y)^2}.$$



- W. Tell's crossbow;
- the event $\not{E}_T$ at LEP/LHC;
- the sum of momenta of the charged particles wrt the jet axis, ...

Solution:

$$\begin{cases} x - m_x = r\cos\theta \\ y - m_y = r\sin\theta \end{cases}; \quad f_{\text{Gauss}}(t|\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{t^2}{2\sigma^2}\right];$$

$$f(x,y) = f(x|\sigma) \times f(y|\sigma) = \frac{1}{2\pi\sigma^2}\exp\left[-\frac{x^2+y^2}{2\sigma^2}\right];$$

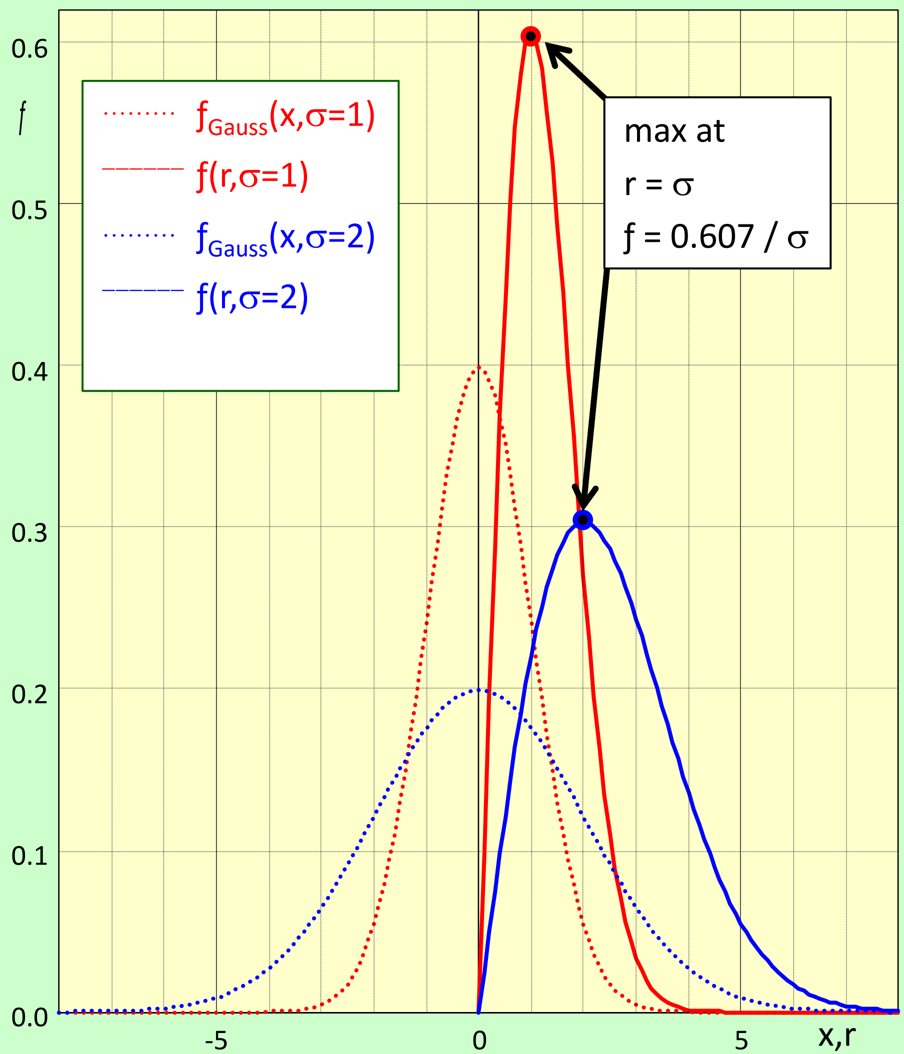
$$J\left(\frac{x,y}{r,\theta}\right) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix} = r;$$

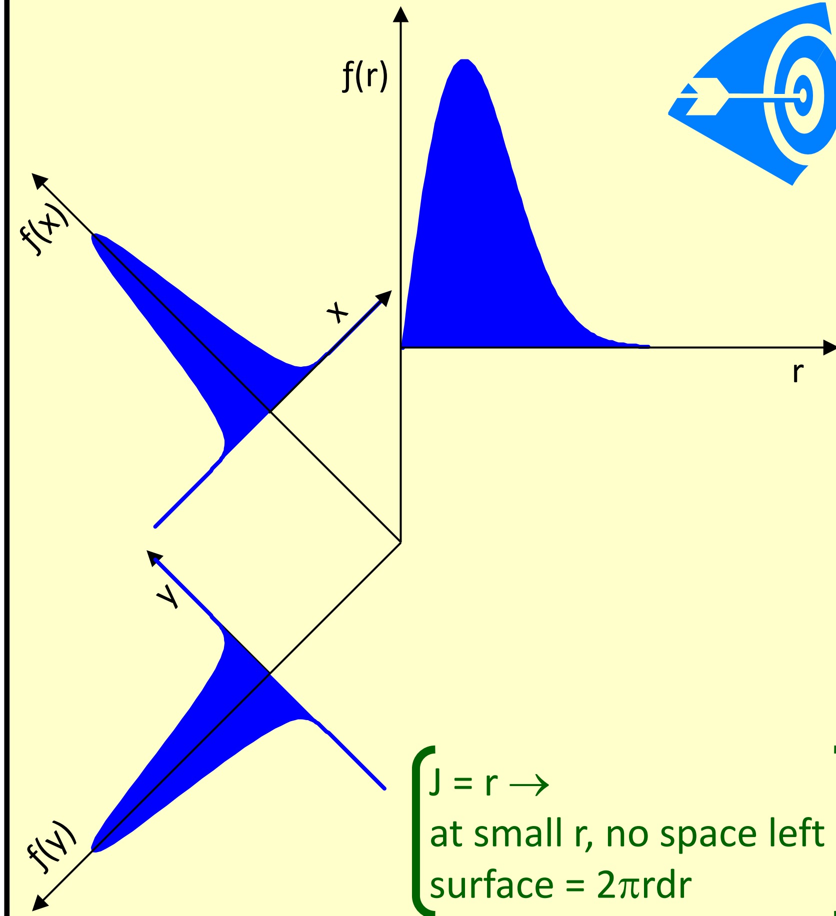
m_x and m_y are translations wrt centre; they do NOT influence the result.

$$f(r,\theta) = f(x,y) \times \left|J\left(\frac{x,y}{r,\theta}\right)\right| = \frac{r}{2\pi\sigma^2}\exp\left[-\frac{r^2}{2\sigma^2}\right]; \quad f(r) = \int_0^{2\pi} d\theta \; f(r,\theta) = 2\pi f(r,\theta) = \frac{r}{\sigma^2}\exp\left[-\frac{r^2}{2\sigma^2}\right].$$

$e^+e^- \to Z \to f\bar{f}$: a W. Tell tale

$$f(r) = r e^{-r^2/(2\sigma^2)} / \sigma^2$$

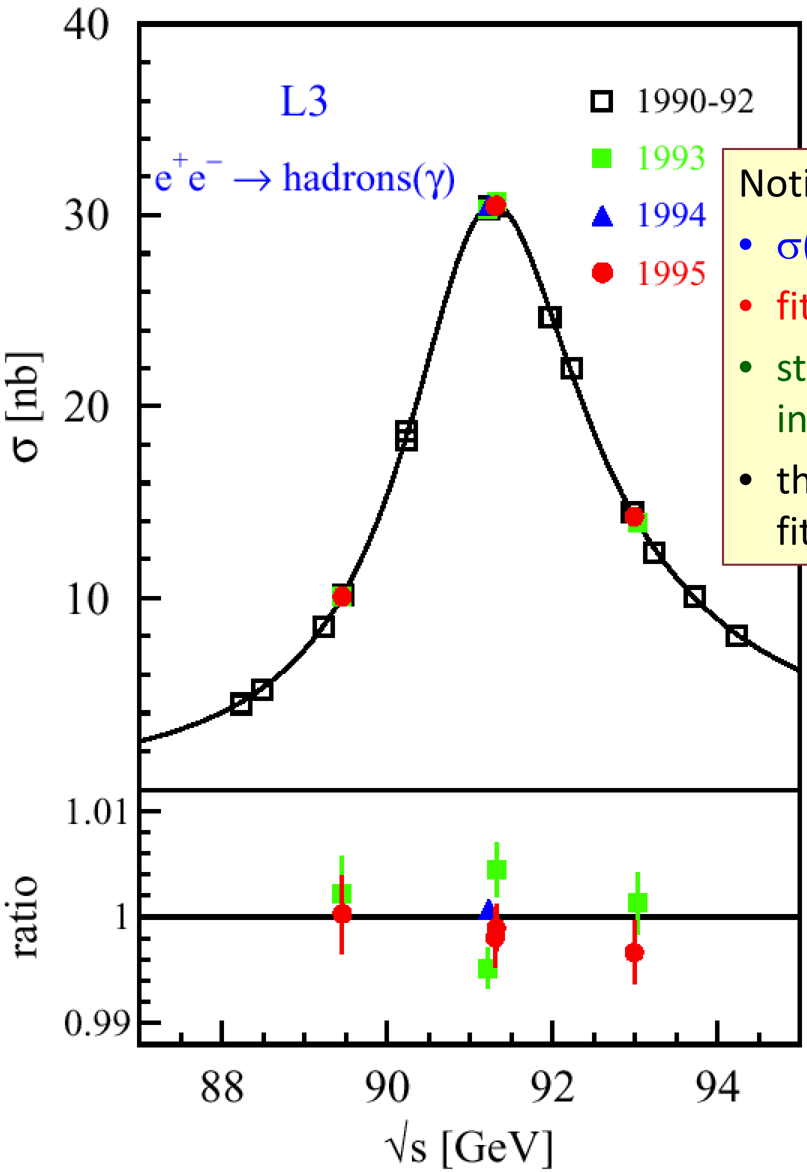




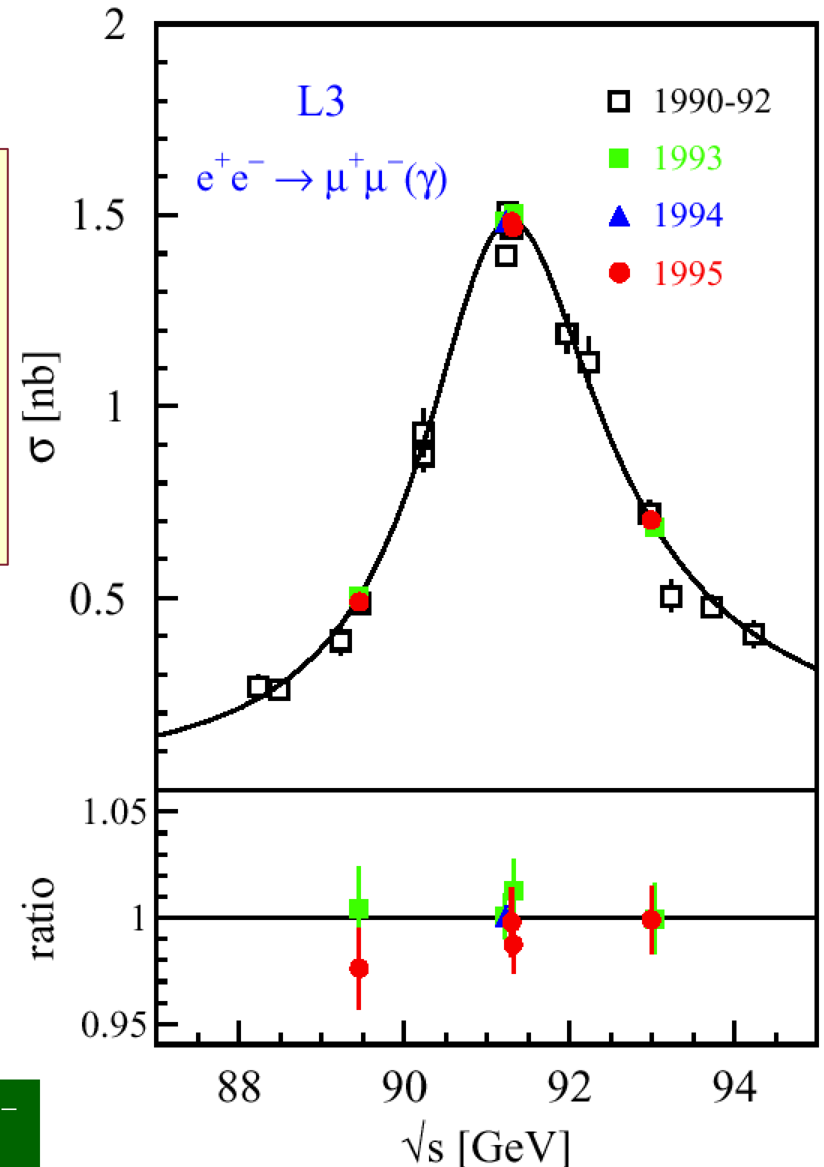
$J = r \to$
at small r, no space left
surface = $2\pi r dr$

next question:
the case $\sigma_x \neq \sigma_y$
[easy, needs only one smart trick]

$e^+e^- \to Z \to f\bar{f}$: lineshape



Notice:
- σ(had) >> σ(μμ);
- fit quality;
- strategy change in 1993;
- the line is the SM fit (see later).



for $e^+e^- \to \tau^+\tau^-$ see later.

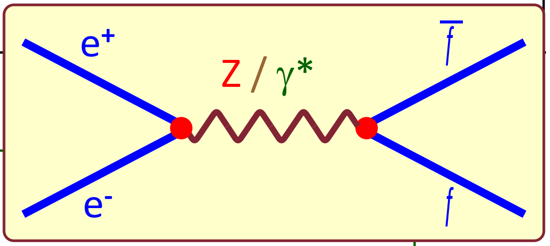
$d\sigma(e^+e^- \to f\bar{f})\ /\ d\Omega$

PDG 2016, 10.31-32-36

Differential cross-section in lowest (Born) order:

$$\frac{d\sigma_{Born}\left(e^+e^- \to f\bar{f}\right)}{d\cos\theta} = \frac{\pi\alpha^2(s)c_f}{2s}\left\{\begin{array}{l}\left(1+\cos^2\theta\right)\times\left[\begin{array}{l}Q_e^2Q_f^2 - 2\boxed{\chi}Q_fQ_eg_V^eg_V^f\cos\delta_R + \\ +\boxed{\chi^2}\left[\left(g_A^e\right)^2+\left(g_V^e\right)^2\right]\left[\left(g_A^f\right)^2+\left(g_V^f\right)^2\right]\end{array}\right] + \\ +2\cos\theta\times\left[-2\boxed{\chi}Q_eQ_fg_A^eg_A^f\cos\delta_R + 4\boxed{\chi^2}g_A^eg_A^fg_V^eg_V^f\right]\end{array}\right\};$$

$$\chi = \frac{G_F}{2\sqrt{2}\pi\alpha(s)}\times\frac{sm_Z^2}{\left(m_Z^2-s\right)^2+m_Z^2\Gamma_Z^2}; \qquad \tan\delta_R = \frac{m_Z\Gamma_Z}{m_Z^2-s};$$



$$\left.\begin{array}{l}A_f^{FB}\left(\sqrt{s}\right) \equiv \dfrac{\sigma\left(\cos\theta>0,\sqrt{s}\right)-\sigma\left(\cos\theta<0,\sqrt{s}\right)}{\sigma\left(\cos\theta>0,\sqrt{s}\right)+\sigma\left(\cos\theta<0,\right)}; \\ A_f^{FB}\left(\sqrt{s}=m_Z,\ Z_{s-channel}\ \text{only}\right) = \\ \qquad = 3\dfrac{g_V^eg_A^e}{\left(g_V^e\right)^2+\left(g_A^e\right)^2}\times\dfrac{g_V^fg_A^f}{\left(g_V^f\right)^2+\left(g_A^f\right)^2};\end{array}\right\}$$

$\boxed{A_f^{FB}}$ is the "forward-backward asymmetry" for $e^+e^- \to f\bar{f}$.

$d\sigma(e^+e^- \to f\bar{f})\ /\ d\Omega$: comments

$$\frac{d\sigma_{\text{Born}}\left(e^+e^- \to f\bar{f}\right)}{d\cos\theta} = \frac{\pi\alpha^2(s)c_f}{2s}\left\{ \begin{aligned} &\left(1+\cos^2\theta\right)\times\left[\begin{aligned} &Q_e^2Q_f^2 - 2\boxed{\chi}Q_fQ_e g_V^e g_V^f \cos\delta_R + \\ &+\boxed{\chi^2}\left[\left(g_A^e\right)^2+\left(g_V^e\right)^2\right]\left[\left(g_A^f\right)^2+\left(g_V^f\right)^2\right]\end{aligned}\right] + \\ &+2\cos\theta\times\left[-2\boxed{\chi}Q_eQ_f g_A^e g_A^f\cos\delta_R + 4\boxed{\chi^2}g_A^e g_A^f g_V^e g_V^f\right]\end{aligned}\right\};$$

$$A_f^{FB}\left(\sqrt{s}\right) \equiv \frac{\sigma\left(\cos\theta>0,\sqrt{s}\right)-\sigma\left(\cos\theta<0,\sqrt{s}\right)}{\sigma\left(\cos\theta>0,\sqrt{s}\right)+\sigma\left(\cos\theta<0,\sqrt{s}\right)} \xrightarrow{\sqrt{s}\to m_Z} 3\,\frac{g_V^e g_A^e}{\left(g_V^e\right)^2+\left(g_A^e\right)^2}\times\frac{g_V^f g_A^f}{\left(g_V^f\right)^2+\left(g_A^f\right)^2}.$$

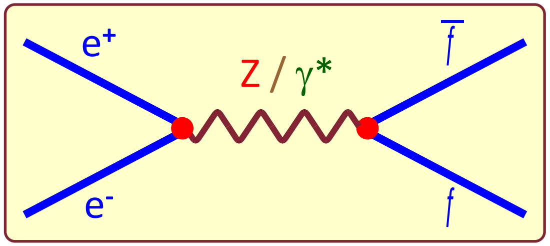
mediators : γ, Z [= $Z_A + Z_V$];

$\mathbb{P}$-cons : $\gamma\gamma$, γZ_V, ZZ [= $Z_A^2 + Z_V^2$];

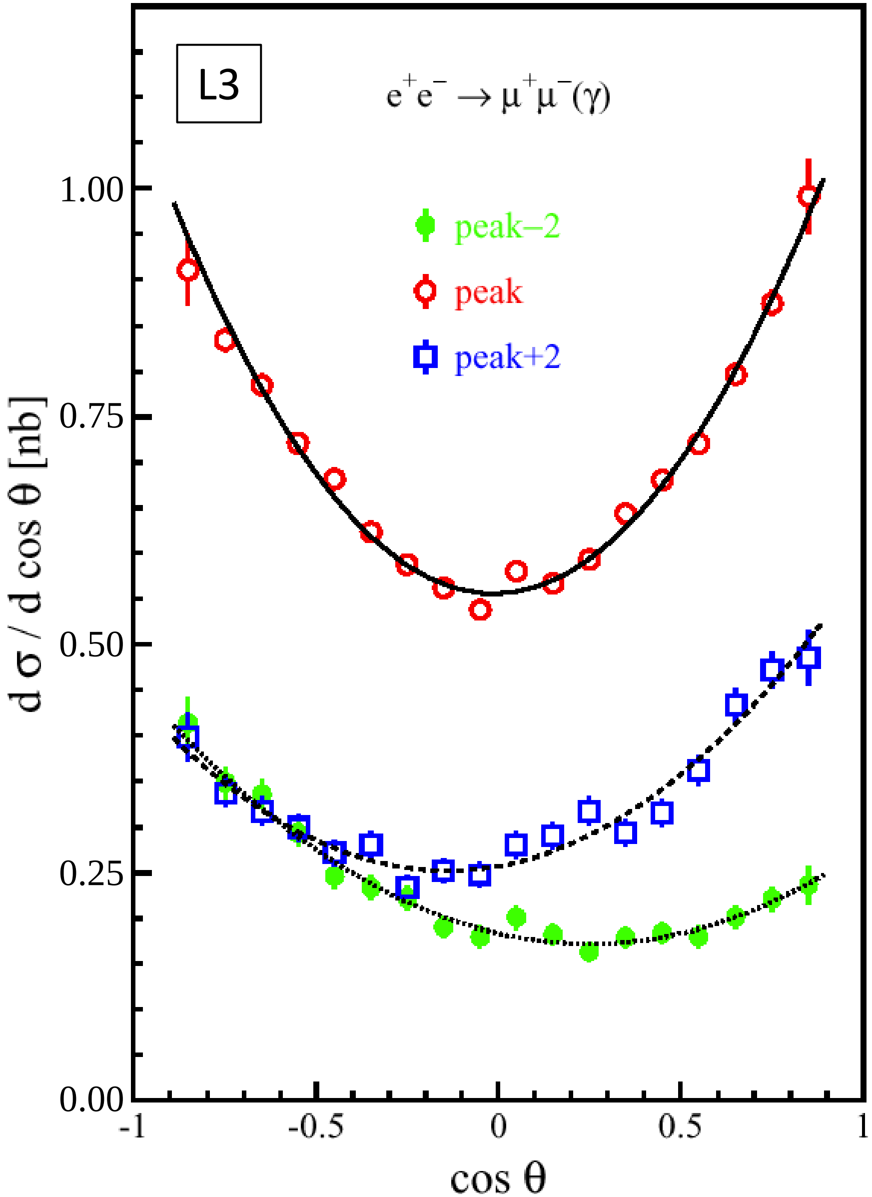
$\mathbb{P}$-viol. : γZ_A, $Z_A Z_V$.

- standard SM computation for $Z_s \oplus \gamma_s$ only (average on initial and sum on final polarization), then sum on φ:
- notice : the term $\propto$ ($\cos\theta$) is anti-symmetric; it does NOT contribute to σ_{tot} ($\int \cos\theta\, d\cos\theta = 0$), but only to the ($\mathbb{P}$-violating) forward-backward asymmetry;
- the $\mathbb{P}$-violation clearly comes from the interference between the vector (γ + Z_V) and axial (Z_A) terms.

- at the pole ($\sqrt{s}=m_Z$) :
  - $\cos\delta_R = 0$;
  - the asymmetry, i.e. the term $\propto \cos\theta$, is $\propto g_V^e$ (very small) for all fermions;
  - for the $\mu^+\mu^-$ case [easily measurable], it is even smaller ($\propto g_V^e g_V^\mu$).

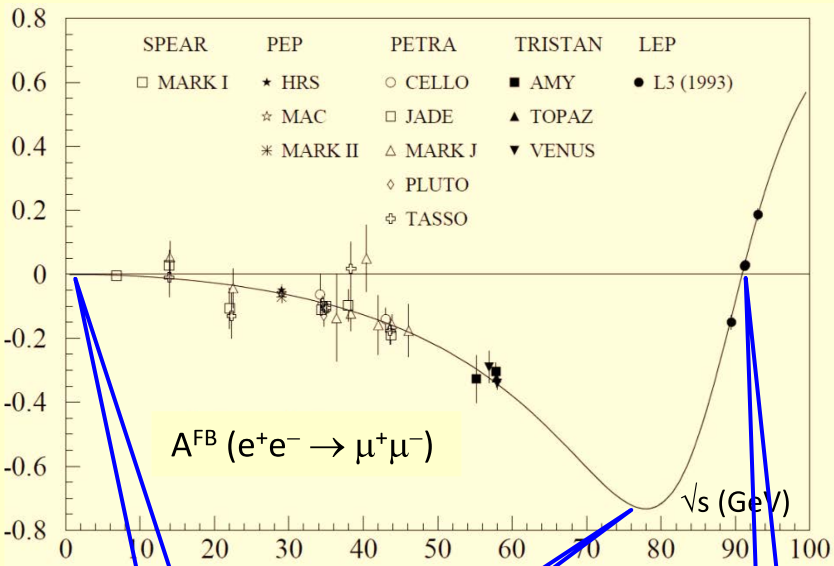


$d\sigma(e^+e^- \to f\bar{f})$ / $d\Omega$: data



- Experimentally, the main problem is the selection $f \leftrightarrow \bar{f}$ (i.e. $\theta \leftrightarrow \pi-\theta$). This is
  - essentially impossible for light quarks $u \leftrightarrow \bar{u}$, $d \leftrightarrow \bar{d}$ (despite heroic efforts based on charge counting);
  - difficult for heavy quarks c,b (based on lepton charge in semileptonic quark decays, e.g. $c \to s\ell^+\nu$, $\bar{c} \to \bar{s}\ell^-\bar{\nu}$);
  - "simple" for $\mu^\pm$ (only problem: wrong sagitta sign because of high momentum);
  - best channel for $d\sigma/d\cos\theta$ and A_{FB}: $e^+e^- \to \mu^+\mu^-(\gamma)$;
- unfortunately, $A_{FB}(\sqrt{s}=m_Z)$ is very small in the $\ell^+\ell^-$ channels, due to the extra small factor g_V^μ;
- notice the asymmetry change for peak ±2 GeV.

$d\sigma(e^+e^- \to f\bar{f}) / d\Omega$: $A_{fb}(\mu^+\mu^-)$

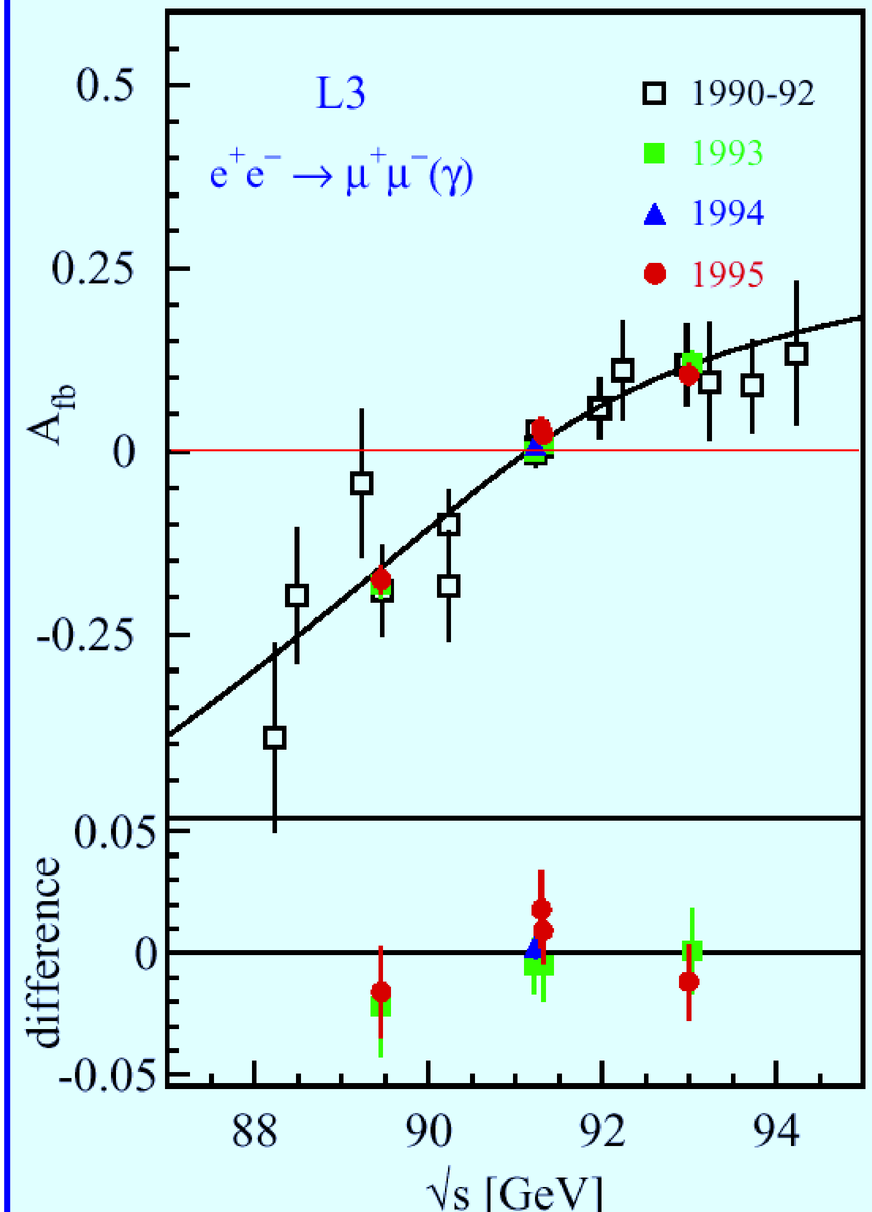


γ only → V only → A^{FB} = 0

Z ≈ A, γ = V → A^{FB} max @ max interference [no exp ever]

√s ≈ m_Z → A dominates → A^{FB} ≈ 0

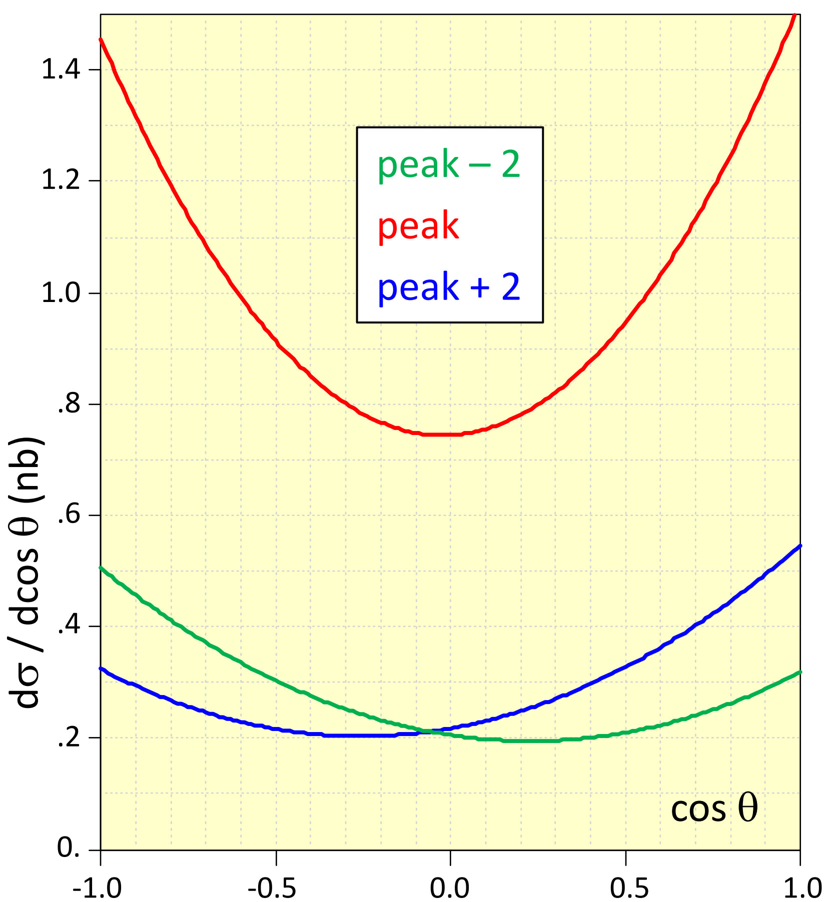
full √s range + SM prediction

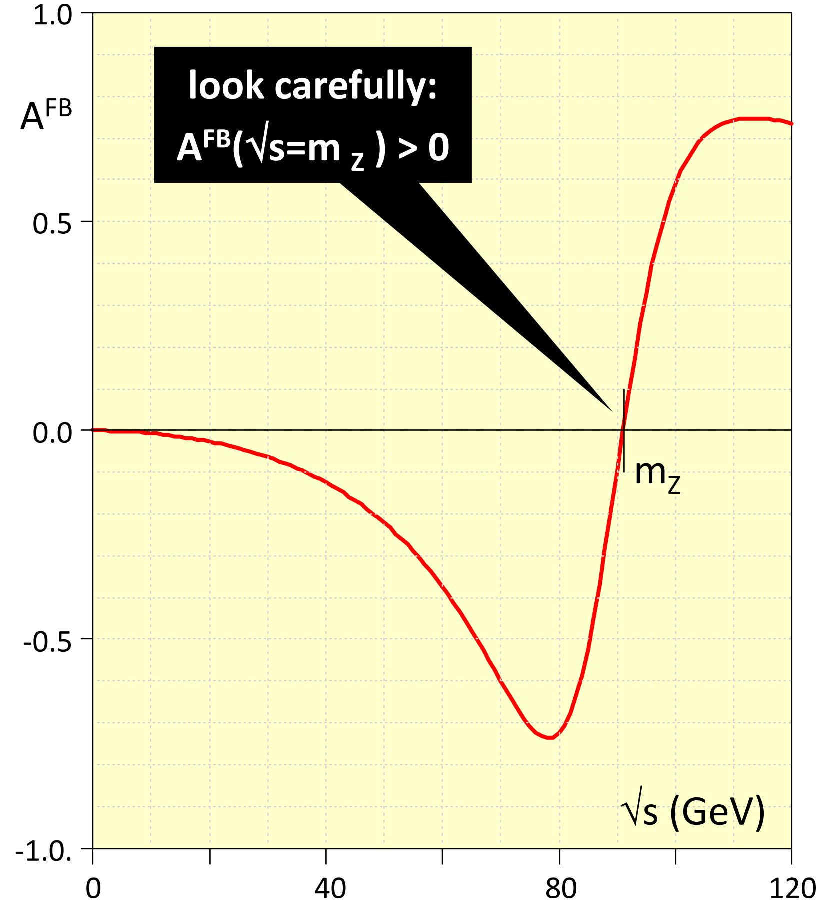


$d\sigma(e^+e^- \rightarrow f\bar{f})$ / $d\Omega$: problem

Problem. Compute $d\sigma/d\cos\theta$ and A^{FB} at lowest order from the formulæ. This is a case where the "tree approx." fails. Explain where and why.

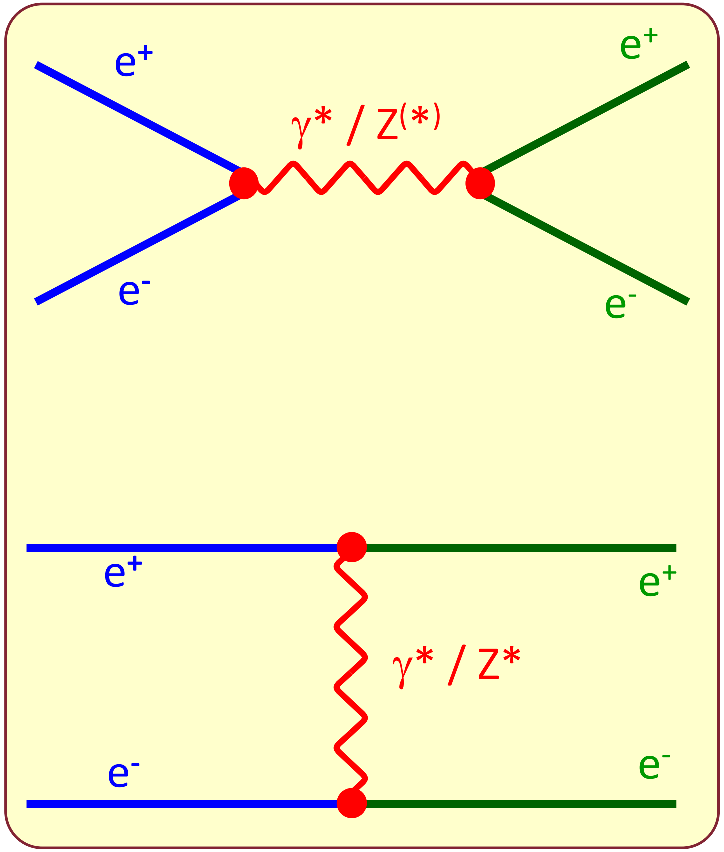
If no success, look to Grünewald, op. cit., pag. 230-232 [simplified explanation: higher orders and selection criteria are important, expecially for peak+2 (→ init. state brem). The correct approach is to use higher orders also in the prediction].



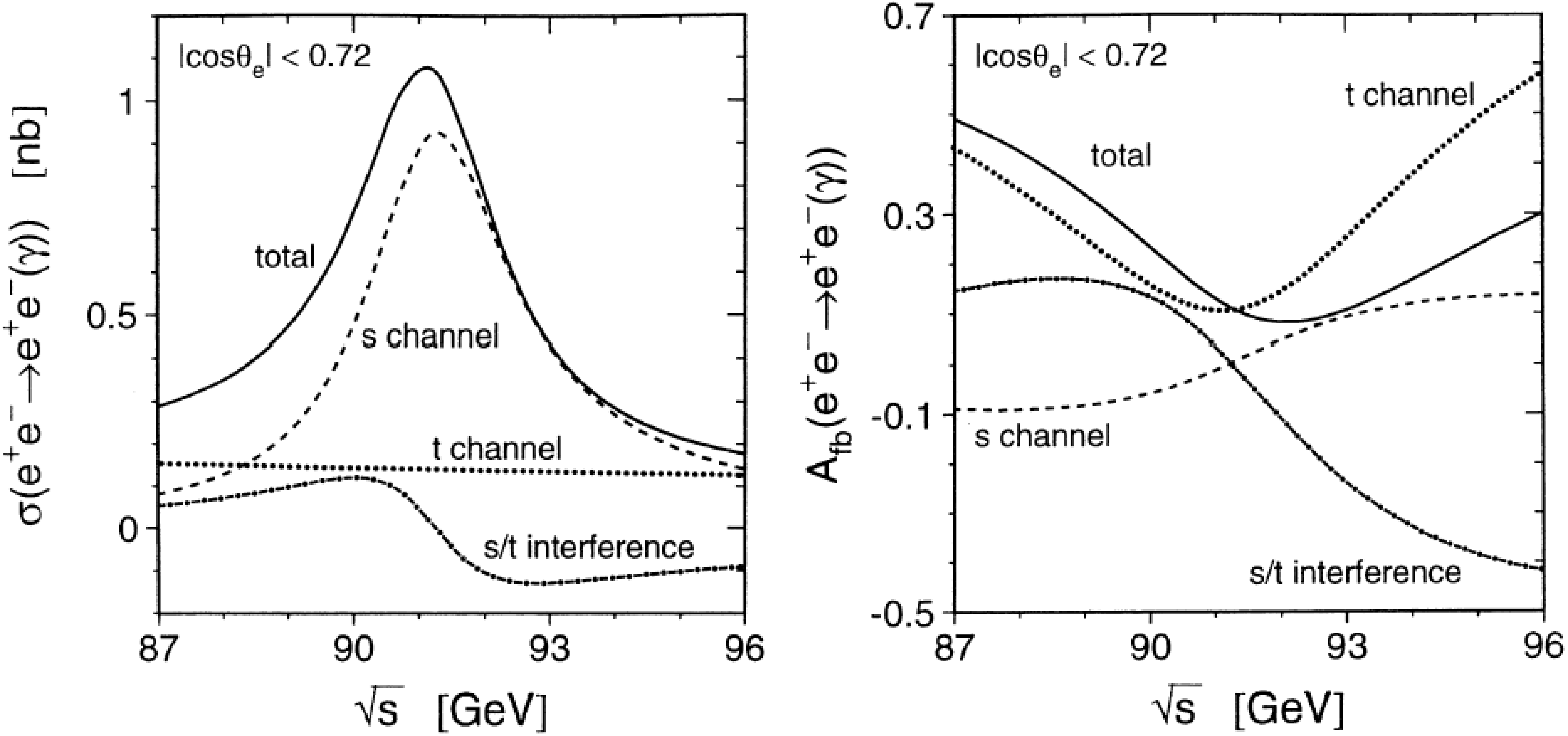


$e^+e^- \rightarrow Z \rightarrow e^+e^-$

- Bhabha scattering is more difficult, due to the presence of another Feynman diagram: the γ^* / Z exchange in the t-channel;
- 4 Feynman diagrams $\rightarrow$ 10 terms :
  - Z s-channel (Z_s);
  - γ^* s-channel (γ_s);
  - Z t-channel (Z_t);
  - γ^* t-channel (γ_t);
  - 6 interferences;
- qualitatively :
  - @ $\sqrt{s} \approx m_Z$ and $\theta >> 0°$, $\mathbf{Z_s}$ dominates.
  - @ $\theta \approx 0°$, $\boldsymbol{\gamma_t}$ dominates for all $\sqrt{s}$;
  - @ $\sqrt{s} << m_Z$ and $\theta >> 0°$, $\boldsymbol{\gamma_s}$ and $\boldsymbol{\gamma_t}$ are both important, while $\mathbf{Z_s}$ is negligible.

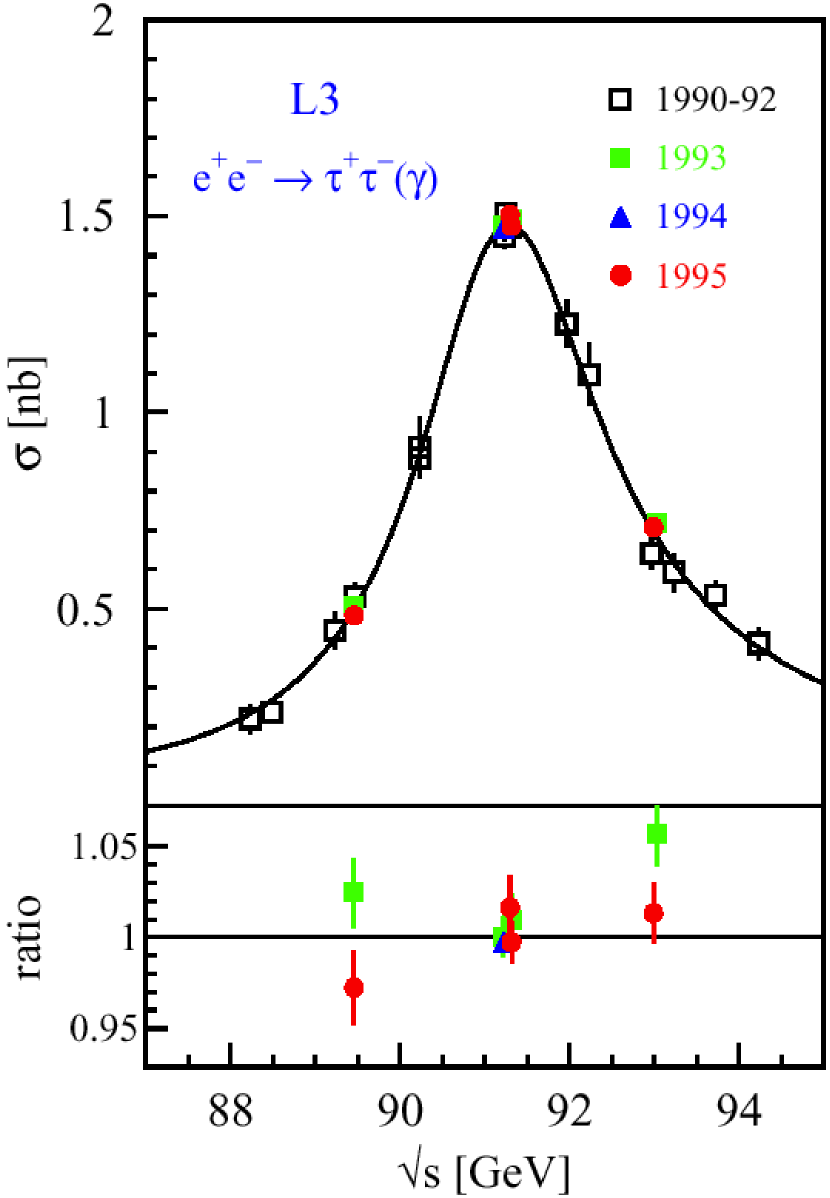


$e^+e^- \rightarrow Z \rightarrow e^+e^-$: σ_{SM}

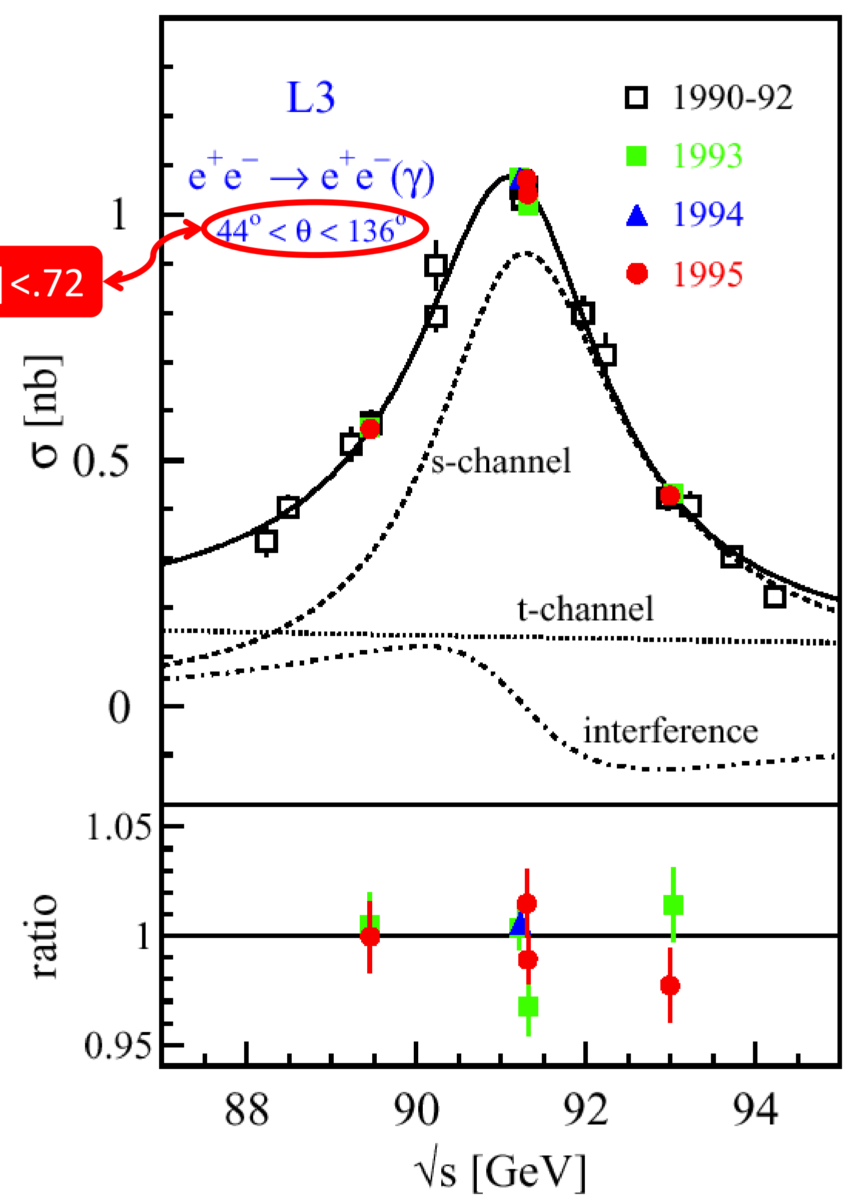


- s, t, interference vs $\sqrt{s}$, with a θ cut ($|\cos\theta| < 0.72$, i.e. $44° < \theta < 136°$);
- data @ $|\cos\theta| \approx 1$ taken, but not used here [used for lumi];
- notice : the cut on $\cos\theta$ is NOT instrumental, but used OFFLINE to enhance Z_s over γ_t, considered as bckgd.

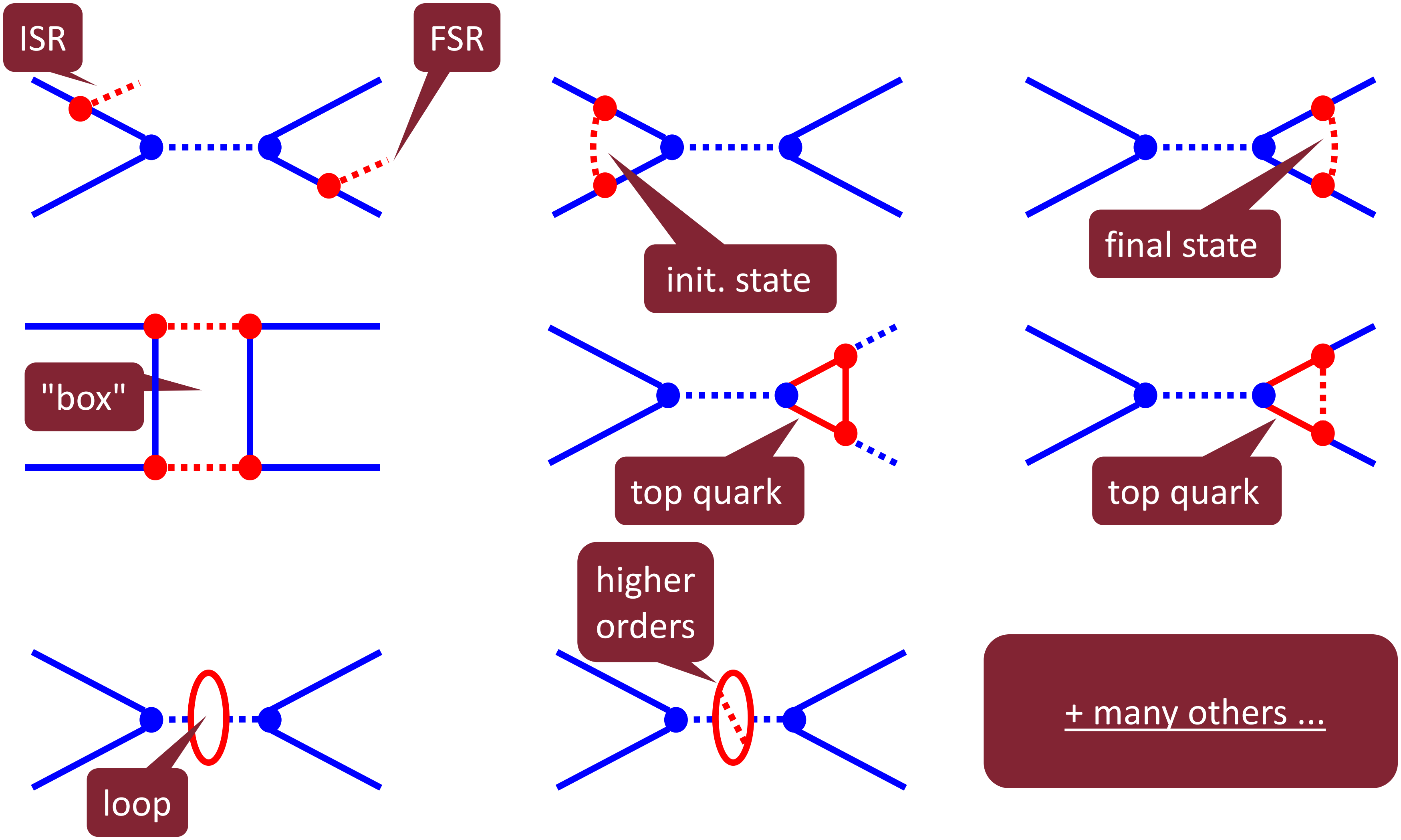
$e^+e^- \to Z \to e^+e^-$: results



|cosθ|<.72



radiative corrections



radiative corrections: what ? why ?

what ?

- higher orders (both SM and bSM);
- dependent on full SM, QCD included;
- conventionally, classified into QED, weak, QCD, bSM (if any);
- ... or initial and final state;
- also particles not kinematically allowed at lower $\sqrt{s}$ (e.g. top, Higgs);

computable ?

- in principle yes, if all parameters known;
- in practice, successive approximations ("order n");

necessary ?

- yes, because required by the measurement accuracy;

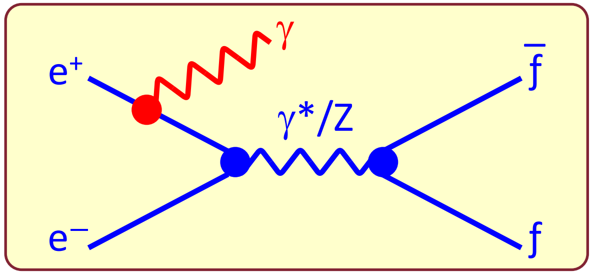
useful ?

- yes, because they give an indirect access to higher energy, by making lower energy observables (like m_z) dependent on higher energy parameters (like m_{top} or m_H);
- i.e., they "raise" the accessible $\sqrt{s}$;
- + more accurate and powerful test of the theory;
- [much work, theses, papers, ...];

how to use the bSM part (e.g. SUSY), both tree-level and higher orders ?

- first, do not include it, and look for discrepancies;
- if disagreement (**εὕρηκα !!!**), include physics bSM and look for agreement;
- if not $\rightarrow$ put a **limit** on physics bSM.

radiative corrections: ISR kinematics



One of the simplest r.c. is the QED brem of a (real) γ from one of the initial state $e^\pm$: **ISR** (Initial State Rad.);

- the kinematics is :

$$e^+e^- (\sqrt{s}, \quad 0, \quad 0);$$
$$\gamma (E_\gamma, \quad E_\gamma \cos\alpha_\gamma, \quad E_\gamma \sin\alpha_\gamma);$$
$$f\bar{f} (\sqrt{s} - E_\gamma, -E_\gamma \cos\alpha_\gamma, -E_\gamma \sin\alpha_\gamma);$$

$$s' \equiv m^2_{f\bar{f}} = \left(\sqrt{s} - E_\gamma\right)^2 - E_\gamma^2 = s\left(1 - 2E_\gamma/\sqrt{s}\right);$$

$$z \equiv s'/s = 1 - 2E_\gamma/\sqrt{s}; \qquad [s' < s \rightarrow z < 1]$$

E_γ is fixed and α-independent:

$$E_\gamma = \frac{\sqrt{s}}{2}\frac{s-s'}{s} = \frac{s-s'}{2\sqrt{s}} = \frac{s-m^2_{f\bar{f}}}{2\sqrt{s}}.$$

- **LEP 1** ($\sqrt{s} < m_z$ + few GeV) :
  - $\sqrt{s'} \approx m_z$, (but Γ_z) $\rightarrow$ large $\Delta E_\gamma/E_\gamma$;
  - α_γ small (brem. dynamics), γ's mostly in the beam pipe;
  - condition : $2m_f \leq \sqrt{s'} \leq \sqrt{s}$;
- **LEP 2** ($\sqrt{s} >> m_z$) :
  - $\sqrt{s'} \approx m_z$ (because of resonance), known as "**return to the Z**";
  - photon is really monochromatic ($\Gamma_z << E_\gamma$) and very energetic;
  - α_γ small (brem. dynamics), γ's mostly in the beam pipe, Z's with high longitudinal momentum, event very unbalanced;
  - events easily removed in the analysis, but it decreases the effective event yield.

radiative corrections: ISR results

Theoretical treatment :

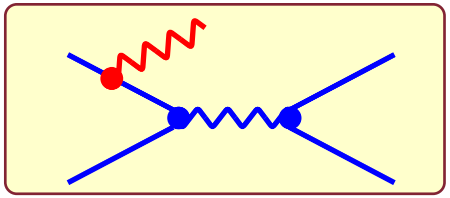
- assume factorization (ISR) $\leftrightarrow$ (Z formation);
- the Z formation at $\sqrt{s'}$ is equivalent to the standard process at $\sqrt{s}$, without ISR :

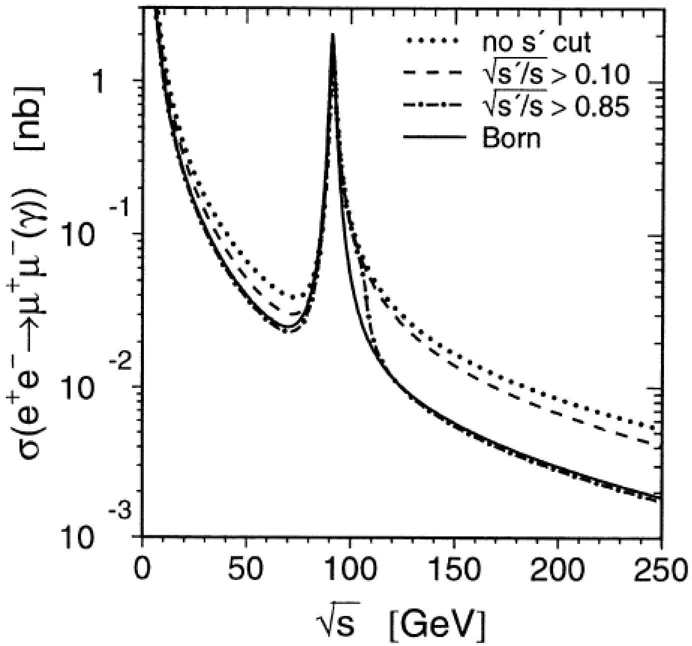
$$\sigma_{ISR}(e^+e^- \to f\bar{f}\gamma;\sqrt{s}) =$$

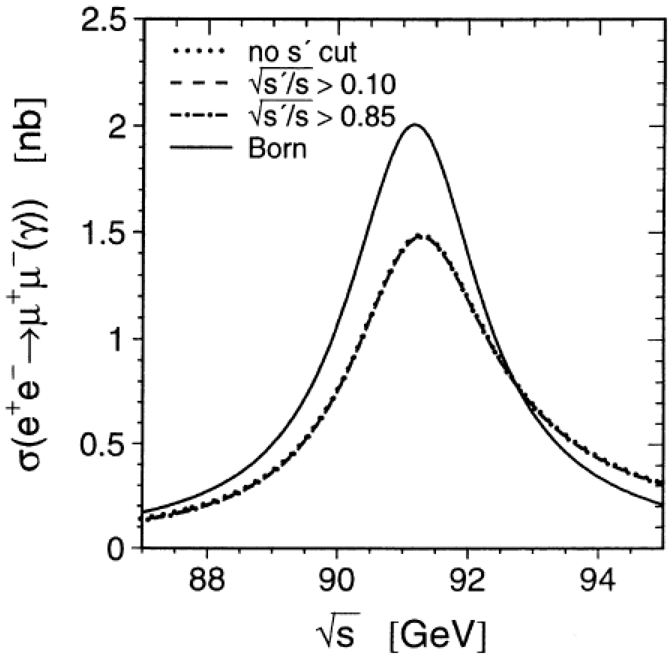
$$= \int_{4m_f^2/s}^{1} dz \begin{pmatrix} R(z,s,\alpha_\gamma) \times \\ \times\ \sigma_{Born}(e^+e^- \to f\bar{f};\sqrt{zs}) \end{pmatrix};$$

- $R(z,s,\alpha_\gamma)$ = ***radiator***, i.e. probability (function of $\sqrt{s}$, z, α_γ) for γ brem;
- R calculable in QED at a given order.

At LEP 2, cut on z ($\approx E_{vis}/\sqrt{s}$), tipically z<0.85).







radiative corrections: results for m_Z

A precise computation requires much tedious work : these values are just for understanding [see fig.] :

- $\sqrt{s}\,|^{max}_{Born} \approx m_Z\,(1+\gamma^2)^{1/4} \approx m_Z\,(1+\frac{1}{4}\gamma^2) \approx$
  $\approx m_Z + 17$ MeV;
  [slightly larger]
- $\sqrt{s}\,|^{max}_{ISR} \approx m_Z\,(1-\frac{1}{4}\gamma^2) + \pi\beta\Gamma_z/8$
  $\approx m_Z + 89$ MeV;
  [slightly larger];
- $\sigma_0^f \equiv \sigma_{Born}(e^+e^- \to f\bar{f};\ \sqrt{s}=m_Z) =$
  $= 12\pi\Gamma_e\Gamma_f\,/\,(m_Z^2\Gamma_Z^2)$;
- $\sigma(e^+e^-\to f\bar{f})\,|^{max}_{Born} \approx \sigma_0^f\,(1+\frac{1}{4}\gamma^2) \approx$
  $\approx \sigma_0^f\,(1+.00019)$
  [slightly larger] ;
- $\sigma(e^+e^-\to f\bar{f})\,|^{max}_{ISR} \approx \sigma_0^f\,\gamma^\beta\,(1+\delta_{sup}) \approx$
  $\approx 0.75\,\sigma_0^f$
  [much smaller];

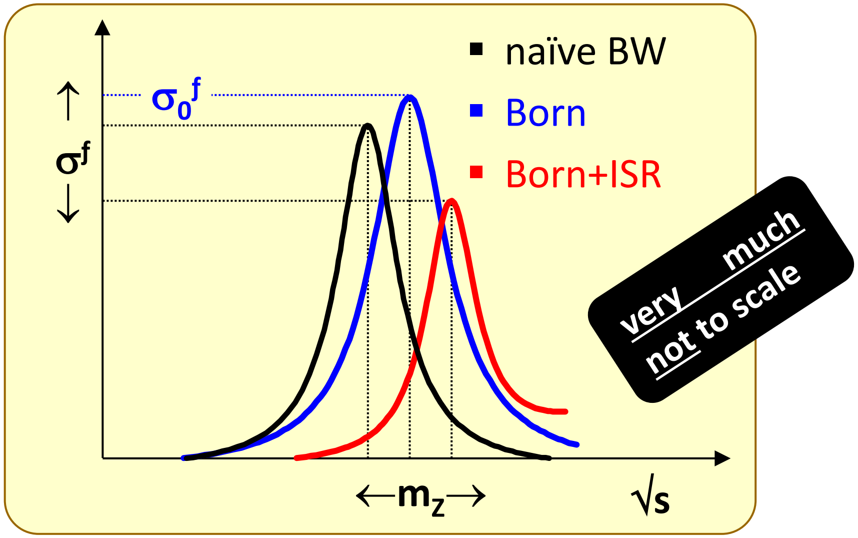
the most important effect

- similar method for Γ_Z :
  - Γ_Z s-dependent : $\Gamma_Z \to s\Gamma_Z\,/\,m_Z^2$;
  - (references);

$\gamma \equiv \Gamma_Z\,/\,m_Z \approx 0.027$;

$\beta \equiv 2\alpha[2\ell n\,(m_Z\,/\,m_e) - 1]/\pi \approx 0.108$;

$\delta_{sup} \equiv$ [soft- and virtual-γ's, calculable].



notice also that the lineshape is dependent on the type of the fermion (e.g., for $e^+e^-\to\nu\bar{\nu}$ no γ in final state).

radiative corrections: parameter Δr

[an example : radiative corrections for $W^\pm$ and Z mass]

- in the SM, m_W and m_Z are related by:

$$m_W^2 \sin^2\theta_W = \frac{\pi\alpha}{\sqrt{2}\,G_F}\ ;\ \sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2};$$

- radiative corrections modify the formulæ;
- ***define*** the parameters Δr (*radiative correction parameter*), $\Delta\alpha$ (*QED rad. corr.*), Δr_w (*weak rad. corr.*) :

$$m_W^2 \sin^2\theta_W \equiv \frac{\pi\alpha}{\sqrt{2}\,G_F} \times \frac{1}{1-\Delta r} \rightarrow$$

$$\Delta r = 1 - \frac{\pi\alpha}{\sqrt{2}\,G_F} \times \frac{m_Z^2}{m_W^2(m_Z^2 - m_W^2)};$$

$$\frac{1}{1-\Delta r} \equiv \frac{1}{1-\Delta\alpha} \times \frac{1}{1-\Delta r_W};$$

- $\Delta\alpha$ is reabsorbed in $\alpha_{(s)}$, *running coupling constant* [*the $_{(s)}$ means "function of $\sqrt{s}$"*] :

$\Delta\alpha_{(s)} = (\alpha_{(s)} - \alpha_{(s=0)}) / \alpha_{(s)}$;

- from QED :

$\Delta\alpha_{(m^2_Z)} \approx 0.07 \rightarrow \alpha_{(m^2_Z)} \approx [128.89 \pm 0.09]^{-1}$;

[error from $\int \sigma(e^+e^- \to \text{hadr.})$ @ $\sqrt{s} \ll m_Z$]

- the equation with $m_w + m_z$ becomes :

$$m_W^2\left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha_{(s=m_Z^2)}}{\sqrt{2}\,G_F} \times \frac{1}{1-\Delta r_W};$$

- [to select top and Higgs terms] expand Δr_w into parts, dependent on m_t ($\propto m_t^2$) and m_H ($\propto \ell n\ m_H$), and the rest ($\Delta\bar{r}_w$) :

$$\Delta r_W = \Delta\bar{r}_W\Big|_{m_t=\hat{m}}^{\text{calc.}} + \frac{\partial\Delta r_W}{\partial m_t}\Bigg|_{m_t=\hat{m}} \delta m_t + \frac{\partial\Delta r_W}{\partial m_H}\delta m_H;$$

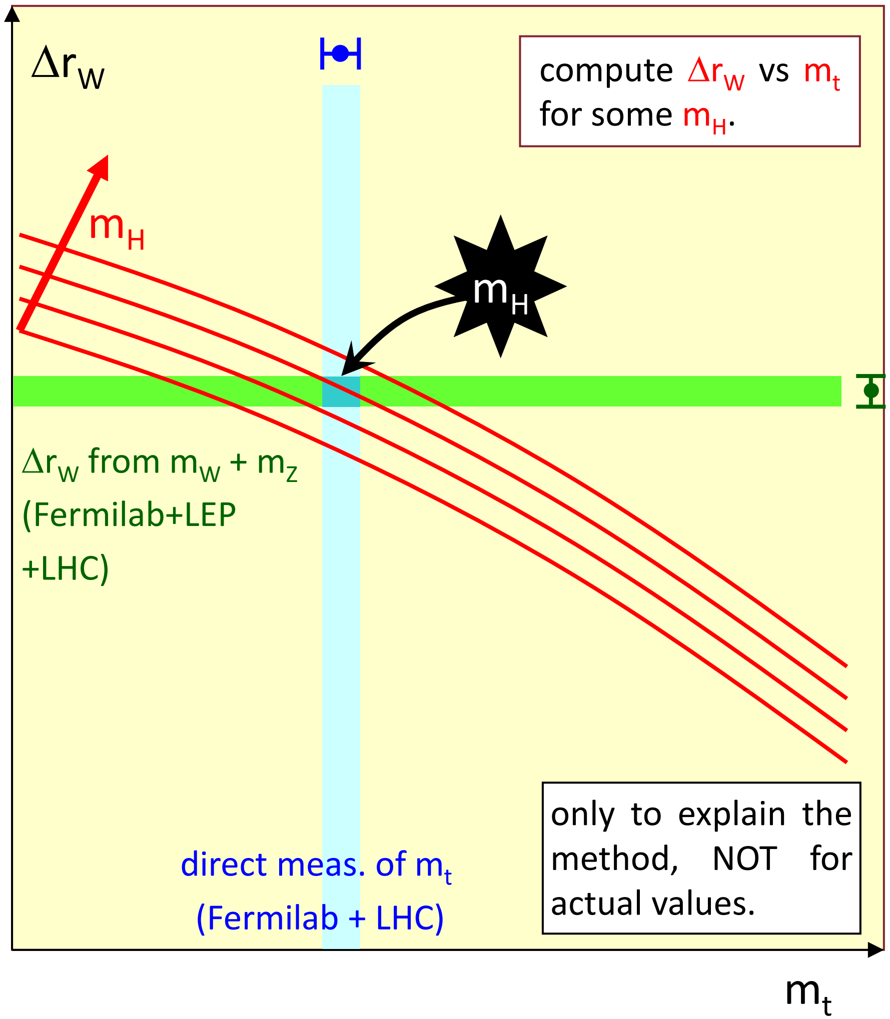
$$[\hat{m} = 175\ \text{GeV}].$$

radiative corrections: method → discovery

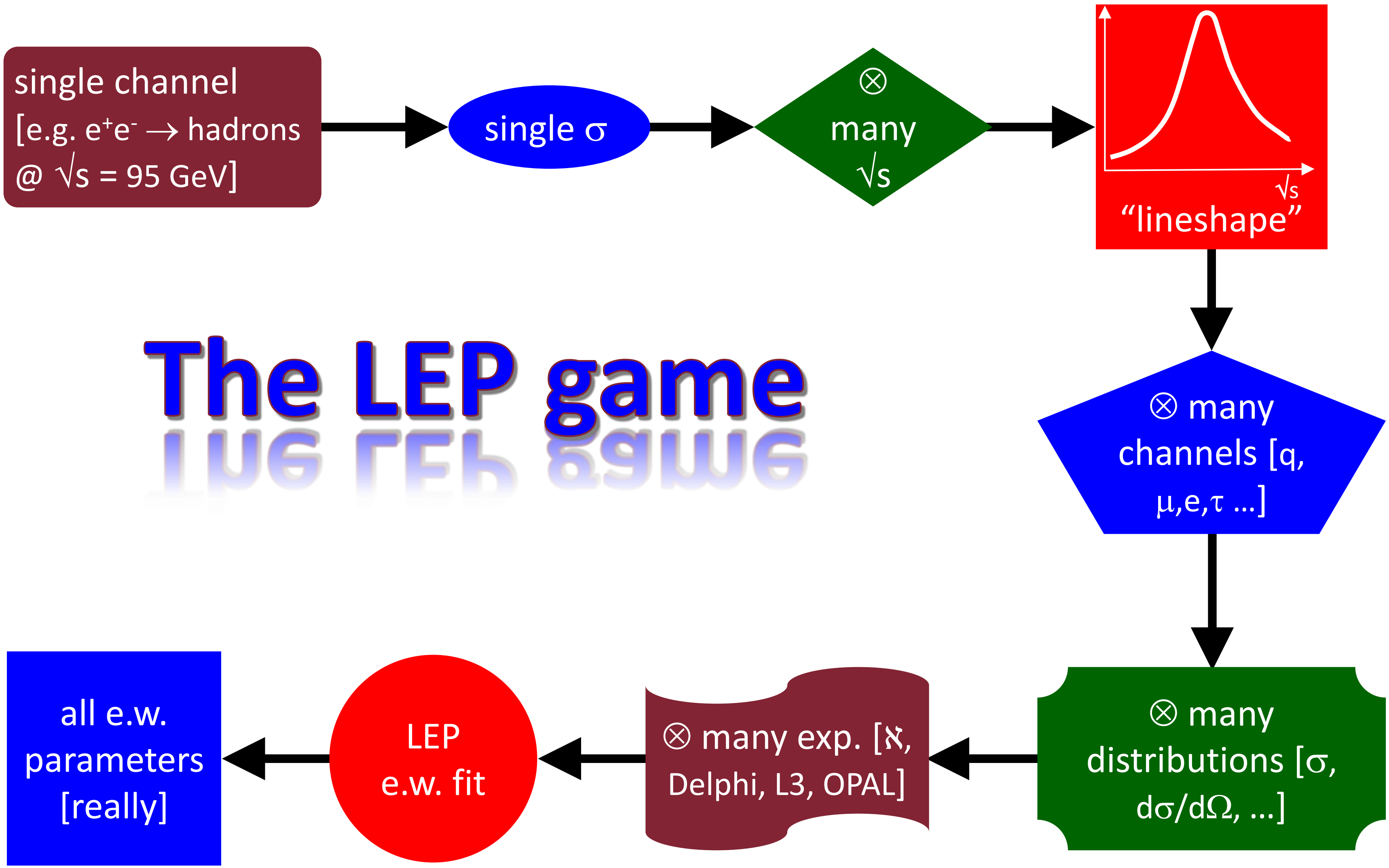
- assume we are in the "post-top, pre-Higgs" era [i.e. 1995-2011] :
- numerically, the sensibility is :

$$\Delta r_W \approx \Delta \bar{r}_W\Big|_{calc.} + \\ -0.0019\left(\frac{m_t}{175\,\mathrm{GeV}}\right)\left(\frac{\delta m_t}{5\,\mathrm{GeV}}\right) + \\ +0.0050\left(\frac{\delta m_H}{m_H}\right);$$

  [the two terms have opposite sign and very different size]
- the meas. of m_W, m_Z, m_t + the calculation of higher orders of SM allow for a "measurement" of m_H á la Hollik;
- in reality, many observables → global fit.



LEP1 SM fit


single channel
[e.g. $e^+e^- \to$ hadrons
@ $\sqrt{s}$ = 95 GeV]
single σ
$\otimes$ many $\sqrt{s}$
$\sqrt{s}$
"lineshape"
$\otimes$ many channels [q, μ,e,τ ...]
$\otimes$ many distributions [σ, $d\sigma/d\Omega$, ...]
$\otimes$ many exp. [ℵ, Delphi, L3, OPAL]
LEP e.w. fit
all e.w. parameters [really]
The LEP game

LEP1 SM fit: explanation

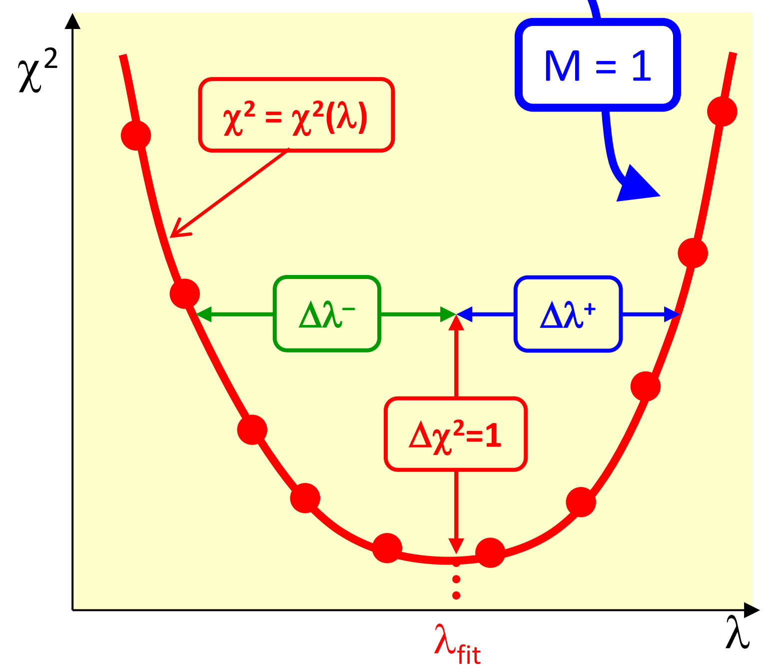
- in the SM, the observables [e.g. σ's, $d\sigma/d\cos\theta$'s, asymmetries, ...] are (functions of few) parameters like m_Z, Γ_Z, Γ_f, θ_W ...;
- in an experiment: **N** observables $\mathbf{t_i}$ (i = 1, ..., N) and **M** SM parameters $\boldsymbol{\lambda_k}$ (k=1,...,M);
- [at LEP 1, N = few$\times$100, M $\leq$ 10, see later);
- [M is fixed, but the choice is free, e.g. one among m_Z, m_W and θ_W is redundant]
- the dependence of t_i from λ_k is known: $\mathbf{t_i = t_i(\lambda_k) \pm \Delta t_i}$ (Δt_i = the theoretical error);
- the N observables are measured : $\mathbf{m_i \pm \Delta m_i}$ (Δm_i = the convolution of stat. and sys.);
- a (difficult) numerical program computes the "best" λ_k's which fit the observations;
- then the same values of λ_k are used for all the computations (shown as the "SM fits").
- [*since N>>M, the dependence of any λ_k on the single i*[th] *meas. is very small.*]
- [*also test the agreement SM $\leftrightarrow$ data.*]

[simplified example with χ^2 :

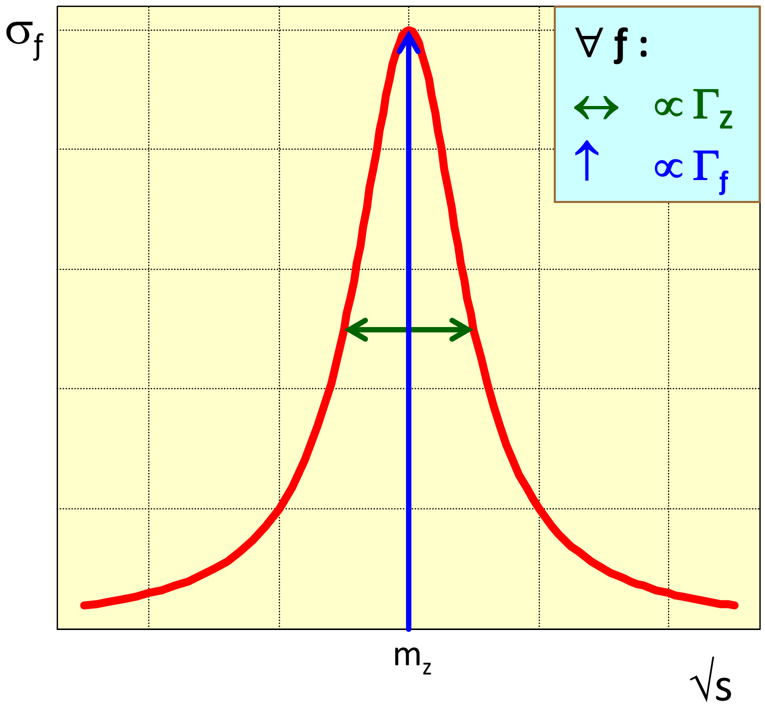
$$\chi^2 = \sum_i \frac{\left[t_i(\lambda_k) - m_i\right]^2}{\Delta t_i^2 + \Delta m_i^2};$$

$$\frac{\partial \chi^2}{\partial \lambda_k} = 0 \text{ (M equations)} \xrightarrow{\text{solve the system}} \boxed{\text{all } \lambda_k\text{'s}}$$

+ errors, correlations, ...]



LEP1 SM fit: σ vs Γ



$$\sigma_{Born}(e^+e^- \to f\bar{f}, \sqrt{s} = m_Z) = \\ = \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2}.$$

- in LEP jargon, "lineshape" means $\sigma(e+e- \to Z \to f\bar{f})$ vs $\sqrt{s}$ (*) for a given fermion pair of type f;
- the lineshape shows the characteristic "bell shape", due to the resonance;
- both the height and the width of the bell depend on the e.w. parameters;
- the strategy is

  a) first, <u>measure</u> mass, full and partial widths of the Z;

  b) then, <u>fit</u> :

  - number of light ν's (= fermion families);
  - electro-weak couplings.

___

(*) *warning : NOT "~~$d\sigma/d\sqrt{s}$~~", which is meaningless.*

LEP1 SM fit: m_Z, Γ_Z

without lepton universality		correlations								
$\chi^2/N_{df} = 32.6/27$		m_Z	Γ_Z	σ_h^0	R_e^0	R_μ^0	R_τ^0	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
m_Z [GeV]	91.1876 ± 0.0021	1.00								
Γ_Z [GeV]	2.4952 ± 0.0023	−.024	1.00							
σ_h^0 [nb]	41.541 ± 0.037	−.044	−.297	1.00						
R_e^0	20.804 ± 0.050	.078	−.011	.105	1.00					
R_μ^0	20.785 ± 0.033	.000	.008	.131	.069	1.00				
R_τ^0	20.764 ± 0.045	.002	.006	.092	.046	.069	1.00			
$A_{FB}^{0,e}$	0.0145 ± 0.0025	−.014	.007	.001	−.371	.001	.003	1.00		
$A_{FB}^{0,\mu}$	0.0169 ± 0.0013	.046	.002	.003	.020	.012	.001	−.024	1.00	
$A_{FB}^{0,\tau}$	0.0188 ± 0.0017	.035	.001	.002	.013	−.003	.009	−.020	.046	1.00

for updated values, check [pdg]

with lepton universality						
$\chi^2/N_{df} = 36.5/31$		m_Z	Γ_Z	σ_h^0	R_ℓ^0	$A_{FB}^{0,\ell}$
m_Z [GeV]	91.1875 ± 0.0021	1.00				
Γ_Z [GeV]	2.4952 ± 0.0023	−.023	1.00			
σ_h^0 [nb]	41.540 ± 0.037	−.045	−.297	1.00		
R_ℓ^0	20.767 ± 0.025	.033	.004	.183	1.00	
$A_{FB}^{0,\ell}$	0.0171 ± 0.0010	.055	.003	.006	−.056	1.00

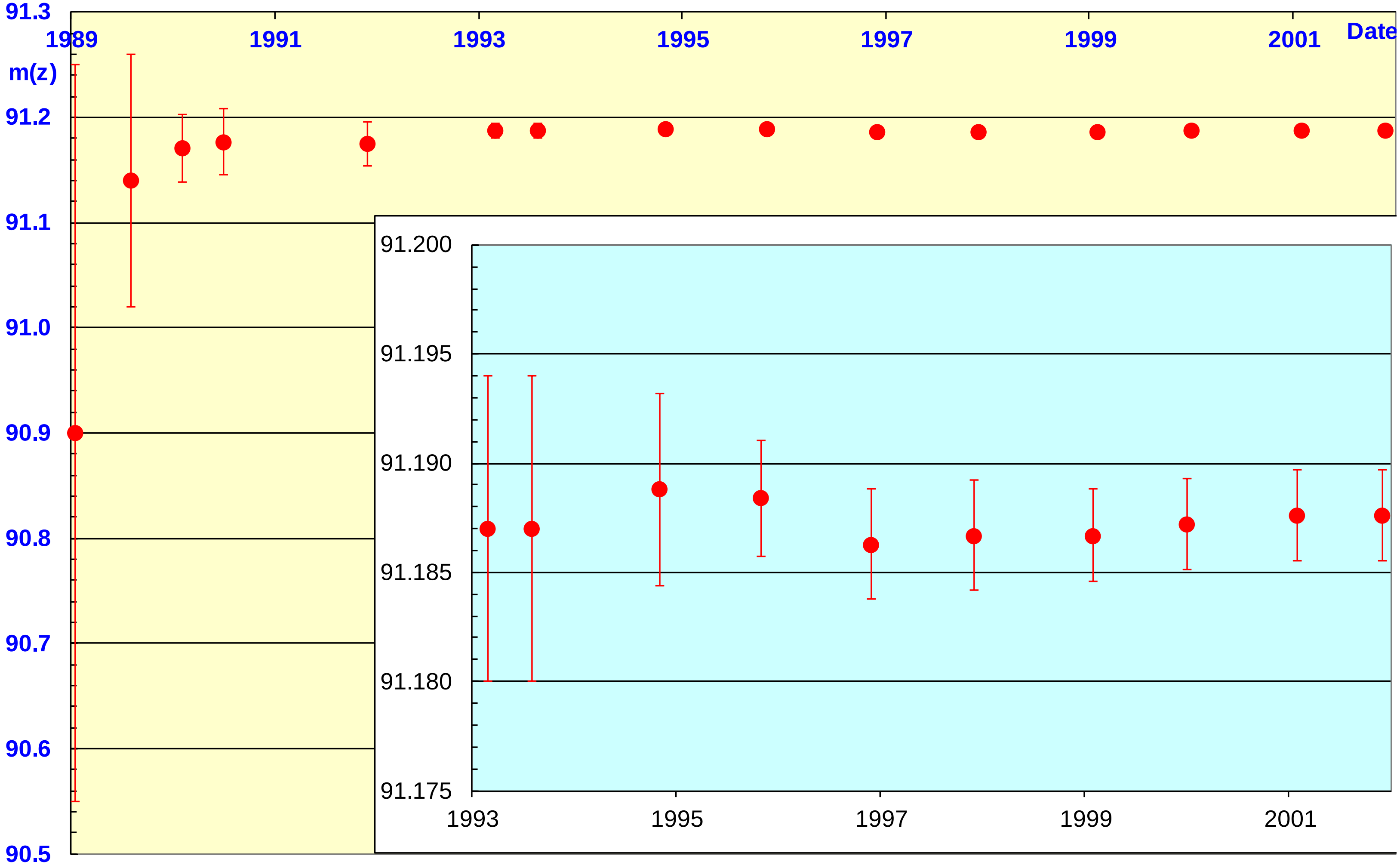
$R_x \equiv \Gamma_{hadr} / \Gamma_x = \sigma_{hadr} / \sigma_x$;
all values computed at the pole.

two fits :

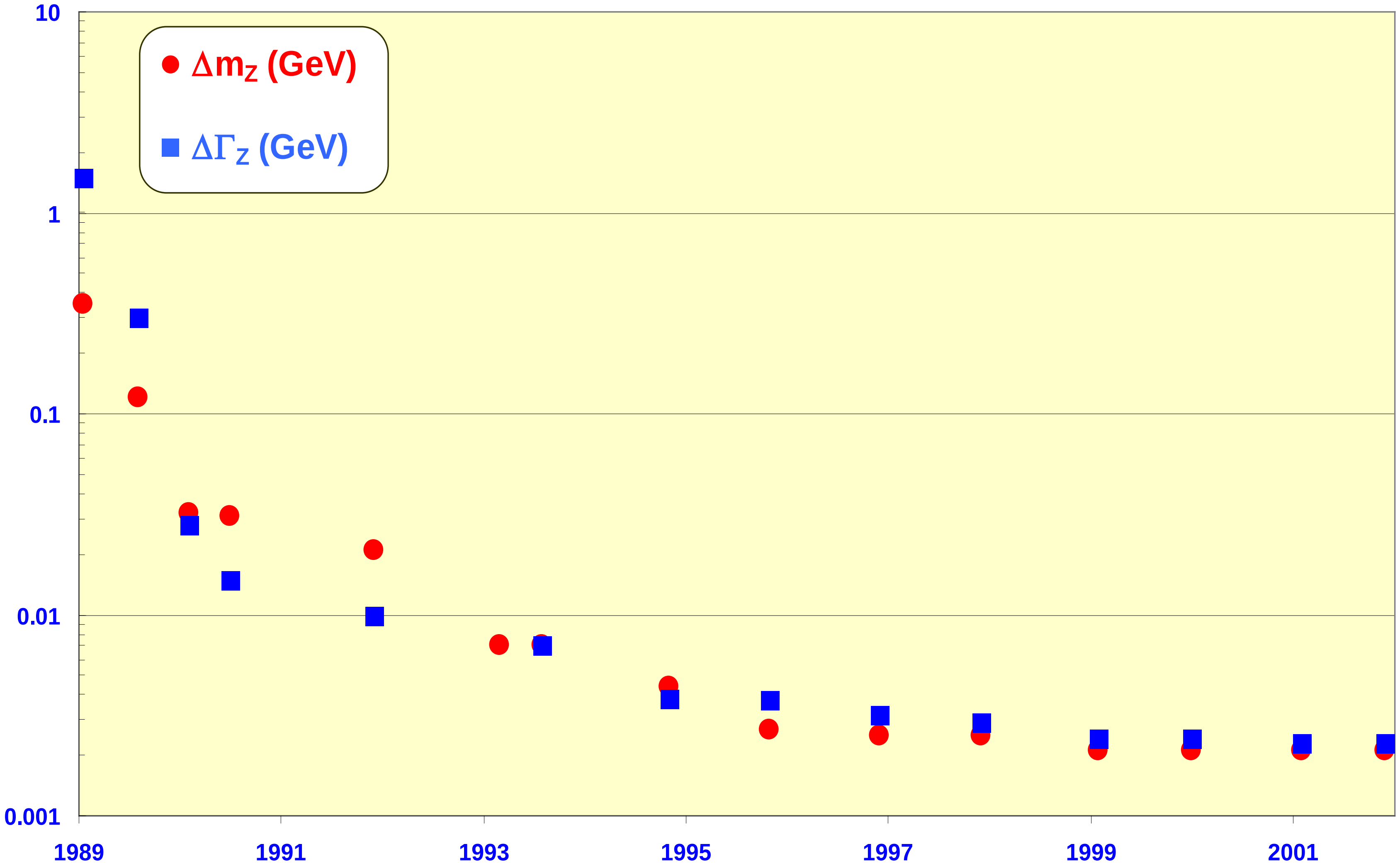
a) "without lepton universality", 9 parameters : larger errors, more general;

b) "with l. u.", 5 parameters, smaller errors, assume lepton universality.

LEP1 SM fit : m_z



LEP1 SM fit: Δm_Z, $\Delta\Gamma_Z$



LEP1 SM fit: n_ν

- Neutrinos are the lightest component of the fermion families [in SM no theor. explanation, just matter of fact];
- assuming this case also for (hypothetical) further families, i.e. additional ν's lightest member of a family;
- the decay $Z \to \nu\bar{\nu}$ is important (~20%), but not observable (but "single γ", not treated here);
- but it contributes to Γ_z (observable);
- indirect detection: measure Γ_z, subtract the contribution of observable decays ("$\Gamma_{visible}$"), get "$\Gamma_{invisible}$" and compute n_ν (more precisely the number of <u>light</u> ν, i.e. $m_\nu < m_z/2$) :

$$\Gamma_{inv} \equiv \Gamma_z - \sum_{j=q,\ell^\pm} \Gamma_j = \Gamma_z - \Gamma_{hadr} - 3\Gamma_{\ell^\pm};$$

$$n_\nu = \frac{\Gamma_{inv}}{\Gamma_\nu^{SM}} = \left(\frac{\Gamma_{inv}^{exp}}{\Gamma_Z^{exp}}\right)\left(\frac{\Gamma_z^{SM}}{\Gamma_\nu^{SM}}\right).$$

- [the last step to decrease stat and syst errors]
- it turns out :

$$n_\nu = 2.9840 \pm 0.0082$$

i.e. n_ν = 3, no other families

[probably the best, most known, most quoted LEP result, see <u>fig on pag. 2</u>].

*NB strictly speaking, n_ν = width of invisible decays normalized to Γ_ν; i.e. it could get contributions from other invisible decays (physics bSM, e.g. neutralino); in such cases, **<u>"n_ν" not an integer</u>**.*

$$\sigma_{Born}(e^+e^- \to f\bar{f}, \sqrt{s} = m_Z) = \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2};$$

$$\Gamma_\nu^{SM} = \frac{G_F m_Z^3 c_f}{12\sqrt{2}\pi}; \qquad \Gamma_z = \sum_i \Gamma_i.$$

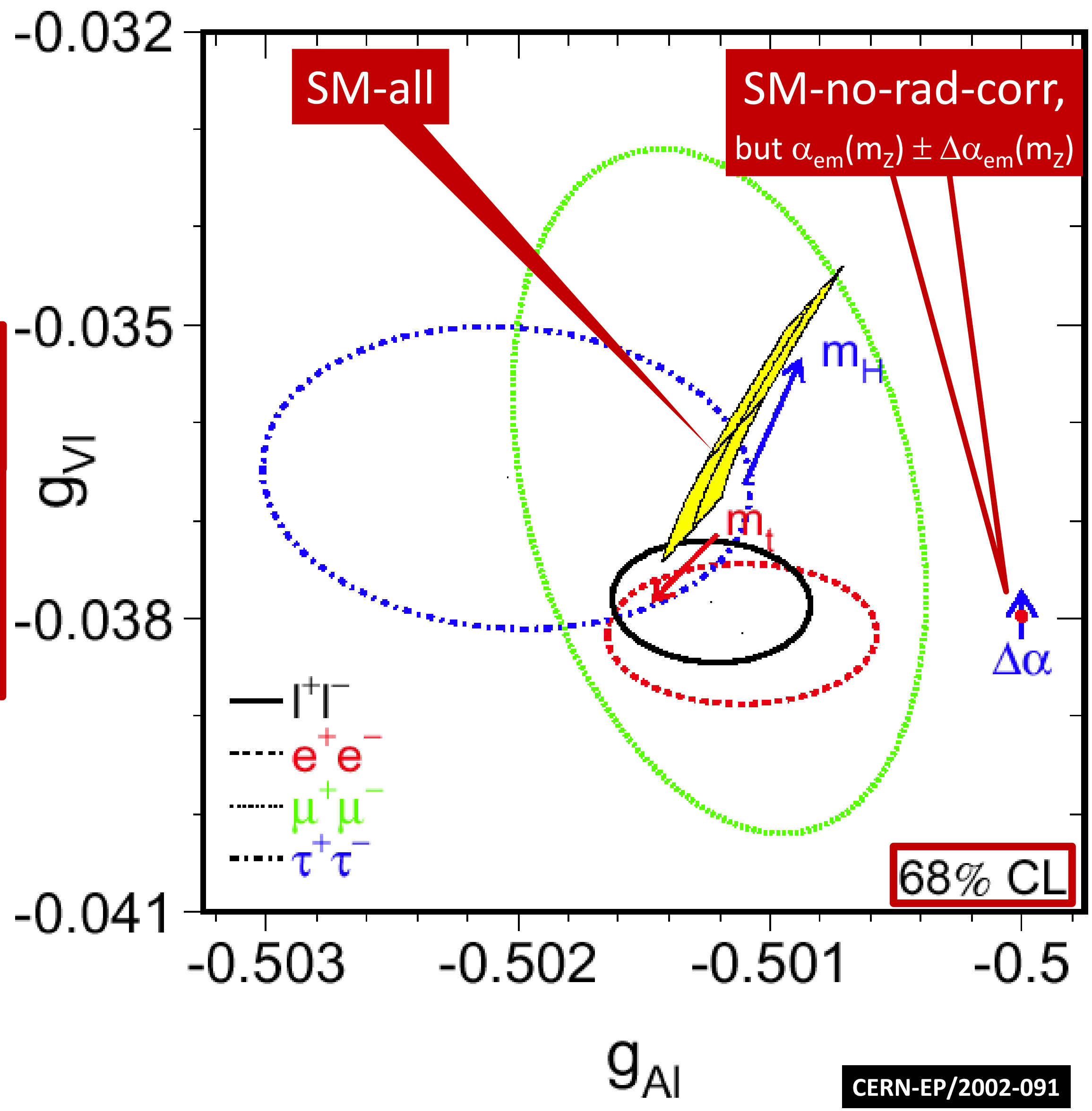
LEP1 SM fit : g_A vs g_V for leptons

Example of global fit result : g_A vs g_V for leptons :

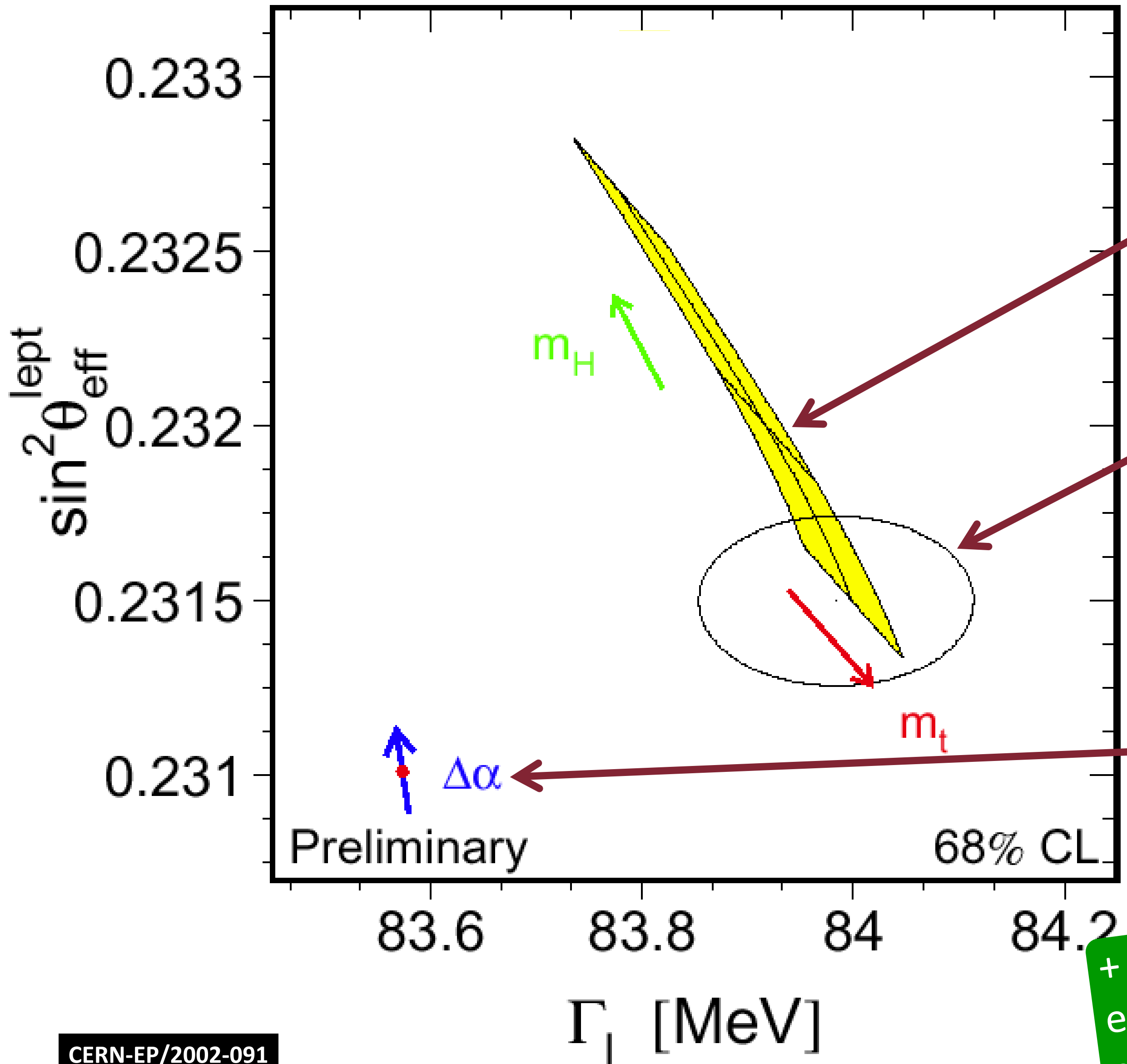
- 68% (i.e. 1 σ) contours;
- computed after top and before Higgs discovery;
- the "$\rightarrow$" shows $\pm$ 1 σ in α_{em}, m_t...
- ... and 114, 300, 1000 GeV for m_H.
- the **red** dot shows the SM Born point, with the QED corr. only (i.e. $\alpha_{em}(m_Z) \approx 1/128$ $\rightarrow$ **weak rad. corr.** are important.

