Particle Physics - Chapter 1
The static quark model

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AA 21-22

last mod.
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1 – The static quark model

Caveats for this chapter:

- *arguments are presented in historical order; some of the results are incomplete, e.g. heavy flavors are not mentioned here (wait a bit);*
- *large overlap with [FNSN, MQR, IE].*

short summary

- in this chapter [*see § 6 for QCD*]:
  - no dynamics, only static classification, i.e. algebraic regularities of the states;
  - only hadrons, no photons / leptons;
- modest program, but impressive results:
  - all hadrons are (*may be classified as*) composites of the same elementary objects, called ***quarks***;
  - the quark dynamics, outside the scope of this chapter, follow simple conservation rules;
  - QM and **group theory** are enough to produce the hadron classification;
  - although quarks have not been observed, their **static** properties can be inferred from the particle spectra;
- does it mean that quarks are "real"? what *really* "real" means? [*???*]

The roadmap:

- operators associated with conserved quantum numbers;
- old attempts of classification;
- first successes (multiplets, Ω^-);
- modern classification:
  - quarks;
  - group theory: flavor-SU(3);
  - color: color-SU(3);
  - symmetries;
- "construction" of mesons and baryons;

→ § 6, QCD.

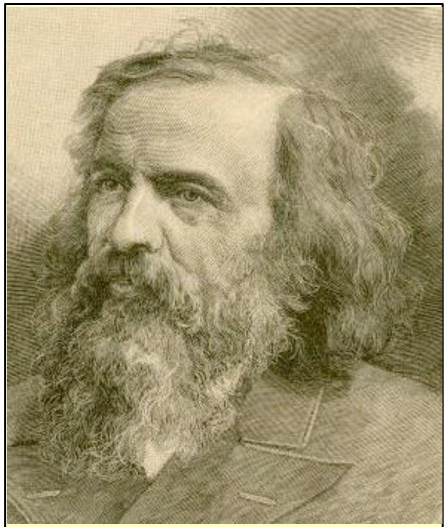
quantum numbers : the Mendeleev way

- Many hadrons exist, with different quantum numbers (qn).
- Some qn show regularities (spin, parity, ...).
- Other qn are more intriguing (mass, ...).
- A natural approach (à la D.I.M. (*)):
  - ➢ investigate in detail the qn:
    - ❑ the associated operators;
    - ❑ the qn conservation;
  - ➢ look for regularities;
  - ➢ classify the states.

(*) even if D.I.M. lived long before the advent of QM.

an example from many years ago [add antiparticles...]

the proliferation of hadrons started in the '50s – now they are few hundreds ...



Dmitri Ivanovich Mendeleev
(Дми́трий Ива́нович Менделе́ев)

Name	$\pi^{\pm}$	π^0	$K^{\pm}$	K^0	η	p	n	Λ	$\Sigma^{\pm,0}$	Δ
Mass (MeV)	140	135	494	498	548	938	940	1116	1190	1232
Charge	±1	0	±1	0	0	1	0	0	±1,0	2,±1,0
Parity	–	–	–	–	–	+	+	+	+	+
Baryon n.	0	0	0	0	0	1	1	1	1	1
Spin	0	0	0	0	0	½	½	½	½	$^3/_2$

many other hadrons

other qn ...

quantum numbers : parity $\mathbb{P}$

for complete definitions and discussion, [FNSN], [MQR], [BJ].

Definition* : $\mathbb{P}\ |\psi(q,\vec{x},t)\rangle = P|\psi(q,-\vec{x},t)\rangle$

- Particles at rest (= in their own ref.sys.) are parity eigenstates:
  $\mathbb{P}\ |\psi(q,\vec{x}=0,t)\rangle = P|\psi(q,\vec{x}=0,t)\rangle$.
- Eigenvalue P : intrinsic parity
  $\mathbb{P}^2 = \mathbb{1}$, P real $\rightarrow$ P = (± 1).
- Dirac equation $\rightarrow$ for spin ½ fermions, P(antiparticle) = -P(particle)
- <u>Convention</u>: P(quarks/leptons) = +1 $\rightarrow$
  $+1 = P_{e-} = P_{\mu-} = P_{\tau-} = P_u = P_d = P_s = \dots$;
  $-1 = P_{e+} = P_{\mu+} = P_{\tau+} = P_{\bar{u}} = P_{\bar{d}} = P_{\bar{s}} = \dots$
- Field theory: for spin-0 bosons $\rightarrow$ P(antiparticle) = +P(particle) :
  $P_{\pi+} = P_{\pi^\circ} = P_{\pi-}$, ...

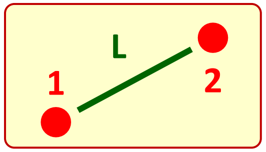
= +1 or –1 ?
... be patient ...

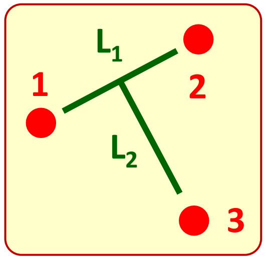
* here and in the following slides :
- q : charges + additive qn;
- $\vec{x},\vec{a}$: polar / axial vectors;
- t : time.

- Gauge theories $\rightarrow$ $P_\gamma = P_g = -1$.

$W^\pm$ and Z do NOT conserve parity in their interactions, so their intrinsic parity is not defined.

- For a many-body system, P is a multiplicative quantum number :
  $\mathbb{P}\psi(\vec{x}_1,\vec{x}_2\dots\vec{x}_n,t) = P_1P_2\dots P_n\psi(-\vec{x}_1,-\vec{x}_2\dots-\vec{x}_n,t)$.
- Particles in a state of orbital angular momentum are parity eigenstates :
  $Y_{km}(\theta,\phi) = (-1)^k\ Y_{km}(\pi-\theta,\phi+\pi) \rightarrow$
  $\mathbb{P}\ |\psi_{km}(\theta,\phi)\rangle = (-1)^k\ |\psi_{km}(\theta,\phi)\rangle$
- Therefore, for a two- or a three-particle system:
  $P_{sys(12)} = P_1P_2\,(-1)^L$;
  $P_{sys(123)} = P_1P_2P_3(-1)^{L1+L2}$.





quantum numbers : charge conjugation $\mathbb{C}$

Definition : $\mathbb{C}$ changes a particle p into its antiparticle $\bar{p}$, leaving untouched the space and time variables :

$\mathbb{C}\ |p,\psi(\vec{x},t)\rangle = C\ |\bar{p}, \psi(\vec{x},t)\rangle$.

- Therefore, under $\mathbb{C}$:

charge	$q \to -q$;	flip
baryon n.	$\mathcal{B} \to -\mathcal{B}$;	
lepton n.	$\mathcal{L} \to -\mathcal{L}$;	
strangeness	$S \to -S$;	
position	$\vec{x} \to \vec{x}$;	no-flip
momentum	$\vec{p} \to \vec{p}$;	
spin	$s \to s$.	

- $\mathbb{C}$ is hermitian; its eigenvalues are ± 1; they are multiplicatively conserved in strong and e.m. interactions. see later

- Only particles (like π^0, unlike K's) which are their own antiparticles, are eigenstates of $\mathbb{C}$, with values C = ($\pm$ 1) :

  C = +1 for π^0, η, η';

  C = –1 for ρ^0, ω, ϕ;

  C = –1 for γ. *for Z, $\mathbb{C}$ and $\mathbb{P}$ are not defined*

- However, few particles are an eigenstate of $\mathbb{C}$; e.g.

  $\mathbb{C}\ |\pi^+\rangle = -\ |\pi^-\rangle$.

- Why define $\mathbb{C}$? E.g. use $\mathbb{C}$-conservation in e.-m. decays:

  $\pi^0 \to \gamma\gamma$: $+1 \to (-1)\ (-1)$ ok;

  $\pi^0 \to \gamma\gamma\gamma$: $+1 \to (-1)\ (-1)\ (-1)$ no.

  BR($\pi^0 \to \gamma\gamma\gamma$) measured to be $\sim 10^{-8}$.

quantum numbers : G-parity $\mathbb{G}$

proposed by Lee and Yang, 1956.

- charge conjugation $\mathbb{C}$ is defined as

  $\mathbb{C}\,|q, \mathscr{B}, L, S> = \pm\,|-q, -\mathscr{B}, -L, -S>$;

- therefore, only states $q = \mathscr{B} = L = S = 0$ may be $\mathbb{C}$-eigenstates (e.g. π^0, η, γ, $[\pi^+\pi^-]$).

Generalization [**G-parity**]: $\mathbb{G} \equiv \mathbb{C}\,\mathbb{R}_2$,
where $\mathbb{R}_2$ = rotation in the isospin space:

$\mathbb{R}_2 \equiv \exp(-i\pi\tau_2)$;

$\mathbb{R}_2|I, I_3> = (-)^{I-I3}\,|I, -I_3>$

$\mathbb{R}_2|q, \vec{x}, t, I, I_3> = (-)^{I-I3}|-q, \vec{x}, t, I, -I_3>$;

- $\mathbb{G}$ has more eigenstates than $\mathbb{C}$; e.g.:

  $\mathbb{C}\ \ |\pi^\pm> \ = -\,|\pi^\mp>$;

  $\mathbb{C}\ \ |\pi^0> \ = +\,|\pi^0>$;

  $\mathbb{R}_2|\pi^\pm> \ = +\,|\pi^\mp>$;

  $\mathbb{R}_2|\pi^0> \ = -\,|\pi^0>$.

- therefore:

  $\mathbb{G}\,|\pi^{\pm,0}> = \mathbb{C}\,\mathbb{R}_2\,|\pi^{\pm,0}> = -\,|\pi^{\pm,0}>$.

- $\mathbb{G}$-parity is multiplicative :

  $\mathbb{G}\,|n\pi^+\, m\pi^-\, k\pi^0> =$

  $= (-)^{n+m+k}\,|n\pi^+\, m\pi^-\, k\pi^0>$;

  $\mathbb{G}\,|q\bar{q}> = (-)^{L+S+I}\,|q\bar{q}> \rightarrow G = (-)^{L+S+I}$;

- $\mathbb{G}$ is useful:
  - ➢ $\mathbb{G}$-parity is conserved <u>only in strong interactions</u> ($\mathbb{C}$ and isospin are valid);
  - ➢ it produces selection rules (e.g. a decay in odd/even number of π's is allowed/forbidden).

- e.g. $\omega(782)$ is $I^G(J^{PC}) = 0^-(1^{--})$:

  BR $(\omega \rightarrow \pi^+\pi^-\pi^0) = (89.2\pm0.7)\%$

  BR $(\omega \rightarrow \pi^+\pi^-) \quad = (\ 1.5\pm0.1)\%$

  opposite to the obvious phase-space predictions (more room for 2π than 3π decay).

- [*see also* J/ψ *decay*].

quantum numbers : some proofs

- See [FNSN, §7].
- Be $|q, \vec{x}, \vec{a}>$ an eigenstate of a generic "flip" operator $\mathbb{K}$ ($\mathbb{K}$ = $\mathbb{C}$, $\mathbb{G}$, $\mathbb{P}$, $\mathbb{S}$ [spin-flip], $\mathbb{T}$ [time-reversal]).
- From their definition:

  $\mathbb{K}^2 = \mathbb{1} \quad \rightarrow \boxed{\mathbb{K}^{-1} = \mathbb{K}.}$
- an eigenstate of $\mathbb{K}$ has eigenvalue K:

  $\mathbb{K}\ |q, \vec{x}, \vec{a}> = K\ |\pm q, \pm\vec{x}, \pm\vec{a}>;$

  $\mathbb{K}^2\ |q, \vec{x}, \vec{a}> = K^2\ |q, \vec{x}, \vec{a}> = |q, \vec{x}, \vec{a}>;$

  $K^2 = 1 \rightarrow \boxed{K = \text{real} = \pm 1.}$
- $\boxed{P(\gamma) = -1}$ (vector potential [FNSN, 7.2.2])

- For a $q\bar{q}$ (or particle-antiparticle) state:

  $\mathbb{S}\ \mathbb{P}\ \mathbb{C}\ \ |q,\vec{x},\vec{s}\quad -q,-\vec{x},-\vec{s}> =$

  $= C\ \mathbb{S}\ \mathbb{P}\ |-q,\vec{x},\vec{s},\ \ q,-\vec{x},-\vec{s}> =$

  $= C\ P\ \mathbb{S}\ \ |-q,-\vec{x},\vec{s},\ \ q,\vec{x},-\vec{s}> =$

  $= C\ P\ \mathcal{S}\ \ |-q,-\vec{x},-\vec{s}, q,\vec{x},\vec{s}> =$

  $= C\ P\ \mathcal{S}\ \ |q,\vec{x},\vec{s},\quad -q,-\vec{x},-\vec{s}>.$

  the spin $\vec{s}$ is an axial vector ($\vec{a}$)

  $\rightarrow \mathbb{S}\ \mathbb{P}\ \mathbb{C} = CP\mathcal{S} = \pm 1;$

  $\rightarrow \boxed{\mathbb{C} = \pm\mathbb{S}^{-1}\ \mathbb{P}^{-1} = \pm\mathbb{S}\ \mathbb{P}.}$

- for such a state [BJ, 335]:

  $\boxed{P = (-1)^{L+1}}$ (see before);

  $\mathcal{S} = (-1)^{S+1}$ (Pauli principle);

  $\rightarrow \boxed{C = P \times \mathcal{S} = (-1)^{L+S};}$

  $\rightarrow \boxed{G = (-1)^{L+S+I}}$ (see before).

  $\mathcal{S}$: eigenvalue of $\mathbb{S}$;
  S : value of the spin.

hadrons : "elementary" or composite ?

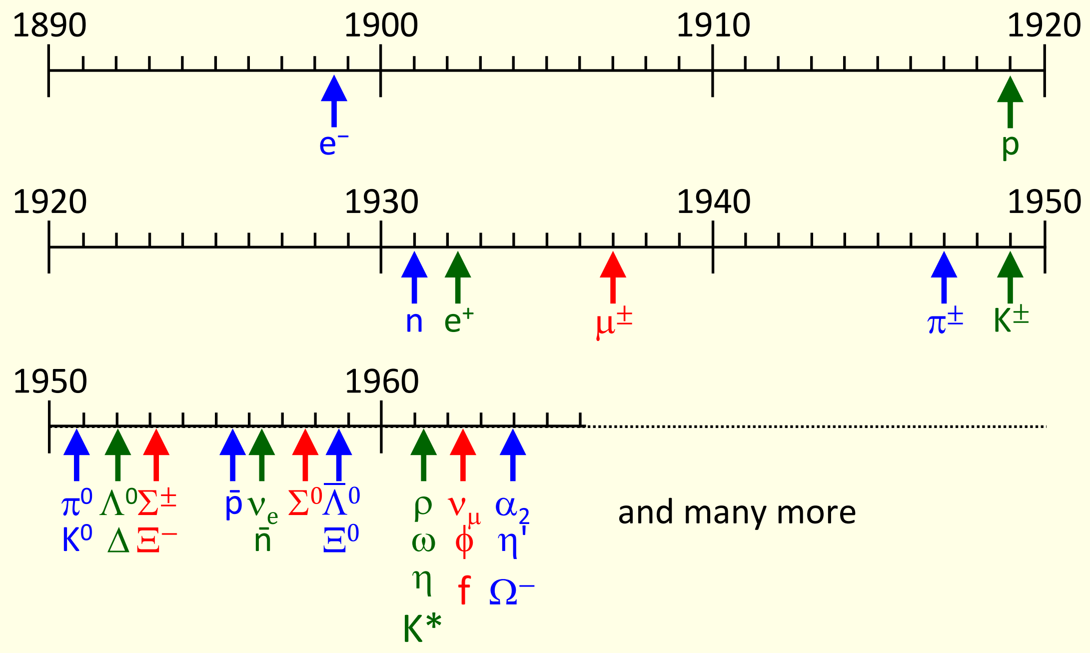
Over time the very notion of "*elementary* (???) particle" entered a deep crisis.