Notice :

- good compatibility among leptons ($\rightarrow$ universality);
- preference for light Higgs (...wow)



LEP1 SM fit : $\sin^2\theta$ vs Γ_ℓ



SM-with-rad-corr:
m_H = 114, 300, 1000 GeV;
m_t = 174.3 ± 5.1 GeV.

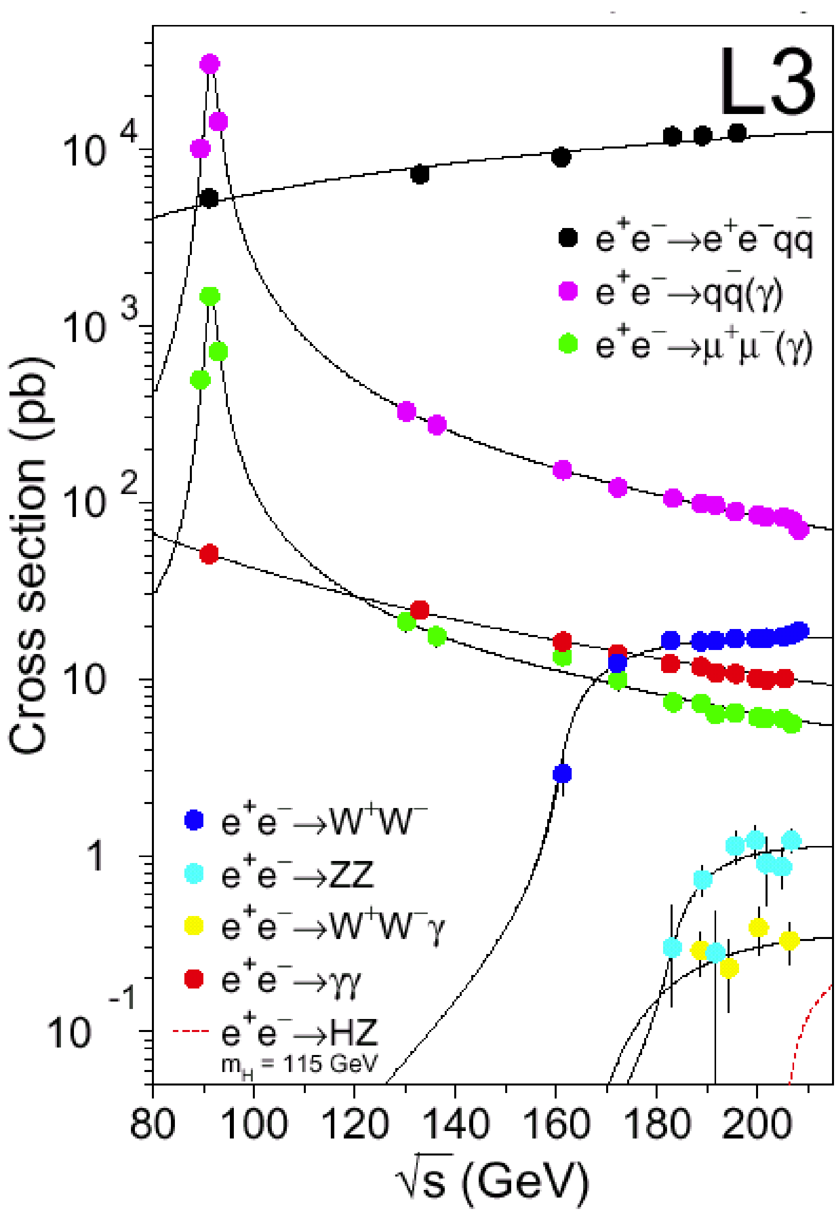
meas. (68%)

SM-no-rad-corr,
but $\alpha_{em}(m_Z) \pm \Delta\alpha_{em}(m_Z)$

CERN-EP/2002-091

+ tons of similar plots, with all the e.w. parameters vs all others, compared with SM → agreement.

$e^+e^- \to W^+W^-$ @ LEP2: LEP2 processes

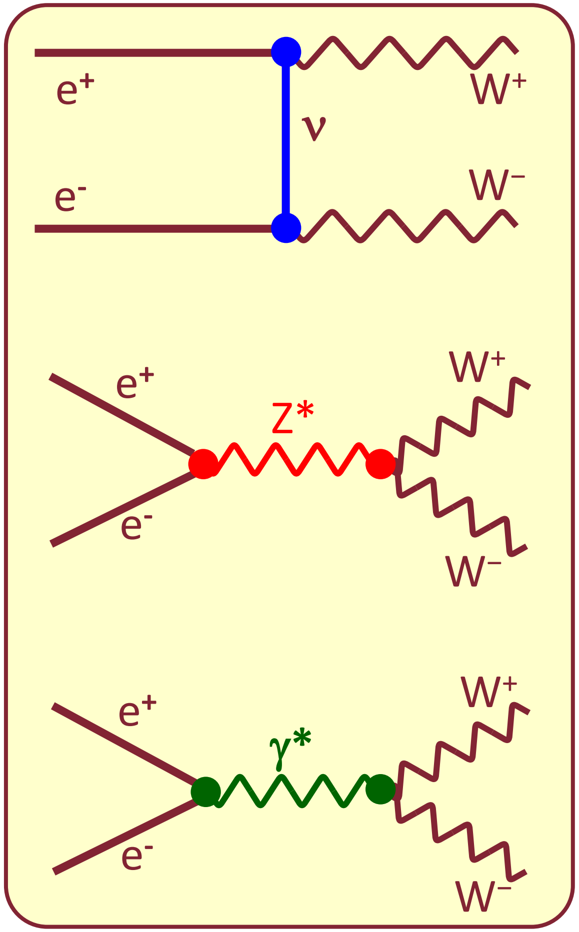


In 1994-2000 LEP gradually $\sqrt{s} = m_z \to 200$ GeV

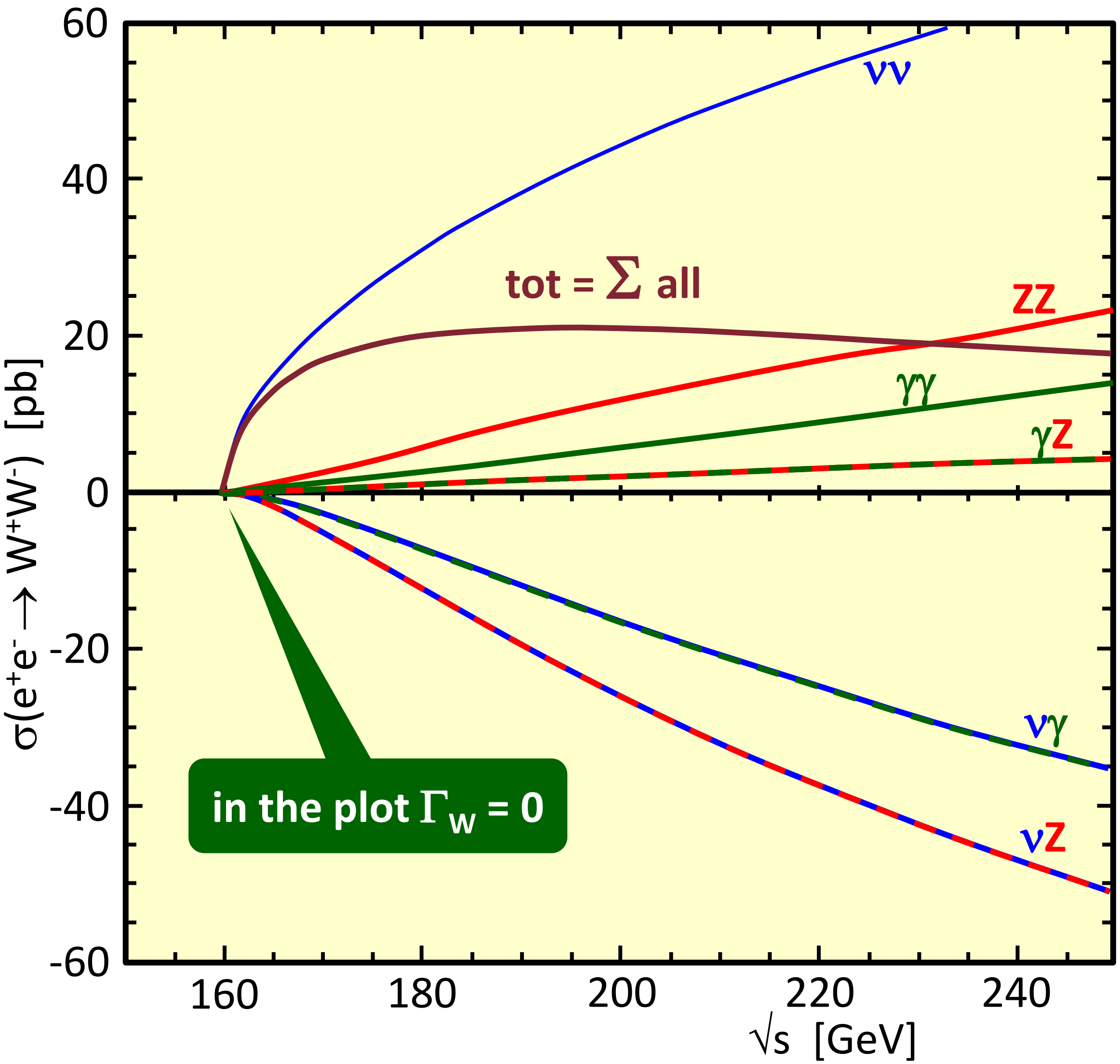
- LEP1 was dominated by the Z pole;
- on the contrary, LEP2 is "democratic";
- many final states :
  - "2 photons", e.g. $e^+e^- \to e^+e^- q\bar{q}$;
  - "2 fermions", e.g. $e^+e^- \to Z^*/\gamma^* \to q\bar{q}$;
  - "4 fermions", e.g. $e^+e^- \to W^+W^- \to q\bar{q}\ q\bar{q}$;
  - $e^+e^- \to \gamma\gamma$;
  - Higgs searches (special case of 4 fermions).
- only W^+W^- and Higgs in these lectures.

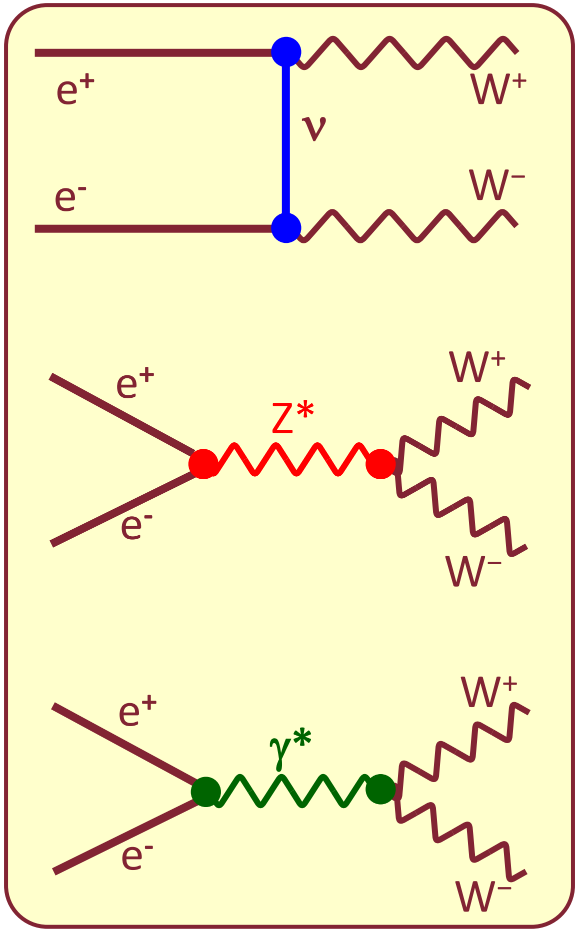
$e^+e^- \to W^+W^-$ @ LEP2: Feynman diagrams

- the process $e^+e^- \to W^+W^- \to f\bar{f}f\bar{f}$ dominates the 4 fermions sample;
- in lowest order, there are three Feynman diagrams;
- all the vertices of the e.w. theory: ffW, ffZ, ffγ, ZWW, γWW;
- the overall (finite) cross section results from delicate cancellations among the 6 terms (3 $|\text{module}|^2$ + 3 interferences) [*next slide*];
- therefore, any possible anomaly (discrepancy wrt SM, e.g. an anomaly in the couplings) would result in evident deviations from the predictions.

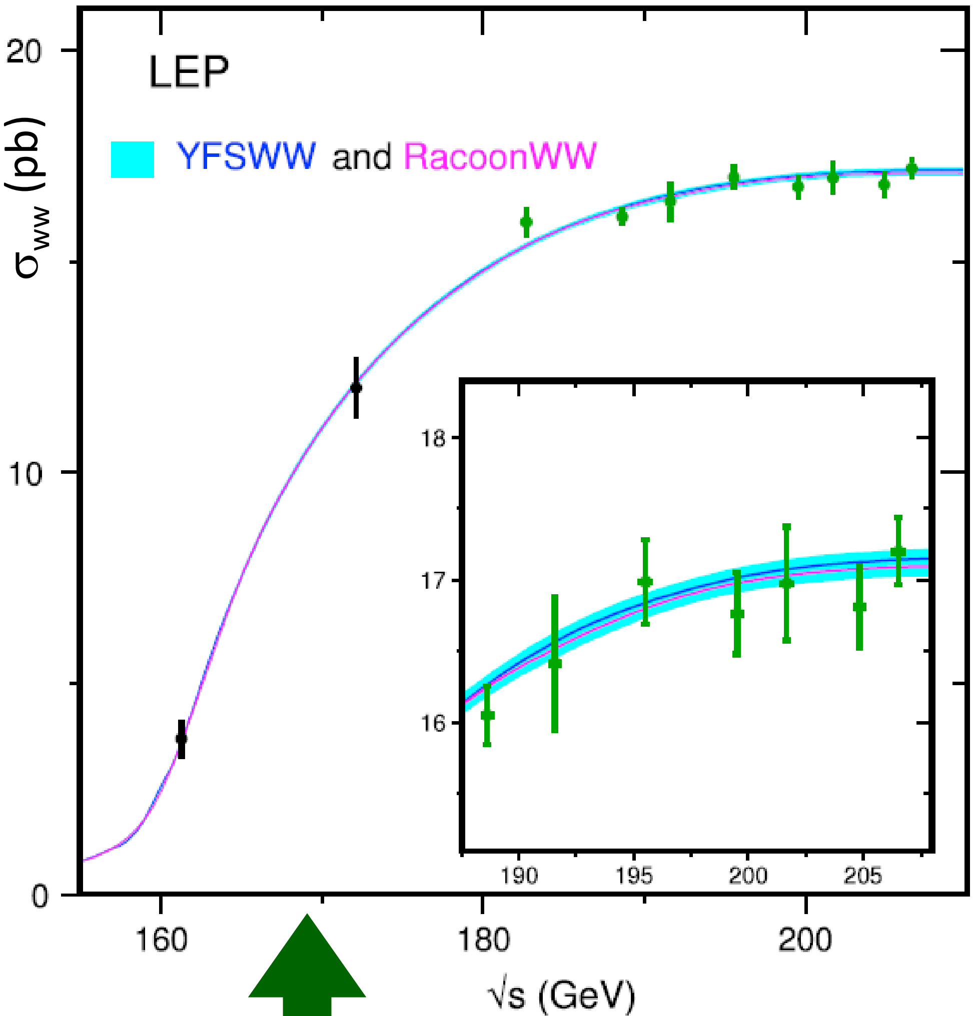


$e^+e^- \to W^+W^-$ @ LEP2: cross section





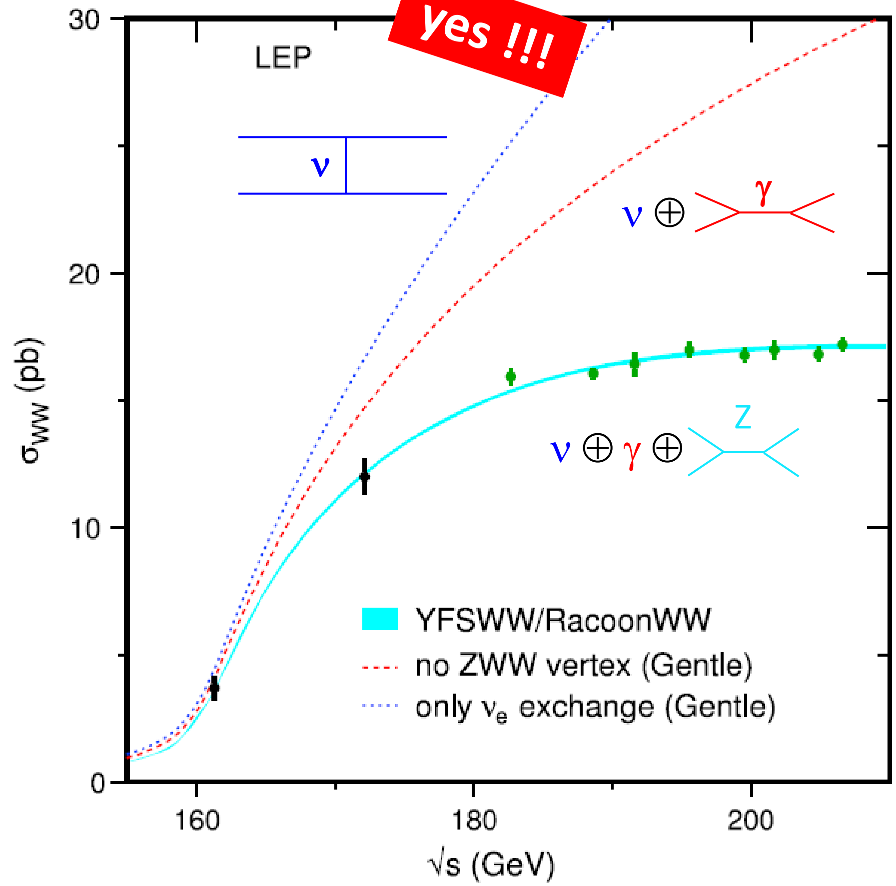
$e^+e^- \to W^+W^-$ @ LEP2: cross section results



all data compared with best SM MC

same data, similar question: are all SM couplings needed ?

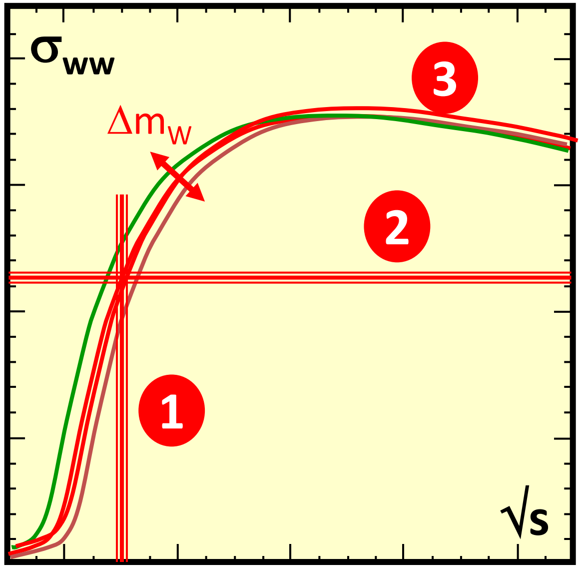
yes !!!

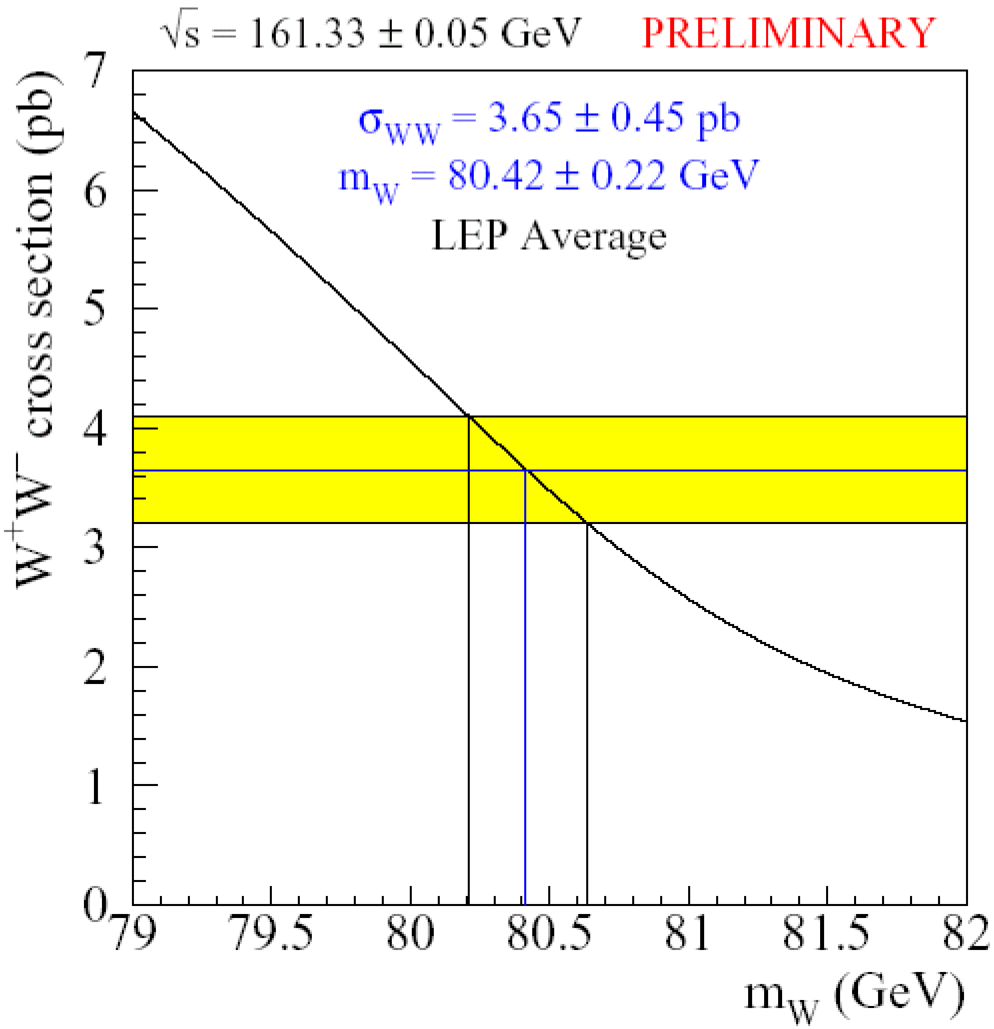


$e^+e^- \to W^+W^-$ @ LEP2: W mass from σ

Technically clever and simple :

- compute $\sigma(e^+e^- \to W^+W^-) = \sigma(m_W)$;
- compute the "best" $\sqrt{s}$, by combining
  - sensitivity ($\partial\sigma/\partial m_W$ = max) $\to \sqrt{s} \approx$ threshold;
  - $(\Delta\sigma^{stat} \downarrow) \to (\sigma \uparrow) \to (\sqrt{s} \uparrow)$;
  - take into account Δ_{theory} and syst.;
- **measure**.

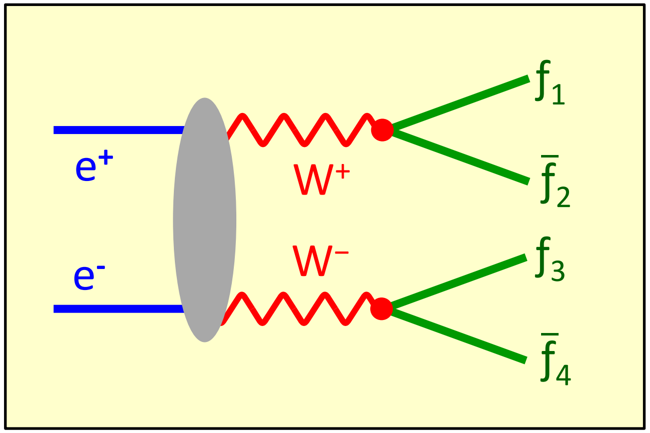




$e^+e^- \to W^+W^-$ @ LEP2: constraints

- kinematical constraints (e.g. 4-mom conservation) help in the analysis :
  - selection criterion (rejection of bad measurements or event classification in other processes);
  - improve resolution (see next);
- this case as an example : likelihood fit to m_W, Γ_W;
- compare analysis/fit on real data wrt same procedure on "pseudo-events" (physics + detector mc);
- Γ_W strongly (anti-)correlated with experimental resolution ["pessimistic" detector mc $\to$ σ_{meas} too large $\to$ Γ_W too small !!!];

- systematics from:
  - ISR/FSR parameterization;
  - reconstruction algorithms (expecially jets, ex. color reconnection, Bose-Einstein correlations);
  - many other sources…
- consistency checks : in this case m_Z , Γ_Z from $e^+e^- \to ZZ$ (with smaller stat).

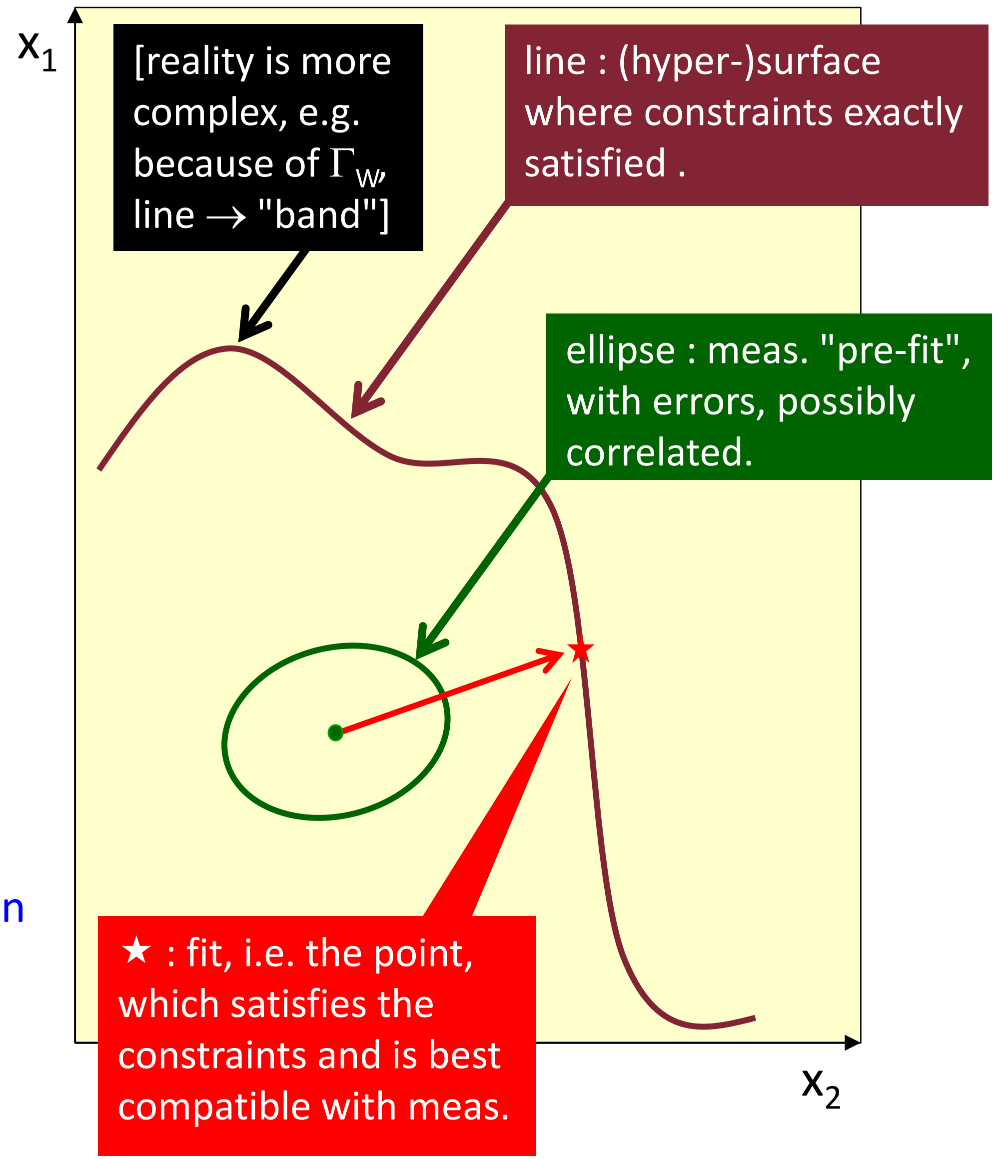


$e^+e^- \rightarrow W^+W^-$ @ LEP2: mass fit

In the parameter space :

- n unkn. = 4 * n_{body} = 16;
- N meas. [e.g. E, $\vec{p}$ for jets / $\ell^\pm$'s];
- K equations [= 4 mom + masses$^{(*)}$];
- C (=N+K-n) constraints;
- E.g. : $e^+e^- \rightarrow W^+W^- \rightarrow f_1\,f_2\,f_3\,f_4$, n=16 :
  - 4 jets : N=16, K=5 $\rightarrow$ **C = 5**;
  - $\ell^\pm\nu jj$: N=12, K=6 $\rightarrow$ **C = 2**;
  - $\ell^+\nu\ell^-\bar{\nu}$: N=8, K=7 $\rightarrow$ **C < 0**;
- If C > 0, a kinematical fit is possible (a simplified sketch in x_1, x_2, i.e. n=2)

[the red arrow "$\rightarrow$" represents a statistical estimate (χ^2, likelihood) and a computation method (e.g. Lagrange multipliers)].



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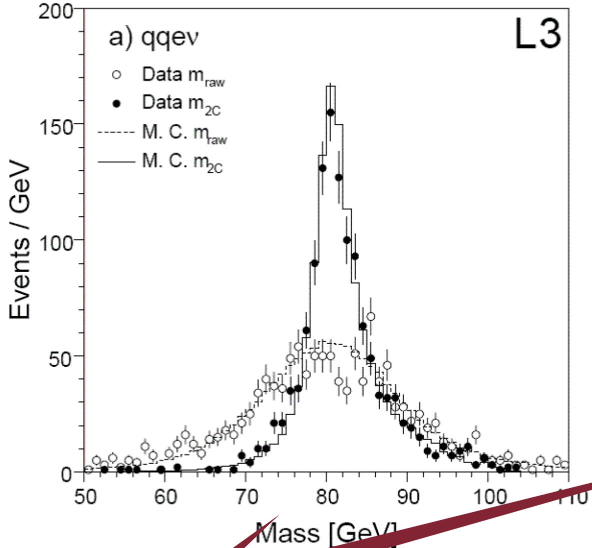
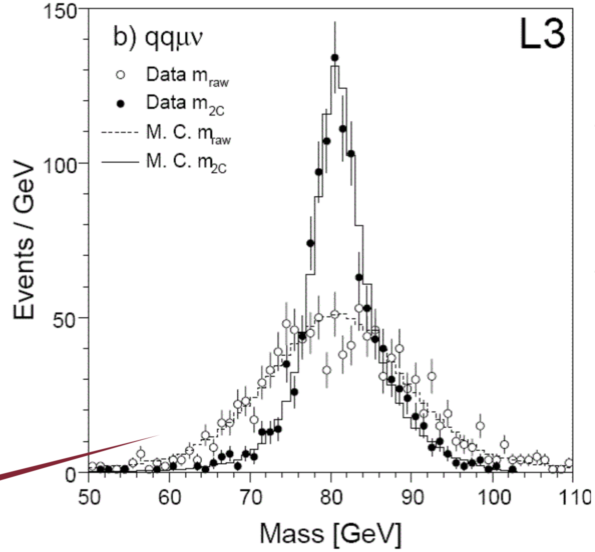
$^{(*)}$ $m_{W+} = m_{W-}$ and $m_\nu \approx 0$.

$e^+e^- \to W^+W^-$ @ LEP2: mass plots

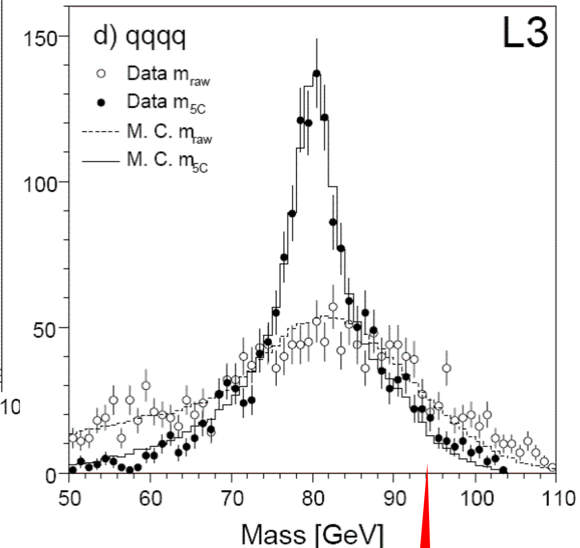
$e^+e^- \to W^+W^- \to q\bar{q}e\nu$



$e^+e^- \to W^+W^- \to q\bar{q}\mu\nu$

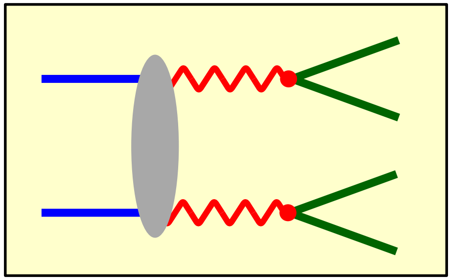


$e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$



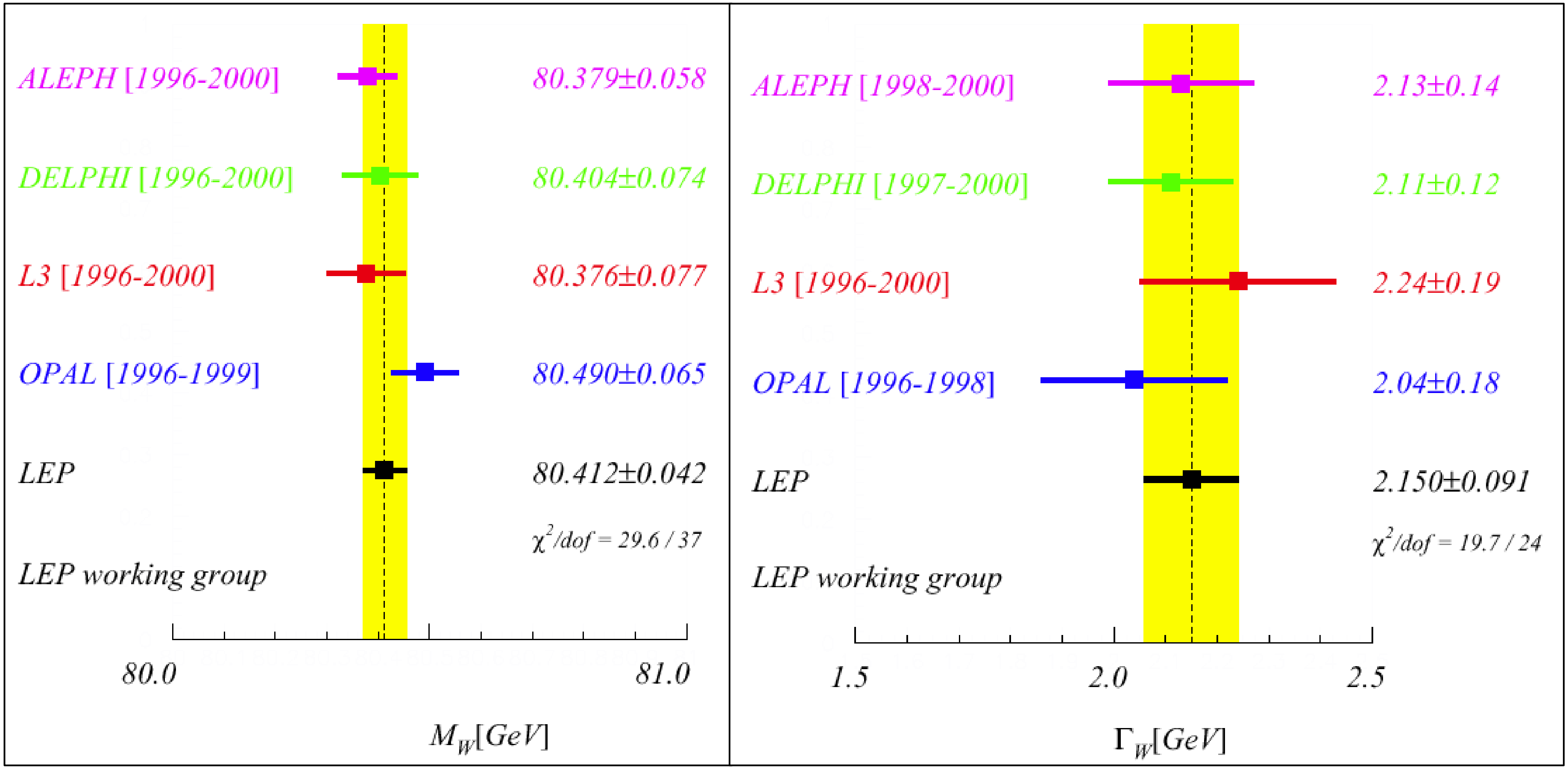
2C

5C



- the effects of kinematical fits :
- "C" (=constraints) from bubble chamber jargon;
- higher C, more constraints, more improvement from fit.

$e^+e^- \to W^+W^-$ @ LEP2: m_W, Γ_W results



$$m_W = 80.412 \pm 0.029 \pm 0.031 \text{ GeV};$$
$$\Gamma_W = 2.150 \pm 0.068 \pm 0.060 \text{ GeV}.$$

NB : 2003 values, WW events only
[no LEP global fit]

$e^+e^- \to W^+W^-$ @ LEP2: $W^\pm$ decay

- in the SM the $W^\pm$ boson decays through CC interactions (V-A);
- therefore the coupling is the same for all $f\bar{f}'$ pairs, providing :
  - $m(f\bar{f}') < m_W$ ($\to$ no t decays);
  - $q\bar{q}$ mixing (à la CKM) must be used;
- ASSUMING (*just for the discussion*) a diagonal CKM matrix, W^+ decays into $e^+\nu$, $\mu^+\nu$, $\tau^+\nu$, $u\bar{d}$, $c\bar{s}$, ($t\bar{b}$ forbidden);
- [if W^-, then corresponding antiparticles];
- ($m_f \ll m_W$ and CKM $\approx$ diagonal) $\to$ same BR for all channels (but color factor);
- the V-A theory gives in lowest order :

  $\Gamma(W\to f\bar{f}') = G_F\, m_W^3 / (6\sqrt{2}\pi) \approx 226$ MeV;
- (3 leptons + 2 quarks × 3 colors = 9) :

$\Gamma_W = \Sigma\,\Gamma_i(W\to f\bar{f}') \approx 9 \times 226$ MeV =

= 2.05 GeV;

$BR(W \to \ell^\pm\nu) \approx 1/9 \approx 0.11$;

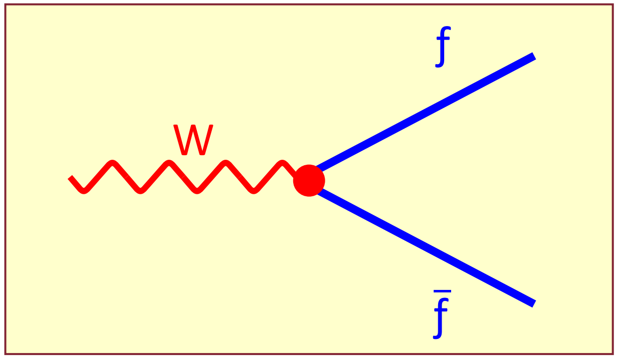
$BR(W^+ \to u\bar{d}) \approx BR(W^+ \to c\bar{s}) \approx 1/3 \approx 0.33$;

- if the correct quark mixing is used, the CKM matrix element $V_{qq'}$ must be considered :

$\Gamma(W\to q\bar{q}') = |V_{qq'}|^2\, G_F\, m_W^3 / (6\sqrt{2}\pi)$;

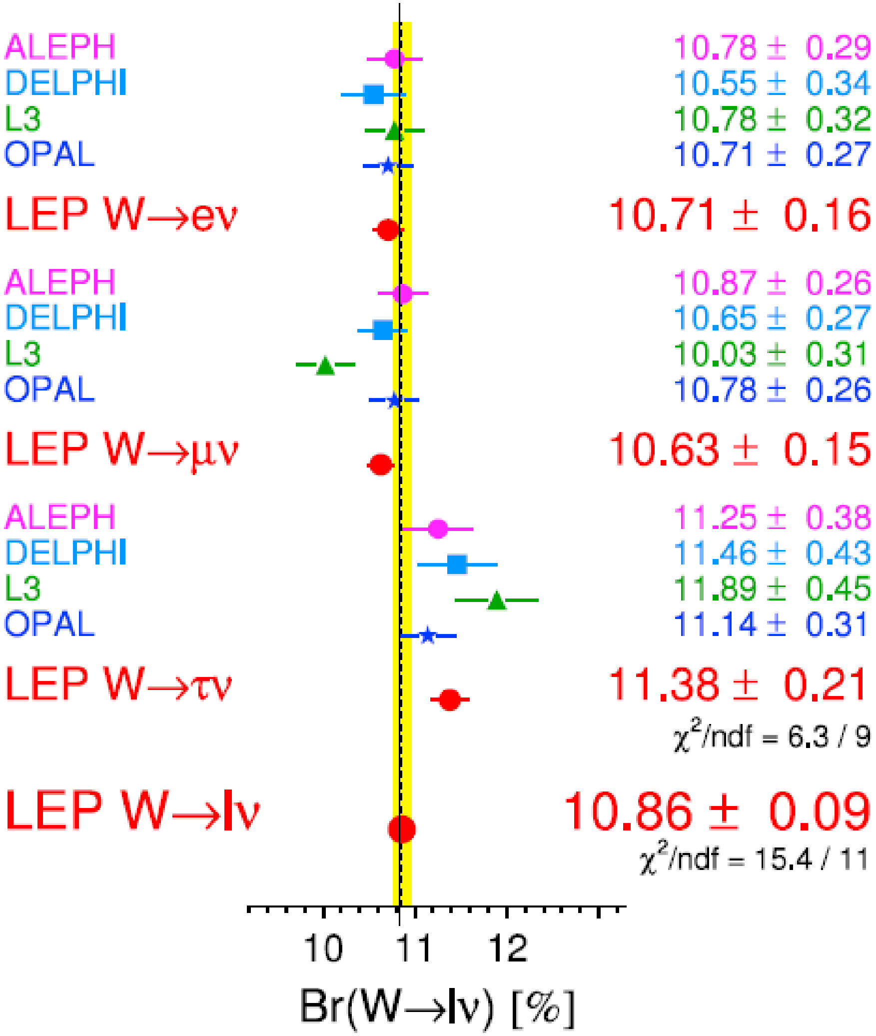
$\Gamma_W = \Sigma\,\Gamma_i(W\to f\bar{f}')$ = unchanged;

$BR(W \to q\bar{q}') \approx |V_{qq'}|^2 / 3$.

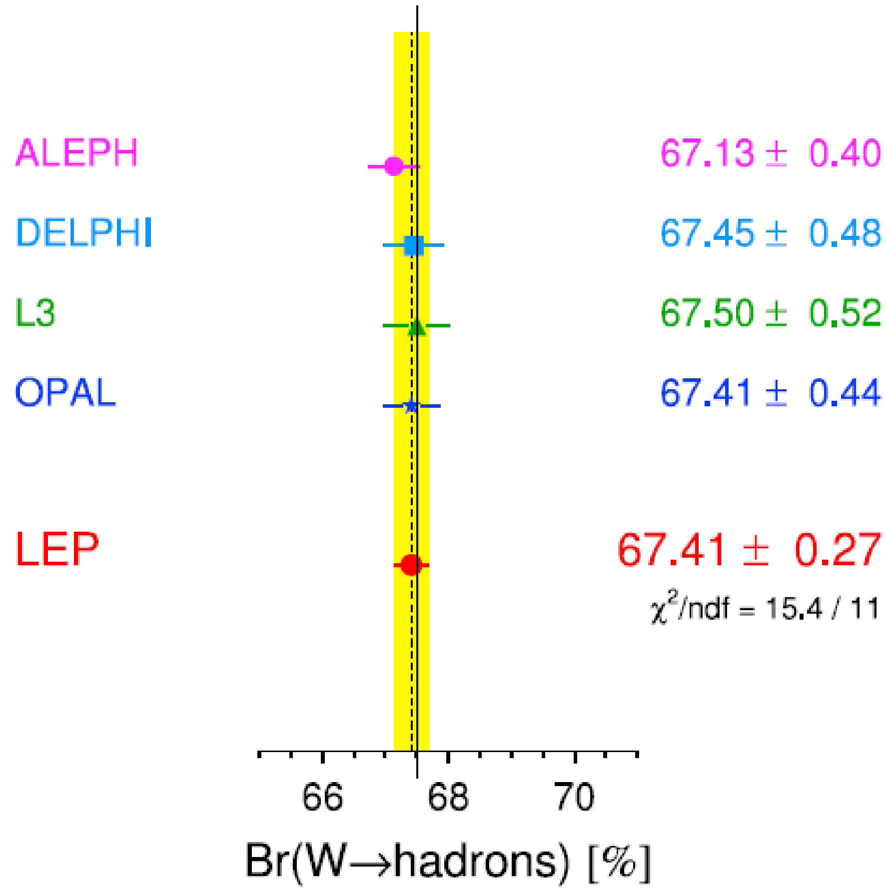


$e^+e^- \to W^+W^-$ @ LEP2: $W^\pm$ decay results

W Leptonic Branching Ratios



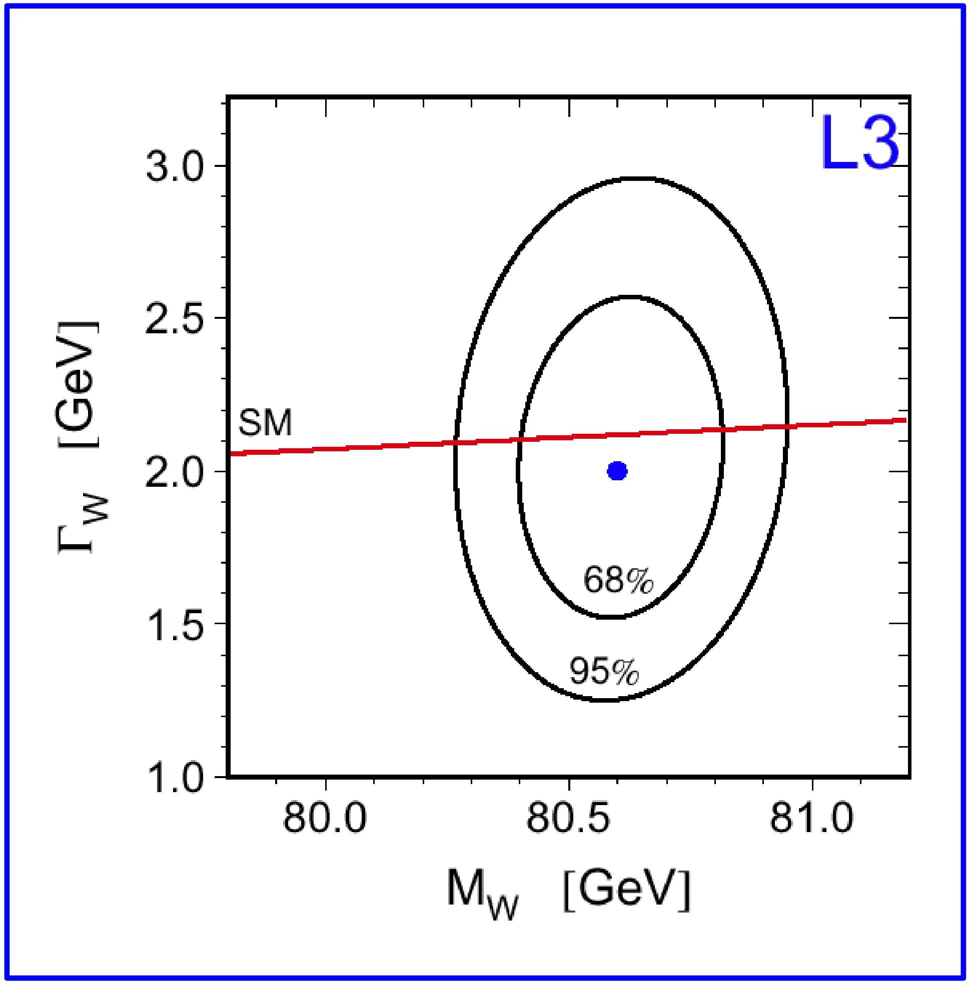
W Hadronic Branching Ratio



$e^+e^- \to W^+W^-$ @ LEP2: m_W vs Γ_W

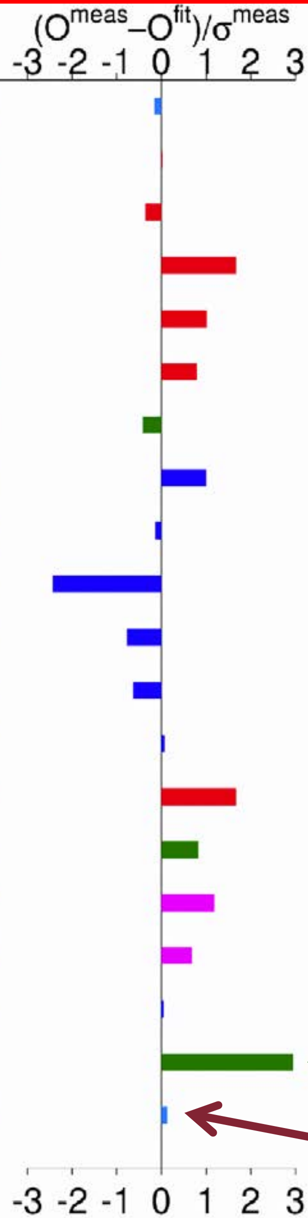
In the SM, m_W and Γ_W are correlated:

- are the previous measurements consistent ?
  - yes, see the plot;
- can do better ? i.e. check the SM with all the LEP measurement ?
  - yes;
- even better ? i.e. add also the other SM non-LEP measurement, i.e. ν's and low-energy ?
  - yes, see next slide;
- is the fit producing a value for the (still) unknown parameters, e.g. m_H ?
  - yes.



global LEP(1+2) fit

	Measurement	Pull
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	-0.16
m_Z [GeV]	91.1875 ± 0.0021	0.02
Γ_Z [GeV]	2.4952 ± 0.0023	-0.36
σ^0_{had} [nb]	41.540 ± 0.037	1.67
R_l	20.767 ± 0.025	1.01
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.79
$A_l(P_\tau)$	0.1465 ± 0.0032	-0.42
R_b	0.21644 ± 0.00065	0.99
R_c	0.1718 ± 0.0031	-0.15
$A^{0,b}_{fb}$	0.0995 ± 0.0017	-2.43
$A^{0,c}_{fb}$	0.0713 ± 0.0036	-0.78
A_b	0.922 ± 0.020	-0.64
A_c	0.670 ± 0.026	0.07
A_l(SLD)	0.1513 ± 0.0021	1.67
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.82
m_W [GeV]	80.426 ± 0.034	1.17
Γ_W [GeV]	2.139 ± 0.069	0.67
m_t [GeV]	174.3 ± 5.1	0.05
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	2.94
$Q_W(Cs)$	-72.83 ± 0.49	0.12



NuTeV $\sigma_{CC,NC}(\nu N)$

parity violation in Cs

circa A.D. 2000, at the end of LEP era.

$$\text{pull} \equiv \frac{\text{experiment - theory}}{\text{error}};$$

expected gaussian, $\mu=0$, $\sigma=1$;

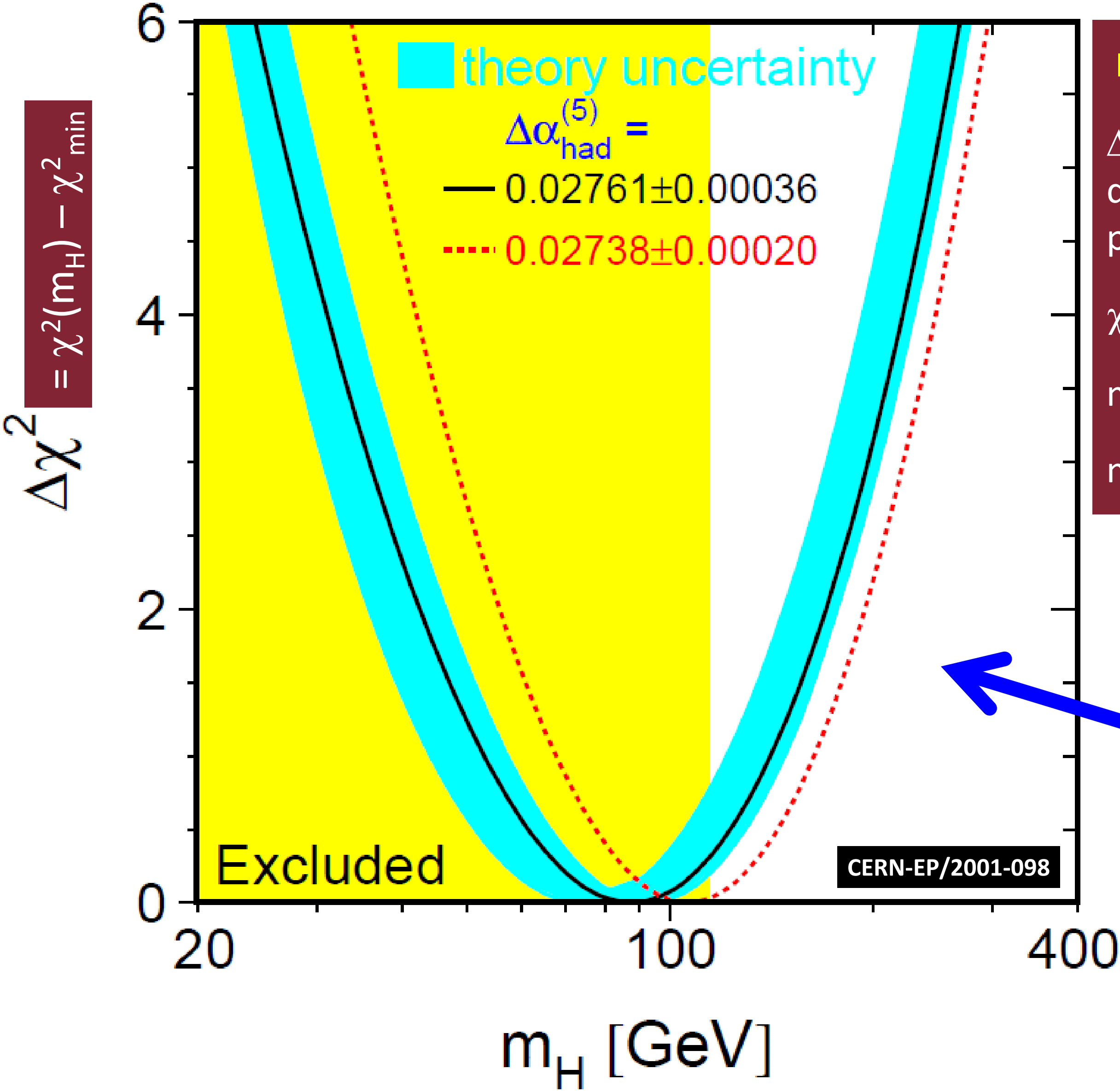
$\chi^2 = \Sigma_i\, (\text{pull}_i)^2$;

χ^2 / dof = 25.5 / 15 $\rightarrow \mathcal{P}(\chi^2)$=4.4%.

This nice agreement was mainly used to:

- claim the quality of the SM (and exp.'s);
- predict the (unknown) mass of the Higgs.

global LEP(1+2) fit : m_H prediction



- excl. by direct search [*see*];

$\Delta\alpha^{(5)}_{had}$: contribution of light quarks to photon vacuum polarization (two computations).

χ^2_{min} / dof = 22.9 / 15

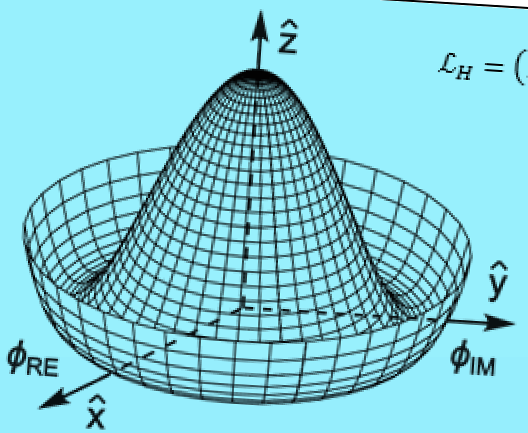
$m_H = 88^{+53}_{-35}$ GeV

m_H < 196 GeV @ 95% CL

Just an example, often remade with small variations before LHC. The 2nd most quoted LEP plot (after n_ν) will disappear soon.

"ℋ" (="ℋistory") in the right corner means "now obsolete" (here and in the following).

iv. Physics 2 : Higgs



$$\mathcal{L}_H = (D_\mu\phi)^* D_\mu\phi - V(\phi) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu\phi = \partial\phi + ieA_\mu\phi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\phi) = \alpha\phi^*\phi + \beta(\phi^*\phi)^2$$

$$\alpha < 0, \qquad \beta > 0$$

- The Higgs boson has been (*very likely*) discovered at LHC, definitely not at LEP.
- Why remember an old and not-so-nice story, like the LEP search of the Higgs ?
- Because it is very instructive – almost all searches are unsuccessful → in practice limits and exclusions are much more frequent than discoveries;
- [*also, in the past fluctuations/mistakes have been rare, but not null*]

- go → § 11, then come back;
- Higgs properties are treated in § LHC [+ RQM + EWI];
- here only an incomplete discussion for Higgs production in e^+e^- at LEP1 & LEP2 energies.

Higgs search @ LEP1

- In the SM the Higgs boson is at the origin of fermion masses;
- at least one H, neutral, spin-0;
- only 1 H → "minimal SM" (MSM, the case discussed in these lectures);
- m_H free parameter of SM (but $m_H < 1$ TeV);
- in the MSM, if m_H is given, the dynamics is completely determined and calculable (couplings, cross sections, BR's, angular distributions, …);
- properties :
  - charge : 0; spin : 0; $J^P = 0^+$;
  - coupling with fermions f :

$$\Gamma(H \to f\bar{f}) = \frac{c_f}{4\pi\sqrt{2}} G_F m_H m_f^2 \beta_f^3;$$

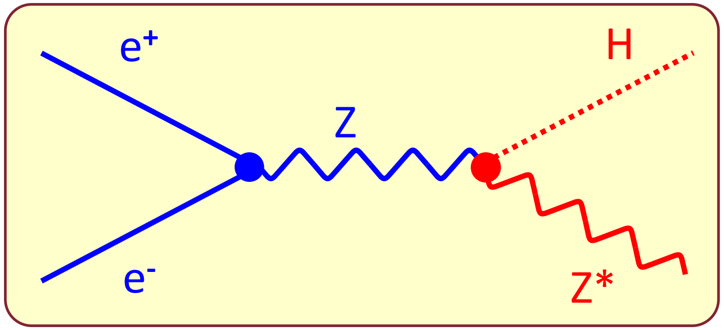
$$\beta_f = \sqrt{1 - 4m_f^2/m_H^2}; \qquad c_f = \begin{cases} 1 & \text{[leptons]} \\ 3 & \text{[quarks]} \end{cases};$$

  - [notice: $\Gamma_f \propto m_f^2$);
  - therefore, H decays mainly in the fermion pair of highest mass kinematically allowed;
  - therefore, if $m_H > 2m_b$ (i.e. > 10 GeV), mainly $H \to b\bar{b}$.
- $Z \not\to HH$ (spin-statistics, *like* $\rho^0 \not\to \pi^0\pi^0$);
- in lowest order only:
  - $Z \not\to H\gamma$ (Z, H neutral !!!) [or $H \not\to Z\gamma$];
  - $H \not\to \gamma\gamma$) (H neutral !!!)

    however, ($H \to \gamma\gamma$) essential for the discovery (see § LHC).
  - $H \not\to gg$ (no strong interactions);
  - but $H \to Z\gamma$, $\gamma\gamma$, gg through higher order processes.

more complete discussion in § LHC, e.g. discussion of $H \to Z$, W decays.

Higgs search @ LEP1: Bjorken process



$e^+e^- \to Z \to HZ^*$
[Bjorken process]

- LEP 1 ($\sqrt{s} \approx m_z$) : $e^+e^- \to Z \to HZ^* \to \left(f\bar{f}\right)\left(f\bar{f}\right)$;

  i.e. the Higgs production is one of the possible Z decays :

$$\frac{1}{\Gamma(Z \to f\bar{f})} \frac{d\Gamma(Z \to Hf\bar{f})}{dx} =$$

$$= \frac{G_F m_Z^2}{24\sqrt{2}\pi^2} \frac{(12 - 12x + x^2 + 8y^2)\sqrt{x^2 - 4y^2}}{(x - y^2)^2};$$

$$x = \frac{2E_H}{m_Z} = \frac{m_Z^2 + m_H^2 - m_{f\bar{f}}^2}{m_Z^2}; \qquad y = \frac{m_H}{m_Z}.$$

- kinematical constraint :

  $\sqrt{s} \approx m_Z > m_{Z*} + m_H \to m_H < m_Z$

- best observable when

  $Z^* \to \ell^+\ell^-$ (no bckgd),

  $H \to b\,\bar{b}$ (BR $\geq$ 80%);

- BR($Z \to H\ell^+\ell^-$) $\approx 10^{-4}$ @ m_H= 8 GeV

  $\approx 10^{-7}$ @ m_H=70 GeV;

- Kinematics not difficult, e.g. $Z^* \to \mu^+\mu^-$,

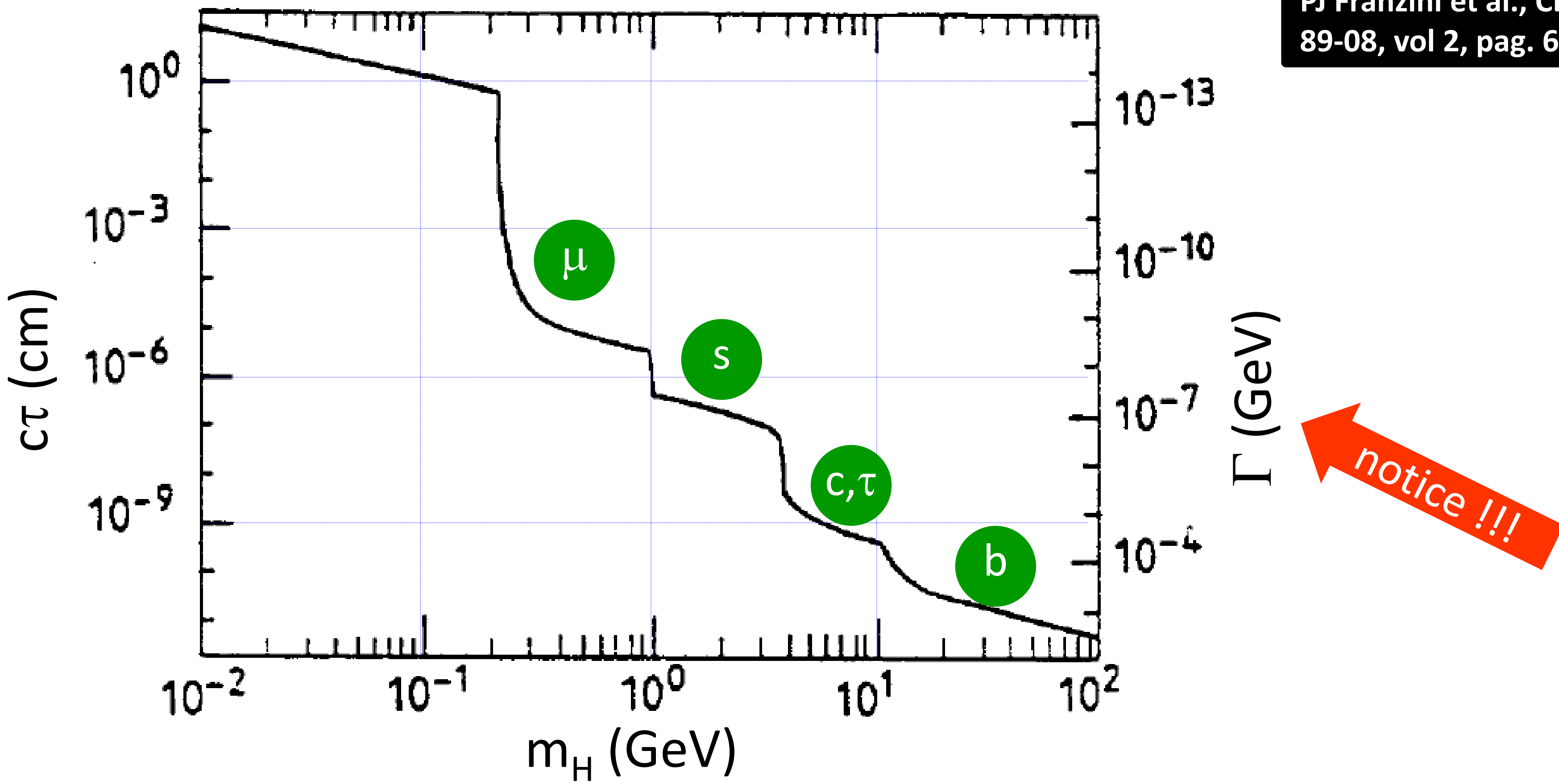
  $m(Z^*) = m_{\mu\mu}$, $E(Z^*) = E_{\mu\mu}$,

  $m_H^2 = s + m_{\mu\mu}^2 - 2\sqrt{s}E_{\mu\mu}$.

ok ?

Higgs search @ LEP1: decay predictions

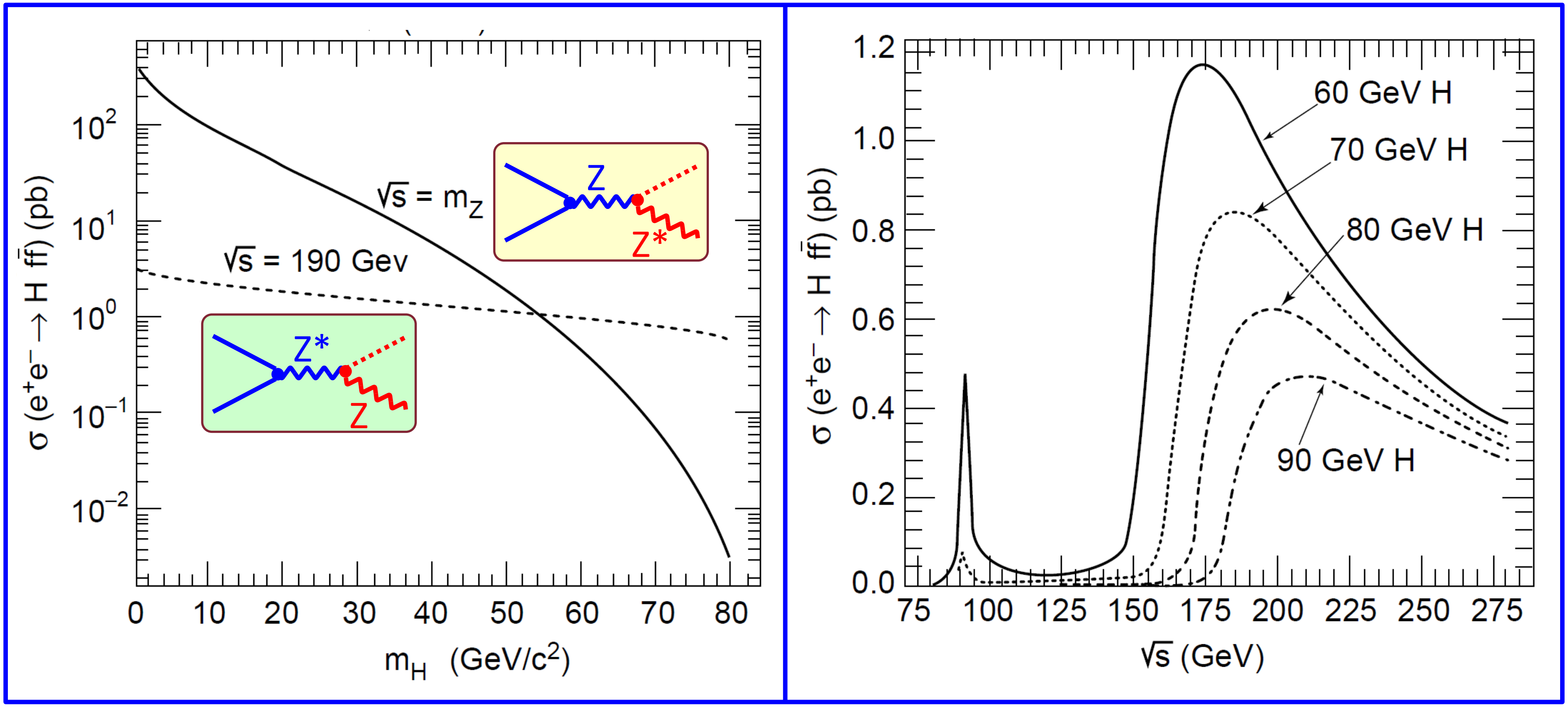
PJ Franzini et al., CERN 89-08, vol 2, pag. 65'.



The main decay product of H is the $f\bar{f}$ of largest mass compatible with m_H: e.g. s means $H \rightarrow s\bar{s}$.

When a new threshold opens up, there is a "step" in $c\tau$ ($\sim 1/\Gamma$), rounded by phase space.

Higgs search @ LEP1: predictions



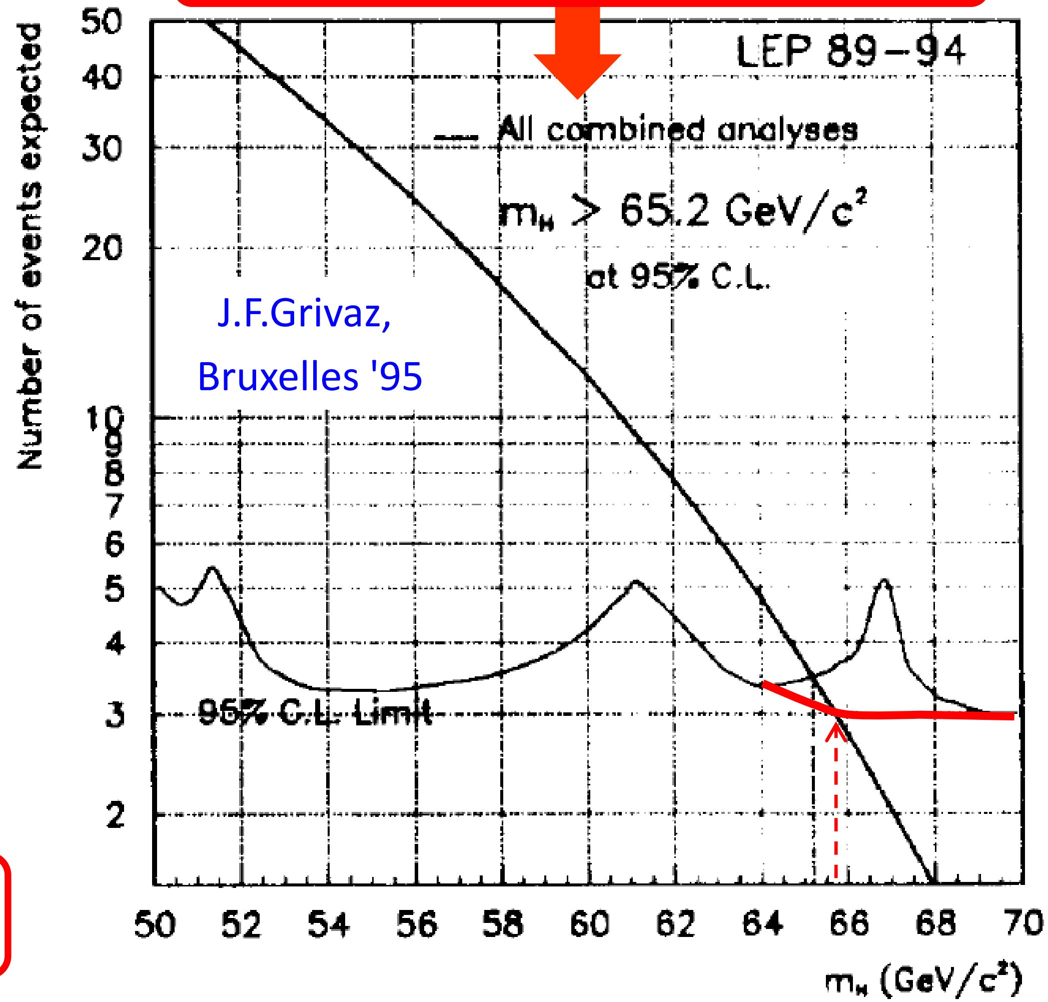
For $\sqrt{s} \approx m_Z$ (real Z) and $m_H << m_Z$, the Bjorken process ($e^+e^- \to Z \to HZ^*$) has a sizeable cross section, but at larger m_H it essentially disappears $\to$ go to larger $\sqrt{s}$.

The predictions at $\sqrt{s} >> m_Z$ come from a similar process ($e^+e^- \to Z^* \to HZ$, virtual Z*), known as "higgs-strahlung" [*next slides*].

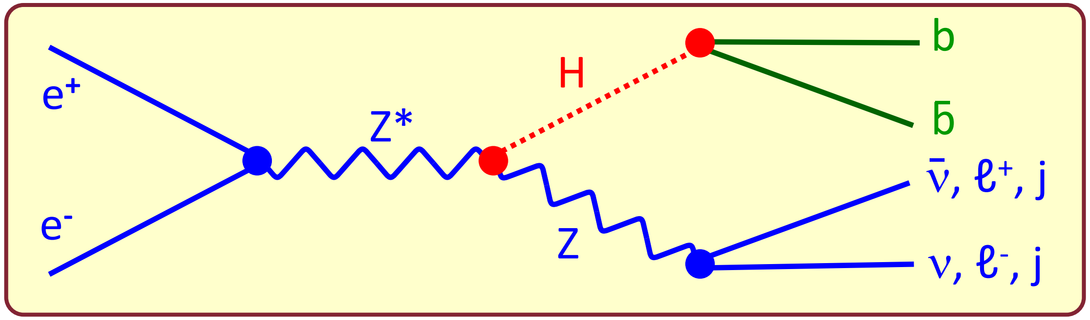
Higgs search @ LEP1: results

- this plot summarizes the limits of the four experiments :
  A : 63.1 GeV
  D : 55.4 "
  L : 60.2 "
  O : 59.1 ";
- the candidate @ m_H = 67 GeV (OPAL) reduces the limit by few × 100 MeV;
- a test case for the method, discussed in § limits; notice :
  - the combined limit is "better" than any single exp.;
  - the "worst" observed limit does not come necessarily from the "worst" exp.;
  - … because it is a random variable;
- conclusion: move to higher √s, i.e. Bjorken process → higgs-strahlung.

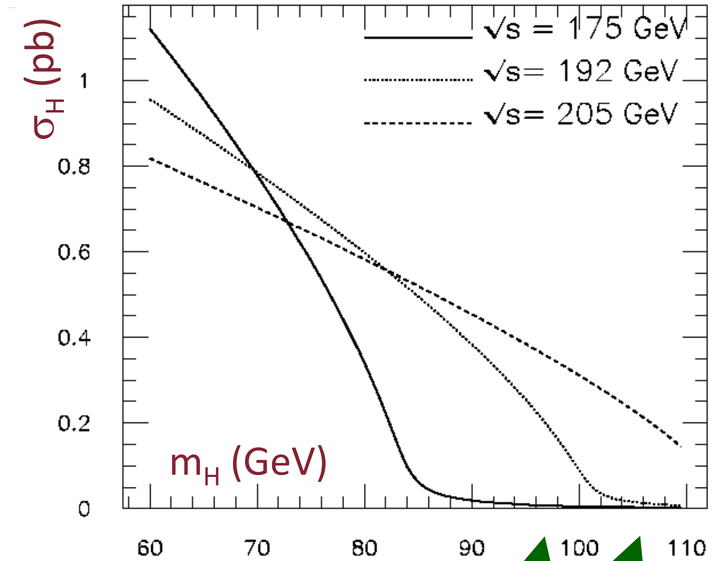
LEP 1, $\sqrt{s} \approx m_Z$:
~3.7 M [Z→ hadrons] / exp in 1989-94;
m_H > 65.2 GeV @ 95% CL



Higgs search @ LEP2



$$e^+e^- \rightarrow Z^* \rightarrow HZ$$
[higgs-strahlung]



- LEP 2 : process of "higgs-strahlung" (= radiative emission of a Higgs boson from a Z*);
- i.e. the higgs production is a 4-fermion final state, mediated by a virtual Z* [like $e^+e^- \rightarrow W^+ W^- \rightarrow 4f$];
- kinematical constraint :

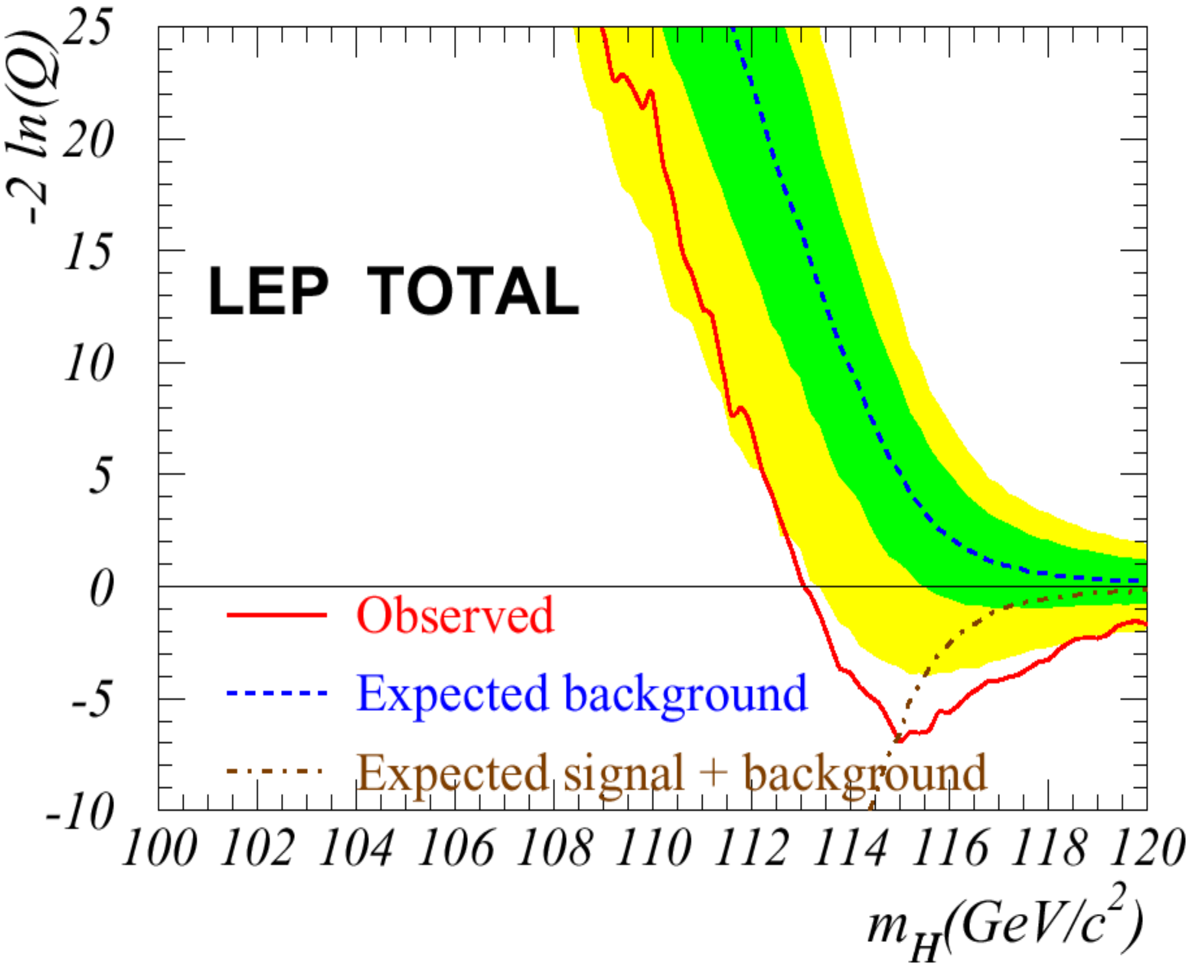
  $$\sqrt{s} = m_{Z^*} > m_Z + m_H$$
- [*no* $\mathfrak{H}$ *here, because of possible future colliders, see later*].

$$\sigma_0(e^+e^- \rightarrow Z^* \rightarrow ZH) = \frac{G_F^2 m_Z^4}{24\pi s}\left[\left(g_V^\ell\right)^2 + \left(g_A^\ell\right)^2\right]\sqrt{\lambda}\,\frac{\lambda + 12 m_Z^2/s}{\left(1 - m_Z^2/s\right)^2};$$

$$\left[\lambda = \left(1 - m_H^2/s - m_Z^2/s\right)^2 - 4 m_H^2 m_Z^2/s^2;\right]$$

$$\frac{1}{\sigma_0}\frac{d\sigma_0}{d\cos\theta} = \frac{\lambda^2 \sin^2\theta + 8 m_Z^2/s}{4\lambda^2/3 + 16 m_Z^2/s} \xrightarrow{s\ \text{large}} \frac{3}{4}\sin^2\theta.$$

Higgs search @ LEP2: LEPC 3/11/2000



- $-2\ell nQ = -2\ell n(\Lambda_s/\Lambda_b)$;
- $-2\ell nQ(m_H=115) = -7$;
- if interpreted as a discovery
  - $m_H = 115^{+1.3}_{-0.9}$ GeV;
  - $1\text{-}CL_b = 4.2\times10^{-3}$;
  - i.e. "2.9 σ";
- if interpreted as a limit :
  - $m_H > 113.5$ GeV @ 95%CL.

???

Higgs search @ LEP2: LEPC 3/11/2000

RECOMMENDATION

Given the consistency for the combined results with the hypothesis of the production of a SM Higgs boson with a mass of 115 GeV, and an observed excess in the combined data set of 2.9σ, a further run with 200 pb^{-1} per experiment at 208 GeV would enable the four experiments to establish a 5σ discovery.

The four experiments consider the search for the SM Higgs boson to be of the highest importance, and CERN should not miss such a unique opportunity for a discovery.

Therefore, we request to run LEP in 2001 to collect $\mathcal{O}(200\ pb^{-1})$ at $\sqrt{s} \geq 208$ GeV.

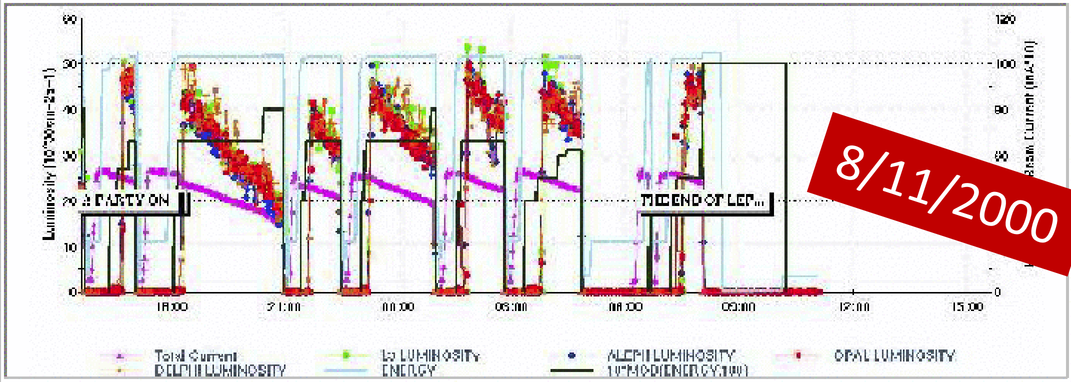
ALEPH, DELPHI, L3, OPAL

The LEP Higgs Working Group

3/11/2000

P. Igo-Kemenes - LEP Seminar - Nov. 3, 2000

LEP shuts down after eleven years of forefront research



8/11/2000

These are the measurements taken of LEP's final beam. The accelerator was switched off for the last time at 8:00 am on 2 November. (Click on photo for enlargement)

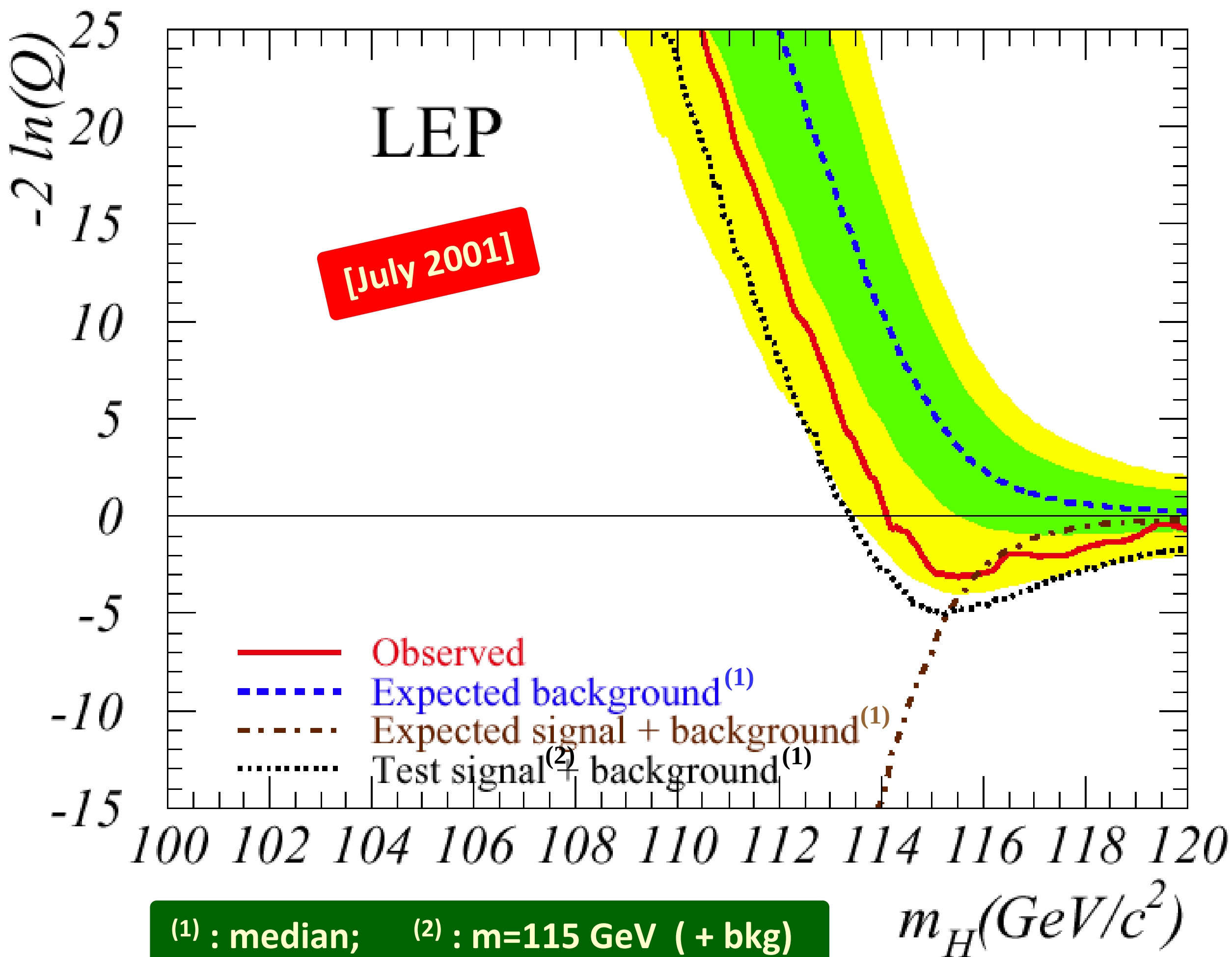
After extended consultation with the appropriate scientific committees, CERN 's Director-General Luciano Maiani announced today that the LEP accelerator had been switched off for the last time. LEP was scheduled to close at the end of September 2000 but tantalising signs of possible new physics led to LEP's run being extended until 2 November. At the end of this extra period, the four LEP experiments had produced a number of collisions compatible with the production of Higgs particles with a mass of around 115 GeV. These events were also compatible with other known processes. The new data was not sufficiently conclusive to justify running LEP in 2001, which would have inevitable impact on LHC construction and CERN's scientific programme. The CERN Management decided that the best

a difficult decision

Higgs search @ LEP2: July 2001

- if intepreted as a discovery:
  - m_H=115.6 GeV:
  - $-2\ell nQ|_{\text{actual events}}$ = -2.9;
  - $1\text{-}CL_b = 3.5\times10^{-2}$;
  - "2.1 σ";
- if interpreted as a limit:
  - m_H > 114.1 GeV @ 95%CL.

????

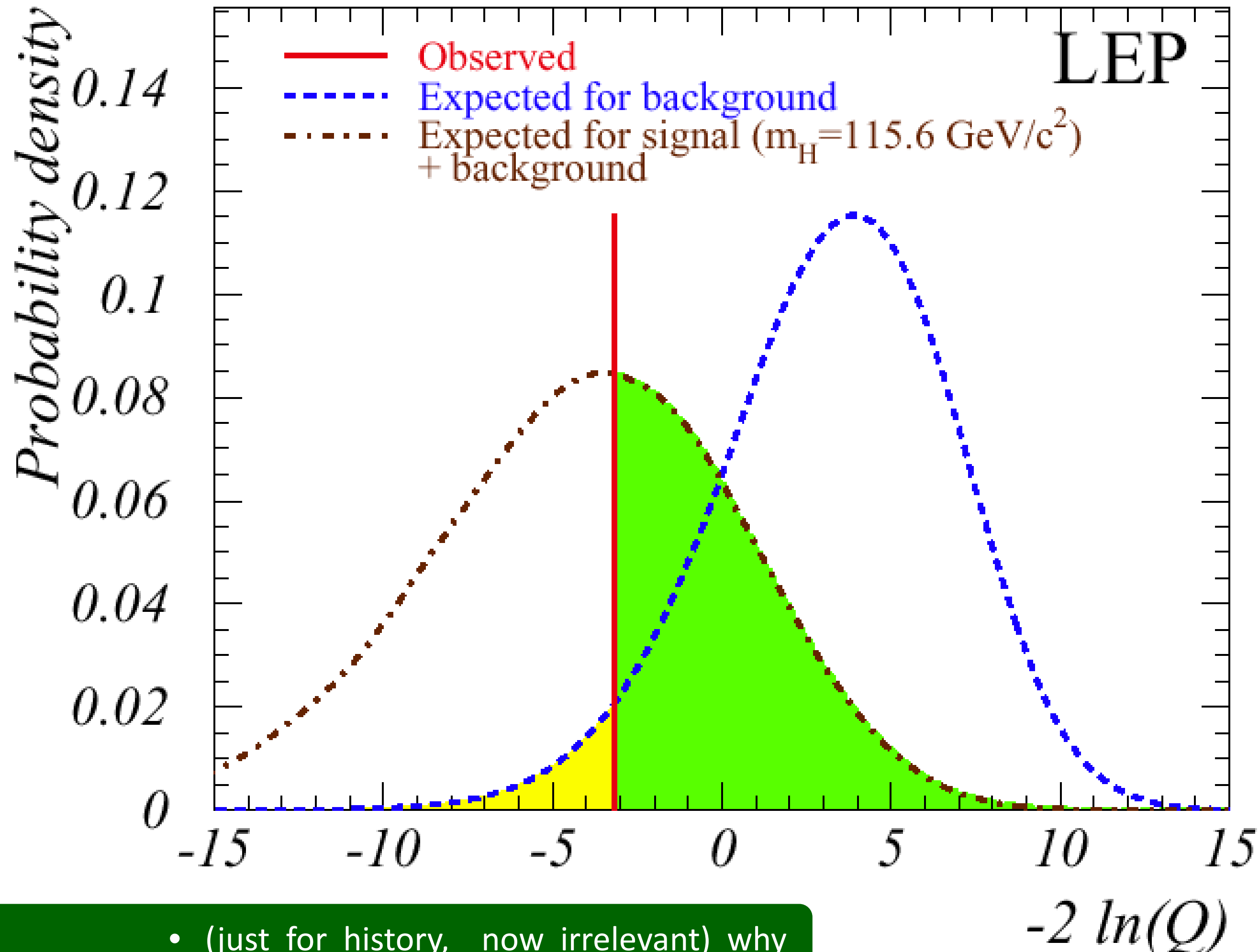


(1) : median; (2) : m=115 GeV (+ bkg)

Higgs search @ LEP2: the end

- method "gedanken-experiment" [i.e. produce via mc many experiments, with the same quality and $\mathcal{L}_{int}$ of the present one] :

- m_H^{test} = 115.6 GeV;
- $\int f_{b,s}\, d(-2\ell nQ) = 1$;
- "◆" = 1-CL_b= 3.5%;
- "◆" = CL_{s+b}= 43%.



Comments/questions (**imho**):

- if this result had been presented in November 2000, there would have been no problem: no one would have claimed the need for further studies.
- (just for history, now irrelevant) why was the first analysis wrong ? well, ... **?**
- why to show it to students ? because it is very instructive, normal classes see only the standard (discovery vs limits).

Higgs search @ LEP2: conclusion

- the "LEPC result" is difficult to explain (NOT only to students) : stat. fluctuations, mistakes, systematics out-of-control, …
- the CERN management (L. Maiani) took the right decision at a high risk;
- the real threat was a delay of LHC, a huge human and economic price;
- instead, the final results are relatively simple to explain: a honest fluctuation at 3.5% does not deserve a discussion;
- the Higgs boson search crossed the ocean, but the TeVatron did not really enter in the game;
- and finally LHC … [*you know*].

Other more personal comments:

- unlike theoretical physics, statistics (and human behavior) require risk evaluation;
- experimental physics lies in the middle;
- you should understand and judge the decisions of the experiments and the management (often they did NOT agree);
- … while the landscape was changing (November '00, July '01, post-LEP, now);
- you might conclude that the "right decision" is a function of role and time (???);
- *… and that searches are risky, not for gutless people.*

the Higgs boson @ LEP : $\sigma(e^+e^- \to HZ)$

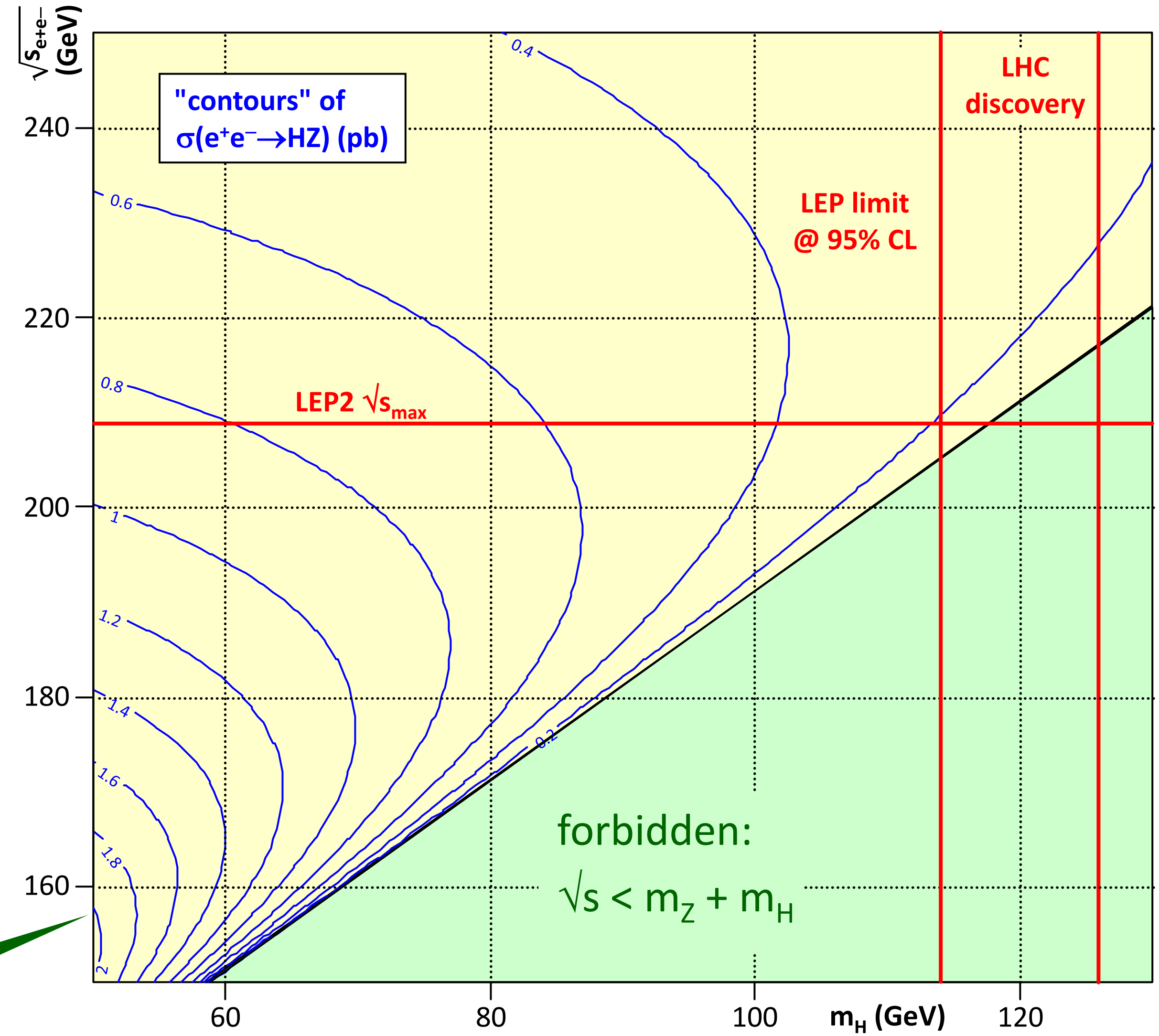
AFTER the LHC discovery:

Q: could LEP see a 126 GeV Higgs ?

plot the cross section:

- σ = 0.2 ÷ 1.8 pb;
- strongly m_H dependent;
- $\mathcal{L}_{int} \approx 200\ pb^{-1}$/year;
- i.e. n = 40÷200 events/y, shared among many decay channels (some undetectable).

A: the plot is very clear: you should be able to judge yourself !



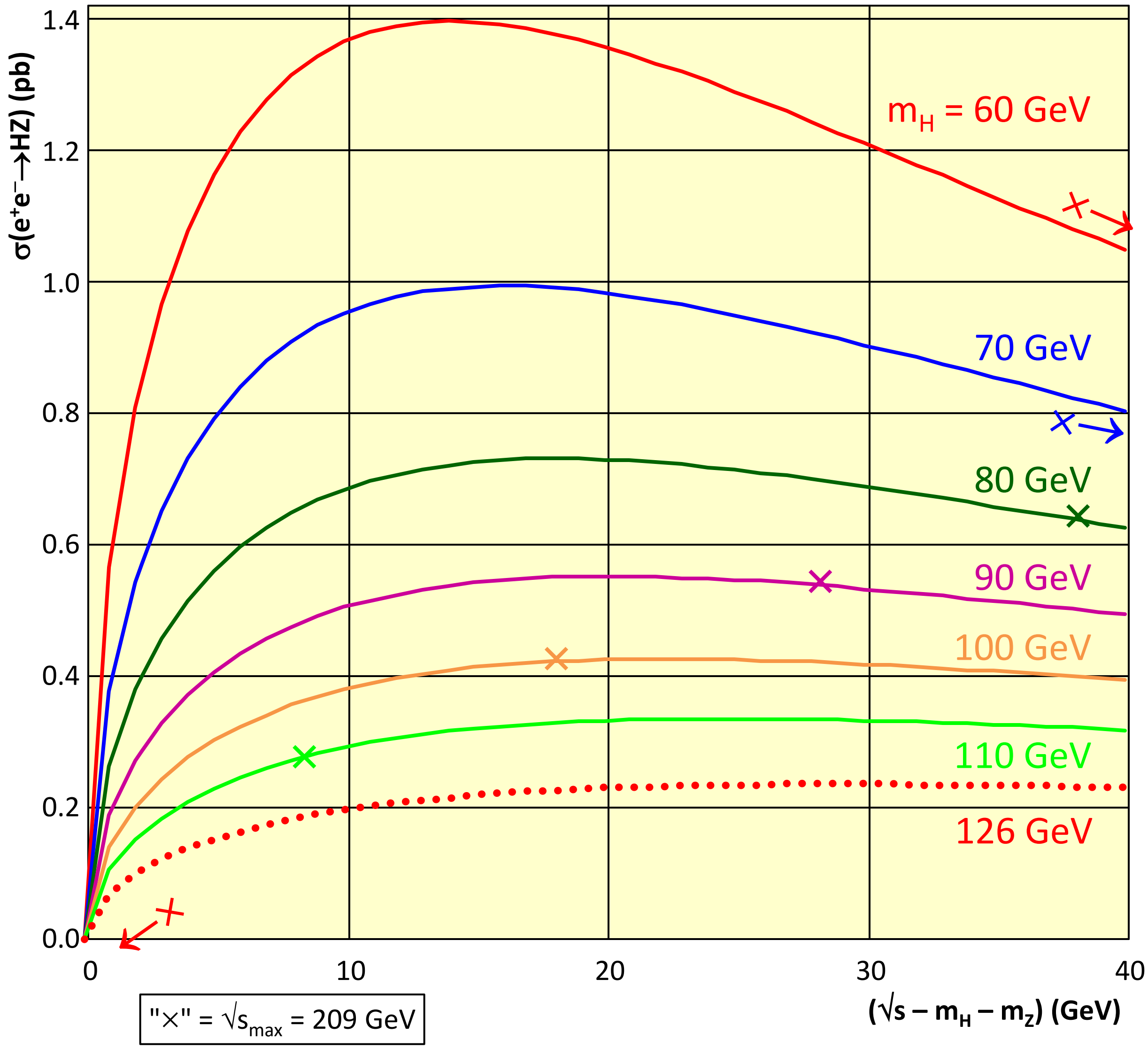
warning:
- tree level,
- $\Gamma_H = \Gamma_Z = 0$;

but ok for discussion.

the Higgs boson @ LEP : higgs-strahlung

Plot $\sigma(e^+e^- \to Z^* \to HZ)$ vs the "kinetic" energy, i.e. ($T = \sqrt{s} - m_H - m_Z$), in the approx. $\Gamma_Z = \Gamma_H = 0$:

- $T \leq 0 \to \sigma = 0$ (obvious);
- the ×'s show $\sqrt{s}$ = 209 GeV;
- $\sigma_{max}(T)$ at $T \approx 15 \div 20$ GeV, slightly increasing with m_H;
- $\sigma_{max}(m_H)$ decreases a lot when m_H increases;
- for $\sqrt{s} >> m_H + m_Z$, $\sigma \propto s^{-1}$ (obvious);
- for m_H > 110 GeV, other processes (not shown), other than higgsstrahlung;
- if m_H = 126 GeV (LHC), H not produced at LEP 2.

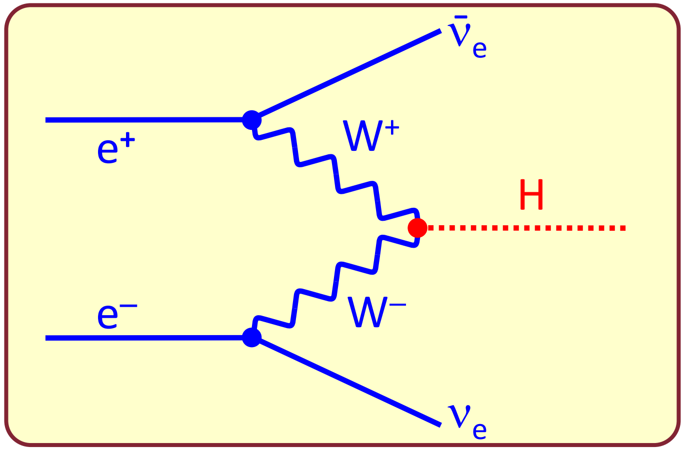


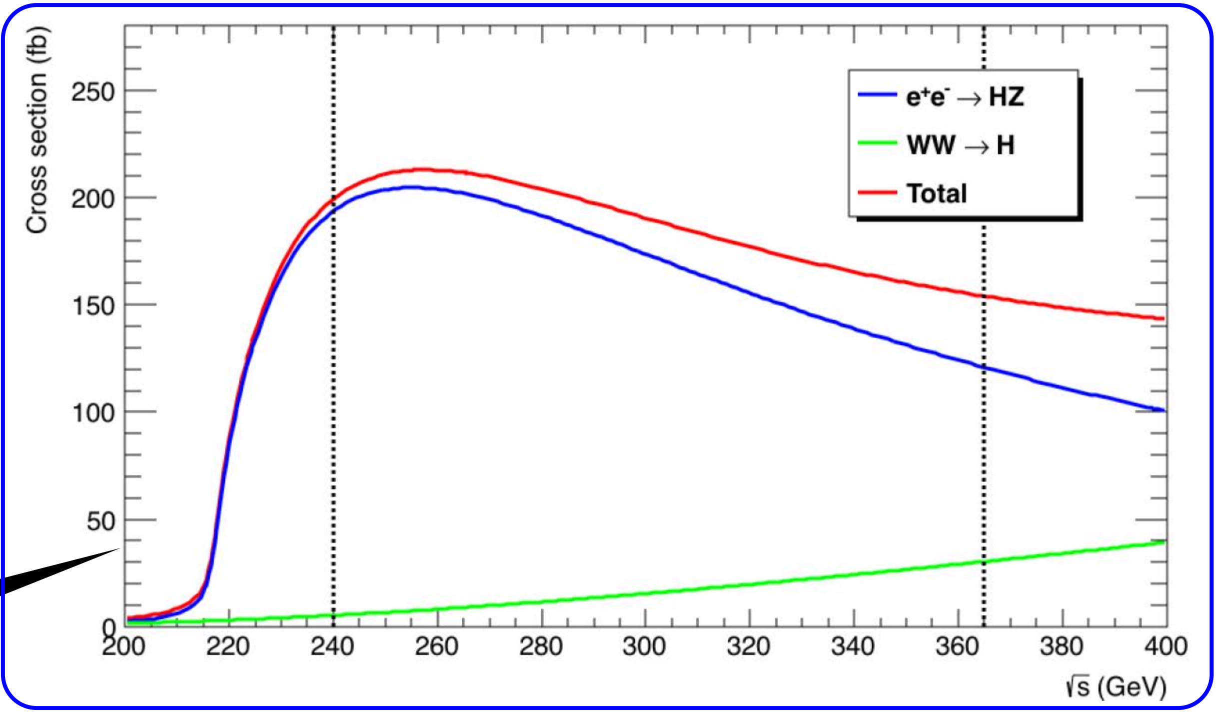
the Higgs boson @ LEP : the future in e^+e^-

In the post-LEP (and post-H-discovery) era, the interest has shifted to a possible higher energy e^+e^- collider (circular or linear).

In this case:

- consider also other processes (e.g. the so called "WW-fusion" $e^+e^- \to H\bar{\nu}_e\nu_e$ [*see*];
- compute the cross-section for m_H=126 GeV, as a function of $\sqrt{s}$ [*see*];
- study the physics of (say) ~1 million H:
  - measure Γ_H *à la* J/ψ;
  - measure all H couplings;
- [*obviously no ℋ here*].





Future Circular Collider Study, CERN 2018

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Jan Brueghel the Elder and Hendrick de Clerck – Abundance and the Four Elements – 1606 – Prado Museum

End of chapter 10

Particle Physics - Chapter 11
Searches and limits

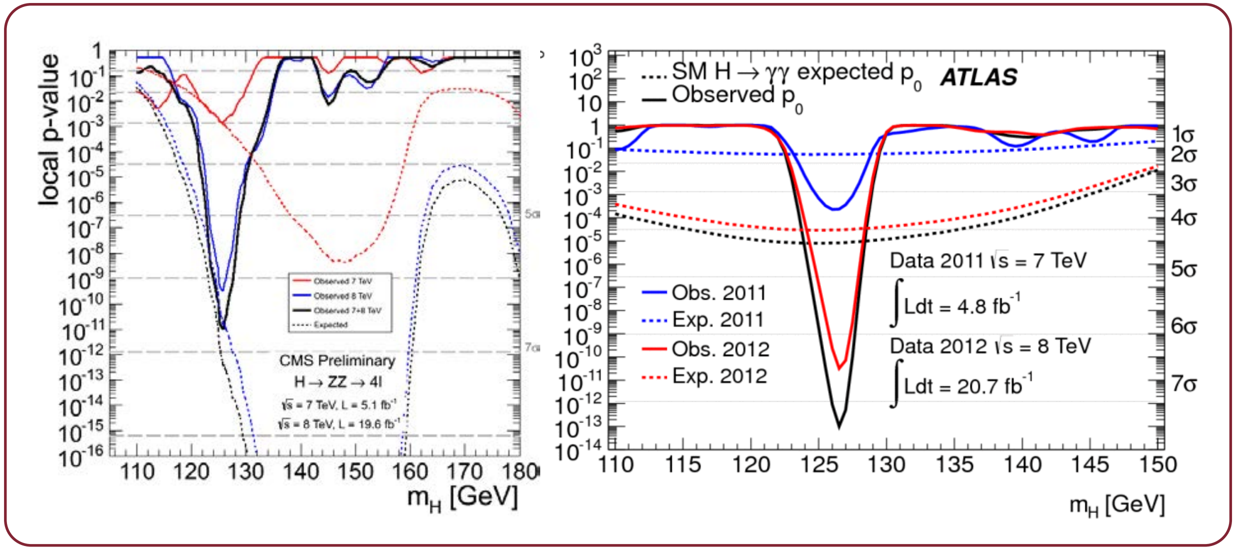
Paolo Bagnaia

SAPIENZA
UNIVERSITÀ DI ROMA

AA 18-19

last mod.
15-May-19

1. Probability
2. Searches and limits
3. Limits
4. Maximum likelihood
5. Interpretation of results



- methods commonly used in all recent searches (e.g. LEP, LHC, gravitational waves);
- also in other lectures (e.g. "Laboratorio di Meccanica", Physics Laboratory);
- but "repetita juvant" (maybe);
- not a well-organized presentation, beyond the scope of present lectures (→ references + next year).

There are three kinds of lies: lies, damned lies, and statistics.

B. Disraeli

probability: a new guest star in the game

- Modern particle physics makes a large use of (relatively) new sciences : **probability** and her sister **statistics**;
- [*we are scientists, not gamblers, and do NOT discuss poker and dice here*];
- in classical physics the resolution function of an observable can be seen as a pdf(*);
- q.m. is probabilistic, at least in its Copenhagen interpretation, since the predictions are distributions, while the experiments produce single values;
- but its use to assess a statement *[e.g. "the probability that we have discovered the Higgs boson in our data"]* is really modern;
- however, we actually think in terms of probability (*risk*, *chance*, *luck* … essentially mean "probability", while *experience*, *past*, *use*, … mean "statistics").

____

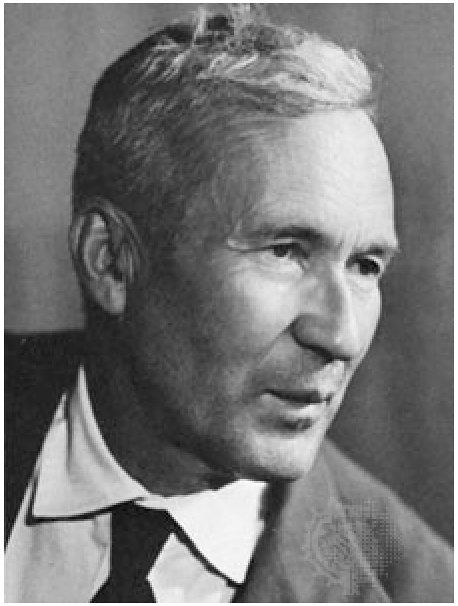
(*) pdf: acronym for *probability distribution function*. (or *probability density function*).

For [some] readers :

- *these lectures avoid carefully to enter in the discussion frequentism ↔ bayesianism;*
- *however, a modern particle physicist must understand and use both;*
- *only, (try to) avoid fights.*

probability: Kolmogorov axioms

Andrei Nikolayevich Kolmogorov [Андрей Николаевич Колмогоров] (1903–1987), a Russian (sovietic) mathematician, in 1933 wrote a fundamental paper on axiomatization of probability; he introduced the space S of the events (A, B, …) and the event probability as a measure $\mathcal{P}$(A) in S.

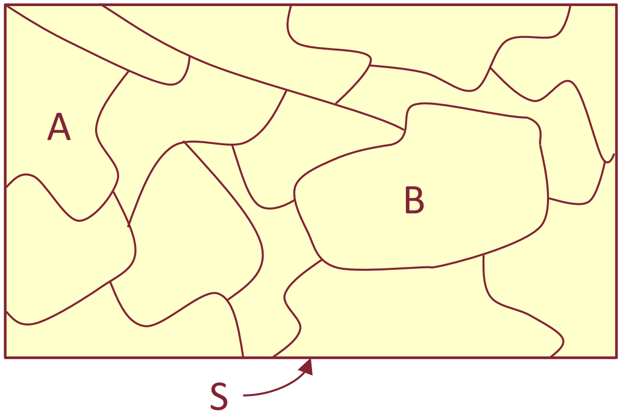


K. axioms are :

1. $0 \le \mathcal{P}(A) \le 1 \;\forall\; A \in S$;
2. $\mathcal{P}(S) = 1$;
3. $A \cap B = \emptyset \Rightarrow \mathcal{P}(A \cup B) = \mathcal{P}(A) + \mathcal{P}(B)$.

Some theorems (easily demonstrated):

- $\mathcal{P}(\bar{A}) = 1 - \mathcal{P}(A)$;
- $\mathcal{P}(A \cup \bar{A}) = 1$;
- $\mathcal{P}(\emptyset) = 0$;
- $A \subset B \Rightarrow \mathcal{P}(A) \le \mathcal{P}(B)$;
- $\mathcal{P}(A \cup B) = \mathcal{P}(A) + \mathcal{P}(B) - \mathcal{P}(A \cap B)$.



searches and limits

- Sometimes, the result of the study is NOT the measurement of an observable x :

  "$x = x_{exp} \pm \Delta x$",

- but, instead, a qualitative "**search**" :

  "the phenomenon $\mathcal{Y}$ does (not) exist",

  or, alternatively :

  "the phenomenon $\mathcal{Y}$ does NOT exist in the parameter range Φ".

- [statements with "not" apply if the effect is not found, and an "**exclusion**" (a "**limit**", when Φ is not full) is established]
- In modern experiments, the searches occupy more than 50% of the published papers, and almost all are negative [*but the Higgs search at LHC, of course*].
- Obviously, a negative result is NOT a failure: if any, it is a failure of the theory under test.
- [but a **discovery** is much more pleasant and rewarding]
- A rigorous procedure, well understood and "easy" to apply, is imperative.
- This method is a major success of the LEP era : it uses math, statistics, physics, common sense and communication skill.
- It MUST be in the panoply of each particle physicist, both theoreticians and experimentalists.

These lectures must remain inside the SM:

- Higgs searches at LEP (negative) and LHC (positive) as examples;
- after the Higgs discovery, the focus has shifted toward "bSM" searches, but the method has not changed (still improving).

searches and limits: definitions

In the next slides :

- $\mathscr{L}_{int}$: integrated luminosity;
- σ_s : cross section of signal (searched for);
- σ_b : cross section of background (known);
- ε_s : efficiency for signal (0÷1, larger is better);
- ε_b : ditto for background (0÷1, smaller is better);
- s : # expected signal events [$s = \mathscr{L}_{int}\varepsilon_s\sigma_s$];
- b : ditto for background [$b = \mathscr{L}_{int}\varepsilon_b\sigma_b$];
- n : # expected events [n = s + b, or n = b];
- N : # found events (N fluctuates around n with Poisson ($\rightarrow$ Gauss) statistics;
- $\mathscr{P}$: probability, according to a given pdf;
- CL : "confidence level", a limit (< 1) in the cumulative probability;
- Λ : likelihood function for signal+bckgd (Λ_s) or bckgd-only (Λ_b) hypotheses;
- μ : parameter defining the signal level [$n = b + \mu s$], used for limit definition;
- p : "p-value", probability to get the same result or another less probable, in the hypothesis of bckgd-only;
- E[x] : expected value of the quantity "x".

a lot of math, but try to understand the underlying physics.

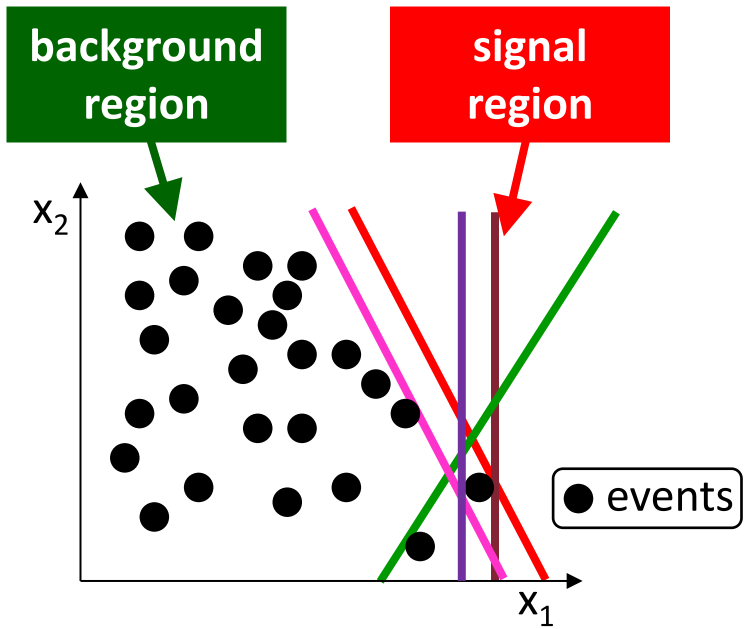
searches and limits: verify/falsify

- A theory (SM, SUSY, …) predicts a phenomenon (a particle, a dynamic effect), e.g. e^+, $\bar{p}$, Ω^-, $W^\pm/Z$, H;
- [in some cases the phenomenon depends on unknown parameter(s), e.g. the Higgs boson mass]
- a new device (e.g. an accelerator) is potentially able to observe the phenomenon [fully or in a range of the parameters space still unexplored];
- therefore, two possibilities:
  - A. **observation**: the theory is "verified" (à la Popper) [and the free parameter(s) are measured];
  - B. **non-observation**: the theory is "falsified" (à la Popper) [or some subspace in the parameter space, e.g. an interval in one dimension, is excluded → a "**limit**" is established];

❖ different approach, nowadays less common ("model independent"): look for unknown effects, without theoretical guidance, e.g. $\mathbb{CP}$ violation, J/ψ.

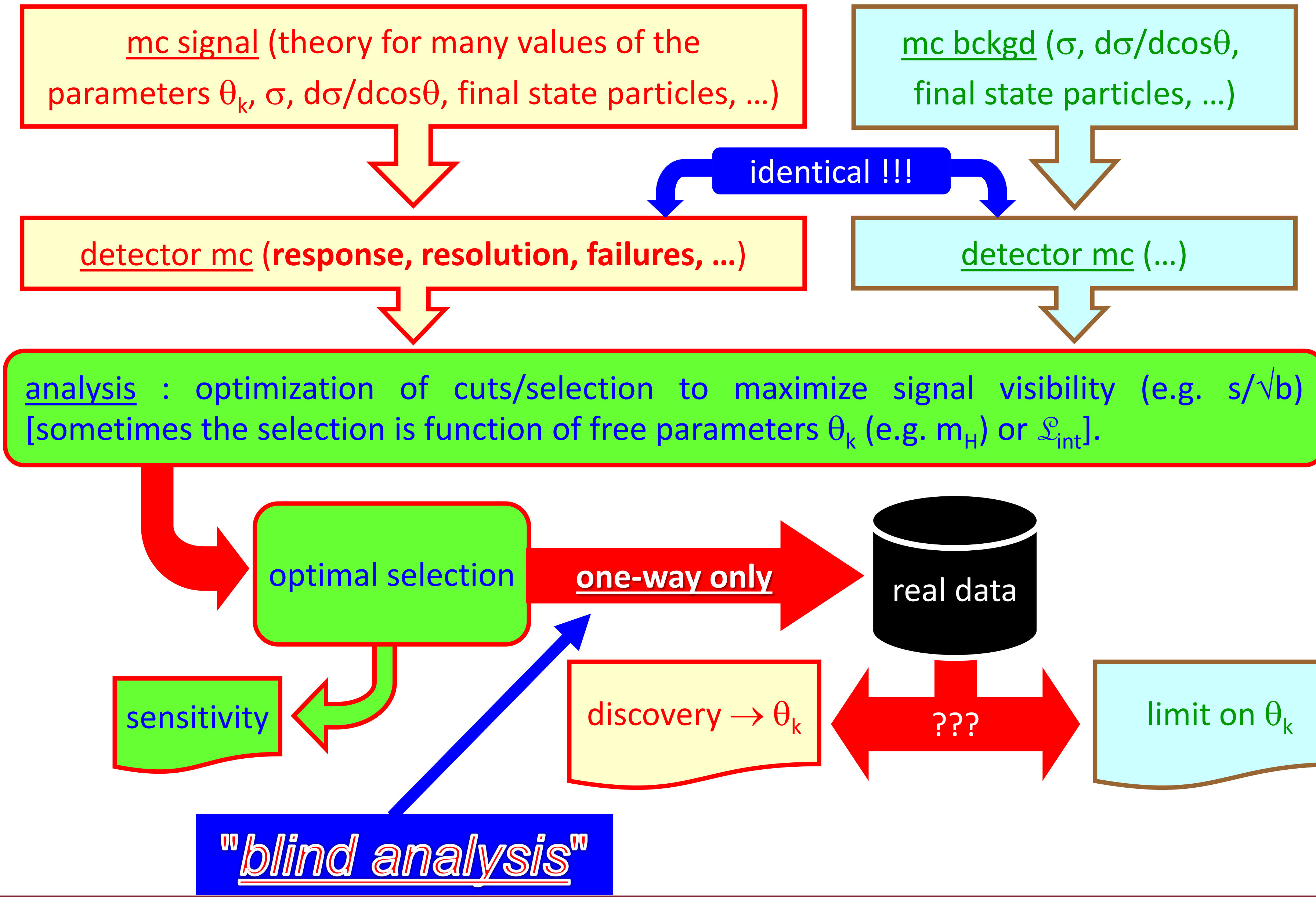
searches and limits: blind analysis

- the key point : usually b≫s, but $f_b(x)$ and $f_s(x)$ are <u>very</u> different → cuts in the event variables (x : mass, angle, …), such that to enhance s wrt b;
- when n is large (n ≫ √n), statistical fluctuations (s.f.) do NOT modify the result;
- usually (not only for impatience) n is small and its s.f. are important;
- small variations in the filter (→ small change in b and s) may correspond to large differences in the result N [*look at the example in two variables: e.g., if b=0.001 after the cuts, when N changes 0 → 1, N=0 or N=1 is totally different*];
- a "neutral" analysis is impossible; <u>a posteriori</u>, it is always easy to justify a little change in the cuts, which strongly affects the results;
- therefore, the only honest procedure consists in defining the selection <u>a priori</u> (e.g. by optimizing the <u>expected</u> visibility on a mc event sample); then, the selection is "blindly" applied to the actual event sample (→ "blind analysis").



which is the "right" cut ?

searches and limits: flowchart



limits

[*in the "good ole times", life was simpler : if the background is negligible, the first observations led to the discovery, as for* e^+, $\bar{p}$, Ω^-, $W^\pm$ *and* Z]

- in most cases, the background (reducible or irreducible) is calculable;
- a discovery is defined as an observation that is incompatible with a +ve statistical fluctuation respect to the expected background alone;
- a limit is established if the observation is incompatible with a –ve fluctuation respect to the expected (signal + background);
- both statements are based on a "reductio ad absurdum"; since all values of N in $[0,\infty]$ are possible, it is compulsory to predefine a CL to "cut" the pdf;
- the CL for discovery and exclusion can be different : usually for the discovery stricter criteria are required;
- a priori the expected signal s can be compared with the fluctuation of the background (in approximation of large number of events, $s \leftrightarrow \sqrt{b}$) : $n_\sigma = s / \sqrt{b}$ is a figure of merit of the experiment;
- a posteriori the observed number (N) is compared with the expected background (b) or with the sum (s + b).

Example. In an exp., expect 100 background events and 44 signal after some cuts; use the "large number" approximation ($\Delta n = \sqrt{n}$) *:*

$b = 100$, $\Delta b = \sqrt{b} = 10$;

$n = s + b = 144$, $\Delta n = 12$.

The pre-chosen confidence level is "3 σ*".*

The discovery corresponds to an observation of

$N > (100+3\times10) = 130$ *events.*

A limit is established if

$N < (144 - 3\times12) = 108$ *events.*

There is no decision if $108 < N < 130$*.*

The values $N < 70$ *and* $N > 180$ *are "impossible".*

limits: problem

Problem (based on previous example) :

compute the factor, wrt to previous luminosity, which allows to avoid the "no-decision" region.

limits: Poisson statistics

- In general, the searches look for processes with VERY limited statistics (*want to discover asap*);
- therefore the limit ("n large", more precisely n >> √n) cannot be used (neither its consequences, like the Gauss pdf);
- searches are clearly in the "Poisson regime": large sample and small probability, such that the expected number of events ("successes") be finite;
- use the Poisson distribution :

$$\mathcal{P}(N|m) = \frac{e^{-m} m^N}{N!}; \quad \langle N \rangle = m; \quad \sigma_N = \sqrt{m};$$

- therefore, in a search, two cases :

a. the signal does exist :

$$\mathcal{P}(N|b+s) = \frac{e^{-(b+s)}(b+s)^N}{N!}; \quad \langle N \rangle = b+s; \quad \sigma_N = \sqrt{b+s};$$

[s may be known or unknown]

b. the signal does NOT exist :

$$\mathcal{P}(N|b) = \frac{e^{-b} b^N}{N!}; \ \langle N \rangle = b; \ \sigma_N = \sqrt{b};$$

- the strategy is : use N (= N^{exp}) to distinguish between case (a) and (b);
- since $\mathcal{P}$ is > 0 for any N in both cases, the procedure is to define a CL **a priori**, and accept the hypothesis (a or b) only if it falls in the **predefined** interval;
- modern (LHC) evolution : define a parameter, usually called "μ" :

$$\mathcal{P}(N|b+\mu s) = \frac{e^{-(b+\mu s)}(b+\mu s)^N}{N!}; \quad \langle N \rangle = b+\mu s; \quad \sigma_N = \sqrt{b+\mu s};$$

clearly, μ = 0 is bckgd only, while μ = 1 means discovery; sometimes results are presented as limits on "μ" [*e.g. exclude* μ = 0 *means "discovery"*].

limits : discovery, exclusion

- the "rule" on the CL usually accepted by experiments is:

> ➢ DISCOVERY : $\mathcal{P}$(b only) $\leq 2.86\times10^{-7}$, [called also "5σ" (1)];
>
> ➢ EXCLUSION : $\mathcal{P}$(s+b) $\leq 5\times10^{-2}$; [called also "95% CL"];

- a priori, the integrated luminosity $\mathcal{L}_{int}$ for discovery / exclusion can be computed :
  - ➢ $\mathcal{L}_{disc}$: $\mathcal{L}_{int}$ min, such that 50% of the experiments(2) (i.e. an experiment in 50% of the times) had $\mathcal{P}$(b only) ≤ $\mathcal{P}_{disc}$;
  - ➢ $\mathcal{L}_{excl}$: $\mathcal{L}_{int}$ min, such that 50% of the experiments(2) (i.e. an experiment in 50% of the times) had $\mathcal{P}$(s+b) ≤ $\mathcal{P}_{excl}$;

NB : this rule corresponds to the median *["an experiment, in 50% of the times…"]*, and it is different from the average *["an experiment, with exactly the expected number of events …"]*.

---

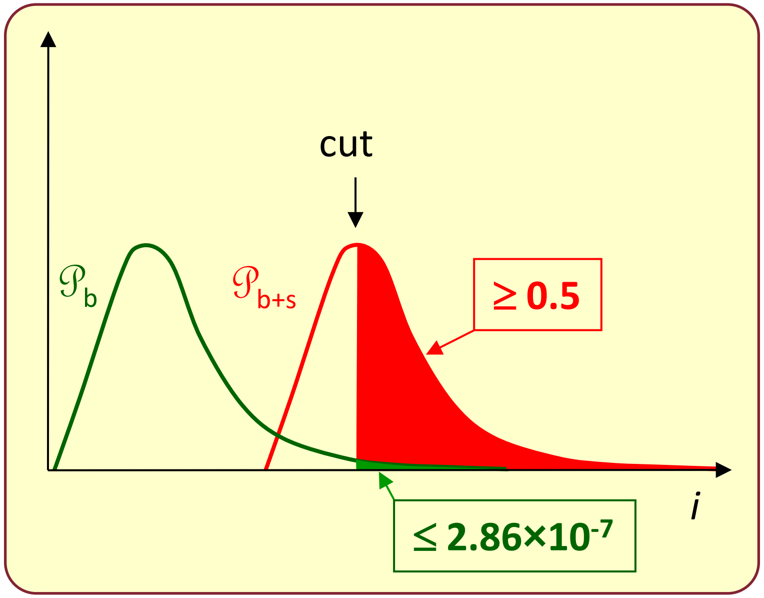
(1) this probability corresponds to 5σ for a gaussian pdf only; but the experimentalists use (*always*) the cut in probability and (*sometimes*) call it "5σ";

(2) for combined studies an "experiment" at LEP [LHC] results from the data of all 4 [2] collaborations; in this case $\mathcal{L}_{int} \to 4(2)\ \mathcal{L}_{int}$.

"A parameter is said to be excluded at xx% confidence level [say 95%] if the parameter itself would yield more evidence than that observed in the data at least 95% of the time in a [pseudo-] set of repeated experiments, all equivalent to the one under consideration."

[CMS web dixit]

limits : Luminosity of discovery, exclusion



- The values of $\mathscr{L}_{disc}$ and $\mathscr{L}_{excl}$ come from the previous equations; compute $\mathscr{L}_{disc}$ ($\mathscr{L}_{excl}$ is similar) :

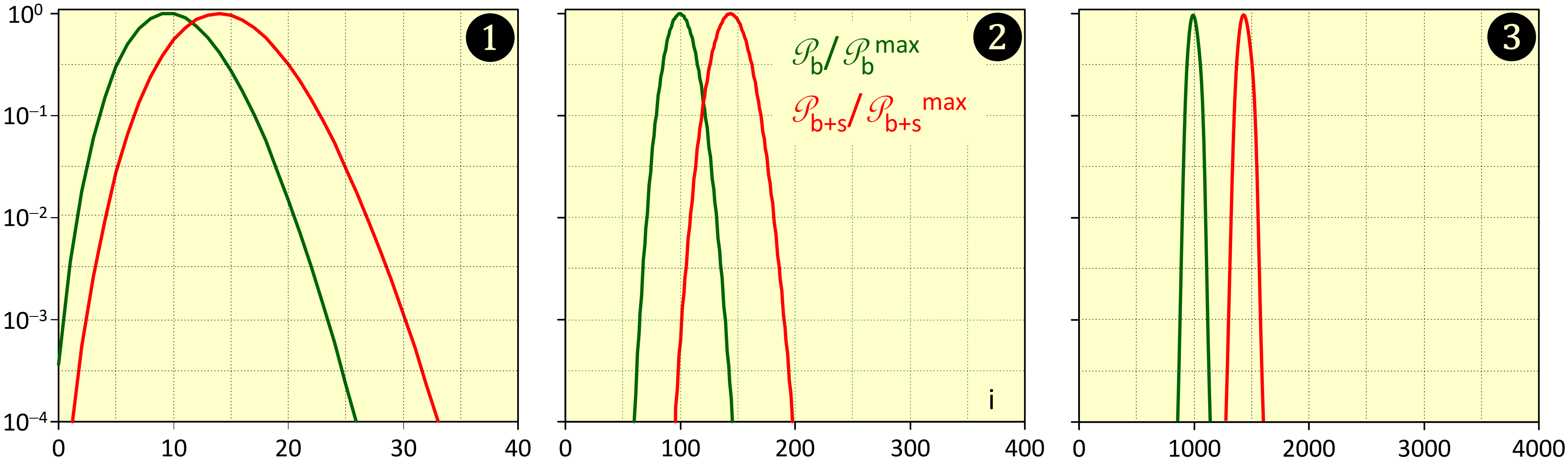
$$"\blacklozenge" = e^{-b} \times \sum_{i=N}^{\infty} \frac{(b)^i}{i!} \leq \mathscr{P}(5\sigma) = 2.86 \times 10^{-7};$$

$$"\blacklozenge" = e^{-(b+s)} \times \sum_{i=N}^{\infty} \frac{(b+s)^i}{i!} \geq 0.5;$$

$$b = \mathscr{L}_{disc} \varepsilon_B \sigma_B; \quad s = \mathscr{L}_{disc} \varepsilon_S \sigma_S.$$

- assume increasing luminosity ($\mathscr{L}_{int}$ = $\mathscr{L}_{disc}$ [$\mathscr{L}_{excl}$]) and constant ε_s, ε_b, σ_s, σ_b;
- assume to start with small $\mathscr{L}_{int}$: the two distributions overlap a lot, no N satisfies the system (i.e. the **green tail** above the **median** is too large);
- when $\mathscr{L}_{int}$ increases, the two distributions are more and more distinct (overlap $\propto 1/\sqrt{\mathscr{L}_{int}}$);
- for a given value of $\mathscr{L}_{int}$, it exists a number of events N, such that the cuts at **2.86×10^{-7}** (**0.5**) in the **first** (**second**) cumulative coincide; this value of $\mathscr{L}_{int}$ correspond to $\mathscr{L}_{disc}$;
- this is the luminosity when, if the signal exists, 50% of the experiments have (at least) 5σ incompatibility with the hypothesis of bckgd only.

limits : Luminosity increase



back to our example:

- b=100, s=44, b+s=144
- show the Poisson distributions for bckgnd only and for bckgnd+signal
- [*notice: log-scale and normalization*]

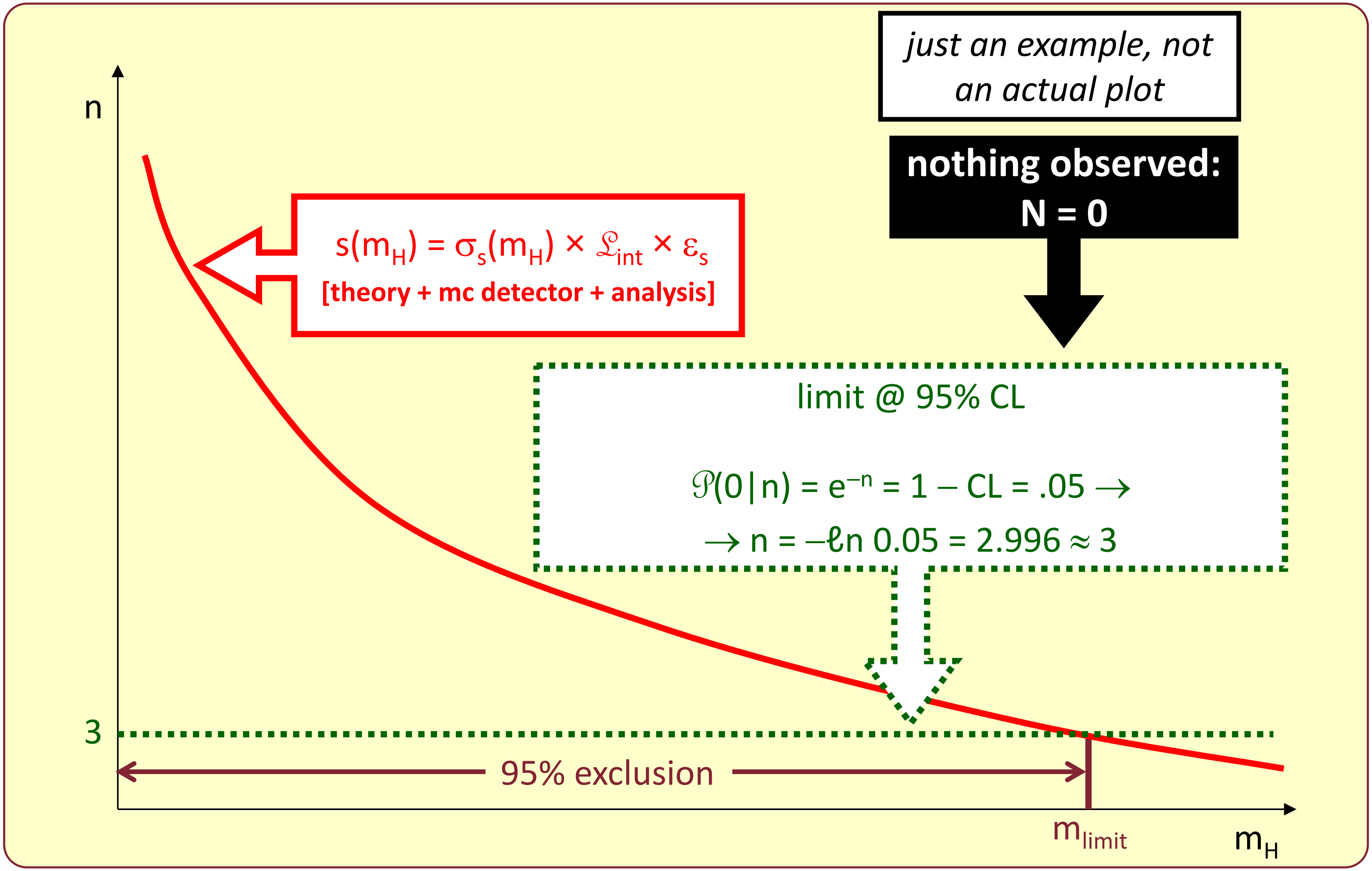
Q in the average case, ok for the 5σ rule ?

A no !!! because b+s (= 144) is at 4.4 σ from b (= 100) → $\mathcal{L}_{int}$ is not sufficient.

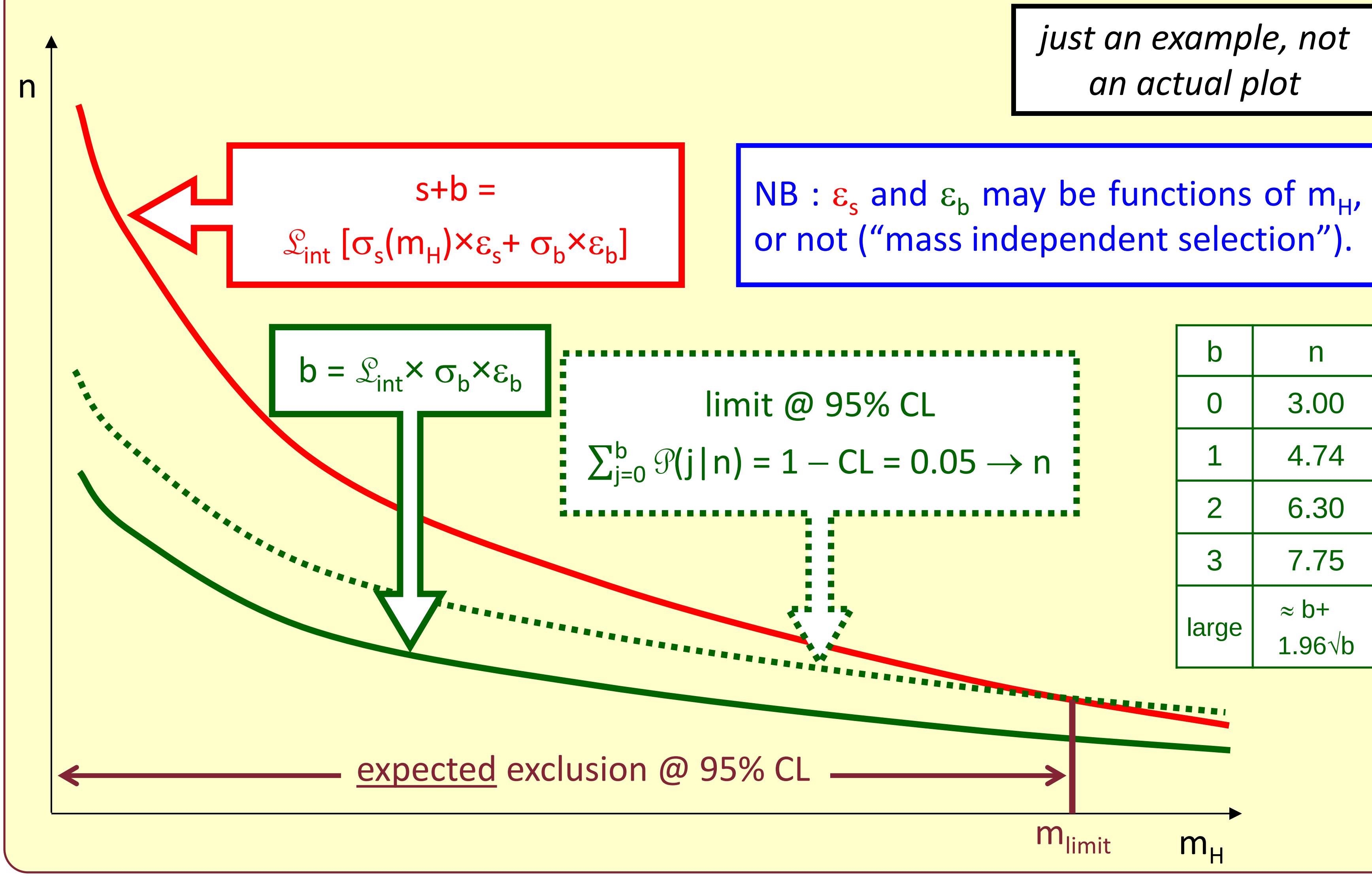
Imagine a real data-taking run:

- at the beginning $\mathcal{L}_{int}$ is small, e.g. b=10, s=4.4, b+s=14.4 (plot n. 1, same axes as other plot);
- then our previous $\mathcal{L}_{int}$ (plot n. 2);
- finally a further increase of 10 in $\mathcal{L}_{int}$ (b=1000, s=440, b+s=1440, plot n. 3);
- in case 3, the 5σ rule is satisfied: ok ! (but long & expensive).

limits : ex. m_H (b=0, N=0)



limits : ex. m_H (a priori, b>0)

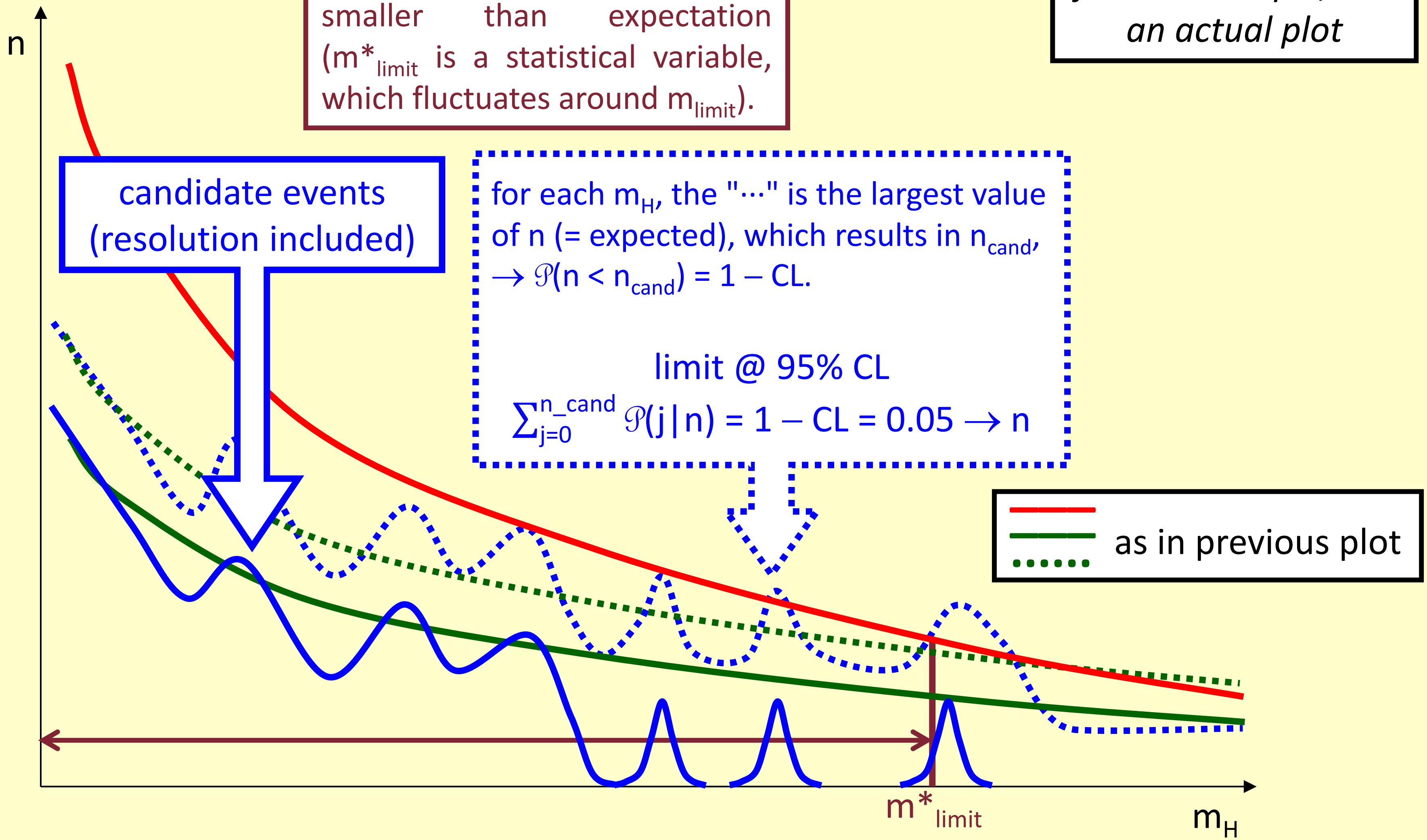


b	n
0	3.00
1	4.74
2	6.30
3	7.75
large	$\approx b + 1.96\sqrt{b}$

limits : ex. m_H (a posteriori, b>0)

NB the result may be larger or smaller than expectation (m^*_{limit} is a statistical variable, which fluctuates around m_{limit}).

just an example, not an actual plot



for each m_H, the "···" is the largest value of n (= expected), which results in n_{cand}, $\rightarrow \mathcal{P}(n < n_{cand}) = 1 - CL$.

limit @ 95% CL

$$\sum_{j=0}^{n\_cand} \mathcal{P}(j|n) = 1 - CL = 0.05 \rightarrow n$$

maximum likelihood: definition

- A random variable x follows a pdf $f(x\,|\theta_k)$;
- the pdf f is a function of some parameters θ_k (k = 1,…,M), sometimes unknown;
- assume a repeated measurement (N times) of x :

  x_j (j = 1,…,N);
- define the likelihood Λ and its logarithm $\ell n(\Lambda)$ [*see box*].

$$\Lambda = \prod_{j=1}^{N} f(x_j\,|\,\theta_k);$$

$$\ell n(\Lambda) = \sum_{j=1}^{N} \ell n[f(x_j\,|\,\theta_k)].$$

Example : observe N decays with (unknown) lifetime τ, measuring the lives t_j, j = 1,…,N.

$$\Lambda = \prod_{j=1}^{N} f(t_j\,|\,\tau) = \prod_{j=1}^{N} \frac{1}{\tau} e^{-t_j/\tau} = \frac{1}{\tau^N} e^{-\sum t_j/\tau};$$

$$\ell n(\Lambda) = \sum_{j=1}^{N} \ell n[\frac{1}{\tau} e^{-t_j/\tau}] = -N\ell n(\tau) - \frac{1}{\tau}\sum_{j=1}^{N} t_j.$$

then look for the value τ^*, which maximizes Λ (or $\ell n\Lambda$).

τ^* is the **max.lik. estimate** of τ.

$$\frac{\partial \ell n(\Lambda)}{\partial \tau} = 0 = -\frac{N}{\tau^*} + \frac{1}{\tau^{*2}}\sum_{j=1}^{N} t_j \Rightarrow$$

$$\tau^* = \frac{1}{N}\sum_{j=1}^{N} t_j = \langle t \rangle.$$

maximum likelihood: parameter estimate

the m.l. method has the following important **asymptotic** properties [*no proof, see the references*]:

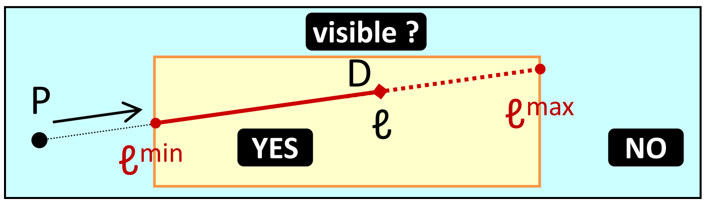
- consistent;
- no-bias;
- result θ^* distributed around θ_{true}, with a variance given by the Cramér-Frechet-Rao limit [*see*];
- "invariant" for a change of parameters, [i.e. the m.l. estimate of a quantity, function of the parameters, is given by the function of the estimates, e.g. $(\theta^2)^* = (\theta^*)^2$];
- such values are also no-bias;
- popular wisdom : "*the m.l. method is like a Rolls-Royce: expensive, but the best*".

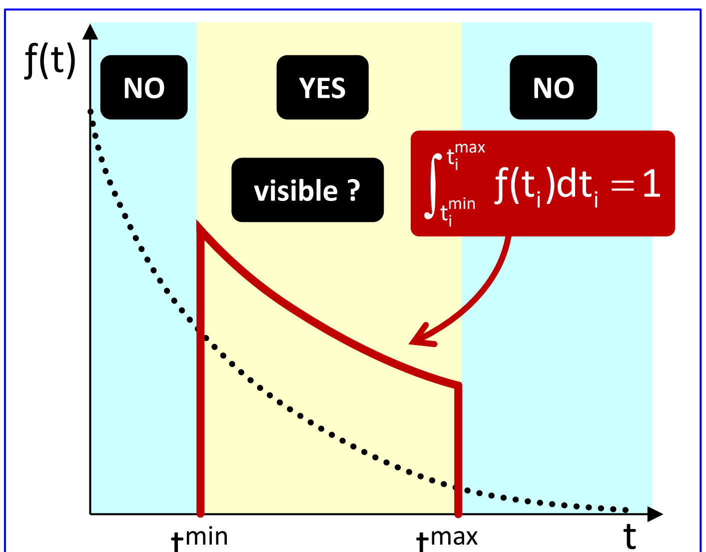
NB. "asymptotically" means : the considered property is valid in the limit $N_{meas} \to \infty$; if N is finite, the property is NOT valid anymore; sometimes the physicists show some "lack of rigor" (say).

maximum likelihood: another example

A famous problem.

We observe a limited region of space (□), with N decays (D) of particles, coming from a point P, possibly external. In all events we measure $\vec{p}$, m, ℓ, ℓ^{min}, ℓ^{max} (minimum and maximum observable lengths), different in every event. Find τ.





Solution

Get t $(=|\vec{p}|\ell/m)$, $t^{min,max}$ $(=|\vec{p}|\ell^{min,max}/m)$. However, t^{min} and t^{max} (and the pdf), are different event by event [*see figure*].

Then:

$$\int_{t_i^{min}}^{t_i^{max}} f(t)dt = 1 \rightarrow f(t) = \begin{cases} 0 & , t < t_i^{min} \\ \dfrac{e^{-t/\tau}/\tau}{e^{-t_i^{min}/\tau} - e^{-t_i^{max}/\tau}} & , t_i^{min} \le t \le t_i^{max} \\ 0 & , t_i^{max} < t \end{cases}$$

$$\ell n\Lambda = \sum_i \left[-\ell n\tau - \frac{t_i}{\tau} - \ell n\left(e^{-t_i^{min}/\tau} - e^{-t_i^{max}/\tau} \right) \right];$$

$$\frac{\partial \ell n\Lambda}{\partial\tau} = 0 = -\frac{N}{\tau} + \frac{1}{\tau^2}\sum_i \left(t_i - \frac{t_i^{min}e^{-t_i^{min}/\tau} - t_i^{max}e^{-t_i^{max}/\tau}}{e^{-t_i^{min}/\tau} - e^{-t_i^{max}/\tau}} \right);$$

$$t_i^{max} = \infty \rightarrow N\tau = \sum_i \left(t_i - t_i^{min} \right) \rightarrow \tau = \frac{1}{N}\sum_i \left(t_i - t_i^{min} \right).$$

otherwise, if $t_i^{max} < \infty \rightarrow$ numerical solution.

maximum likelihood: m_H at LEP

Our problem: use the full LEP statistics for the search of the Higgs boson. Define:

- "channel c", c=1,...,C : (one experiment) × (one $\sqrt{s}$) × (one final state) [e.g. (L3) – ($\sqrt{s}$ = 204 GeV) – ($e^+e^- \to HZ \to b\bar{b}\mu^+\mu^-$)] (actually C > 100);
- "m = m_H, test mass" : the mass under study ("the hypothesis"), which must be accepted/rejected (a grid in mass, with interval ~ mass resolution);
- for each c(hannel) and each m_H, (in principle) a different analysis → sets of $\{\sigma_S, \sigma_B, \varepsilon_S, \varepsilon_B, \mathcal{L}\}_{c,m}$ [$\mathcal{L}\,\varepsilon_S\,\sigma_S = s_{c,m}$, $\mathcal{L}\,\varepsilon_B\,\sigma_B = b_{c,m}$, $b_{c,m} + s_{c,m} = n_{c,m}$, all $f(m_H)$];
- therefore for each c and each m_H → a set of N_c candidates (= events surviving the cuts); event j has kinematical variables (e.g. 4-momenta of particles) $\vec{x}_{jc}$ [event j of channel c];
- [actually an event of a channel should be a candidate for few similar m_H;]
- the mc samples (both signal and bckgd) allow to define $f^S_{c,m}(\vec{x})$ and $f^B_{c,m}(\vec{x})$, the pdf for signal and bckgd of all the variables, after cuts and fits;
- other variables (e.g. reconstructed masses, secondary vertex probability, ...) are properly computed;
- for each m_H, define the total number of candidates $M_m \equiv \sum_c N_{c,m}$;
- notice that, generally speaking, all variables are correlated [e.g. $m_j = m_{jm} = m_j(m_H)$, because efficiency, cuts and fits do depend on m_H].

Then, start the statistical analysis...

maximum likelihood: hypothesis test

- The likelihood function [PDG] is the product of the pdf for each event, calculated for the observed values;
- for searches, it is the Poisson probability for observing N events times the pdf of each single event [*see box*];
- since there are two hypotheses (b only and b+s), there are two pdf's and therefore two likelihoods;
- both are functions of the parameter(s) of the phenomenon under study (e.g. m_H);
- the **likelihood ratio Q** is a powerful (**the most powerful**) test between two hypotheses, mutually exclusive;
- the term "-2 ℓn …" is there only for convenience [both for computing and because $-2\ell n(\Lambda) \to \chi^2$ for n large];

- in the box [*see previous slide*]:
  - "c=1,…C" refers to different channels;
  - $f^{s,b}$ are the pdf (usually from mc) of the kinematical variables $\vec{x}$ for event j_c :

$$\text{given } f_c^s(\vec{x}),\ f_c^b(\vec{x}),\ f_c^{b+s}(\vec{x}) = \frac{s_c f^s(\vec{x}_c) + b_c f^b(\vec{x}_c)}{s_c + b_c} :$$

$$\Lambda_s = \Lambda_s(m_H) = \prod_{c=1}^{C} \left\{ \begin{array}{l} \mathcal{P}_{\text{poisson}}(N_c \mid b_c + s_c) \times \\ \prod_{j_c=1}^{N_c} \left[f_c^{b+s}(\vec{x}_{j_c}) \right] \end{array} \right\};$$

$$\Lambda_b = \Lambda_b(m_H) = \prod_{c=1}^{C} \left\{ \begin{array}{l} \mathcal{P}_{\text{poisson}}(N_c \mid b_c) \times \\ \prod_{j_c=1}^{N_c} \left[f_c^{b}(\vec{x}_{j_c}) \right] \end{array} \right\};$$

$$-2\ell n Q = -2\ell n\left(\frac{\Lambda_S}{\Lambda_B}\right) = 2\left(\ell n\Lambda_B - \ell n\Lambda_S\right).$$

maximum likelihood: m_H at LEP - formulæ

… and therefore →

[once again, remember that everything is an implicit function of the test mass m_H].

$$\Lambda_S = \prod_{c=1}^{C}\left\{\frac{e^{-(s_c+b_c)}(s_c+b_c)^{n_c}}{n_c!}\times\prod_{j=1}^{N_c}\left[\frac{s_c f^S(\vec{x}_{jc})+b_c f^B(\vec{x}_{jc})}{s_c+b_c}\right]\right\} =$$

$$= \prod_{c=1}^{C}\left\{\frac{e^{-(s_c+b_c)}}{n_c!}\times\prod_{j=1}^{N_c}\left[s_c f^S(\vec{x}_{jc})+b_c f^B(\vec{x}_{jc})\right]\right\};$$

$$\Lambda_B = \prod_{c=1}^{C}\left\{\frac{e^{-b_c}\times b_c^{\,n_c}}{n_c!}\times\prod_{j=1}^{N_c} f^B(\vec{x}_{jc})\right\} =$$

$$= \prod_{c=1}^{C}\left\{\frac{e^{-b_c}}{n_c!}\times\prod_{j=1}^{N_c} b_c f^B(\vec{x}_{jc})\right\};$$

$$-2\ell n Q = -2\ell n\left(\frac{\Lambda_S}{\Lambda_B}\right) = -2\ell n\left(\frac{\prod_{c=1}^{C}\left\{\frac{e^{-(s_c+b_c)}}{n_c!}\times\prod_{j=1}^{N_c}\left[s_c f^S(\vec{x}_{jc})+b_c f^B(\vec{x}_{jc})\right]\right\}}{\prod_{c=1}^{C}\left\{\frac{e^{-b_c}}{n_c!}\times\prod_{j=1}^{N_c} b_c f^B(\vec{x}_{jc})\right\}}\right) =$$

$$= 2\sum_{c=1}^{C} s_c - 2\sum_{c=1}^{C}\left[\sum_{j=1}^{N_c}\ell n\left(1+\frac{s_c f^S(\vec{x}_{jc})}{b_c f^B(\vec{x}_{jc})}\right)\right].$$

interpretation of results: discovery plot

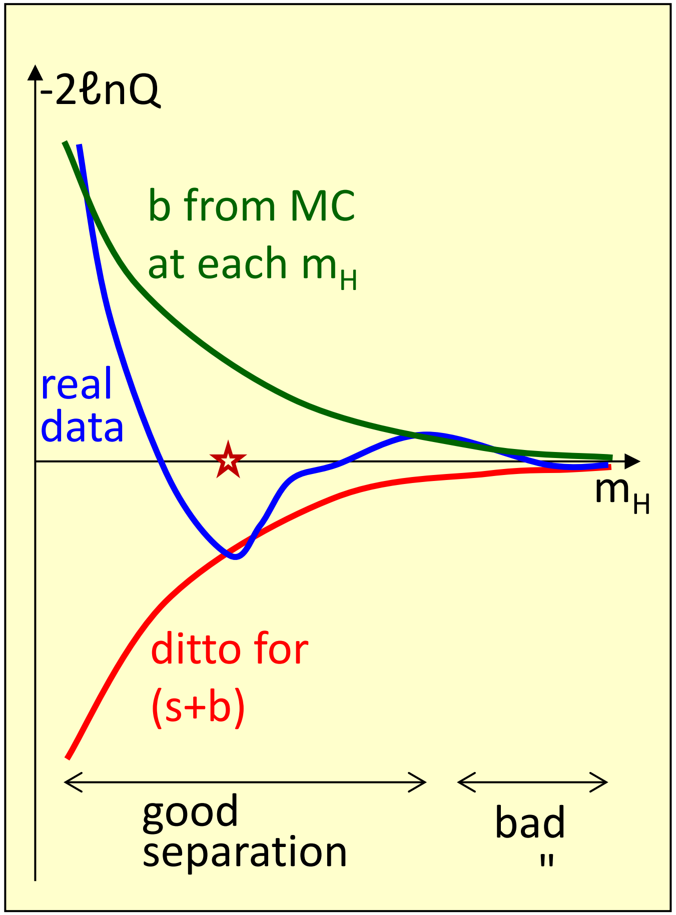
- the likelihood is expected to be larger when the correct pdf is used;
- then the result of the test can be easily guessed (and translated into χ^2):

$$-2\,\ell n\,Q = -2\,\ell n(\Lambda_s/\Lambda_b) \approx \chi_s^2 - \chi_b^2$$

	b true	(s+b) true
Λ_b	+large	+small
Λ_s	+small	+large
Λ_s/Λ_b	<< 1	>> 1
$\ell n(\Lambda_s/\Lambda_b)$	–large	+large
$-2\ell nQ$	+large	–large

the plot is a little cartoon of an ideal situation (e.g. Higgs search at LEP2), that never happened :

- the cross-section decreases when m_H increases → for large m_H no discovery.
- ~~look the blue line → discovery !!!~~



unfortunately, for the H at LEP **it did NOT happen**

interpretation of results: parameter μ

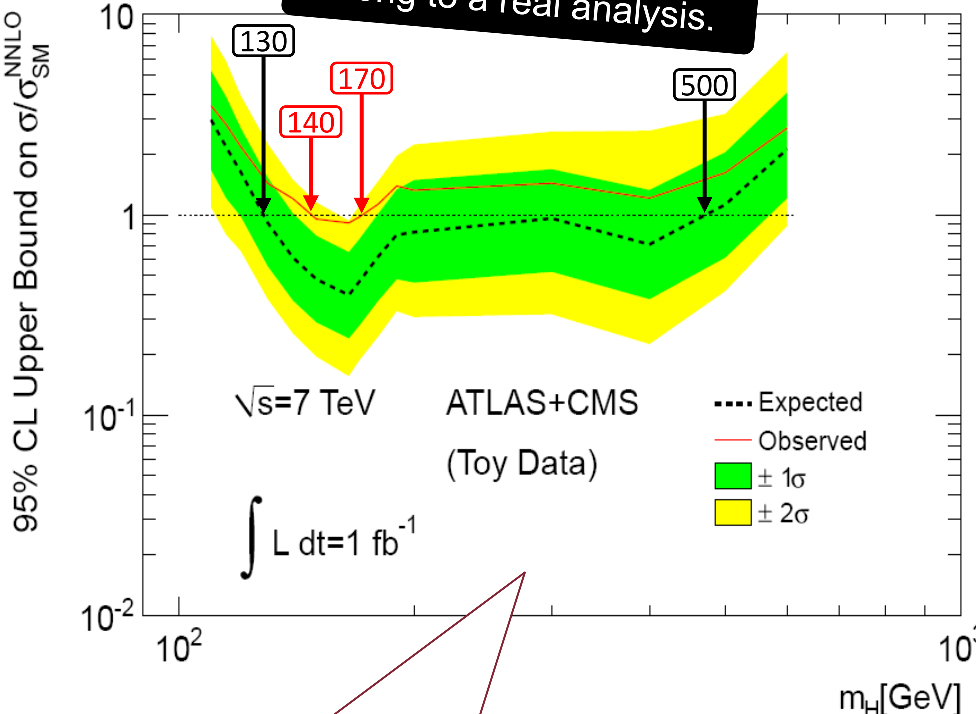
- put : $\sigma = \sigma^b + \mu\, \sigma_{SM}{}^s$;
  [i.e. $n = b + \mu\, s$];
- plot : horizontal : m_H.
  vertical : μ [$=(\sigma^{exp}-\sigma^b)/\sigma_{SM}{}^s$];

➢ the lines show, with a given $\mathscr{L}_{int}$ and analysis, the expected limit (--), and the actual observed limit (–), i.e. the μ value excluded at 95% CL;

➢ the band ♦ (♦) shows the fluctuations at $\pm 1\sigma$ ($\pm 2\sigma$) of the "bckgd only" hypothesis.

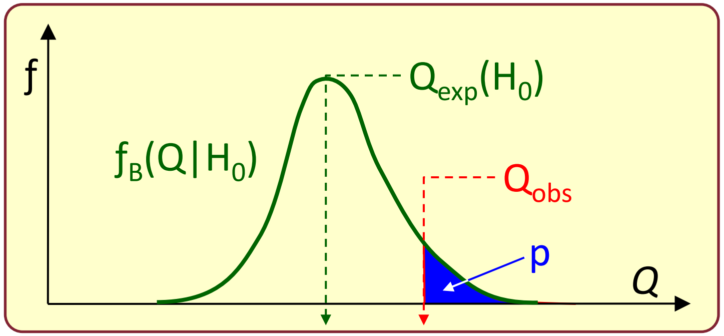
- the case $\mu \neq 0,1$ has no well-defined physical meaning (= *a theory identical to the SM, but with a scaled cross section*);
- if the lines are at $\mu > 1$, the "distance" respect to μ=1 reflects the $\mathscr{L}_{int}$ necessary to get the limit in the SM.

the plot comes from a toy simulation, it does NOT belong to a real analysis.



in this hypothetical case, the region 140 < m_H < 170 GeV is excluded at 95% CL, while the expected limit was 130÷500 GeV (either bad luck or hint of discovery).

interpretation of results: p-value



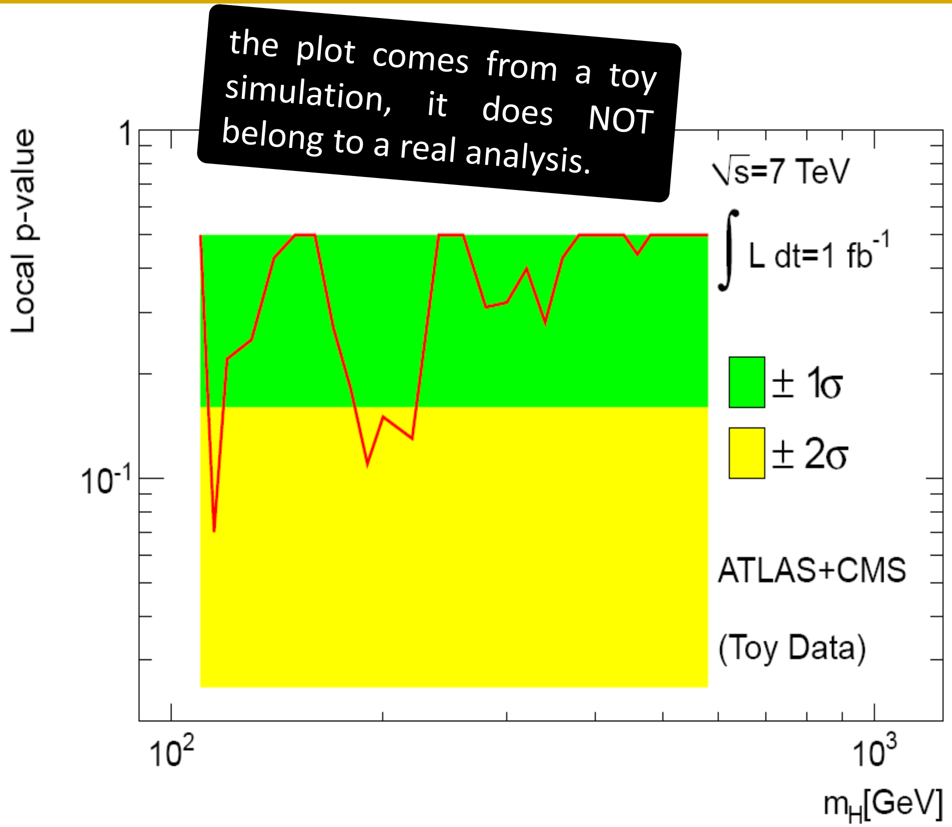
$$p \equiv \int_{x_{obs}}^{\infty} f(x|H_0)dx$$

- the "p-value" is the probability to get the same result or another less probable, in the hypothesis of bckgd only.
- x = "statistics" (e.g. likelihood ratio);
- H_0 = "null hypothesis" (i.e. bckgd only);

i.e.

p small $\rightarrow$ H_0 NOT probable

$\rightarrow$ discovery !!!



- vertical : *p*-value;
- horizontal : m_H.
- the band ♦ (♦) shows the fluctuations at 1σ (2σ).

NB the discovery corresponds to the red line below 5σ (or 2.86×10^{-7}), not shown in this fake plot.

References

1. classic textbook : Eadie et al., Statistical methods in experimental physics;
2. modern textbook : Cowan, Statistical data analysis;
3. simple, for experimentalists : Cranmer, Practical Statistics for the LHC, [arXiv:1503.07622v1]; (also CERN Academic Training, Feb 2-5, 2009);
4. bayesian : G.D'Agostini, YR CERN-99-03.
5. statistical procedure for Higgs : A.L.Read, J. Phys. G: Nucl. Part. Phys. 28 (2002) 2693;
6. mass limits : R.Cousins, Am.J.Phys., 63 (5), 398 (1995).
7. [PDG explains everything, but very concise]

bells are related to dramatic events even outside particle physics

End of chapter 11

Particle Physics - Chapter 12a
LHC – machine and detectors

Paolo Bagnaia
SAPIENZA
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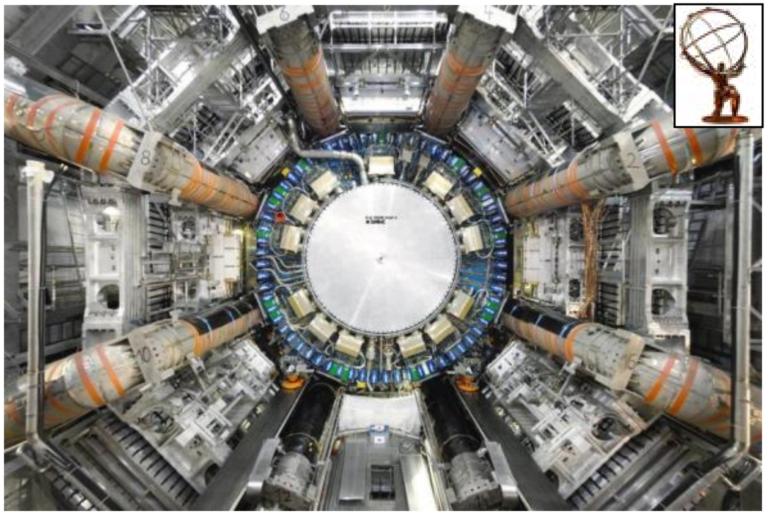
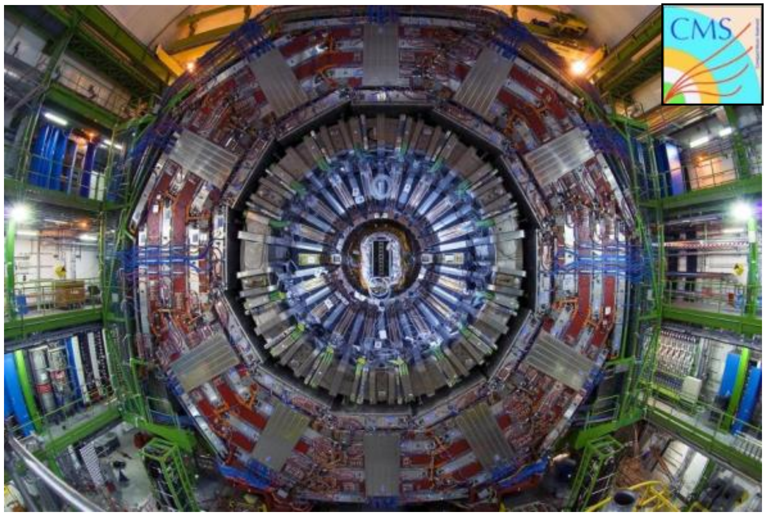
AA 18-19

last mod.
23-May-19

12 – LHC – machine and detectors

1. LHC physics
2. The LHC Collider
3. The luminosity
4. LHC operations
5. The ATLAS detector
6. The CMS detector
7. Detectors comparison
8. Detector performances
9. LHC events





LHC physics

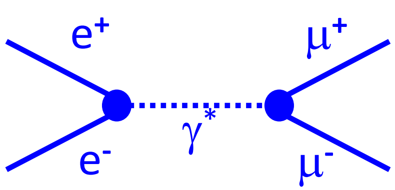
- the LHC physics programme still has a long story ahead;
- [the heavy ion programme is outside the scope of the lectures – see ALICE talks]
- until now, its results can be broadly divided into three categories :
    a. "bread and butter", i.e. quantitative improvements on soft & SM physics;
    b. the discovery of the Higgs boson [*still a tiny probability that the "bump" is NOT the Higgs boson of the SM*];
    c. searches of physics beyond the SM;
- (a) contains beautiful and intelligent results, from soft physics to jets, from $W^{\pm}$ / Z to top;
- however, they are too fresh [*imho*] to be part of an institutional course;
- [*we all hope that*] (c) will be the most interesting;
- however, it is outside the scope of these lectures;
- therefore, this chapter includes two parts :
    1. a general discussion of the method of analysis of LHC, mainly the problems caused by the high $\mathscr{L}$;
    2. a report of the Higgs discovery [*noblesse oblige*];
- the other parts are left to the next semester, your Thesis and (hopefully) your individual research activity.

Enjoy it !

LHC physics: why 10^{34} cm^{-2} s^{-1}

Such a large $\mathscr{L}$ is a must or a luxury ?
Compute two toy processes :

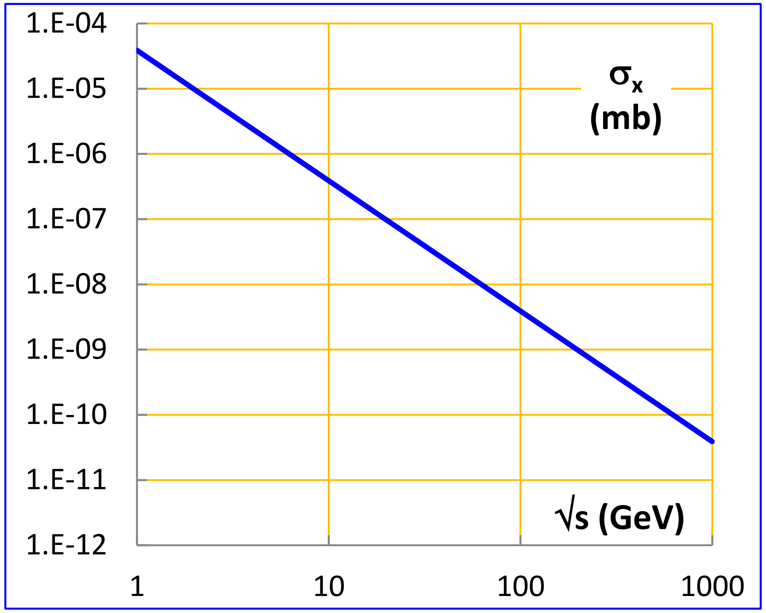
- cross section for a s-channel process :
  - $\sigma \approx K g^2 / s$;
    Ex. $\sigma[e^+e^- \to \gamma^* \to \mu^+\mu^-] = {}^4/_3\, \pi\, \alpha^2 / s$.
  - K : adimensional factor ~1 (e.g. $4\pi/3$);
  - g : coupling constant < 1 (it depends on the dynamics);
  - s : $(\text{energy})^2$ in CM sys;

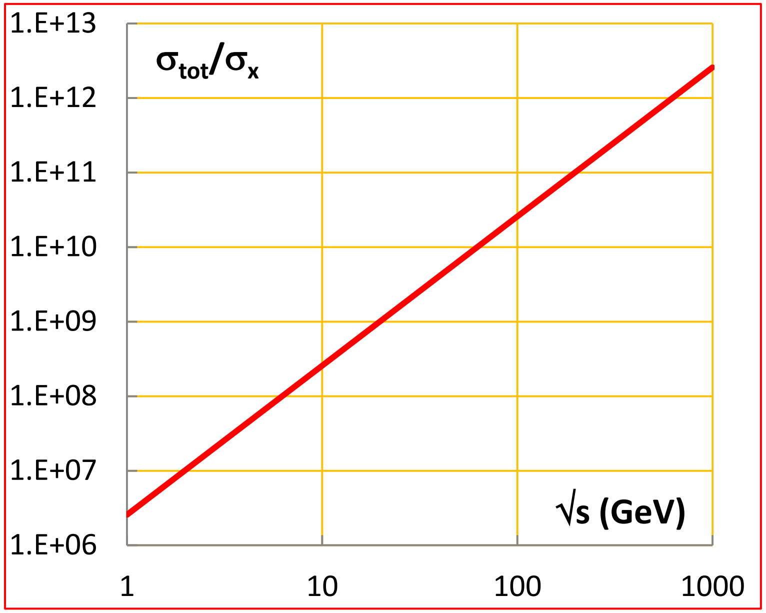


- formation of a resonance (s-channel) [e.g. $\sqrt{s} = m_x = 100$ GeV]:
  - g $\sim 10^{-2}$;
  - $m_x \sim 100$ GeV;
  - $\sigma \approx K g^2 / m_x^2 =$
    $= [0.389 \text{ GeV}^2 \text{ mb}] \times 10^{-4} / 10^4 \approx$
    $= 4 \times 10^{-36}$ cm^2;

[*of course, it is too simplistic : parton structure functions (pdf), decay BR, detector acceptance, analysis inefficiencies are neglected; but all these effects DECREASE the yield or the identification of the effects.*]

LHC physics: plots for 10^{34} cm^{-2} s^{-1}



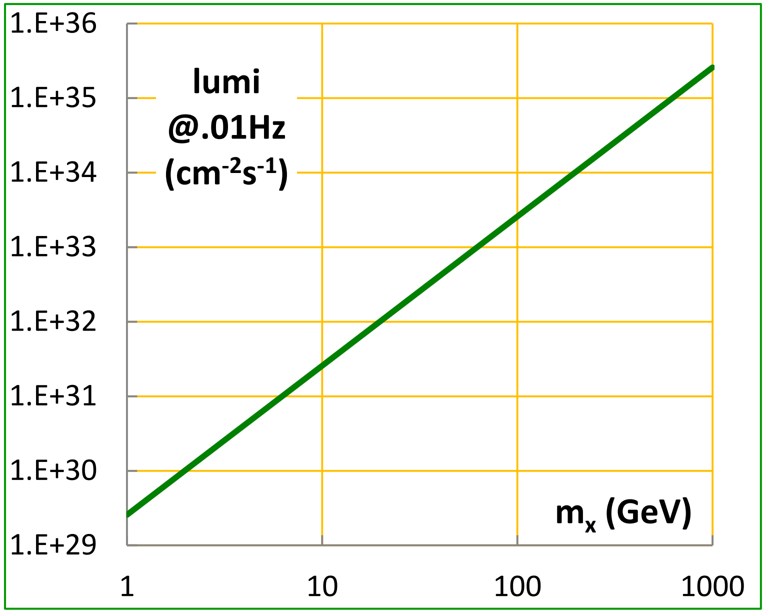


these plots show the trend vs $\sqrt{s}$ of :

- σ_x : s-channel cross section just defined;
- σ_{tot}/σ_x : if $\sigma_{tot} \approx$ 100 mb, ratio between number of events and interesting ones;
- lumi@.01Hz : $\mathscr{L}$ to get a rate of .01 Hz for the m_x just defined;

∴ obvious, but concerning →

high $\mathscr{L}$ is a must.



LHC physics: events at 10^{34} cm^{-2} s^{-1}

How many (interesting) events? an estimate of the order of magnitude:

- "average year" $\sim 10^7$ s;
- $\mathcal{L}_{max} \approx 10^{34}$ cm^{-2} s^{-1};
  - ➢ $\mathcal{L}_{int} \approx 10^{41}$ cm^{-2} = 100 fb^{-1};
- last column roughly includes the detection efficiencies;
- clearly, it is NOT possible to record all these events ($\rightarrow$ act on trigger/selection).

Process	σ (pb)	rate (@ 10^{34} cm^{-2} s^{-1})	events / year
collisions (bc)	---	4×10^7	4×10^{14}
events	1×10^{11}	1×10^9	10^{16}
$W \rightarrow e\nu$	1.5×10^4	150	10^9
$Z \rightarrow e^+e^-$	1.5×10^3	15	10^8
$t\,\bar{t}$	800	8	10^8
$b\,\bar{b}$	5×10^8	5×10^6	10^{13}
$\tilde{g}\,\tilde{g}$ (SUSY) [m_g =1 TeV]	1	0.01	10^5
Higgs [m_H=125 GeV]	20	0.2	2×10^6
QCD jets [p_T>200 GeV]	10^5	1000	10^{10}

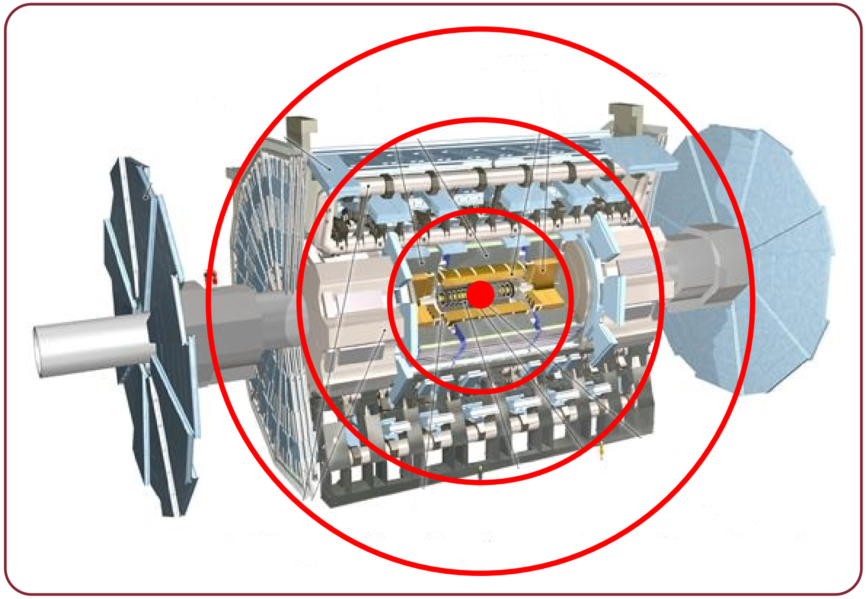
LHC physics: DAQ at 10^{34} cm^{-2} s^{-1}

[σ_{tot}(pp) is a fundamental parameter of the Nature; however, here we study it only as an obstacle to observe high-p_T collisions]

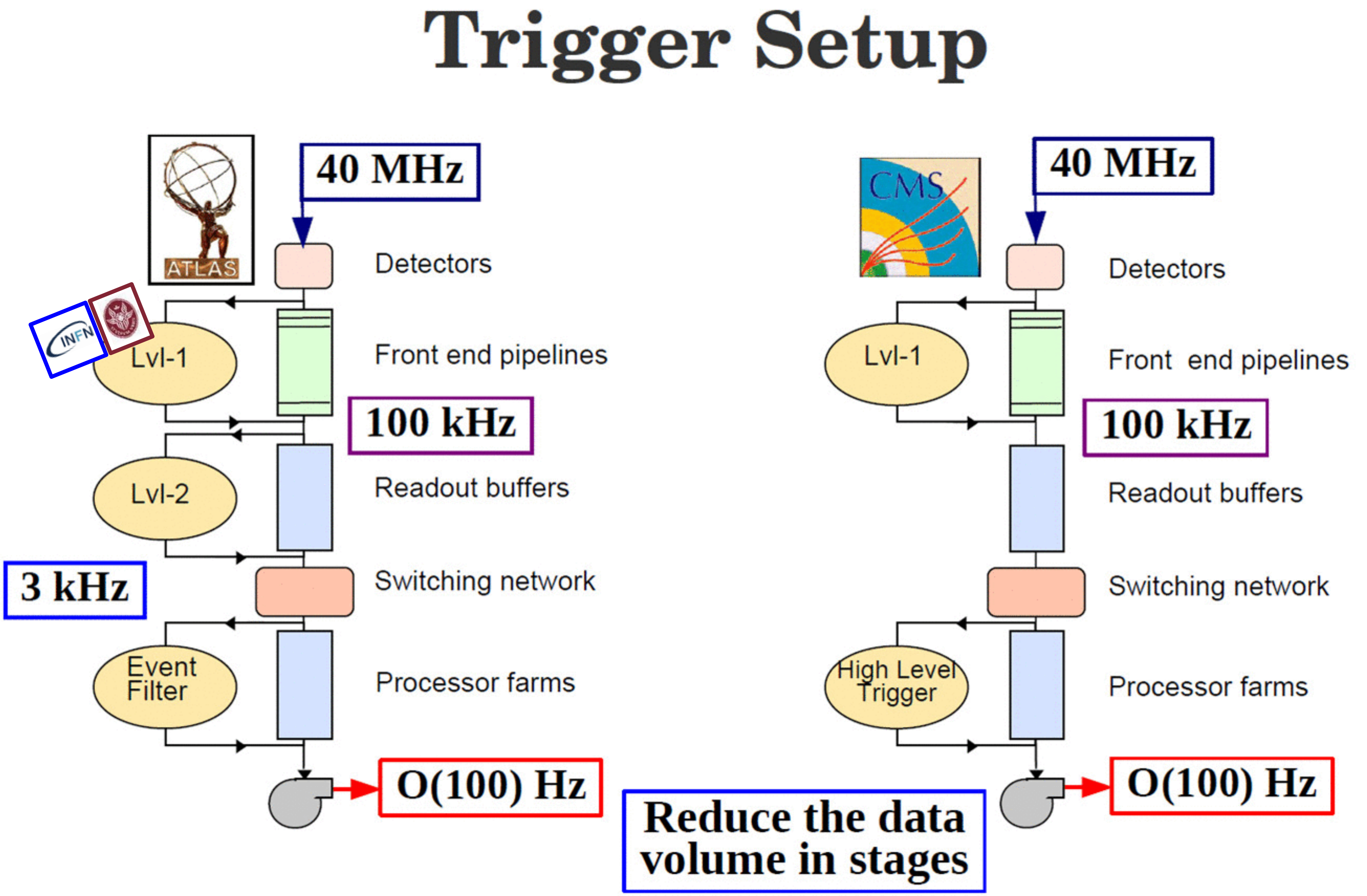
- $\mathcal{L} \approx 10^{34}$ cm^{-2}s^{-1} (actually higher);
- τ_{bc} = 25 ns
- $f_{bc} = 1/\tau_{bc}$ = 40 MHz;
- $\sigma_{tot} \approx$ 100 mb (= 10^{-25} cm^2);
- therefore :
  - $R_{int} \approx$ 1 GHz;
  - $\mathcal{L}_{bc}$ = 2.5 ×10^{26} cm^{-2};
  - n_{bc} = 25 events / bc;
  - $n_{inelast} \approx$ 20 events / bc;
  - $N^{\pm}_{partic.} \approx$ 1000 / bc;
  - $dN^{\pm}/d\eta \approx$ 100 / bc;
  - $W_{detect.} \approx$ 3 kW;
  - Δs_{bc} = 25 ns × c = 7.5 m;

do you see the paradox ?

- i.e. there are "waves" of ~1000 $\pi^{\pm}$ (+ as many γ's) every 25 ns;
- the waves are on concentric spheres at 7.5 m each other (e.g. at the same time the muon chambers "see" previous bc's respect to the inner detector);
- the detectors must have an adequate bandwidth to cope with it (and the necessary radiation resistance !!!).