The existence of (too) many hadrons was seen as a contradiction with the elementary nature of the fundamental component of matter.

It was natural to interpret the hadrons as consecutive resonances of elementary components.

The main problem was then to measure the properties of the components and possibly to observe them.

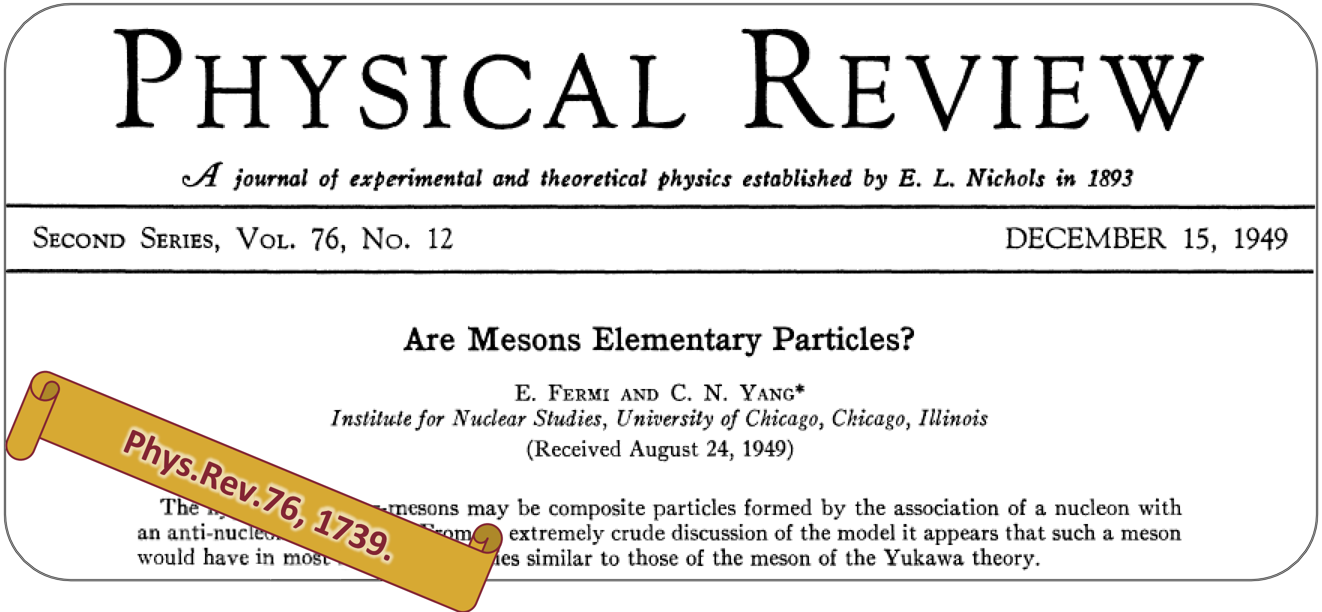
[and the leptons ? ... see later]



too many hadronic states: resonances ?

the figure shows the particle discoveries from 1898 to the '60s; their abundance and regularity, as a function of quantum numbers like charge and strangeness, were suggesting a possible sequence, similar to the Mendeleev table [FNSN].

hadrons : "elementary" or composite ?


PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893
SECOND SERIES, VOL. 76, NO. 12 DECEMBER 15, 1949

Are Mesons Elementary Particles?

E. FERMI AND C. N. YANG*
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received August 24, 1949)

The [illegible]-mesons may be composite particles formed by the association of a nucleon with an anti-nucleo[illegible] From [illegible] extremely crude discussion of the model it appears that such a meson would have in most [illegible]ies similar to those of the meson of the Yukawa theory.

1949 : E.Fermi and C.N. Yang proposed that ALL the resonances were bound state p-n.

1956 : Sakata extended the Fermi-Yang model including the Λ, to account for strangeness : all hadronic states were then composed by (p, n, Λ) and their antiparticles.

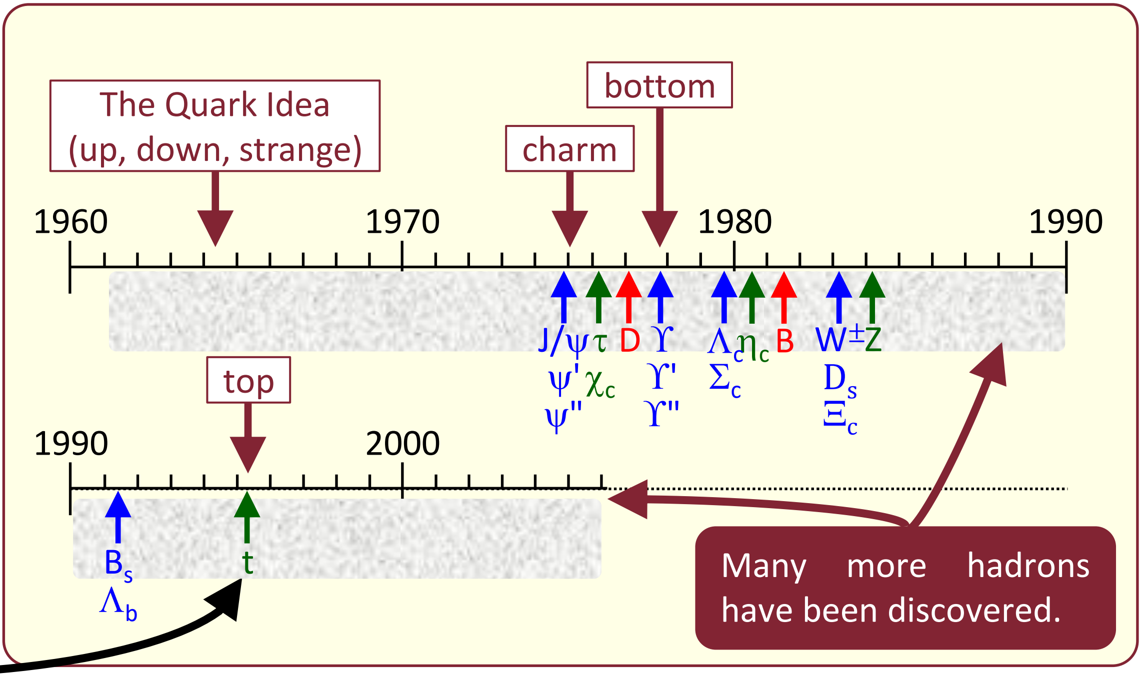
Enrico Fermi	Chen-Ning Yang (杨振宁 - 楊振寧, *Yáng Zhènníng*)	Shoiki Sakata (坂田 昌一, *Sakata Shōichi*)
		

hadrons : "elementary" or composite ?

1961 : M. Gell-Mann and Y. Ne'eman (independently) proposed a new classification, the **Eightfold Way**, based on the symmetry group SU(3). The classification did **NOT** explicitly mention an **internal structure**. The name was invented by Gell-Mann and comes from the "eight commandments" of the Buddhism.

Murray Gell-Mann

Yuval Ne'eman (יובל נאמן)



Warning : "t" is a quark, not a hadron (in modern language).

the Eightfold Way: 1961-64

FNSN, 7

All hadrons (known in the '60s) are classified in the plane **(I_3 – Y)**, (Y = strong hypercharge):

$I_3 = I_z$ = third component of isospin;
$Y = \mathscr{B} + S$ **[baryon number + strangeness]**.

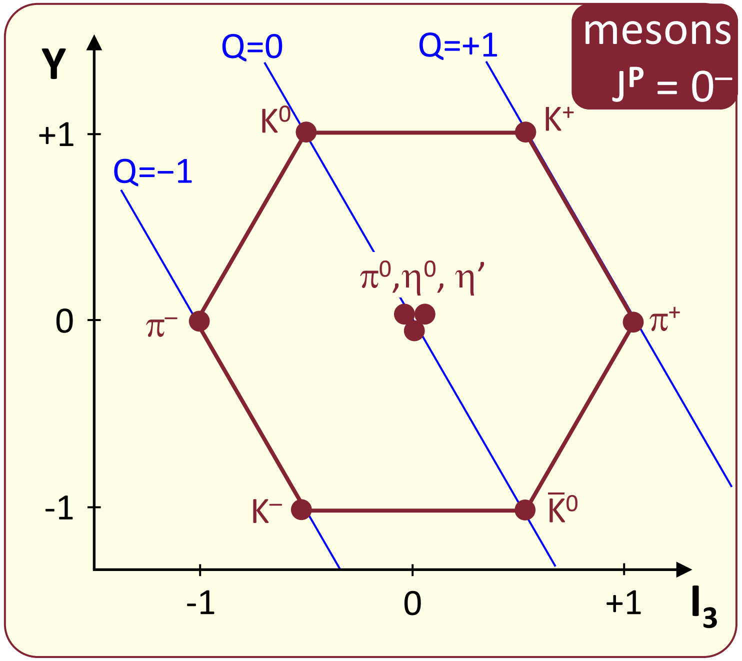
The strangeness ***S***, which contributes to ***Y***, had the effect to enlarge the isospin symmetry group **SU(2)** to the larger **SU(3): Special Unitarity group**, with dimension=3.

The Gell-Mann – Nishijima formula (1956) was :

$$Q = I_3 + ½(\mathscr{B}+S)$$

including heavy flavors [$\mathscr{B}$:baryon, B:bottom] :

$$Q = I_3 + ½(\mathscr{B}+S+C+B+T)$$



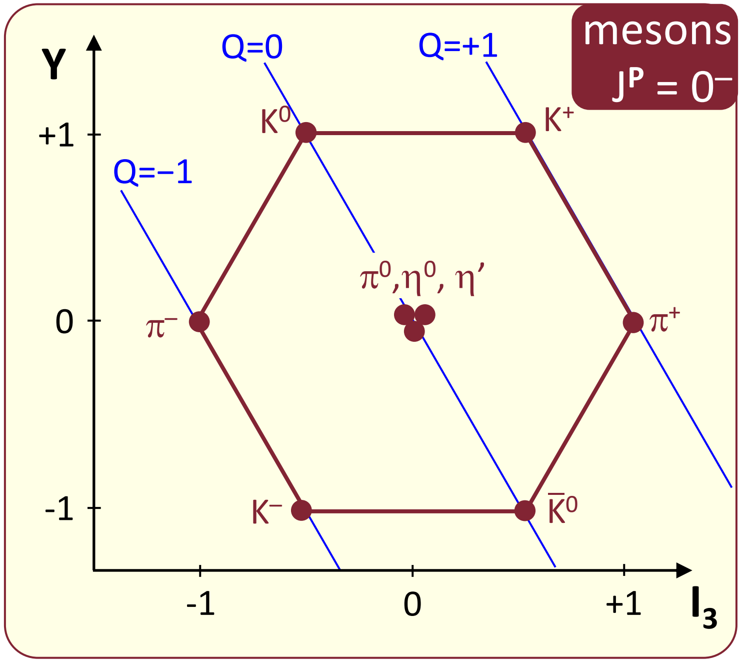
This symmetry is now called "**flavor SU(3) [SU(3)$_F$]**", to distinguish it from the "**color SU(3) [SU(3)$_C$]**", which is the exact symmetry of the strong interactions in QCD. *see later*

the Eightfold Way: SU(3)

The particles form the multiplets of **SU(3)$_F$**. Each multiplet contains particles that have the same spin and intrinsic parity. The basic multiplicity for mesons is nine ($3\times\bar{3}$), which splits in two SU(3) multiplets: (octet + singlet). For baryons there are octects + decuplets.

The gestation of SU(3) was long and difficult. It both explained the multiplets of known particles/resonances, and (more exciting) predicted new states, before they were actually discovered (*really a triumph*).

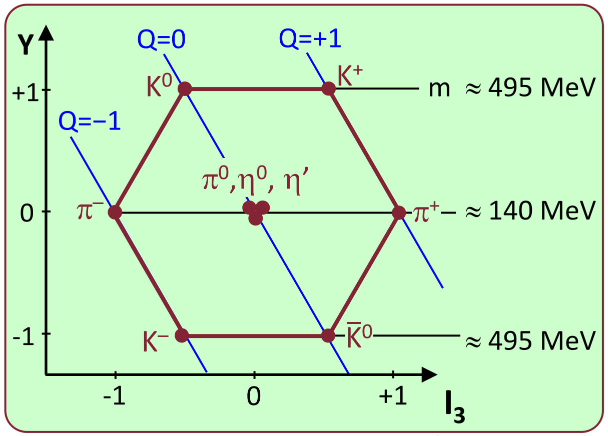
However, the mass difference p – n (or $\pi^\pm - \pi^0$) is < few MeV, while the π – K (or p – Λ) is much larger. Therefore, while the isospin symmetry **SU(2)** is almost exact, the symmetry **SU(3)$_F$**, grouping together strange and non-strange particles, is substantially violated.