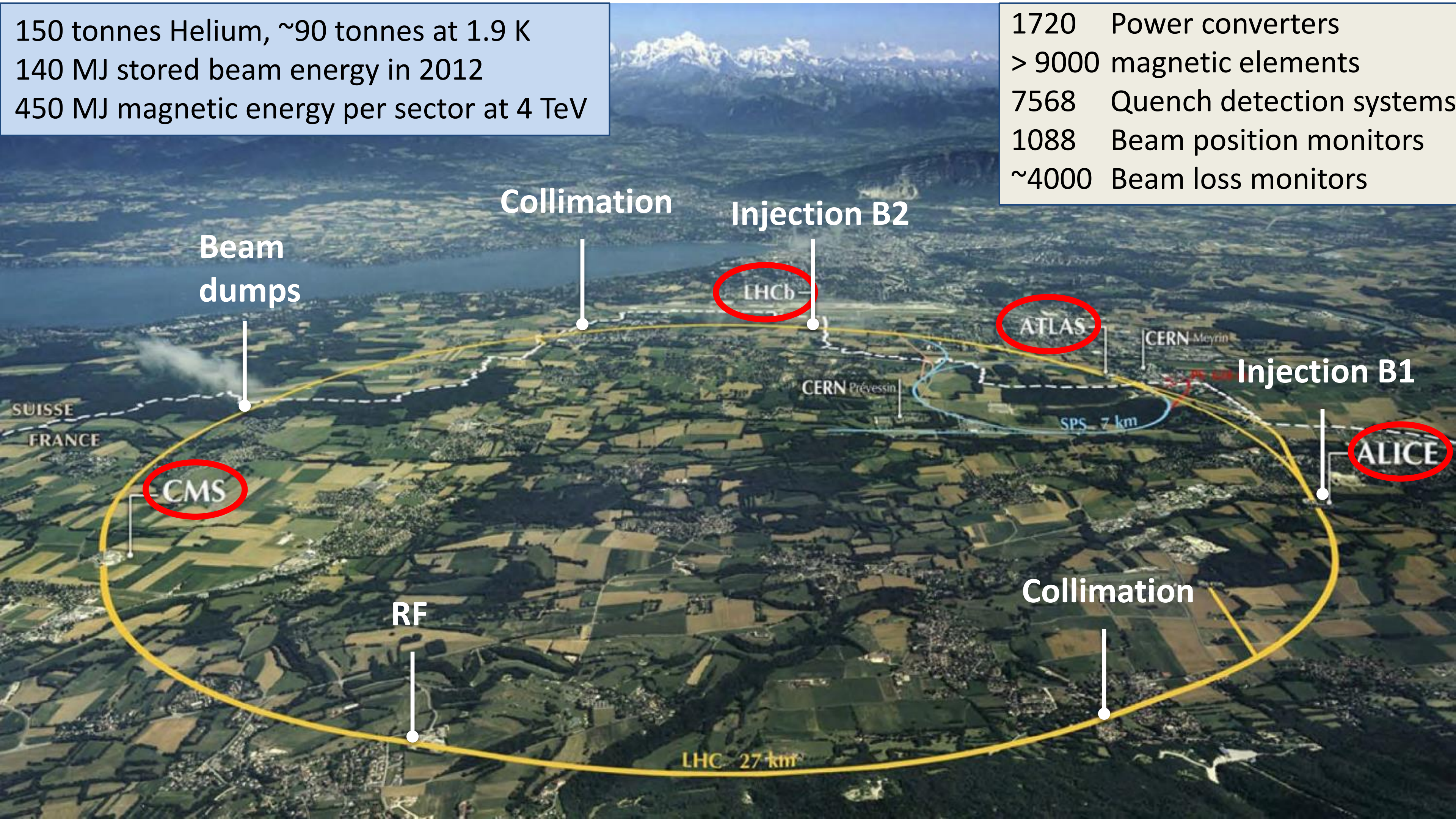
LHC physics: trigger at 10^{34} cm^{-2} s^{-1}



The LHC Collider

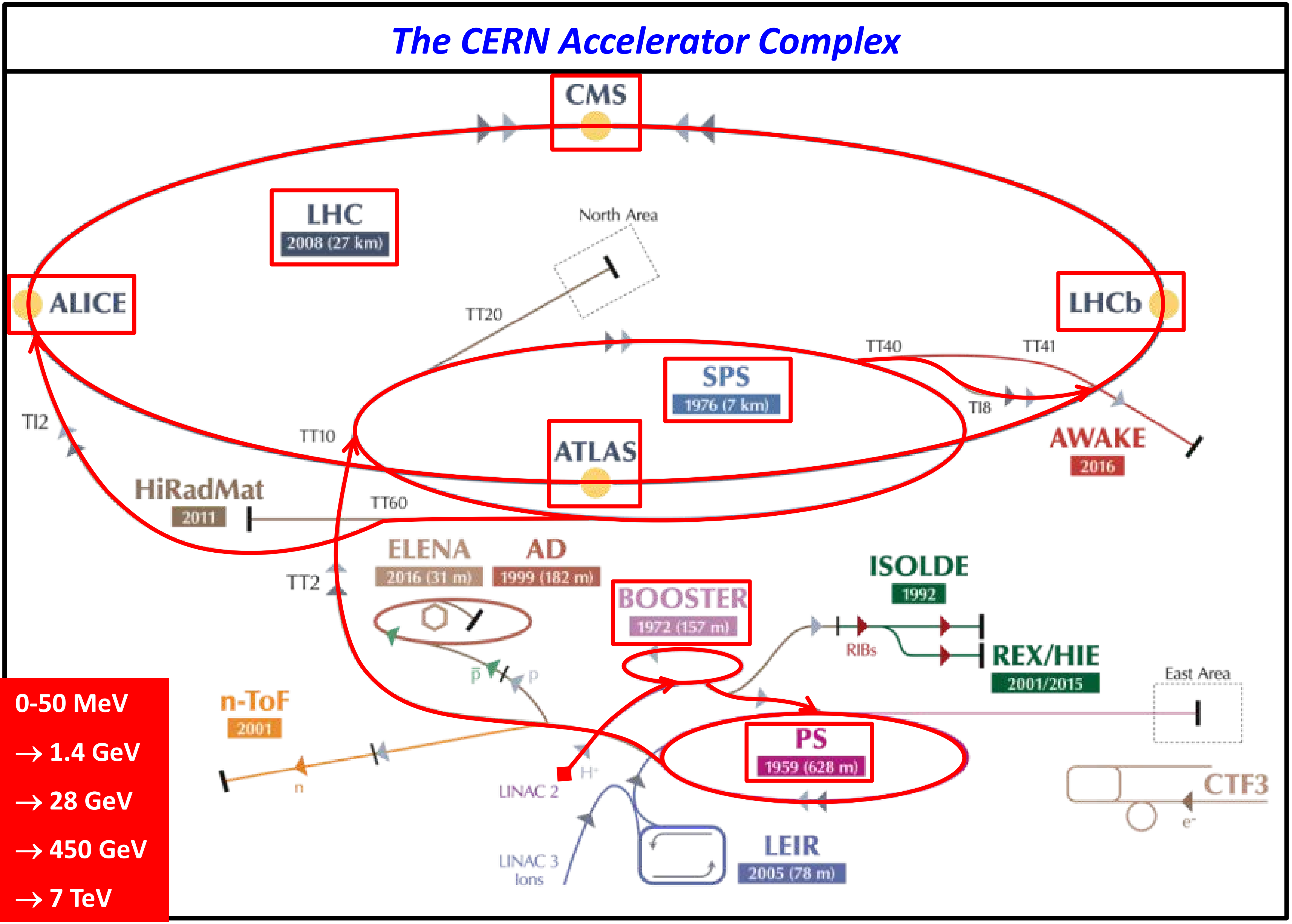
150 tonnes Helium, ~90 tonnes at 1.9 K
140 MJ stored beam energy in 2012
450 MJ magnetic energy per sector at 4 TeV

1720	Power converters
> 9000	magnetic elements
7568	Quench detection systems
1088	Beam position monitors
~4000	Beam loss monitors



The LHC Collider: a view

The LHC Collider: the complex



Linac2	0-50 MeV
PSB	→ 1.4 GeV
PS	→ 28 GeV
SPS	→ 450 GeV
LHC	→ 7 TeV

The LHC Collider : parameters

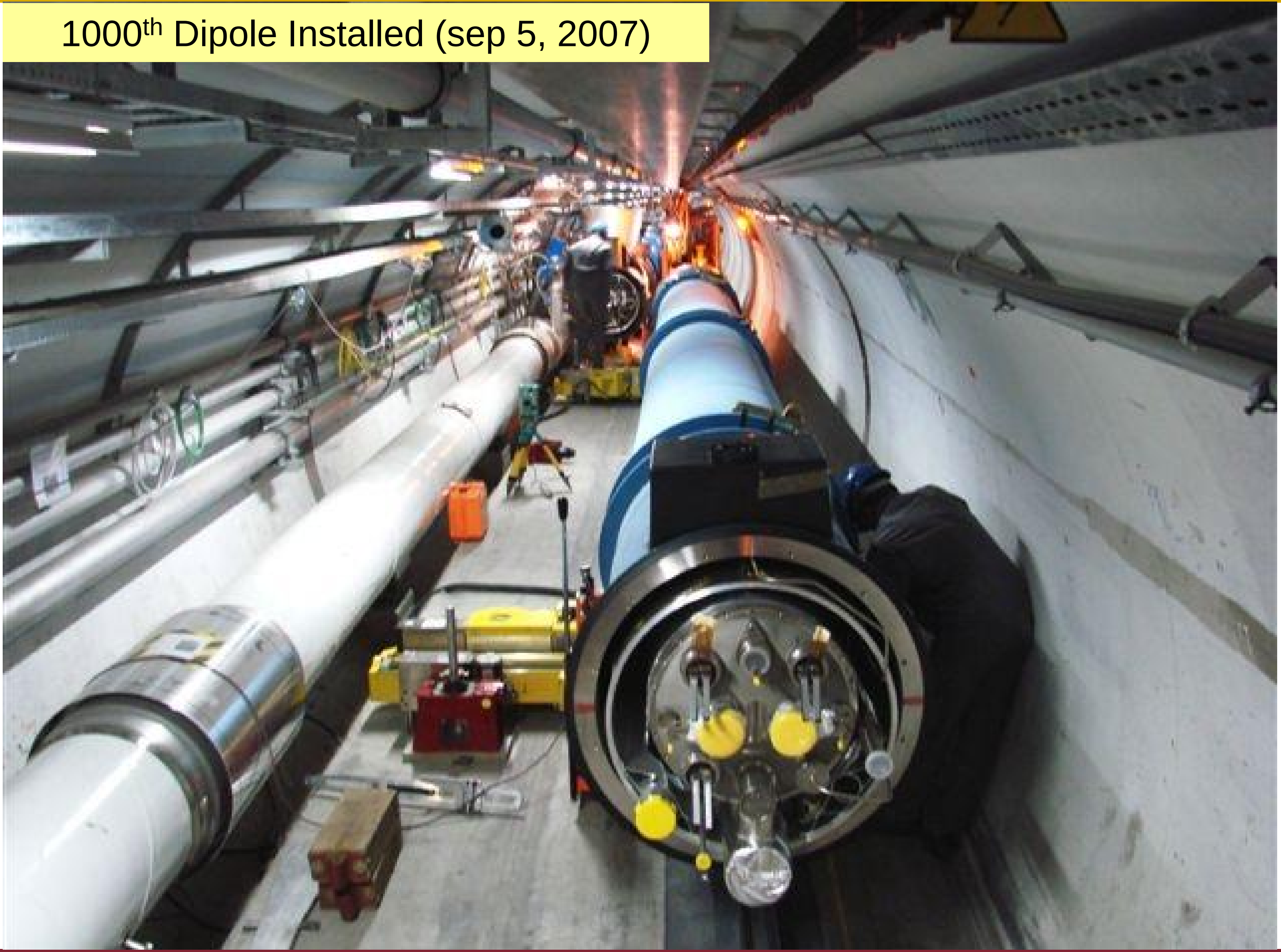
Date	2009	2012	2015	nomin.
Maximum beam energy (TeV) ↑	3.5	4	6.5	7
Delivered integrated luminosity (fb^{-1}) ↑	up to 5.6	23.3	4	—
Luminosity $\mathcal{L}$ (10^{33} $cm^{-2}s^{-1}$) ↑	3.7	7.7	5.2	>10
Time between collisions τ_{bc} (ns) ↔	49.90	49.90	24.95	24.95
Full crossing angle (μ rad) ↔	240	≈ 300		≈ 300
Energy spread $\Delta E/E$ (units 10^{-3}) ↓	0.116	0.116		0.113
Bunch length (cm) ↔	9	9		7.5
Beam radius (10^{-6} m) ↓	26	20		16.6
Initial luminosity decay time, $-\mathcal{L}/(d\mathcal{L}/dt)$ (hr) ↑	8	8		14.9
Transverse emittance ($10^{-9}\pi$ rad-m) ↓	0.7	0.6		0.5
β^*, ampl. function @ i.p. (m) ↓	1	0.6	0.4	0.55
Beam-beam tune shift / crossing (10^{-4})	23	60		34
Particles per bunch (10^{10}) ↑	15	15		11.5
Bunches per ring per species ↑	1380	1380	2244	2808
Average beam current / species (mA) ↑	374	374		584

Parameter	Value
Circumference	26.659 km
Interaction regions	4 total, 2 high $\mathcal{L}$
Free space at interaction point	38 m
Magnetic length of dipole	14.3 m
Length of standard cell	106.9 m
Phase advance per cell	90°
Dipoles in ring	1232 main dipoles
Quadrupoles in ring	482 2-in-1 + 24 1-in-1
Magnet type	s.c. 2 in 1 cold iron
Peak magnetic field	8.3 T
Injection energy	450 GeV
RF frequency	400.8 MHz

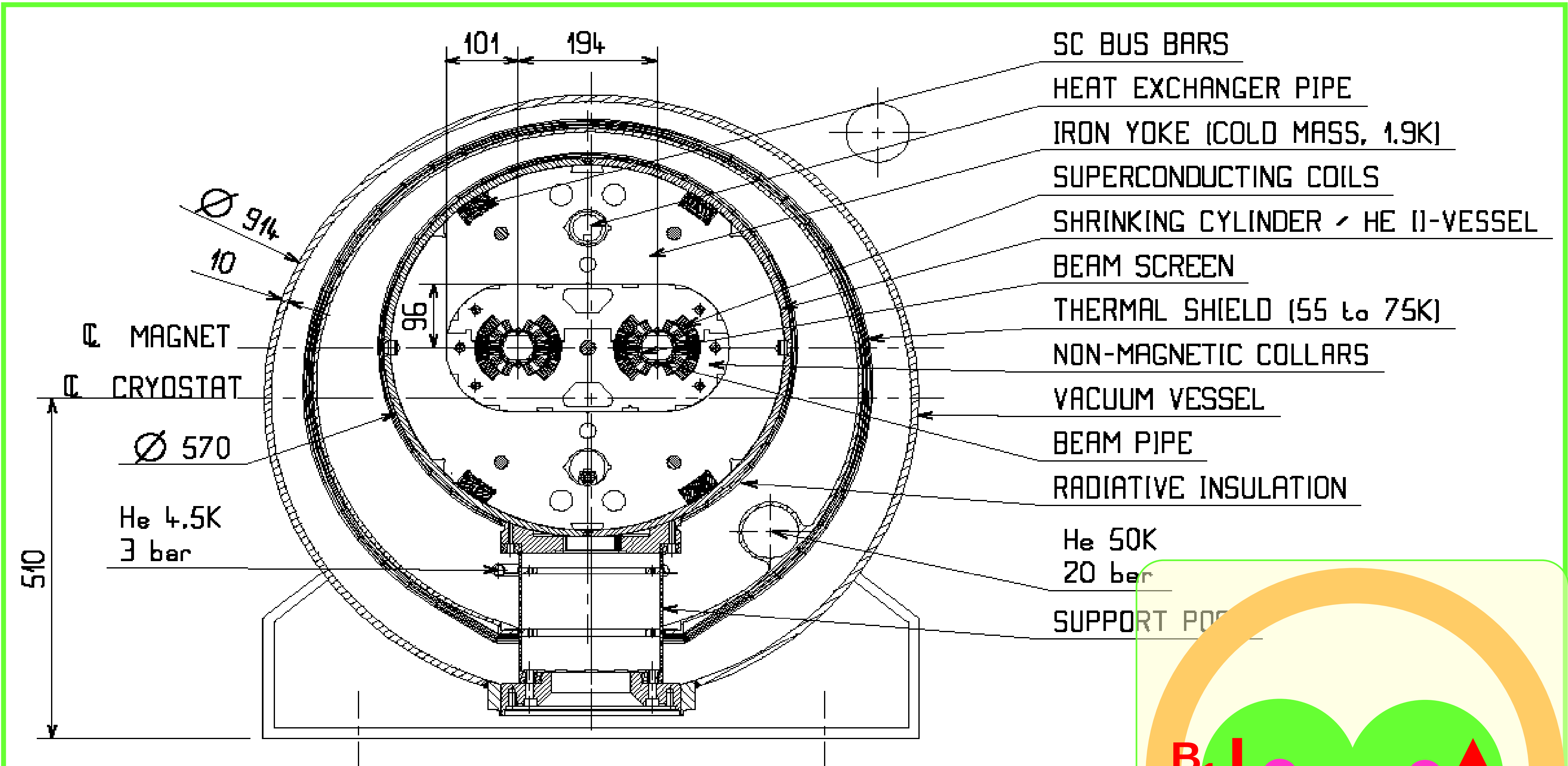
from [PDG]

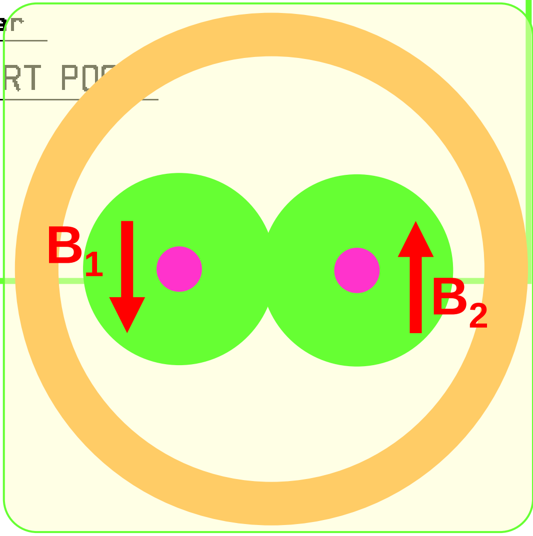
The LHC Collider: dipoles

1000th Dipole Installed (sep 5, 2007)



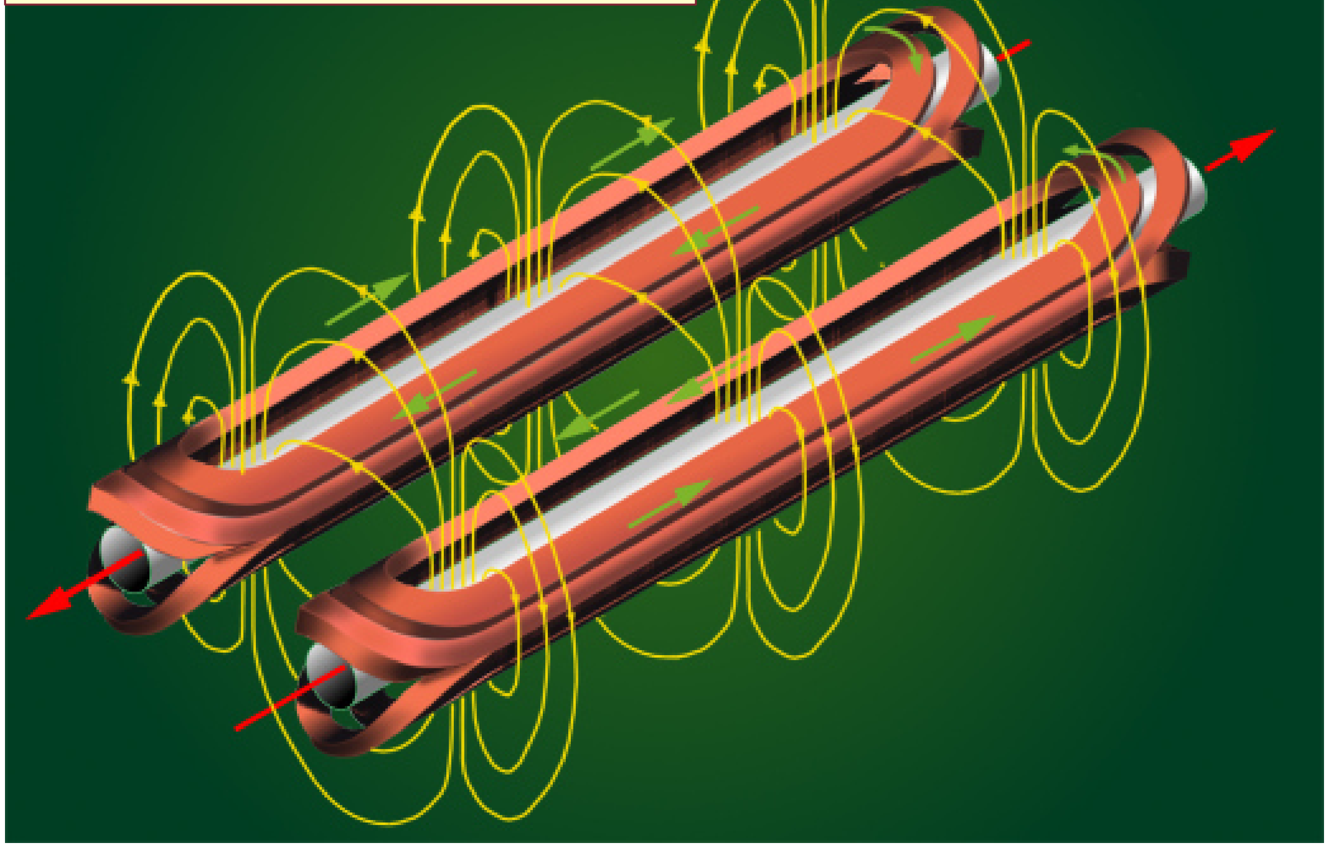
The LHC Collider: dipole structure





The LHC Collider: dipole operations

Mike Lamont – CERN academic training – sep 2013
(read it ! it's wonderful !!!)



Dipoles

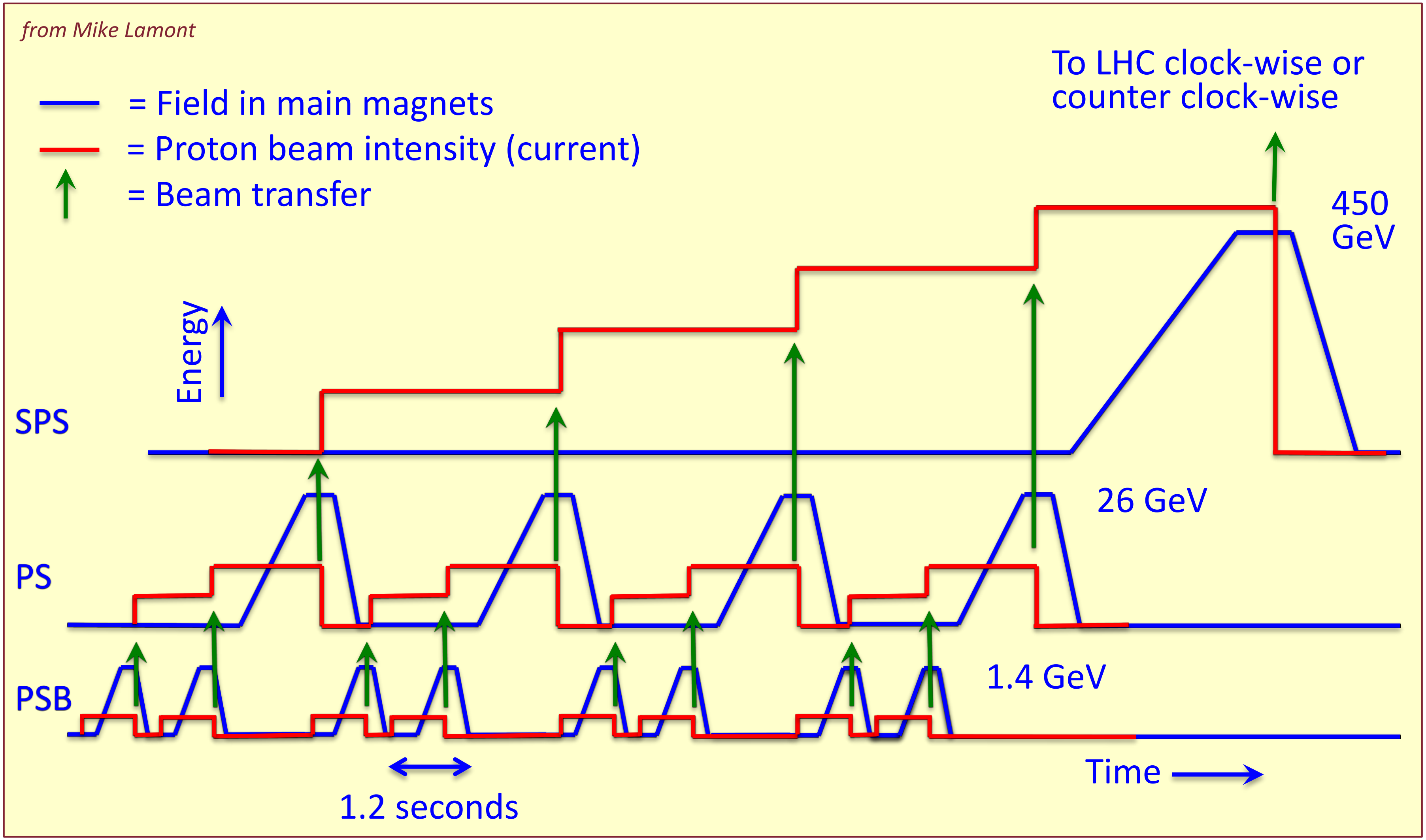
- Number 1232
- Field (450 GeV) 0.535 T
- Field (7 TeV) 8.33 T
- Bending radius 2803.95 m
- Main Length 14.3 m

Horizontal force component per quadrant (nominal field) 1.7 MN/m.

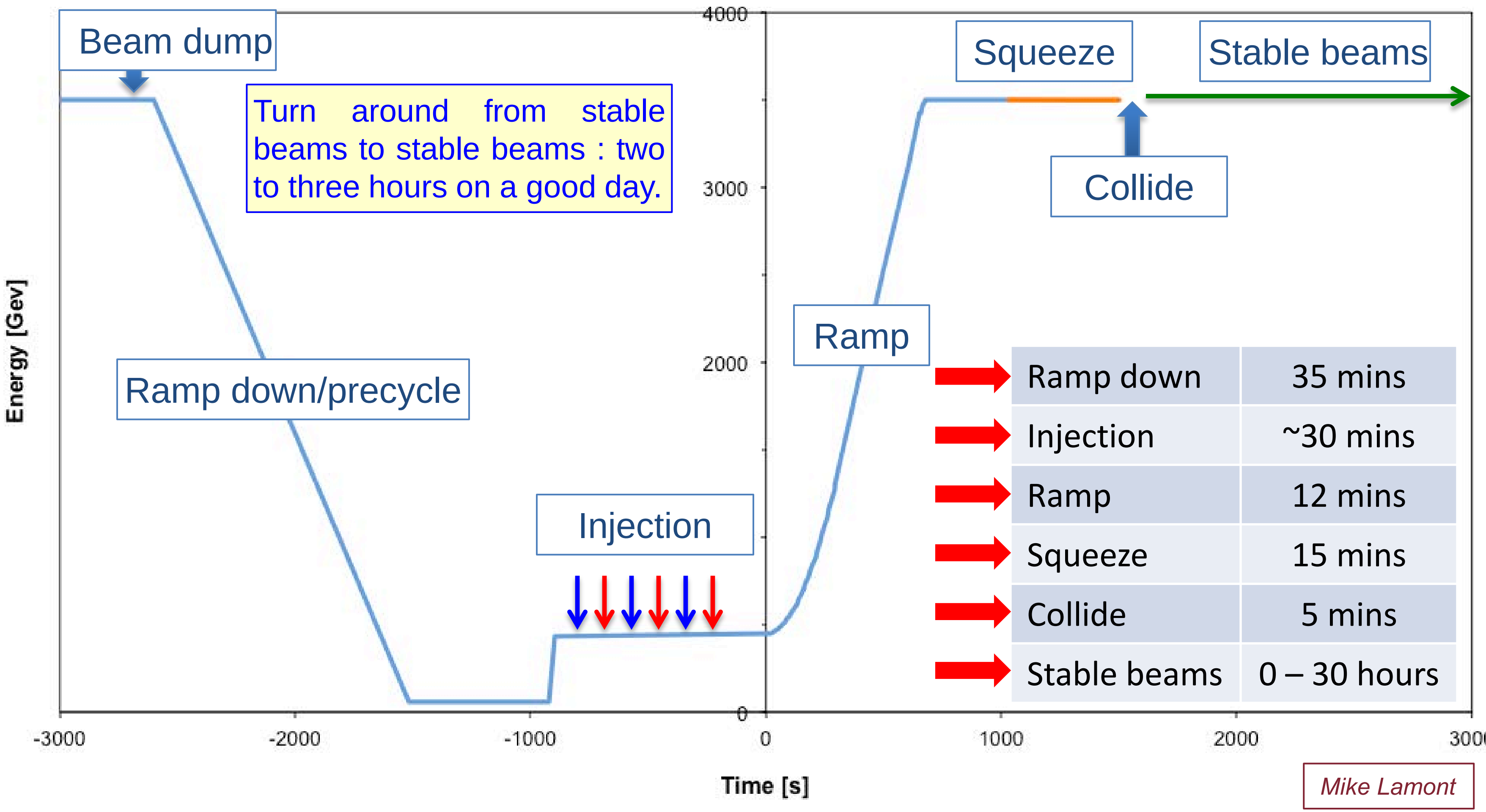
Force tends to "open" the magnet, hence the Austenitic steel collars.

[more info : http://lhc-machine-outreach.web.cern.ch/lhc-machine-outreach/]

The LHC Collider: injection cycle



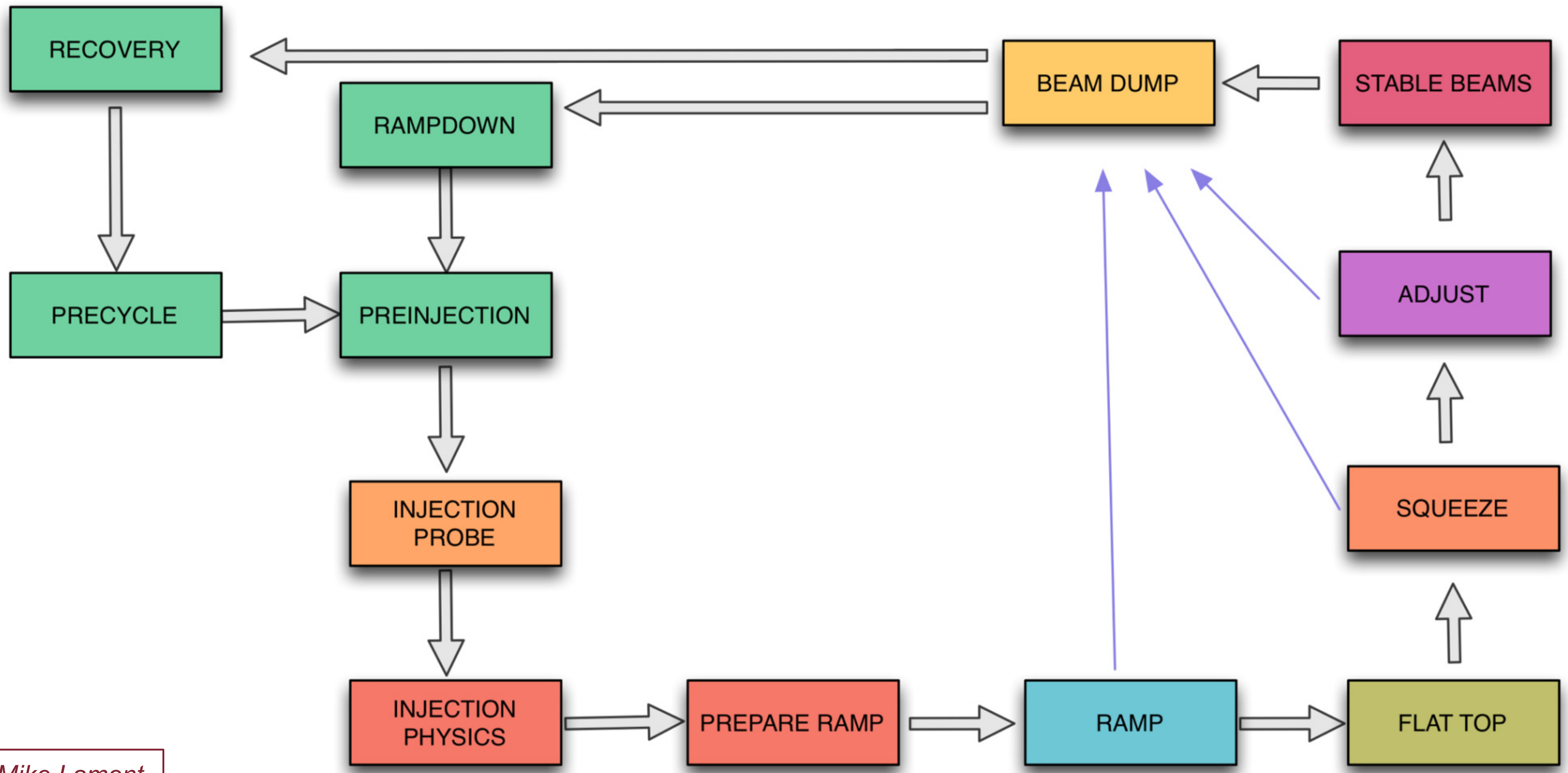
The LHC Collider: data-taking cycle



Ramp down	35 mins
Injection	~30 mins
Ramp	12 mins
Squeeze	15 mins
Collide	5 mins
Stable beams	0 – 30 hours

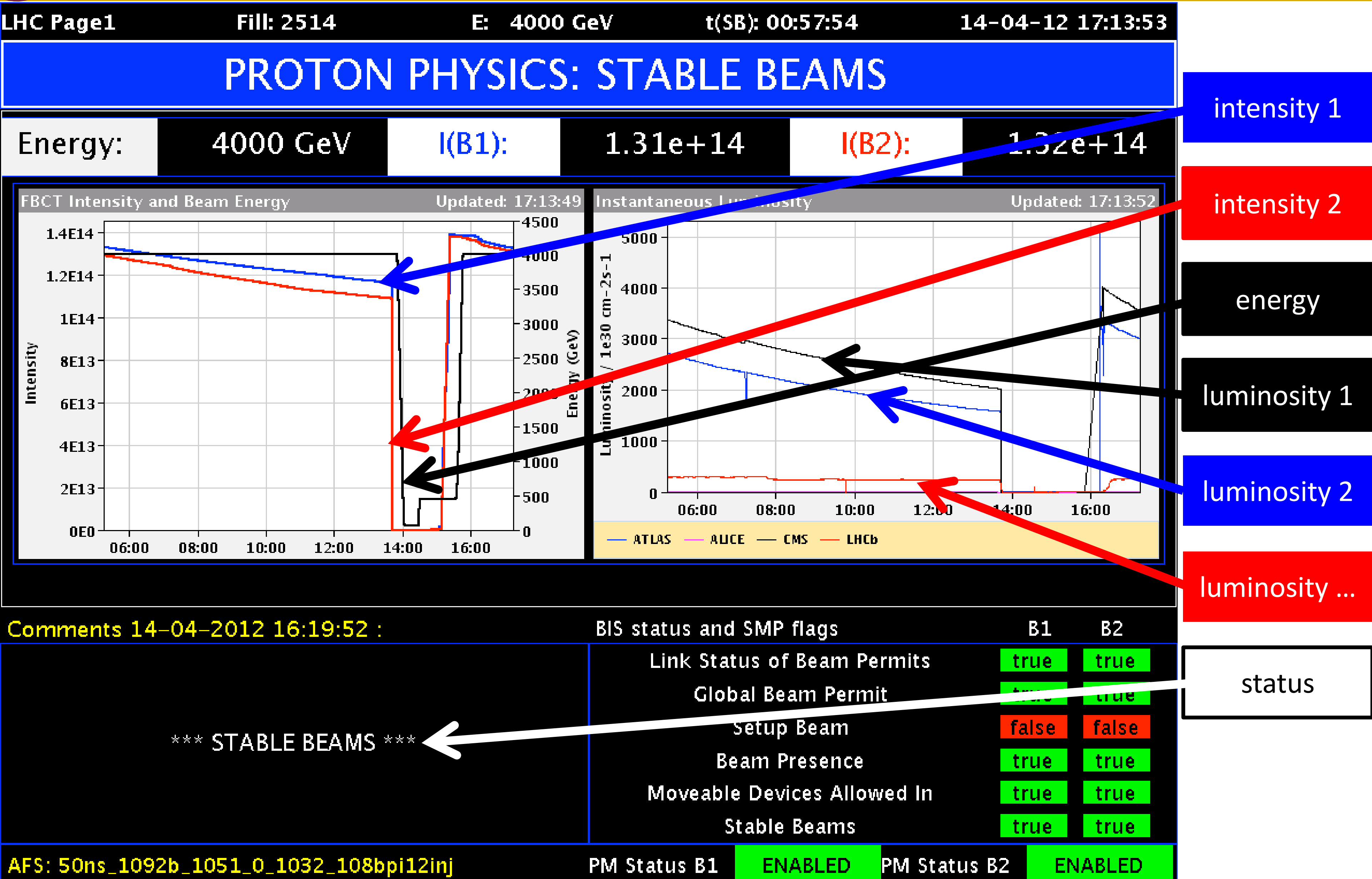
The LHC Collider: nominal cycle

Globally the machine state is fairly well described by machine mode/beam mode combination

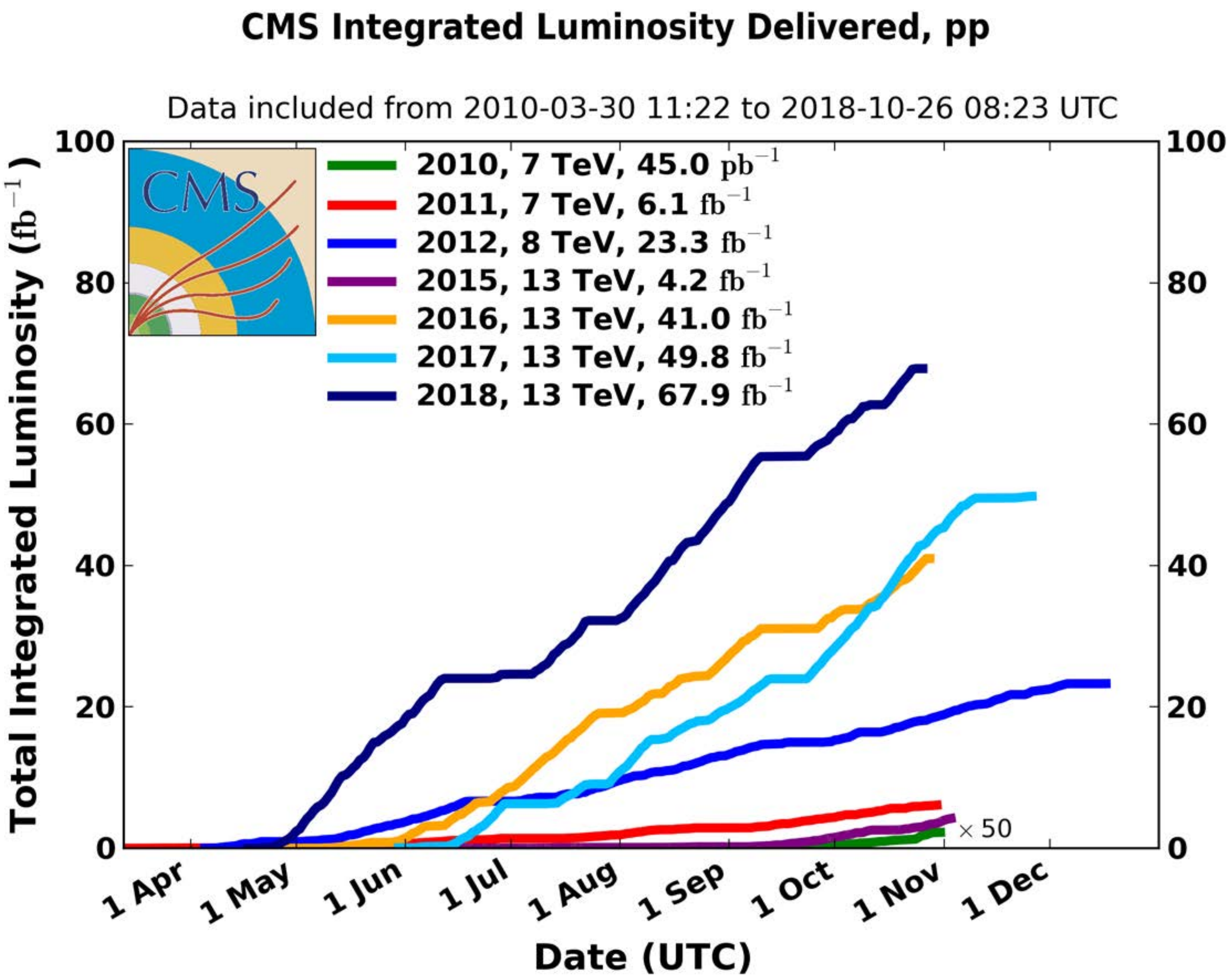


Mike Lamont

The LHC Collider: the user view

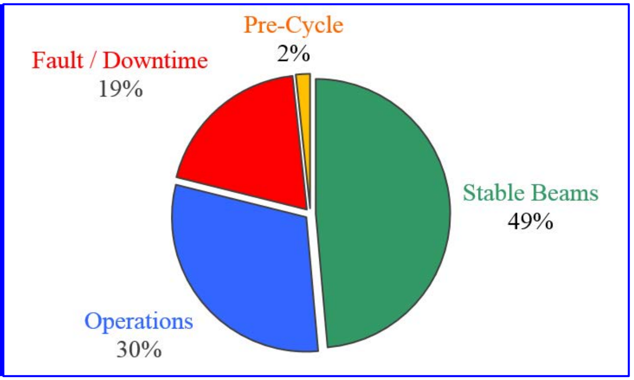


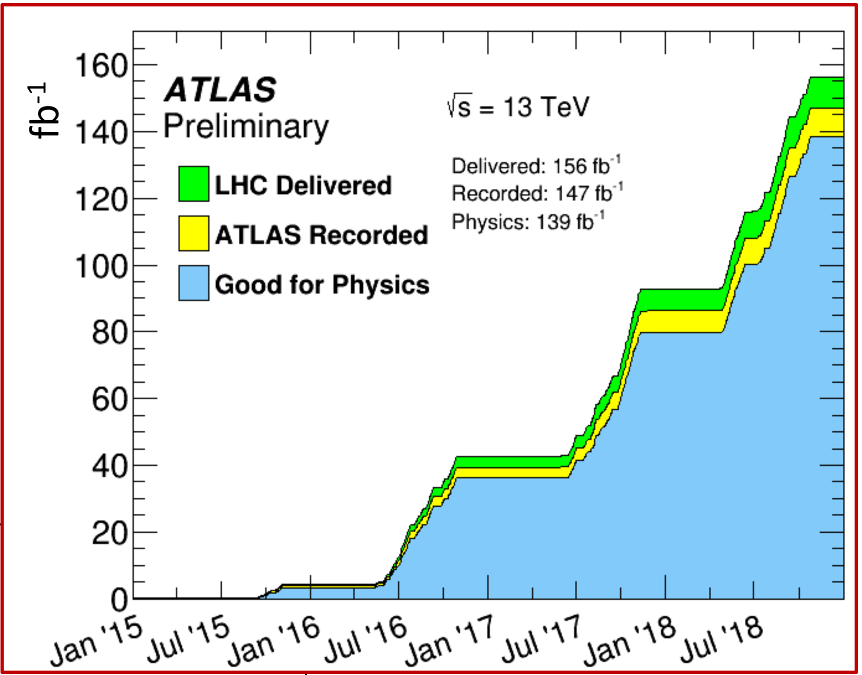
The luminosity: $\mathcal{L}_{int}$ vs time



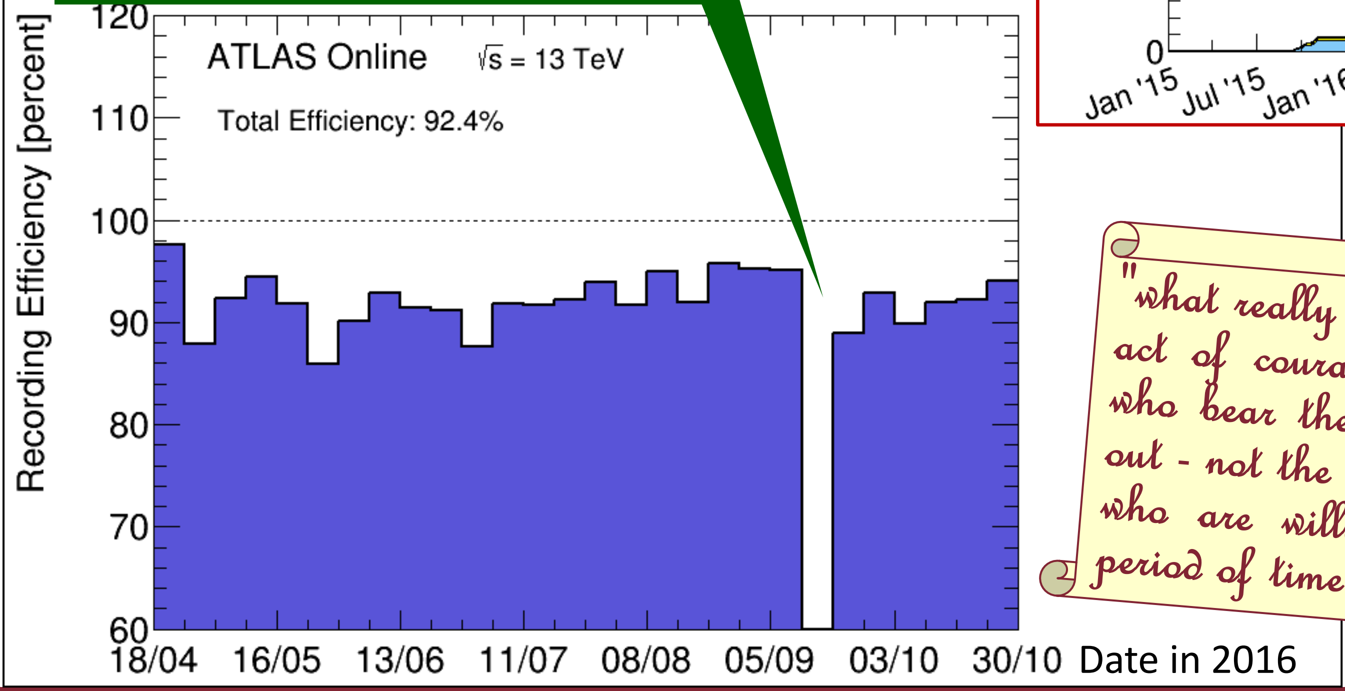
The luminosity: $\mathscr{L}_{int}$ vs time

LHC perfor-mances 2017





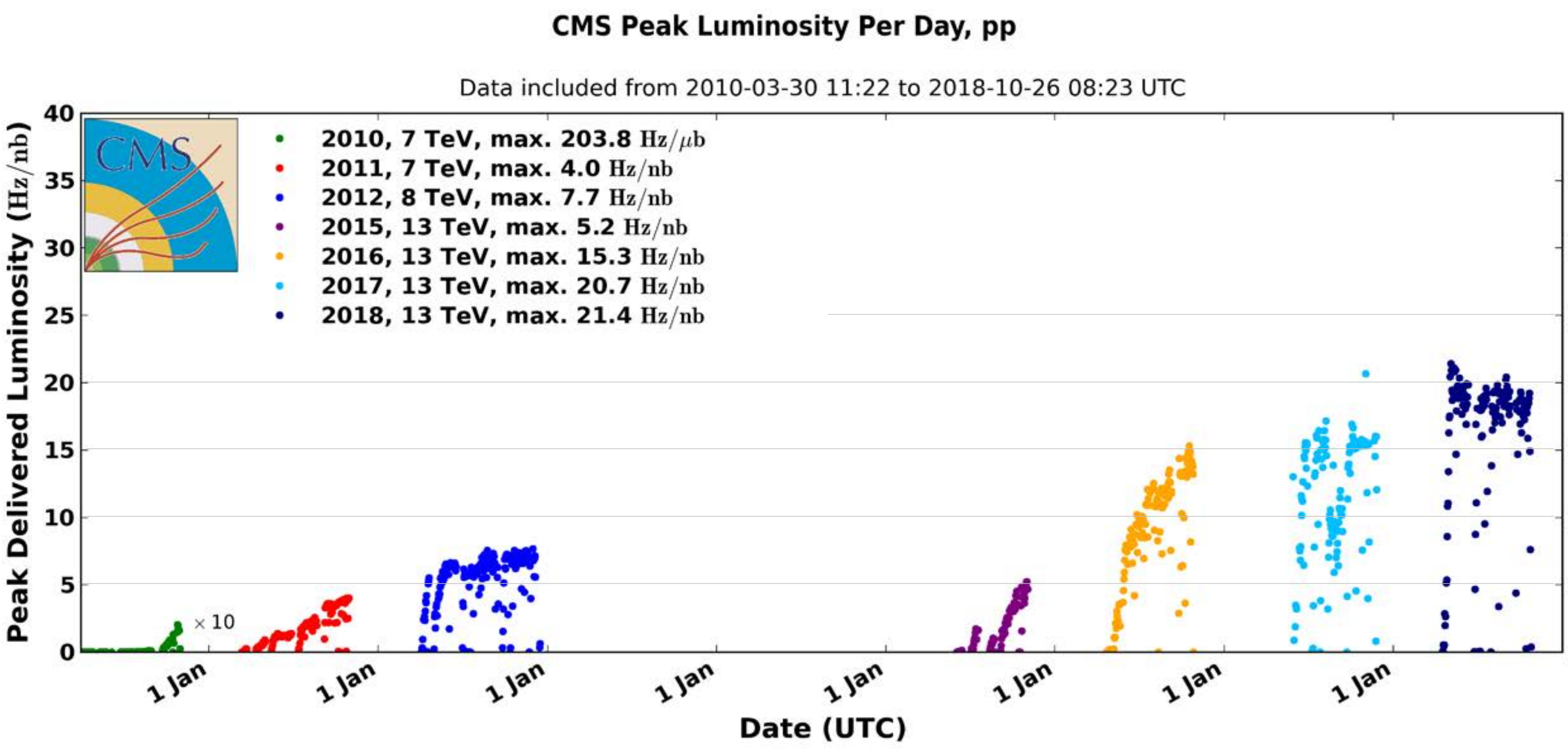
An almost impossible achievement: efficiency > 90% for many months [the hole is just a period of LHC maintenance].



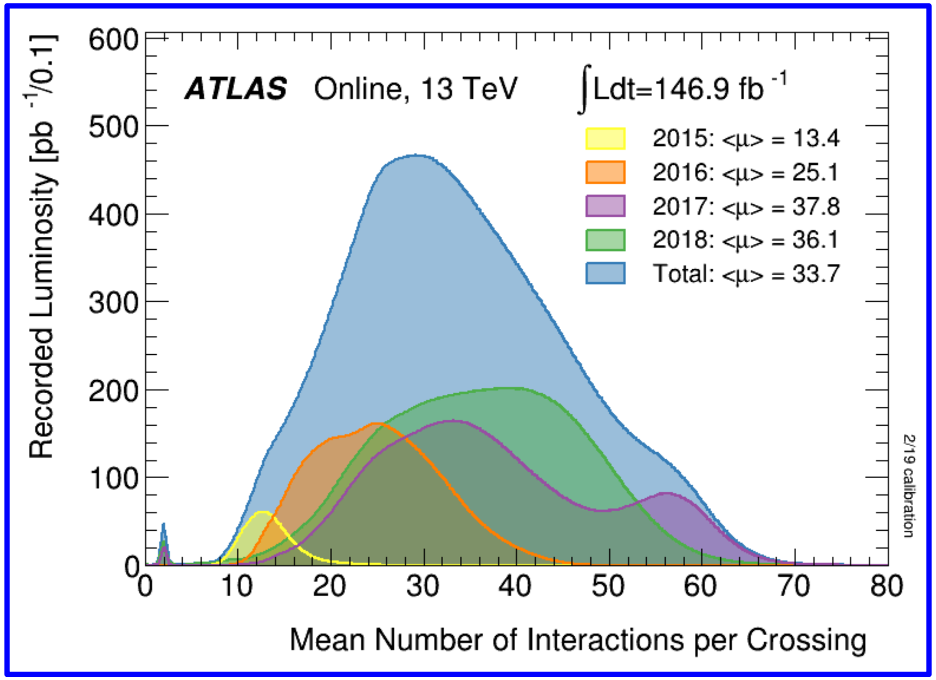
"what really counts is not the immediate act of courage or of valor, but those who bear the struggle day in and day out - not the sunshine patriots but those who are willing to stand for a long period of time", John F. Kennedy.

The luminosity: $\mathscr{L}_{peak}$

- In 2016 LHC has achieved the luminosity foreseen in the project, i.e. $\mathscr{L} = 10^{34}$ cm^{-2}s^{-1} …
- … and in 2017-18 it doubled it ($\mathscr{L} = 2 \times 10^{34}$ cm^{-2}s^{-1});
- for $\sqrt{s}$ = 14 TeV, wait another couple of years.
- [1 Hz/nb = 10^{33} cm^{-2}s^{-1}]



The luminosity : $\langle n_{int}\rangle$



$\mu = \langle n_{int}\rangle = \mathscr{L}\tau_{bc}\sigma_{inel}$ = [e.g. ≈]
$\approx (10^{+34})\times(25\times10^{-9})\times(8\times10^{-26}) \approx 20.$

see § 8

Pros and cons of the value of μ at LHC:

- ☺ for fixed τ_{bc}, $\mu \propto \mathscr{L}$, so large μ necessary for rare processes, like Higgs;
- ☺ for fixed $\mathscr{L}$, $\mu \propto \tau_{bc}$; so a decrease in μ is payed by a decrease in τ_{bc}, the processing time for the trigger and DAQ (now 25 ns, the bare minimum);
- ☹ large $\mu \to$ many overlapping events
  - → systematics in trigger thresholds;
  - → systematics in vertex reconstruction;
  - → systematics in calo calibrations and reconstruction;
  - → mistakes in assignment of heavy flavors, jets, muons to event;
  - → (… many other problems …)
- ☺ some of the LHC data have been taken with a different τ_{bc} (50 ns instead of 25 ns); for the same $\mathscr{L}$, this fact doubles μ (→ 25 ns is better than 50 ns, but …)
- ☺ anyway, **large μ is necessary**, so you better learn to survive with it.

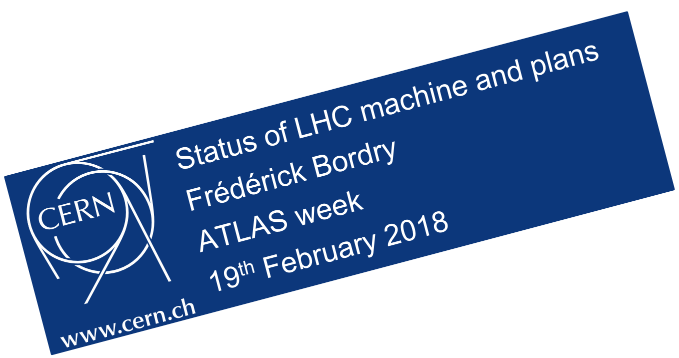
LHC operations: 2018…

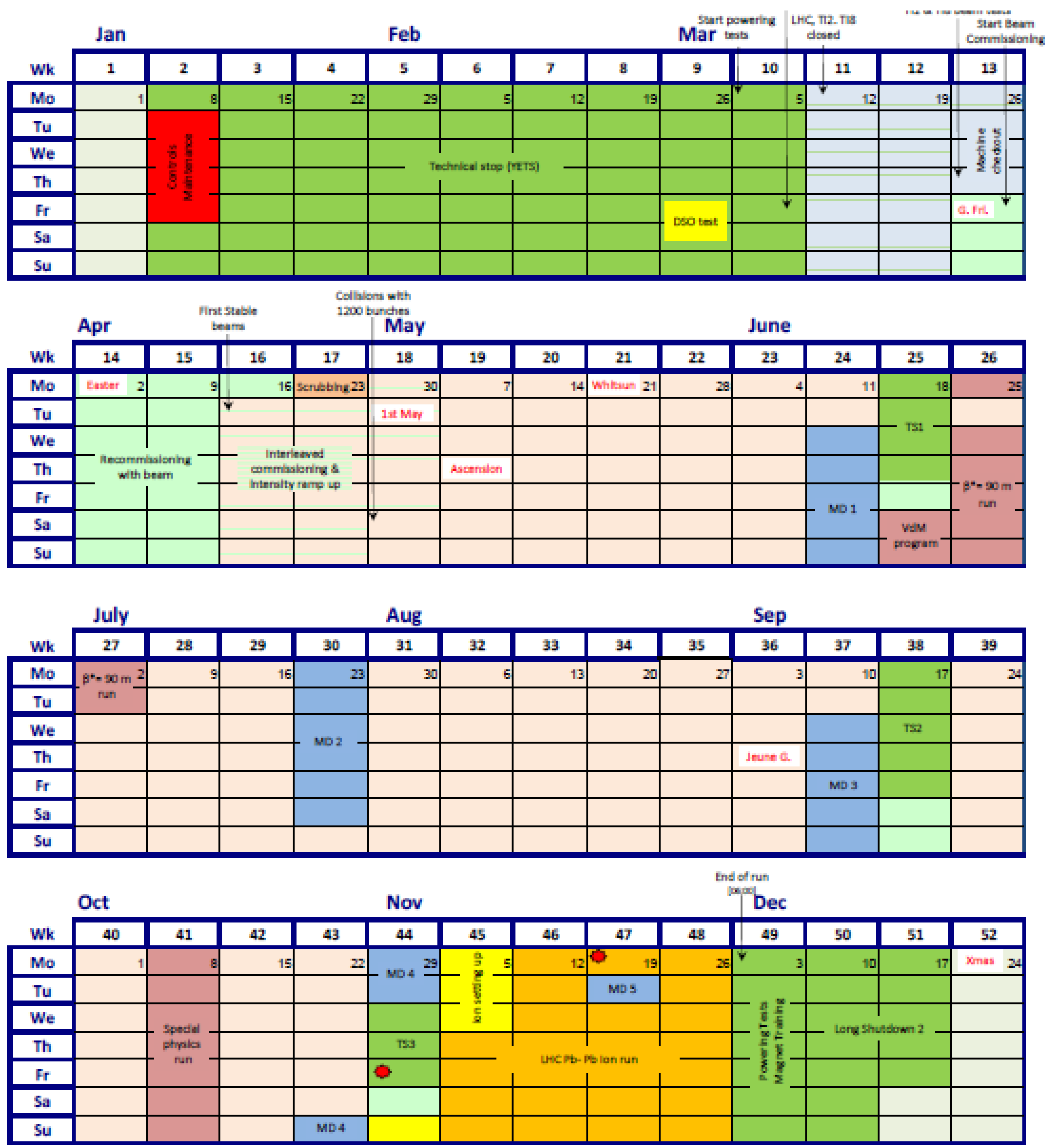
LHC schedule 2018

(the last year of operations before LS2, see later)

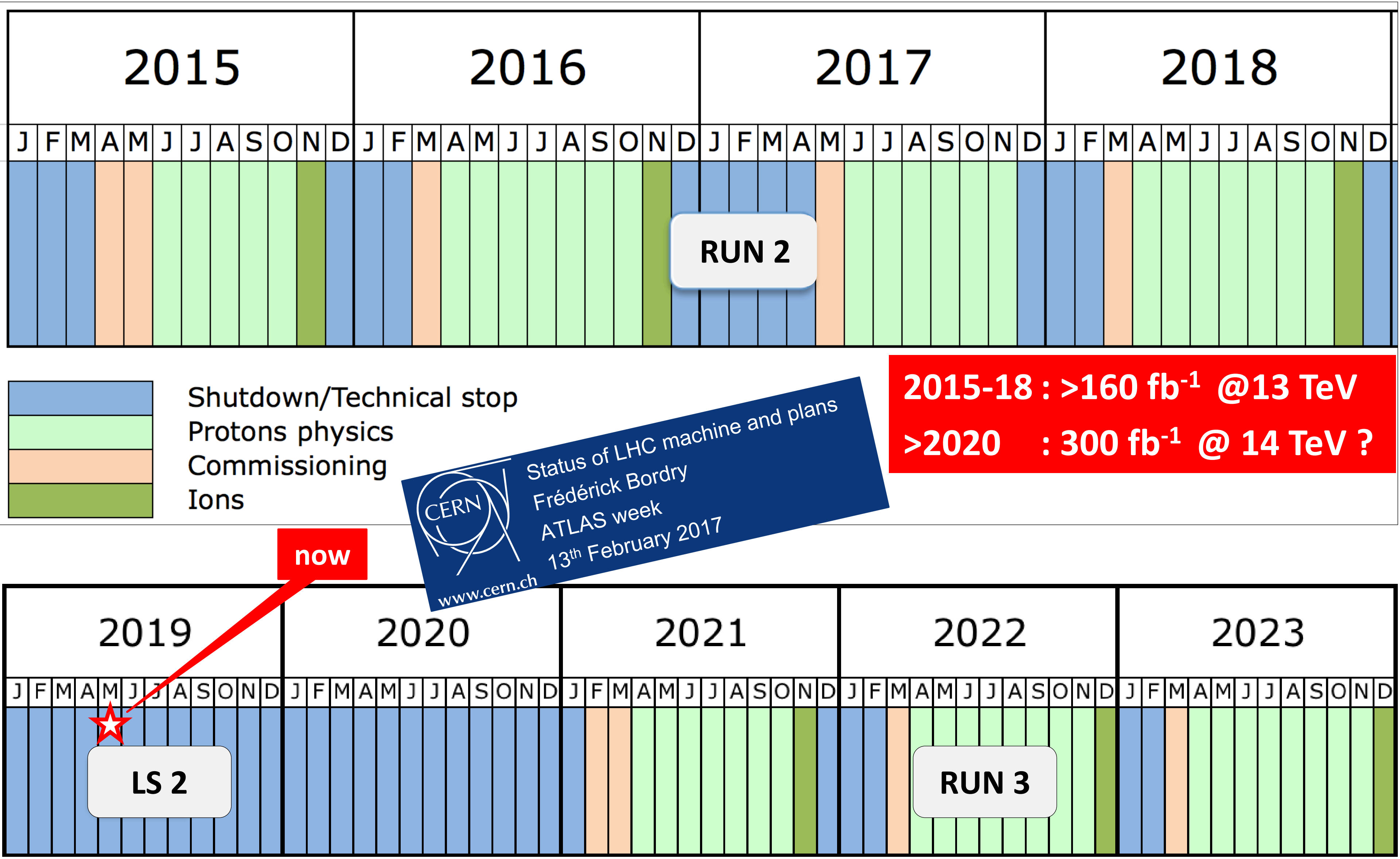
$> 65\ fb^{-1}$

keeping the LHC availability close to 50% (stable beams)





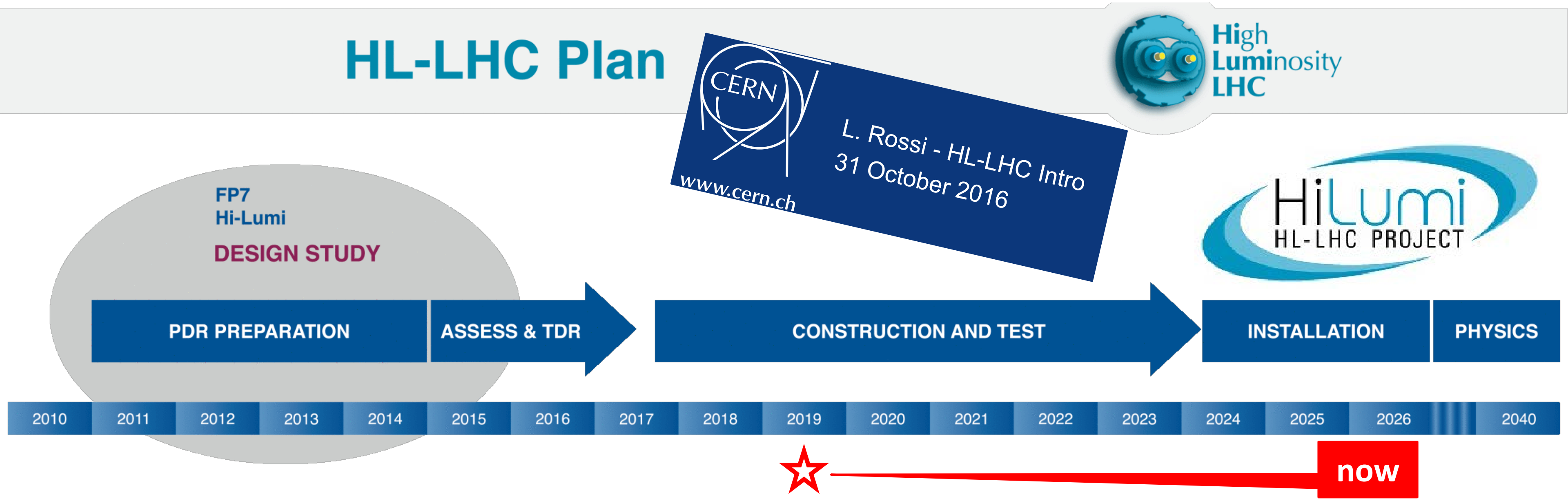
LHC operations: 2015-2023



2015-18 : >160 fb^{-1} @13 TeV

>2020 : 300 fb^{-1} @ 14 TeV ?

LHC operations: HL-LHC



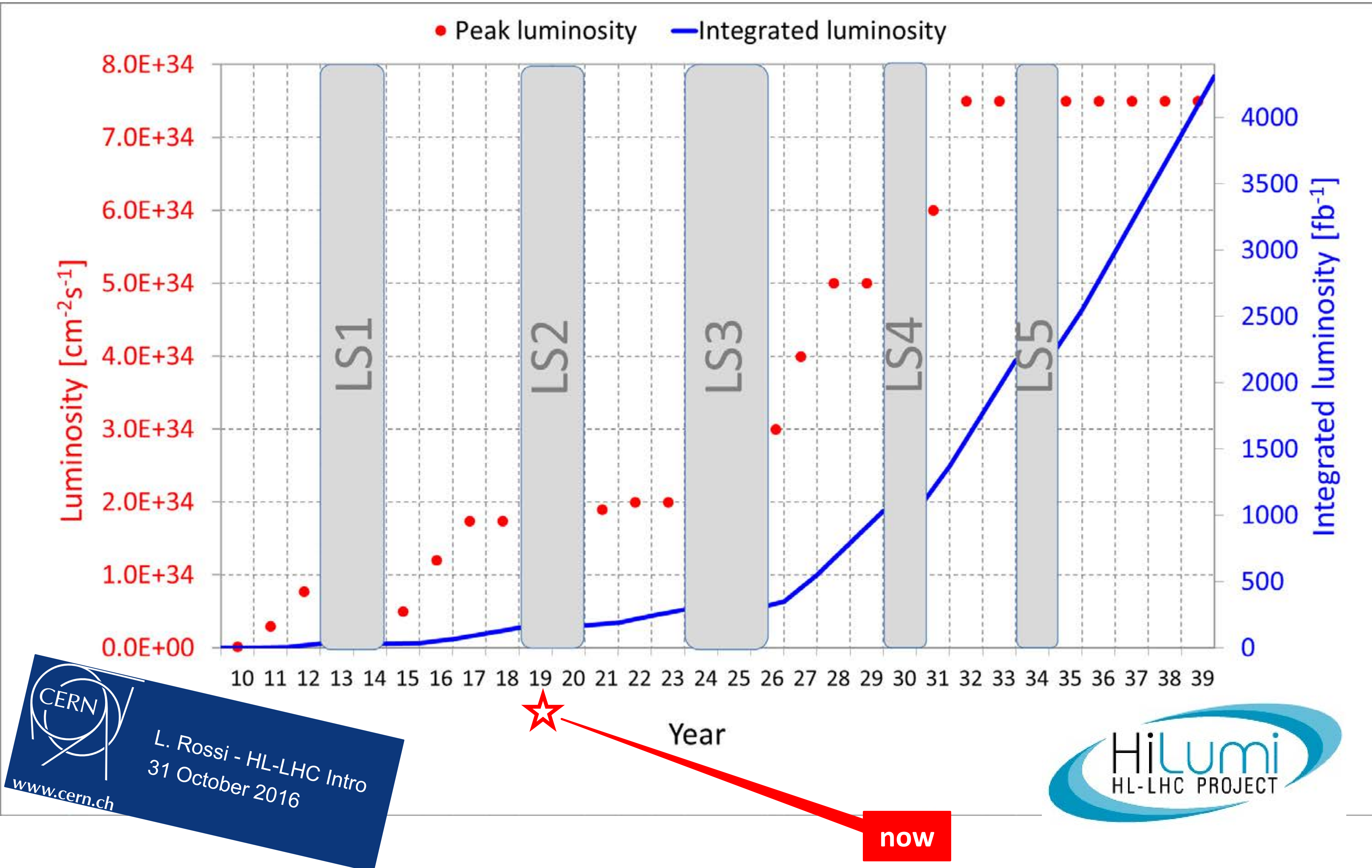
- March 2016: HL-LHC included in the ESFRI (European Strategy Forum on Research Infrastructures) roadmap as "landmark project" in March 2016.
- June 2016: HL-LHC project formally approved by CERN's Council.

- "Full exploitation of the LHC physics potential with the HL-LHC phase is the top priority of the ESPP [*European Strategy for Particle Physics*] and the highest near-term large-project priority of the US P5 roadmap."

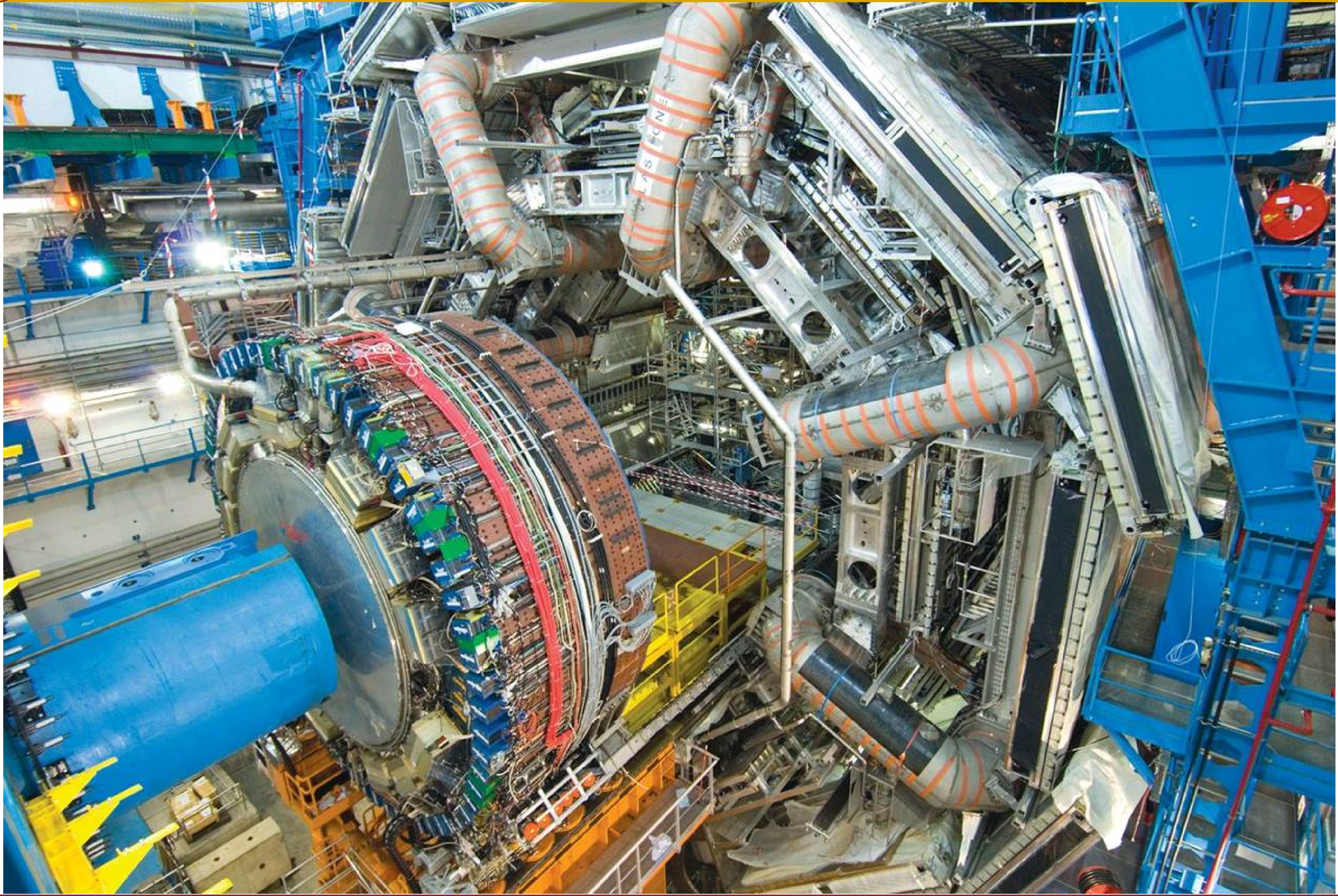
  "LHC/HL-LHC is CERN's flagship project for the next 20 years."

  [Fabiola Gianotti, CERN's Scientific Strategy, ECFA HL-LHC Experiments Workshop, Aix-Les-Bains, 3/10/2016].

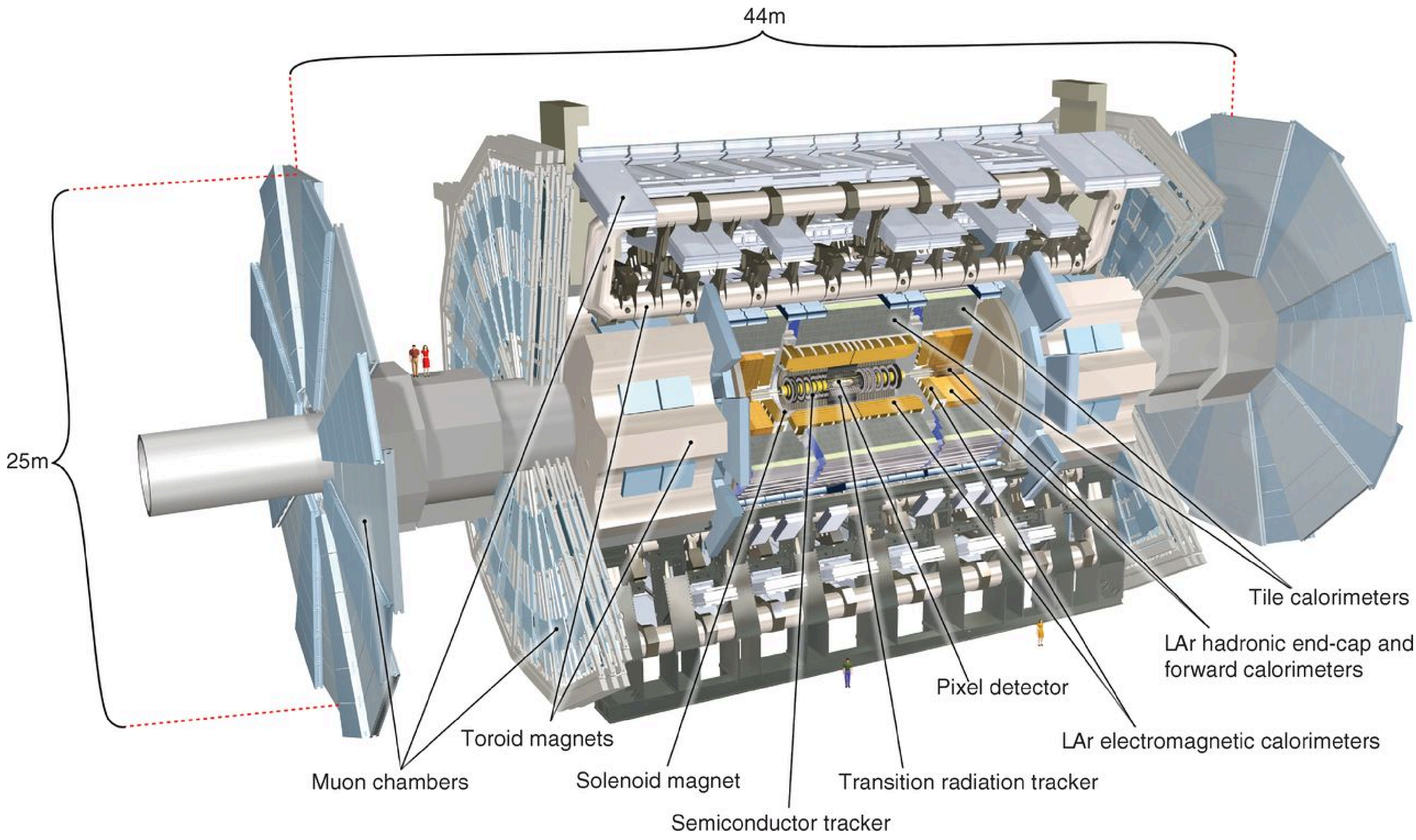
LHC operations: HL-LHC performances



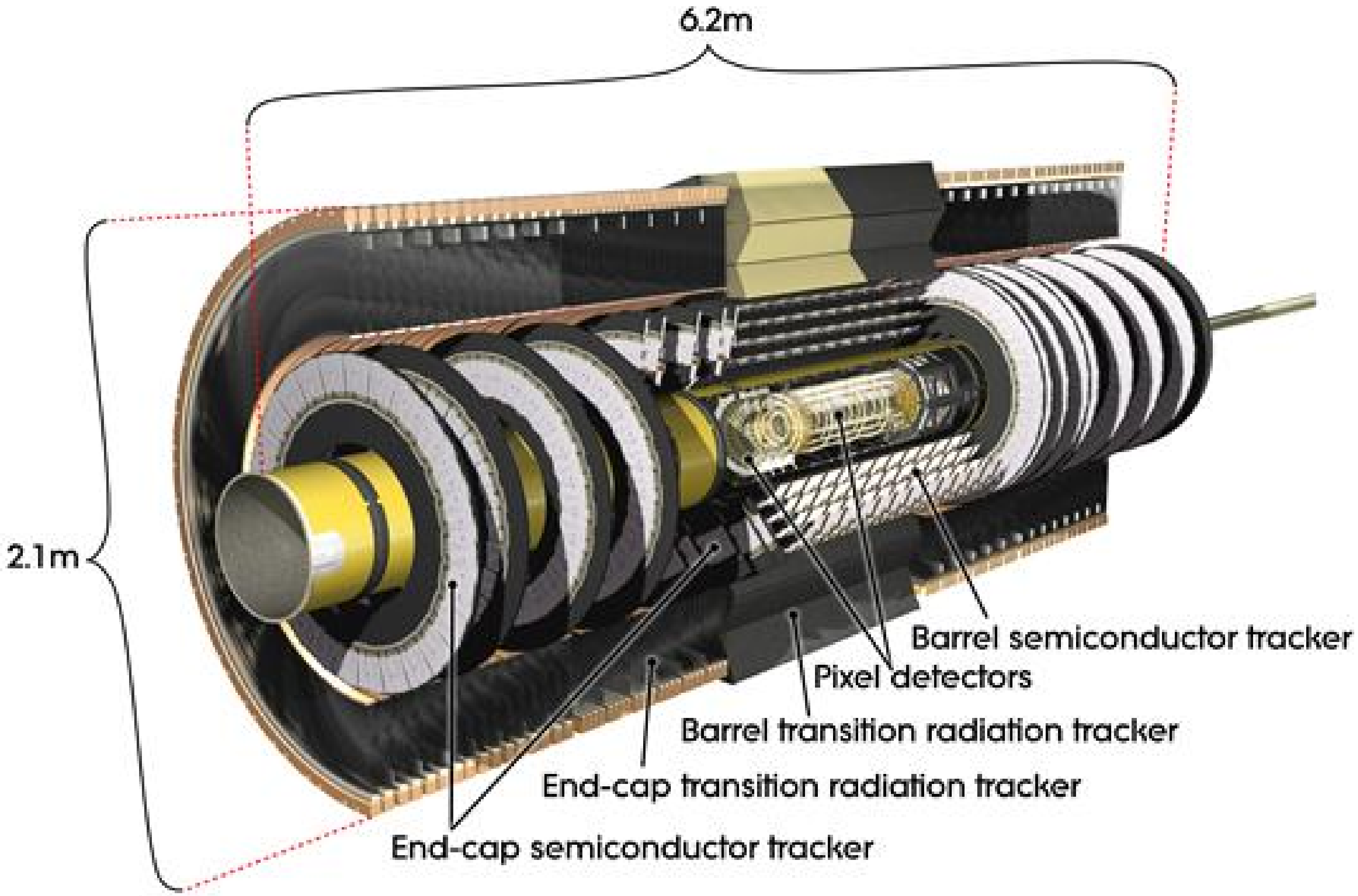
The ATLAS detector



The ATLAS detector: scheme

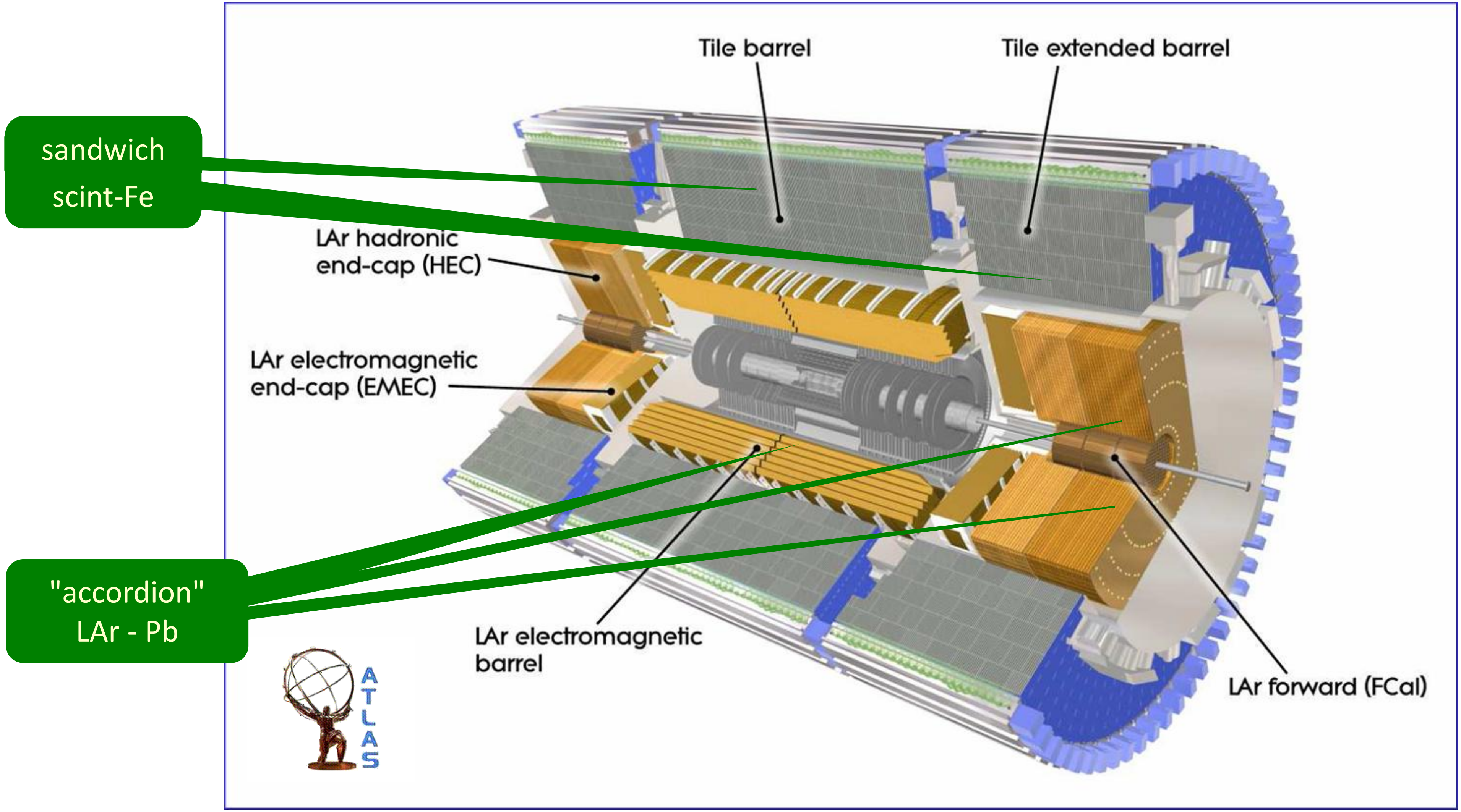


The ATLAS detector: inner tracker

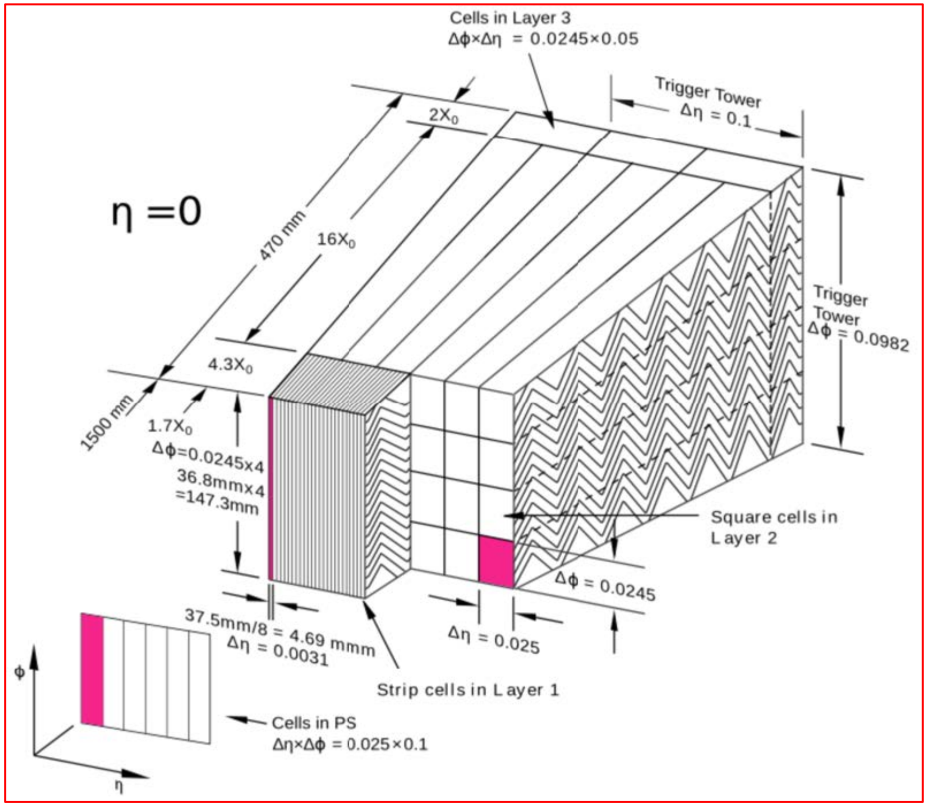


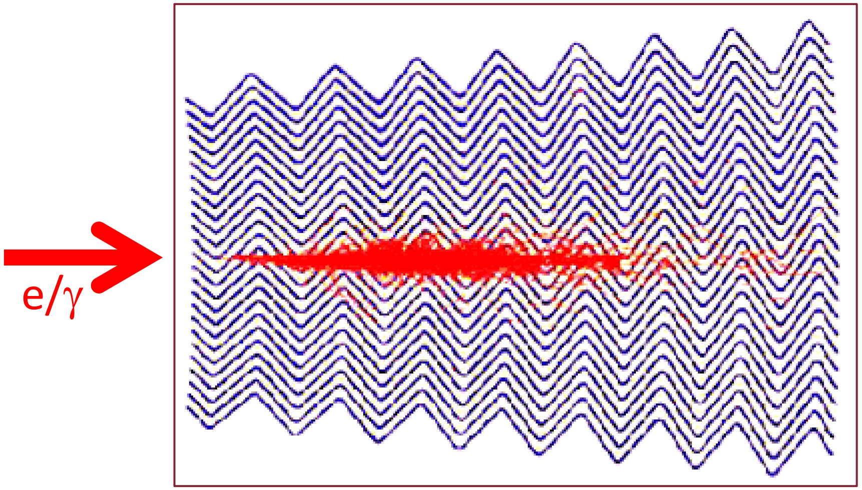
Pixel	SCT	TRT
3 cylindrical layers	4 cylindrical layers	73 straw planes
2×3 disks	2×9 disks	160 straw planes

The ATLAS detector: calorimeters



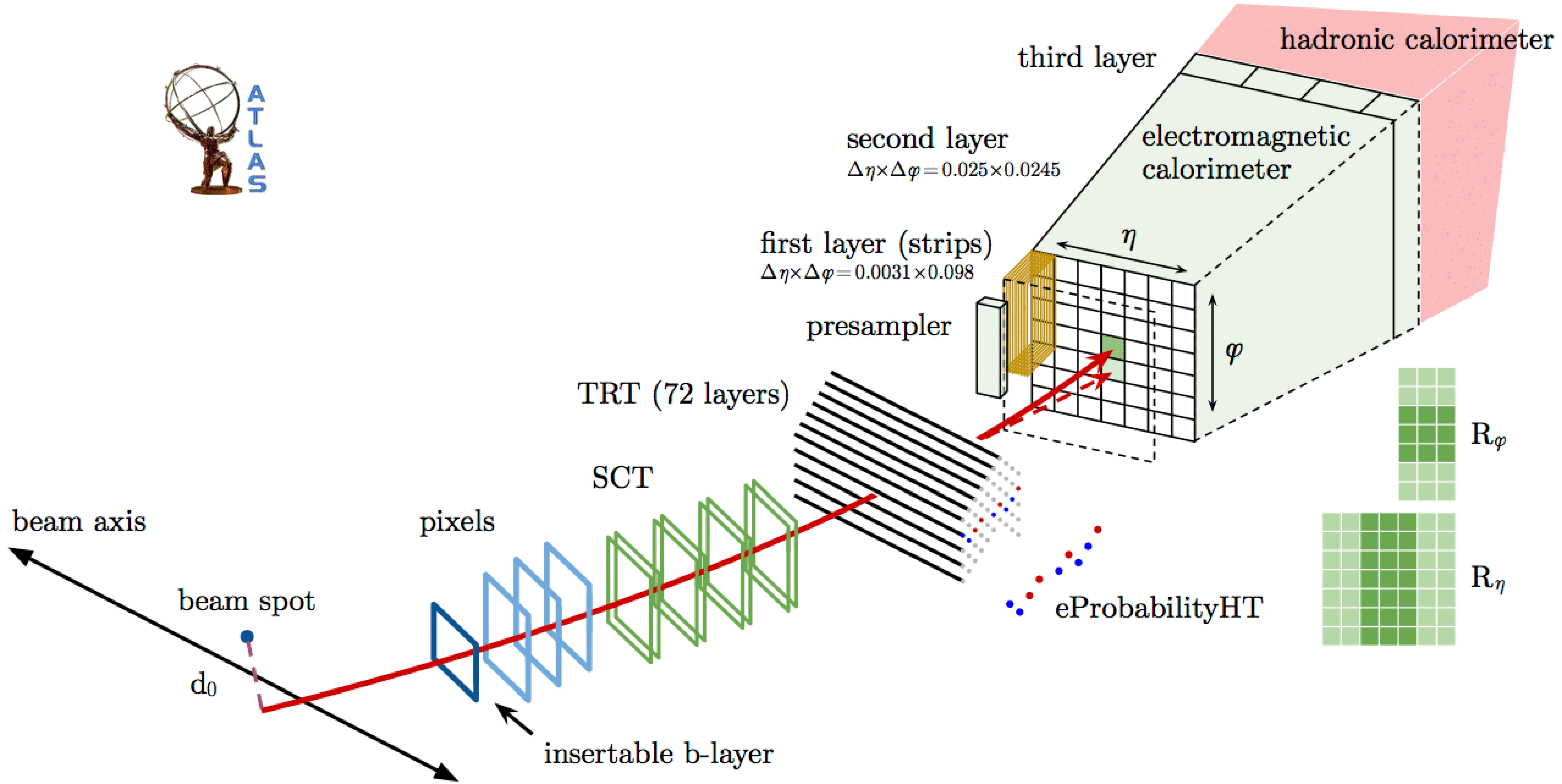
The ATLAS detector: e.m. calo





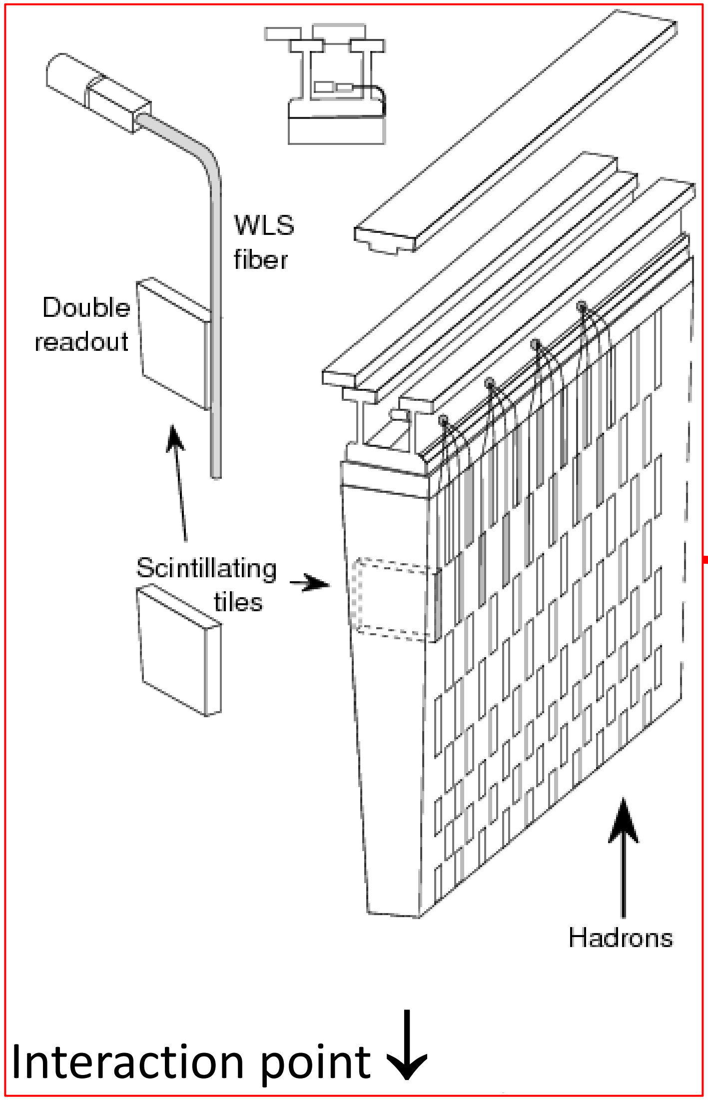
- "accordion" LAr – Pb
- cryogenic
- hermetic
- longitudinal + radial segmentation

The ATLAS detector: $e^{\pm}$ id and measure

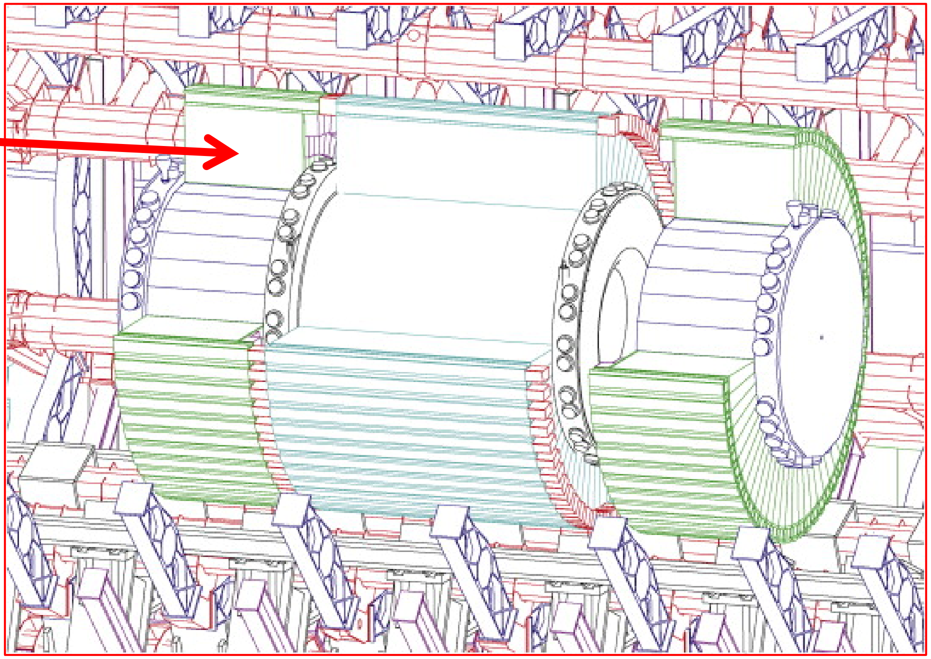


an electron is detected many (>> 100) times after the interaction point; even the non-detection in the had. calo is important (cfr a γ in the pixels/SCT/TRT).

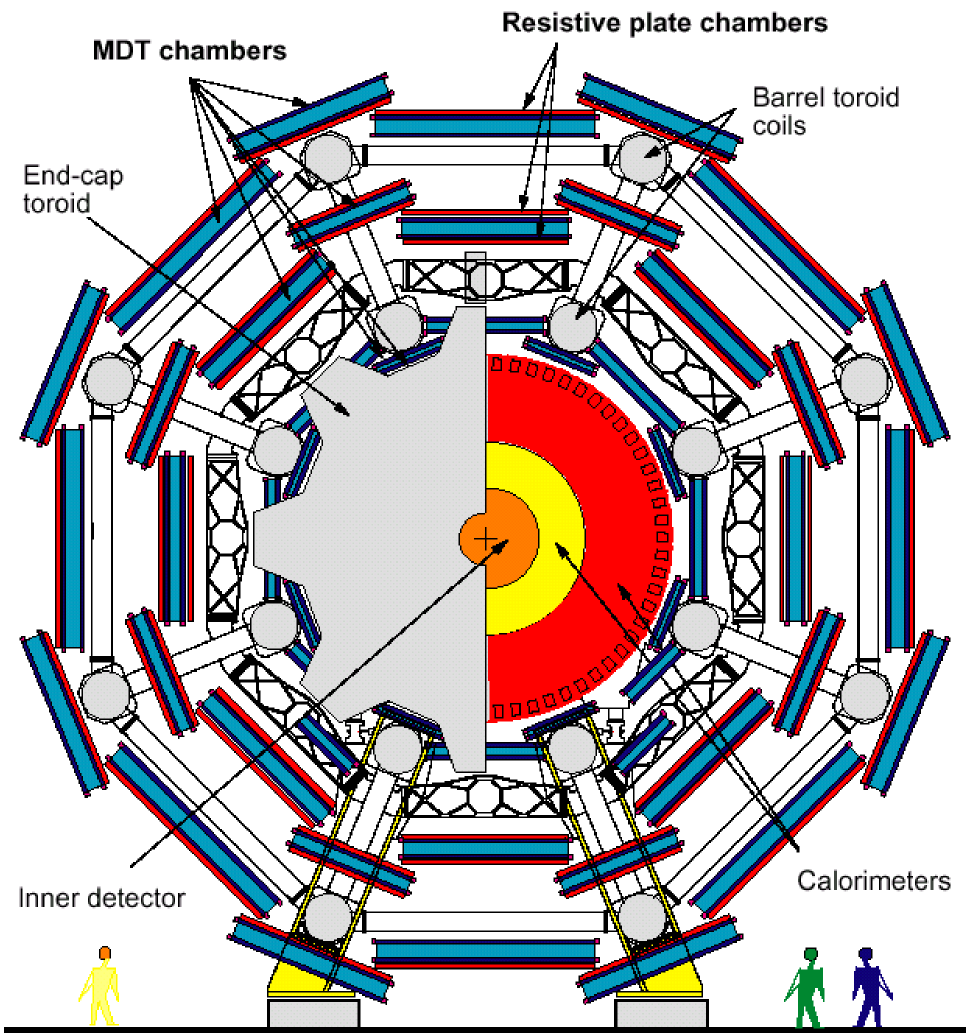
The ATLAS detector: had. calo

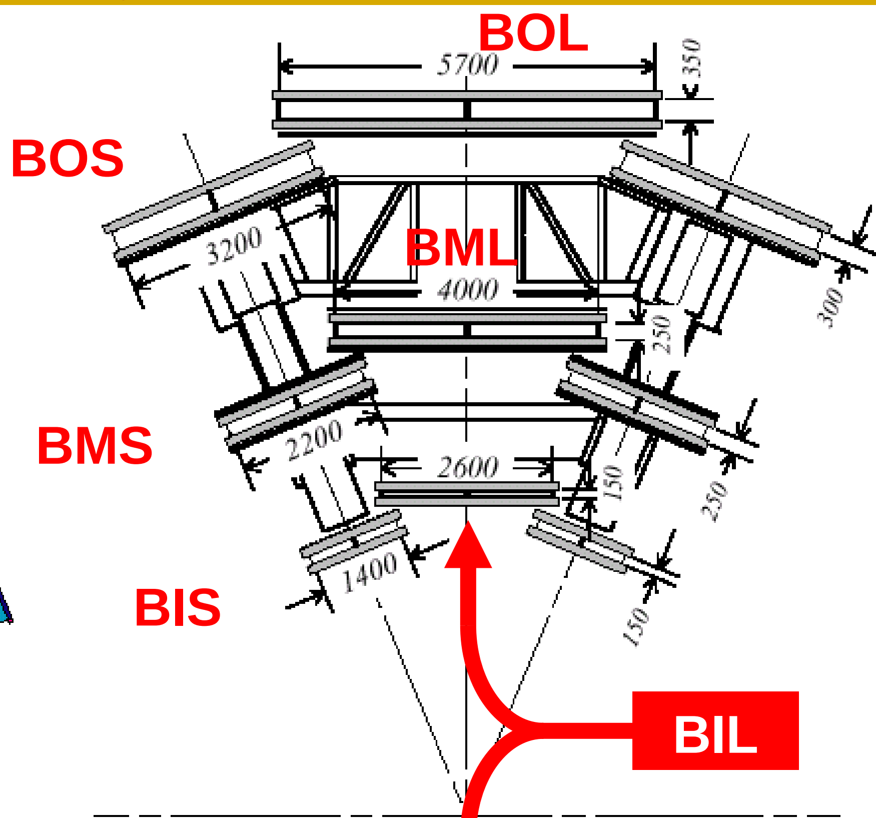


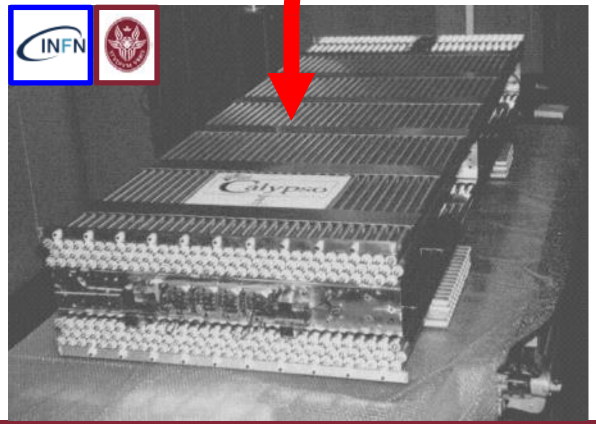
- "tiles" Fe – Scintillator
- WLS readout
- hermetic
- high segmentation



The ATLAS detector: μ spectrometer



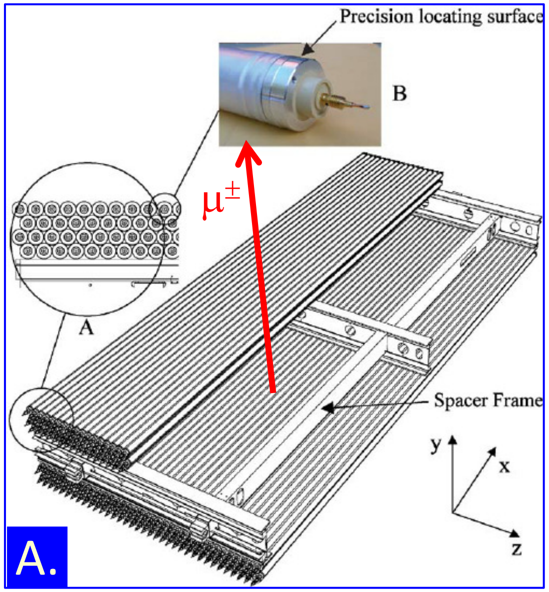


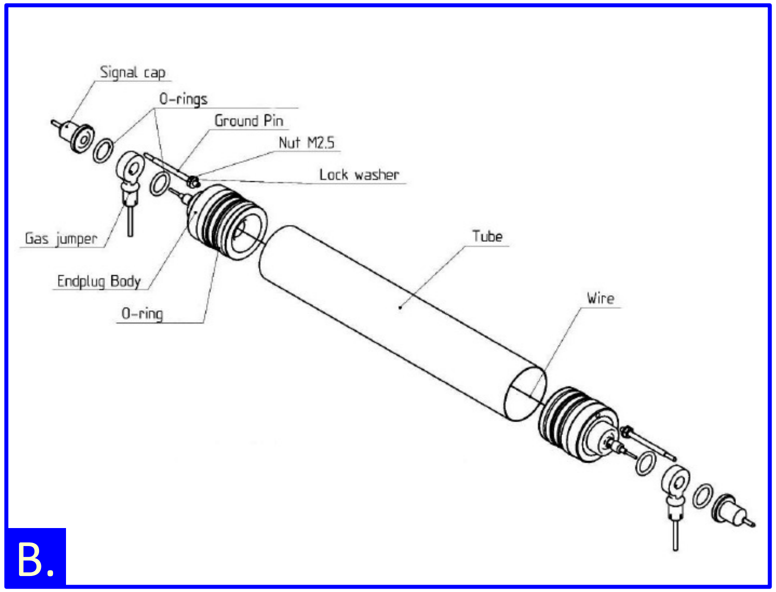


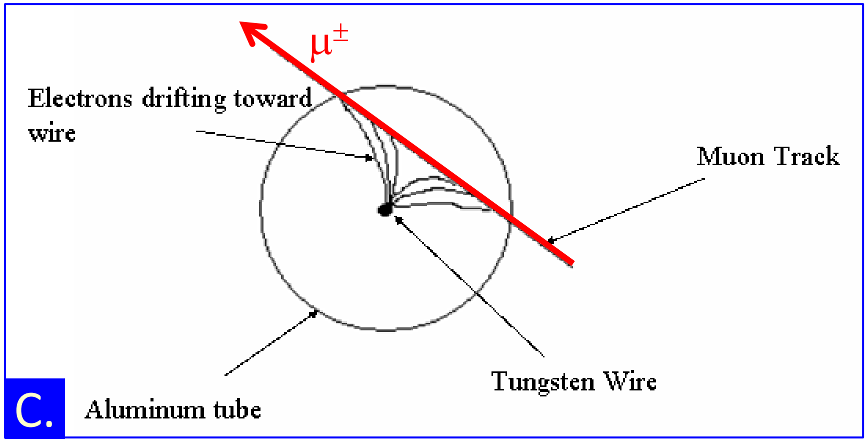
The ATLAS detector: μ chambers

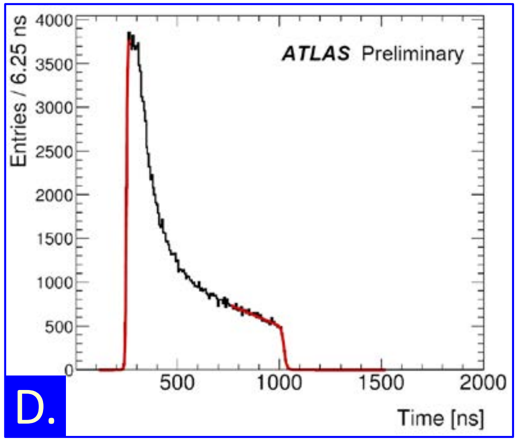
Schematic view of:

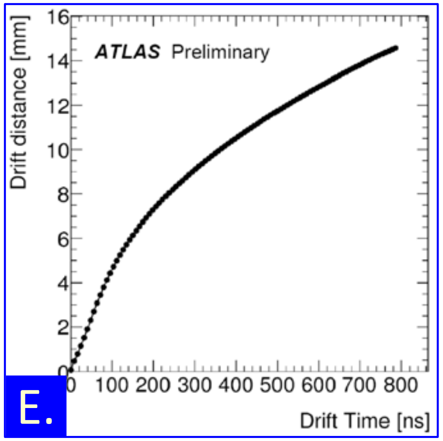
A. a chamber of drift tubes
B. a single tube
C. a muon hitting a tube
D. the hit time distribution
E. the r-t relation



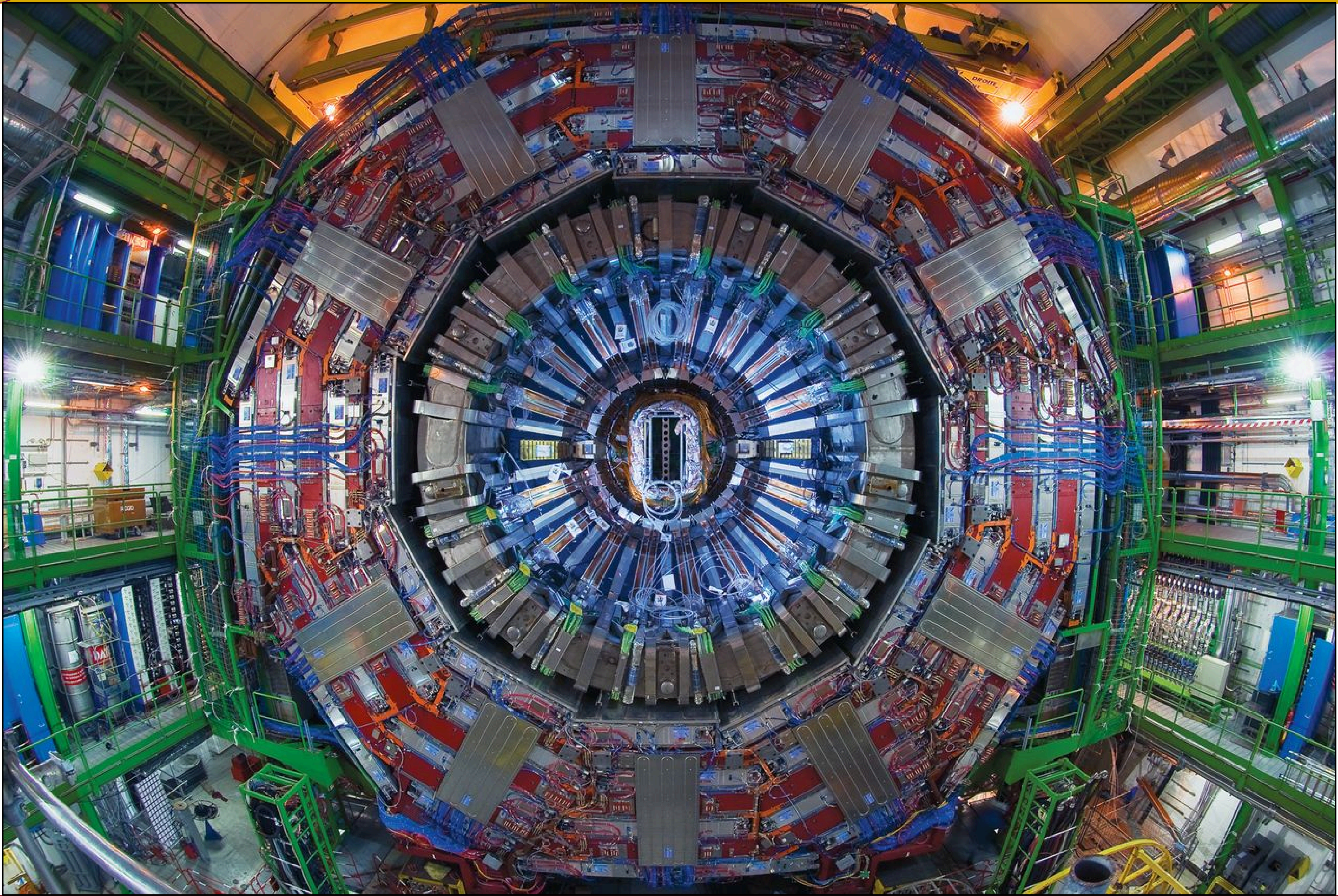




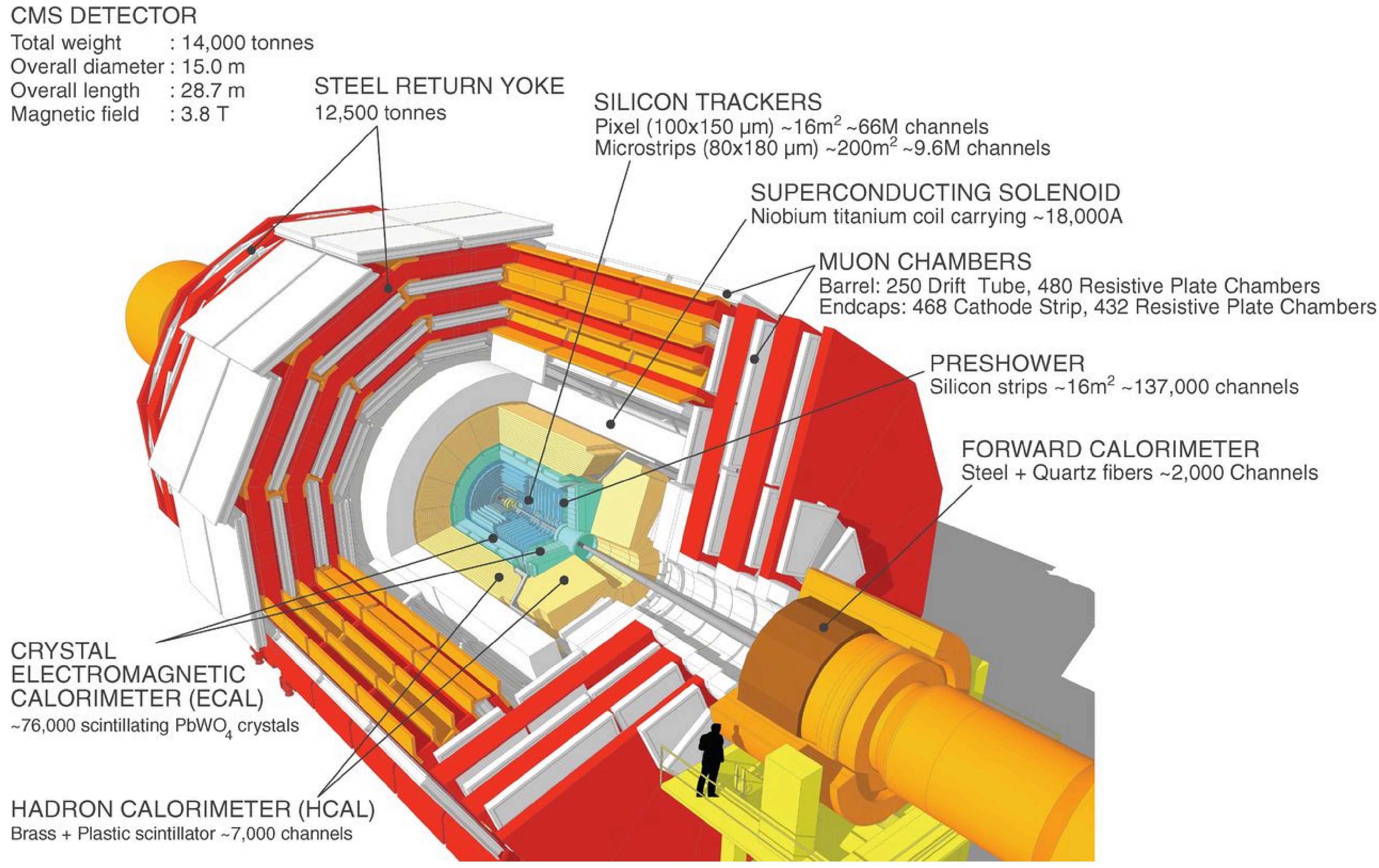




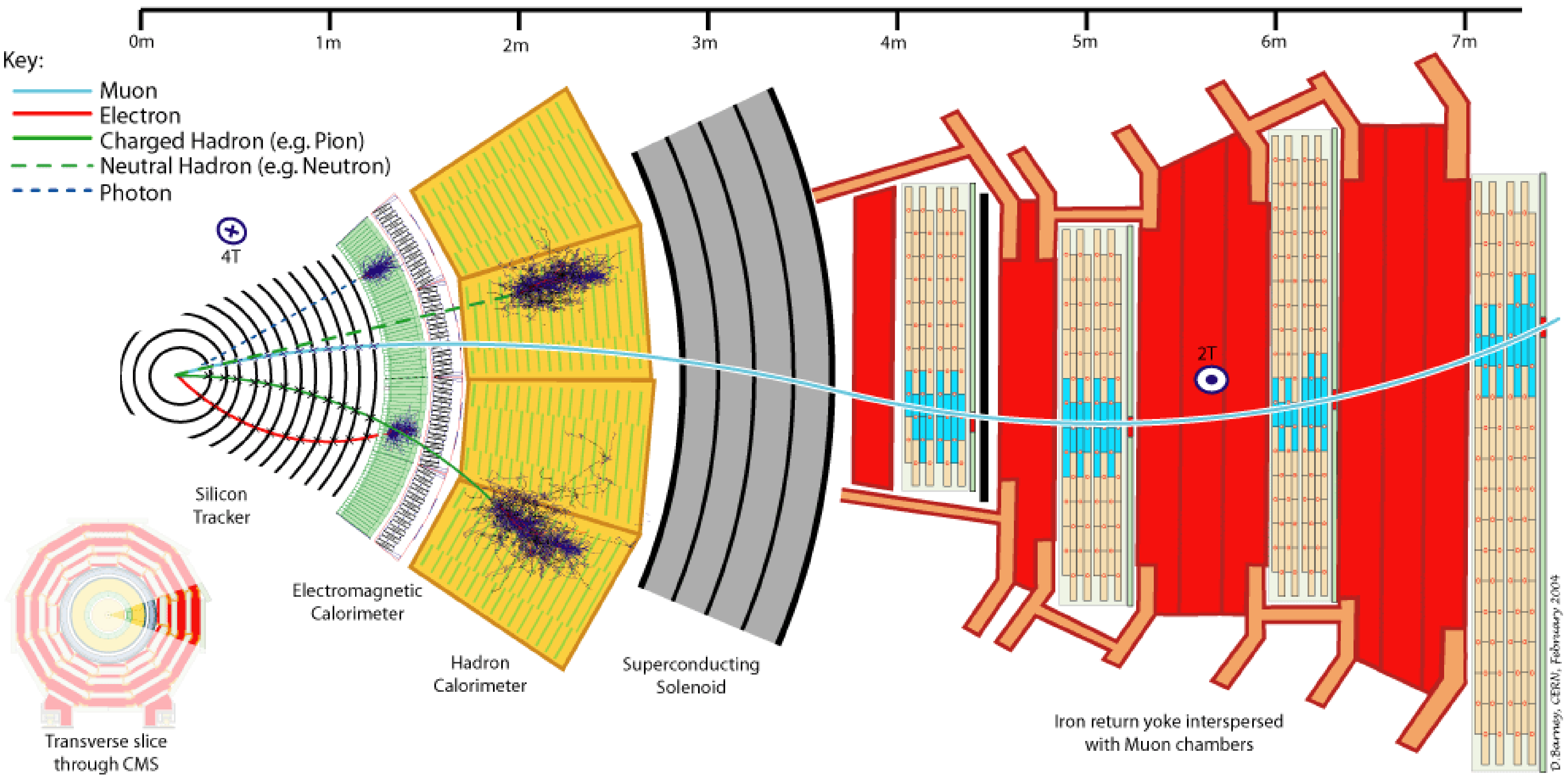
The CMS detector



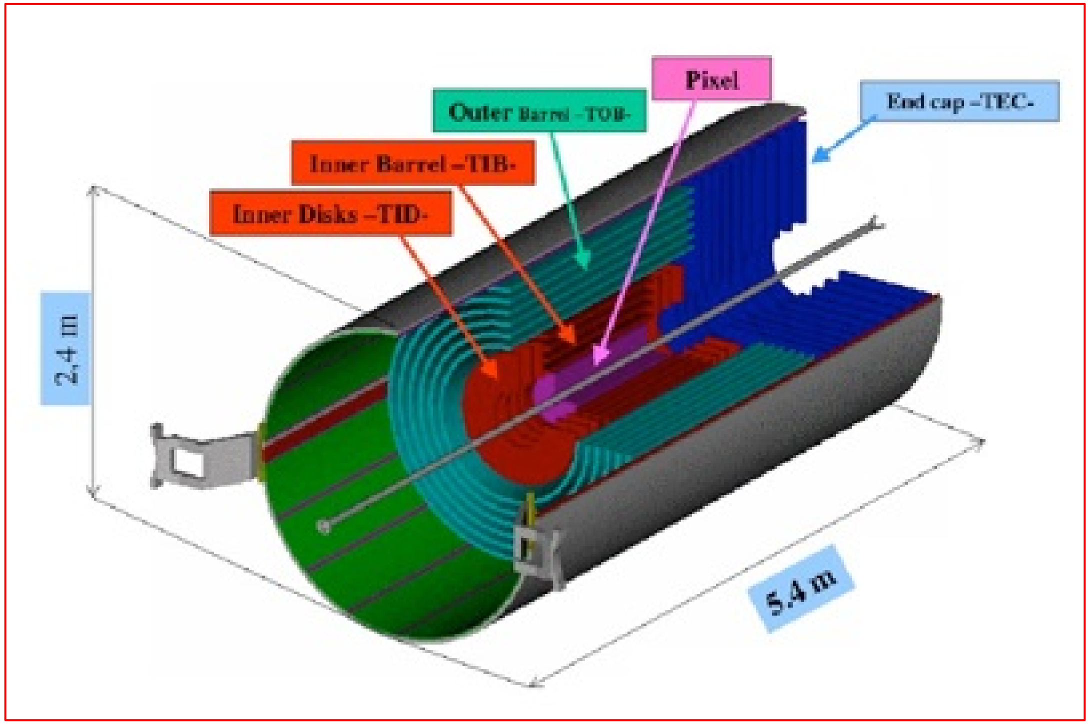
The CMS detector: view

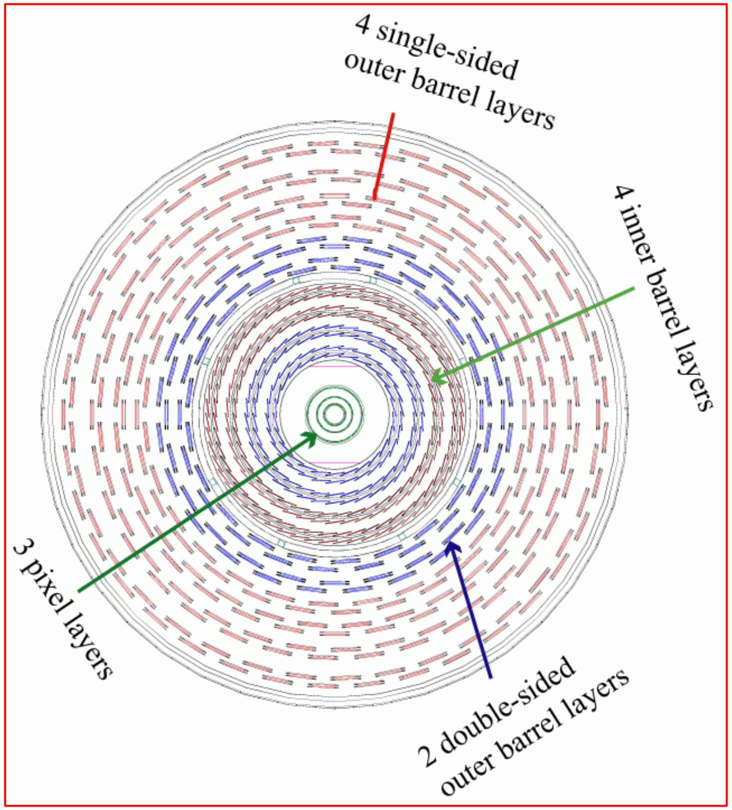


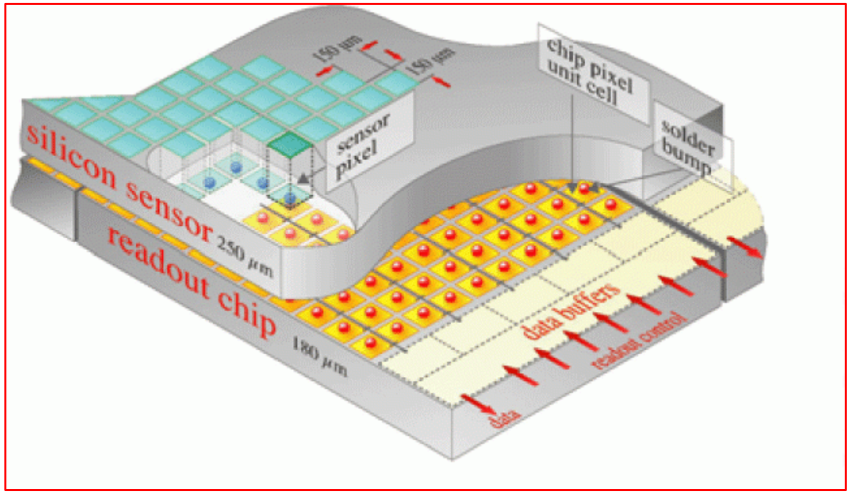
The CMS detector: scheme



The CMS detector: inner tracker

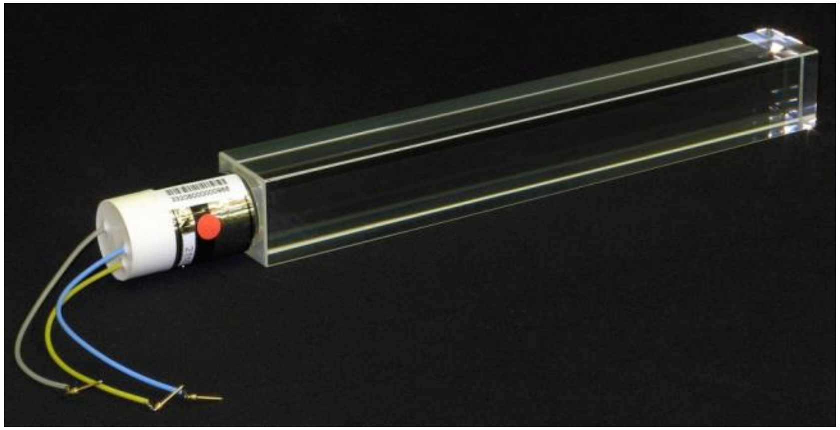


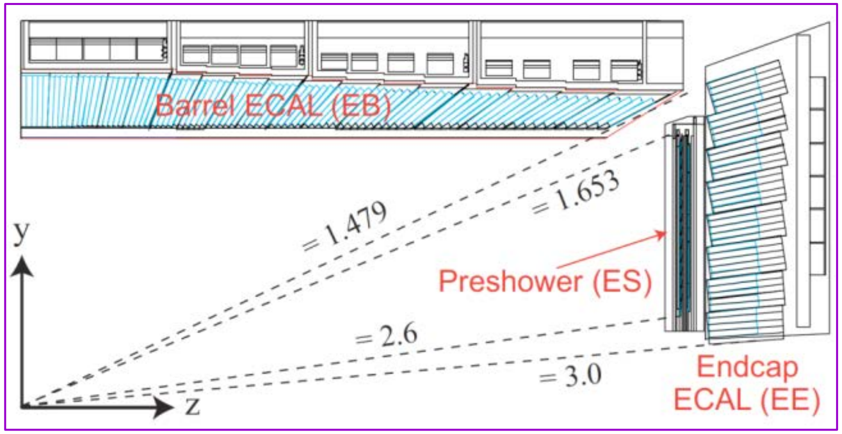


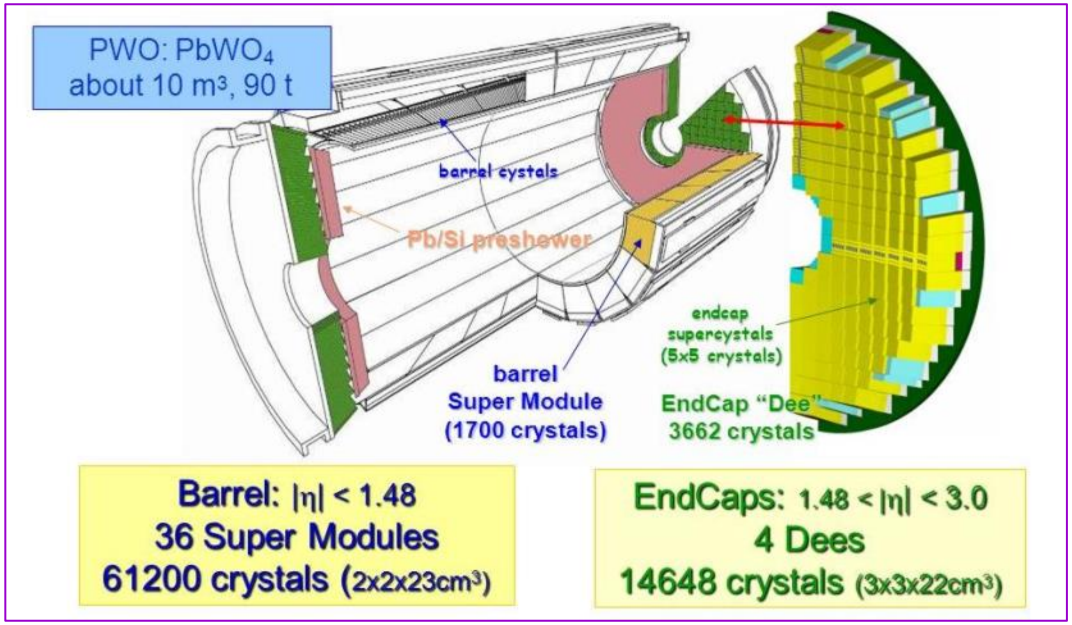


Si pixel + strip detector

The CMS detector: e.m. calo







e.m. calo:
$PbWO_4$ crystals

The CMS detector: had. calo

<u>HCAL (tower structure)</u>:

- Barrel (HB): $|\eta| < 1.4$, 2304 towers
- End caps (HE): $1.3 < |\eta| < 3.0$, „ towers
- Outside coil (HO): $|\eta| < 1.26$ (tail catcher)

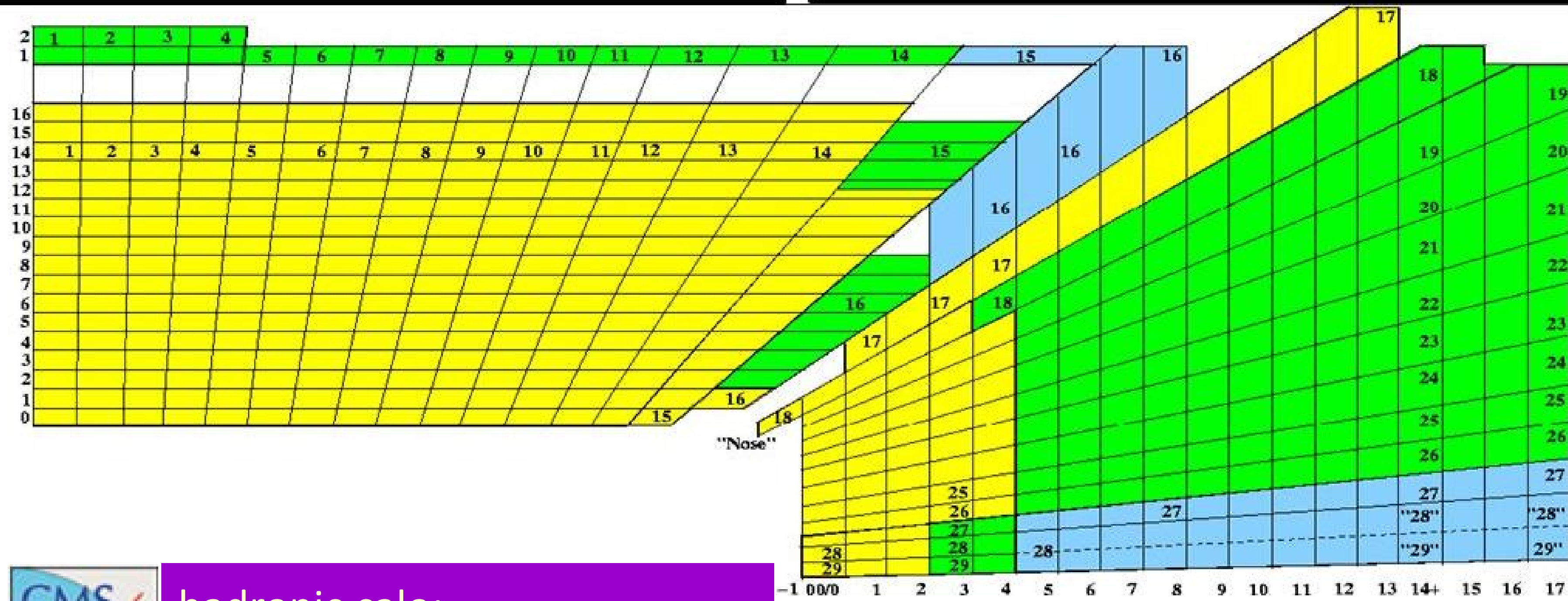
→ 4608 towers (Plastic scintillator tiles, ≈ 10 λ_N)
→ $\Delta\eta \times \Delta\varphi \approx 0.087 \times 0.087 \to 0.350 \times 0.175$

- Forward (HF): $2.9 < |\eta| < 5.0$ (not shown)

→ 2 x 900 towers (Quartz fibers, ≈ 10 λ_N)
→ $\Delta\eta \times \Delta\varphi \approx 0.111 \times 0.175 \to 0.302 \times 0.350$

<u>CASTOR calorimeter</u> (not shown):

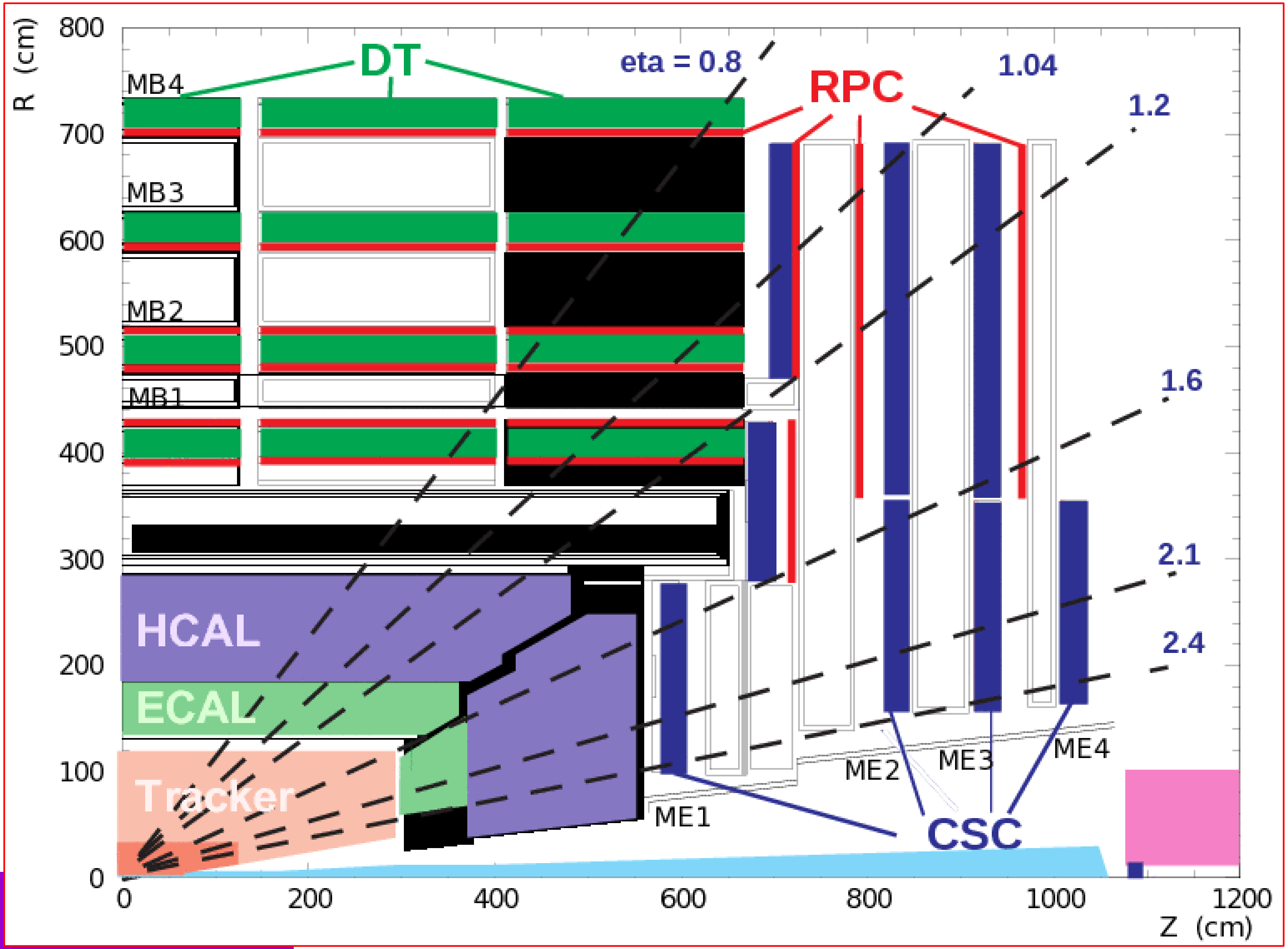
- $5.1 < |\eta| < 6.5$, ≈ 22 X_0, ≈ 10 λ_N



hadronic calo:

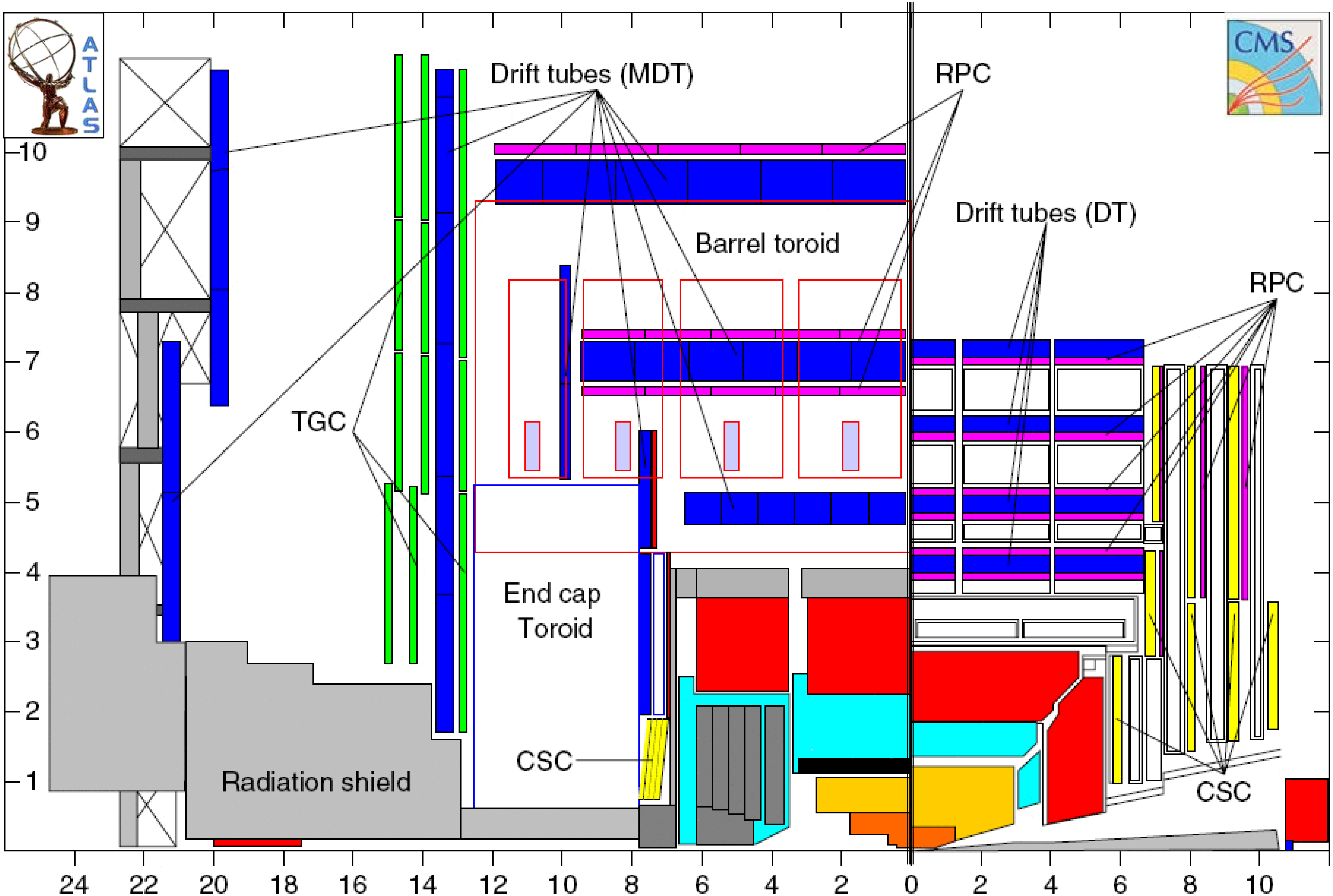
Brass/scintillator/wls readout

The CMS detector: μ system



muon system:
drift tube (DT) chambers

Detectors comparison : ATLAS vs CMS



Detectors comparison : structure

thanks to Anna Colaleo

	ATLAS	CMS
Magnet(s)	Air-core toroids + Solenoid in inner cavity Calorimeters outside field 4 magnets	Solenoid Calorimeters inside field 1 magnet
Tracker/ Inner Detector	Silicon pixels, Silicon strips, Transition Radiation Tracker. 2T magnetic field	Silicon pixels, Silicon strips. 4 T magnetic field
Electro-magnetic calorimeter	Lead plates as absorbers with liquid argon as the active medium	Lead tungstate (PbW04) crystals both absorb and respond by scintillation
Hadronic calorimeter	Iron absorber with plastic scintillating tiles as detectors in central region, copper and tungsten absorber with liquid argon in forward regions.	Stainless steel and copper absorber with plastic scintillating tiles as detectors
Muon detector	Large air-core toroid magnets with muon chamber form outer part of the whole ATLAS	Muons measured already in the central field, further muon chambers inserted in the magnet return yoke

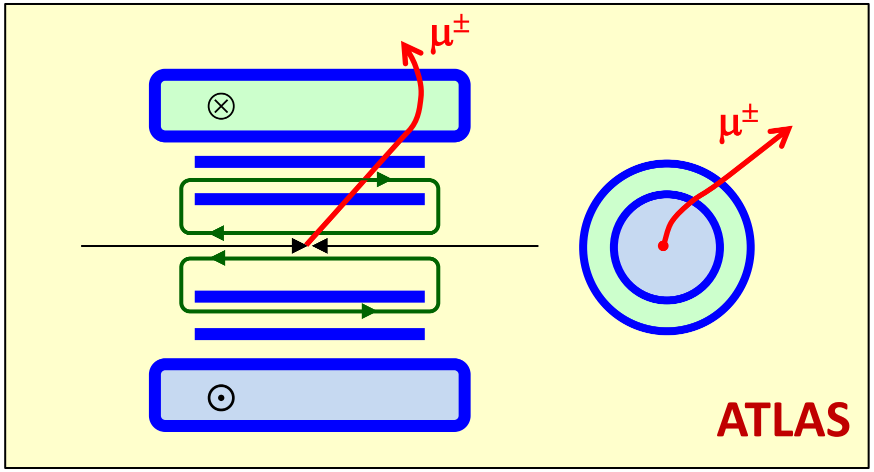
Detectors comparison : performances

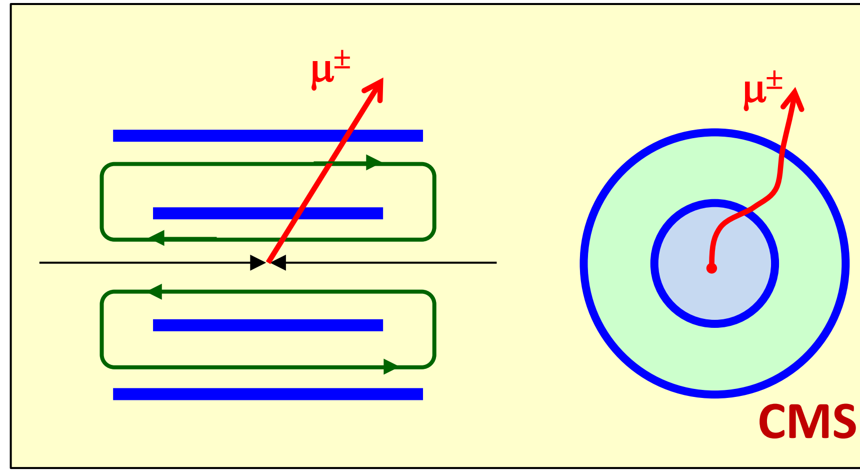
thanks to Anna Colaleo

	ATLAS	CMS
Tracker/ Inner Detector	TRD → particle identification $\sigma/p_T \approx 5 \times 10^{-4} p_T$ (GeV) $\oplus$ 0.01	No particle identification $\sigma/p_T \approx 1.5 \times 10^{-4} p_T$ (GeV) $\oplus$ 0.005
Electro-magnetic calorimeter	$\sigma/E \approx 10\%/\sqrt{E}$ (GeV) Longitudinal segmentation	$\sigma/E \approx (2 \div 5)\ \%/\sqrt{E}$ (GeV) No longitudinal segmentation
Hadronic calorimeter	$> 10\ \lambda$ $\sigma/E \approx 50\%/\sqrt{E}$ (GeV) $\oplus$ 0.03	$> 5.8\ \lambda$ + tail catcher $\sigma/E \approx 65\%/\sqrt{E}$ (GeV) $\oplus$ 0.05
Muon detector	air $\sigma/p_T \approx 7\%$ @ 1 TeV (spectrometer alone)	Fe $\sigma/p_T \approx 5\%$ @ 1 TeV (combining spectrometer + tracker)

imho (*common, but not unanimous*):
- two complementary strategies almost everywhere;
- … with different optimizations (e.g. resolution vs robustness);
- a textbook example of "guided" detector design;
- … to guarantee optimal results (→ not miss major discoveries).

Detectors comparison : mag. spectrometers





ATLAS:

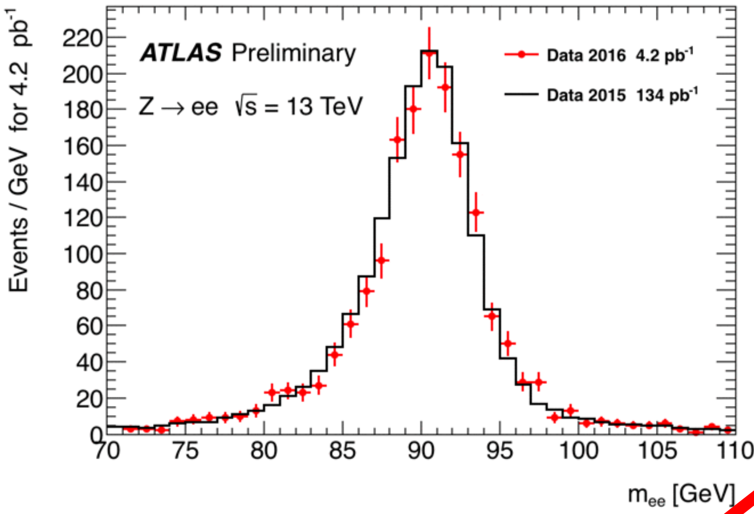
- main magnet: <u>toroid</u> B = 0.7 T;
- bending in (r,z);
- straight tracks in (r,φ);
- at small r, a solenoid B = 2 T $\rightarrow$ bending also in (r,φ);
- less precise in extrapolating to main vtx;
- μ-system in air $\rightarrow$ no multiple scatt. for μ's;
- larger bending for μ at large η $\rightarrow$ more precise.

CMS:

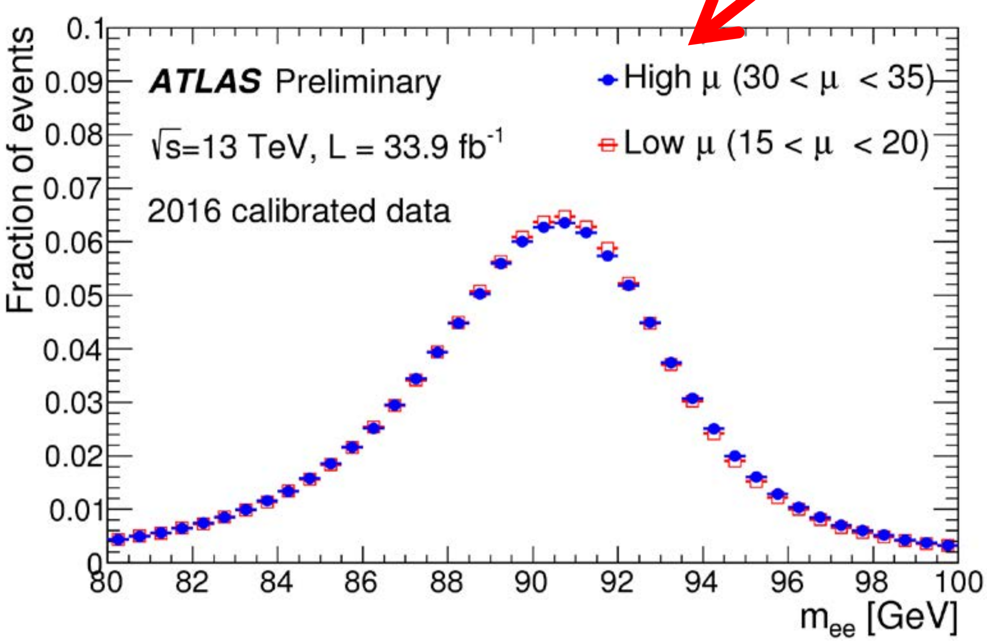
- main magnet: <u>solenoid</u> B = 4 T;
- bending in (r,φ);
- straight tracks in (r,z);
- more precise in extrapolating to main vtx;
- μ-system in Fe $\rightarrow$ large multiple scatt. for μ's;
- less bending for μ's at large η.

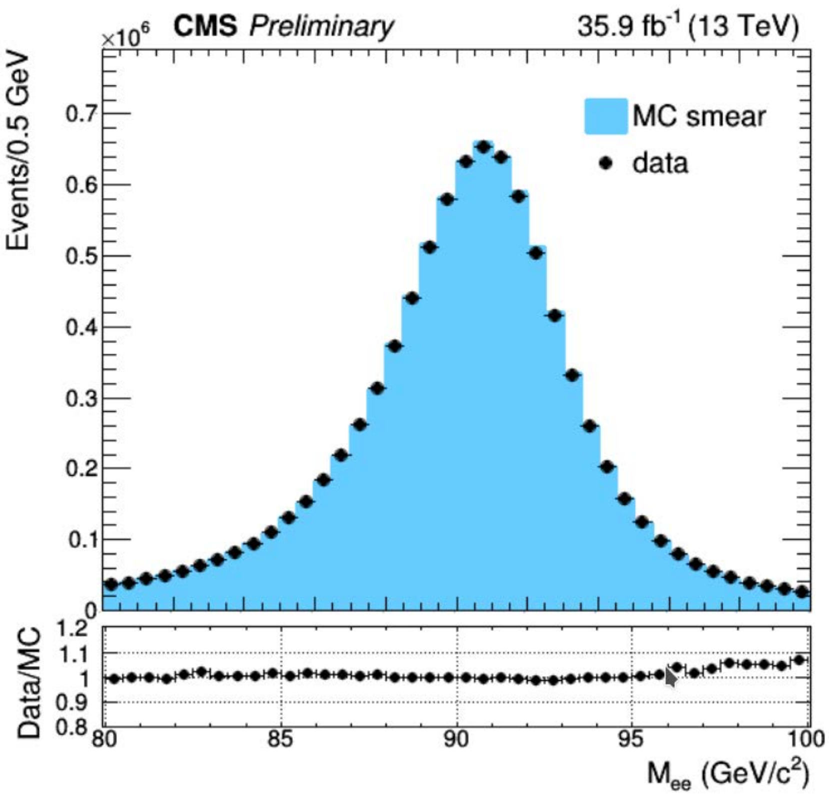
thanks to Anna Colaleo

Detector performances : $Z \rightarrow e^+e^-$

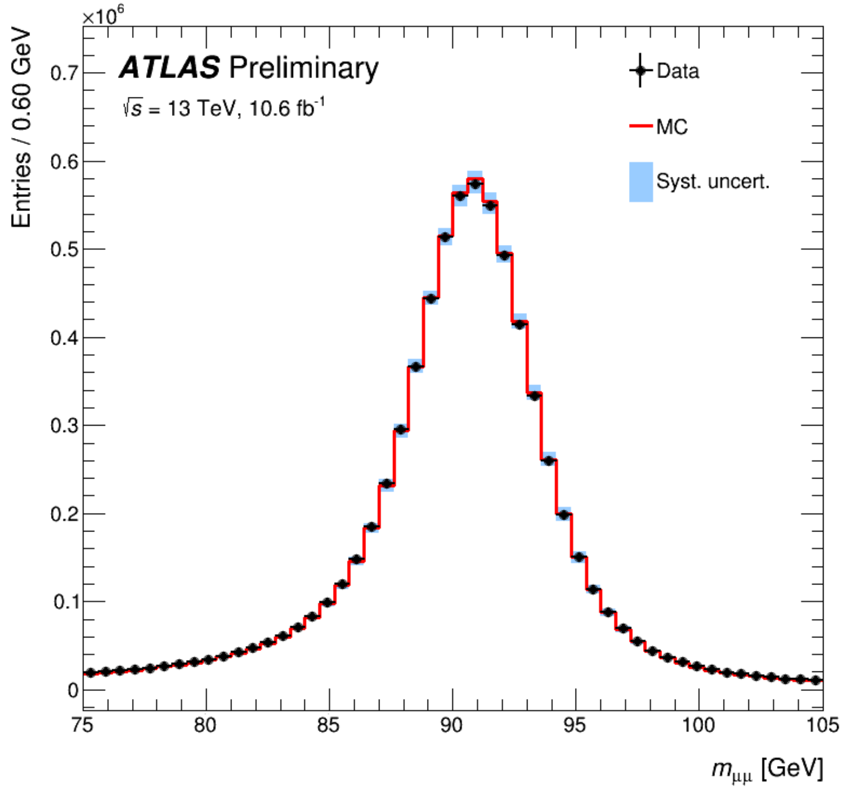


- the classic benchmark of tracker + e.m. calo.;
- no way to improve wrt LEP, used only for detector debug/calibration.

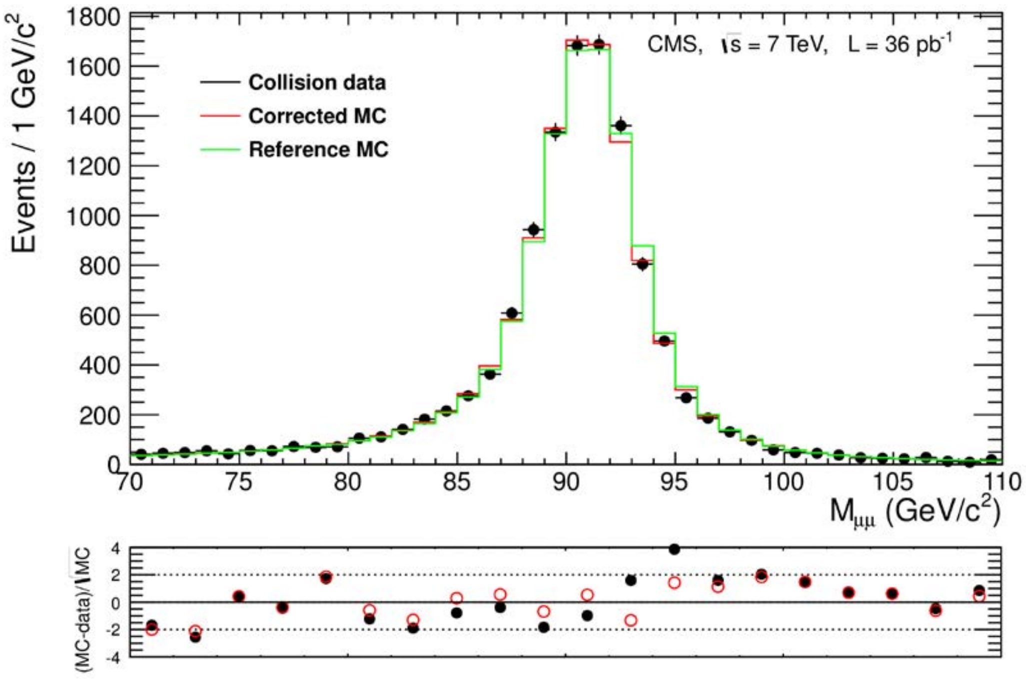




Detector performances : $Z \to \mu^+\mu^-$

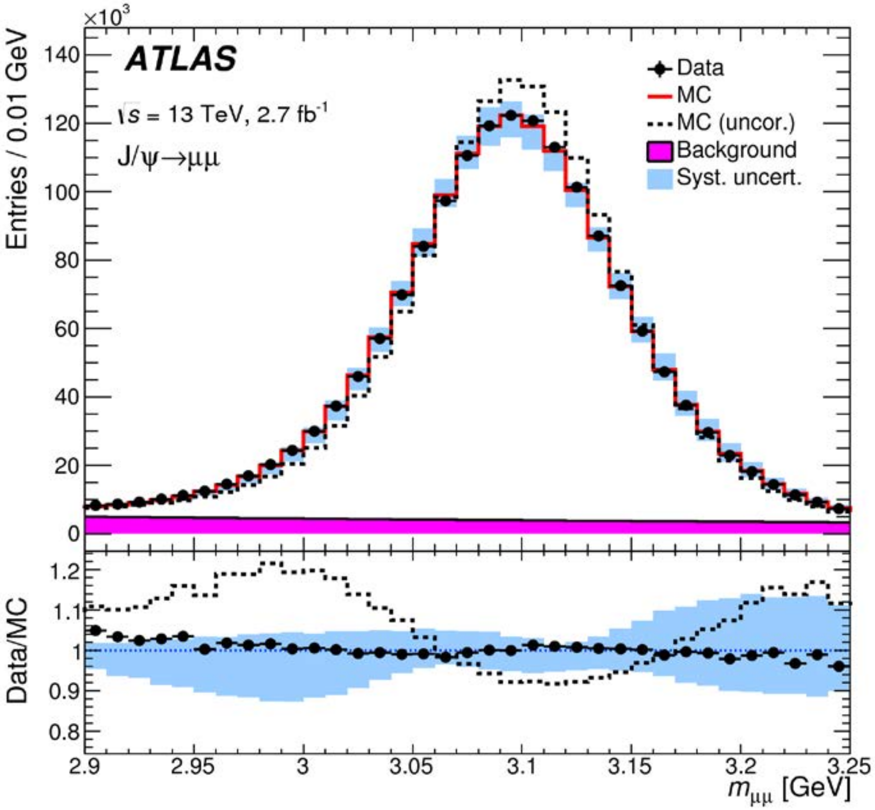


- the classic benchmark of tracker + muon chambers;
- [no way to improve wrt LEP, used only for detector debug/calibration.]



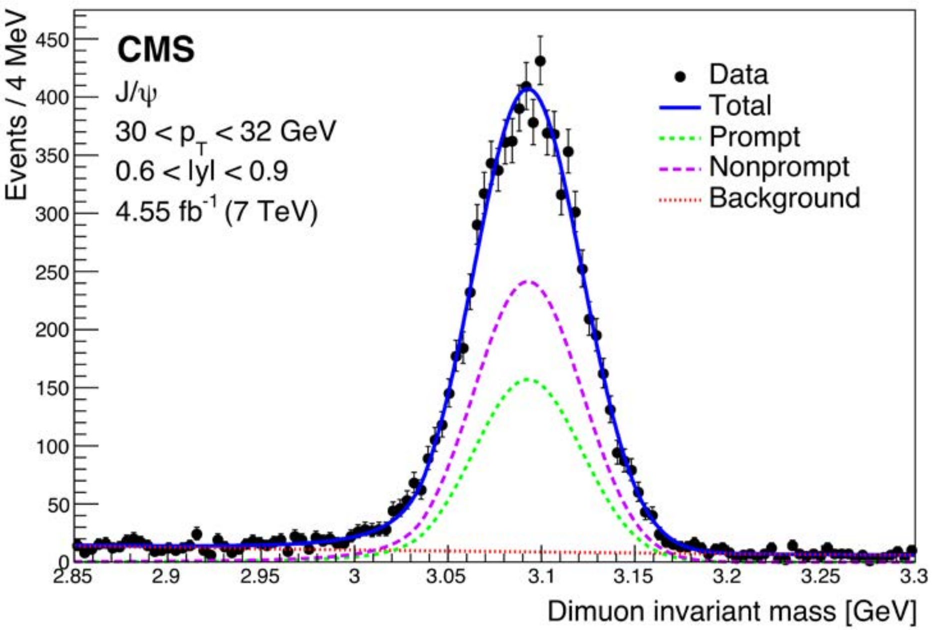
[time evolution of physics analyses : discovery → precision meas → detector study → background]

Detector performances : $J/\psi \to \mu^+\mu^-$

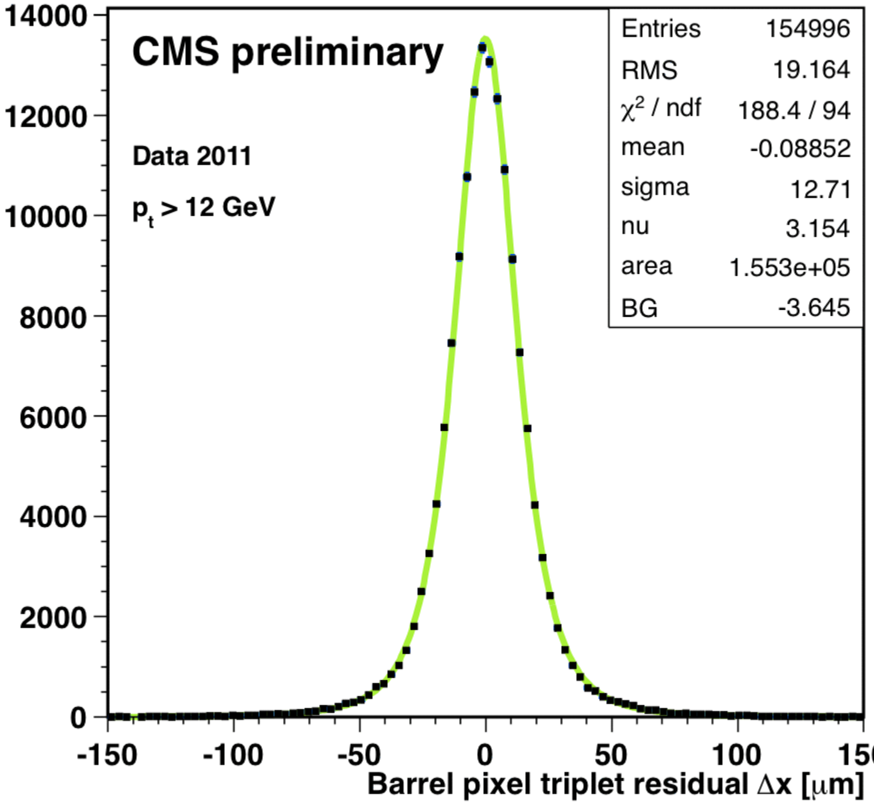


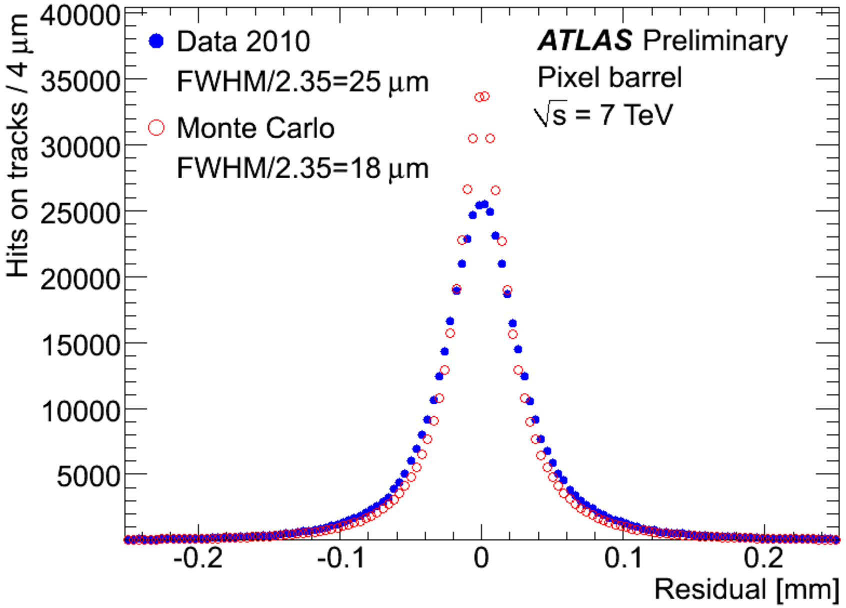
$Z \to \mu^+\mu^-$ and J/ψ $Z \to \mu^+\mu^-$ are ideal channels for μ studies :

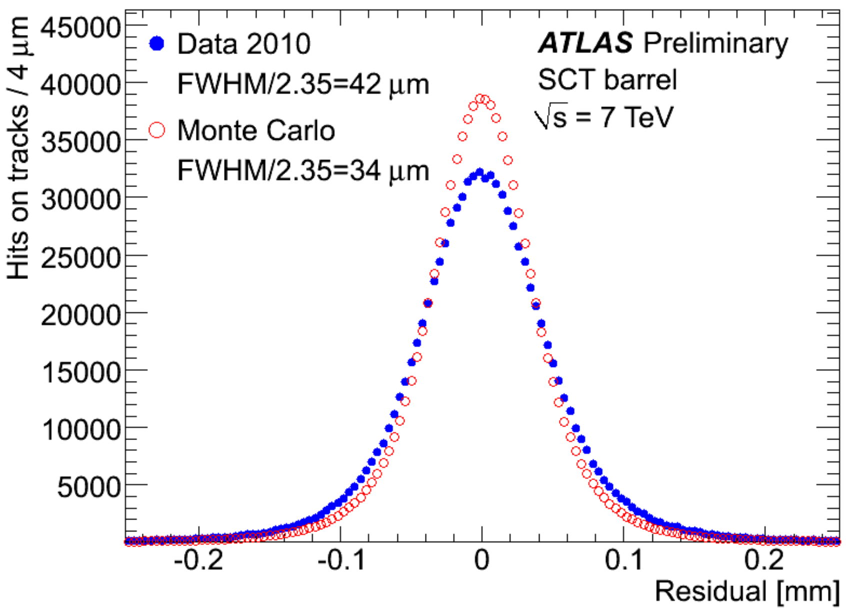
- inner detector + muon spectrometer;
- agreement (MC $\leftrightarrow$ data) $\to$ confidence in analysis (including errors !).



Detector performances : silicon trackers



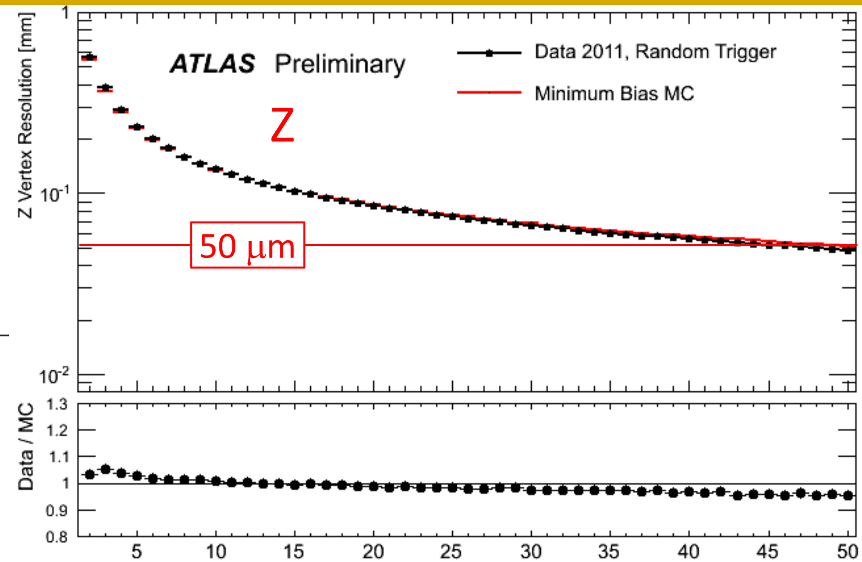


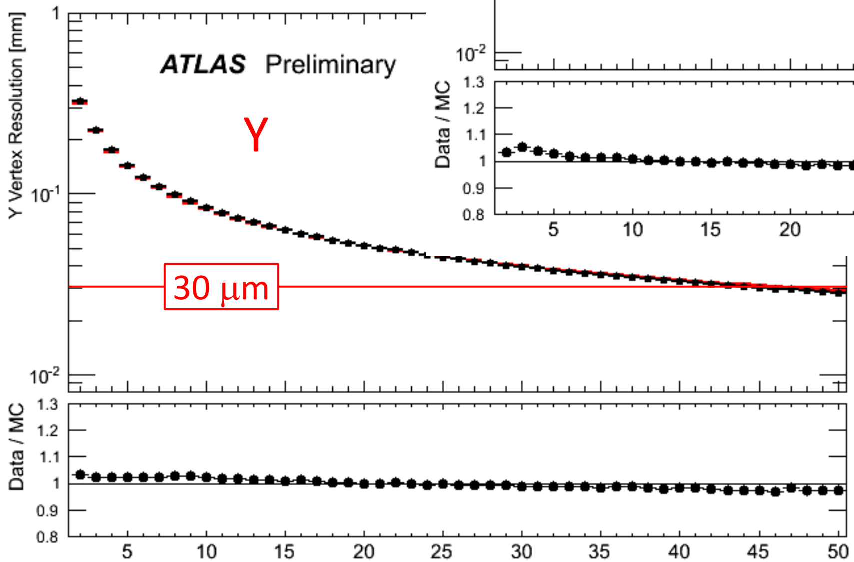


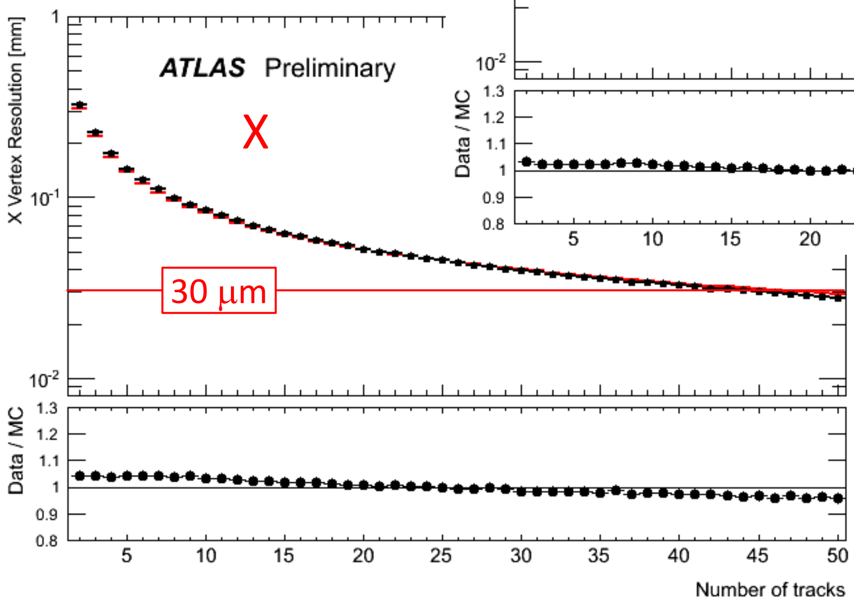
- resolution of few μm necessary for impact parameter → identification of secondary verteces → heavy flavors → higgs;
- agreement (MC ↔ data) → confidence in analysis (including errors !).

Detector performances : vertex resolution

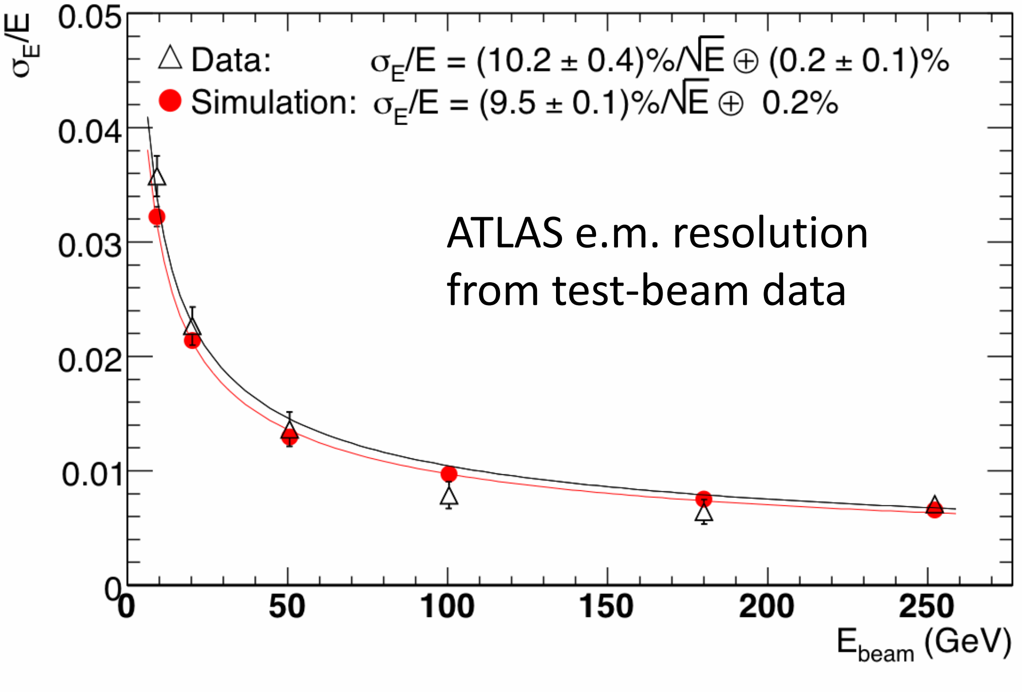
- Vertex resolution as a function of the its track multiplicity.
- Expect $\sigma \approx \sigma_0/\sqrt{N} \oplus \sigma_1$;
- σ_0 and σ_1 constants, different for xy (transverse) and z (beam axis).





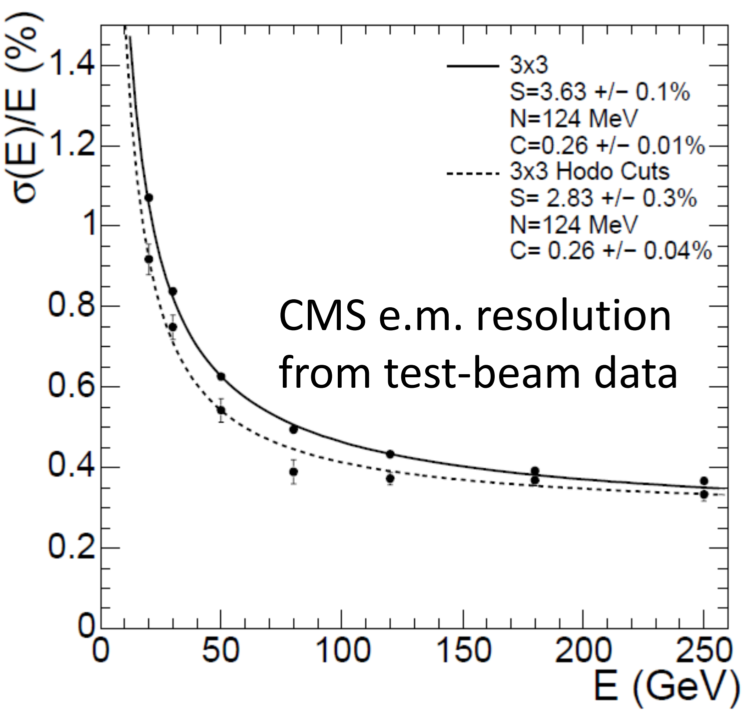


Detector performances : e.m. calo



→ test these expectations with real particles:

- Z (previous slides);
- π^0, η, ... (next slides)

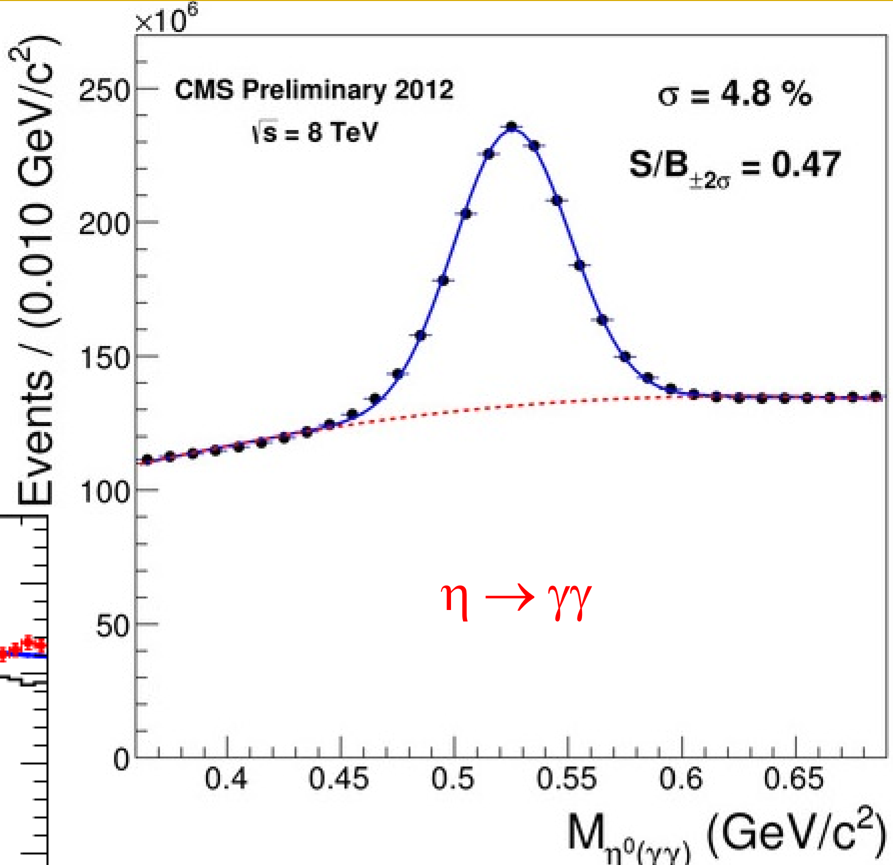


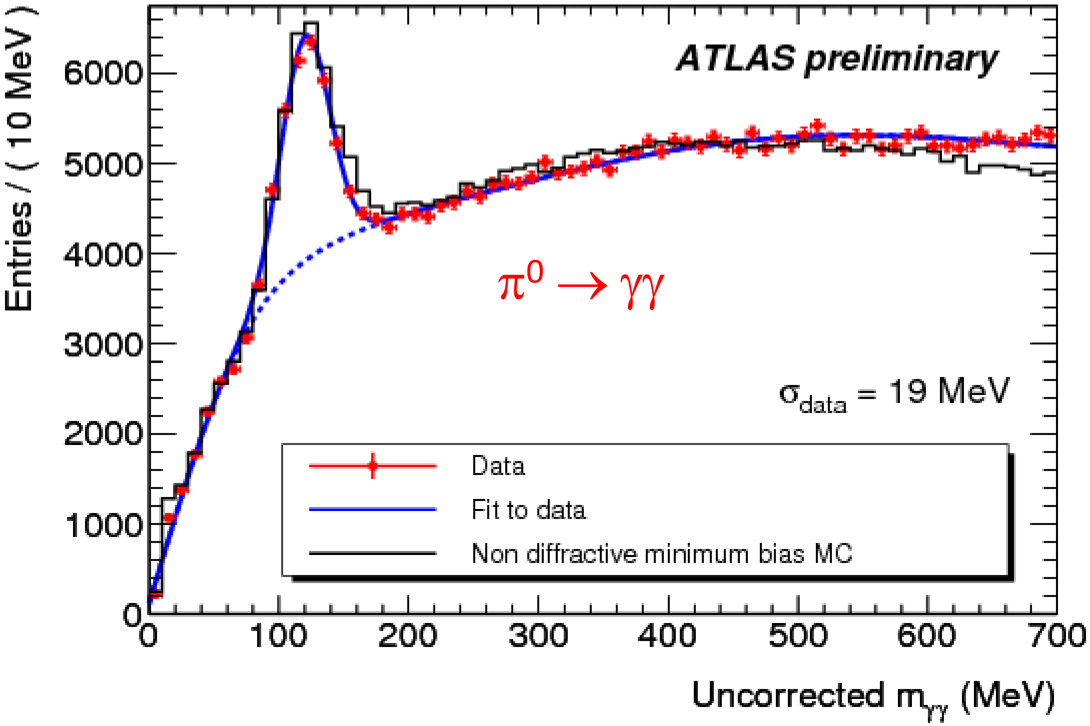
- although real life is somewhat different (sys from cell-to-cell calib, control of temperature, etc), test-beam results are impressive;
- expect $\sigma/E \approx \sigma_1/E \oplus \sigma_2 / \sqrt{E} \oplus \sigma_3$;
- σ_1 looks negligible, while σ_3 dominates at high E.

Detector performances : π^0, $\eta \to \gamma\gamma$

The π^0 and η widths are a measurement of the electro calo resolution in a difficult environment (inside jets or in high multiplicity events).

Notice the good (almost perfect) agreement with MC predictions.

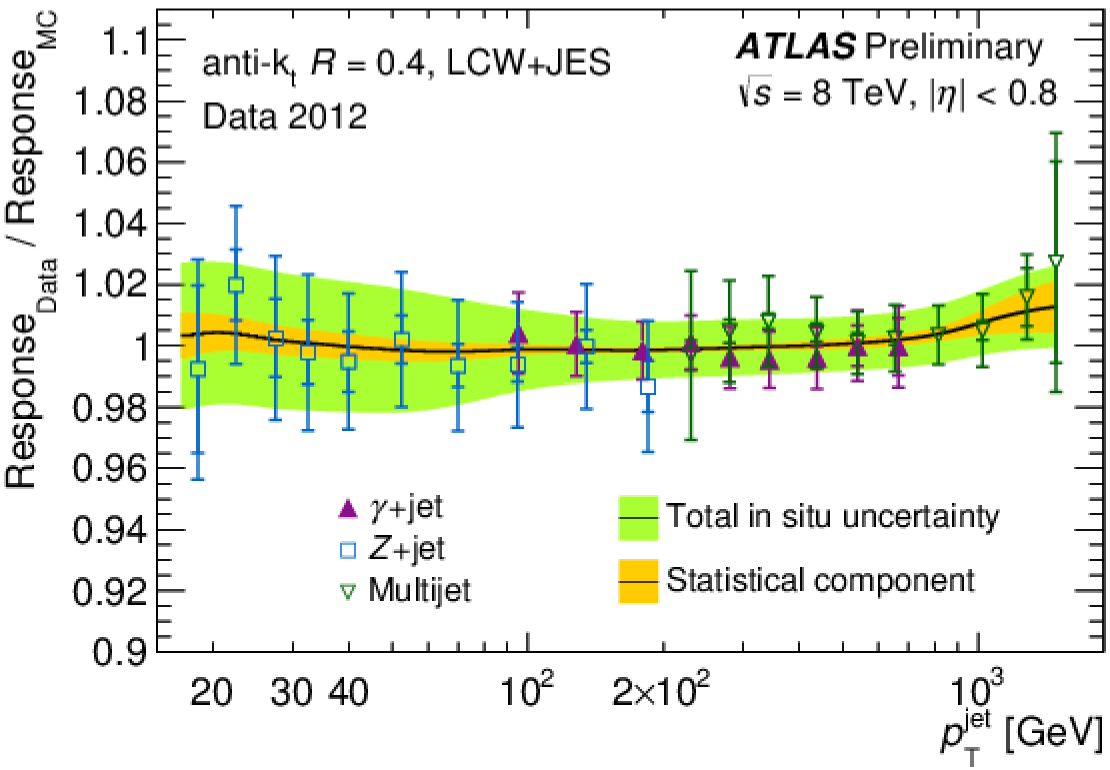




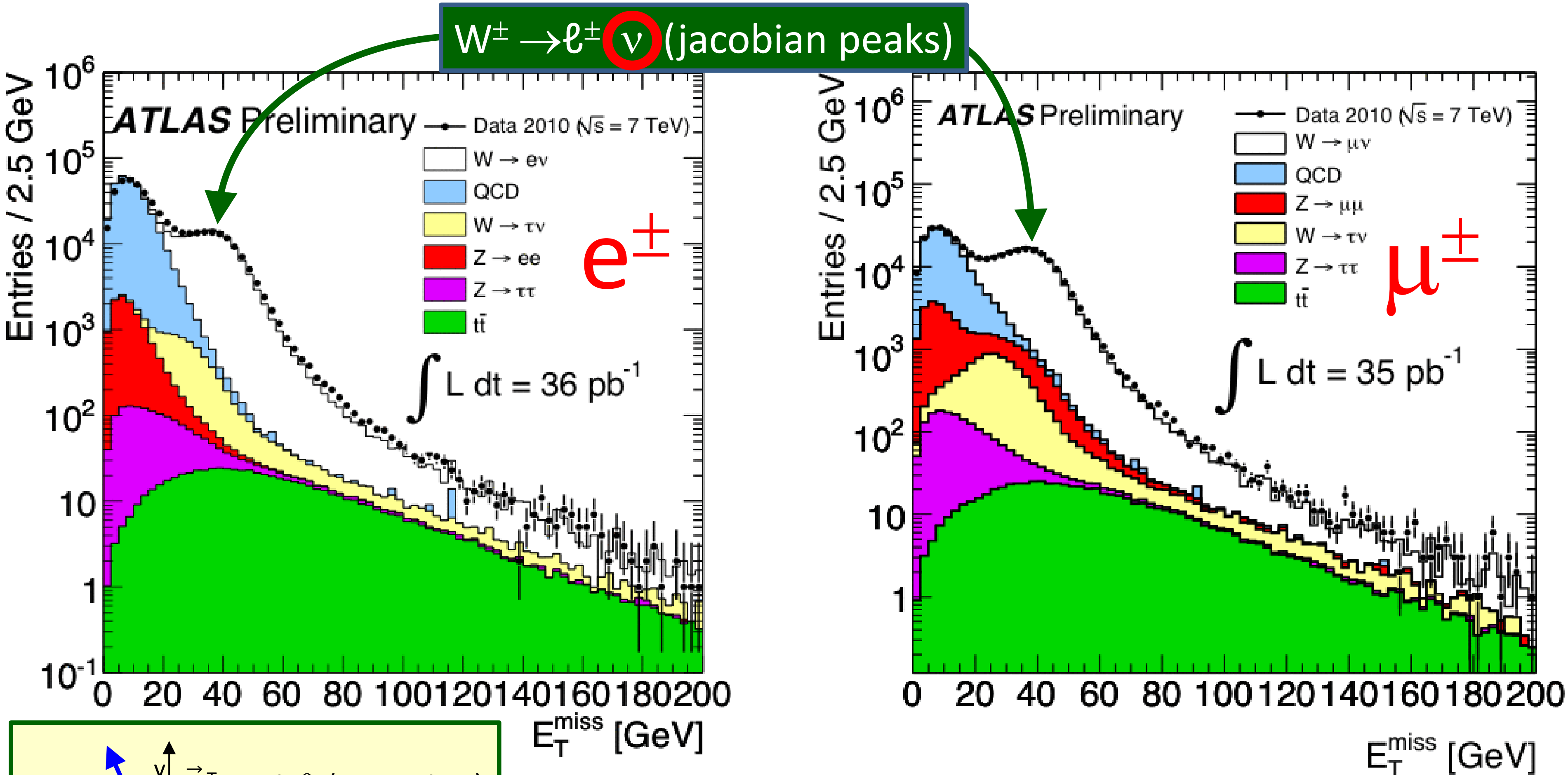
Detector performances : jet response

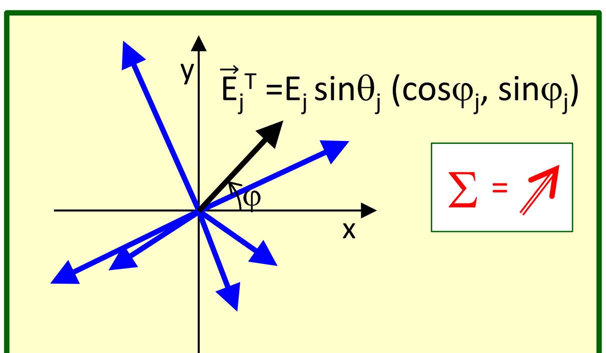
jet resolution as a function of p_T^{jet} :

- measured for different event types;
- stat and (mainly) syst uncertainty 2%, almost independent on p_T.



Detector performances : $\not{E}_T$



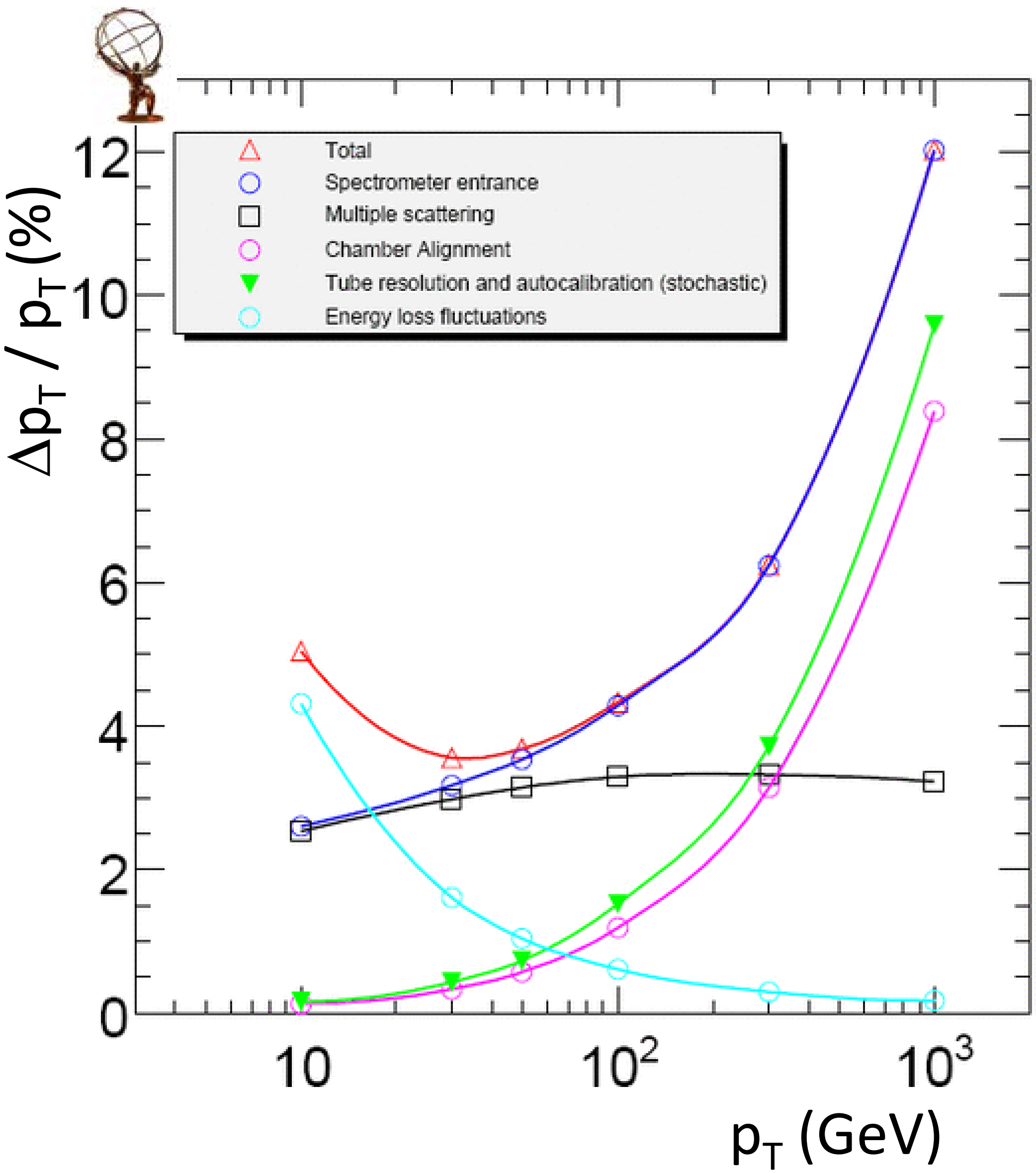


$\not{E}_T$ [= $|\sum \vec{E}_j^T|$] for events with $e^\pm$ or $\mu^\pm$ with p_T > 20 GeV :

- **great** agreement with predictions;

→ **reliable** measurement

Detector performances: ATLAS $\mu^{\pm}$



$\Delta p_T/p_T$ vs p_T [project, low η] :

▼ meas. error + calib ($\propto p_T$);

○ chamber alignment ($\propto p_T$);

□ multiple scattering ($\propto \approx$const);

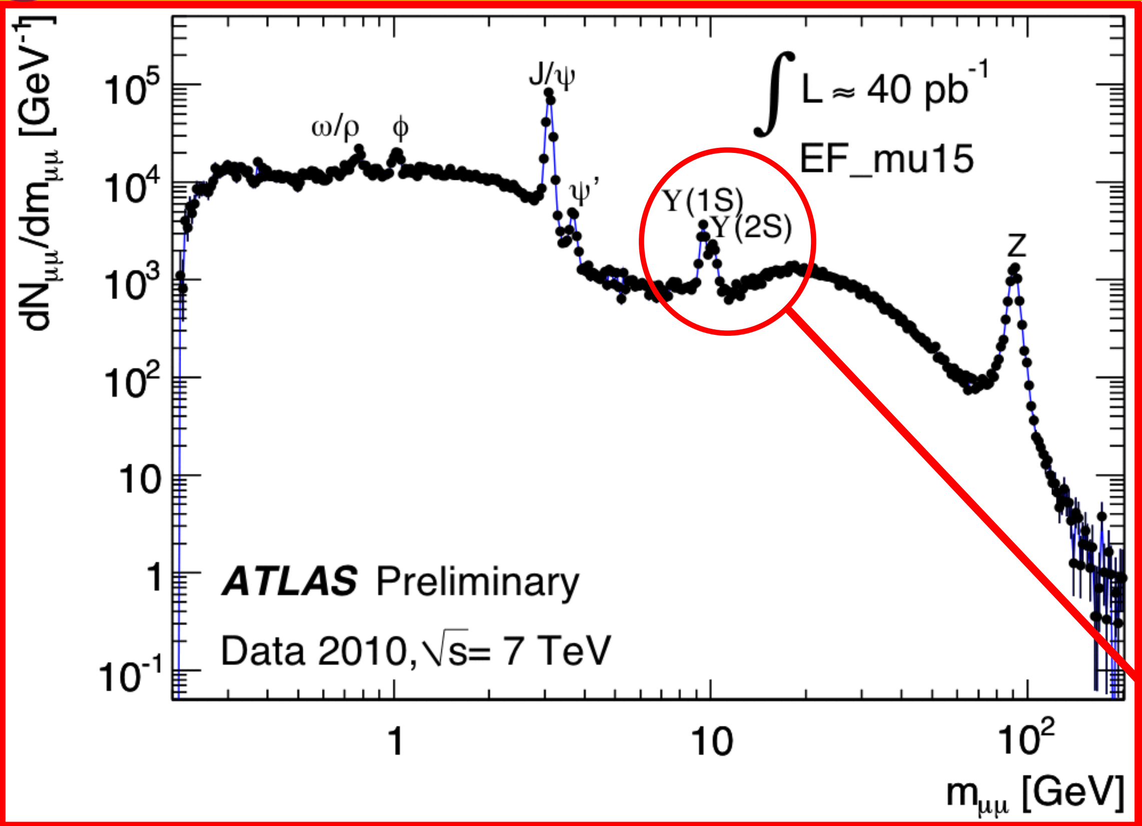
○ ΔE_μ(calo) fluctuations (tail at high loss measurable from brem shower);

○ at spectrometer entrance (= ▼ ⊕ ○ ⊕ □);

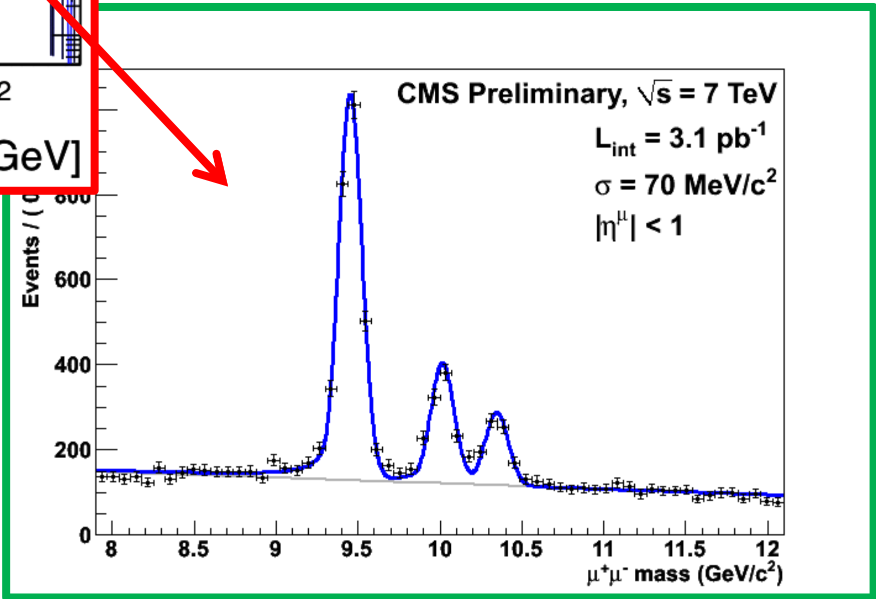
△ total at main vertex (= ○ ⊕ ○).

- at low p_T (p_T < 200 GeV) vtx extrapolation (○) and scattering (□) give the main contributions;
- at high p_T the accuracy of the spectrometer (▼⊕○) dominates;
- at fixed p_T and high η (not shown), Δp_T gets worse.