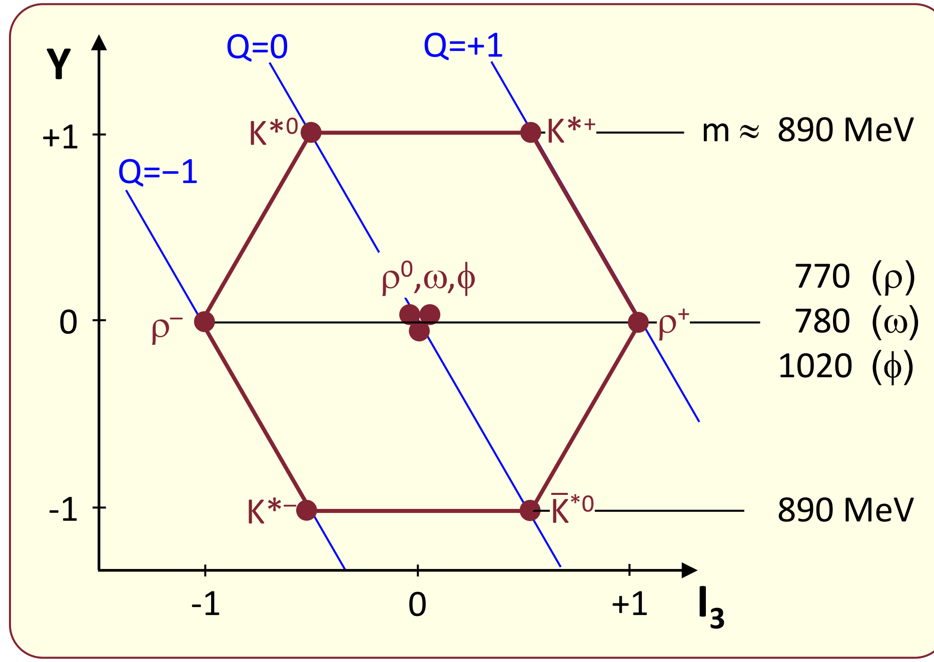
In principle, in a similar way, the discovery of heavier flavors could be interpreted with higher groups (e.g. **SU(4)$_F$** to incorporate the charm quark, and so on). However, these higher symmetries are broken even more, as demonstrated by the mass values. Therefore, **SU(6)$_F$** for all known mesons $J^P = 0^-$ is (almost) never used.

the Eightfold Way: mesons $J^P=1^-$

Another example of a multiplet: the octet of vector mesons :

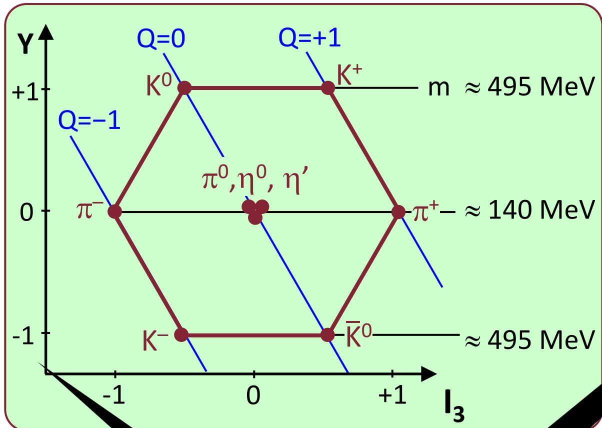


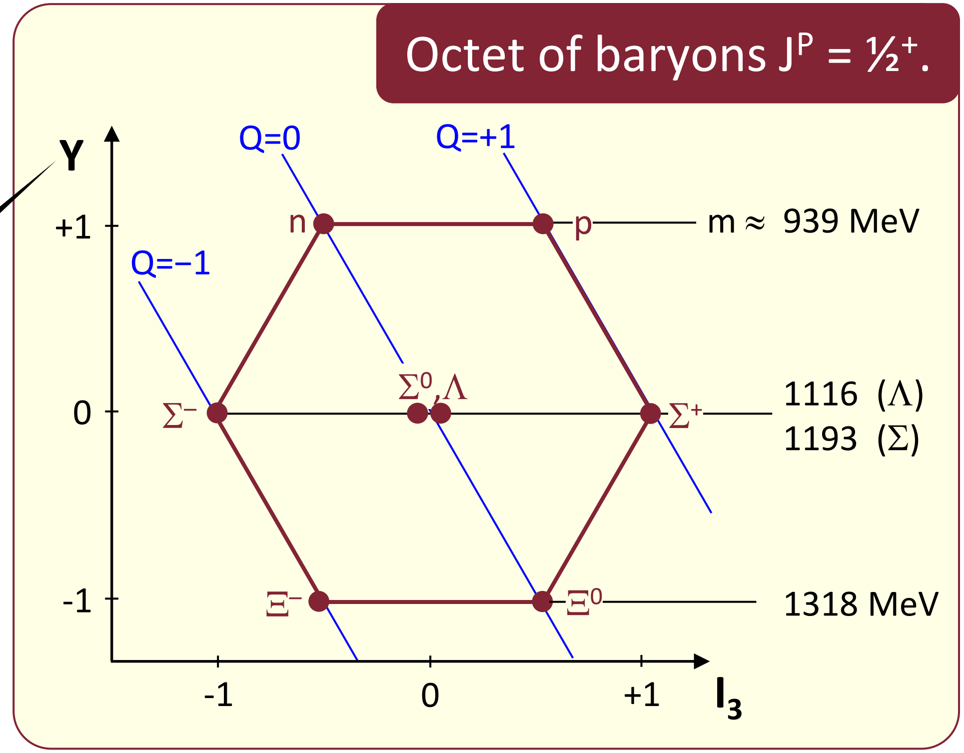
[mesons $J^P = 0^-$]



meson resonances $J^P = 1^-$ (all discovered by 1961).

the Eightfold Way: baryons $J^P=½^+$





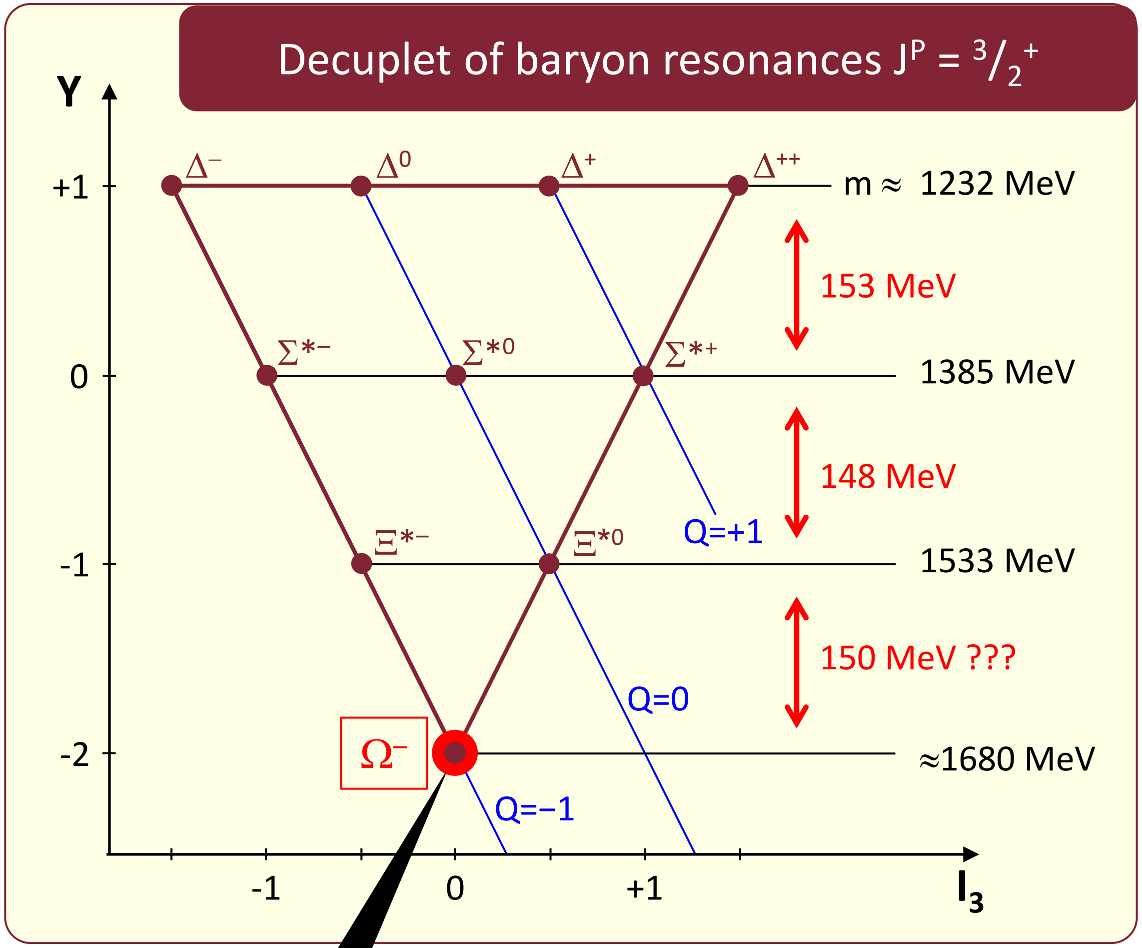
mesons: Y = S

baryons: Y = S + $\mathscr{B}$

notice the masses: for mesons, because of $\mathbb{CPT}$ (K ↔ K̄) the masses of an octet are symmetric wrt (S=0, I_3=0), while for baryons the mass increases as –S

[*because the s-quark (S= –1) is heavier than u/d, but they did not know it*]

the Eightfold Way: baryons $J^P = {}^3/_2{}^+$



when the Eightfold Way was first proposed, this particle (now called Ω⁻) was not known → see next slide.

The next multiplet of baryons is a decuplet $J^P = {}^3/_2{}^+$.

When the E.W. was proposed, they knew only 9 members of the multiplet, but can predict the last member:

- it is a decuplet, because of E.W.;
- the state $Y = -2$, $I_3 = 0$ ($\rightarrow Q = -1$, $S = -3$, $\mathscr{B}=1$) must exists;
- call it Ω^-;
- look the mass differences vs Y:
- mass linear in Y $\rightarrow m_{\Omega^-} \approx 1680$ MeV (*NOT an E.W. requirement, but a reasonable assumption*);
- the conservation laws set the dynamics of production and decay of the Ω^-.

the discovery of the Ω^-

The particle Ω^-, predicted (★) in 1962, was discovered in 1964 by N.Samios et al., using the 80-inch hydrogen bubble chamber at Brookhaven (next slide).

The Ω^- can only decay weakly to an S = –2 final state [(1)] :

$\Omega^- \to \Xi^0 \pi^-$; $\to \Xi^- \pi^0$; $\to \Lambda^0 K^-$;

[a posteriori confirmed by the measurement $\tau_{\Omega^-} \cong 0.82 \times 10^{-10}$ s]

———————————————

(1) Since the electromagnetic and strong interactions conserve the strangeness, the lightest (non-weak) S- and $\mathcal{B}$- conserving decay is :

$\Omega^- \to \Xi^0 K^-$ [S : –3 → –2 –1, $\mathcal{B}$: +1 → +1 +0]

which is impossible, because

$m(\Omega) \approx 1700$ MeV < $m(\Xi) + m(K) \approx 1800$ MeV.

Therefore the Ω^- must decay via strangeness-violating weak interactions : <u>the Ω^- lifetime reflects its weak (NOT strong NOR e.m.) decay</u>.

(★) ***<u>From a 1962 report:</u>***

Discovery of Ξ^* resonance with mass ~1530 MeV is announced [...].

[*As a consequence,*] **Gell-Mann and Ne'eman** [...] predicted a new particle and all its properties:

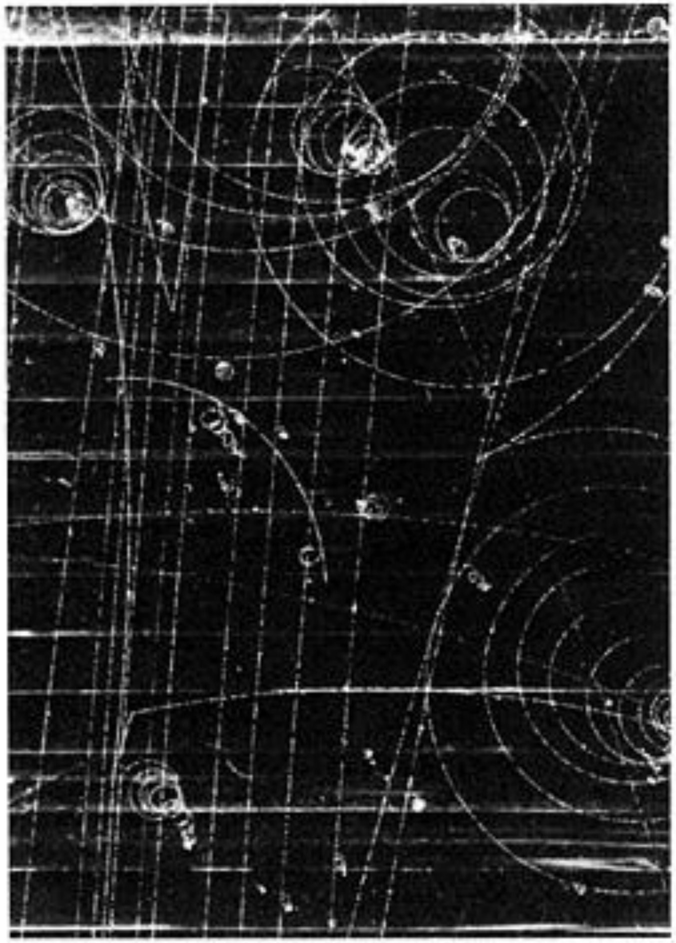
- Name = Ω^- (*Omega* because this particle is the last in the decuplet);
- Mass ≈ 1680 MeV (the masses of Δ, Σ^* and Ξ^* are about equidistant ~150 MeV);
- Charge = –1;
- Spin = $^3/_2$;
- Strangeness = –3, Y = –2;
- Isospin = 0 (no charge-partners);
- Lifetime ~10^{-10} s, because of its weak decay, since strong decay is forbidden**[(1)]**;
- Decay modes: $\Omega^- \to \Xi^0 \pi^-$ or $\Omega^- \to \Xi^- \pi^0$.