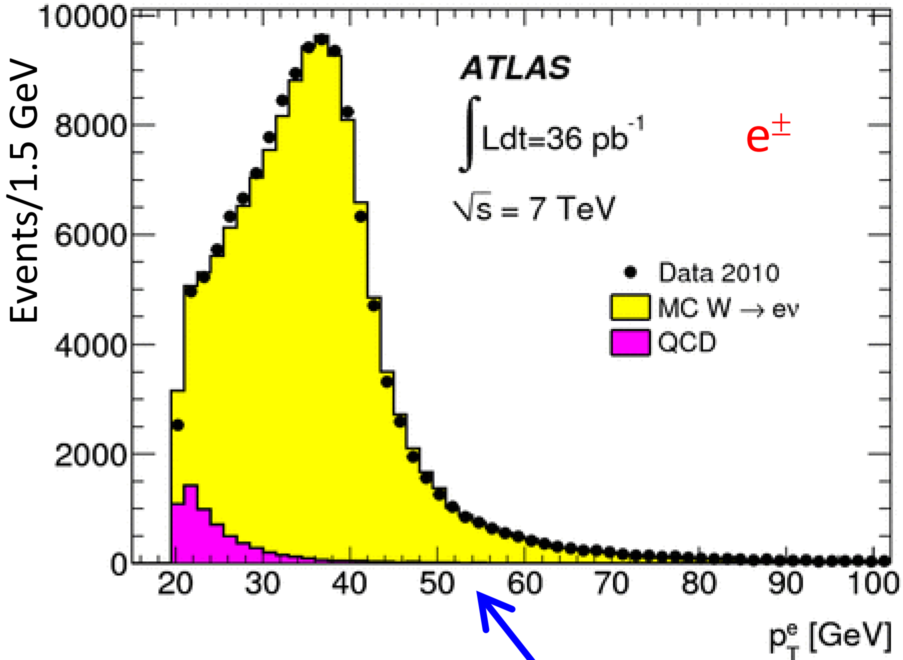
Detector performances : mass($\mu^+\mu^-$)

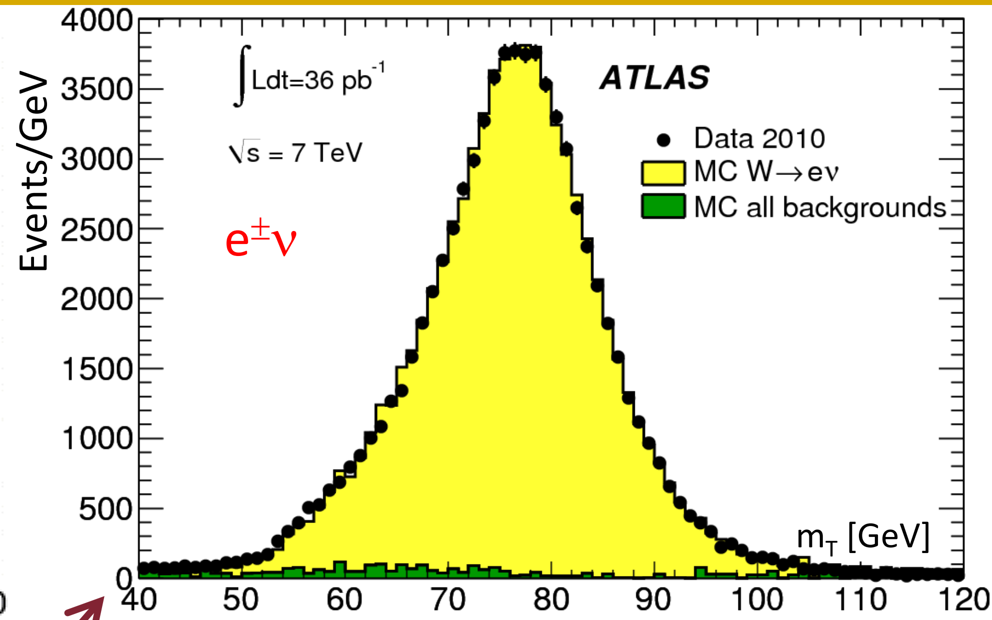


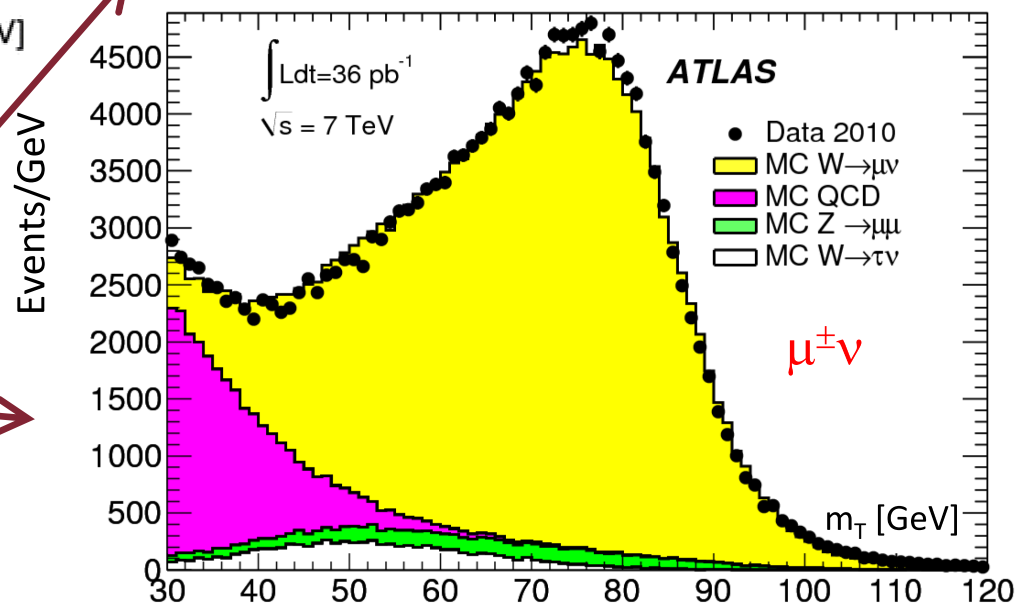
- at low $m_{\mu\mu}$, low trigger thresholds from low $\mathscr{L}$ runs;
- also a technical challenge.



Detector performances: $W^{\pm} \to \ell^{\pm}\nu$



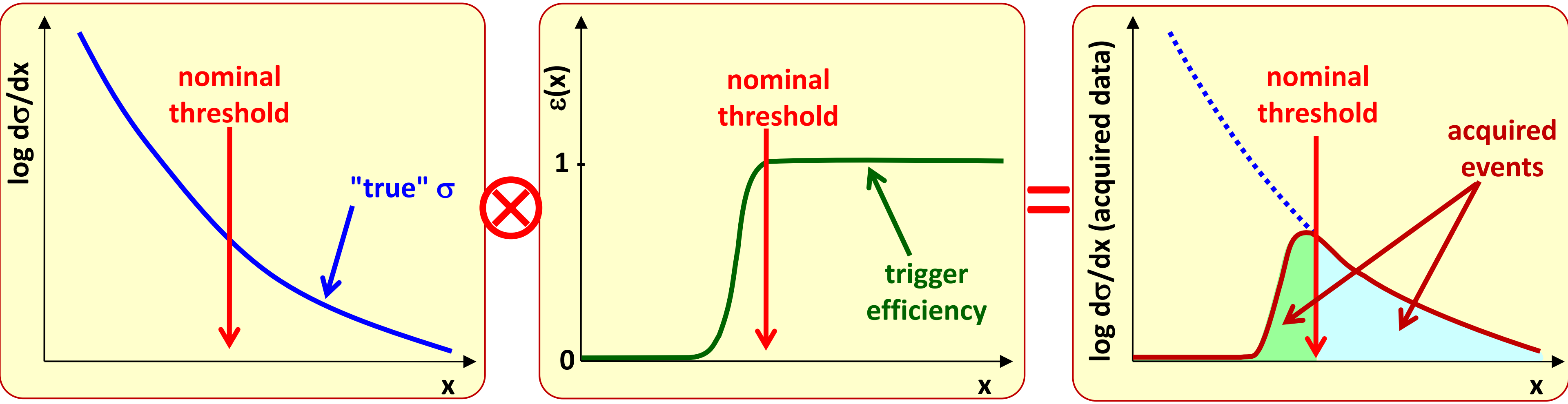




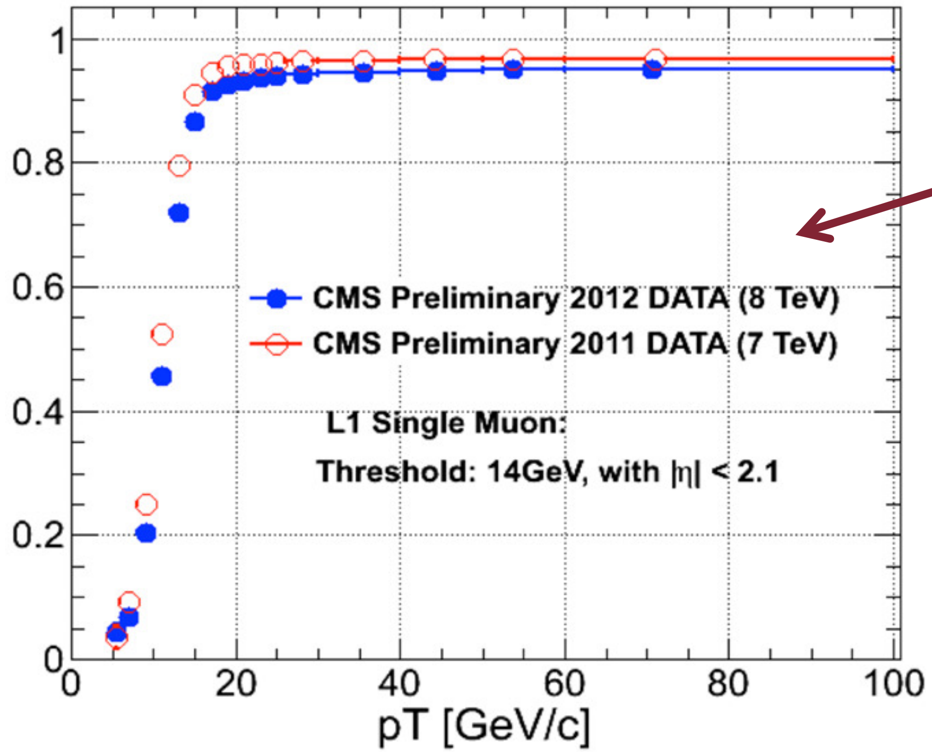
- the jacobian peak of the $e^{\pm}$;
- the transverse mass for $e^{\pm}\nu$ and $\mu^{\pm}\nu$.

Detector performances: trigger thresholds

- e^+e^- : small cross section → [R = $\mathscr{L}\sigma$ ≈ few Hz] → event trigger, i.e. trigger on single bunch crossing, if it contains an event candidate; @ LEP, $1-\varepsilon \approx 10^{-3}$, negligible dead time;
- $pp(\bar{p}p)$: high hadronic total cross section → [R = $\mathscr{L}\sigma \approx 10^6$ - 10^9 Hz] → rates too big (and uninteresting events) → physics trigger, i.e. select a (tiny) fraction of events, which exhibit peculiar characteristics (i.e. high-p_T, multileptons, high $\not{E}_T$ ...); use cuts (i.e. thresholds), user defined in kinematical variables;
- the thresholds are applied on a kinematical variable "x" (e.g. p_T^{lepton}), measured in a rough and fast way by the trigger detector(s); therefore the experimenters have to compromise among rejection, efficiency, dead time, bandwidth ... and physics.

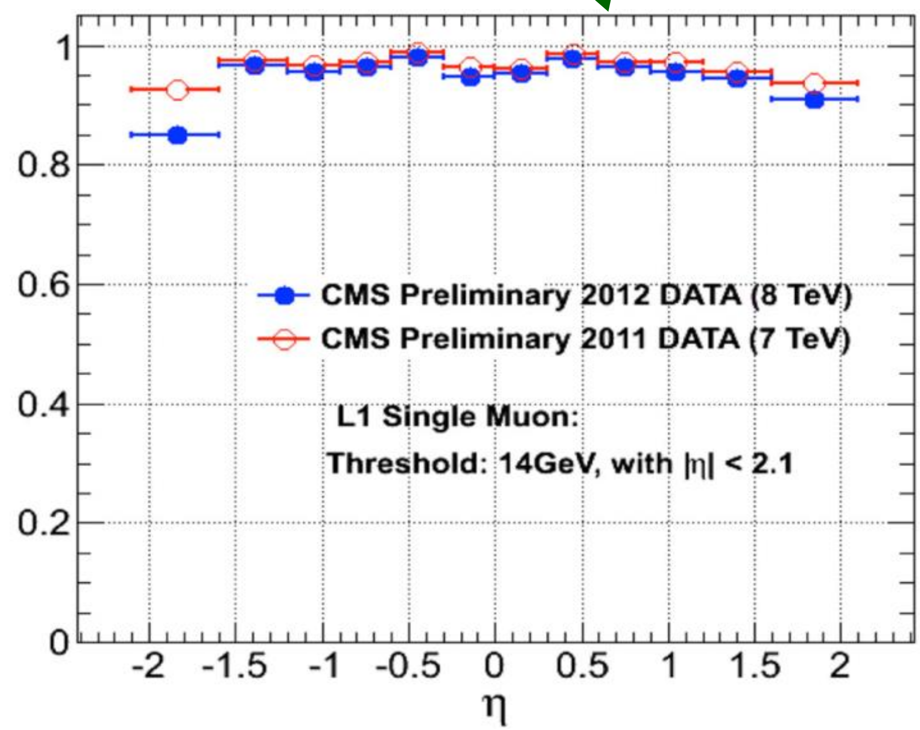


Detector performances: μ-trigger lvl-1



Efficiency ε at level 1 :

- vs p_T : notice the "size" of the threshold;
- vs η, integrated for p_T > 14 GeV : notice the flatness.



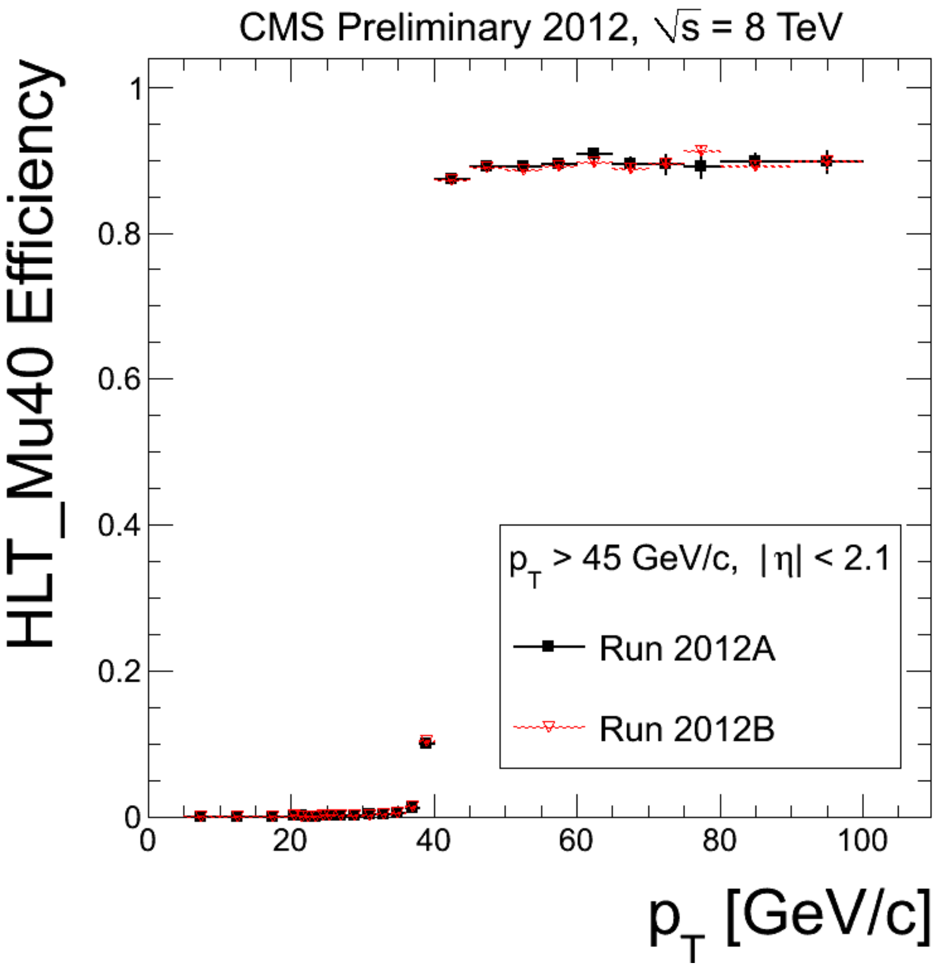
NB the effective yield $N_{obs} = N_{produced} * \varepsilon$:

- the bulk of the data is near p_T threshold
- ... where ε is varying;
- ... and the physics less interesting.

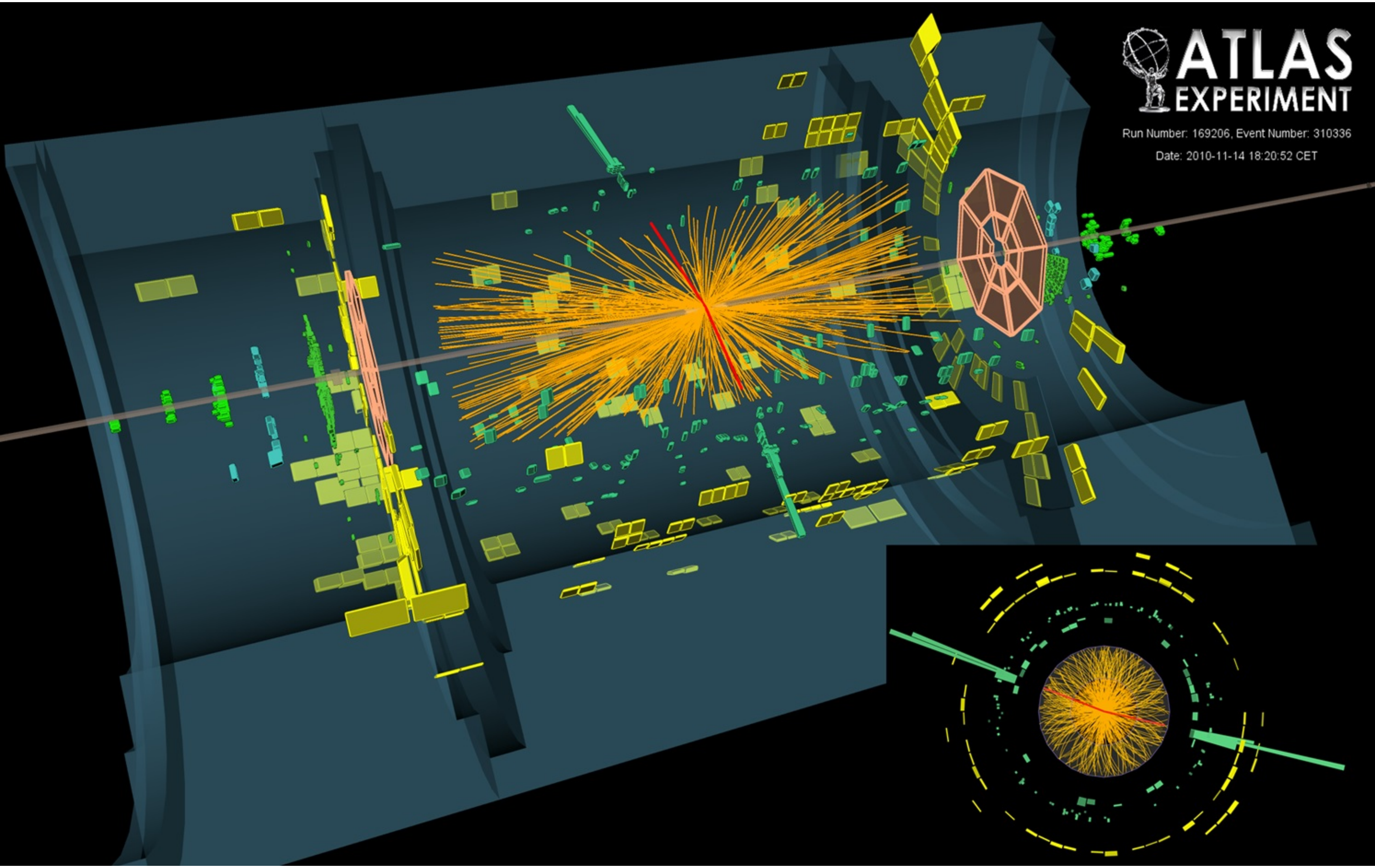
Detector performances: μ-trigger HLT

Efficiency ε vs p_T at the highest trigger level (HLT):

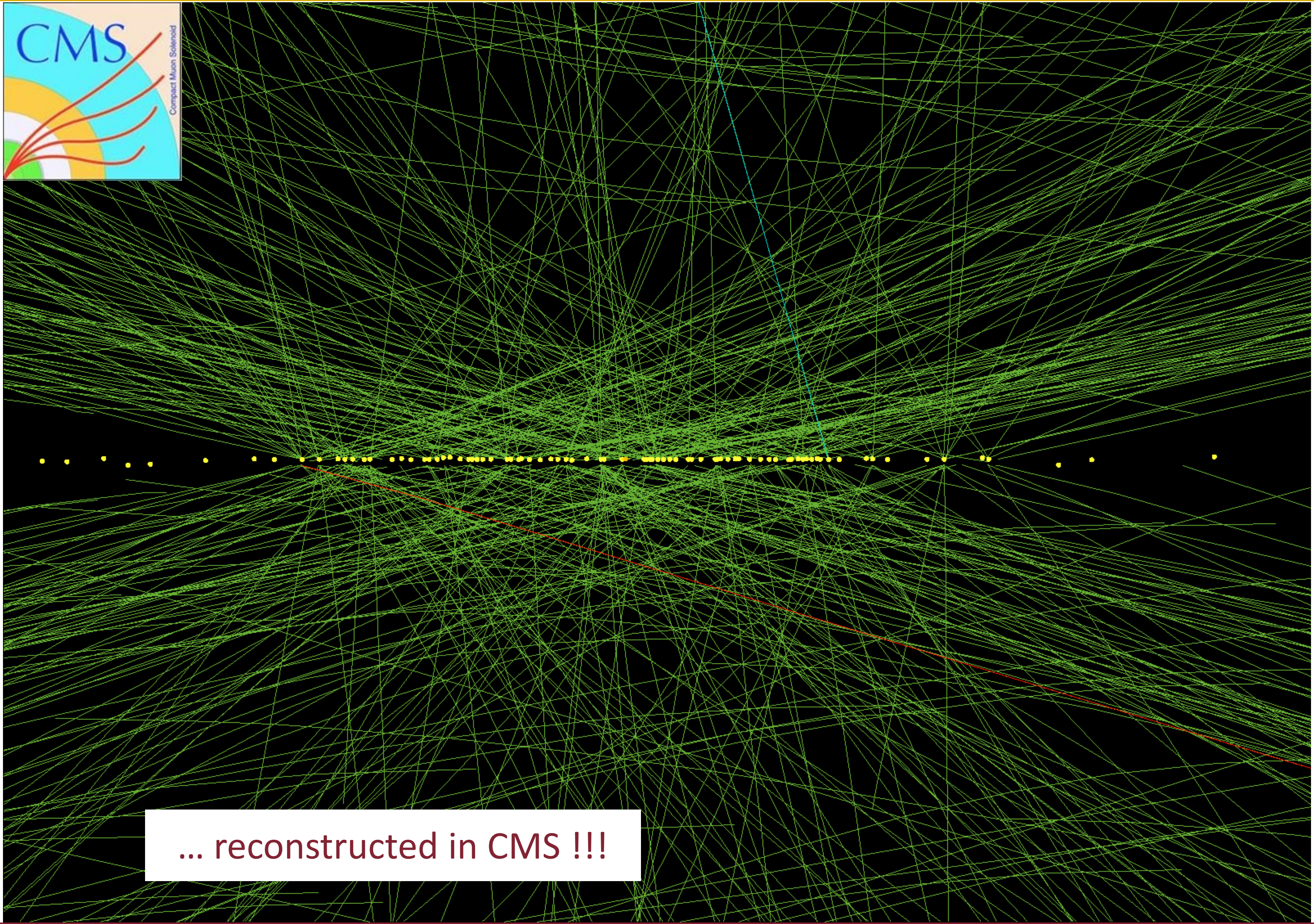
- notice the sharper "size" of the threshold (→ less useless data);
- … at the price of a much higher threshold (→ no recovery of events lost in lvl1);
- … with the advantage of (much) smaller rates :
  O(10 KHz) @ lvl-1 → *O*(10 Hz).



LHC events : Pb Pb $\rightarrow$ Z X $\rightarrow$ e^+e^- X

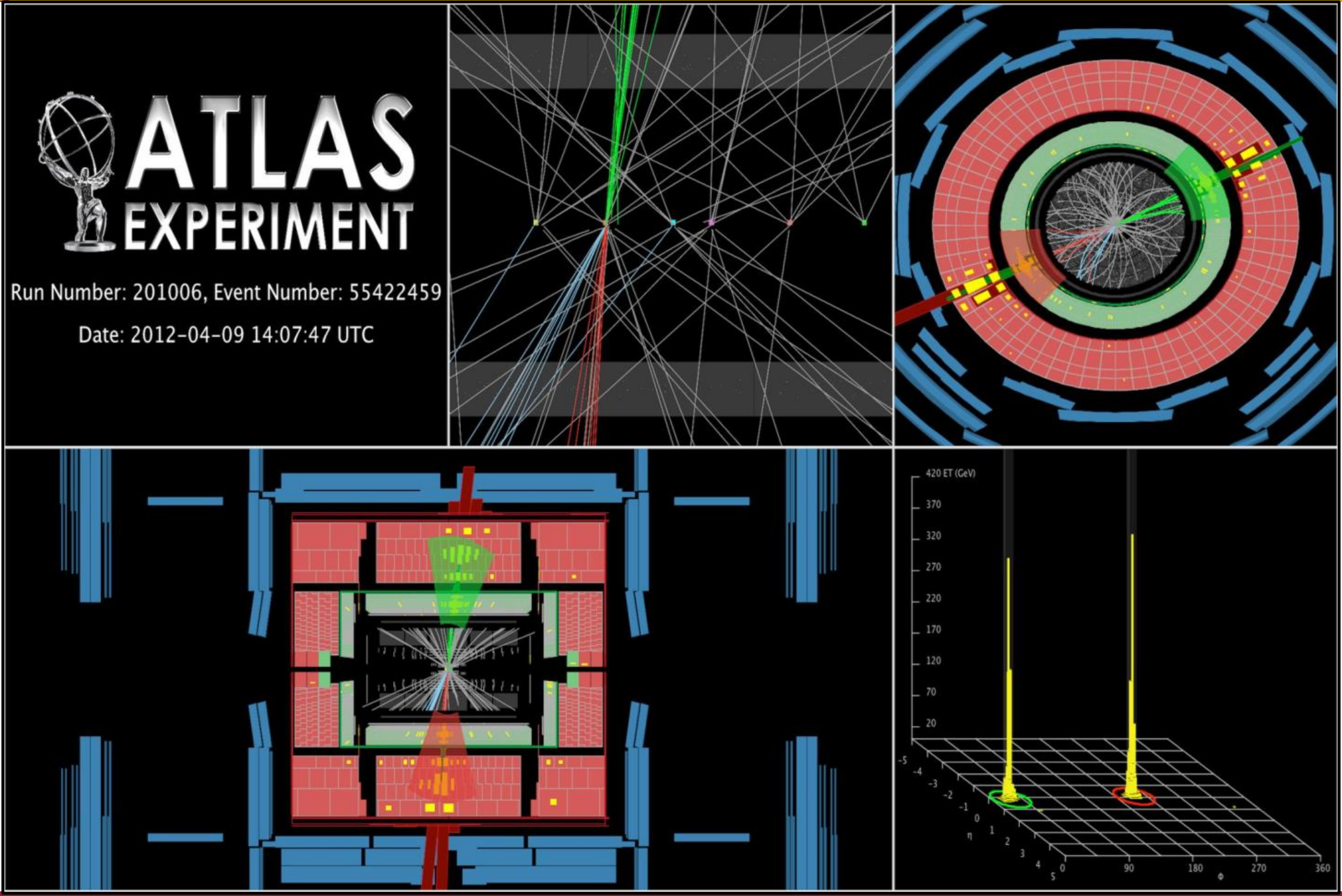


LHC events : 78 primary interactions

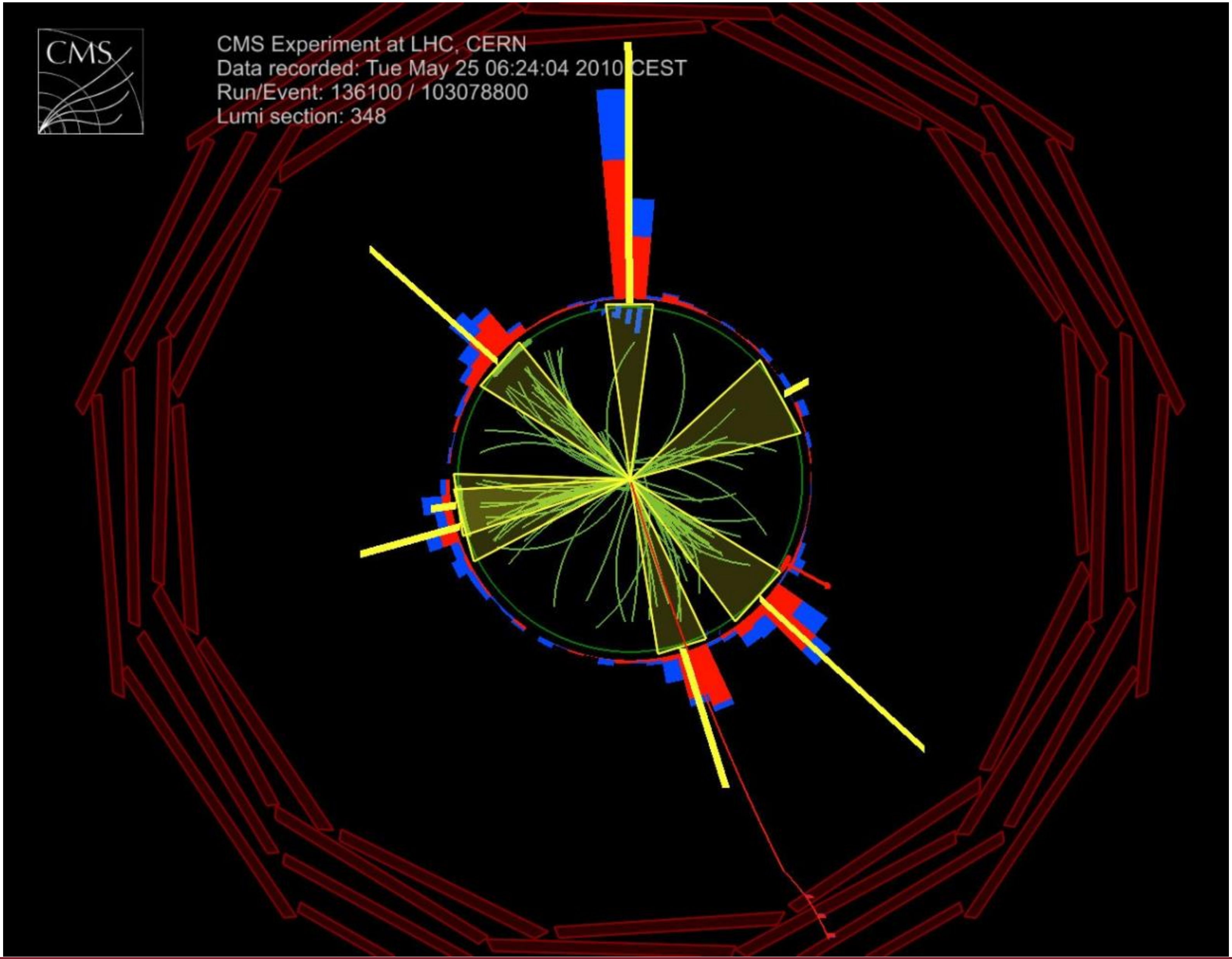


… reconstructed in CMS !!!

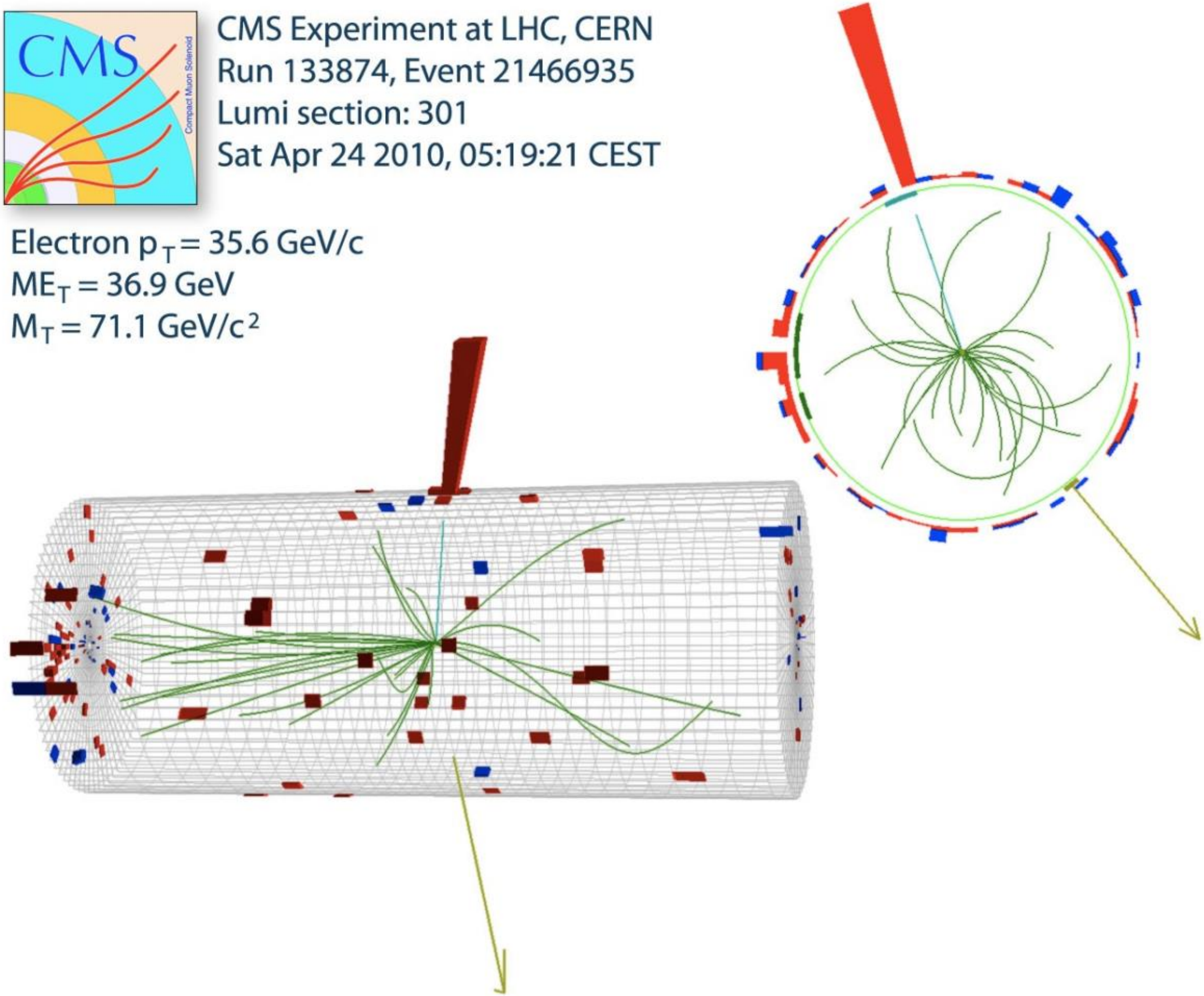
LHC events : 2 jets, $p_T \approx 2$ TeV



LHC events : a multijet event



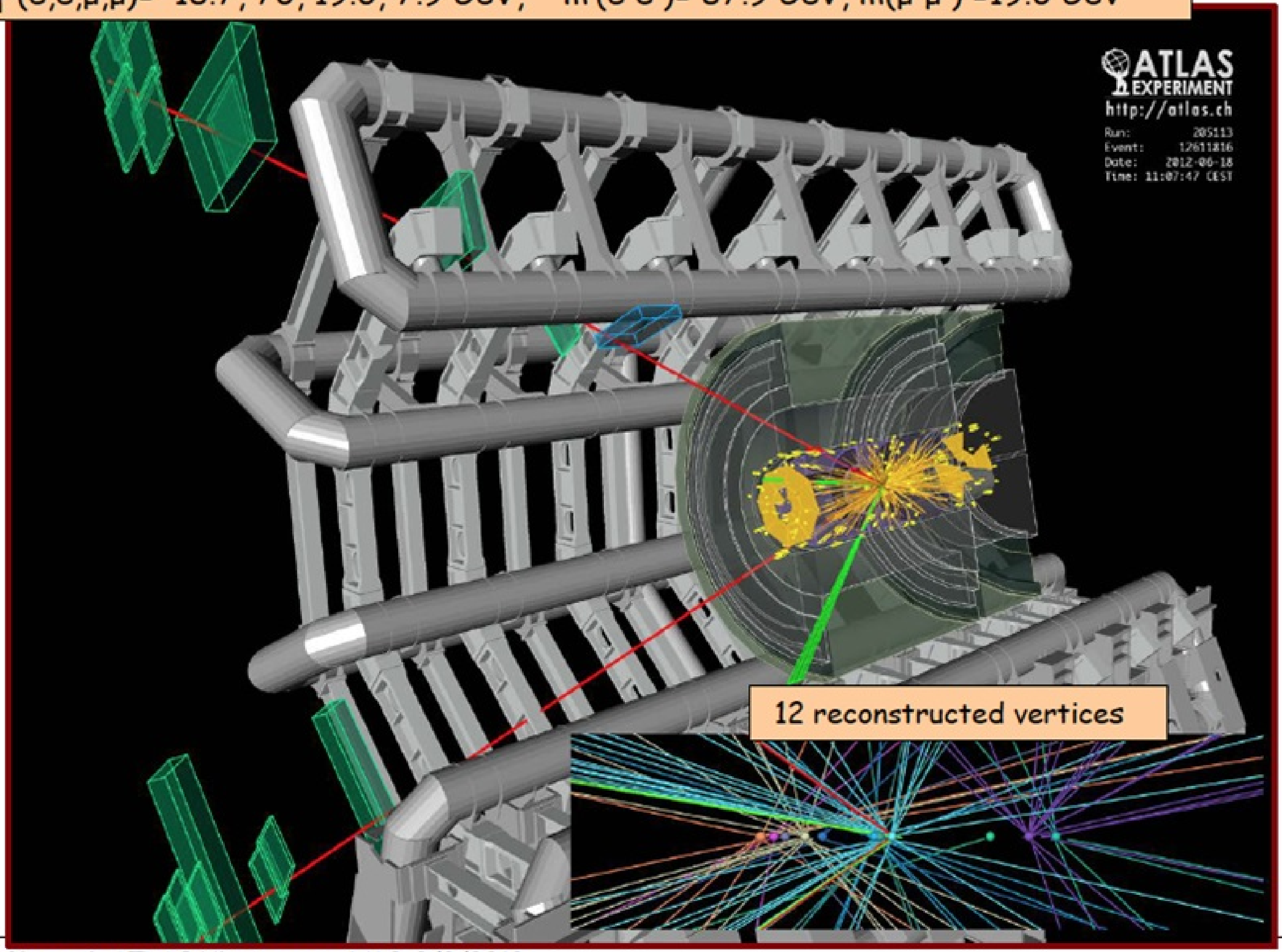
LHC events : W $\rightarrow$ eν



LHC events : $H \to ZZ^* \to (e^+e^-)(\mu^+\mu^-)^*$

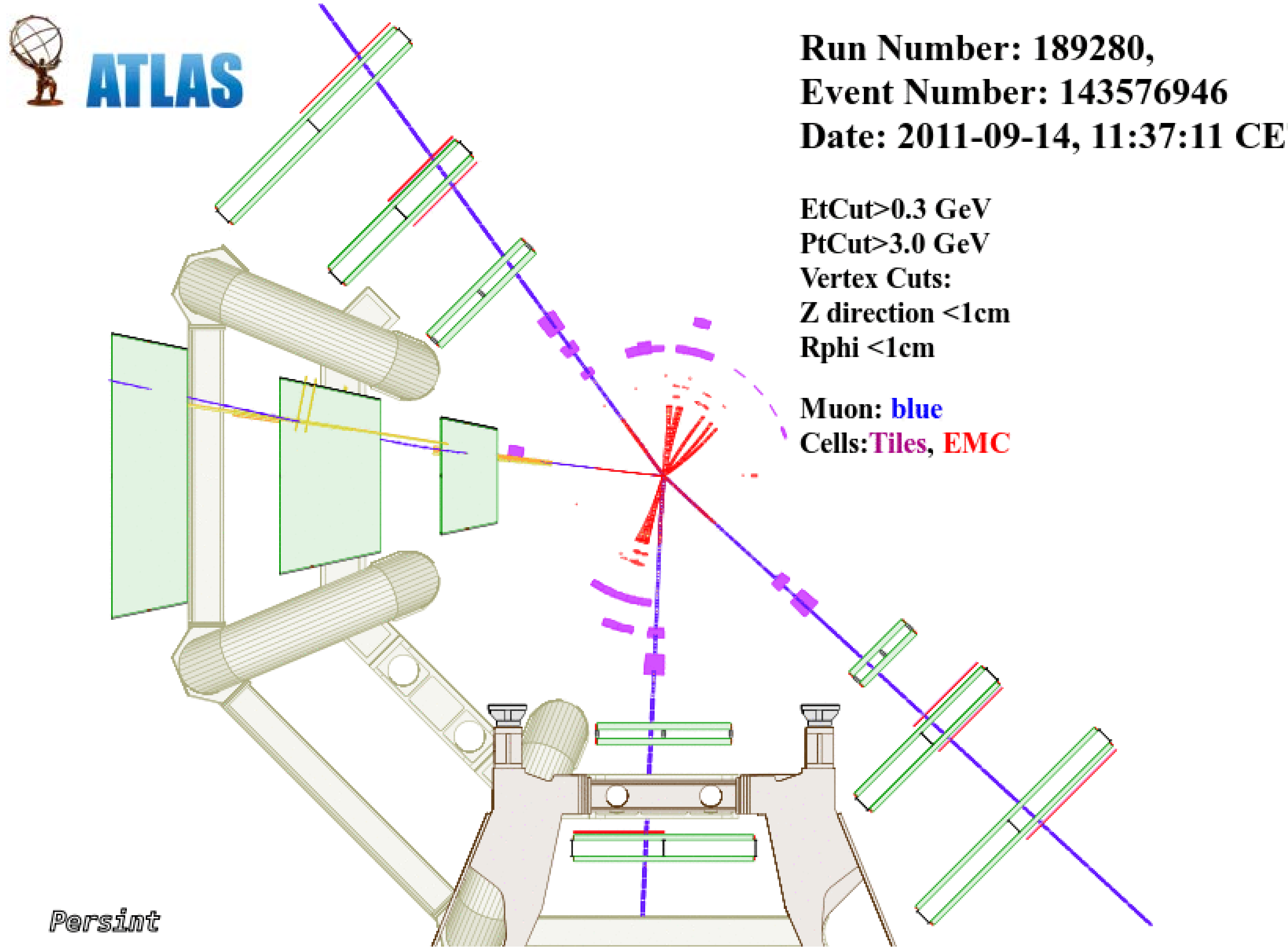
2e2μ candidate with $m_{2e2\mu}$= 123.9 GeV

p_T (e,e,μ,μ)= 18.7, 76, 19.6, 7.9 GeV, m (e^+e^-)= 87.9 GeV, m($\mu^+\mu^-$) =19.6 GeV

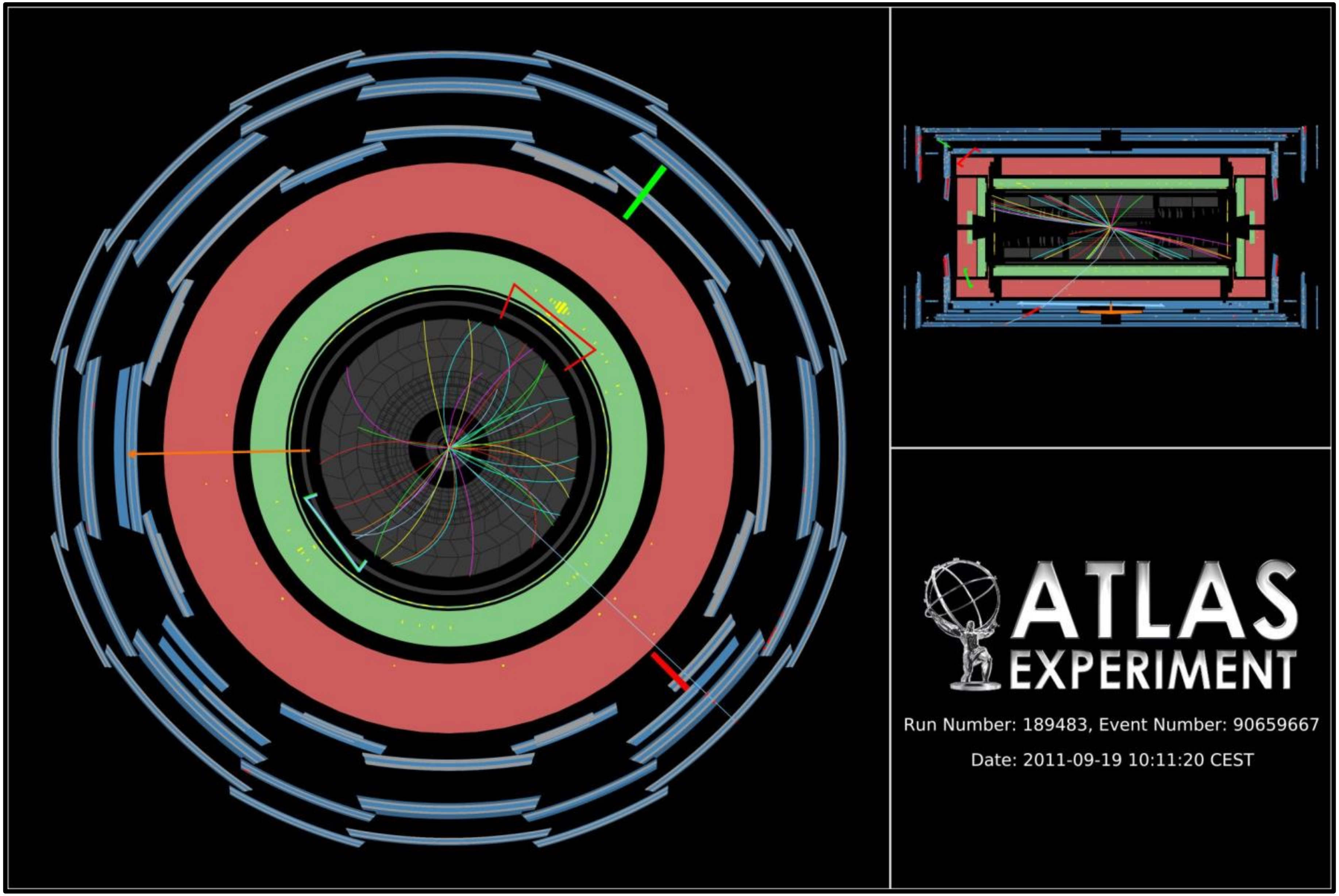


F. Gianotti, ATLAS Higgs paper, LMC, 8/8/2012

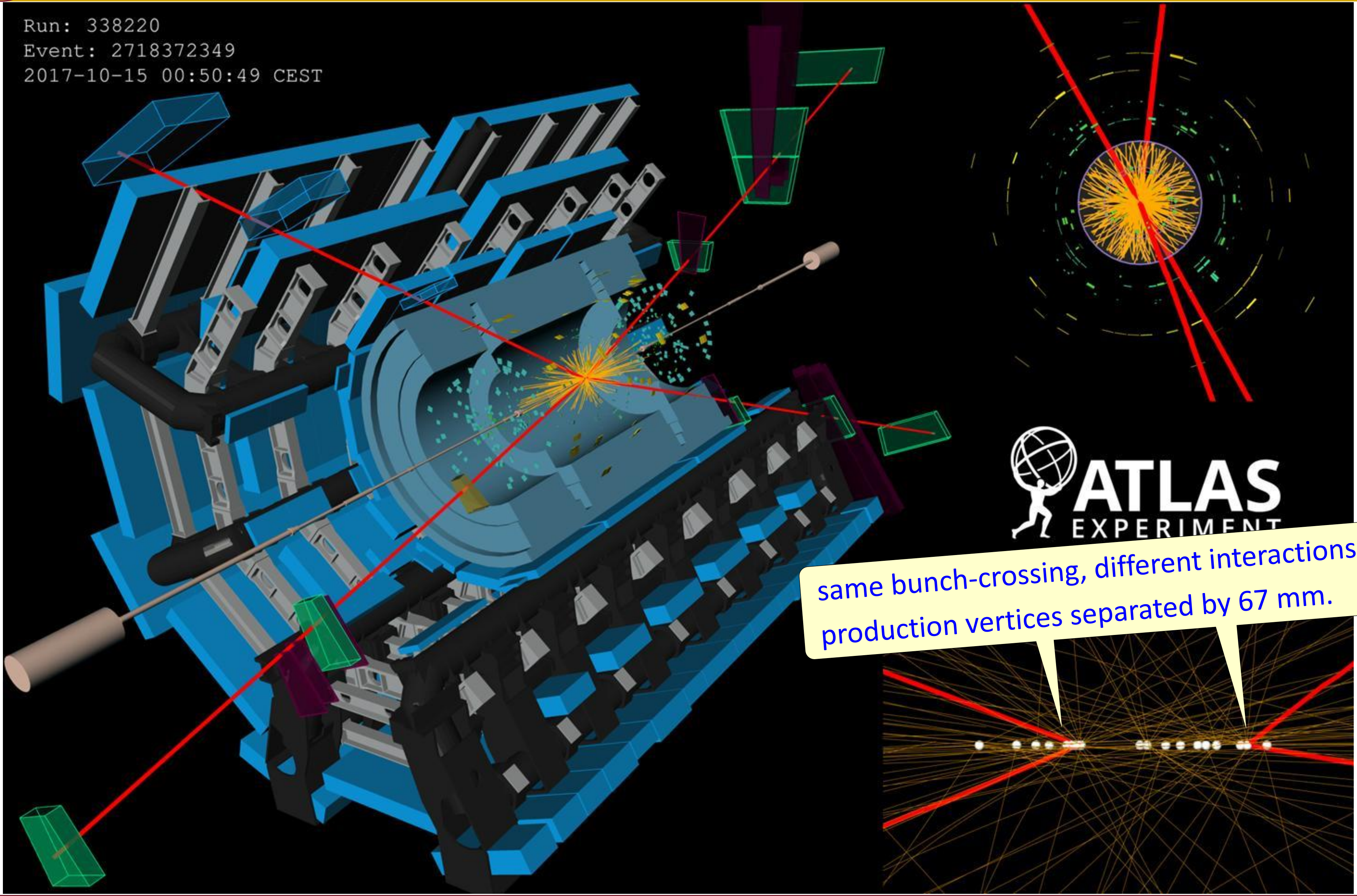
LHC events : $H \rightarrow ZZ^* \rightarrow (\mu^+\mu^-)(\mu^+\mu^-)^*$



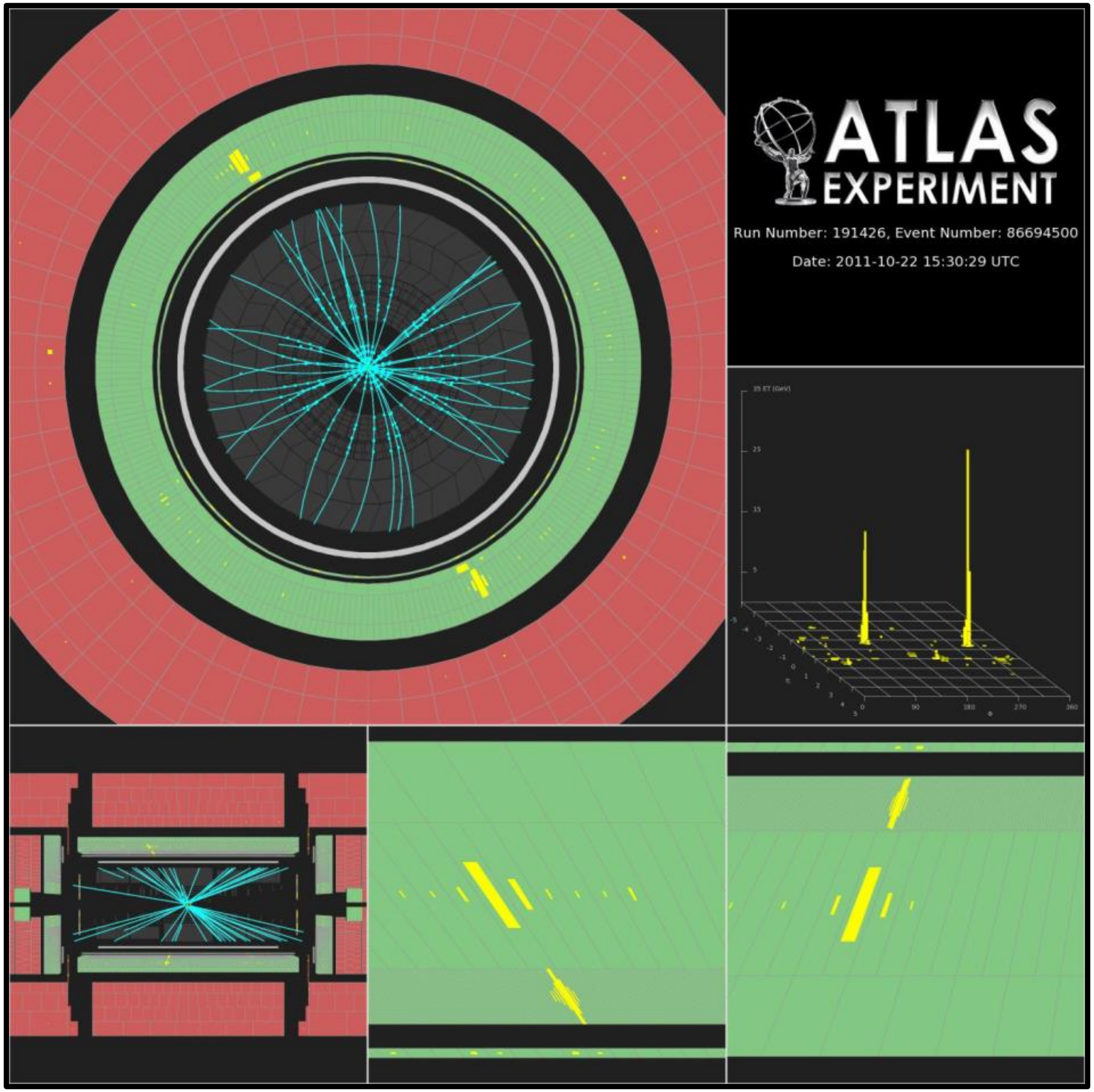
LHC events : $H \to W^+W^- \to e^+\nu\mu^-\nu$



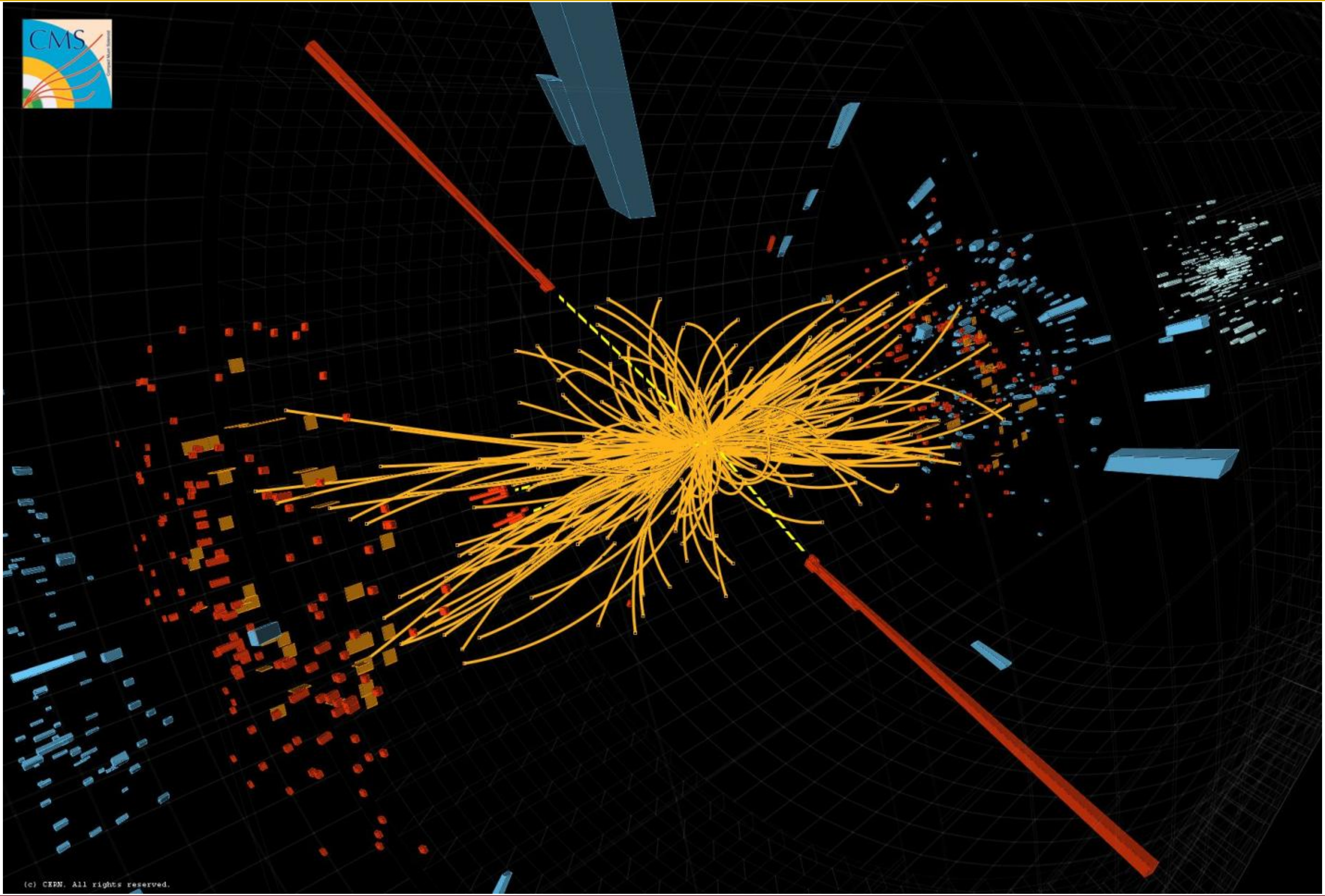
LHC events : $Z \to \mu^+\mu^-$, $Z \to \mu^+\mu^-$



LHC events : H $\rightarrow \gamma\gamma$



LHC events : H $\rightarrow \gamma\gamma$



References: collider & experiments

1. LHC : JINST 3 (2008) S08001.
2. LHC : L.Evans, Ann. Rev. Nucl. Part. Sci. 2011. 61:435–66.
3. LHC (recent) : J. Wenninger, PoS (Charged 2018) 001.
4. ATLAS detector : JINST 3 (2008) S08003.
5. ATLAS events : https://twiki.cern.ch/twiki/bin/view/AtlasPublic/EventDisplayPublicResults/
6. CMS detector : JINST 3 (2008) S08004.
7. CMS events : https://cdsweb.cern.ch/
8. [*see also references on results*]

Jacopo Robusti (Tintoretto) - The Forge of Vulcan, ca. 1578
Palazzo Ducale - Venice

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End of chapter 12

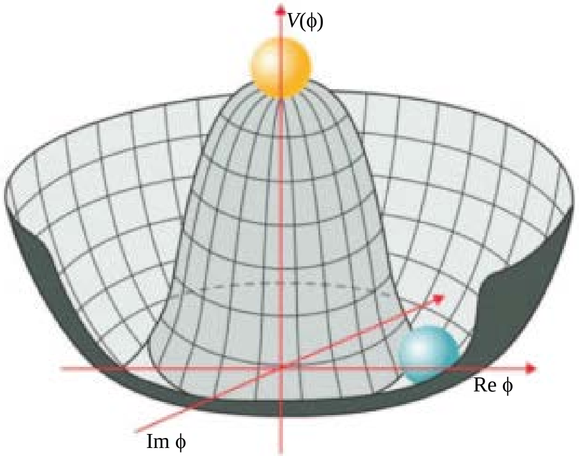
Particle Physics - Chapter 12b
LHC – Higgs discovery

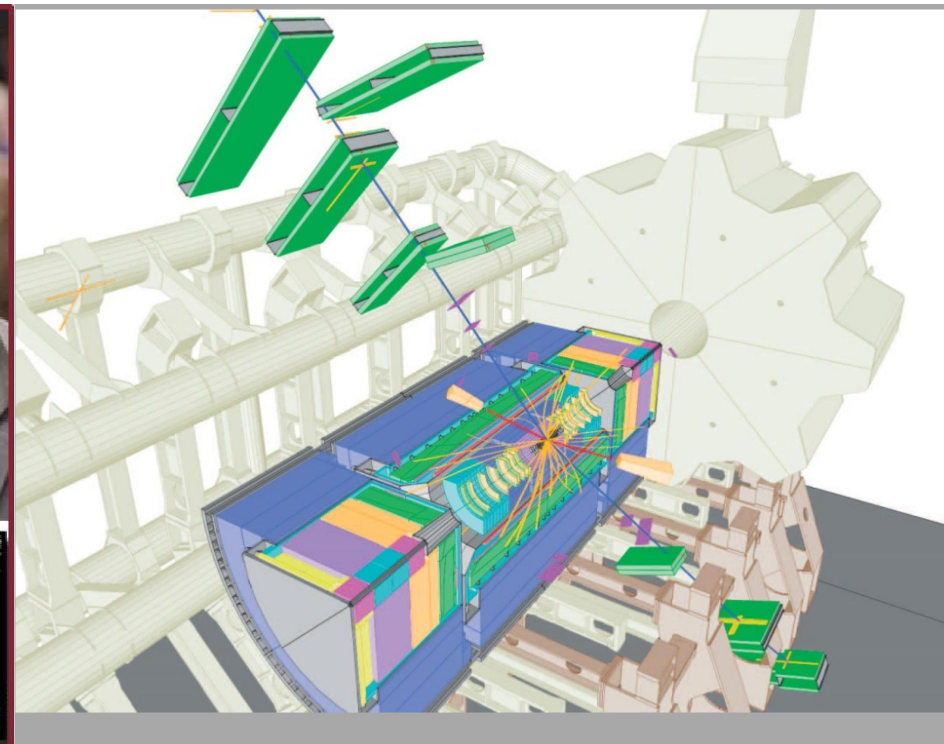
Paolo Bagnaia
SAPIENZA
UNIVERSITÀ DI ROMA

AA 18-19

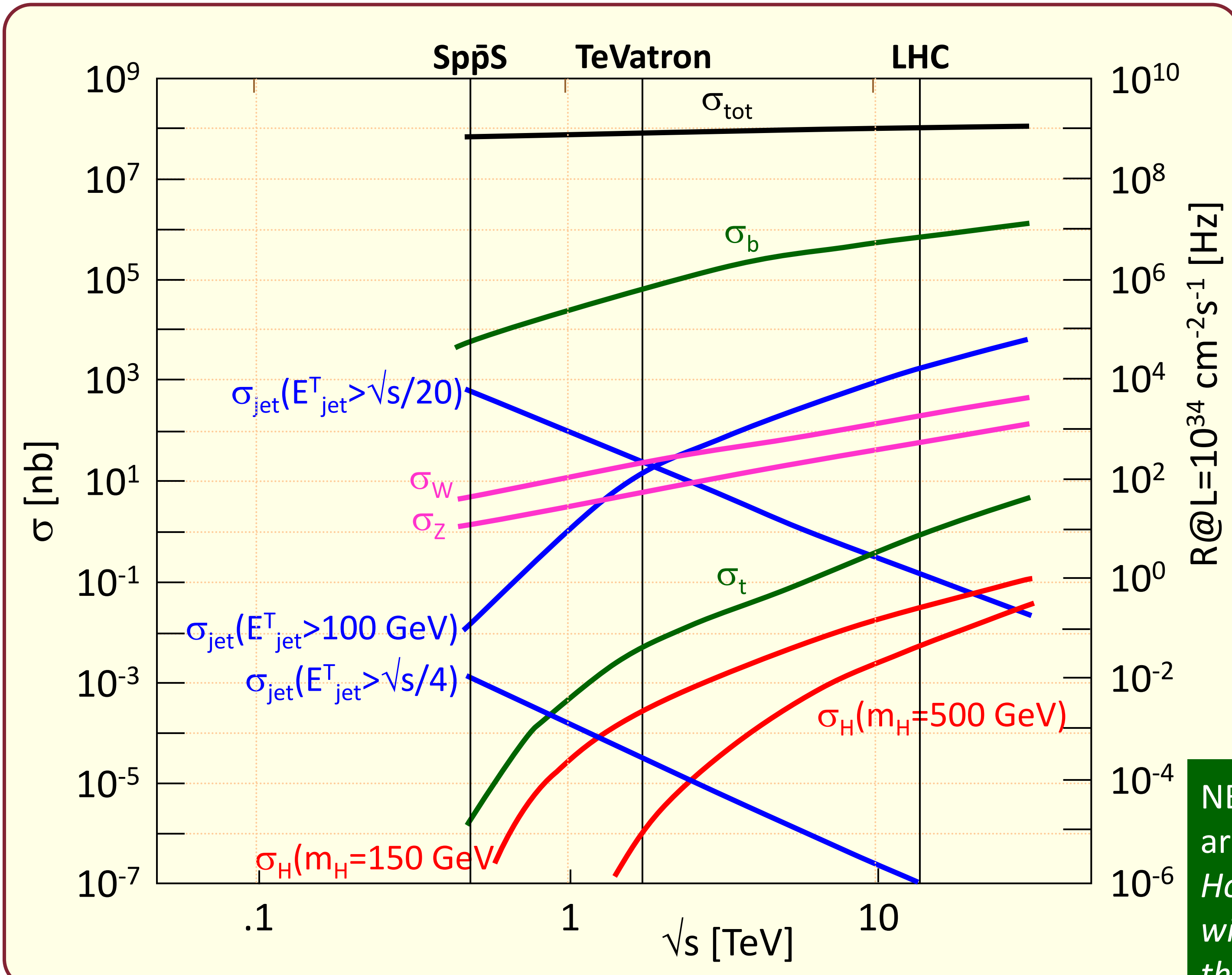
last mod.
31-May-19

12 – LHC – Higgs discovery





LHC results

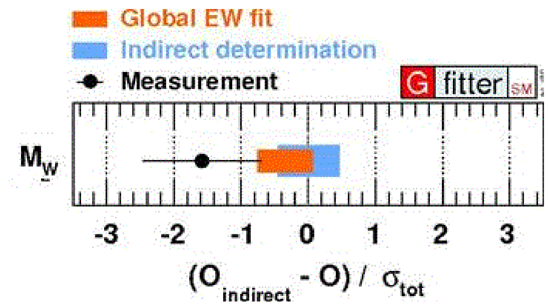


- some examples only;
- only show the results;
- no unfair comparison ATLAS ↔ CMS;
- analyses in progress, no attempt to follow the frequent updates.

NB. Sp$\bar{p}$S and Tevatron are $\bar{p}$p, LHC is pp. *However, no difference within the accuracy of this plot.*

LHC results: Rosetta stone of the SM fit

M. Baak et al., arXiv: 1407.3792 [hep-ph] : "Comparison of the fit results with the indirect determination in units of the total uncertainty, defined as the uncertainty of the direct measurement and that of the indirect determination added in quadrature. The indirect determination of an observable corresponds to a fit without using the corresponding direct constraint from the measurement".



I.e. (see the example for M_W) :

- O_{exp} : exp. measurement;
- O_{fit} : result of the complete e.w. fit *;
- $O_{indirect}$: e.w. fit, with all meas, BUT the plotted one;
- σ_{exp} : error on O_{exp} (stat $\oplus$ sys $\oplus$ theo);
- σ_{tot} : $\sigma_{exp} \oplus \sigma_{indirect}$.

$= 0 \pm \sigma_{indirect}/\sigma_{tot}$

Then, for all quantities:

- blue strip : $(O_{indirect} - O_{indirect}) / \sigma_{tot} \pm \sigma_{indirect}/\sigma_{tot}$;
- orange strip : $(O_{indirect} - O_{fit}) / \sigma_{tot} \pm \sigma_{fit}/\sigma_{tot}$;
- points : $(O_{indirect} - O_{exp}) / \sigma_{tot} \pm \sigma_{exp}/\sigma_{tot}$.

roughly speaking:

- blue width : error of indirect fit;
- orange displacement : how much a point moves its fit;
- orange width : error of full fit;
- points : uncorrelated wrt blue;
- points + err : pull.

[a lot of info, main result:

SM = ok $\rightarrow$ all within errors]

"$\oplus$" = "in quadrature";

* the e.w. fit gets (using higher orders) m_H, m_z, couplings, fermion masses; then all e.w. quantities can be computed.

LHC results: SM fits

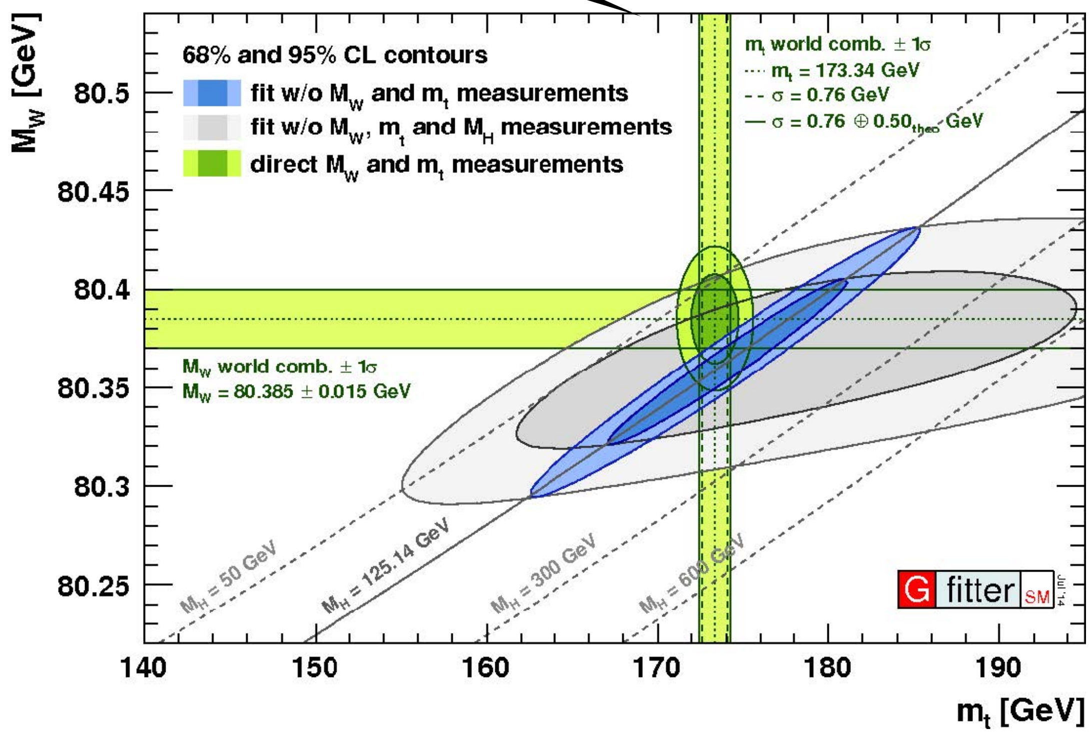
- in the past the main interest was:
  - possible deviations from SM;
  - predict unseen particles (top, Higgs) via rad. corrections;
- now the fit is over-constrained: look for bad pulls → physics bSM;

- perfect agreement (see figs.), textbook-like.

In simpler words :

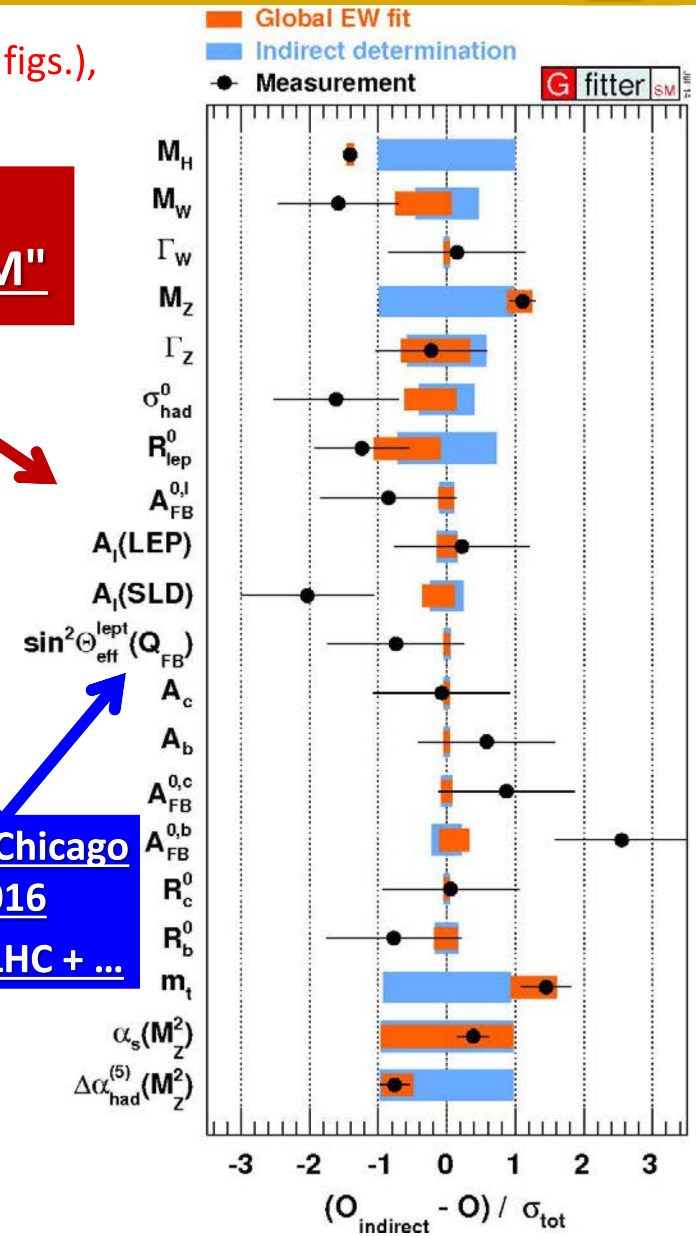
"triumph of the SM"

the glorious end of a 25-years story



ICHEP Chicago 2016

LEP + LHC + ...



LHC results: jet spectrum

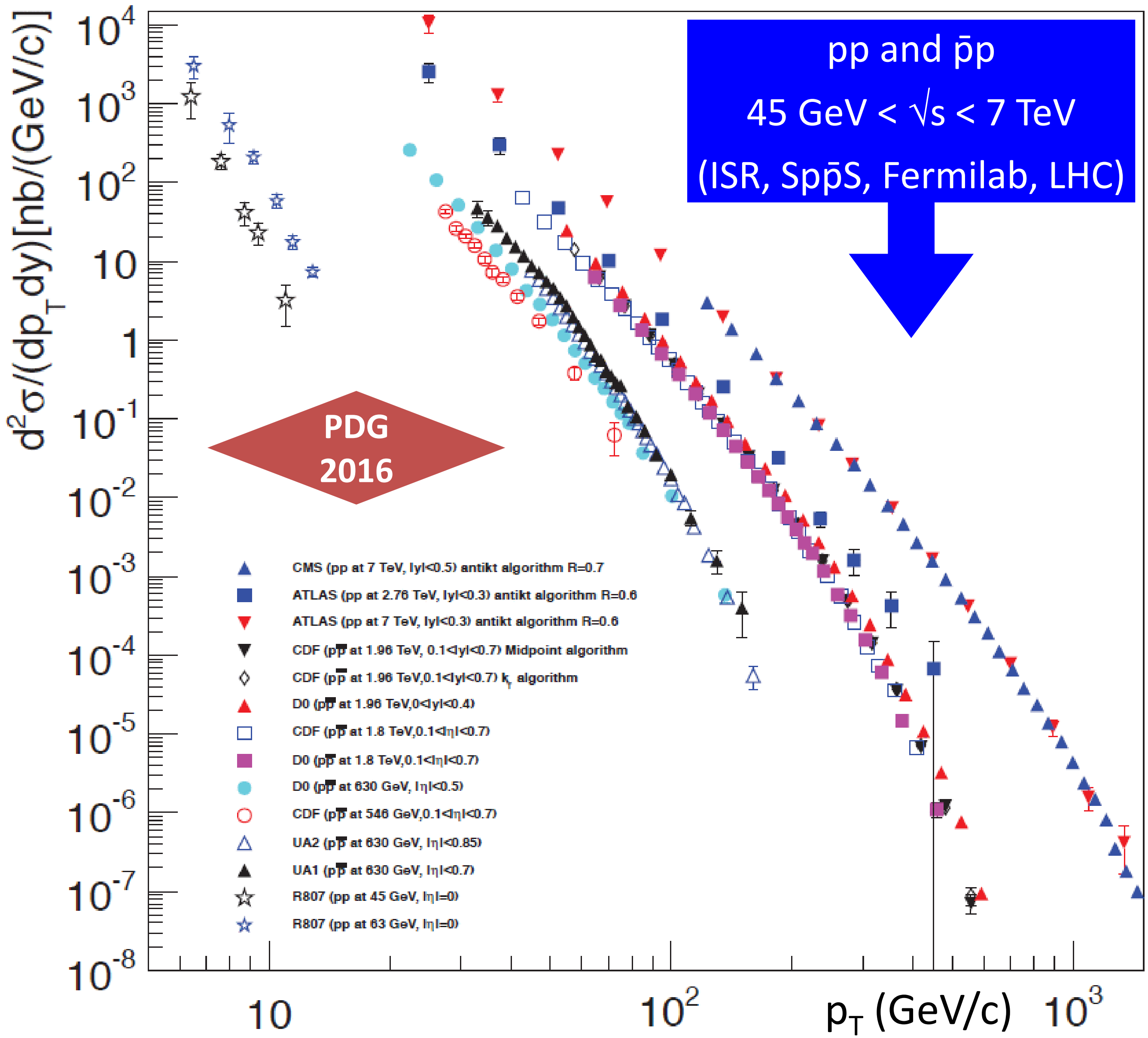
"Simple" explanation:

Inclusive differential jet cross sections, in the central rapidity region, plotted as a function of the jet transverse momentum.

Results earlier than from the Tevatron Run 2 used transverse energy rather than transverse momentum and pseudo-rapidity η rather than rapidity y, but p_T and y are used for all results shown here for simplicity. The error bars plotted are in most cases the experimental stat. and syst. errors added in quadrature.

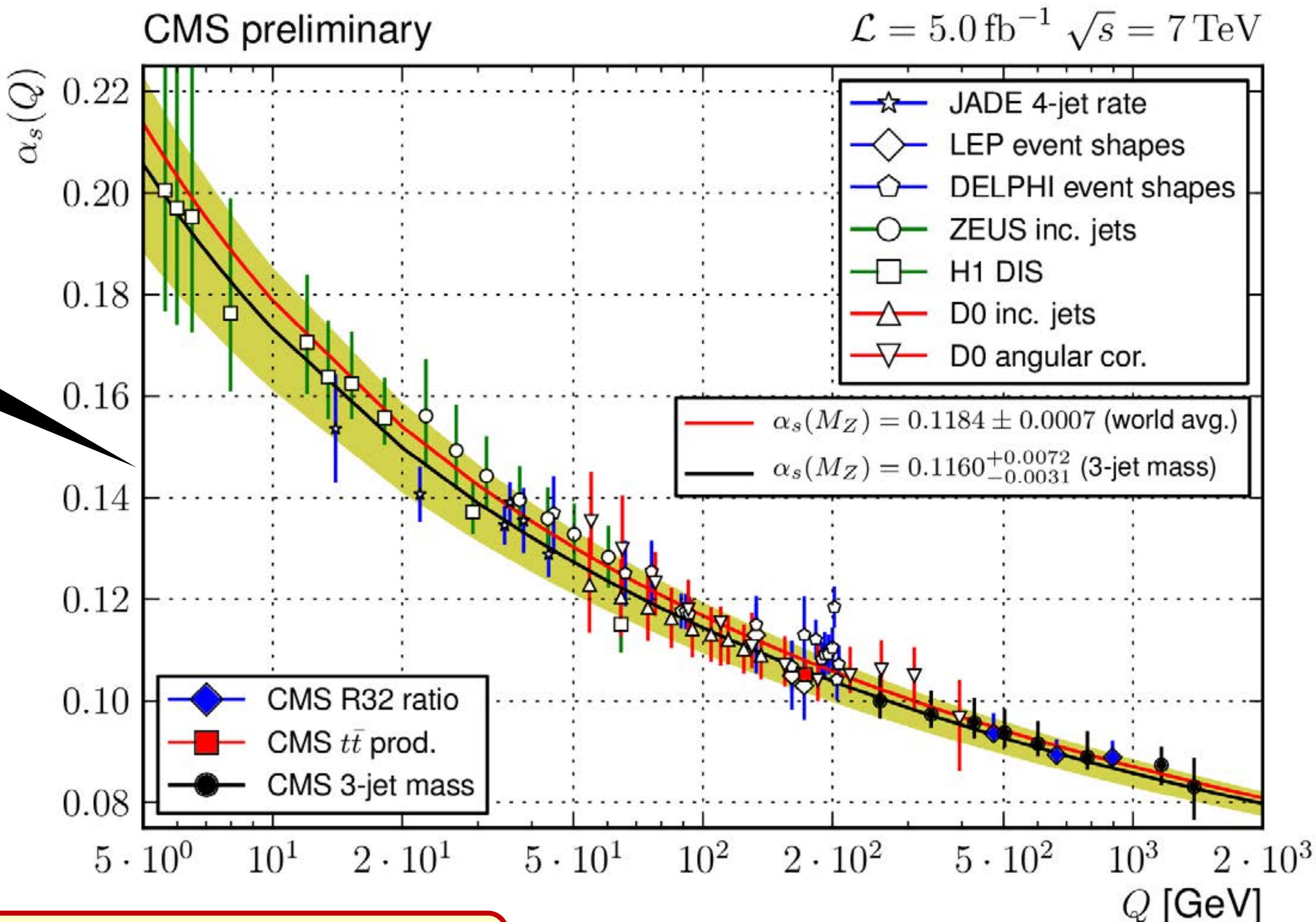
The CDF and D0 measurements use jet sizes of 0.7 (JetClu for CDF Run 1, and Midpoint and kT for CDF Run 2, a cone algorithm for D0 in Run 1 and the Midpoint algorithm in Run 2). The ATLAS results are plotted for the antikT algorithm for R=0.4, while the CMS results also use antikT, but with R=0.5. NLO QCD predictions in general provide a good description of the Tevatron and LHC data; the Tevatron jet data in fact are crucial components of global PDF fits, and the LHC data are starting to be used as well.

Comparisons with the older cross sections are more difficult due to the nature of the jet algorithms used.



LHC results: α_s running

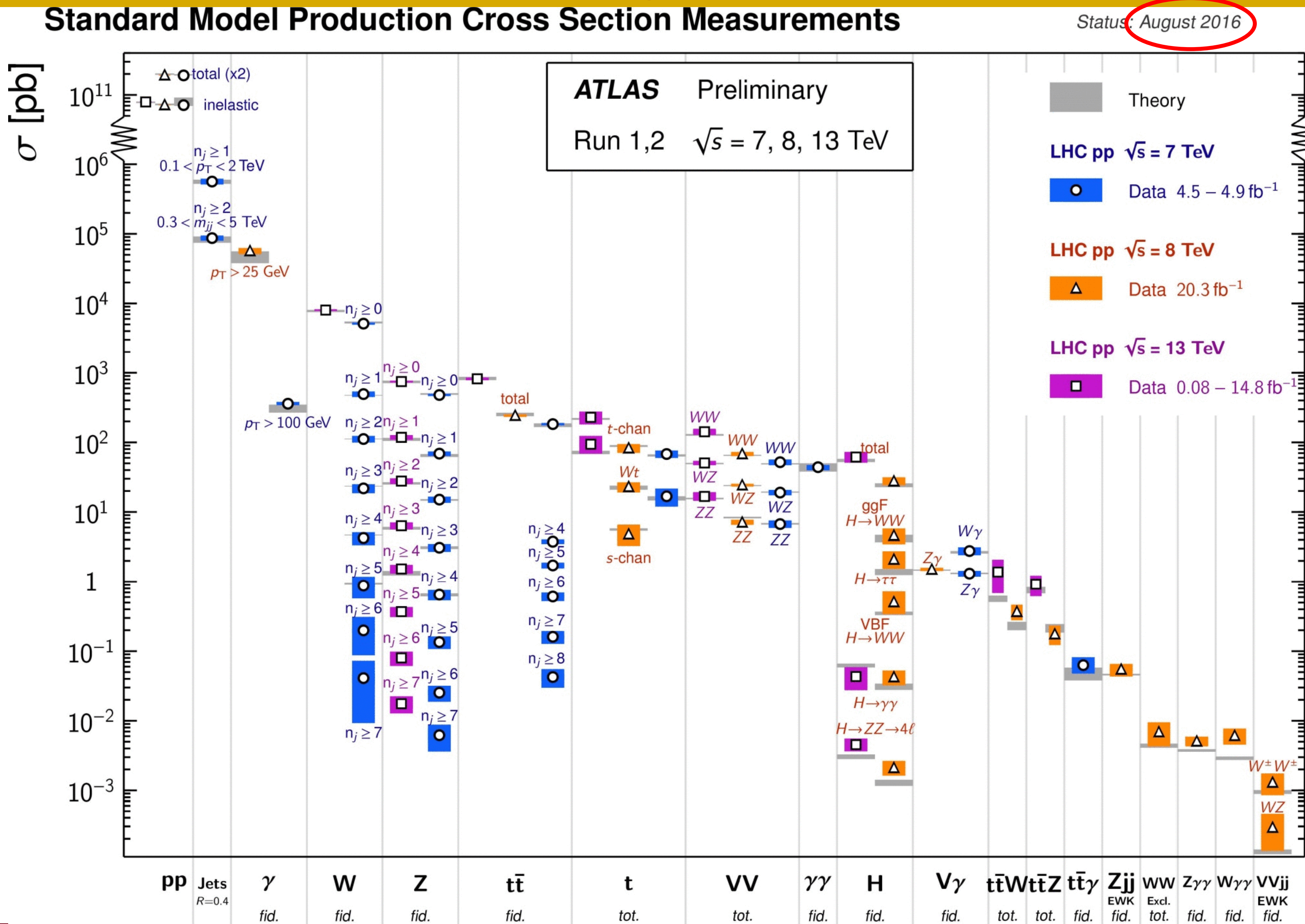
an update of the plot shown in § 6, with many more points.



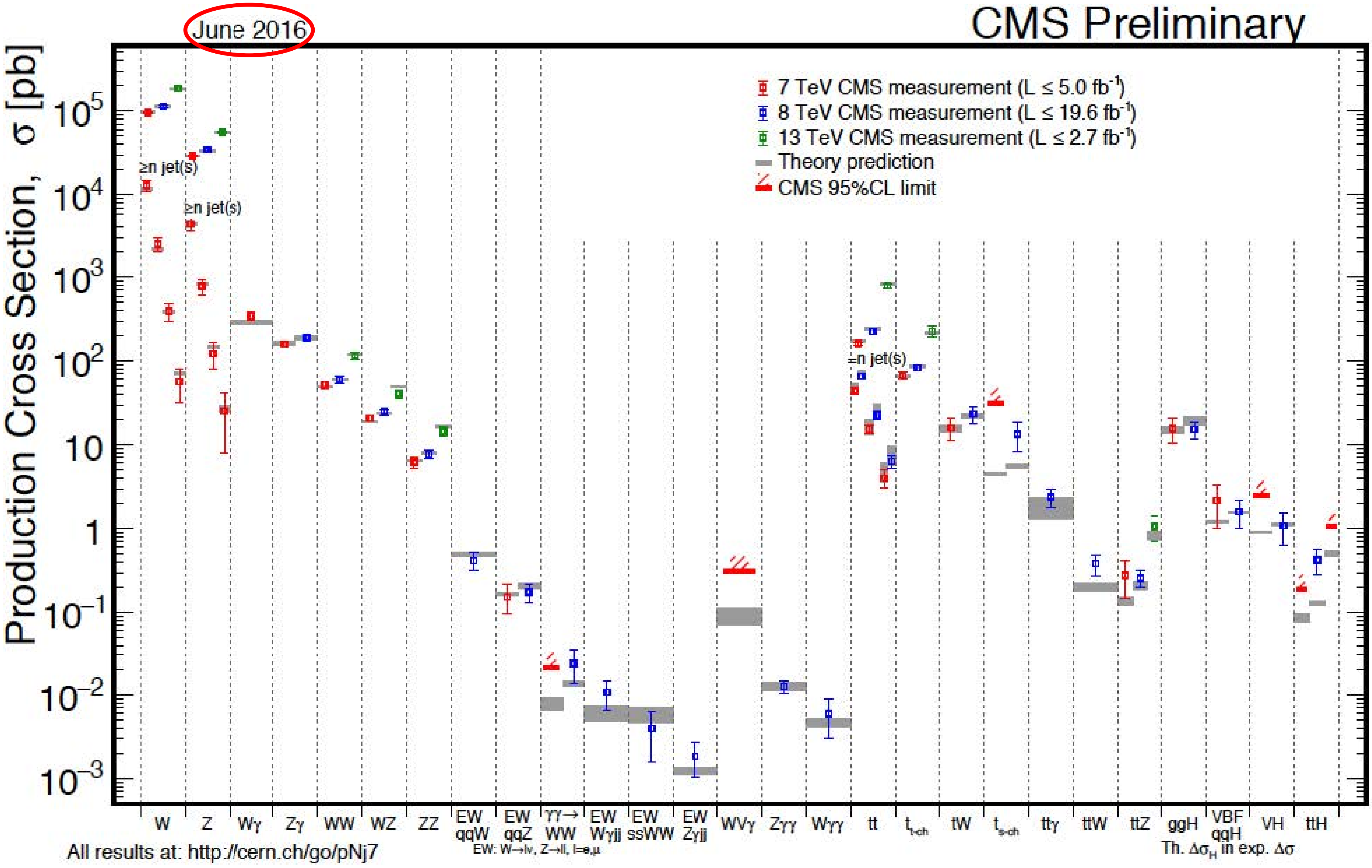
$$\alpha_s\left(Q^2\right) = \alpha_s\left(Q_0^2\right) \Big/ \left[1 + \alpha_s\left(Q_0^2\right) \frac{11N_c - 2N_f}{12\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right];$$

$N_c = 3;\quad N_f = 3 \to 4 \to 5 \to 6.$

LHC results: SM processes (ATLAS)

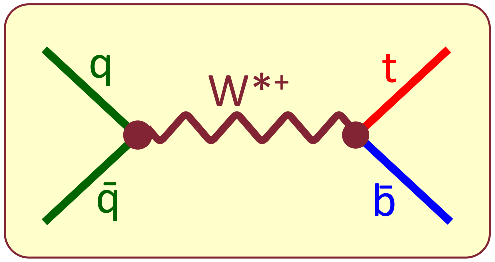


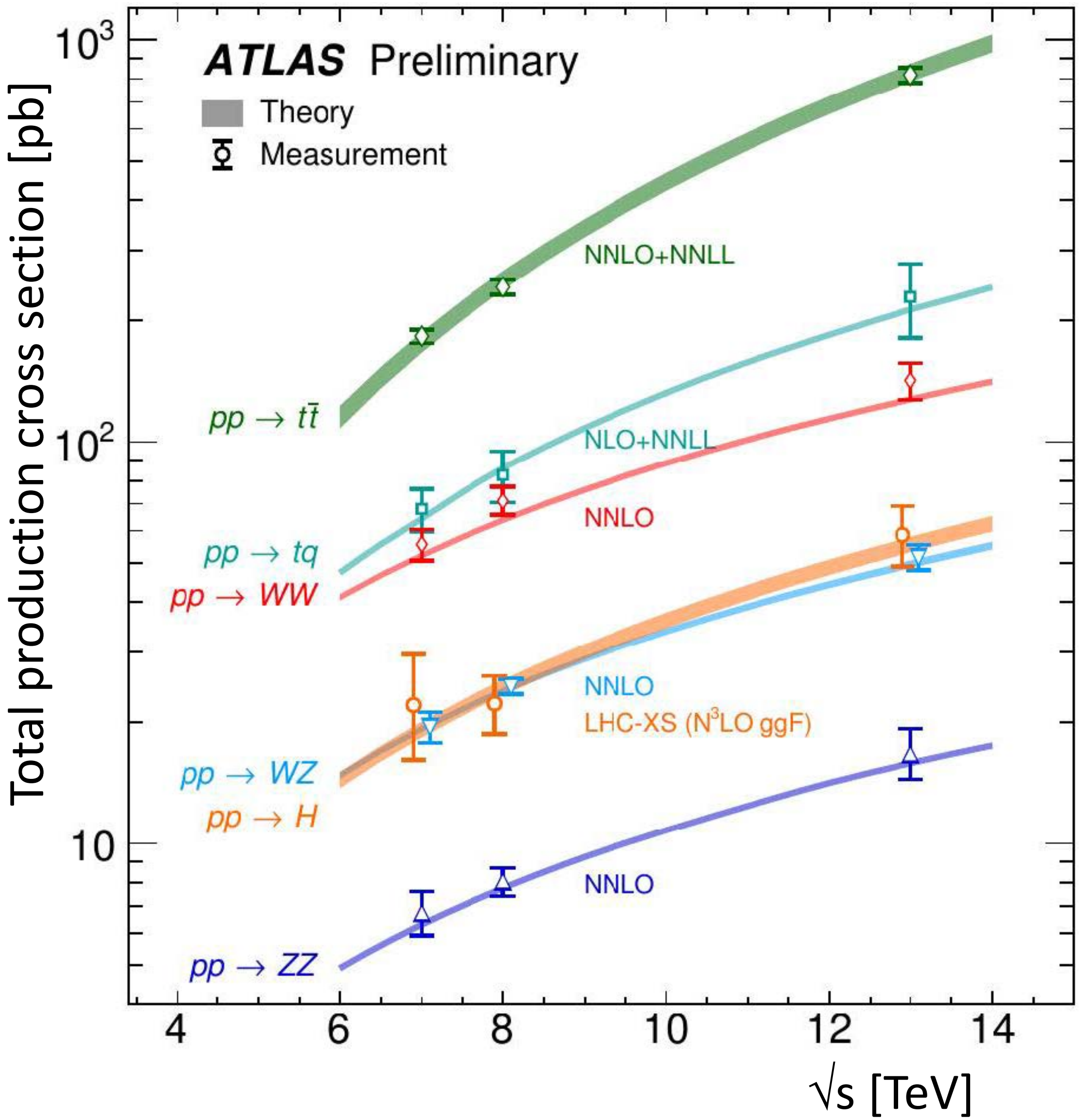
LHC results: SM processes (CMS)



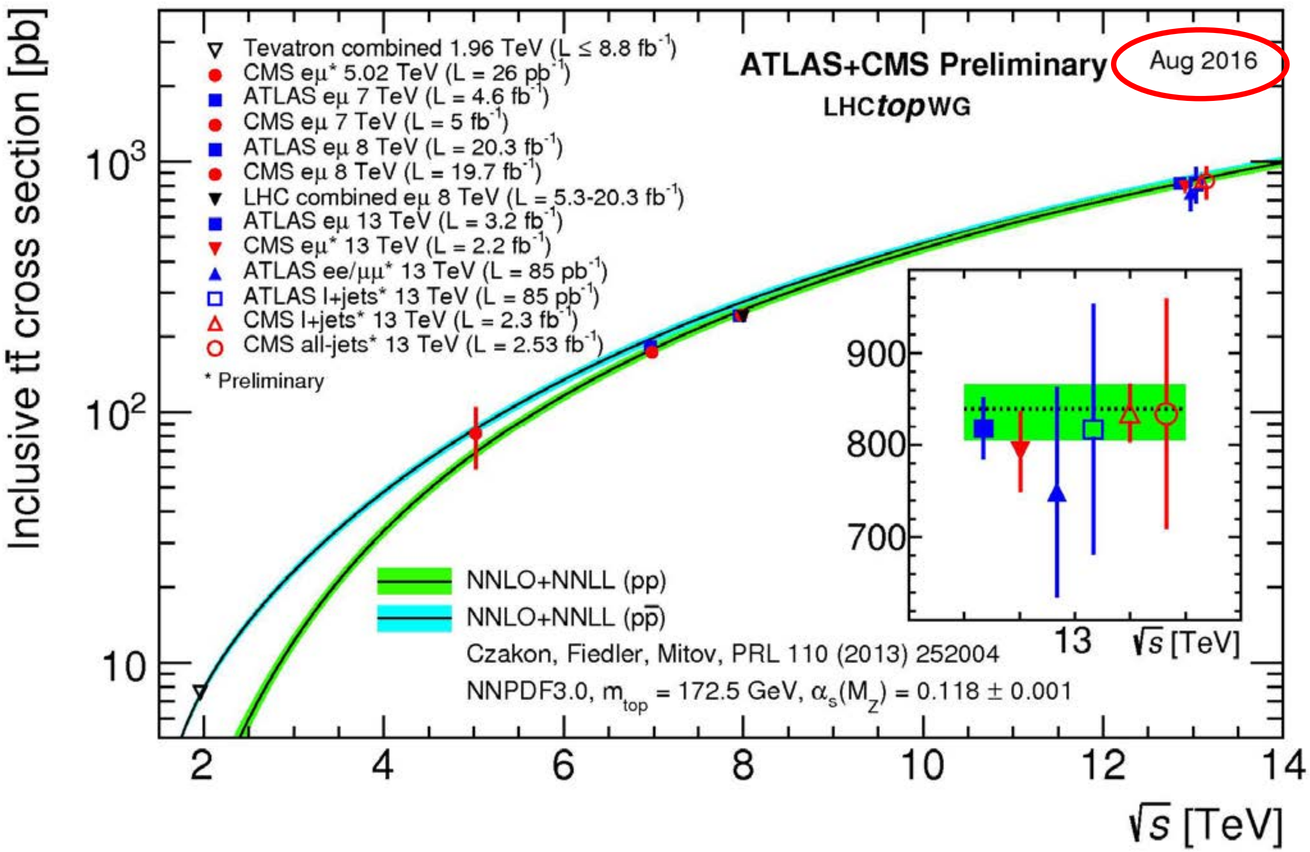
LHC results: small-σ processes

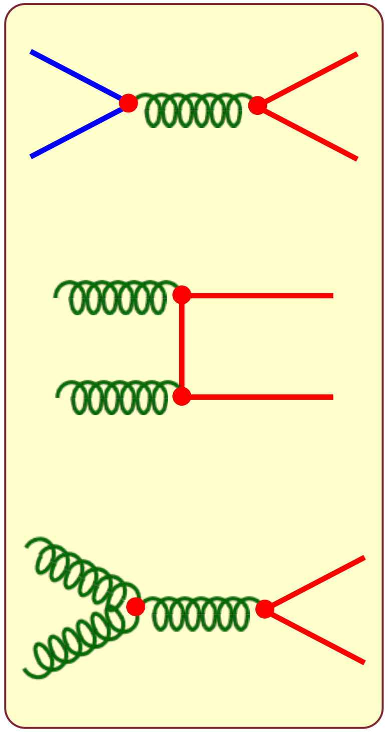
- the "heavy flavor/boson" sector:
  - ➢ $t\bar{t}$ (QCD);
  - ➢ single top (ew) [*example below*];
  - ➢ WW, WZ, ZZ (ew);
  - ➢ H (ew);
- shown vs √s;
- lessons:
  - ➢ LHC "sees" well at the pb level;
  - ➢ H is not very different from ZW / WW / ZZ channels, neither as mass, nor as σ, nor as √s dependence;
- as usual, SM (QCD+ew) works well.