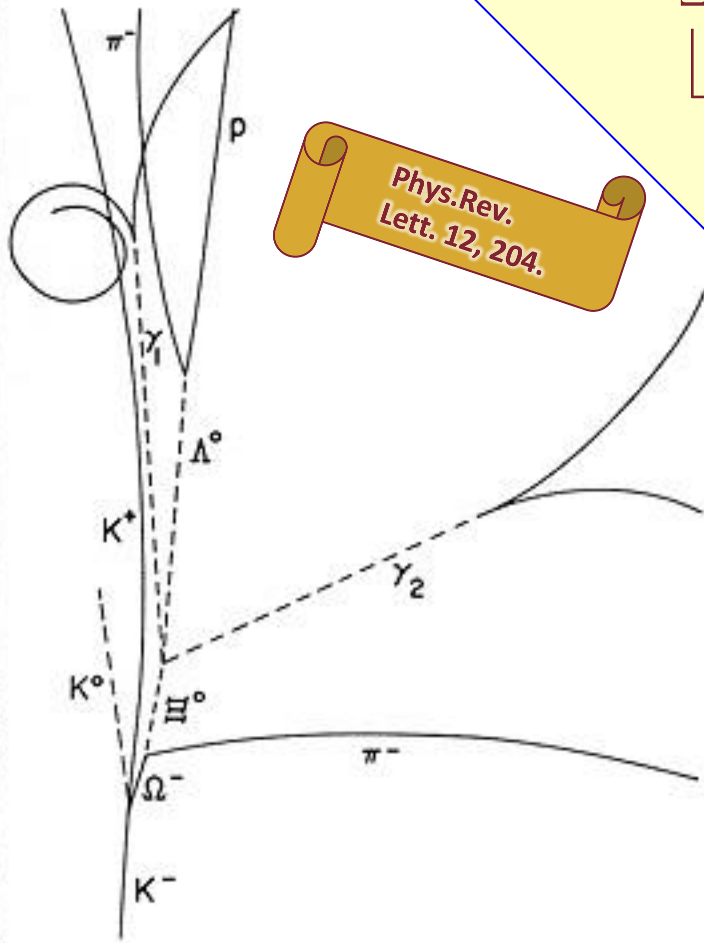
the discovery of the Ω^-: the event

the Ω^- observation required both genius and luck (e.g. compute the probability of the two γ conversions in H_2):

$K^- + p \rightarrow \Omega^- + K^+ + K^0$

- $\hookrightarrow \Xi^0 + \pi^-$ ($\Delta S = 1$ weak decay)
  - $\hookrightarrow \pi^0 + \Lambda$ ($\Delta S = 1$ w.d.)
    - $\Lambda \hookrightarrow \pi^- + p$ ($\Delta S = 1$ w.d.)
    - $\pi^0 \hookrightarrow \gamma + \gamma$ (e.m. decay)
      - $\hookrightarrow e^+e^-$
      - $\hookrightarrow e^+e^-$





Phys.Rev. Lett. 12, 204.

Brookhaven National Laboratory 80-inch hydrogen bubble chamber - 1964

Nick Samios

the static quark model

In 1964 M. Gell-Mann and G. Zweig proposed independently that all the hadrons are composed of three constituents, that Gell-Mann called[1] **quarks**.

This model, enriched by both extensions (other quarks) and dynamics (electroweak interactions and QCD) is still the basis of our understanding of the elementary particles, the **Standard Model**[2].

In this chapter we consider only the static properties of the three original quarks. Sometimes, in the literature, it is referred as the *naïve quark model*.

1969 : Gell-Mann is awarded Nobel Prize "*for his contributions and discoveries concerning the classification of elementary particles and their interactions*".

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[1] The name so whimsical was taken from the (now) famous quote "*Three quarks for Muster Mark !*", from James Joyce's novel "*Finnegans Wake*" (book 2, chapt. 4).

[2] At that time it was not clear whether the quark hypothesis was a mathematical convenience or reality. Today, as shown in the following, our understanding is clearer, but complicated: the quarks are real (to the extent that all QM particles are), but they cannot be seen as isolated single objects.

the static quark model: u d s

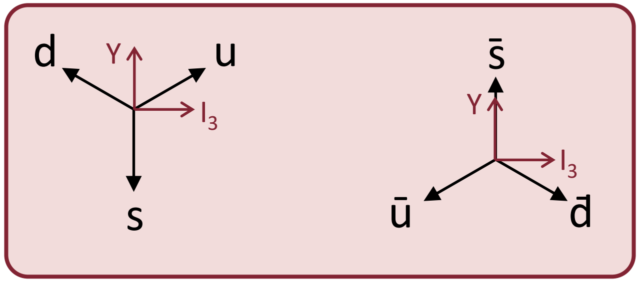
The hypothesis:

- three quarks **u**, **d**, and **s** (up, down, strange);
- quarks (q): standard Dirac fermions with spin ½ and fractional charge (±⅓e ±⅔e);
- antiquarks (q̄): according to Dirac theory, the q-antiparticles;
- baryons: combinations qqq (e.g. uds, uud);
- antibaryons: three antiquarks (e.g ūūđ);
- mesons: pairs qq̄ (e.g uū, uđ, sū);
- "*antimesons*": a q̄q pair: the mesons are their own antiparticles, i.e. "anti-mesons" = mesons.

	u	d	s	c	b	t
$\mathcal{B}$ baryon	⅓	⅓	⅓			
J spin	½	½	½			
I isospin	½	½	0			
I_3 3rd i-spin	½	−½	0			
S strang.	0	0	−1			
Y $\mathcal{B}$+S	⅓	⅓	−⅔			
Q $I_3 + \frac{1}{2}Y$	⅔	−⅓	−⅓			

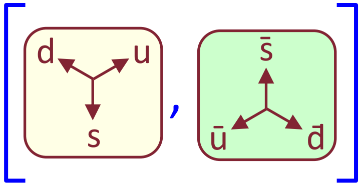
c, **b**, **t** not yet discovered in the '60 !!! see § 3

The quarks form a triplet, which is a basic representation of the group SU(3). Quarks may be represented in a vector shape in the plane $I_3 - Y$; their combinations (= hadrons) are the sums of such vectors.

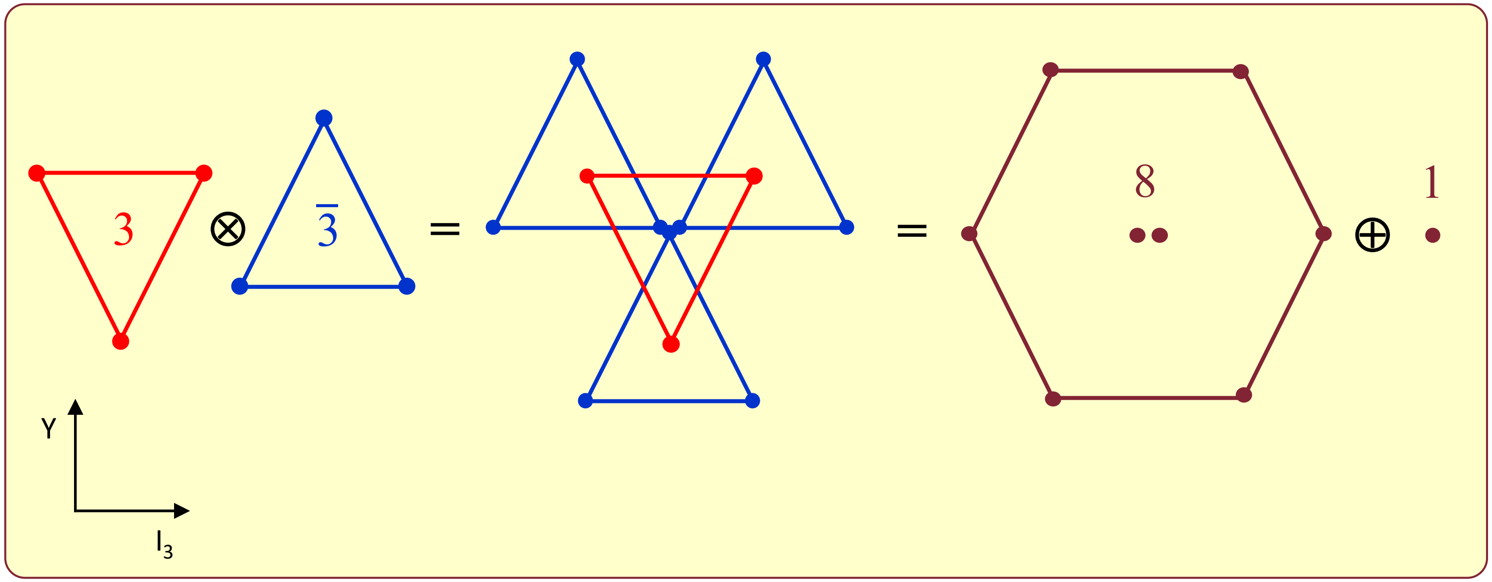


The mesons $q\bar{q}$

"Build" the mesons $q\bar{q}$ with these rules :



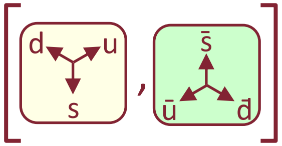
- in the space I_3 - Y, sum "vectors" (i.e. quarks and antiquarks) to produce $q\bar{q}$ pairs, i.e. mesons;
- all the combinations are allowed:



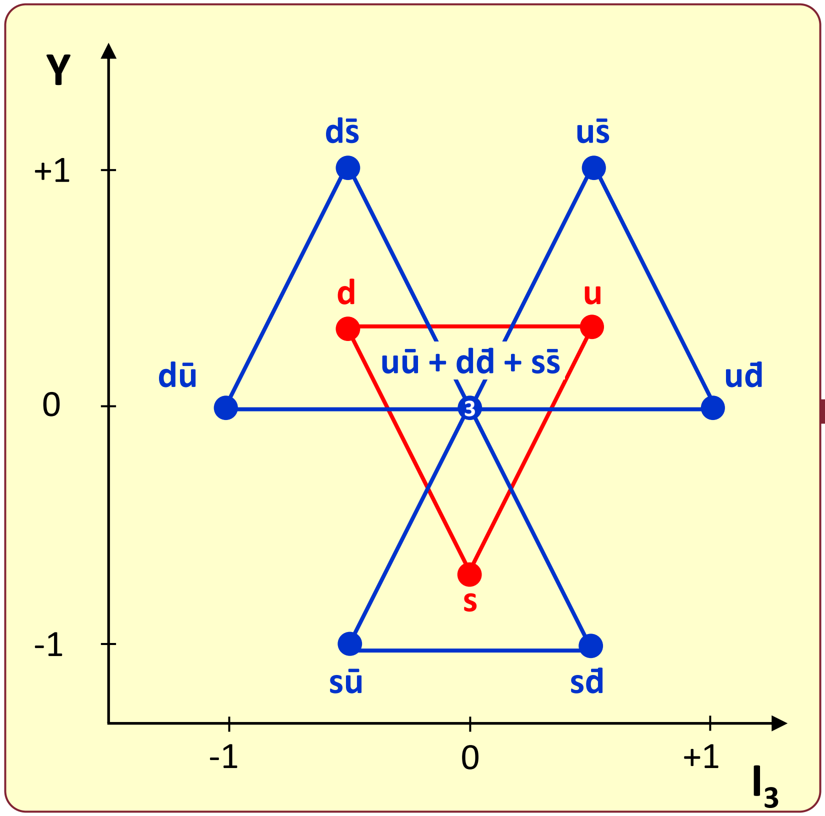
- the pseudoscalar mesons ($J^P=0^-$) are $q\bar{q}$ states in s-wave with opposite spins (⇑⇓).

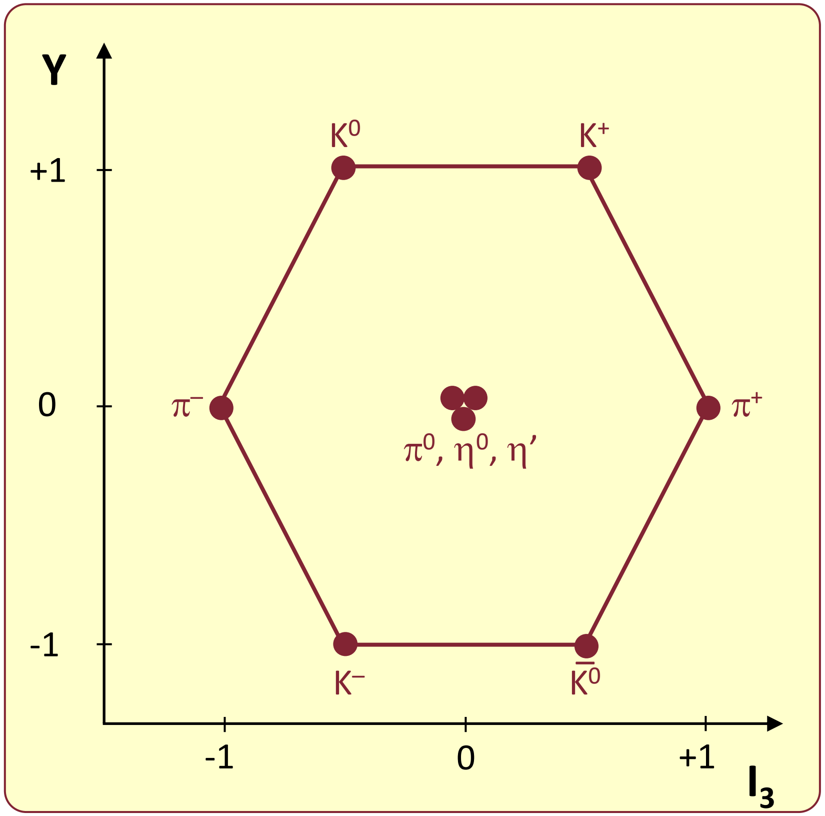
The mesons: $J^{PC}=0^{-+}$

More specifically, with s-wave ($J^{PC}=0^{-+}$), we get the "pseudoscalar" nonet :



Notice that π^0, η, η' are combinations (mixing) of the three possible $q\bar{q}$ states (for the mixing parameters see later) :

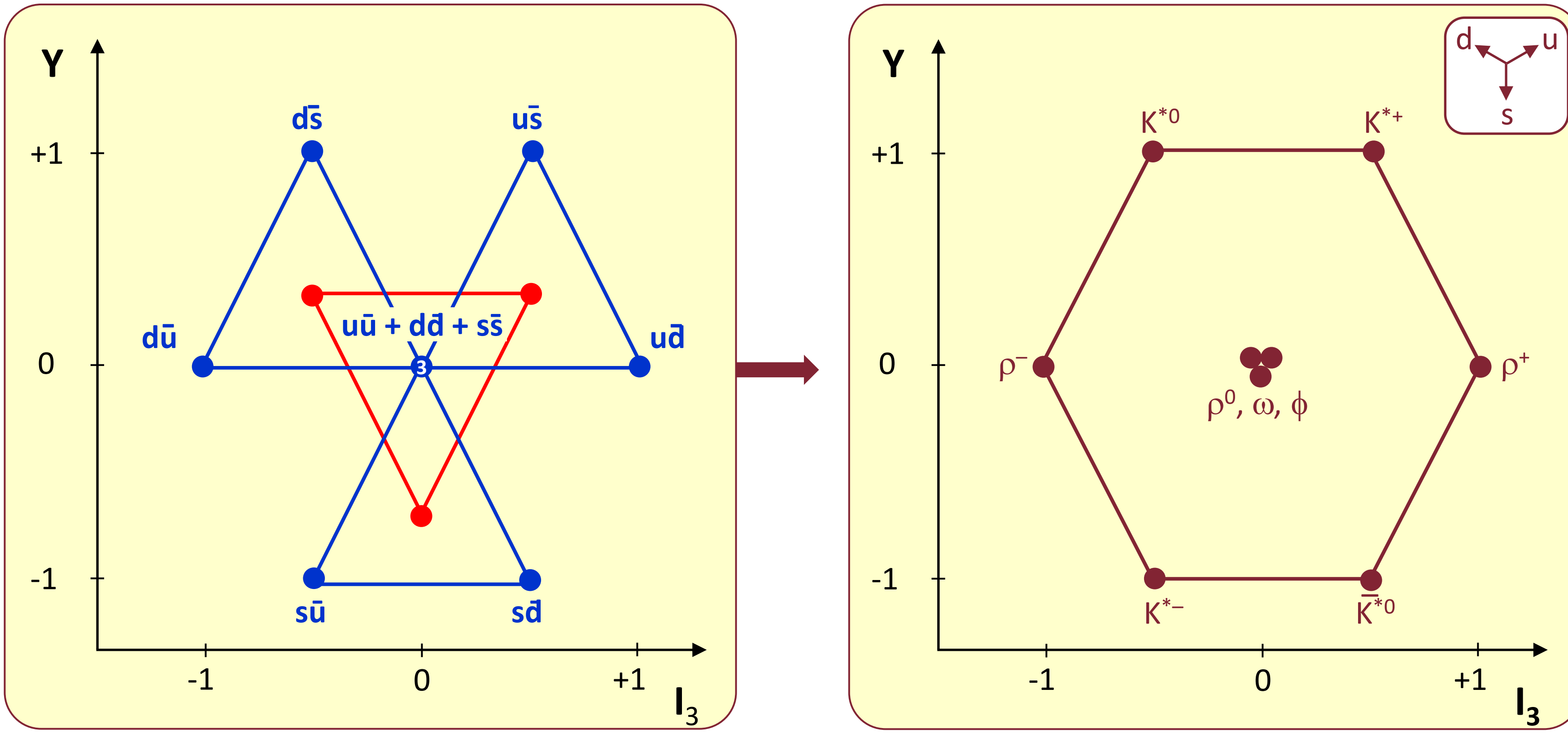




The mesons: $J^{PC}=1^{--}$

If $J^{PC} = 1^{--}$ (i.e. spin ⇈), the "vector" nonet :