LHC results: $\sigma_{t\bar{t}}$ vs $\sqrt{s}$



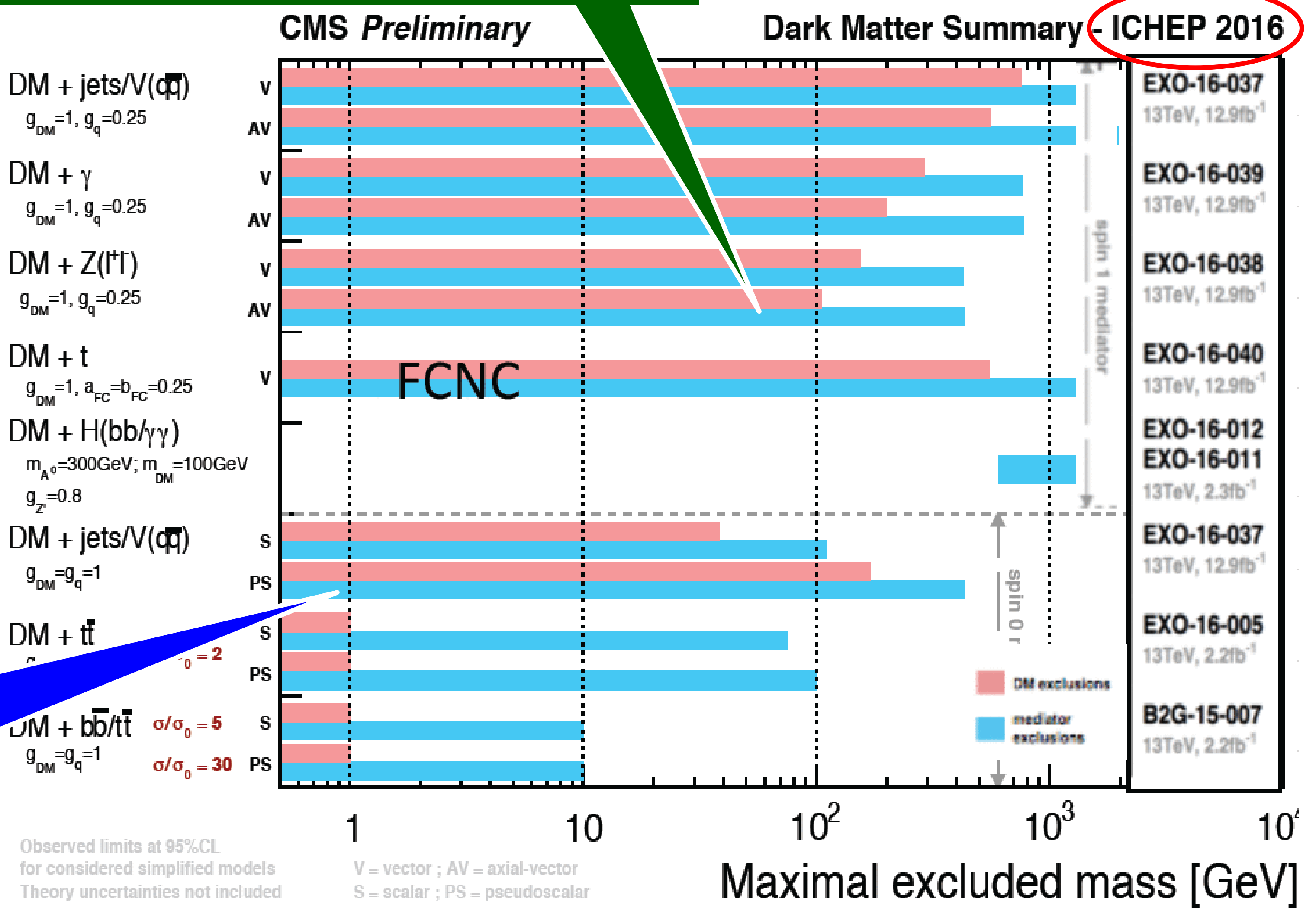


- technically a difficult analysis (secondary verteces + leptons + multijets + $\not{E}_T$);
- agreement ATLAS ↔ CMS and QCD ↔ data;
- [as seen in § 3] p̄p larger at small √s, but pp equivalent when √s increases, due to gluon dominance in PDF at small x;
- another perfect agreement, textbook-like.

LHC results: bSM (CMS DM)

just as examples, "CMS dark matter" and "ATLAS supersymmetry" searches

[ATLAS-DM and CMS-SUSY are not too different]



nothing found until now, but we know that something is hidden somewhere, so please continue …

LHC results: bSM (ATLAS SUSY)

ATLAS SUSY Searches* - 95% CL Lower Limits

Status: August 2016

ATLAS Preliminary

$\sqrt{s}$ = 7, 8, 13 TeV

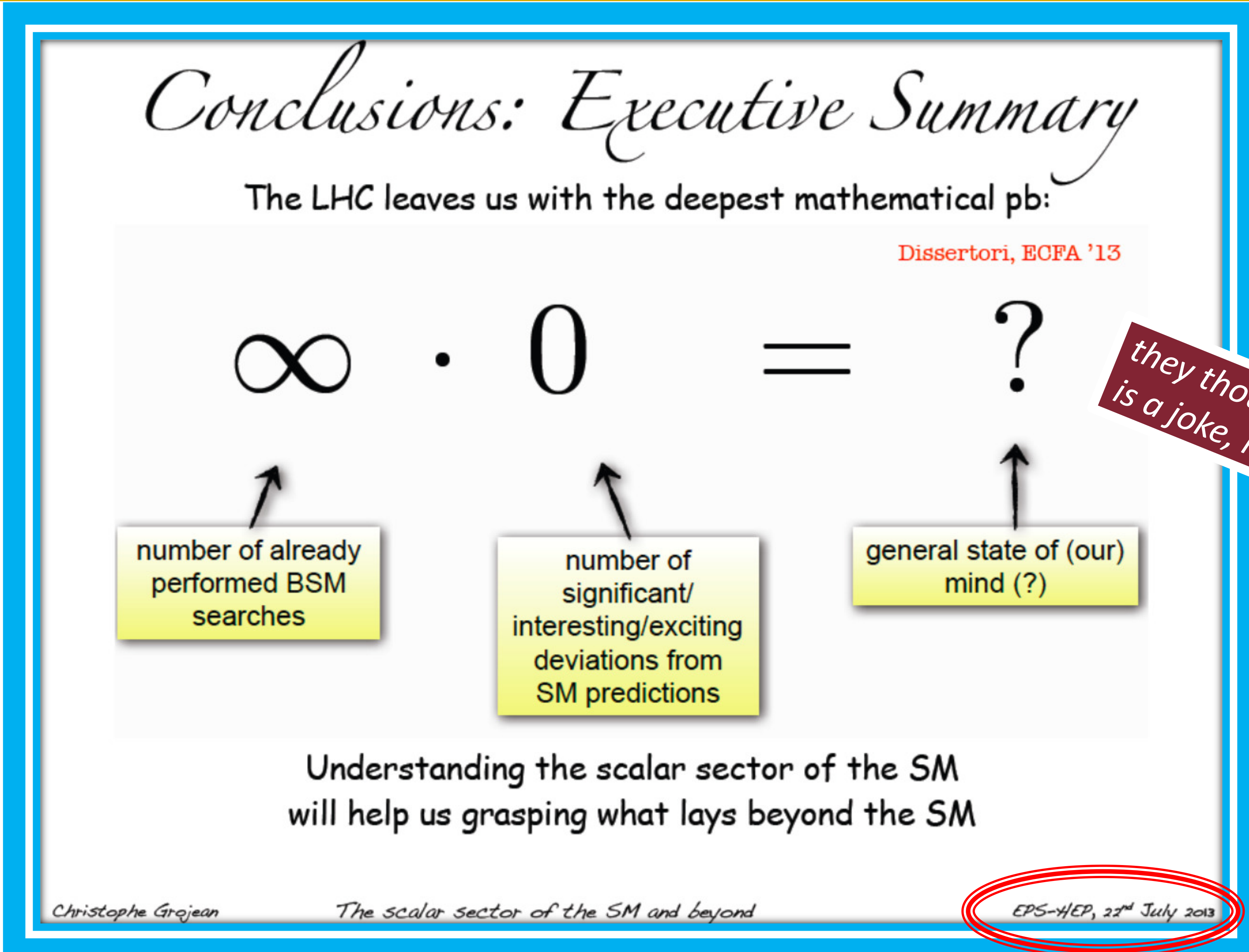
$\sqrt{s}$ = 7, 8 TeV (green) — $\sqrt{s}$ = 13 TeV (blue)

	Model	e,μ,τ,γ	Jets	E_T^{miss}	$\int\mathcal{L}\,dt$ [fb^{-1}]	Mass limit		Reference
Inclusive Searches	MSUGRA/CMSSM	0-3 e,μ/1-2 τ	2-10 jets/3 b	Yes	20.3	$\tilde{q},\tilde{g}$ 1.85 TeV	m($\tilde{q}$)=m($\tilde{g}$)	1507.05525
	$\tilde{q}\tilde{q}$, $\tilde{q}\to q\tilde{\chi}^0_1$	0	2-6 jets	Yes	13.3	$\tilde{q}$ 1.35 TeV	m($\tilde{\chi}^0_1$)<200 GeV, m(1st gen. $\tilde{q}$)=m(2nd gen. $\tilde{q}$)	ATLAS-CONF-2016-078
	$\tilde{q}\tilde{q}$, $\tilde{q}\to q\tilde{\chi}^0_1$ (compressed)	mono-jet	1-3 jets	Yes	3.2	$\tilde{q}$ 608 GeV	m($\tilde{q}$)-m($\tilde{\chi}^0_1$)<5 GeV	1604.07773
	$\tilde{g}\tilde{g}$, $\tilde{g}\to q\bar{q}\tilde{\chi}^0_1$	0	2-6 jets	Yes	13.3	$\tilde{g}$ 1.86 TeV	m($\tilde{\chi}^0_1$)=0 GeV	ATLAS-CONF-2016-078
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qq\tilde{\chi}^\pm_1\to qqW^\pm\tilde{\chi}^0_1$	0	2-6 jets	Yes	13.3	$\tilde{g}$ 1.83 TeV	m($\tilde{\chi}^0_1$)<400 GeV, m($\tilde{\chi}^\pm$)=0.5(m($\tilde{\chi}^0_1$)+m($\tilde{g}$))	ATLAS-CONF-2016-078
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qq(\ell\ell/\nu\nu)\tilde{\chi}^0_1$	3 e,μ	4 jets	-	13.2	$\tilde{g}$ 1.7 TeV	m($\tilde{\chi}^0_1$)<400 GeV	ATLAS-CONF-2016-037
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qqWZ\tilde{\chi}^0_1$	2 e,μ (SS)	0-3 jets	Yes	13.2	$\tilde{g}$ 1.6 TeV	m($\tilde{\chi}^0_1$) <500 GeV	ATLAS-CONF-2016-037
	GMSB ($\tilde{\ell}$ NLSP)	1-2 τ + 0-1 ℓ	0-2 jets	Yes	3.2	$\tilde{g}$ 2.0 TeV		1607.05979
	GGM (bino NLSP)	2 γ	-	Yes	3.2	$\tilde{g}$ 1.65 TeV	$c\tau$(NLSP)<0.1 mm	1606.09150
	GGM (higgsino-bino NLSP)	γ	1 b	Yes	20.3	$\tilde{g}$ 1.37 TeV	m($\tilde{\chi}^0_1$)<950 GeV, $c\tau$(NLSP)<0.1 mm, μ<0	1507.05493
	GGM (higgsino-bino NLSP)	γ	2 jets	Yes	13.3	$\tilde{g}$ 1.8 TeV	m($\tilde{\chi}^0_1$)>680 GeV, $c\tau$(NLSP)<0.1 mm, μ>0	ATLAS-CONF-2016-066
	GGM (higgsino NLSP)	2 e,μ (Z)	2 jets	Yes	20.3	$\tilde{g}$ 900 GeV	m(NLSP)>430 GeV	1503.03290
	Gravitino LSP	0	mono-jet	Yes	20.3	$F^{1/2}$ scale 865 GeV	m($\tilde{G}$)>1.8 × 10^{-4} eV, m($\tilde{g}$)=m($\tilde{q}$)=1.5 TeV	1502.01518
3rd gen. $\tilde{g}$ med.	$\tilde{g}\tilde{g}$, $\tilde{g}\to b\bar{b}\tilde{\chi}^0_1$	0	3 b	Yes	14.8	$\tilde{g}$ 1.89 TeV	m($\tilde{\chi}^0_1$)=0 GeV	ATLAS-CONF-2016-052
	$\tilde{g}\tilde{g}$, $\tilde{g}\to t\bar{t}\tilde{\chi}^0_1$	0-1 e,μ	3 b	Yes	14.8	$\tilde{g}$ 1.89 TeV	m($\tilde{\chi}^0_1$)=0 GeV	ATLAS-CONF-2016-052
	$\tilde{g}\tilde{g}$, $\tilde{g}\to b\bar{t}\tilde{\chi}^+_1$	0-1 e,μ	3 b	Yes	20.1	$\tilde{g}$ 1.37 TeV	m($\tilde{\chi}^0_1$)<300 GeV	1407.0600
3rd gen. squarks direct production	$\tilde{b}_1\tilde{b}_1$, $\tilde{b}_1\to b\tilde{\chi}^0_1$	0	2 b	Yes	3.2	$\tilde{b}_1$ 840 GeV	m($\tilde{\chi}^0_1$)<100 GeV	1606.08772
	$\tilde{b}_1\tilde{b}_1$, $\tilde{b}_1\to t\tilde{\chi}^\pm_1$	2 e,μ (SS)	1 b	Yes	13.2	$\tilde{b}_1$ 325-685 GeV	m($\tilde{\chi}^0_1$)<150 GeV, m($\tilde{\chi}^\pm_1$)= m($\tilde{\chi}^0_1$)+100 GeV	ATLAS-CONF-2016-037
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to b\tilde{\chi}^\pm_1$	0-2 e,μ	1-2 b	Yes	4.7/13.3	$\tilde{t}_1$ 117-170 GeV, 200-720 GeV	m($\tilde{\chi}^\pm_1$) = 2m($\tilde{\chi}^0_1$), m($\tilde{\chi}^0_1$)=55 GeV	1209.2102, ATLAS-CONF-2016-077
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to Wb\tilde{\chi}^0_1$ or $t\tilde{\chi}^0_1$	0-2 e,μ	0-2 jets/1-2 b	Yes	4.7/13.3	$\tilde{t}_1$ 90-198 GeV, 205-850 GeV	m($\tilde{\chi}^0_1$)=1 GeV	1506.08616, ATLAS-CONF-2016-077
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to c\tilde{\chi}^0_1$	0	mono-jet	Yes	3.2	$\tilde{t}_1$ 90-323 GeV	m($\tilde{t}_1$)-m($\tilde{\chi}^0_1$)=5 GeV	1604.07773
	$\tilde{t}_1\tilde{t}_1$ (natural GMSB)	2 e,μ (Z)	1 b	Yes	20.3	$\tilde{t}_1$ 150-600 GeV	m($\tilde{\chi}^0_1$)>150 GeV	1403.5222
	$\tilde{t}_2\tilde{t}_2$, $\tilde{t}_2\to\tilde{t}_1+Z$	3 e,μ (Z)	1 b	Yes	13.3	$\tilde{t}_2$ 290-700 GeV	m($\tilde{\chi}^0_1$)<300 GeV	ATLAS-CONF-2016-038
	$\tilde{t}_2\tilde{t}_2$, $\tilde{t}_2\to\tilde{t}_1+h$	1 e,μ	6 jets + 2 b	Yes	20.3	$\tilde{t}_2$ 320-620 GeV	m($\tilde{\chi}^0_1$)=0 GeV	1506.08616
EW direct	$\tilde{\ell}_{L,R}\tilde{\ell}_{L,R}$, $\tilde{\ell}\to\ell\tilde{\chi}^0_1$	2 e,μ	0	Yes	20.3	$\tilde{\ell}$ 90-335 GeV	m($\tilde{\chi}^0_1$)=0 GeV	1403.5294
	$\tilde{\chi}^+_1\tilde{\chi}^-_1$, $\tilde{\chi}^+_1\to\tilde{\ell}\nu(\ell\tilde{\nu})$	2 e,μ	0	Yes	20.3	$\tilde{\chi}^\pm_1$ 140-475 GeV	m($\tilde{\chi}^0_1$)=0 GeV, m($\tilde{\ell},\tilde{\nu}$)=0.5(m($\tilde{\chi}^\pm_1$)+m($\tilde{\chi}^0_1$))	1403.5294
	$\tilde{\chi}^+_1\tilde{\chi}^-_1$, $\tilde{\chi}^+_1\to\tilde{\tau}\nu(\tau\tilde{\nu})$	2 τ	-	Yes	20.3	$\tilde{\chi}^\pm_1$ 355 GeV	m($\tilde{\chi}^0_1$)=0 GeV, m($\tilde{\tau},\tilde{\nu}$)=0.5(m($\tilde{\chi}^\pm_1$)+m($\tilde{\chi}^0_1$))	1407.0350
	$\tilde{\chi}^\pm_1\tilde{\chi}^0_2\to\tilde{\ell}_L\nu\tilde{\ell}_L\ell(\tilde{\nu}\nu)$, $\ell\tilde{\nu}\tilde{\ell}_L\ell(\tilde{\nu}\nu)$	3 e,μ	0	Yes	20.3	$\tilde{\chi}^\pm_1,\tilde{\chi}^0_2$ 715 GeV	m($\tilde{\chi}^\pm_1$)=m($\tilde{\chi}^0_2$), m($\tilde{\chi}^0_1$)=0, m($\tilde{\ell},\tilde{\nu}$)=0.5(m($\tilde{\chi}^\pm_1$)+m($\tilde{\chi}^0_1$))	1402.7029
	$\tilde{\chi}^\pm_1\tilde{\chi}^0_2\to W\tilde{\chi}^0_1Z\tilde{\chi}^0_1$	2-3 e,μ	0-2 jets	Yes	20.3	$\tilde{\chi}^\pm_1,\tilde{\chi}^0_2$ 425 GeV	m($\tilde{\chi}^\pm_1$)=m($\tilde{\chi}^0_2$), m($\tilde{\chi}^0_1$)=0, $\tilde{\ell}$ decoupled	1403.5294, 1402.7029
	$\tilde{\chi}^\pm_1\tilde{\chi}^0_2\to W\tilde{\chi}^0_1h\tilde{\chi}^0_1$, $h\to b\bar{b}/WW/\tau\tau/\gamma\gamma$	e,μ,γ	0-2 b	Yes	20.3	$\tilde{\chi}^\pm_1,\tilde{\chi}^0_2$ 270 GeV	m($\tilde{\chi}^\pm_1$)=m($\tilde{\chi}^0_2$), m($\tilde{\chi}^0_1$)=0, $\tilde{\ell}$ decoupled	1501.07110
	$\tilde{\chi}^0_2\tilde{\chi}^0_3$, $\tilde{\chi}^0_{2,3}\to\tilde{\ell}_R\ell$	4 e,μ	0	Yes	20.3	$\tilde{\chi}^0_{2,3}$ 635 GeV	m($\tilde{\chi}^0_2$)=m($\tilde{\chi}^0_3$), m($\tilde{\chi}^0_1$)=0, m($\tilde{\ell},\tilde{\nu}$)=0.5(m($\tilde{\chi}^0_2$)+m($\tilde{\chi}^0_1$))	1405.5086
	GGM (wino NLSP) weak prod.	1 $e,\mu+\gamma$	-	Yes	20.3	$\tilde{W}$ 115-370 GeV	$c\tau$<1 mm	1507.05493
	GGM (bino NLSP) weak prod.	2 γ	-	Yes	20.3	$\tilde{W}$ 590 GeV	$c\tau$<1 mm	1507.05493
Long-lived particles	Direct $\tilde{\chi}^+_1\tilde{\chi}^-_1$ prod., long-lived $\tilde{\chi}^\pm_1$	Disapp. trk	1 jet	Yes	20.3	$\tilde{\chi}^\pm_1$ 270 GeV	m($\tilde{\chi}^\pm_1$)-m($\tilde{\chi}^0_1$)~160 MeV, τ($\tilde{\chi}^\pm_1$)=0.2 ns	1310.3675
	Direct $\tilde{\chi}^+_1\tilde{\chi}^-_1$ prod., long-lived $\tilde{\chi}^\pm_1$	dE/dx trk	-	Yes	18.4	$\tilde{\chi}^\pm_1$ 495 GeV	m($\tilde{\chi}^\pm_1$)-m($\tilde{\chi}^0_1$)~160 MeV, τ($\tilde{\chi}^\pm_1$)<15 ns	1506.05332
	Stable, stopped $\tilde{g}$ R-hadron	0	1-5 jets	Yes	27.9	$\tilde{g}$ 850 GeV	m($\tilde{\chi}^0_1$)=100 GeV, 10 μs<τ($\tilde{g}$)<1000 s	1310.6584
	Stable $\tilde{g}$ R-hadron	trk	-	-	3.2	$\tilde{g}$ 1.58 TeV		1606.05129
	Metastable $\tilde{g}$ R-hadron	dE/dx trk	-	-	3.2	$\tilde{g}$ 1.57 TeV	m($\tilde{\chi}^0_1$)=100 GeV, τ>10 ns	1604.04520
	GMSB, stable $\tilde{\tau}$, $\tilde{\chi}^0_1\to\tilde{\tau}(\tilde{e},\tilde{\mu})+\tau(e,\mu)$	1-2 μ	-	-	19.1	$\tilde{\chi}^0_1$ 537 GeV	10<tanβ<50	1411.6795
	GMSB, $\tilde{\chi}^0_1\to\gamma\tilde{G}$, long-lived $\tilde{\chi}^0_1$	2 γ	-	Yes	20.3	$\tilde{\chi}^0_1$ 440 GeV	1<τ($\tilde{\chi}^0_1$)<3 ns, SPS8 model	1409.5542
	$\tilde{g}\tilde{g}$, $\tilde{\chi}^0_1\to eev/e\mu\nu/\mu\mu\nu$	displ. $ee/e\mu/\mu\mu$	-	-	20.3	$\tilde{\chi}^0_1$ 1.0 TeV	7 <$c\tau$($\tilde{\chi}^0_1$)< 740 mm, m($\tilde{g}$)=1.3 TeV	1504.05162
	GGM $\tilde{g}\tilde{g}$, $\tilde{\chi}^0_1\to Z\tilde{G}$	displ. vtx + jets	-	-	20.3	$\tilde{\chi}^0_1$ 1.0 TeV	6 <$c\tau$($\tilde{\chi}^0_1$)< 480 mm, m($\tilde{g}$)=1.1 TeV	1504.05162
RPV	LFV $pp\to\tilde{\nu}_\tau+X$, $\tilde{\nu}_\tau\to e\mu/e\tau/\mu\tau$	$e\mu,e\tau,\mu\tau$	-	-	3.2	$\tilde{\nu}_\tau$ 1.9 TeV	λ'_{311}=0.11, $\lambda_{132/133/233}$=0.07	1607.08079
	Bilinear RPV CMSSM	2 e,μ (SS)	0-3 b	Yes	20.3	$\tilde{q},\tilde{g}$ 1.45 TeV	m($\tilde{q}$)=m($\tilde{g}$), $c\tau_{LSP}$<1 mm	1404.2500
	$\tilde{\chi}^+_1\tilde{\chi}^-_1$, $\tilde{\chi}^+_1\to W\tilde{\chi}^0_1$, $\tilde{\chi}^0_1\to ee\nu, e\mu\nu, \mu\mu\nu$	4 e,μ	-	Yes	13.3	$\tilde{\chi}^\pm_1$ 1.14 TeV	m($\tilde{\chi}^0_1$)>400GeV, $\lambda_{12k}\neq0$ (k = 1, 2)	ATLAS-CONF-2016-075
	$\tilde{\chi}^+_1\tilde{\chi}^-_1$, $\tilde{\chi}^+_1\to W\tilde{\chi}^0_1$, $\tilde{\chi}^0_1\to\tau\tau\nu_e, e\tau\nu_\tau$	3 $e,\mu+\tau$	-	Yes	20.3	$\tilde{\chi}^\pm_1$ 450 GeV	m($\tilde{\chi}^0_1$)>0.2×m($\tilde{\chi}^\pm_1$), $\lambda_{133}\neq0$	1405.5086
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qqq$	0	4-5 large-R jets	-	14.8	$\tilde{g}$ 1.08 TeV	BR(t)=BR(b)=BR(c)=0%	ATLAS-CONF-2016-057
	$\tilde{g}\tilde{g}$, $\tilde{g}\to qq\tilde{\chi}^0_1$, $\tilde{\chi}^0_1\to qqq$	0	4-5 large-R jets	-	14.8	$\tilde{g}$ 1.55 TeV	m($\tilde{\chi}^0_1$)=800 GeV	ATLAS-CONF-2016-057
	$\tilde{g}\tilde{g}$, $\tilde{g}\to\tilde{t}_1t$, $\tilde{t}_1\to bs$	2 e,μ (SS)	0-3 b	Yes	13.2	$\tilde{g}$ 1.3 TeV	m($\tilde{t}_1$)<750 GeV	ATLAS-CONF-2016-037
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to bs$	0	2 jets + 2 b	-	15.4	$\tilde{t}_1$ 410 GeV, 450-510 GeV		ATLAS-CONF-2016-022, ATLAS-CONF-2016-084
	$\tilde{t}_1\tilde{t}_1$, $\tilde{t}_1\to b\ell$	2 e,μ	2 b	-	20.3	$\tilde{t}_1$ 0.4-1.0 TeV	BR($\tilde{t}_1\to be/\mu$)>20%	ATLAS-CONF-2015-015
Other	Scalar charm, $\tilde{c}\to c\tilde{\chi}^0_1$	0	2 c	Yes	20.3	$\tilde{c}$ 510 GeV	m($\tilde{\chi}^0_1$)<200 GeV	1501.01325

Mass scale [TeV]: 10^{-1} — 1

*Only a selection of the available mass limits on new states or phenomena is shown.

LHC results: bSM results

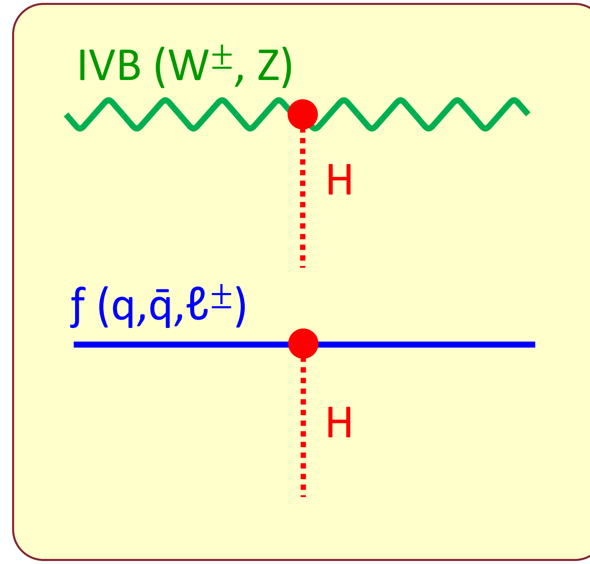


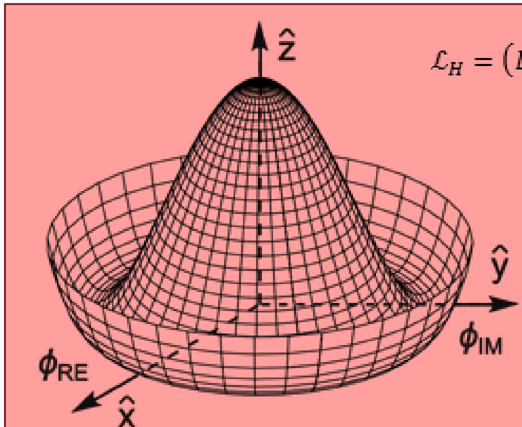
they thought it is a joke, really

the MSM Higgs boson

$\cancel{m}_H$

- [*the symbol* $\cancel{m}_H$ *means that in the slide the value of the mass of the Higgs may vary:*
  - *for didactic reasons,*
  - *because the analysis is still in progress,*
  - *because of a possible larger H sector*]
- [at least] one H boson in SM;
- just one Higgs in "minimal standard model" MSM [*MSM assumed in the following*];
- [> 1 in theories bSM, e.g. in SUSY: h, H, A, $H^{\pm}$]
- charge : 0; spin : 0; $J^P = 0^+$ [other H may have different q.n.];
- in MSM directly coupled with all massive particles, i.e. all but γ, g, ν's (if massless);
- it behaves like a normal particle (with exotic couplings): it is produced, it decays, etc etc.
- [*more on this subject later in the chapter*]





$$\mathcal{L}_H = (D_\mu\phi)^* D_\mu\phi - V(\phi) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu\phi = \partial\phi + ieA_\mu\phi$$

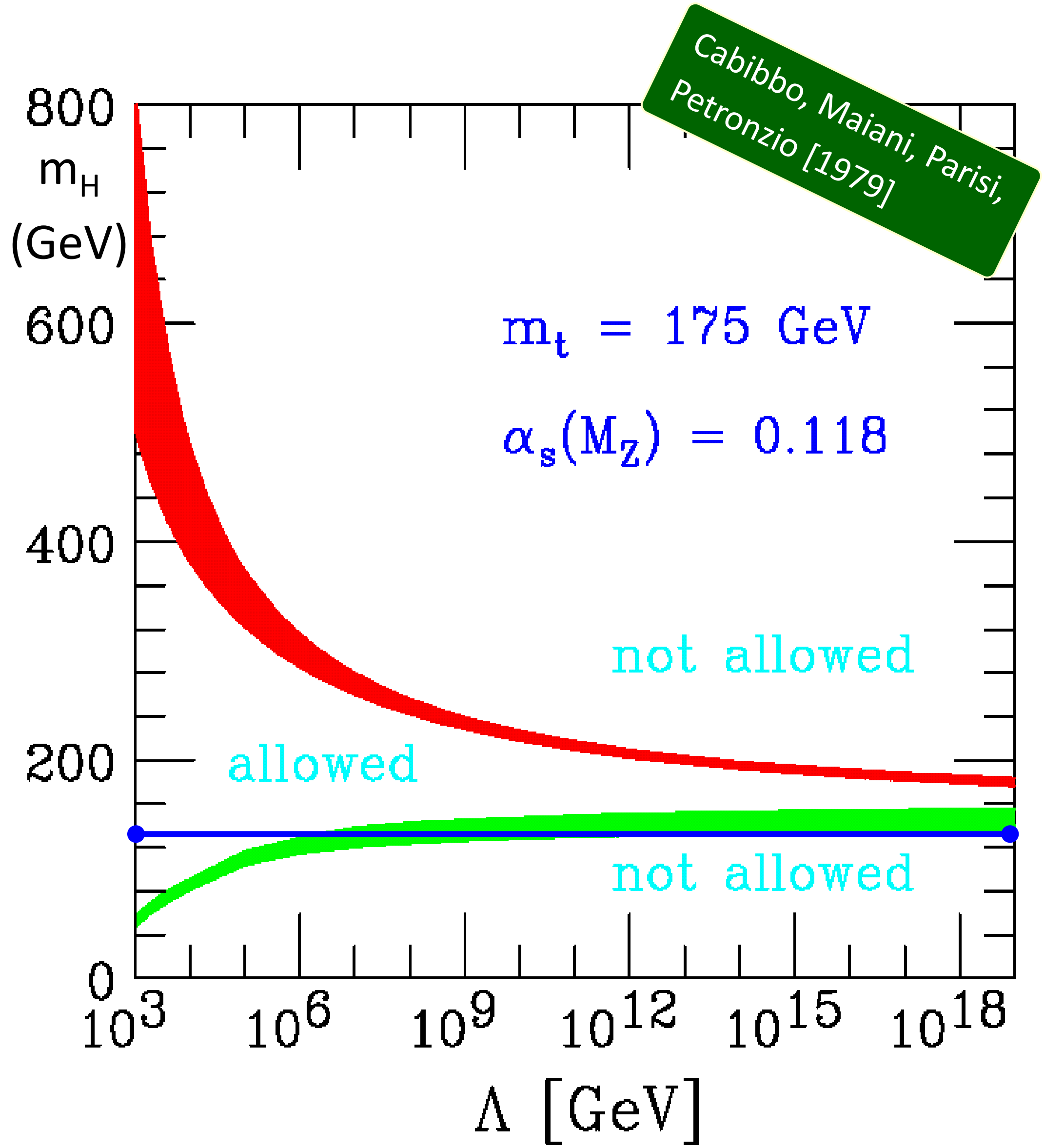
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\phi) = \alpha\phi^*\phi + \beta(\phi^*\phi)^2$$

$$\alpha < 0, \qquad \beta > 0$$

the MSM Higgs boson: mass limits

- the Higgs mass is a free parameter of the SM [sometimes another correlated parameter chosen as "fundamental"];
- however, the non-violation of the unitarity puts a limit $m_H \leq$ 1 TeV (approx.);
- the further demand that the SM be consistent up to a given scale Λ (triviality bound) puts another limit on m_H, function of Λ **(red line)**;
- the vacuum stability also limits m_H (stability bound, **green line**);
- considering all together, $\Lambda = m_{Planck}$ $\rightarrow$ 130 < m_H <180 GeV;
- the blue line corresponds to m_H = 125 GeV [*quite puzzling*].

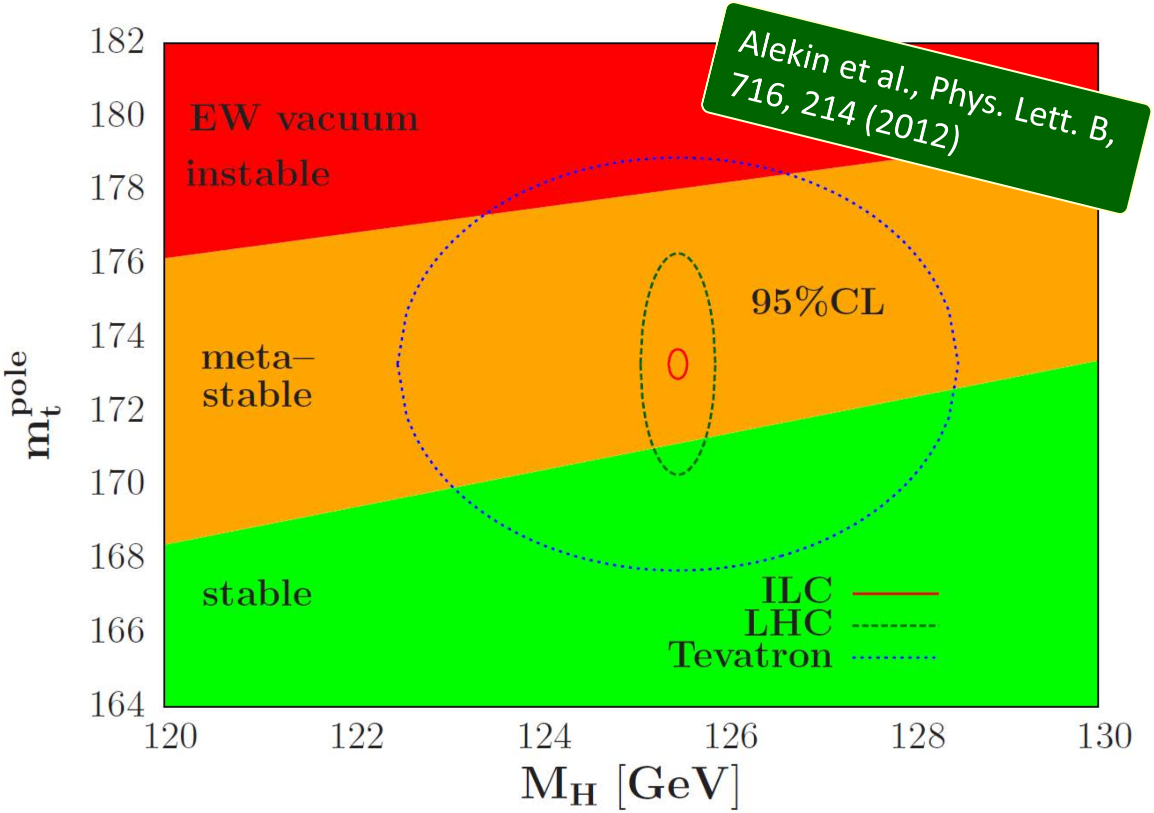


the MSM Higgs boson: vacuum stability

this page should appear after the discussion of the Higgs discovery, but here it is easier.

Assume the Higgs has been found at ~125 GeV:

- according to the previous argument, the universe is **stable**, **meta-stable**, or **in-stable** ?
- even with the MSM assumption (particle found at LHC = MSM Higgs), present error ("LHC") does not solve the question;
- only a future, more precise measurement ("ILC"), will solve it;
- notice in the plot:
  - the value of the top quark mass is VERY important;
  - the "ILC" value is arbitrarily put at the LHC/ TeVatron measurement: only look to the size of the error;



- however, if one takes the LHC measurement at face value, the universe is **metastable**, but its lifetime may exceed its age (~ 10^{10} years);
- so, do not panic, but **improve the measurement** !!!

the MSM Higgs boson: potential V_H

Define the SM parameters (PDG 2016 §10):

$$V_H(\phi) = -\mu^2\phi^\dagger\phi + \lambda^2(\phi^\dagger\phi)^2/2;$$

$$V_H = 0 \to \phi(V_H = 0) \equiv \upsilon = \frac{\sqrt{2}\mu}{\lambda} = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246 \text{ GeV};$$

ⓘ other definitions:
$\lambda^2/2 \to \lambda, \lambda^2, \lambda/2, \lambda/4, \ldots$
$-\mu^2 \to \mu^2, \ldots$

$$m_H = \lambda\upsilon; \quad m_W = \frac{1}{2}g\upsilon;$$

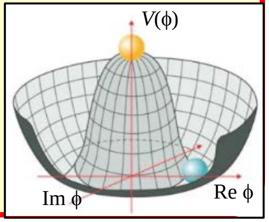
$$m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\,\upsilon = \frac{m_W}{\cos\theta_W}; \quad m_\gamma = 0;$$

$$g = \frac{2m_W}{\upsilon} \approx 0.65; \quad g' = \frac{2\sqrt{m_Z^2 - m_W^2}}{\upsilon} \approx 0.35;$$

$$\mathcal{L}^{\text{coupling}}_{\text{Higgs}} = -g_f\bar{\psi}_f\psi_f H + \delta_V g_V V_\mu V^\mu H + [\ldots];$$

$$[V = W/Z; \quad \delta_W = 1; \quad \delta_Z = ½]$$

$$g_f = \frac{m_f}{\upsilon}; \qquad g_V = \frac{2m_V^2}{\upsilon};$$



$$m_H^{\text{exp}} \approx 125 \text{ GeV} \to$$

$$\lambda^{\text{exp}} = \frac{m_H}{\upsilon} \approx 0.51;$$

$$\mu^{\text{exp}} = \frac{m_H}{\sqrt{2}} \approx 88 \text{ GeV}.$$

imho (λ, μ) neither theor. clear, nor exp clean.

To test the SM couplings, redefine g_f and g_V with "modifiers" κ's:

$$g_f = \frac{m_f}{\upsilon} \to \kappa_f\frac{m_f}{\upsilon};$$

$$g_V = \frac{2m_V^2}{\upsilon} \to \kappa_V\frac{2m_V^2}{\upsilon};$$

and adjust κ_f and κ_W to the data;

$$\kappa_j^2 = \frac{\sigma_j}{\sigma_j^{SM}}; \quad \text{or} \quad \kappa_j^2 = \frac{\Gamma_j}{\Gamma_j^{SM}};$$

$$\text{SM} \to \kappa_f = \kappa_V = 1.$$

the MSM Higgs boson: function V(ϕ)

Study the function V=V(ϕ):

$$V_H(\phi) = -\mu^2\phi^\dagger\phi + \frac{\lambda^2}{2}(\phi^\dagger\phi)^2 =$$

$$\boxed{= m_H^2\left(-\frac{1}{2}|\phi|^2 + \frac{1}{\sqrt{2}}G_F|\phi|^4\right);}$$

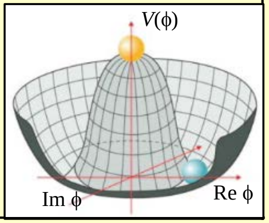
$$\left[\mu = \frac{m_H}{\sqrt{2}}; \quad \lambda = m_H\sqrt{\sqrt{2}G_F}\right]$$

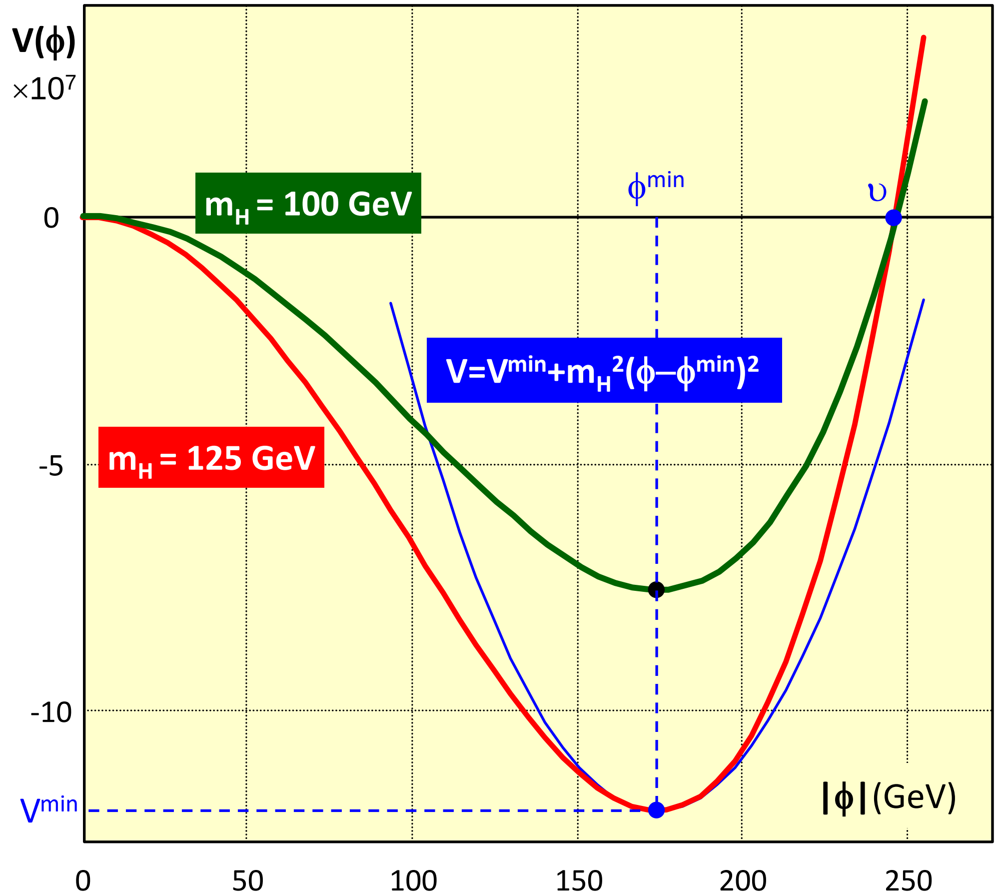
$$\upsilon = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246 \text{ GeV};$$

$$\left.\frac{\partial V_H}{\partial\phi}\right|_{\phi=\phi^{min}} = 0 \rightarrow \phi^{min} = \sqrt{\frac{1}{2\sqrt{2}G_F}} \approx 174 \text{ GeV};$$

$$V_H^{min} = V_H\left(\phi^{min}\right) = \frac{-m_H^2}{8\sqrt{2}G_F};$$

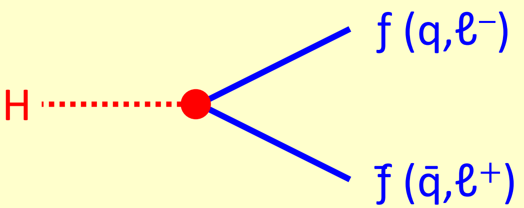
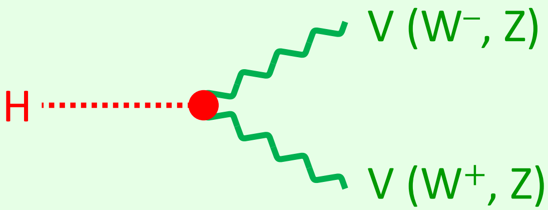
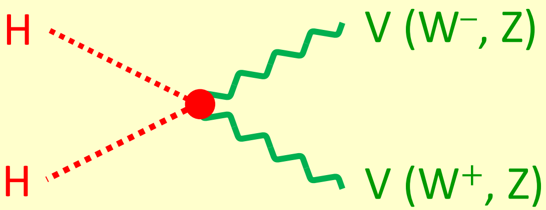
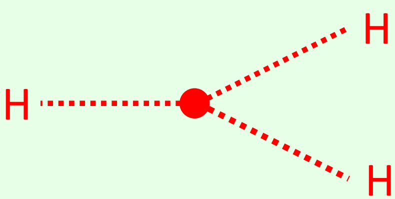
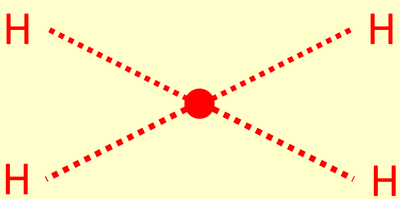
$$\boxed{\left.\frac{\partial^2 V_H}{\partial\phi^2}\right|_{\phi=\phi^{min}} = 2m_H^2.}$$





- the horizontal shape of V_{Higgs} (e.g. ϕ^{min}, υ) does NOT depend on m_H;
- the vertical shape is $\propto m_H^2$ (show m_H = 100 / 125 GeV);
- the parabola at ϕ_{min} represents a particle of mass m_H = the Higgs boson !

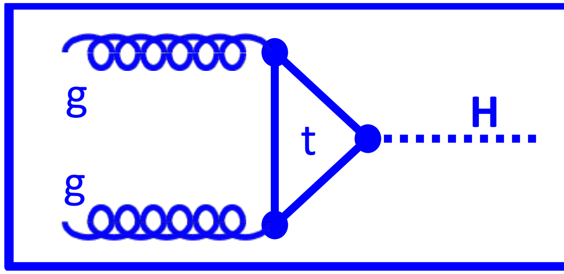
the MSM Higgs boson: all SM couplings

	**Hff**	g_{Hff}	$= m_f / \upsilon = (\sqrt{2}\, G_F)^{1/2}\, m_f$	$\times (-i)$
	**HVV**	g_{HVV}	$= 2m_V^2/\upsilon = 2(\sqrt{2}\, G_F)^{1/2}\, m_V^2$	$\times (ig_{\mu\nu})$
	**HHVV**	g_{HHVV}	$= 2m_V^2/ \upsilon^2 = 2\sqrt{2}\, G_F\, m_V^2$	$\times (ig_{\mu\nu})$
	**HHH**	g_{HHH}	$= 3m_H^2 / \upsilon = 3(\sqrt{2}\, G_F)^{1/2}\, m_H^2$	$\times (-i)$
	**HHHH**	g_{HHHH}	$= 3m_H^2 / \upsilon^2 = 3\sqrt{2}\, G_F\, m_H^2$	$\times (-i)$

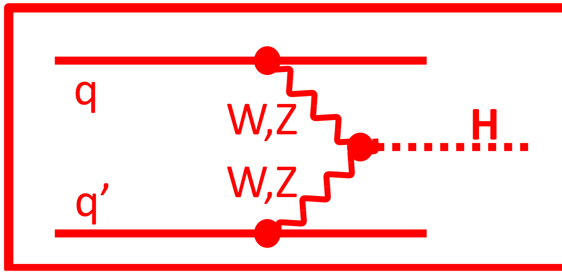
A. Djouadi, Phys. Rep., 457 (2008) 1.

Higgs properties: production dictionary

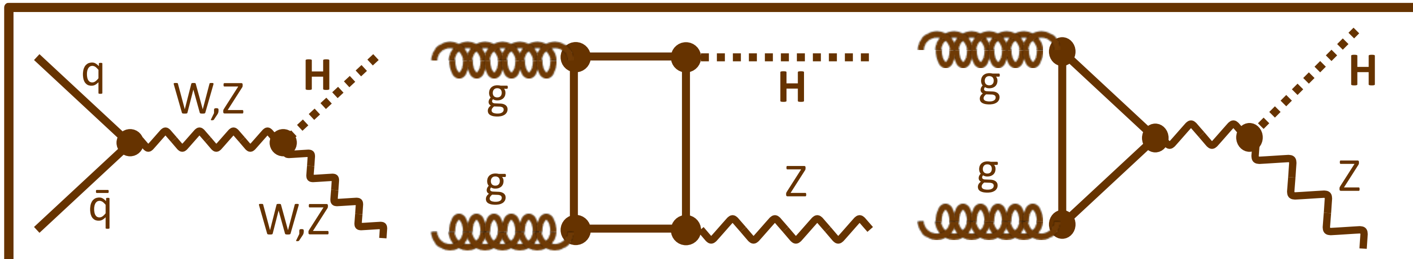
- Higgs production processes in hadron colliders, with their usual names;
- only main diagrams, many others less important (e.g. single top);
- emphasis on detectability → some particles in final state may help;
- in the following, W and W* both appear as W [same for Z/Z*].



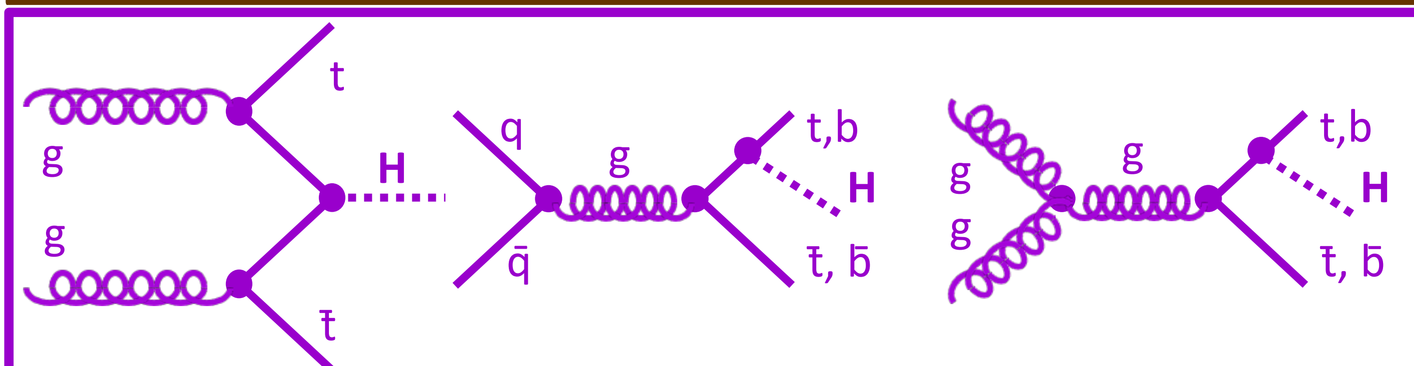
ggF — gluon gluon fusion



VBF — vector boson fusion



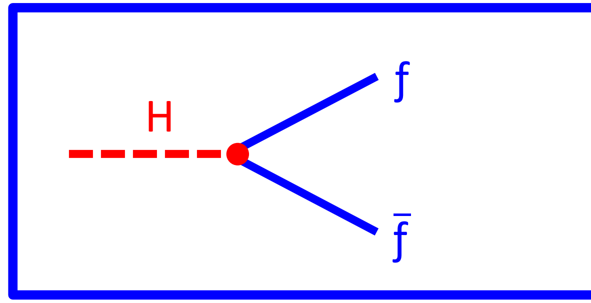
VH [= WH + ZH] — VH production



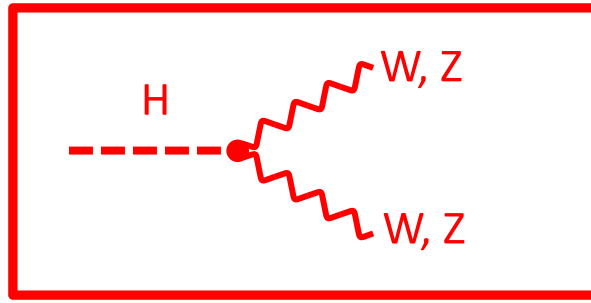
tt̄H — tt̄H process

Higgs properties: decay dictionary

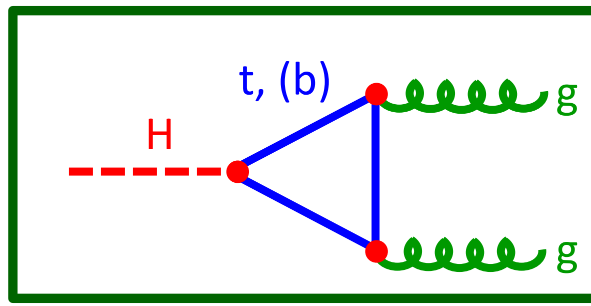
- Higgs decay modes;
- in the diagrams, "f" represents any fermion; however the coupling (and therefore the BR) is strongly dependent on its mass;
- here W and W* both appear as W [same for Z/Z*].



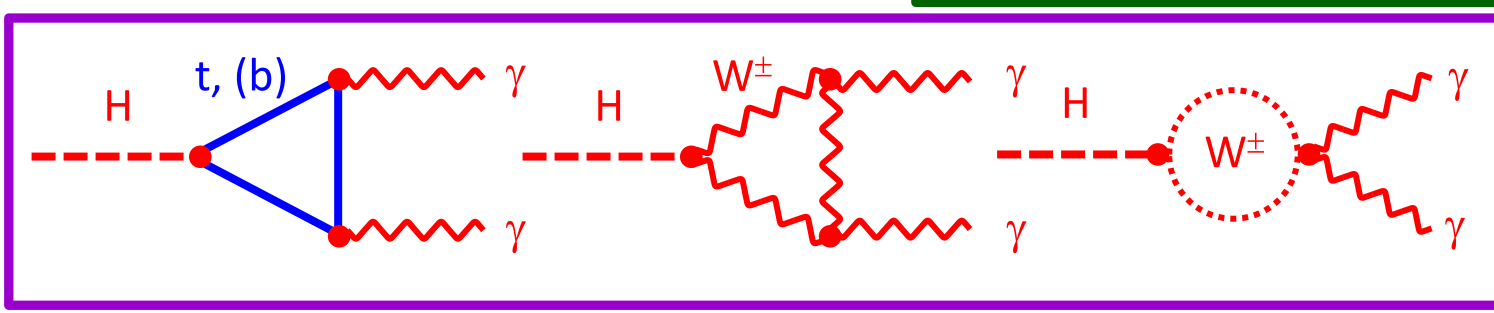
$H \to f\bar{f}$



$H \to WW / ZZ$



$H \to gg$



$H \to \gamma\gamma$

Higgs properties: decays f, W, Z

- at "tree level" the partial width for the Higgs decay into a pair of real fermions (f=quarks, leptons) or real gauge bosons (V = W, Z) is given by :

$$\Gamma(H \to f\bar{f}) = \frac{c_f}{4\pi\sqrt{2}} G_F m_H m_f^2 \beta_f^3;$$

$$\beta_f = \sqrt{1 - \frac{4m_f^2}{m_H^2}}\ ; \quad c_f = \begin{cases} 1\ [\text{leptons}] \\ 3\ [\text{quarks}] \end{cases};$$

$$\Gamma(H \to VV) = \delta_V \frac{G_F m_H^3}{64\pi\sqrt{2}} \beta_V (4 - 4x_V + 3x_V^2);$$

$$\beta_V = \sqrt{1 - \frac{4m_V^2}{m_H^2}}\ ;\ x_V = \frac{4m_V^2}{m_H^2};\ \delta_V = \begin{cases} 2\ [W^\pm] \\ 1\ [Z] \end{cases};$$

- therefore, for m_H small (m_H < 110 GeV), $H \to b\bar{b}$ dominates (see § LEP);

- if m_H > 2 $m_{W,Z}$, the largest BR would be for $H \to W^+W^-$, $H \to ZZ$;

- in the region m_H = 110 ÷ 180 GeV, the decays into W*W and Z*Z are important (also because of their detectability); but the formula with β_V assumes real W/Z; when virtual W*/Z* are required, the computation is different; for m_H=125 GeV, results are reported below;

- when m_H increases, new decay channels open; moreover, the partial widths also increase; therefore Γ_{tot} is a strong function of m_H :

$$\Gamma_{tot}(m_H) = \sum_j \Gamma(H \to f_j \bar{f}_j) + \sum_k \Gamma(H \to V_k^{(*)} V_k);$$

$$BR(H \to X) = \Gamma(H \to X) / \Gamma_{tot} = BR(m_H);$$

both Γ_{tot} and BR function of m_H.

Higgs properties: decays gg, γγ

- in addition, also few "higher order" decays (γγ, Zγ, gg);
- the decays H→gg and H→γγ (much less H→Zγ) are important for the discovery :
  - ➢ the decay H→gg is large, although not easy to identify (→ 2 jets, large QCD bckgd);
  - ➢ the decay H→γγ is rare, but has high efficiency and little bckgd (see later);
- complete formulas in references :

$$\Gamma(H\rightarrow gg)=\frac{1}{36\pi^3\sqrt{2}}\alpha_s^2 G_F m_H^3 \left|I_{gg}\right|^2;$$

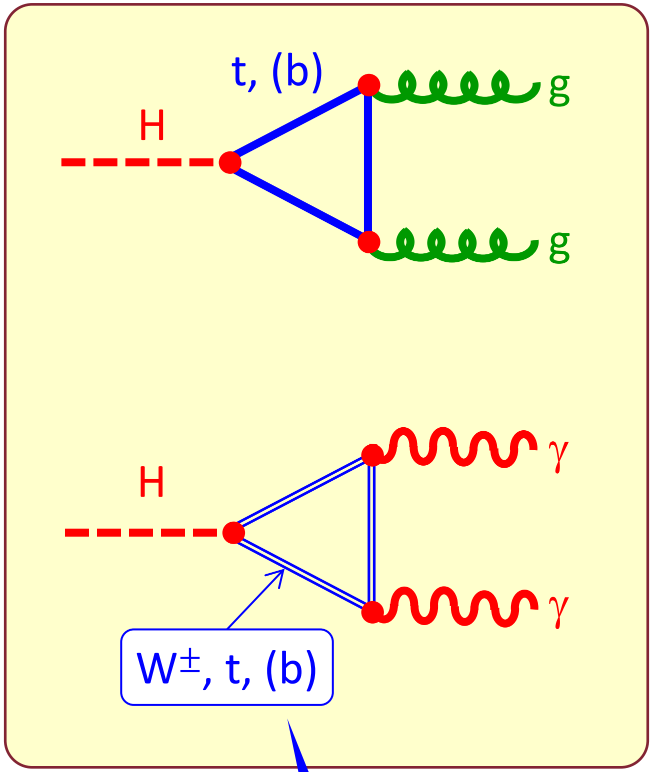
$$I_{gg}=\sum\nolimits_q I_q(m_q^2/m_H^2)=f(m_H)\sim 0.1\div 1;$$

(sum over quarks, important for q=t);

$$\Gamma(H\rightarrow \gamma\gamma)=\frac{1}{8\pi^3\sqrt{2}}\alpha_{em}^2 G_F m_H^3 \left|I_{\gamma\gamma}\right|^2;$$

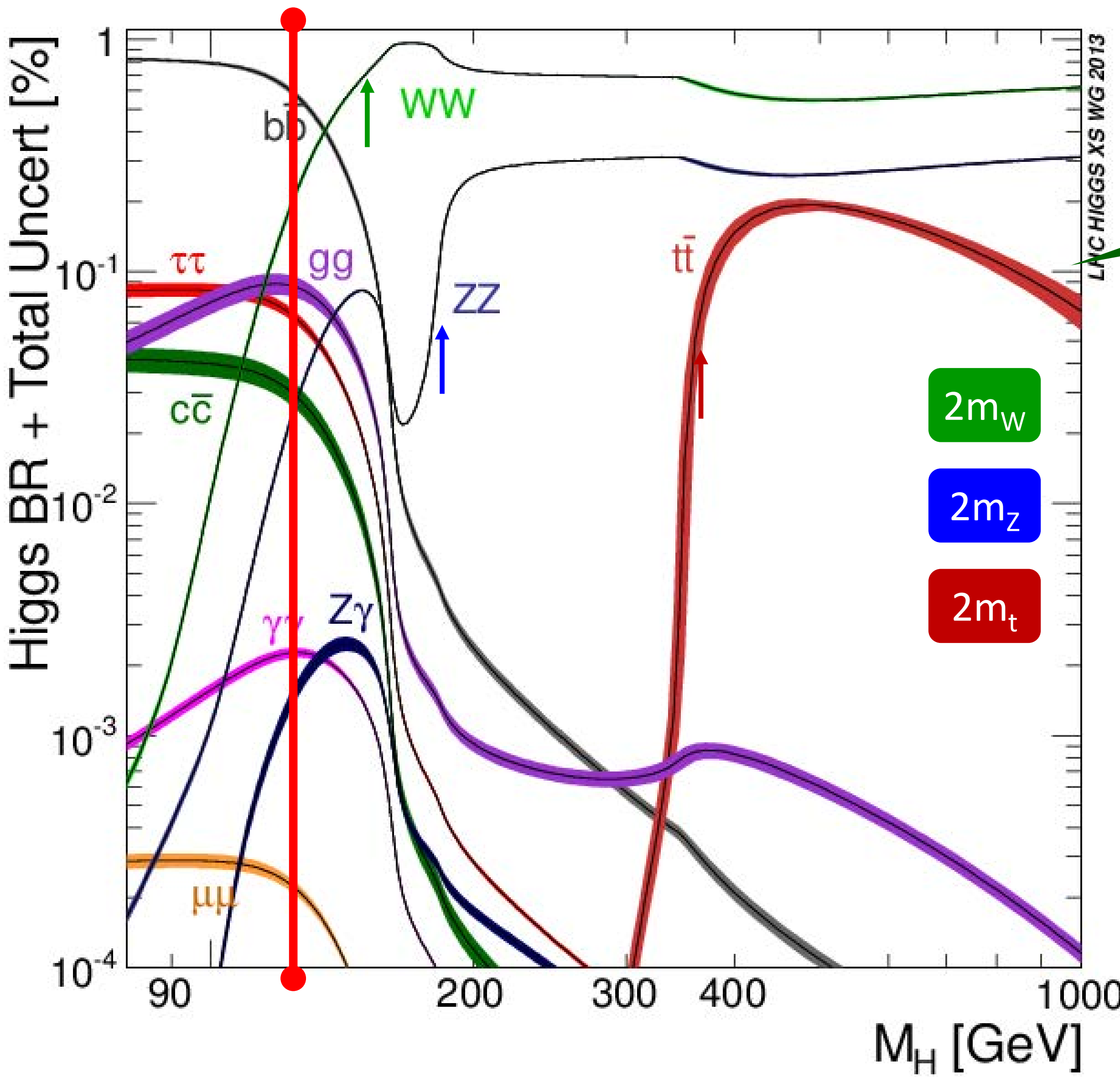
$$I_{\gamma\gamma}=\sum\nolimits_f c_f q_f^2 I_f(m_f^2/m_H^2)+I_W=f(m_H)\sim 1\div 10;$$

[sum over charged fermions f, c_f=1($\ell^\pm$) or 3(q)].



for γγ, interference W ↔ t large and negative (a nice test of the SM).

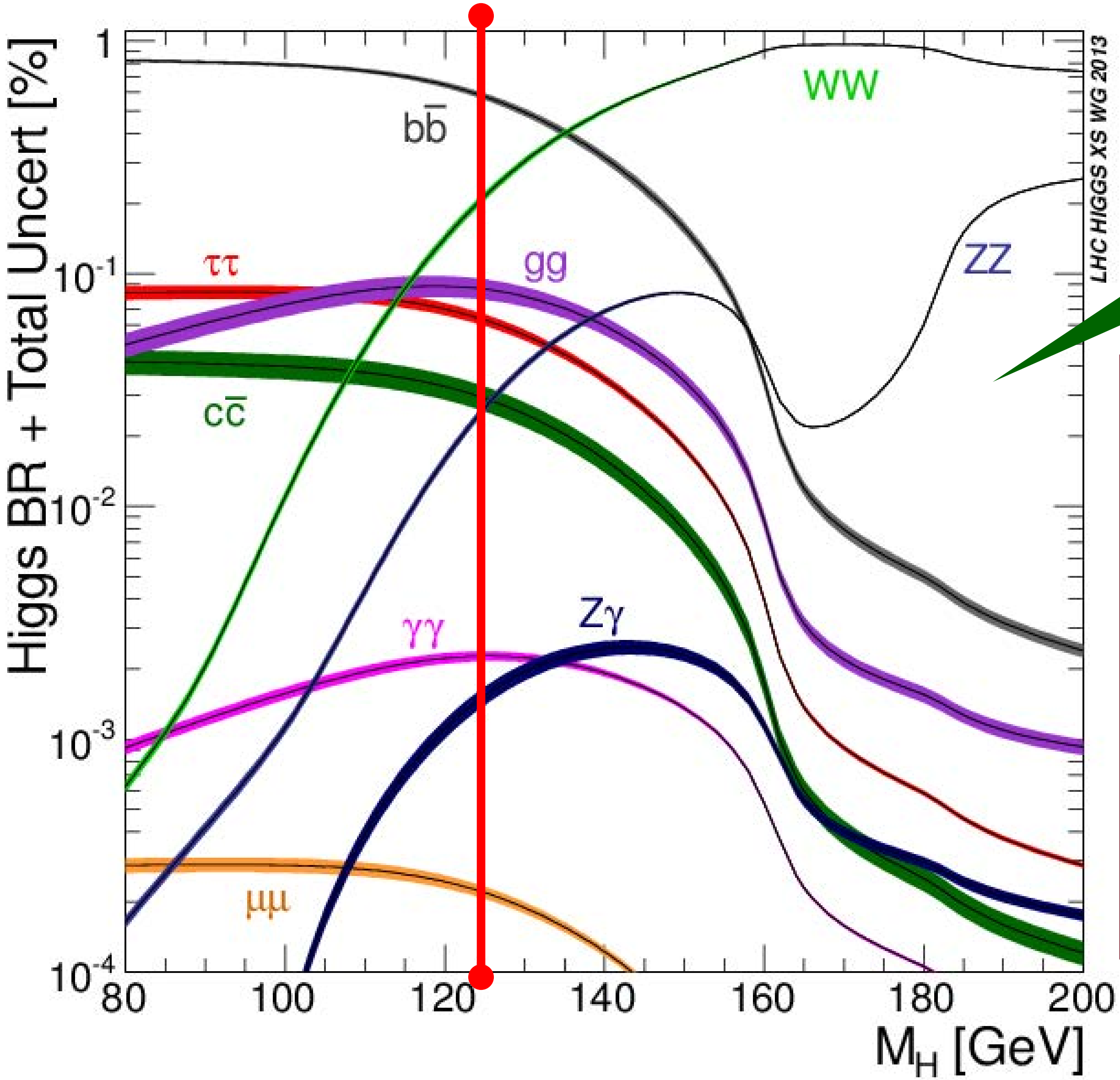
Higgs properties: decay BR vs H mass



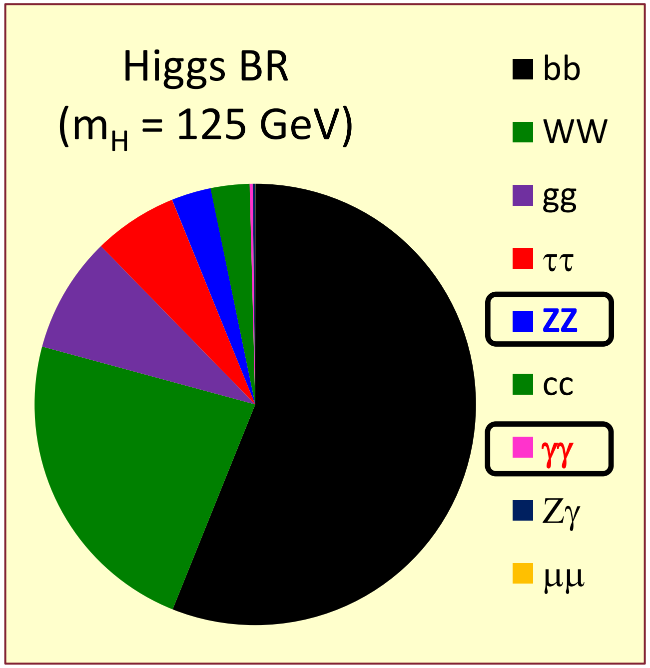
Unlike LEP2, in the LHC energy range, the Higgs boson decay mode is highly variable
→ challenge for the experiments.

m_H = 125 GeV [*CERN-2013-004*]		
decay mode	Γ (MeV)	BR (%)
$b\bar{b}$	2.35	57.5
$W^{\pm}W^{\mp *}$	0.886	21.7
gg	0.349	8.6
$\tau^+\tau^-$	0.257	6.3
ZZ*	**0.110**	**2.7**
$c\bar{c}$	0.118	2.9
γγ	**9.3E-03**	**0.23**
Zγ	6.3E-03	0.15
$\mu^+\mu^-$	8.9E-04	0.022
sum	4.08	100

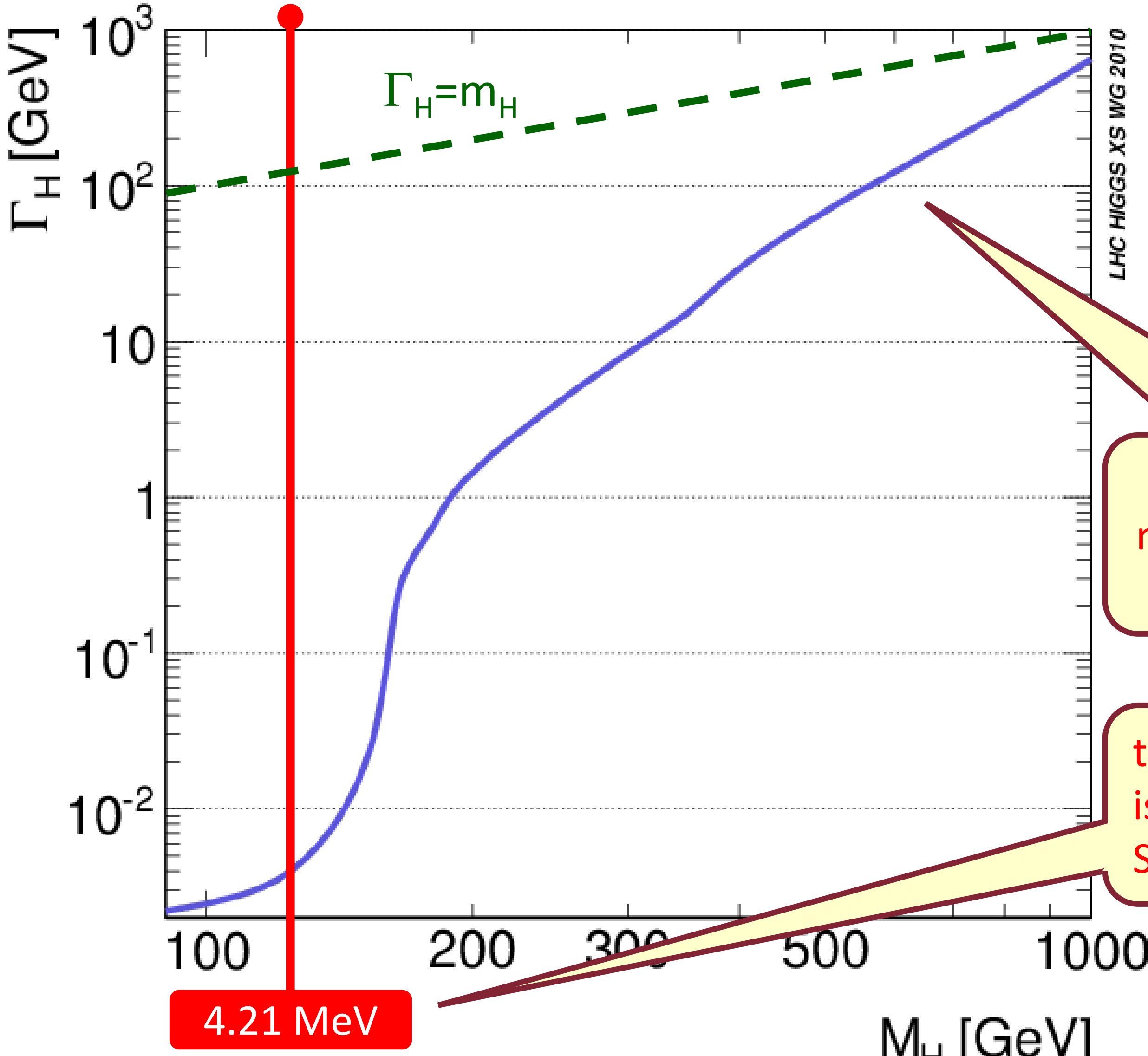
Higgs properties: BR($80 < m_H < 200$ GeV)



enlarge for $80 < m_H < 200$ GeV.



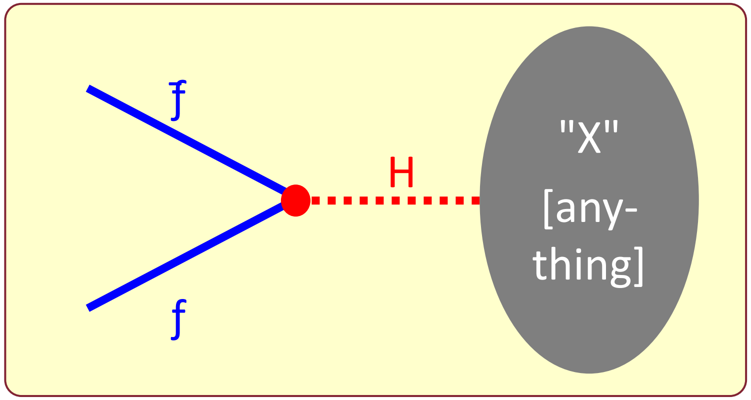
Higgs properties: full width vs m_H



roughly $\propto m_H^3$
$m_H \approx 1.4$ TeV $\rightarrow \Gamma_{tot} \approx m_H$,
not anymore a particle

the direct measure of Γ_H is a powerful test of the SM: ideas are welcome

Higgs properties: formation in $\ell^+\ell^-$



Question (for a lepton collider, not for LHC): what about the direct formation ($f\bar{f} \to H \to X$) in the s channel ?

Answer: it is depressed by the H coupling with low-mass fermions ($\Gamma_f \propto m_f^2$).

Compute it for a hypotetical $\mu^+\mu^-$ machine:

$$\sigma(f\bar{f} \to H \to X) = \frac{4\pi\Gamma_f\Gamma_X}{\left(s-m_H^2\right)^2+\Gamma_H^2 m_H^2} = \frac{4\pi\Gamma_f\Gamma_X}{\Gamma_H^2 m_H^2}\left[\frac{\Gamma_H^2 m_H^2}{\left(s-m_H^2\right)^2+\Gamma_H^2 m_H^2}\right]$$

$$\xrightarrow[X=\text{all}]{} \frac{4\pi}{m_H^2}\frac{\Gamma_f}{\Gamma_H}\left[\frac{\Gamma_H^2 m_H^2}{\left(s-m_H^2\right)^2+\Gamma_H^2 m_H^2}\right] \xrightarrow[f\bar{f}=\mu^-\mu^+,\ \sqrt{s}=m_H=125\text{ GeV}]{} 64\text{ pb.}$$

see § 3 [quoted for $e^+e^- \to J/\psi$] :

$$\sigma(ab \to J/\psi \to f\bar{f}, \sqrt{s}) = \frac{16\pi}{s}\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{f\bar{f}}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

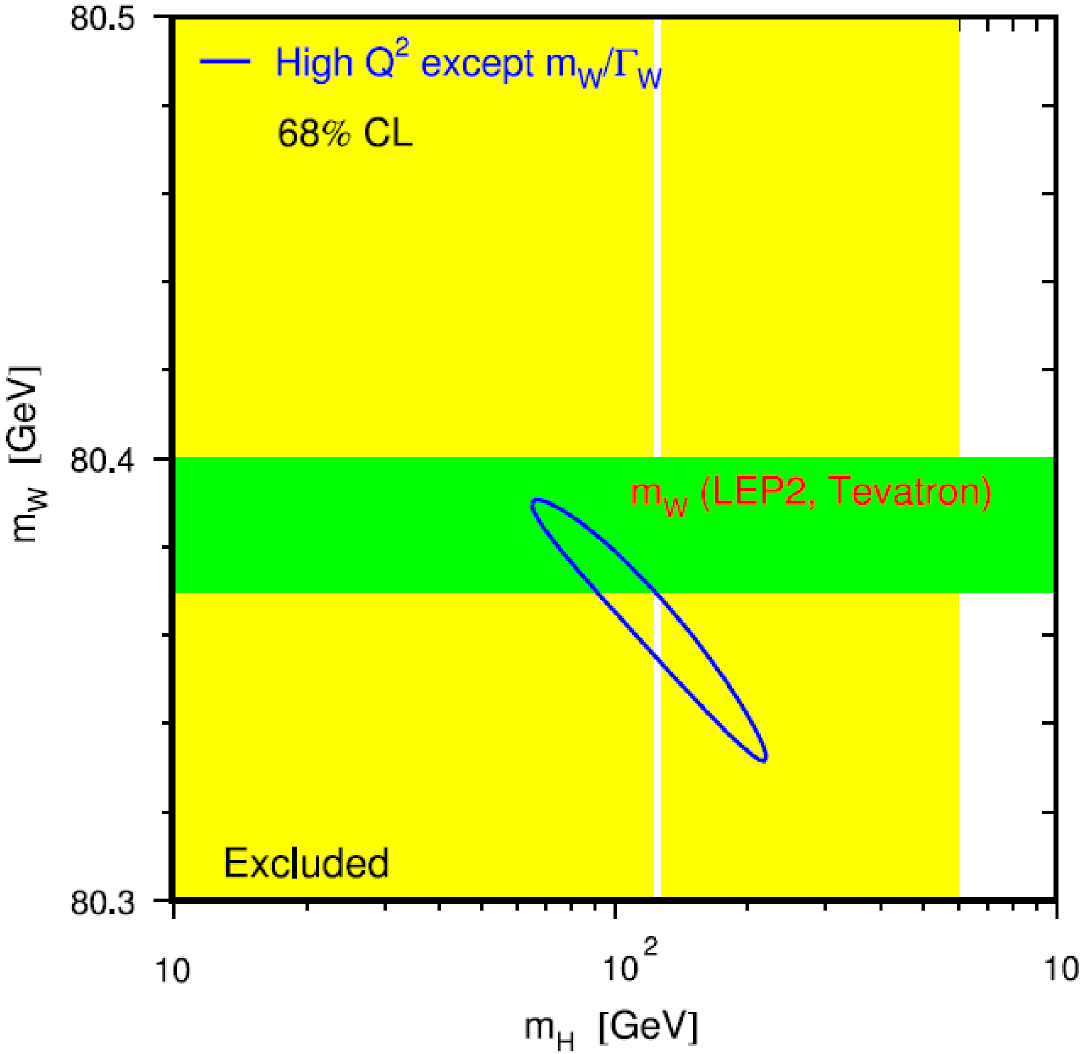
for e^+e^-, factor $(m_e/m_\mu)^2 \approx 1/40{,}000$:

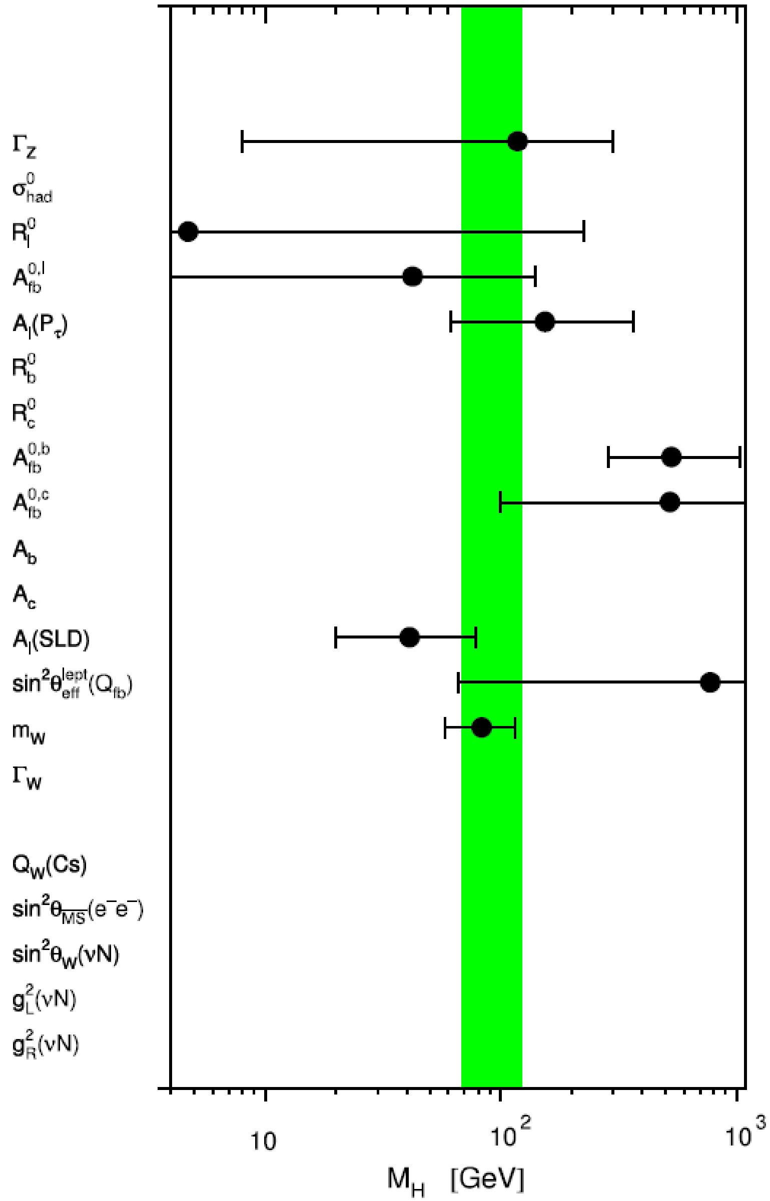
- $\to$ impossible for electron colliders;
- $\to$ one of the main motivations for muon colliders.

Higgs — pre-LHC : LEP legacy

bottom : m_W vs m_H (strong correlation);

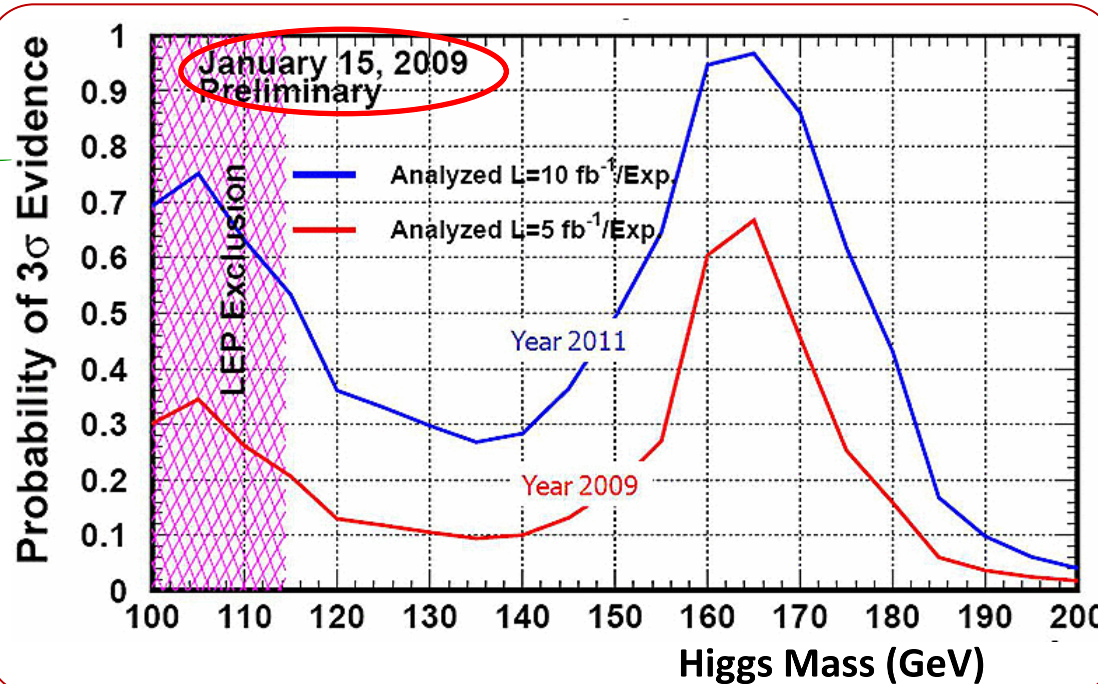
right : individual meas. contribution;

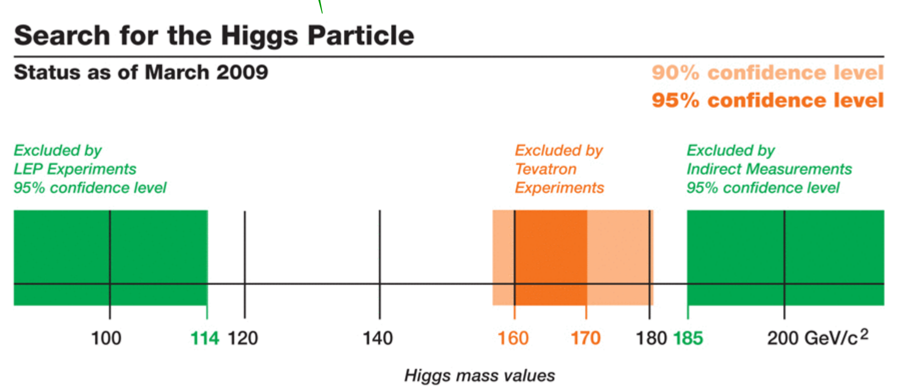




Higgs — pre-LHC : Tevatron legacy (1)

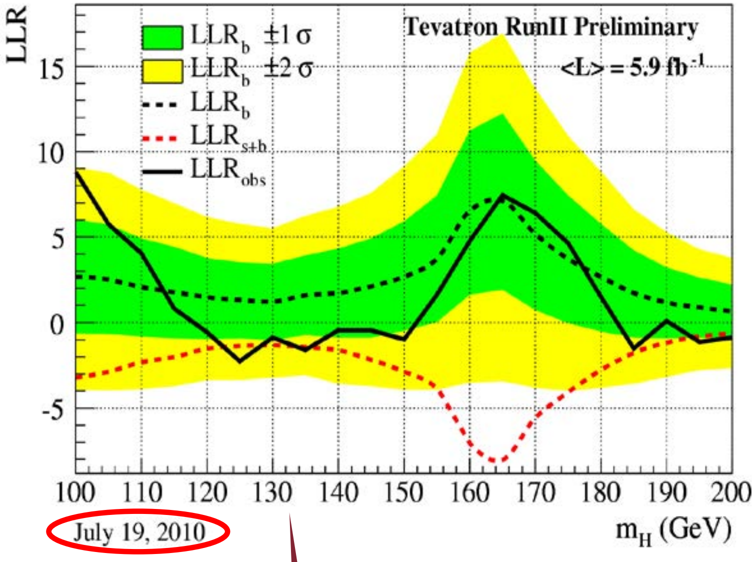
CDF + D0 combined





"3σ evidence" has **never** been used at LEP or at LHC; although well defined and politically understandable, it is a useless complication.

Higgs — pre-LHC : Tevatron legacy (2)

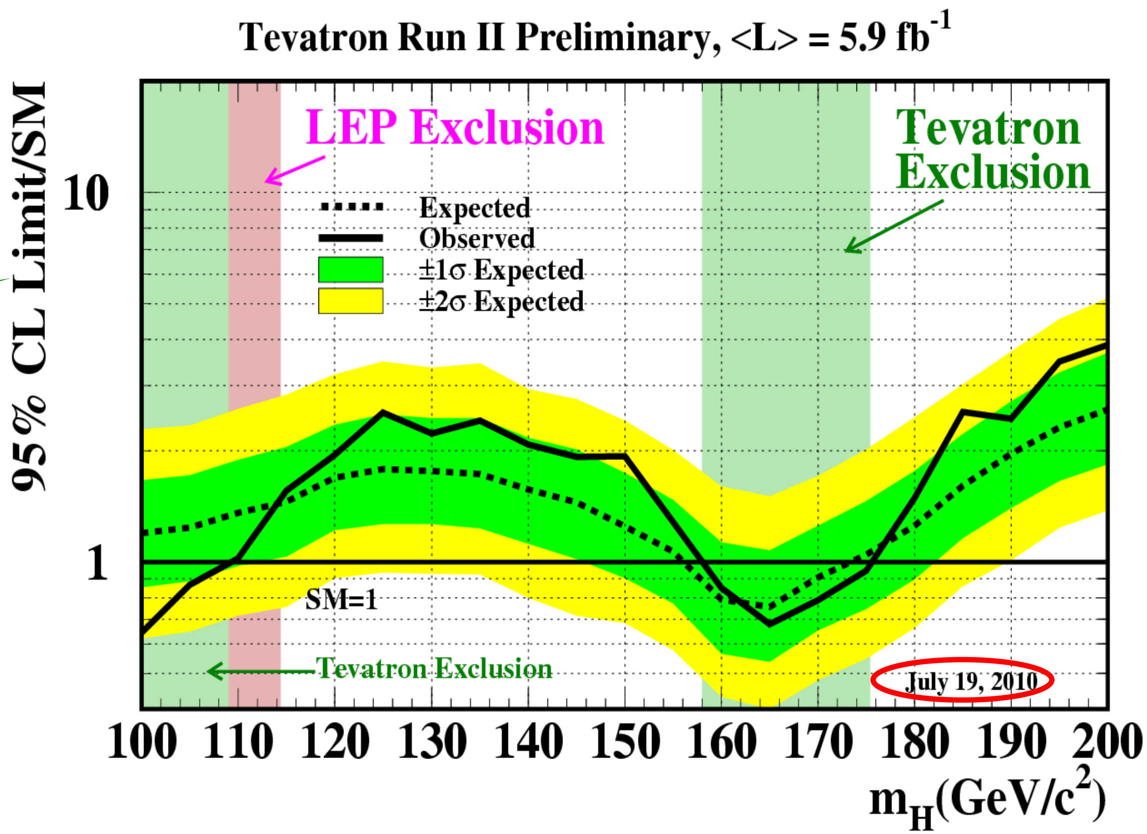


at LEP, for m_H < 115 GeV, the value of n (= $\mathscr{L}_{int}\ \varepsilon_s\ \sigma_s$) was monotonic and strongly decreasing with m_H;

on the contrary, for higher m_H, due to the different decay modes with different efficiency, n has various maxima; the exclusion interval breaks accordingly.