Notice that ρ^0, ω, φ are combinations (mixing) of the three possible $q\bar{q}$ states :

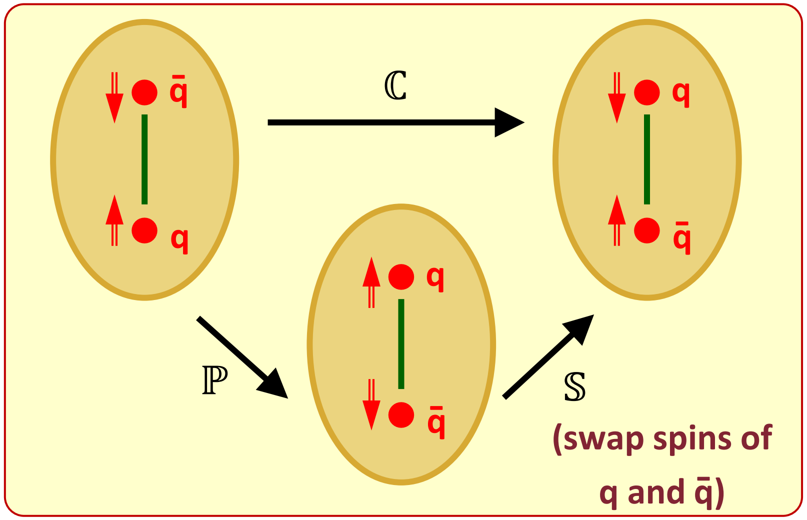


Meson quantum numbers: J^{PC}

- Parity : the quarks and the antiquarks have opposite P :

  $P_{q\bar{q}} = P_1 P_2 (-1)^L = -1 (-1)^L = (-1)^{L+1}$.

Charge conjugation : for mesons, which are also ℂ eigenstates, ℂ = ℙ𝕊, parity followed by spin swap (see before).



the "intrinsic spin" $\vec{s}$ of a meson (seen as an elementary object) becomes the "full angular momentum" $\vec{J}$ of a $q\bar{q}$ composite.

- Therefore we can apply (see before):

  $P = (-1)^{L+1}$;

  $C = P \times \mathcal{S} = (-1)^{L+S}$;

  $G = (-1)^{L+S+I}$ (only for the eigenstates of $\mathbb{G}$, e.g. $\pi^{\pm 0}/\eta$, NOT K^0).

- Conclusion: the first multiplets are (+ many others):

L	S	J=L⊕S	P	C	I	G
0	0	0	–	+	0	+
					1	–
	1	1	–	–	0	–
					1	+
1	0	1	+	–	0	–
					1	+
	1	0,1,2	+	+	0	+
					1	–

$J^{PC} = 0^{-+}, 1^{--}, 1^{+-}, 0^{++}, 1^{++}, 2^{++}, \ldots$

Meson quantum numbers : multiplets

- For the lowest state nonets, these are the quantum numbers :

L	S	J^{PC}	$^{2s+1}L_J$	I=1 state
0	0	0^{-+}	$^1S_0$	$\pi(140)$
	1	1^{--}	$^3S_1$	$\rho(770)$
1	0	1^{+-}	$^1P_1$	$b_1(1235)$
	1	0^{++}	$^3P_0$	$a_0(1450)$
		1^{++}	$^3P_1$	$a_1(1260)$
		2^{++}	$^3P_2$	$a_2(1320)$

- all these multiplets have main qn n = 1;
- as of today ~20 meson multiplets have been (partially) discovered [PDG].
- important activity from the '50 to the '70; still some addict;

- method (mainly bubble chambers) :
  - measure (zillions of) events; e.g. :

    $\bar{p}p \to \pi^+ \pi^+ \pi^- \pi^- \pi^0;$

  - look for "peaks" in final state combined mass, e.g. $m(\pi^+ \pi^- \pi^0)$;
  - the peaks are associated with high mass resonances, decaying via strong interactions (width $\to \Gamma \to$ strength);
  - the scattering properties (e.g. the angular distribution) and decay modes identify the other quantum numbers;
- result : an overall consistent picture;
- Great success !!!

"If I could remember the names of all these particles, I'd be a botanist."
Enrico Fermi

Meson quantum numbers : example

C. Alff et al. Phys. Rev. Lett., 9:322, 1962.

read it !!! only 3 pages !!!

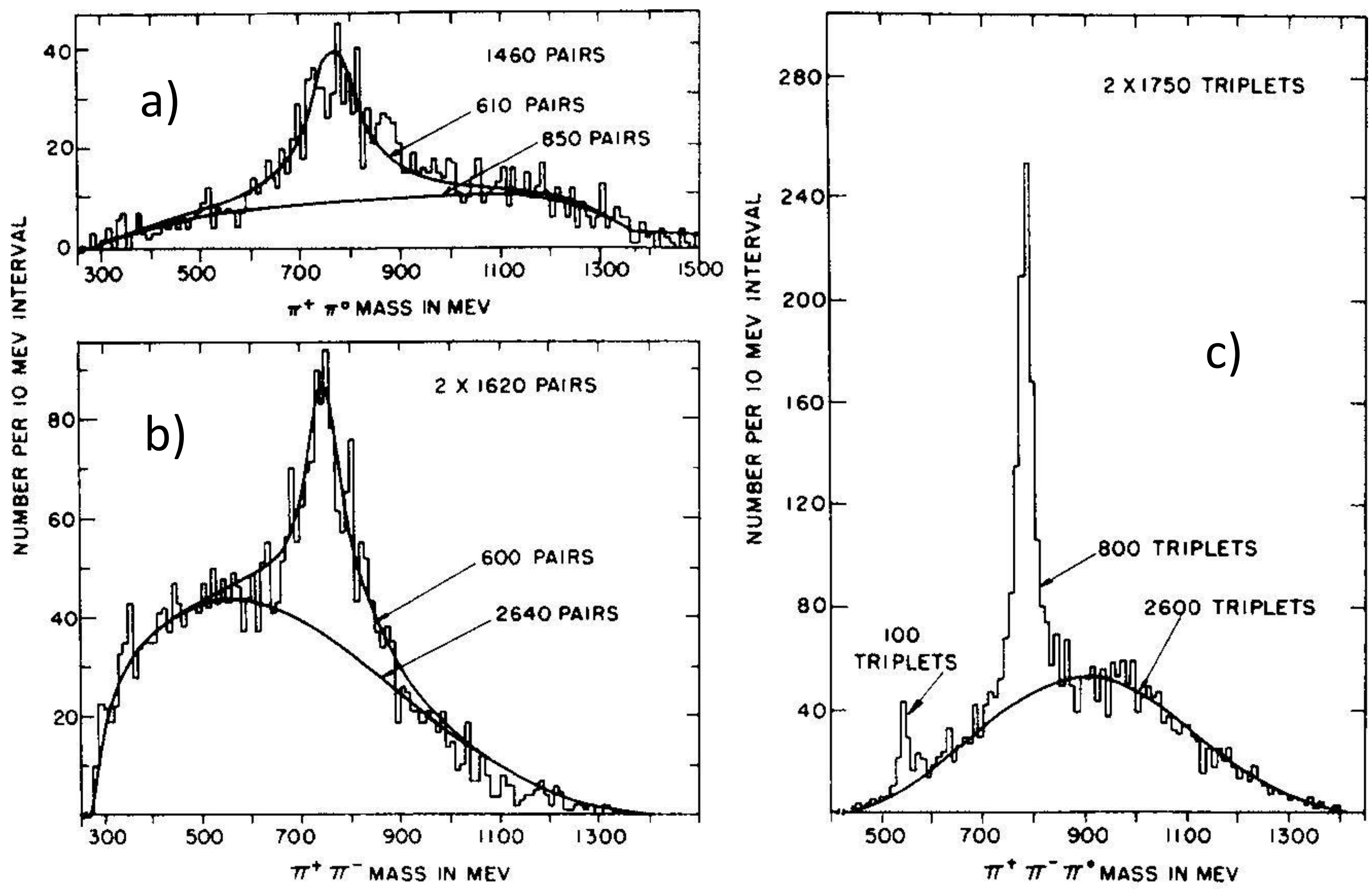
Three examples in $\pi^+ p \to X$

$|\vec{p}_{\pi^+}|$ = 2.3-2.9 GeV

a) $m(\pi^+\pi^0)$ for $X = \pi^+\pi^0 p$

b) $m(\pi^+\pi^-)$ for $X = \pi^+ \pi^+ \pi^- p$

c) $m(\pi^+ \pi^- \pi^0)$ for $X = \pi^+\pi^+\pi^-\pi^0 p$



Q: which resonances ?

a) $\rho^+(770) \to \pi^+\pi^0$; b) $\rho^0(770) \to \pi^+\pi^-$; c) $\eta(548) \to \pi^+\pi^-\pi^0$ $\omega(782) \to \pi^+\pi^-\pi^0$.

why not the ρ^0 ?

Meson quantum numbers : $\rho^0 \not\to \pi^0\pi^0$

Problem: $\rho^0 \to \pi^0\pi^0$ is allowed ? **NO**, because of :

a) ℂ-parity

$C(\rho^0) = -1; C(\pi^0) = +1$

therefore, since the initial state is a C-eigenstate,

$-1 = (+1) \times (+1) \to$ **NO**

b) Clebsch-Gordan coeff. in isospin space

$|\rho^0\rangle = |I{=}1, I_3{=}0\rangle;$

$|\pi^0\rangle = |1, 0\rangle;$

therefore the decay is

$\langle\pi^0\pi^0|\rho^0\rangle = \langle j_1 j_2 m_1 m_2 | J\ M\rangle =$
$= \langle 1\ 1\ 0\ 0\ |\ 1\ 0\ \rangle = 0;$

$\to$ **NO**.

[PDG, § 44 :

1⊗1		...	1
		...	0
...	...	...	...
0	0	...	**0**

c) Spin-statistics

[Povh, problem 15-1]

- $S(\rho^0) = 1$, $S(\pi^0) = 0$ $\to L(\pi^0\pi^0) = 1;$
- ρ^0 is a boson $\to$ wave function symmetric;
- the π^0's are two equal bosons $\to$ space wave function symmetric;
- L=1 makes the wave function anti-symmetric

$\to$ **NO**.

NB. A general rule : "a vector cannot decay into two equal (pseudo-)scalars".

But (a) and (b) do not hold for weak decays. Instead (c) is due to statistics + angular momentum conservation, and is valid for all interactions.

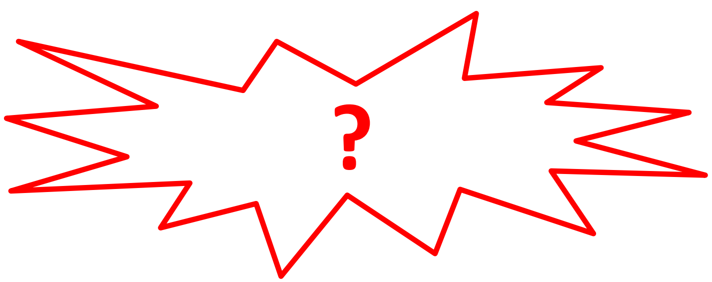
[(c) also forbids **Z → HH**]

Meson quantum numbers : decays

General comments on particle decays ($\mathcal{D}$):

- $\mathcal{D}$ exist if no selection rule $\mathbf{S}$ forbids them;
- only $\mathbf{S}$ valid for ALL interactions count;
- [*e.g. strangeness conservation, which is violated in weak interactions is* NOT *included*;]
- [*be careful: some* $\mathbf{S}$ *are "non-obvious", but still valid, e.g.* NO $\rho^0 \to \pi^0\pi^0$;]
- the $\mathbf{S}$ include conservation of "numbers" (charge, baryon n., lepton n.), of 4-momentum, ang. momentum, …;
- e.g. $\rho^0 \to \pi^+\pi^-$, $\rho^0 \to e^+e^-$, $\pi^0 \to \gamma\gamma$, $\Lambda \to p\pi^0$;
- some strong $\mathcal{D}$ (e.g. $\rho^0 \to \pi^+\pi^-$), some e.m. ($\rho^0 \to e^+e^-$), some weak ($\Lambda \to p\pi^0$);
- for each $\mathcal{D}$ there is a matrix element $\mathcal{M}_j$ and a partial width Γ_j;
- in Γ_j the phase-space factor may be important (e.g. $\phi \to K^+K^-$);
- all these $\mathcal{D}$ actually exist and contribute to the particle $\Gamma_T = \Sigma_j \Gamma_j$ and $\tau = 1/\Gamma_T$;
- for a particle the partial Γ_j's may vary a lot, mainly because of their couplings, e.g. $\Gamma(\rho^0 \to \pi^+\pi^-) >> \Gamma(\rho^0 \to e^+e^-)$;
- in practice, when strong $\mathcal{D}$ exist, they are dominant;
- the other $\mathcal{D}$ have small, but non-zero probability, e.g. $BR(\rho^0 \to e^+e^-) \approx 4.7 \times 10^{-5}$.

NB *the decay mode(s) and the* Γ_T, Γ_j *are specific to the process and* *__not necessarily__* *similar for similar particles; e.g.* $\Gamma_T(\pi^\pm) << \Gamma_T(\pi^0)$ *are very different, as* p *(stable !) and* n ($\tau_n \approx 879$ s).



Meson mixing

Light mesons	$q\bar{q}$	J^{PC} (1)	I	I_3	S	Q (1)	mass (MeV)	$q\bar{q}$ of I_3=0 (2)
π^+, π^0, π^-	$u\bar{d}$, $q\bar{q}^{(2)}$, $d\bar{u}$	0^{-+}	1	1, 0, -1	0	1, 0, -1	140	$\sim(u\bar{u}-d\bar{d})/\sqrt{2}$
η	$q\bar{q}^{(2)}$	0^{-+}	0	0	0	0	550	$\sim(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$
η'	$q\bar{q}^{(2)}$	0^{-+}	0	0	0	0	960	$\sim(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$
K^+, K^0 (3)	$u\bar{s}$, $d\bar{s}$	0^-	½	½,-½	+1	1, 0	495	
$\bar{K}^0, K^-$ (3)	$s\bar{d}$, $s\bar{u}$	0^-	½	½, -½	-1	0, -1	495	
ρ^+, ρ^0, ρ^-	$u\bar{d}$, $q\bar{q}^{(2)}$, $d\bar{u}$	1^{--}	1	1, 0, -1	0	1, 0, -1	770	$\sim(u\bar{u}-d\bar{d})/\sqrt{2}$
ω	$q\bar{q}^{(2)}$	1^{--}	0	0	0	0	780	$\sim(u\bar{u}+d\bar{d})/\sqrt{2}$
ϕ	$q\bar{q}^{(2)}$	1^{--}	0	0	0	0	1020	$\sim s\bar{s}$
K^{*+}, K^{*0} (3)	$u\bar{s}$, $d\bar{s}$	1^-	½	½,-½	+1	1, 0	890	
$\bar{K}^{*0}, K^{*-}$ (3)	$s\bar{d}$, $s\bar{u}$	1^-	½	½, -½	-1	0, -1	890	

Notes :

(1) (L=0, $\mathscr{B}$=0) → $P = (-)^{L+1} = -$; $C = (-)^{L+S} = (-)^S$; $Q = I_3 + ½Y = I_3 + ½S$;

(2) The mesons π^0, η, η', ρ^0, ω, ϕ are mixing of $u\bar{u} \oplus d\bar{d} \oplus s\bar{s}$ (see next);

(3) States with strangeness ≠ 0 are NOT eigenstates of C; since they have I=½, no I_3=0 exists.

Meson mixing: $J^P = 0^-, 1^-$

Mesons are bound states $q\bar{q}$. Consider only uds quarks (+ $\bar{u}\bar{d}\bar{s}$) in the nonets ($J^P = 0^-\ 1^-$, the *pseudo-scalar* and *vector* nonets) :

- the states ($\pi^+=u\bar{d}$, $\pi^-=d\bar{u}$, $K^+=u\bar{s}$, $K^0=d\bar{s}$, $K^-=s\bar{u}$, $\bar{K}^0=s\bar{d}$) have no quark ambiguity;
- but ($u\bar{u}\ d\bar{d}\ s\bar{s}$) have the same quantum numbers and the three states ($\psi_{8,0}\ \psi_{8,1}\ \psi_1$) mix together ($\rightarrow$ 2 angles per nonet);
- the physical particles (π^0, η, η' for 0^-, ρ^0, ω, ϕ for 1^-) are linear combinations $q\bar{q}$;
- ($\psi_{8,1}$) decuples ($\pi^0\ \rho^0$) ($\rightarrow$ 1 angle only);
- θ_{ps} and θ_v are computed from the mass matrices* [PDG, §15.2];
- notice: the vector mixing $\theta_v \approx 36° \approx \tan^{-1}(1/\sqrt{2})$, i.e. the ϕ meson is almost $s\bar{s}$ only [i.e. $\phi \rightarrow K\bar{K}$, see KLOE exp.];

(... continue)

$$\left.\begin{aligned} \psi_{8,1} &= (u\bar{u} - d\bar{d})/\sqrt{2} \\ \psi_{8,0} &= (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6} \\ \psi_1 &= (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \end{aligned}\right\} \begin{array}{c} \psi_{\text{multi},I} \\ \text{ideal case} \end{array}$$

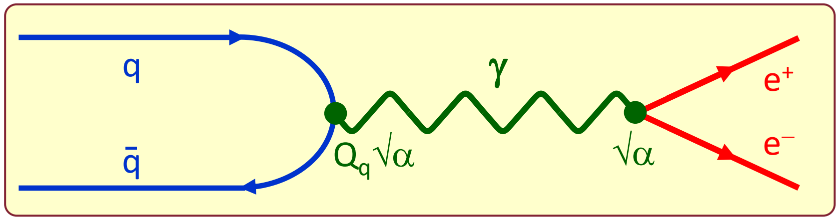
$$\left.\begin{aligned} \pi^0(140) &\approx \psi^{ps}_{8,1} = (u\bar{u} - d\bar{d})/\sqrt{2} \\ \eta(550) &= \psi^{ps}_{8,0}\cos\theta_{ps} - \psi^{ps}_1 \sin\theta_{ps} \\ \eta'(960) &= \psi^{ps}_{8,0}\sin\theta_{ps} + \psi^{ps}_1 \cos\theta_{ps} \end{aligned}\right\} \begin{array}{c} J^P = 0^-, \\ \theta_{ps} \approx -25°; \end{array}$$

$$\left.\begin{aligned} \rho^0(770) &\approx \psi^{v}_{8,1} = (u\bar{u} - d\bar{d})/\sqrt{2} \\ \phi(1020) &= \psi^{v}_{8,0}\cos\theta_{v} - \psi^{v}_1 \sin\theta_{v} \approx s\bar{s} \\ \omega(780) &= \psi^{v}_{8,0}\sin\theta_{v} + \psi^{v}_1 \cos\theta_{v} \approx \\ &\approx (u\bar{u} + d\bar{d})/\sqrt{2} \end{aligned}\right\} \begin{array}{c} J^P = 1^-, \\ \theta_{vect} \approx 36°. \end{array}$$

* in principle, both the mass spectra and the mixing angles can be computed from QCD lagrangian $\mathscr{L}_{QCD}$ ... waiting for substantial improvements in computation methods.

Meson mixing: $J^P = 1^-$

The decay amplitudes in the e.m. channels may be computed, up to a common factor, and compared to the experiment;



Few problems :

- the values are small*, e.g. $BR(\rho^0 \to e^+e^-) \approx 4.7\times10^{-5}$;
- the phase-space factor is important, especially for ϕ, which is very close to the $s\bar{s}$ threshold ($m_\phi - 2\ m_K$ = few MeV).

However, the overall picture is clear: the theory explains the data ***very well***.

---

* warning: the dominant $\rho^0\omega\phi$ decay modes are strong; however, the e.m. decays $\rho^0\omega\phi \to e^+e^-$, with a much smaller BR, are detectable $\to \Gamma_{e.m.}$ measurable $\to$ quark charges compared.

$$\left.\begin{array}{l} \rho^0(770) = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}); \\ \omega(780) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}); \\ \phi(1020) = s\bar{s}; \end{array}\right\} \to \mathcal{M}_{fi}(\rho^0\omega\phi \to e^+e^-) \propto \alpha \sum_j Q_q^j;$$

$$\to \begin{cases} \Gamma(\rho^0 \to e^+e^-) \propto \left[\frac{1}{\sqrt{2}}\left(\frac{2}{3} - \frac{-1}{3}\right)\right]^2 = \frac{1}{2}; \\ \Gamma(\omega \to e^+e^-) \propto \left[\frac{1}{\sqrt{2}}\left(\frac{2}{3} + \frac{-1}{3}\right)\right]^2 = \frac{1}{18}; \\ \Gamma(\phi \to e^+e^-) \propto \left[\frac{1}{3}\right]^2 = \frac{1}{9}; \end{cases}$$

$$\to \Gamma_\rho : \Gamma_\omega : \Gamma_\phi = \begin{cases} 9 : 1 : 2 & \text{(theo)} \\ 8.8 \pm 2.6 : 1 : 1.7 \pm 0.4 & \text{(exp).} \end{cases}$$

The baryons qqq

The construction looks complicated, but in fact is quite simple :

- add the three quarks one after the other;
- count the resultant multiplicity.

In group's theory language :

$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8' \oplus 1$

i.e. a decuplet, two octects and a singlet.

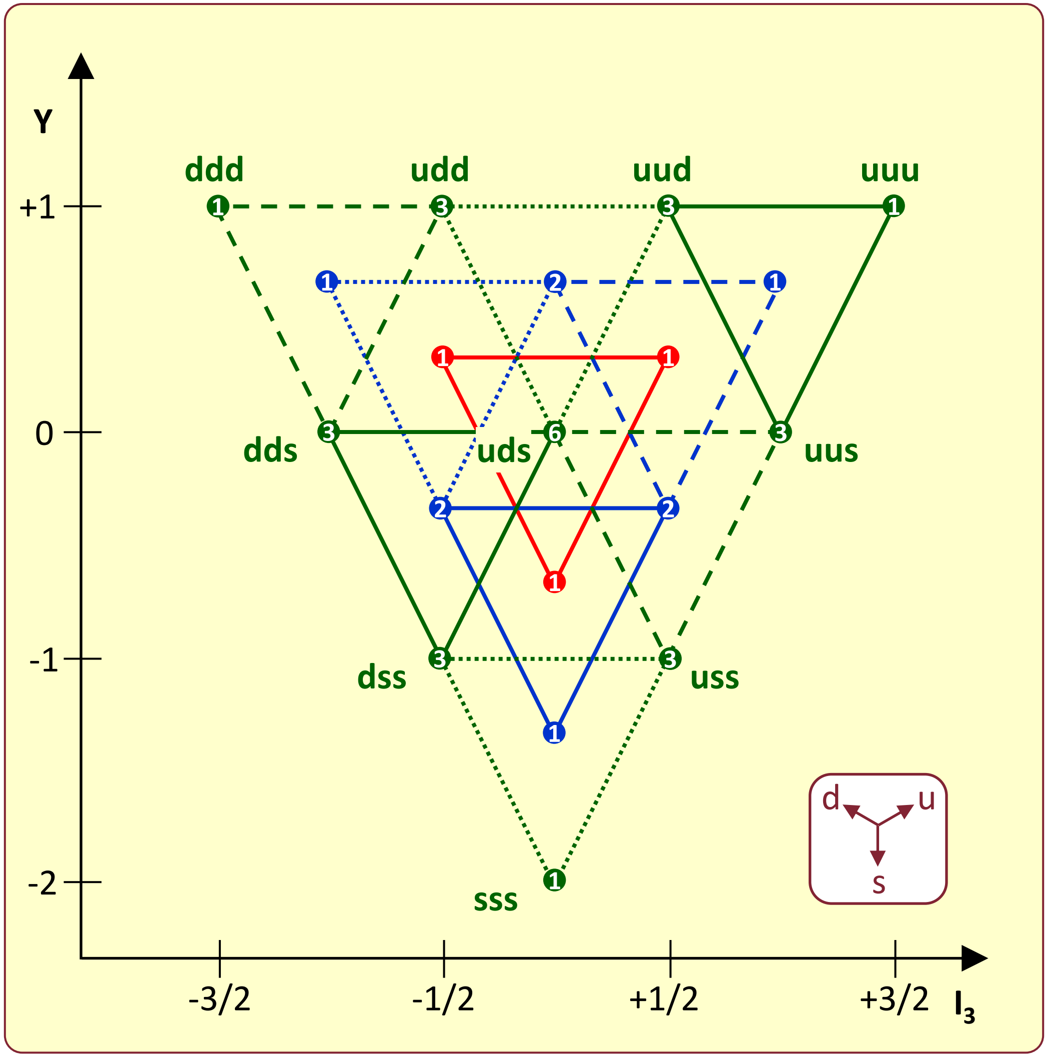
[proof. :

$(3 \otimes 3) \otimes 3 = (6 \oplus \bar{3}) \otimes 3;$

$6 \otimes 3 = 10 \oplus 8;$

$\bar{3} \otimes 3 = 8 \oplus 1.$ q.e.d.]

Both for 10, 8, 8' and 1 the three quarks have L = 0.



The baryons: quantum numbers

Baryons	qqq	J^P	I	I_3	S	$Q^{(1)}$	mass (MeV)
p, n	uud, udd	$\frac{1}{2}^+$	½	½, -½	0	1, 0	940
Λ	uds	$\frac{1}{2}^+$	0	0	-1	0	1115
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	$\frac{1}{2}^+$	1	1, 0, -1	-1	1, 0, -1	1190
Ξ^0, Ξ^-	uss, dss	$\frac{1}{2}^+$	½	½,-½	-2	1, 0	1320
$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	uuu, uud, udd, ddd	$\frac{3}{2}^+$	$^3/_2$	$^3/_2$, $^1/_2$, $-^1/_2$, $-^3/_2$	0	2, 1, 0, -1	1230
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	uus, uds, dds	$\frac{3}{2}^+$	1	1, 0, -1	-1	1, 0, -1	1385
Ξ^{*0}, Ξ^{*-}	uss, dss	$\frac{3}{2}^+$	½	½,-½	-2	1, 0	1530
Ω^-	sss	$\frac{3}{2}^+$	0	0	-3	-1	1670

10
⇑⇑⇑

8
⇑⇑⇓

Notes :

(1) $Q = I_3 + \frac{1}{2}Y = I_3 + \frac{1}{2}(\mathscr{B} + S);\ \mathscr{B} = 1.$