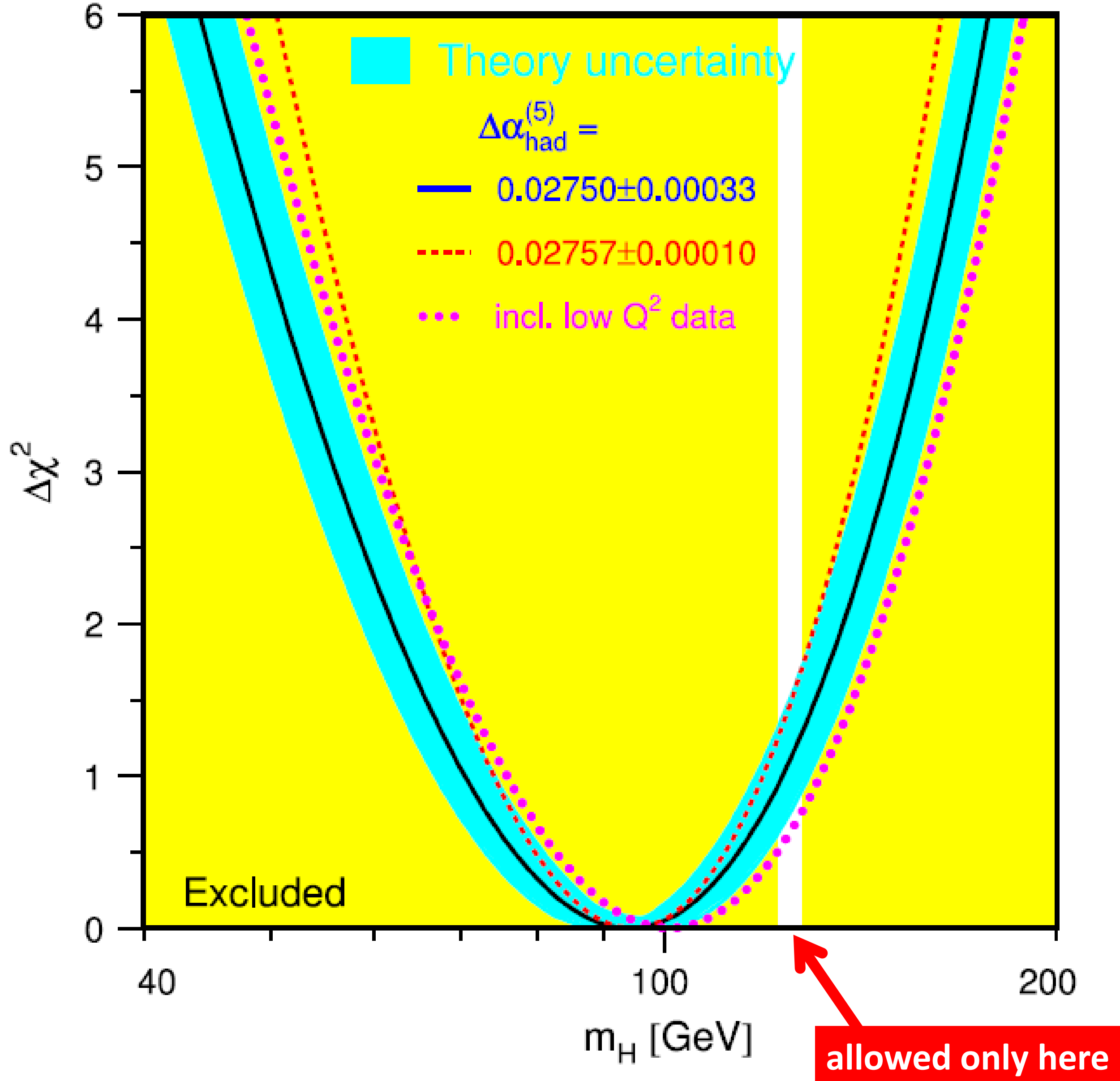
i.e. "μ"

"LLR" ≡ log likelihood ratio = $-2\ell n(\Lambda_s/\Lambda_b)$



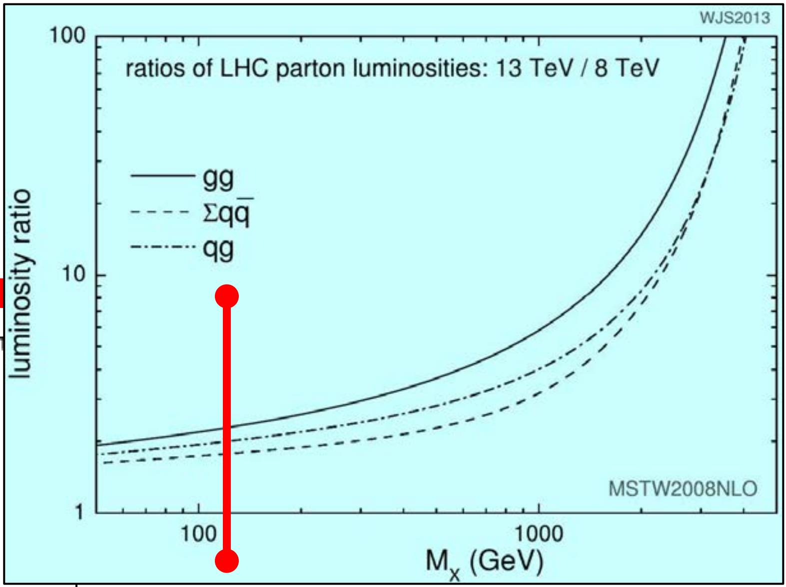
Higgs — pre-LHC : complete legacy

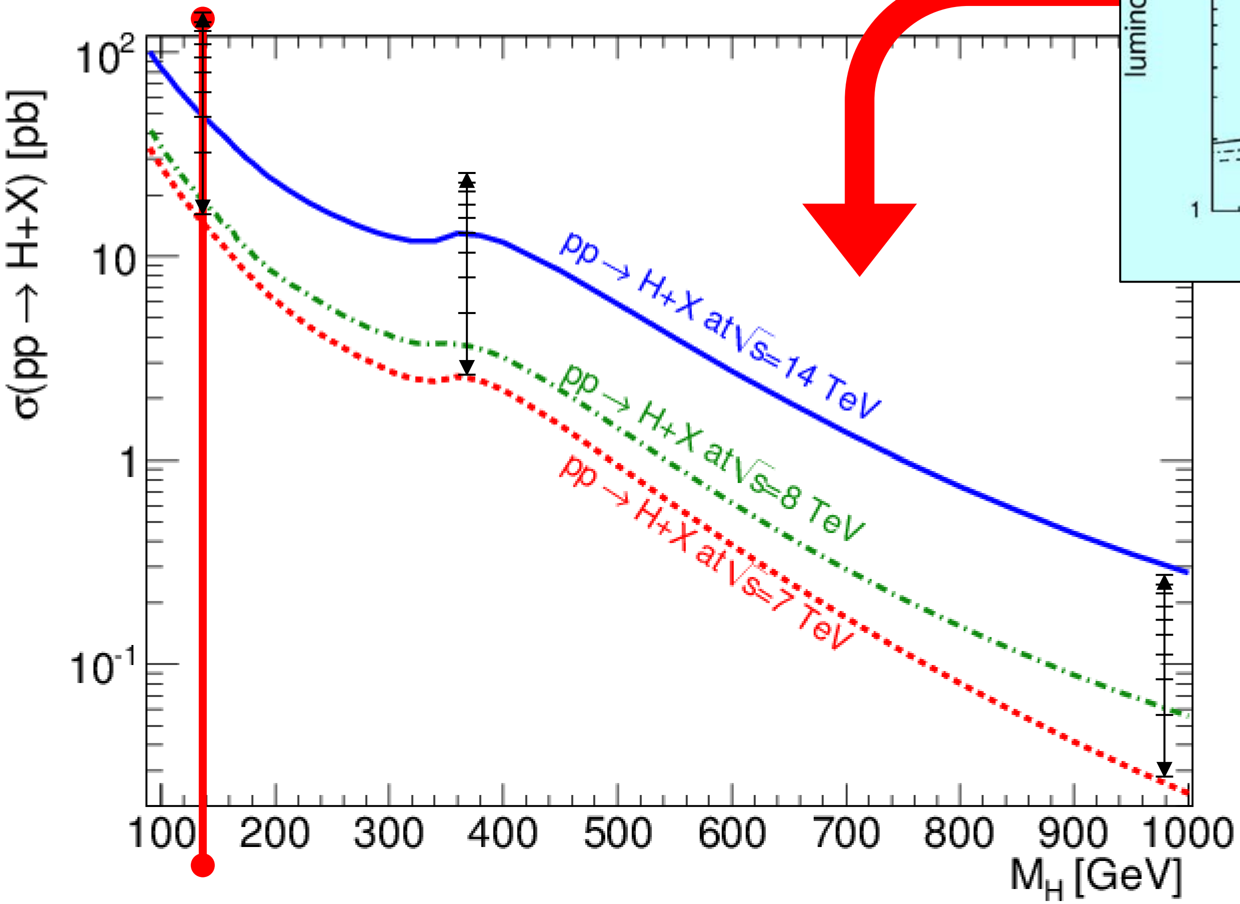
- the (in)famous "blueband", already discussed, wants a light Higgs; it includes all the known info, BUT the direct search at LEP, Tevatron and LHC, shown separately;
- instead, the yellow bands represent the result of the direct searches [NB : no experimental correlation with the blueband];
- the yellow bands varied a lot with time; the present figure refers to just before 2012; it includes TeVatron (160-170 GeV excluded) and the first LHC data;
- everything is now ready to show the direct LHC search.



Higgs — LHC predictions : production

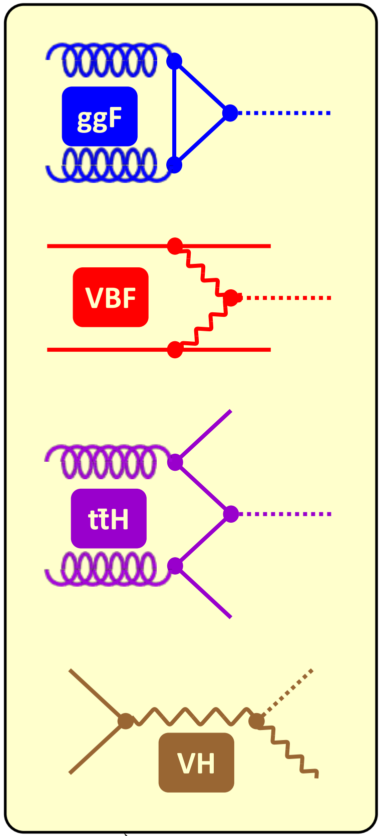
- the real process is at partonic level;
- parton densities (= PDF) at fixed x, depend on √s;
- they are strong functions of $\hat{s}$ (= M_x).



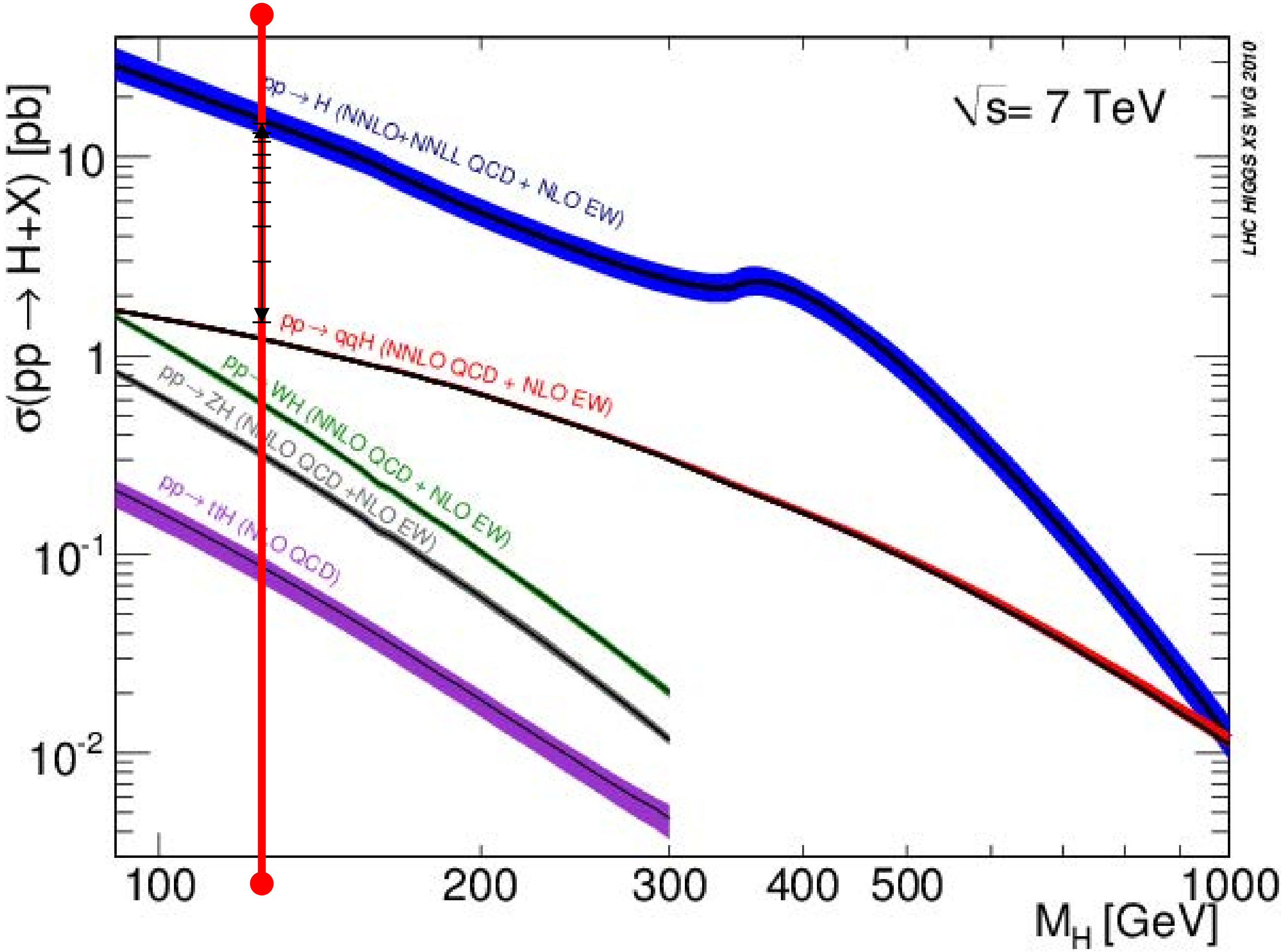


- σ_{125} = few × 10 pb;
- many BR decays, some unmeasurable;
- only observables $\sigma \times BR_i$;
- notice the scales.

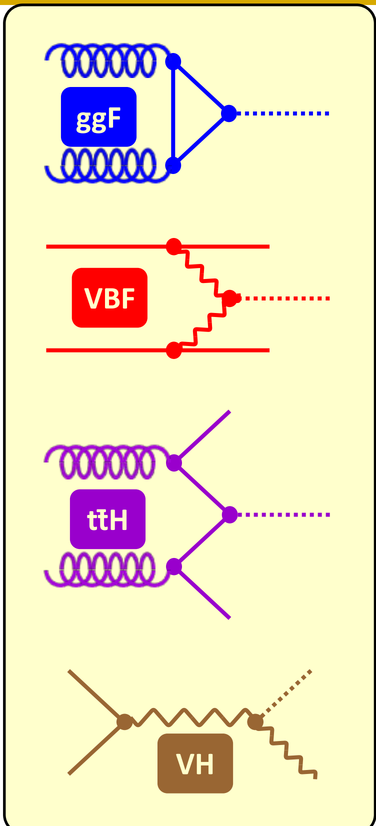
Higgs – LHC predictions : σ_H @ 7 TeV



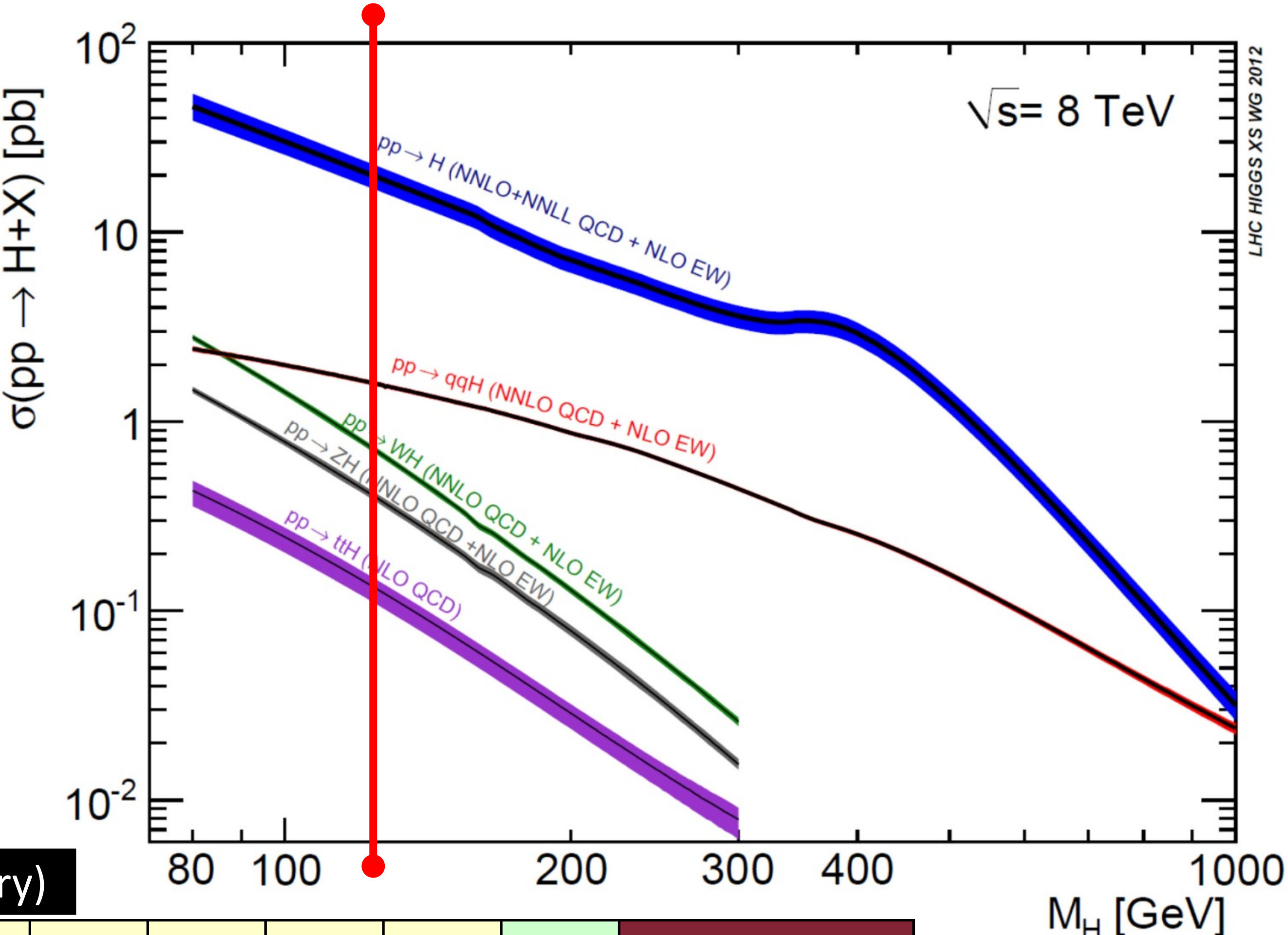
(see dictionary)



Higgs — LHC predictions : σ_H @ 8 TeV

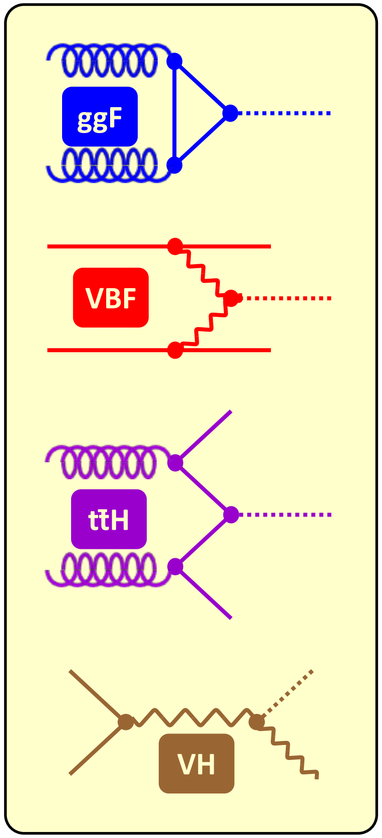


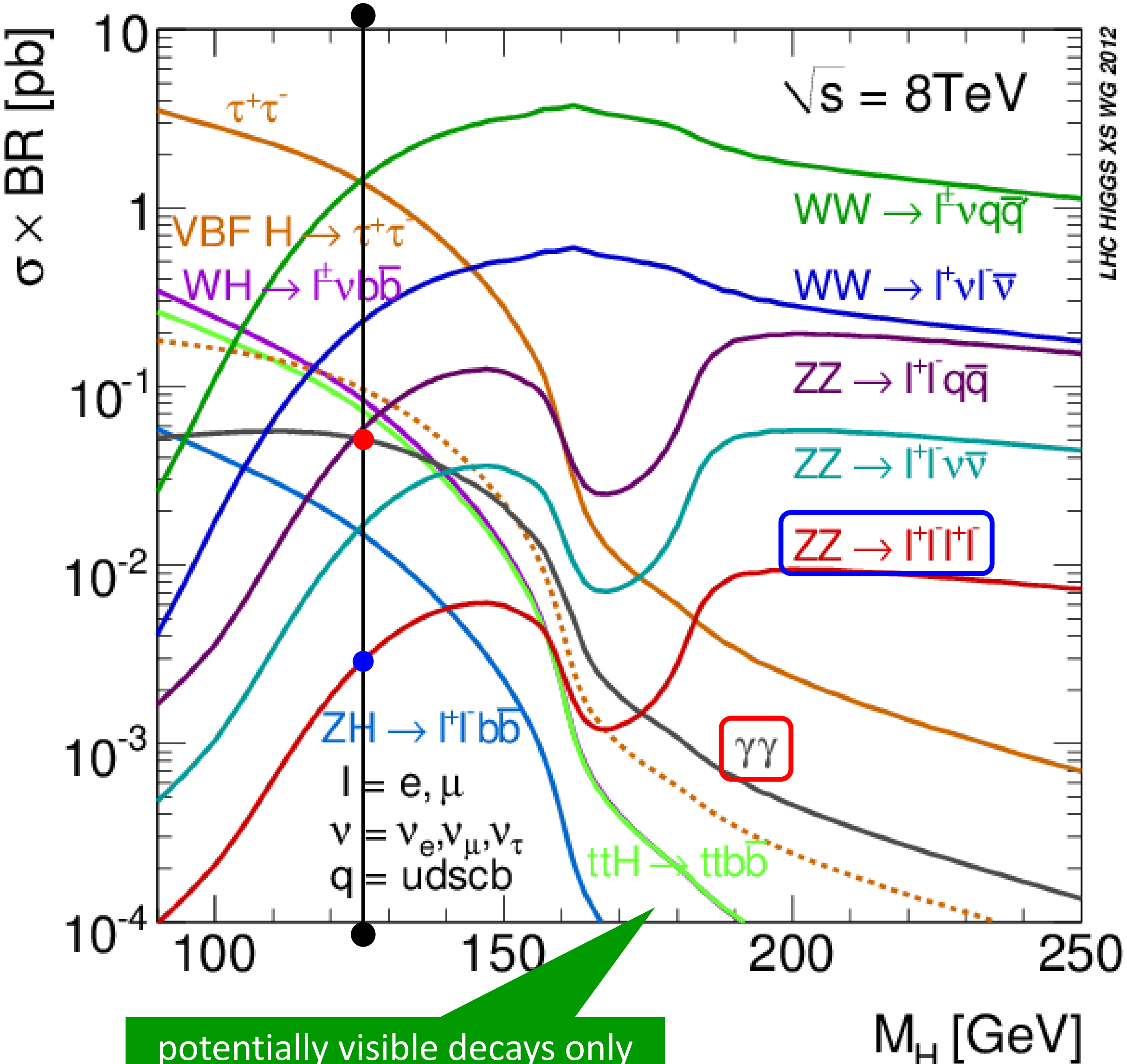
(see dictionary)



√s (TeV)	ggF	VBF	WH	ZH	tt̄H	(...)	Sum	σ(pp → HX) (pb) [computed for m_H = 125 GeV]
7	15.0	1.22	0.58	0.33	0.09	0.2	17.4	
8	19.2	1.58	0.70	0.41	0.13	0.3	22.3	
14	49.2	4.15	1.47	0.86	0.59	…	56.3	

Higgs — LHC predictions : $\sigma_H \times BR$

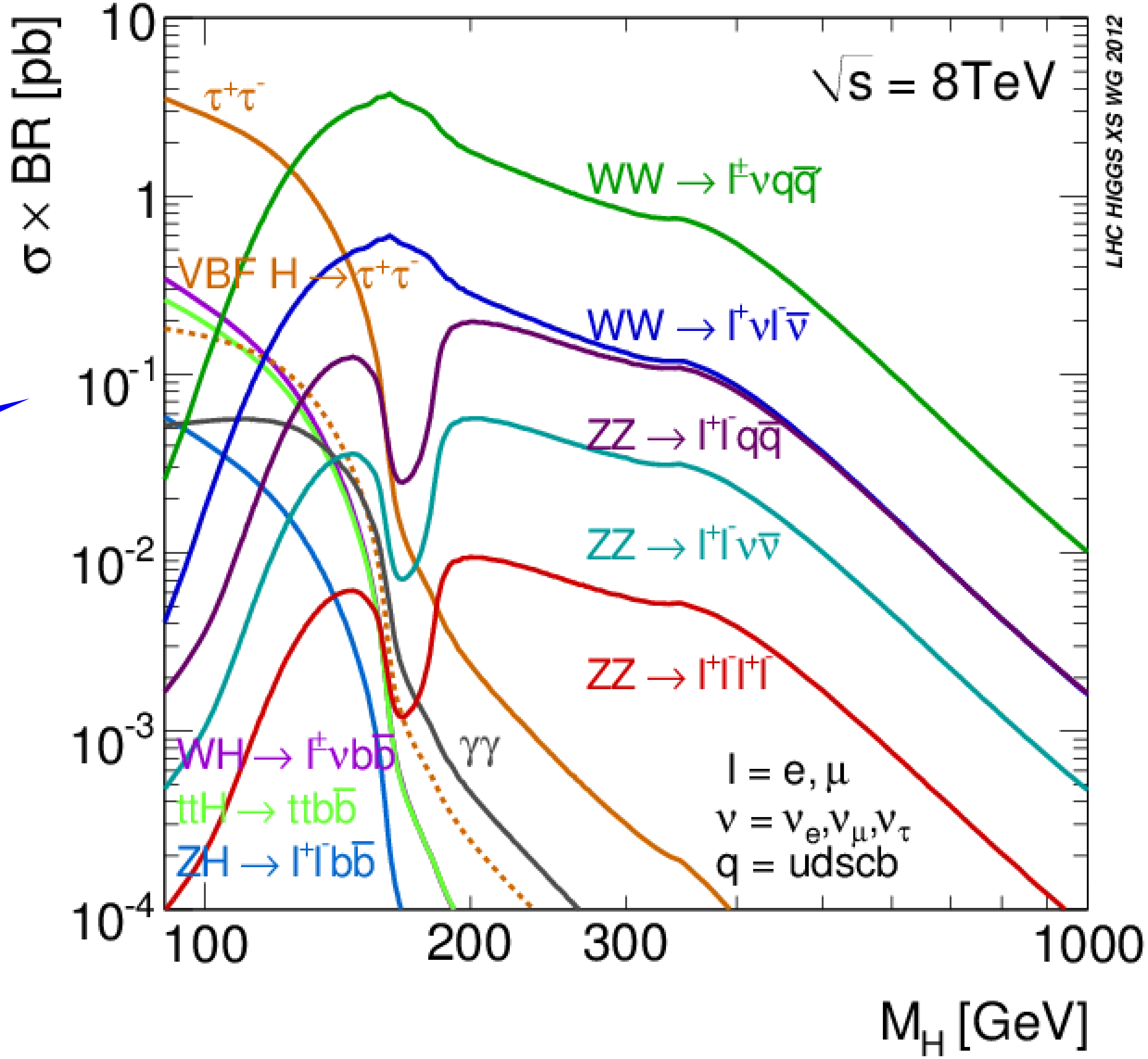




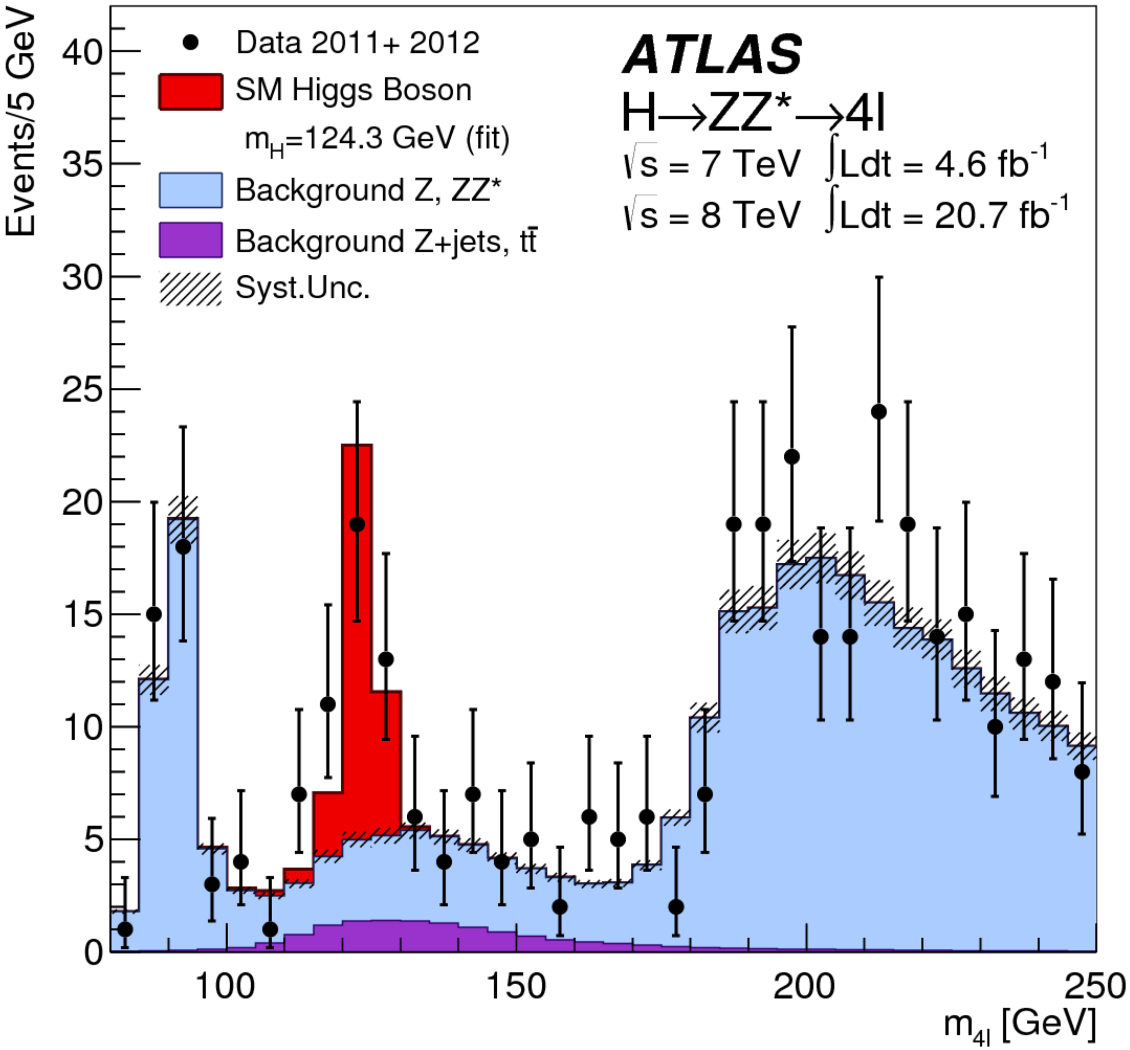
potentially visible decays only
[e.g. WW → $q\bar{q}q\bar{q}$ is missing]

Higgs — LHC predictions : $\sigma_H \times BR$

same plot, different scale, to show the high m_H behavior (not irrelevant, even after Higgs discovery, because of possible extensions of the Higgs sector, even in **non-minimal SM**).



Higgs discovery : H → ZZ* - ATLAS



looking for the Higgs boson !!!

$H \rightarrow ZZ^* \rightarrow \ell^+\ell^-\ell^+\ell^-$

Test mass ~ 125 GeV

(exact values from mass fits, small variations – within errors)

more results in >2013, some shown below.

1. ATLAS animated gifs: https://twiki.cern.ch/twiki/bin/view/AtlasPublic/HiggsPublicResults#Animations
2. ditto for CMS: https://twiki.cern.ch/twiki/bin/view/CMSPublic/Hig13002TWiki

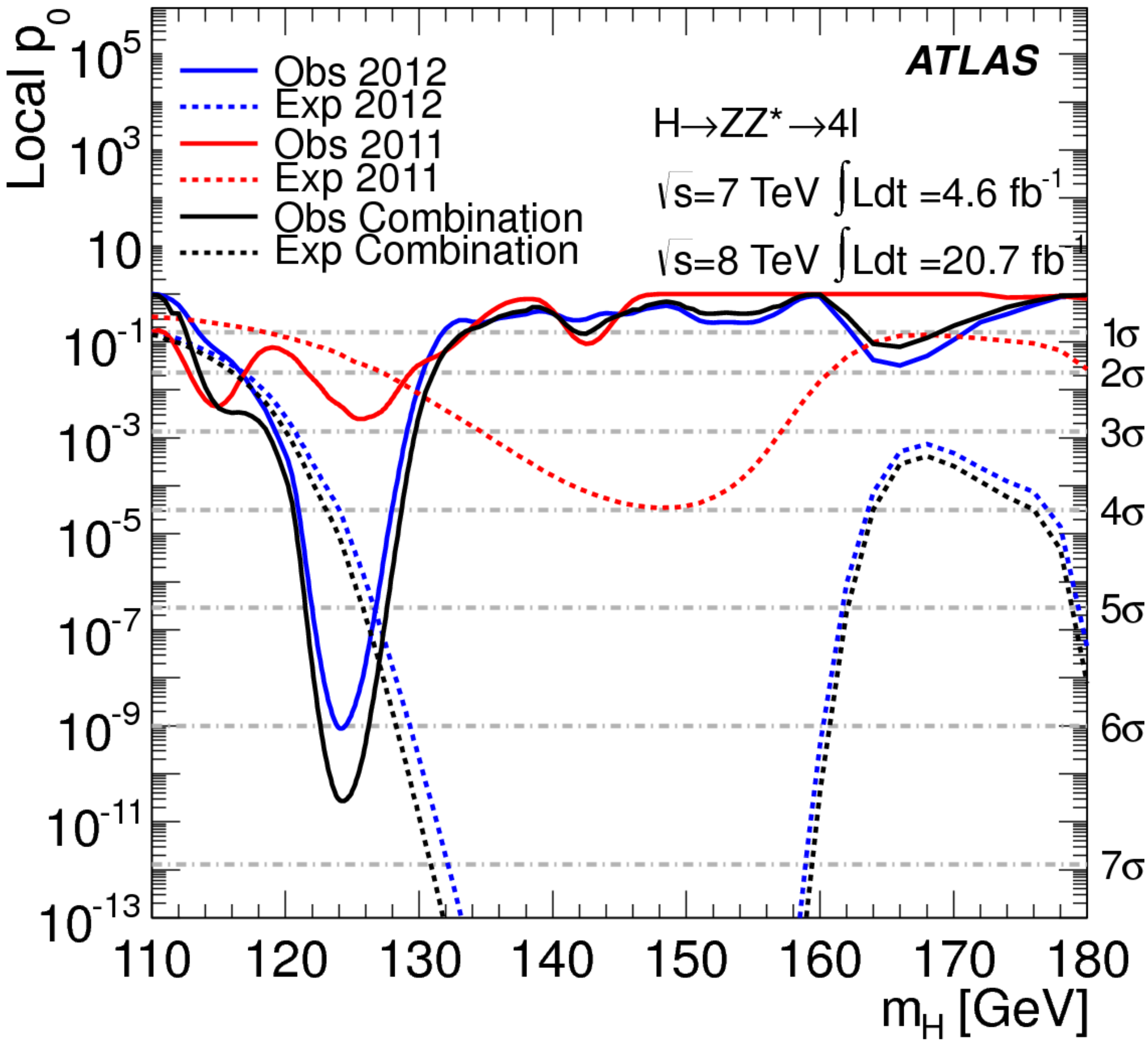
Higgs discovery : H → ZZ* - ATLAS p-value

ATLAS 4 ℓ±

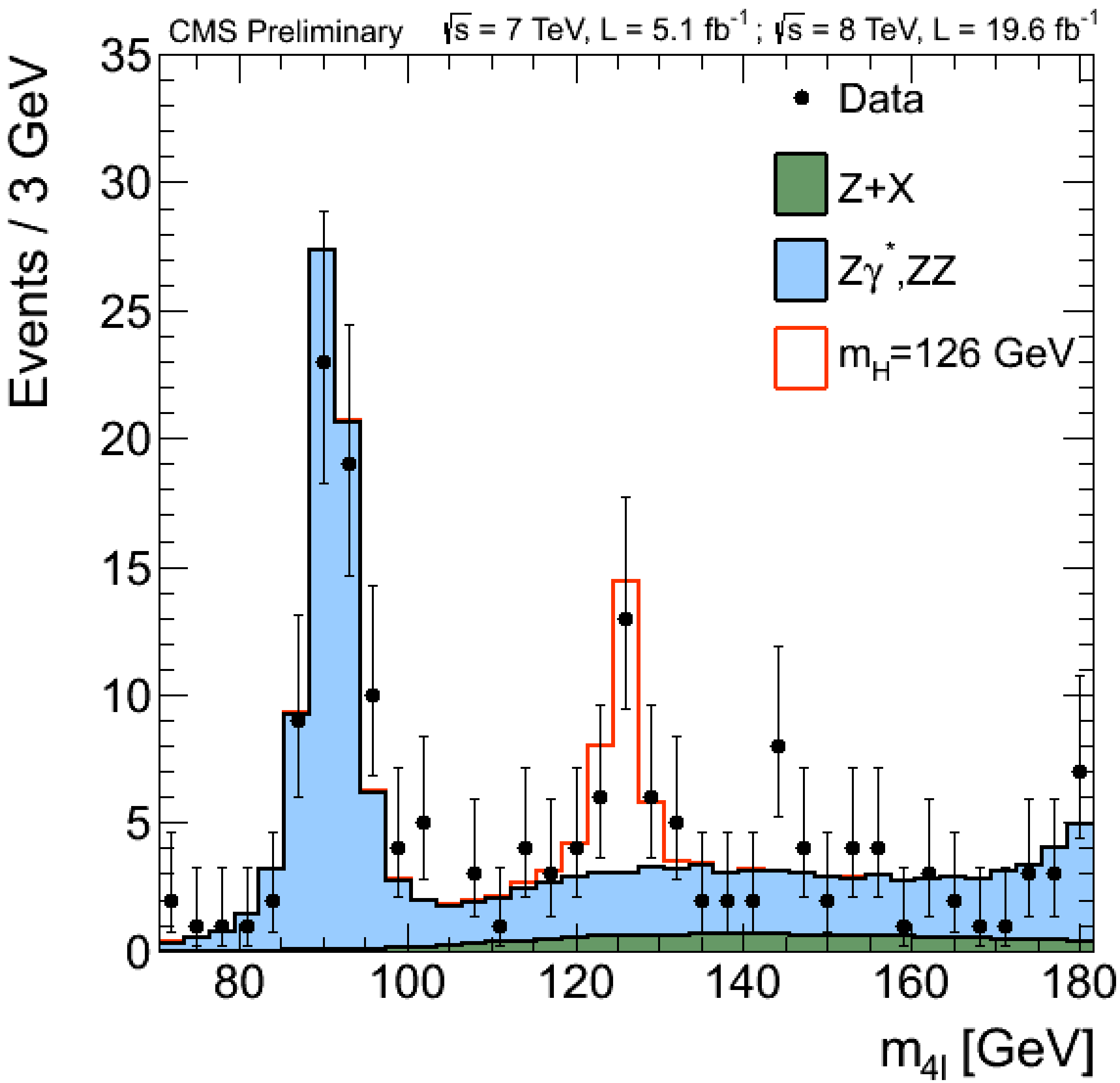
- 2011 : some excess, below 3σ;
- **2012 : ~ 6 σ;**
- combined : between 6 and 7 σ.

more than expected, but **not incompatible**.

NB. obs (–) and exp (- -) are expected to agree ONLY at m_H^{obs}.



Higgs discovery : H → ZZ* - CMS

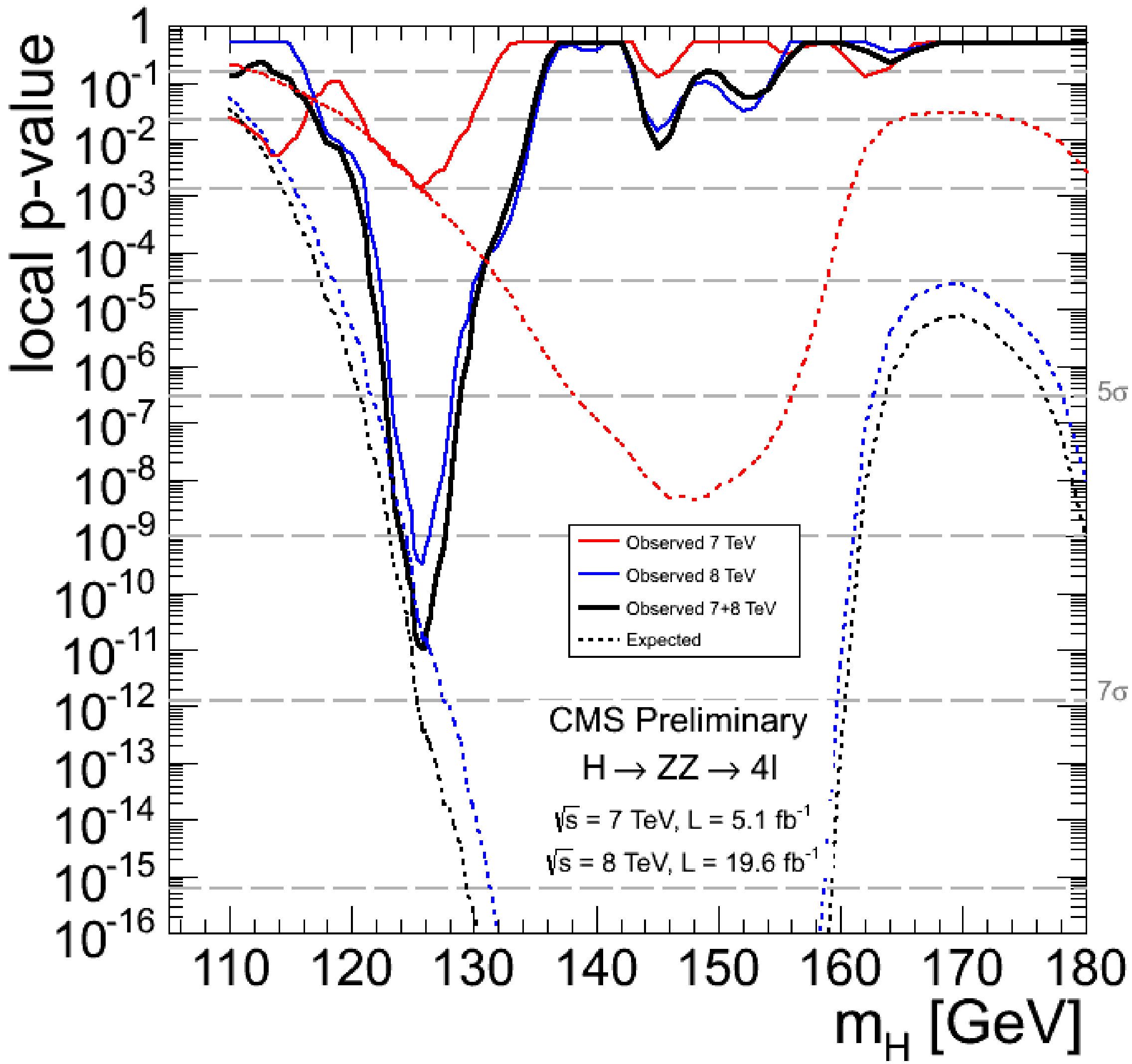


$H \to ZZ^* \to \ell^+\ell^-\ell^+\ell^-$

Test mass ~ 125 GeV

(exact values from mass fits, small variations – within errors)

Higgs discovery : H → ZZ* - CMS p-value



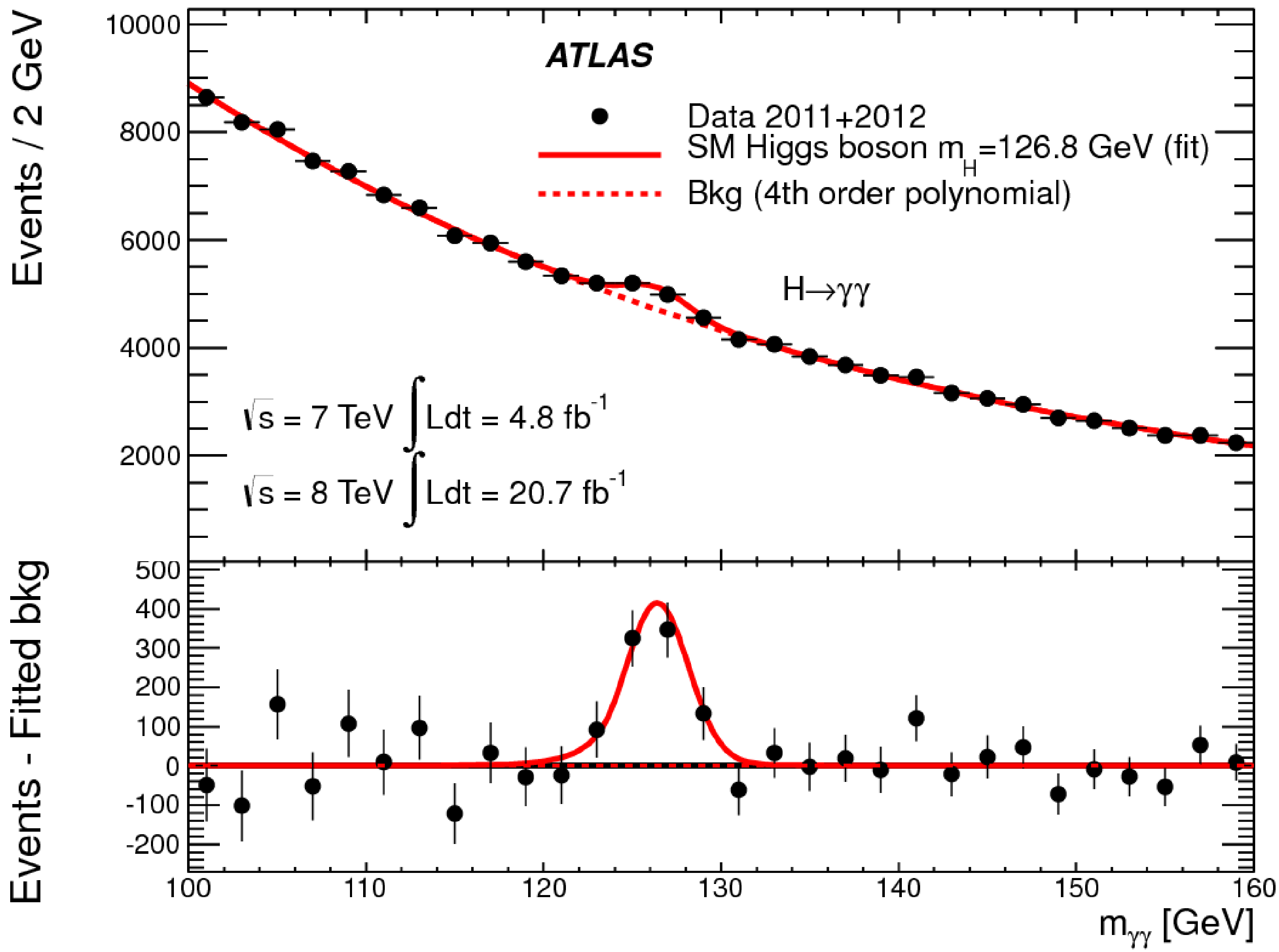
CMS 4 ℓ±

- 2011 : some excess, ~3 σ;
- **2012 : > 6 σ;**
- combined : between 6 and 7 σ.

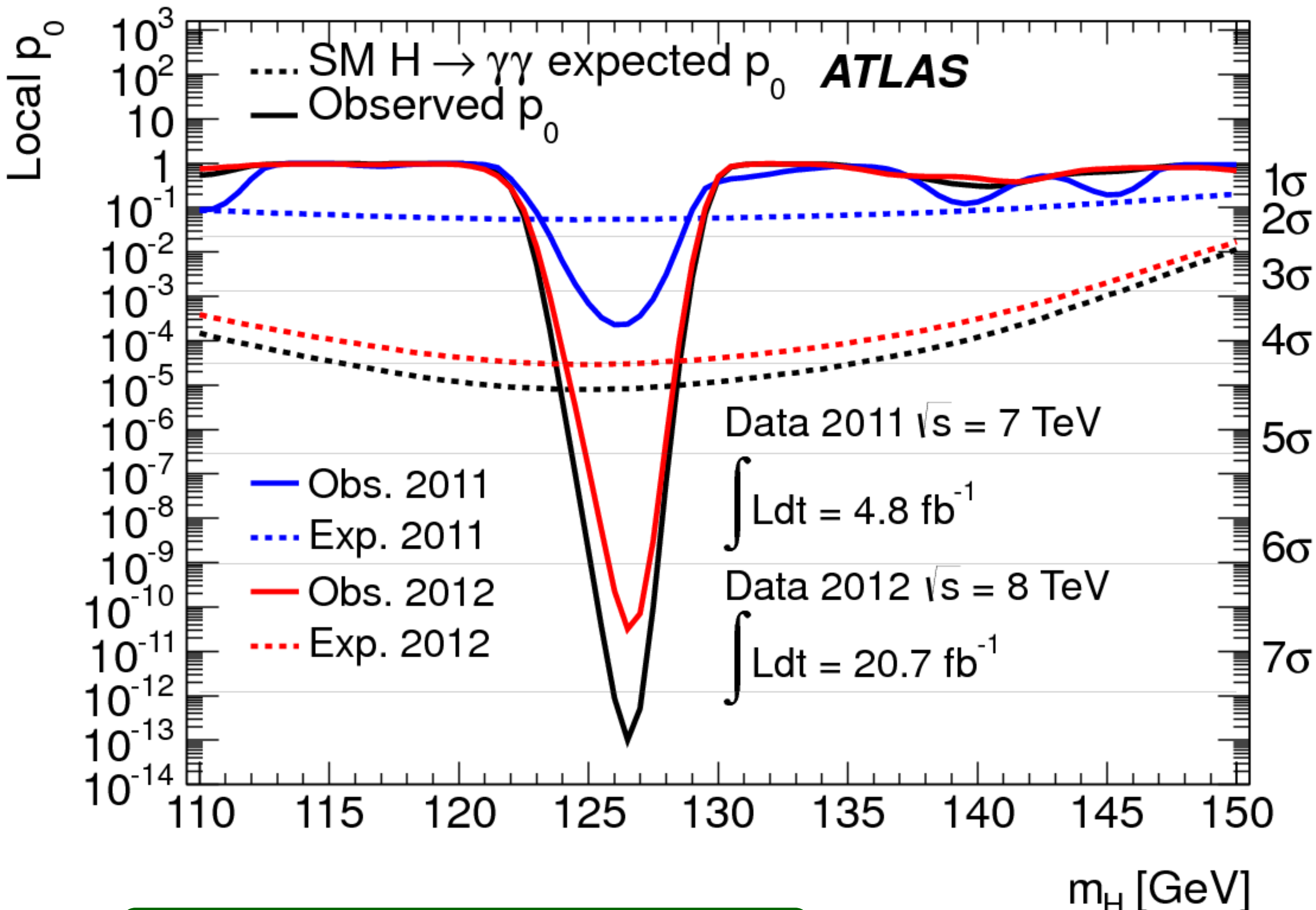
well compatible with expected.

NB. obs (–) and exp (- -) are expected to agree ONLY at m_H^{obs}.

Higgs discovery : $H \rightarrow \gamma\gamma$ - ATLAS



Higgs discovery : H → γγ - ATLAS p-value



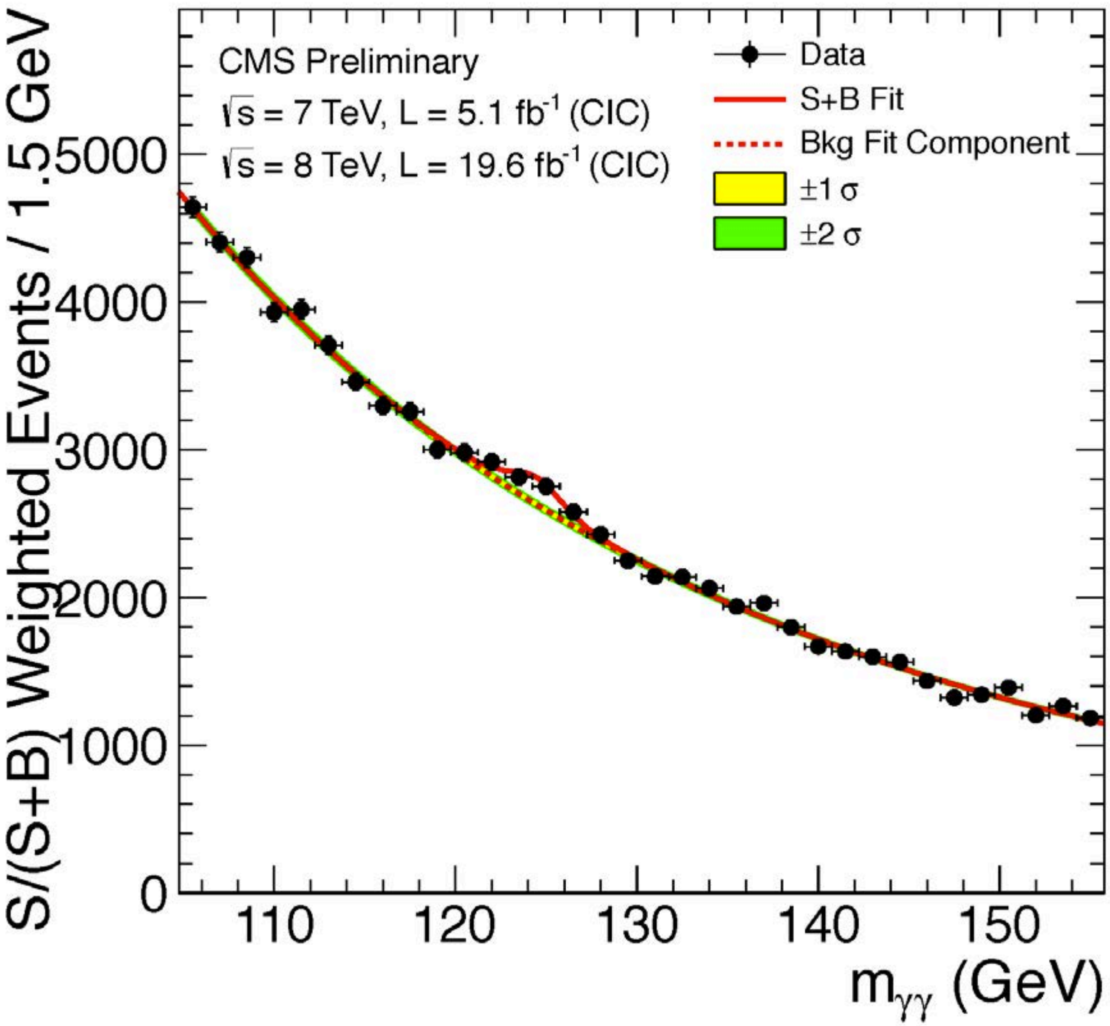
ATLAS γγ

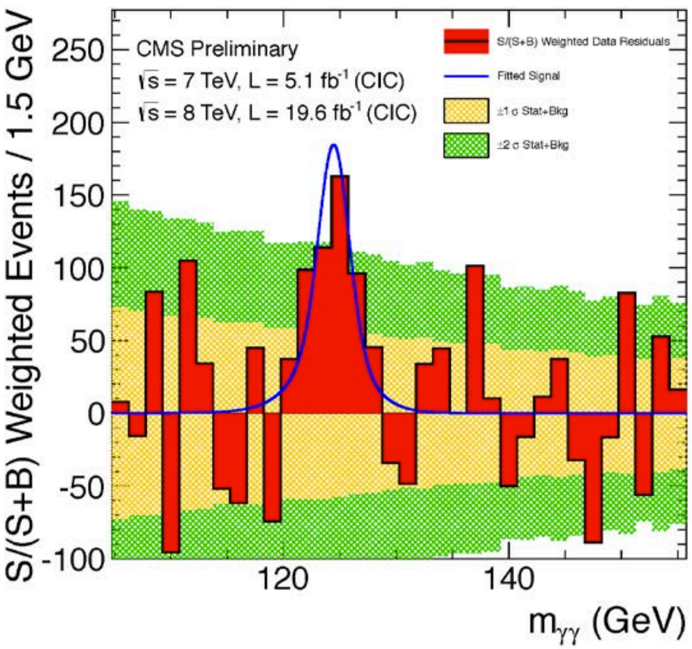
- 2011 : some excess, >3 σ;
- **2012 : > 6 σ;**
- combined : >7 σ.

more than expected, but **not incompatible**.

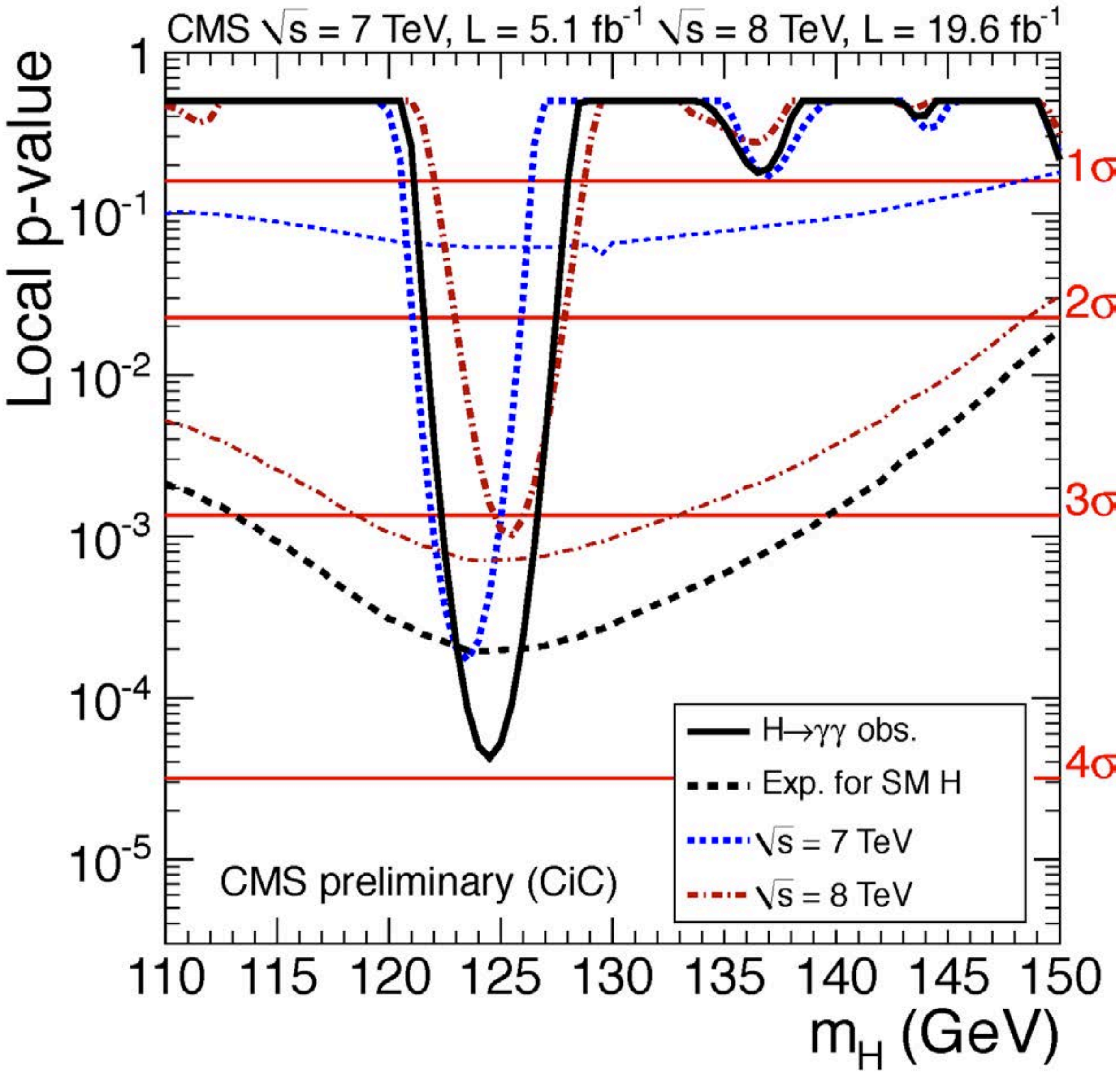
NB. obs (–) and exp (- -) are expected to agree ONLY at m_H^{obs}.

Higgs discovery : $H \to \gamma\gamma$ - CMS





Higgs discovery : H → γγ - CMS p-value



CMS γγ

- 2011 : some excess, >3 σ;
- **2012 : > 3 σ;**
- combined : ~4 σ.

well compatible with expected.

NB. obs (−) and exp (- -) are expected to agree ONLY at m_H^{obs}.

Higgs current status

After discovery, what next ?

[*no possibility for stat. fluctuations, but maybe it is* ***NOT*** *the SM Higgs*]

Strategy :

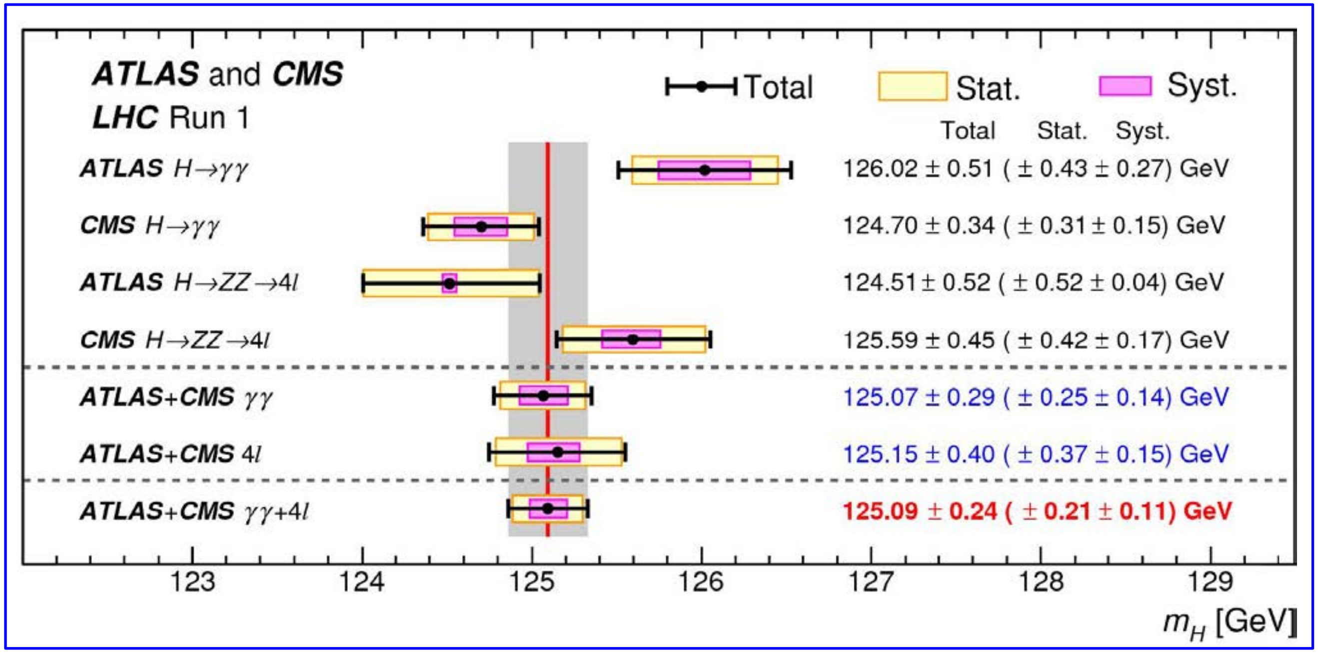
- measure as many as possible H properties :
  - mass (→ masses in all decays);
  - production rates (also vs $\sqrt{s}$);
  - decay BR's;
  - couplings;
  - decay angular distributions;
- compare with SM predictions and check (***hope***) for discrepancies;
- look for the rest of the m_H range, searching for a richer Higgs spectrum;
- [*the same for any other bSM theory*];
- [*also with model-independent analyses*].

Warning:

- neither a standard textbook explanation nor a report of present state-of-art results, but an attempt to show the strategy of the current studies;
- best effort to produce updated results and plots, but no guarantee (updates almost daily);
- few properties only (e.g. skip the interesting but complicated attempt to measure H width);
- no discussion of bSM analyses (actually most studies, but none successful, until now…)
- a neverending work in progress …

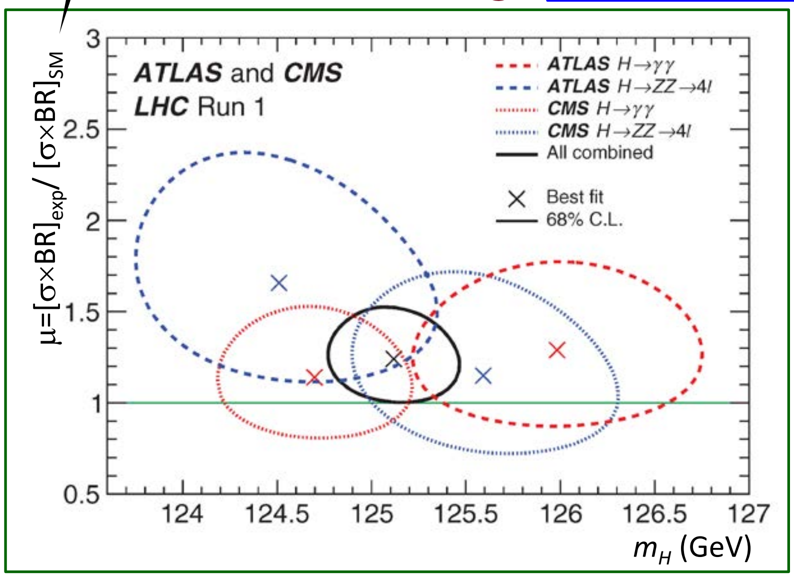
Higgs current status: mass(es)

Data 2011-12	7 TeV	8 TeV
ATLAS	5 fb^{-1}	20 fb^{-1}
CMS	5 fb^{-1}	20 fb^{-1}



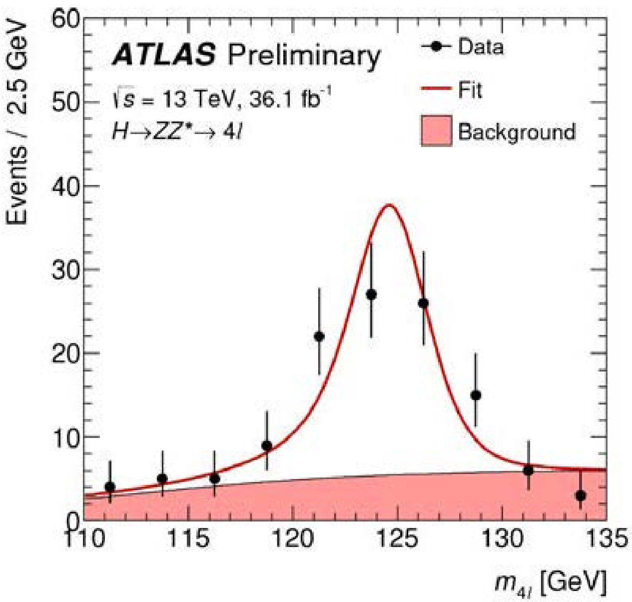
As usual, normalize to SM: "$\mu=[\sigma\times BR]_{exp}/ [\sigma\times BR]_{SM}$"

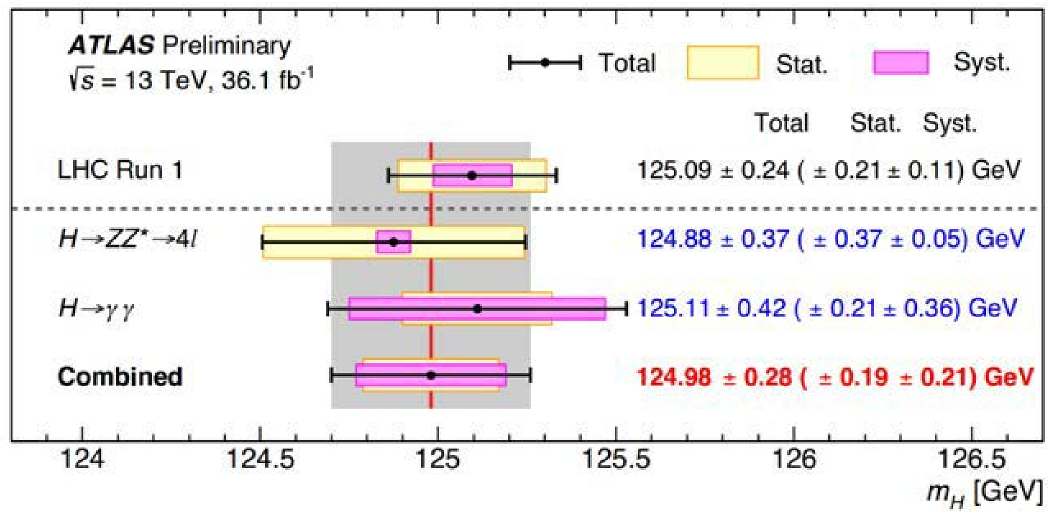
Phys.Rev.Letters, 114, 191803 (2015).

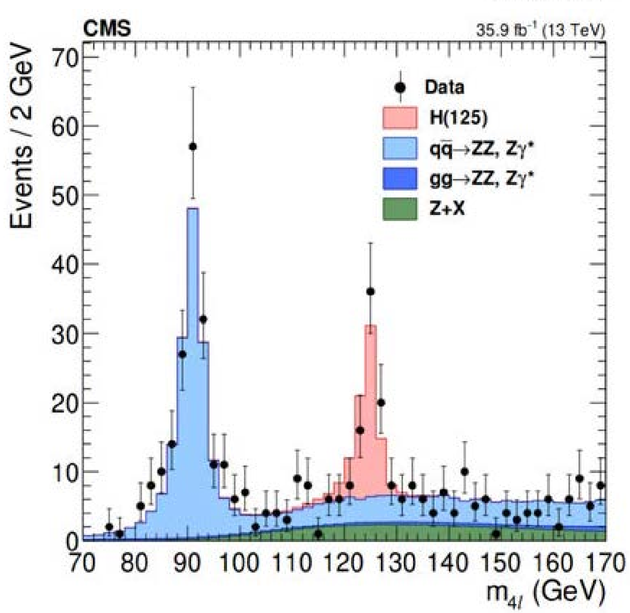


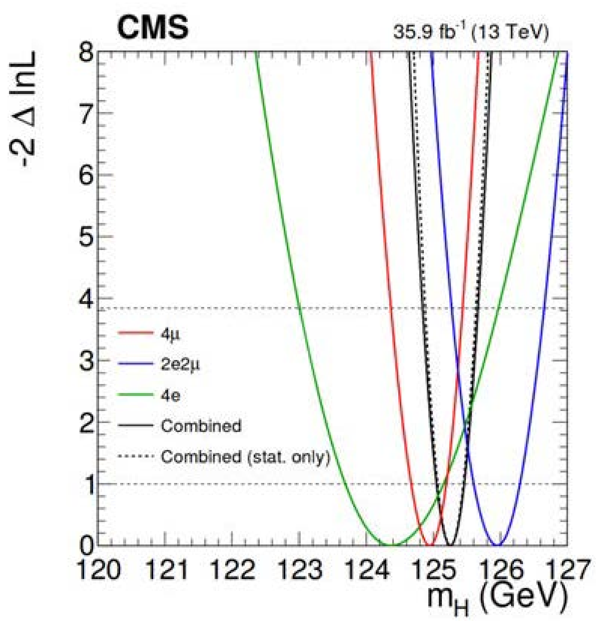
- if it is NOT the SM boson → m(γγ) and m(4ℓ) could be different;
- … and their yield uncorrelated wrt H_{SM};
- but in the data their mass is compatible;
- and their strength is (a bit too large, but still) ok for a SM Higgs of ~125 GeV;
- and ATLAS and CMS are fully compatible. ☺

Higgs current status: mass(es) in 2018









	CMS	ATLAS
m_H	125.26	124.98
±	0.21	0.28
[stat]	0.20	0.19
[sys]	0.08	0.21

Moriond February 2018

Higgs current status: Γ's

- $\Gamma(H \to \text{fermions/IVBs/...}) \equiv \Gamma_{ff/WW/ZZ/...}$ completely specified in SM, once m_H fixed [see table before and IE, § 14];
- measurable from H production and decay (difficult because of higher orders, loops, ...);
- <u>strong function of m_f / m_{IVB}</u>;
- [$\mathscr{L}_{INT}$ up → more events → smaller m_f probed];
- wonderful agreement with theory [*as usual ...* ☺ ... ☹];
- **powerful test of SM : improve accuracy for better test → discrepancies** [*hope* ... ☹ ... ☺].

$$\Gamma_{f\bar{f}} = \frac{c_f}{4\pi\sqrt{2}} G_F m_H m_f^2 \beta_f^3; \text{ [see before]}$$

$$\beta_f = \sqrt{1 - \frac{4m_f^2}{m_H^2}}; \qquad c_f = \begin{cases} 1 & \text{[leptons]} \\ 3 & \text{[quarks]} \end{cases};$$

for Γ_{ww^*/zz^*} take into account $m_H < 2m_{w,z}$:

$$\Gamma_{ww} = \frac{3G_F^2 m_H m_w^4}{2\pi^3} \times J_w\left(\frac{m_w}{m_H}\right);$$

$$J_w(m_H = 125 \text{ GeV}) \simeq 0.0227;$$

$$\Gamma_{zz} = \frac{3G_F^2 m_H m_z^4}{2\pi^3} \times g_z\left(\sin^2\theta_w\right) \times J_z\left(\frac{m_w}{m_H}\right);$$

$$g_z(x^2) = \frac{7}{12} - \frac{10}{9}x^2 + \frac{40}{27}x^4;$$

$$J_z(m_H = 125 \text{ GeV}) \simeq 0.00366;$$

$\Gamma_{\gamma\gamma}$ and Γ_{gg} : see before.

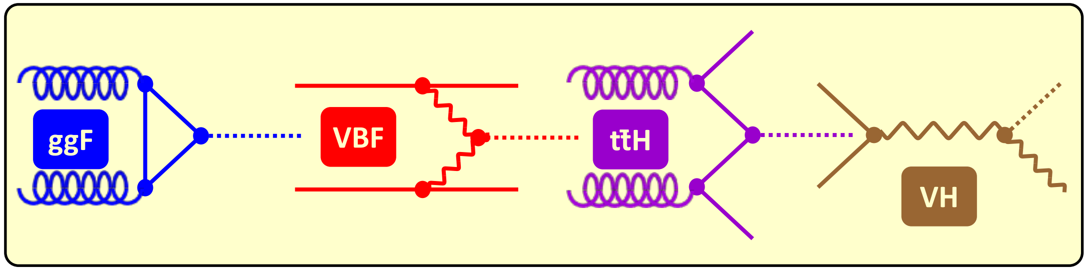
Higgs current status: σ × BR

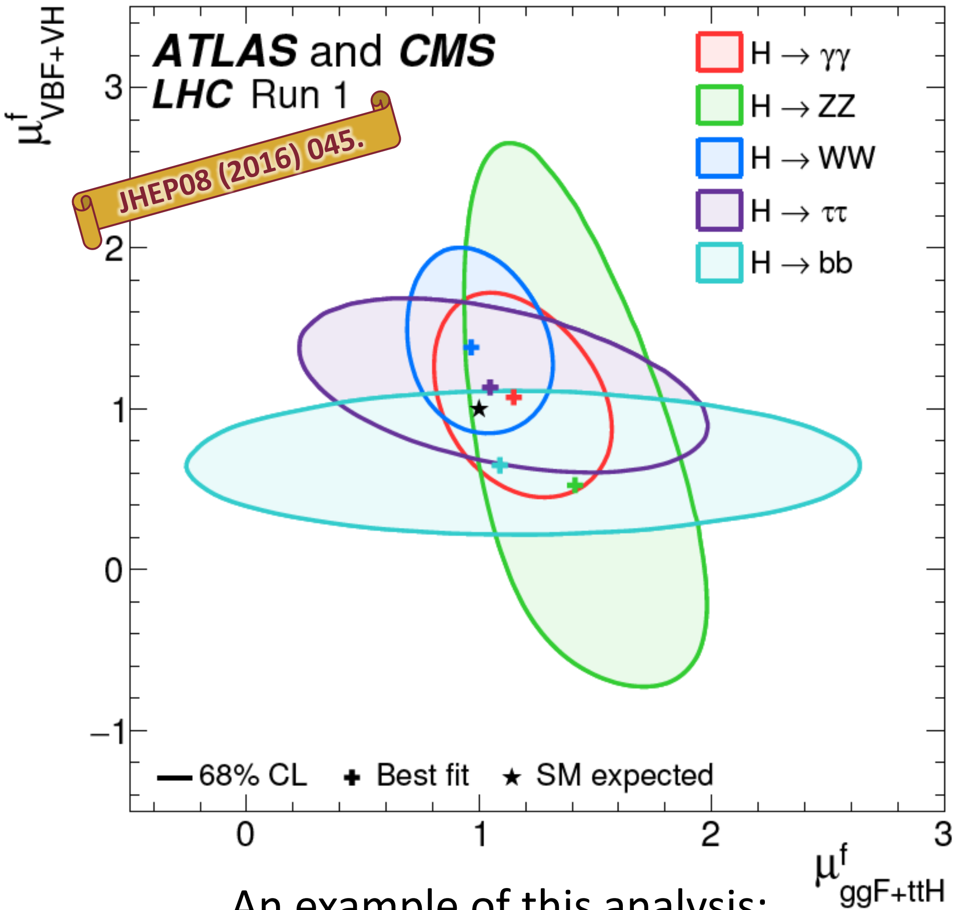
Define :

- $\mu_i = \dfrac{\sigma_i}{\sigma_i^{SM}} \begin{cases} \text{cross-section for H production} \\ i = \text{ggF, VBF, WH, ZH, ttH} \end{cases};$
- $\mu^f = \dfrac{BR^f}{BR^f_{SM}} \{\text{branching ratios for H decay};$
- $\mu_i^f = \dfrac{\sigma_i \times BR^f}{\sigma_i^{SM} \times BR^f_{SM}} \begin{cases} \text{only quantity accessible} \\ \text{to experiments} \end{cases};$

interpretation of result for μ :

- μ = 0 → no signal in this channel;
- μ = 1 → compatible with SM → SM ok;
- μ > 1 → too much signal → SM wrong.





An example of this analysis:

- for all the decays (5 × "f");
- group (VBF+VH) and (ggF+tt̄H), i.e. bosons vs fermions;
- OK !!!

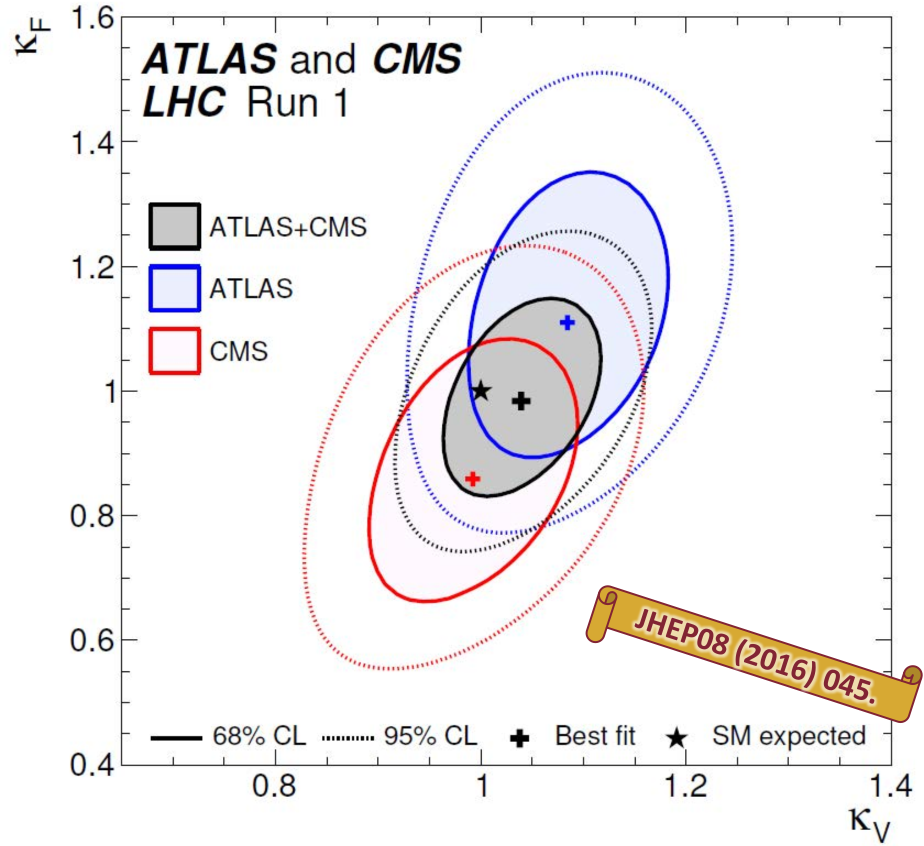
Higgs current status: couplings

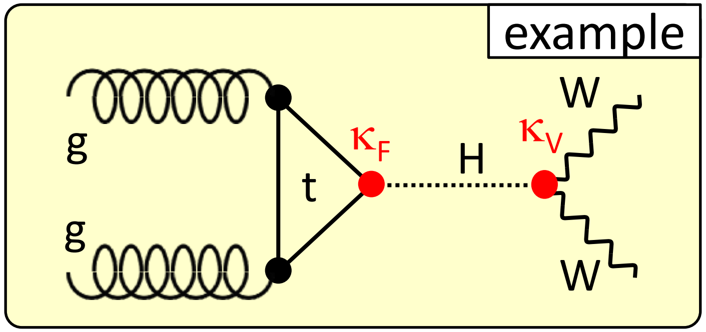
The modifiers κ already defined; in practice:

- $\kappa_j^2 = \dfrac{\Gamma^j}{\Gamma^j_{SM}}$ [j = all particles (f / V)];
- [simpler: only two: $\begin{cases} \kappa_f & \text{for fermions} \\ \kappa_V & \text{for IVBs} \end{cases}$;]

"κ_j" is the "non-SM"-ness of the j-coupling, e.g. :

- $\kappa_j = 0$ → no coupling observed in case "j";
- $\kappa_j = 1$ → compatible with SM → SM ok;
- $\kappa_j \gg 1$ → too large coupling → SM wrong;
- $\kappa_f \neq \kappa_V$ → fermions and IVBs different.





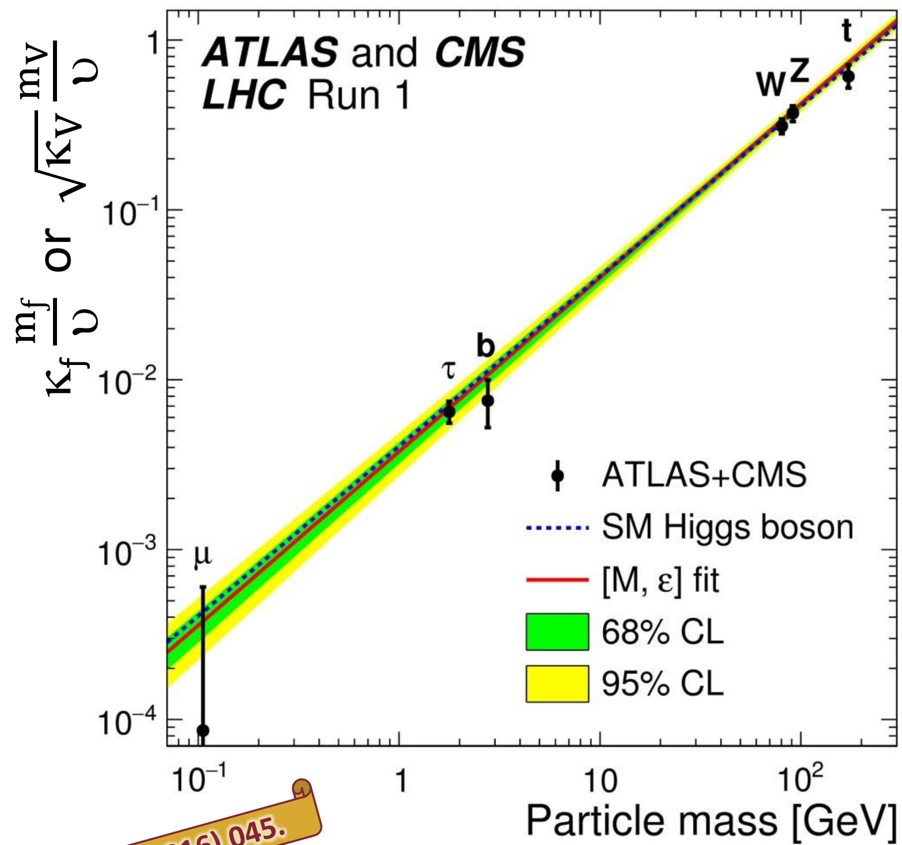
An example of this analysis:

- κ_F vs κ_V (i.e. fermions vs IVBs);
- large errors, but compatible with $\kappa_F = \kappa_V = 1$;
- agreement ATLAS ↔ CMS.

Higgs current status: couplings vs m_f/m_V

Higgs couplings (measured vs SM):

- plot together couplings (including κ_f, κ_V) vs mass of fermions and IVBs;
- clearly compatible with SM ($\kappa_f = \kappa_V = 1$);
- agreement ATLAS ↔ CMS;
- impressive, from m_μ to m_t → more than 3 orders of magnitude.



JHEP08 (2016) 045.

The "[M, ε] fit" is another approach:

- redefine g_f and g_V:

$$g_f = \frac{m_f}{\upsilon} \rightarrow \left(\frac{m_f}{M}\right)^{1+\varepsilon};$$

$$g_V = \frac{2m_V^2}{\upsilon} \rightarrow \frac{2m_V^{2(1+\varepsilon)}}{M^{(1+2\varepsilon)}};$$

- fit M and ε from the data:

SM $\rightarrow \varepsilon = 0$, $M = \upsilon = 246$ GeV.

fit →

$\varepsilon = 0.023+0.029-0.027$

$M = 233+13-12$ GeV

Higgs current status: other meas

Current status (not in these lectures, but PDG § 11, ICHEP 2016, Moriond '17/'18):

- $(H \to \gamma\gamma)$ and $(H \to ZZ^* \to 4\ell)$ golden;
- $(H \to WW^* \to \ell\nu\ell\nu)$ solid;
- $(H \to \tau\tau\ /\ b\bar{b})$ less significant;
- also ttH / tH, ($\to$ tH coupling);
- $H \to Z\gamma$, $c\bar{c}$, $\mu\mu$ next in line (?);
- spin-parity: $J^P = 0^+$ (established);
- couplings (some results shown)

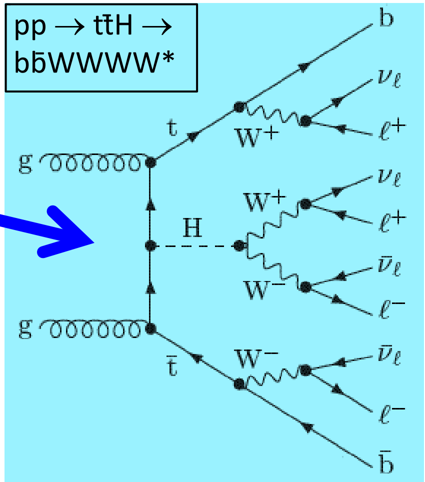
$\to$ until now, all **compatible with SM**.

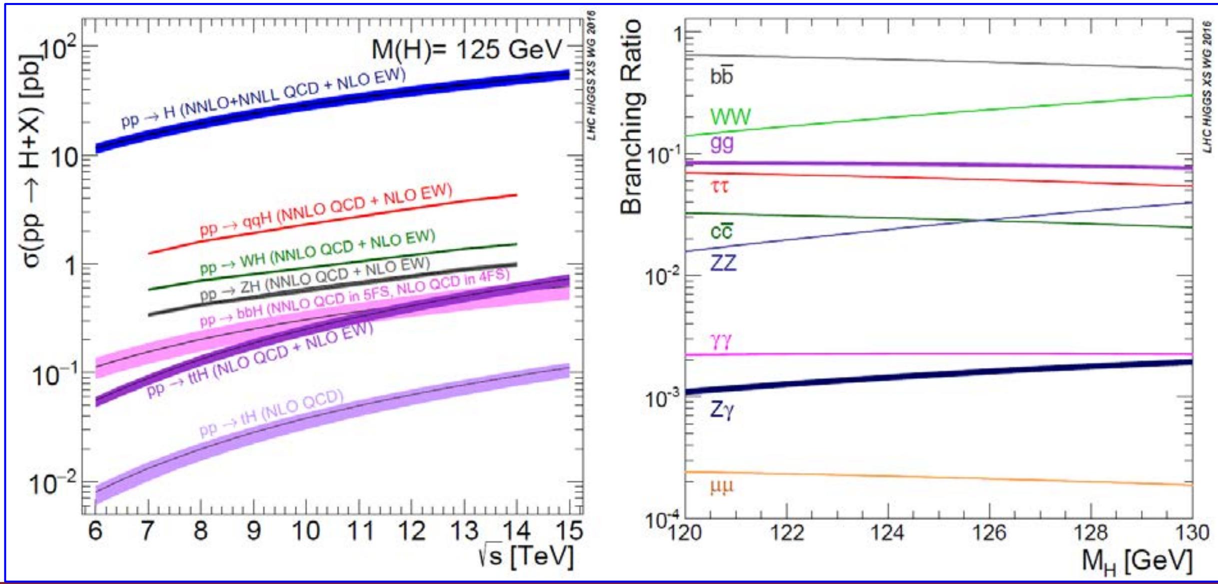
Next (limits on exotica already shown):

- rarer decays;
- HH (wait for HL-LHC);
- "violating" decays (e.g. lepton flavor);
- decays $\to$ dark matter;
- decays bSM (e.g. $\to$ SUSY);
- bSM Higgs (e.g. SUSY higgsinos);
- …

nice process:

- $\geq 4\ \ell^\pm$ (e.g. 3μ $1e$);
- $\geq 4\ \nu$ ($\not{E}_T$);
- 4 sec. vtx;





Higgs current status: conclusion (1)

… finally (PDG 2018, §11.VIII, slightly simplified):

The discovery of the Higgs boson [*H*] is an important milestone in the history of particle physics. Five years after its discovery, a significant number of measurements probing its nature have been made. They are revealing an increasingly precise profile of the H.

The LHC has delivered in Run 2 a luminosity of more than 36 fb^{-1} of data collected by fully operational ATLAS and CMS. Milestone measurements have been performed: (i) H decay to $\tau^+\tau^-$ (CMS); (ii) H decay to $b\bar{b}$ (ATLAS+CMS); (iii) evidence for the production of the H through the $t\bar{t}H$ mechanism (ATLAS+CMS). These and all other experimental measurements are consistent with the EWSB [*ElectroWeak Symmetry Breaking*] mechanism of the SM.

New theoretical calculations are still occurring. With these improvements in the state-of-the-art in theory predictions and the increase in luminosity and energy, Higgs physics has definitively entered a precision era.

Since the discovery of the H, new ideas have emerged to probe its rare decays and production modes, as well as indirectly measure the H width. The H has now become part of the searches for new physics.

Many extensions of the SM at higher energies call for an enlargement of the EWSB sector. Hence, direct searches for additional scalar states can provide valuable insights on the dynamics of the EWSB mechanism. The ATLAS and CMS experiments have searched for additional H's, and imposed constraints in broad ranges of mass and couplings for various extended Higgs scenarios.

The landscape of Higgs physics has been extended extraordinarily since the discovery. The current dataset is approximately one percent of the total dataset foreseen for the High Luminosity-LHC. This perspective brings new challenges to increase further the reach in precision and it also widens the possibilities of unveiling the nature of the EWSB.

(continue …)

Higgs current status: conclusion (2)

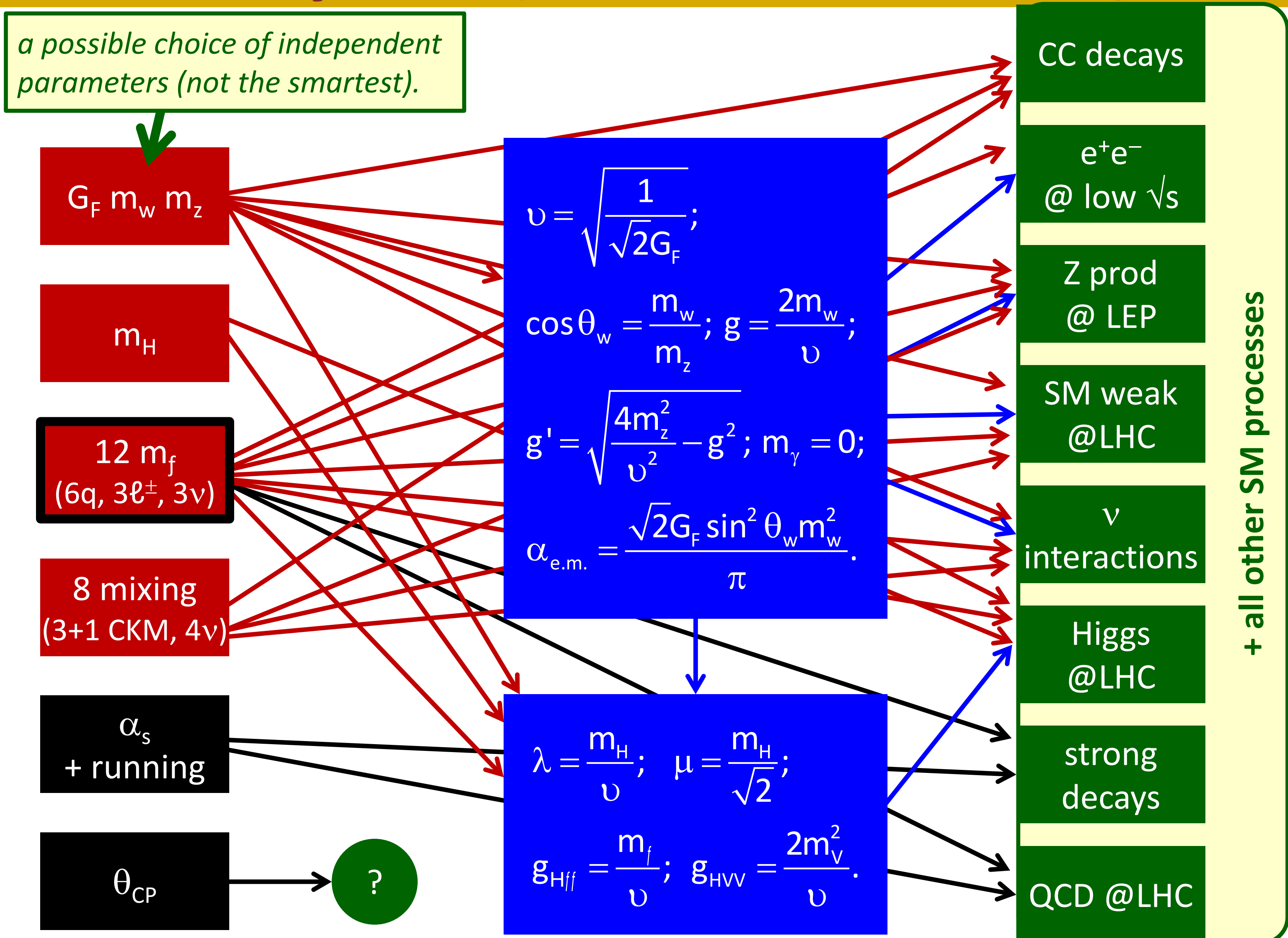
(… continue)

Outlook – The unitarization of the vector boson scattering (VBS) amplitudes was a determining consideration in the building of the accelerator and the detectors. It motivated the existence of a H or the observability of manifestations of strong dynamics at TeV scale. Now that a H has been found and its couplings to gauge bosons are consistent with the SM predictions, perturbative unitarity is preserved to a large amount with the sole exchange of the H, and without the need for any additional states. VBS is, however, still an important channel to investigate further in order to better understand the nature of the Higgs sector and the possible completion of the SM at the TeV scale.

The H couplings are not dictated by any local gauge symmetry. Thus, in addition to a new particle, the LHC has also discovered a new force, different in nature from the other fundamental interactions since it is non-universal and distinguishes between the three families of quarks and leptons. The existence of the H embodies the problem of an unnatural cancellation among the quantum corrections to its mass if new physics is present at scale significantly higher than the EW scale. The non-observation of additional states which could stabilize the H mass is a challenge for natural scenarios like supersymmetry or models with a new strong interaction in which the H is not a fundamental particle. This increasingly pressing paradox starts questioning the principle of naturalness.

The search for the H has occupied the particle physics community for the last 50 years. Its discovery has shaped and sharpened the physics programs of the LHC and of prospective future accelerators. The experimental data together with the progress in theory mark the beginning of a new era of precision H measurements.

SM today: a simple tree-level flow-diagram



a change of perspective

For the first time after (maybe) the birth of quantum mechanics, elementary particle physics is in an uncommon state:

- no major (nor minor) prediction awaits confirmation (e^+, $\bar{p}$, …, $W^\pm$, Z, t, …, H all discovered);
- no major (nor minor) observed phenomenon awaits explanation (strong interactions have been tamed, CP violation is under control);
- exceptions : **dark energy + dark matter**;
- hope in the few missing pieces (ν masses and mixing, Higgs precision measurements, QCD @ low Q^2, …);

→ **(personal) conclusion**:

Either we are at the borders of a big desert, or some new physics (e.g. SUSY, extra-dimensions, …) is just above the present limits, but has not given us the slightest hint of a presence:

... however, much indirect evidence that this story has more chapters ...

why the SM is not final

THE SM DOES NOT EXPLAIN [*SNOWMASS 2013, Energy frontier summary*]:

a. dark matter/energy [*85% of the matter in the universe is "dark" - neutral, weakly interacting*];

b. excess of baryons over antibaryons in the universe [*the SM contains a mechanism to generate baryon number in the early universe, baryon number violation, CP violation, and a phase transition in cosmic history; however it predicts a baryon-antibaryon asymmetry that is too small by ten orders of magnitude*];

c. grand unification [*the quantum numbers of the quarks and leptons under the gauge symmetry SU(3)×SU(2)×U(1) of the SM suggests that these symmetry groups are unified into a larger grand unification group, like SU(5) or SO(10); however, the results of precision measurements of the strengths of the gauge couplings is inconsistent with this hypothesis*];

d. ν masses [*the SM could account for Dirac ν's with few new parameters – technically simple, but intriguing*];

e. fermion mixing [*the pattern of weak interaction mixing among neutrinos is completely different from that observed for quarks*];

f. gravity [*no quantum theory of gravity is incorporated in the SM*].

These difficulties are not equally important [*I am particularly impressed by (a) and (f)*] – However, all together largely justify the claim that the present SM is not the last word of the story.

Thanks for attending

Best wishes !

P.B.

References: results

1. Science, 338 (2012) 1560, 1569, 1576 [simple, divulgative];
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6. H production + decay : ATLAS+CMS, JHEP08 (2016) 045.
7. https://twiki.cern.ch/twiki/bin/view/LHCPhysics ;
8. https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CombinedSummaryPlots/ ;
9. https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicCollisionPlots.

Évrard d'Espinques - The knights and kings of the Round Table experiencing a vision of the Holy Grail, miniature tirée du "Lancelot en prose" a.d. 1474 [French National Library].

References: gif's

1. ATLAS animated gifs: https://twiki.cern.ch/twiki/bin/view/AtlasPublic/HiggsPublicResults#Animations
2. ditto for CMS: https://twiki.cern.ch/twiki/bin/view/CMSPublic/Hig13002TWiki

Caravaggio (Michelangelo Merisi) – I bari – ca 1594
Kimbell Art Museum, Fort Worth

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End of chapter 12