The baryons: the octet $J^P = ½^+$

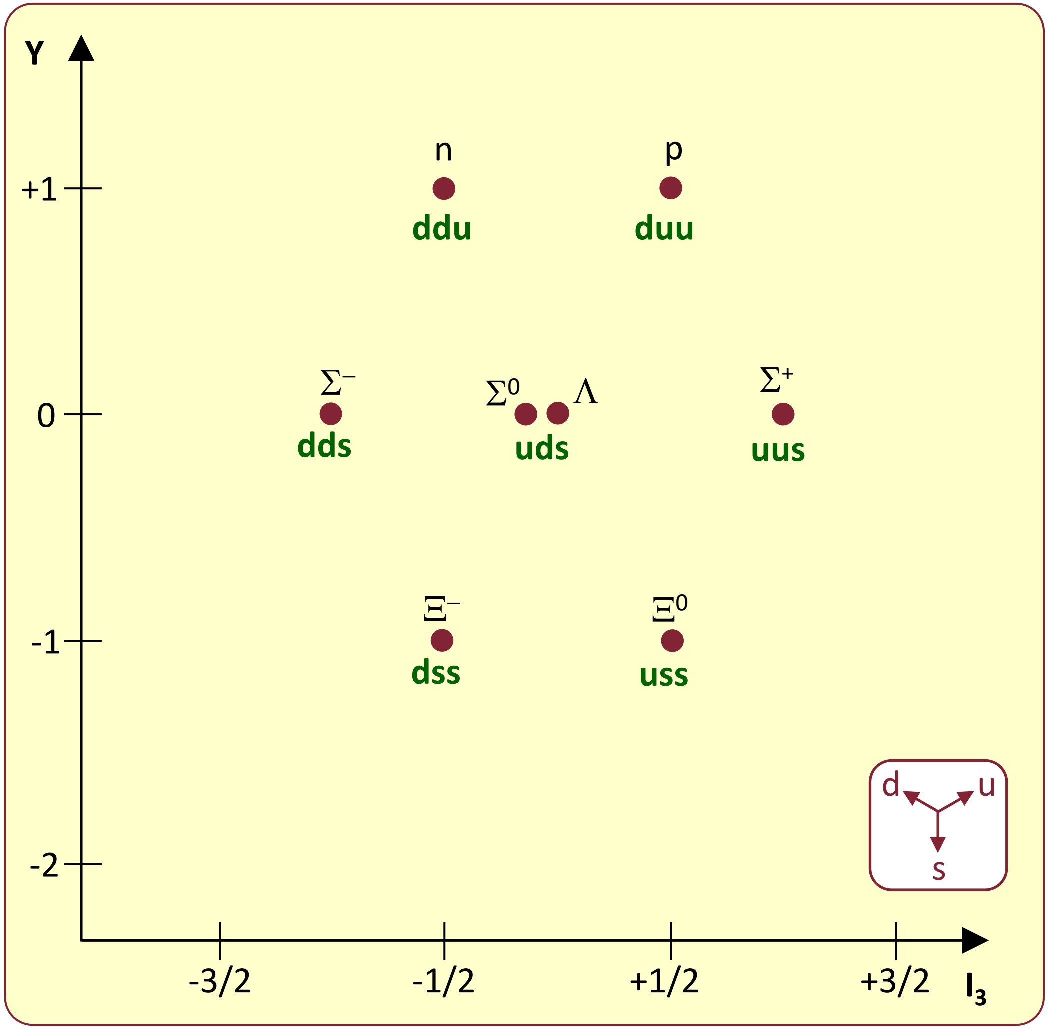
The lowest mass multiplet is an octet, which contains the familiar p and n, a triplet of S=-1 (the Σ's) a singlet S=-1 (the Λ) and a doublet of S=-2 (the Ξ's, sometimes called "cascade baryons").

The three quarks have ℓ = 0 and spin (⇑⇑⇓), i.e. a total spin of ½.

The masses are :

- ~ 940 MeV for p and n;
- ~1115 MeV for the Λ;
- ~1190 MeV for the Σ's;
- ~1320 MeV for the Ξ's;

(difference of < few MeV in the isospin multiplet, due to e-m interactions.)



The baryons: the decuplet $J^P = {}^3/_2{}^+$

The decuplet is rather simple (*but there is a spin/statistics problem, see later*). The spins are aligned (⇑ ⇑ ⇑), to produce an overall $J={}^3/_2$.

The masses, at percent level, are :

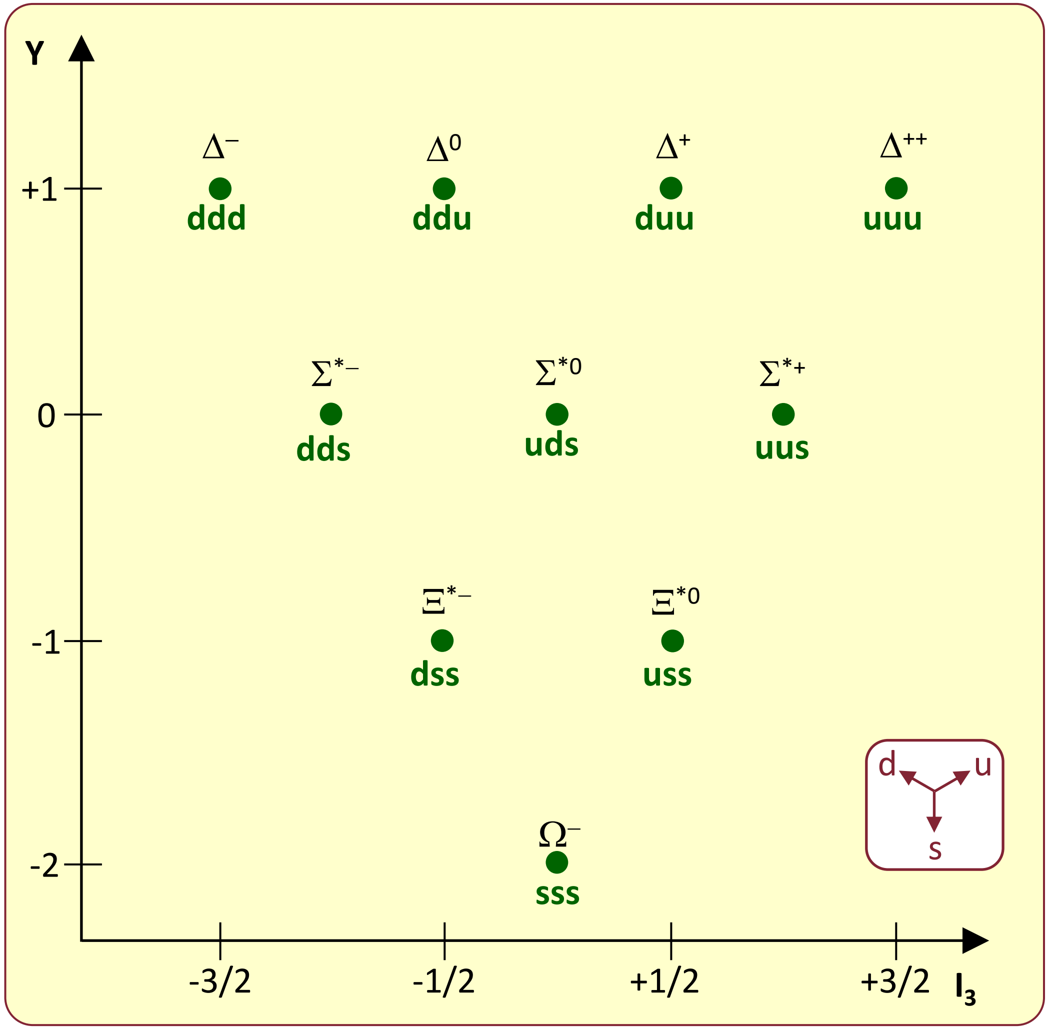
~ 1230 MeV for the Δ's;

~ 1385 MeV for the Σ^*'s,

~ 1530 MeV for the Ξ^*'s

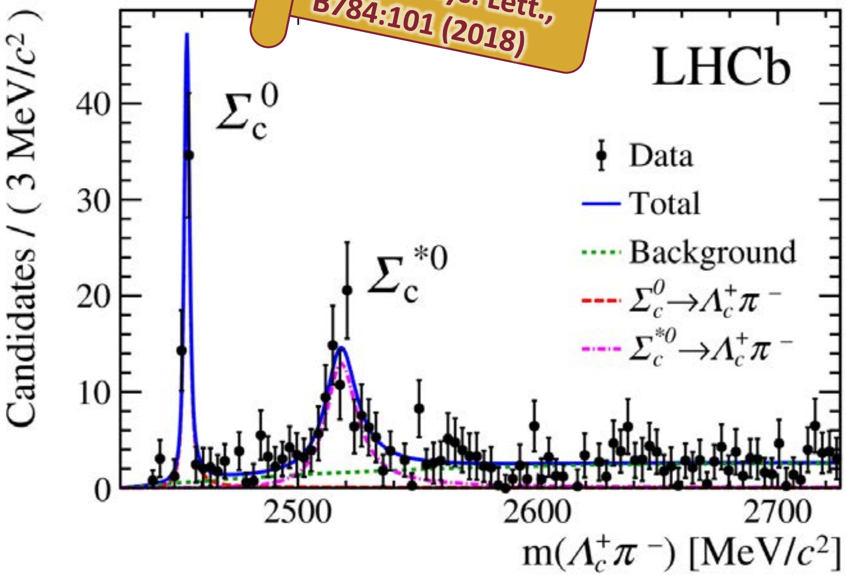
~ 1670 MeV for the Ω^-.

Notice that the mass split among multiplets is very similar, ~150 MeV (important for the Ω^- discovery, lot of speculations, no real explanation).



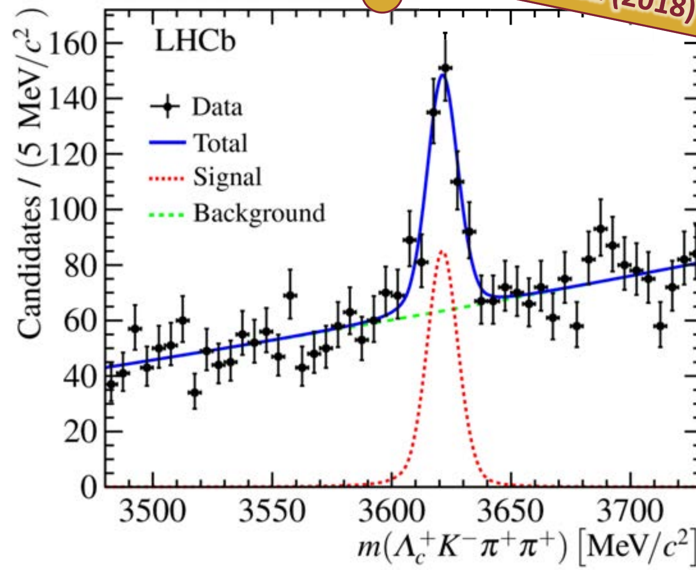
The baryons: example

LHCb - Phys. Lett., B784:101 (2018)



A - $\Sigma_c^0, \Sigma_c^{0*} \to \Lambda_c^+\pi^-$

LHCb - Phys. Rev. Lett., 121:162002 (2018)



B - $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$

Recently, the LHCb Collaboration at LHC has realized a nice search for baryons made with heavy quarks.

[these two examples should stay in § 3, because they contain the c quark]

quark content:

Ξ_{cc}^{++} : ucc;
$\Sigma_c^0, \Sigma_c^{0*}$: ddc;
Λ_c^+ : udc;
K^- : $\bar{u}$s;
$\pi^\pm$: u$\bar{d}$, $\bar{u}$d.

Check conservation of:

✓✓ a. baryon n.
✓✓ b. charge
✓✗ c. charm
– ✗ d. strangeness

Write the Feynman diagrams of the decays → after § 6

SU(3)

- For the SU(2) symmetry, the generators are the Pauli matrices. The third one is associated to the conserved quantum number I_3.

- For SU(3), the Gell-Mann matrices T_j (j=1-8) are defined (next page).

- The two diagonal ones are associated to the operators of the third component of isospin (T_3) and hypercharge (T_8).

- The eigenvectors $|u\rangle$ $|d\rangle$ $|s\rangle$ are associated with the quarks (u, d, s).

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \psi_1^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \psi_1^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix};$$

$$\sigma_2 = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \quad \psi_2^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}; \quad \psi_2^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix};$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \psi_3^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \psi_3^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix};$$

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = -i\sigma_1\sigma_2\sigma_3 = I;$$

$$\left[\sigma_i, \sigma_j\right] = 2i\sum_k \varepsilon_{ijk}\sigma_{k.}$$

Pauli matrices and eigenvectors

in the following, some of the properties of SU(3) in group theory: no rigorous math, only results useful for our discussions.

Demonstrations (some trivial) are in [IE], [BJ 10] or [YK1 G]. A discussion of the group theory, applied to elementary particle physics, in [IE, app. C].

And we have separate – optional – courses.

SU(3) : Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \lambda_2 = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad \lambda_5 = i\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

$$\lambda_7 = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}; \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Gell-Mann matrices λ_i

$$T_i = \frac{1}{2}\lambda_i$$

$$\sum_{j=1}^{8} \lambda_j^2 = \frac{16}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ diagonal.}$$

$$U = 1 + \frac{i}{2}\sum_{j=1}^{8} \varepsilon_j \lambda_j \text{ unitary matrix, } \det U = 1.$$

SU(3) : eigenvectors

Definition of I_3, Y, quark eigenvectors and related relations :

$$\hat{T}_3 = \frac{1}{2}\lambda_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad Y = \frac{1}{\sqrt{3}}\lambda_8 = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix};$$

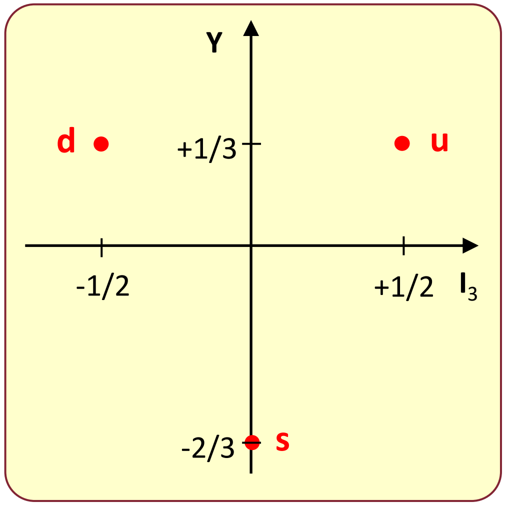
$$|u\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad |d\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad |s\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};$$

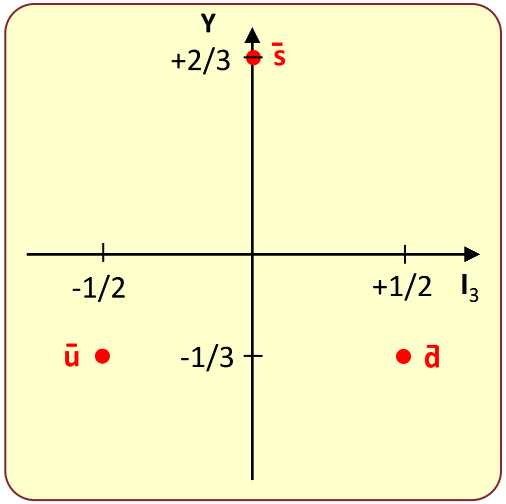
$$\hat{T}_3|u\rangle = +\frac{1}{2}|u\rangle; \quad \hat{T}_3|d\rangle = -\frac{1}{2}|d\rangle; \quad \hat{T}_3|s\rangle = 0;$$

$$\hat{Y}|u\rangle = +\frac{1}{3}|u\rangle; \quad \hat{Y}|d\rangle = +\frac{1}{3}|d\rangle; \quad \hat{Y}|s\rangle = -\frac{2}{3}|s\rangle;$$

$$\hat{T}_3|\bar{u}\rangle = -\frac{1}{2}|\bar{u}\rangle; \quad \hat{T}_3|\bar{d}\rangle = +\frac{1}{2}|\bar{d}\rangle; \quad \hat{T}_3|\bar{s}\rangle = 0;$$

$$\hat{Y}|\bar{u}\rangle = -\frac{1}{3}|\bar{u}\rangle; \quad \hat{Y}|\bar{d}\rangle = -\frac{1}{3}|\bar{d}\rangle; \quad \hat{Y}|\bar{s}\rangle = +\frac{2}{3}|\bar{s}\rangle.$$





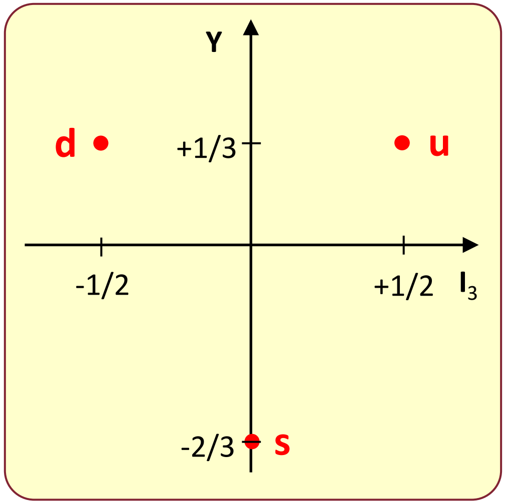
SU(3) : operators

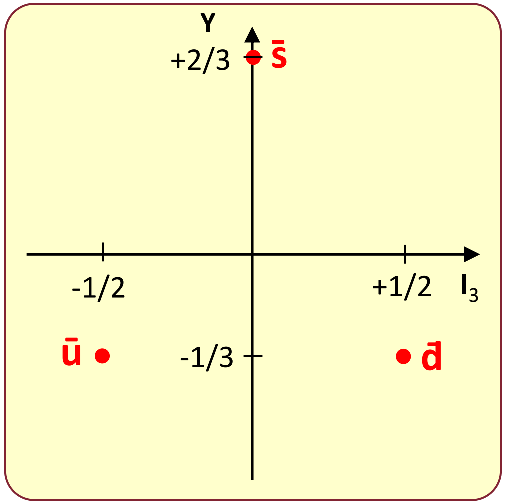
The ladder operators $T_\pm$, $U_\pm$, $V_\pm$:

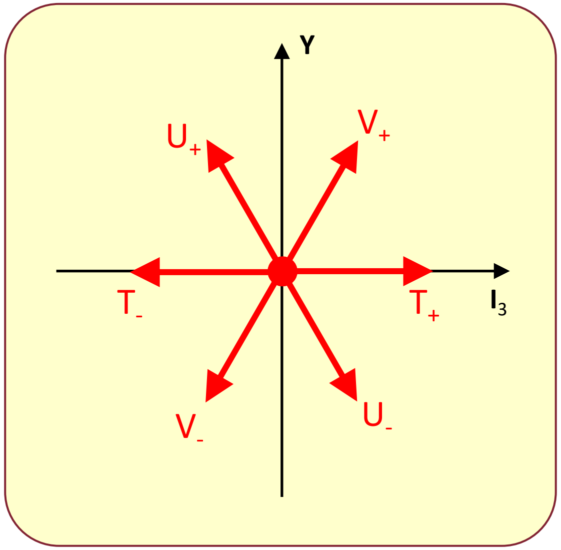
$$T_\pm = T_1 \pm iT_2; \quad U_\pm = T_6 \pm iT_7; \quad V_\pm = T_4 \pm iT_5;$$

As an example, take V_+ :

$$V_+ = T_4 + iT_5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{cases} V_+|\bar{u}\rangle = -|\bar{s}\rangle; \\ V_+|\bar{d}\rangle = 0; \\ V_+|\bar{s}\rangle = 0; \end{cases}$$





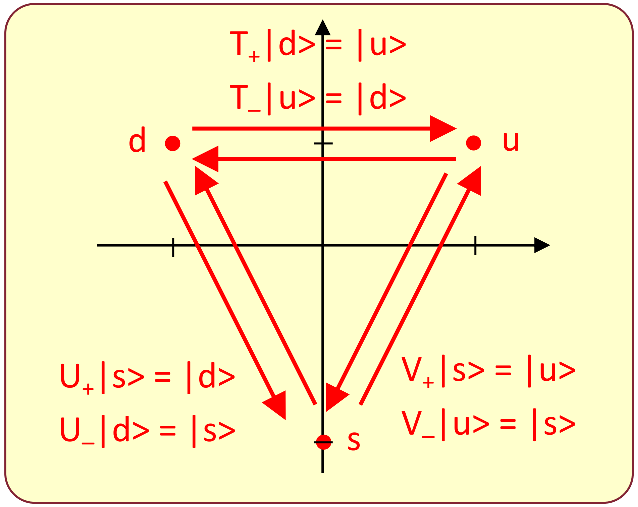


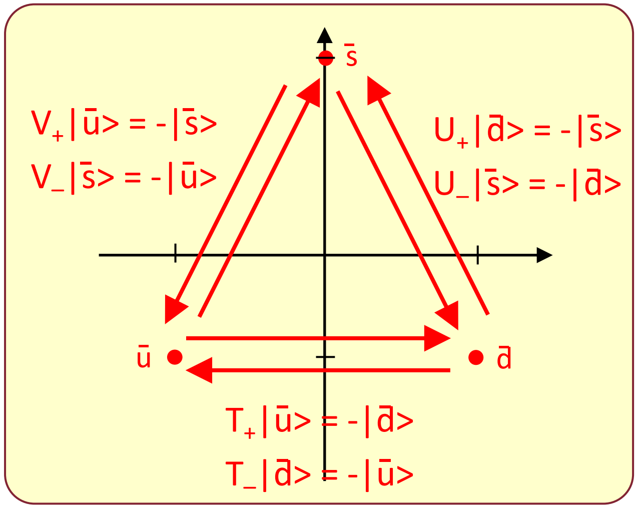
$$V_+|u\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0;$$

$$V_+|d\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0;$$

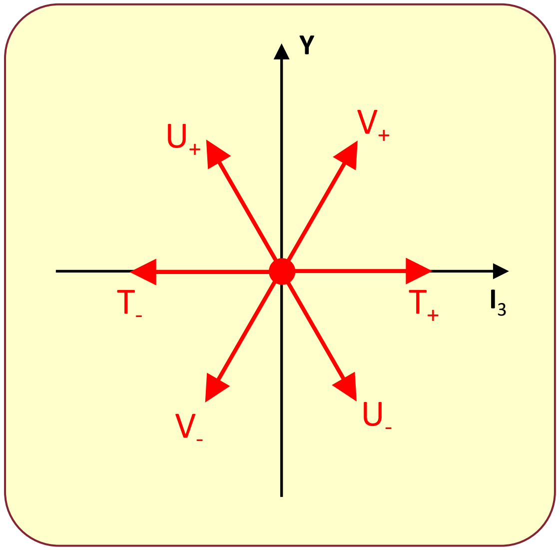
$$V_+|s\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = |u\rangle.$$

SU(3) : ladder operators





The ladder operators $T_{\pm}$, $U_{\pm}$, $V_{\pm}$.



§ QCD

Color : a new quantum number

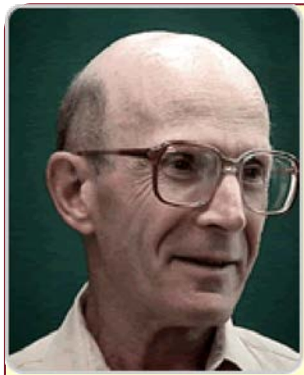
Consider the Δ^{++} resonance:

- $J^P = 3/2^+$ (<u>measured</u>);

$\rightarrow$ quark/spin content [*no ambiguity*]:
$|\Delta^{++}\rangle = | u\Uparrow u\Uparrow u\Uparrow \rangle$

- wave function :

$\psi(\Delta^{++}) = \psi_{space} \times \psi_{flavor} \times \psi_{spin}$ NO !!!


Oskar W. Greenberg


Moo-Young Han (한무영)


Yoichiro Nambu (南部 陽一郎, *Nambu Yōichirō*)

But from symmetry considerations:

the Δ^{++} is lightest uuu state
$\rightarrow$ L = 0;
$\rightarrow$ ψ_{space} symmetric;
$\rightarrow$ ψ_{flavor} and ψ_{spin} symmetric;

$\rightarrow$ $\psi(\Delta^{++})$ = sym.$\times$sym.$\times$sym. = sym.

$\rightarrow$ the Δ^{++} is a fermion, i.e. asym.

$\rightarrow$ <u>**NO**</u> !!!

Anomaly : the Δ^{++} is a spin $^3/_2$ fermion and its function MUST be <u>antisymmetric</u> for the exchange of two quarks (Pauli principle). However, this function is the product of three symmetric functions, and therefore is <u>symmetric</u> $\rightarrow$ **???**.

The solution was suggested in 1964 by Greenberg, later also by Han and Nambu. They introduced a <u>**new quantum number**</u> for strongly interacting particles, composed by quarks : the **COLOR**.

Color : why's and how's

The idea [*see §6, the following is quite naïve*] :

1. quarks exist in three colors (say Red, Green and Blue, like the TV screen(*));
2. they sum like in a TV-screen : e.g. when RGB are all present, the screen is white;
3. the "anticolor" is such that, color + anticolor = white (e.g. $\bar{R}$ = G + B);
4. anti-quarks bring ANTI-colors (see previous point);
5. Mesons and Baryons, which are made of quarks, are white and have no color: they are a "**color singlet**".

Therefore, we have to include the color in the complete wave function; e.g. for Δ^{++} :

$$\psi(\Delta^{++}) = \psi_{space} \times \psi_{flavor} \times \psi_{spin} \times \psi_{color}$$

$$\psi_{color} = (1/\sqrt{6})\, (u_r^1 u_g^2 u_b^3 + u_g^1 u_b^2 u_r^3 + u_b^1 u_r^2 u_g^3 - u_g^1 u_r^2 u_b^3 - u_r^1 u_b^2 u_g^3 - u_b^1 u_g^2 u_r^3)$$

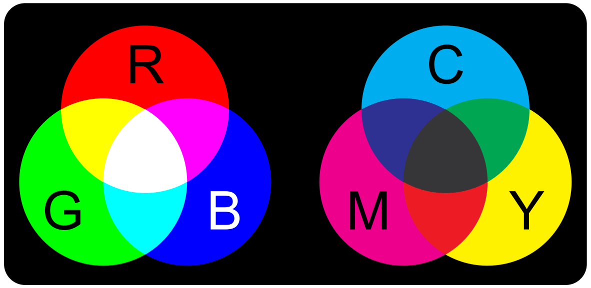
(where u_r, u_g, u_b are the color functions for u quarks of red, green, blue type).

Then ψ_{color} is antisymmetric for the exchange of two quarks and so is the global wave function.

The introduction of the color has many other experimental evidences and theoretical implications, *which we will discuss in the following*.

____

(*) *however, these colors are in no way similar to the real colors; therefore the names "red-green-blue" are totally irrelevant.*



Summary: Symmetries and Multiplets

for a complete discussion, [BJ 10].

1. Since the strong interactions conserve isotopic spin ("$\mathbb{I}$"), hadrons gather in I-multiplets. Within each multiplet, the states are identified by the value of I_3.

2. If no effect breaks the symmetry, the members of each multiplet would be mass-degenerate. The electromagnetic interactions, which do not respect the I-symmetry, split the mass degeneration (at few %) in I-multiplets.

3. Since the strong interactions conserve I, $\mathbb{I}$-operators must commute with the strong interactions Hamiltonian ("$\mathbb{H}_s$") and with all the operators which in turn commute with $\mathbb{H}_s$.

4. Among these operators, consider the angular momentum $\mathbb{J}$ and the parity $\mathbb{P}$. As a result, all the members of an isospin multiplet must have the same spin and the same parity.

5. $\mathbb{H}_s$ is also invariant with respect to unitary representations of SU(2). The quantum numbers which identify the components of the multiplets are as many as the number of generators, which can be diagonalized simultaneously, because are mutually commuting. This number is the *rank* of the Group. In the case of SU(2) the rank is 1 and the operator is $\mathbb{I}_3$.

6. Since $[\mathbb{I}_j , \mathbb{I}_k] = i\varepsilon_{jkm}\mathbb{I}_m$, each of the generators commutes with $\mathbb{I}^2$:

   $\mathbb{I}^2 = \mathbb{I}_1^2 + \mathbb{I}_2^2 + \mathbb{I}_3^2$.

   Therefore $\mathbb{I}^2$, obviously hermitian, can be diagonalized together with $\mathbb{I}_3$.

(continue ...)

Summary: Symmetries and Multiplets

7. The eigenvalues of $\mathbb{I}$ and $\mathbb{I}_3$, can "tag" the eigenvectors and the particles.
8. This fact gives the possibility to regroup the states into multiplets with a given value of *I*.
9. We can generalize this mechanism from the isospin case to any operator : if we can prove that $\mathbb{H}$ is invariant for a given kind of transformations, then:
   a. look for an appropriate symmetry group;
   b. identify its irreducible representations and derive the possible multiplets,
   c. verify that they describe physical states which actually exist.
10. This approach suggested the idea that Baryons and Mesons are grouped in two octets, composed of multiplets of isotopic spin.
11. In reality, since the differences in mass between the members of the same multiplet are ~20%, the symmetry is "broken" (i.e. approximated).
12. Since the octets are characterized by two quantum numbers (I_3 and Y), the symmetry group has rank = 2, i.e. two of the generators commute between them.
13. We are interested in the "irreducible representations" of the group, such that we get any member of a multiplet from everyone else, using the transformations.

(… continue …)

Summary: Symmetries and Multiplets

14. The non-trivial representation (non-trivial = other than the Singlet) of lower dimension is called "Fundamental representation".

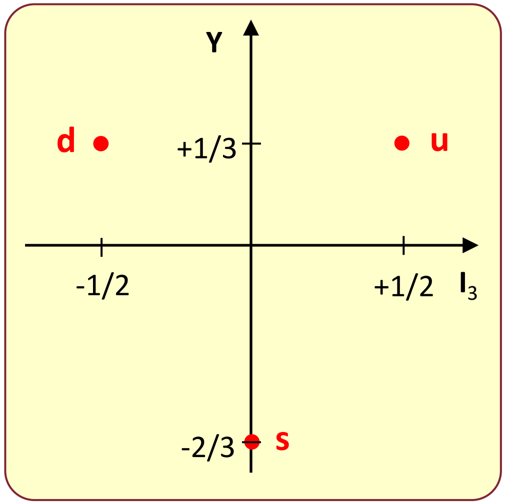
15. In SU(3) there are eight symmetry generators. Two of them are diagonal and associated to I_3 and Y.

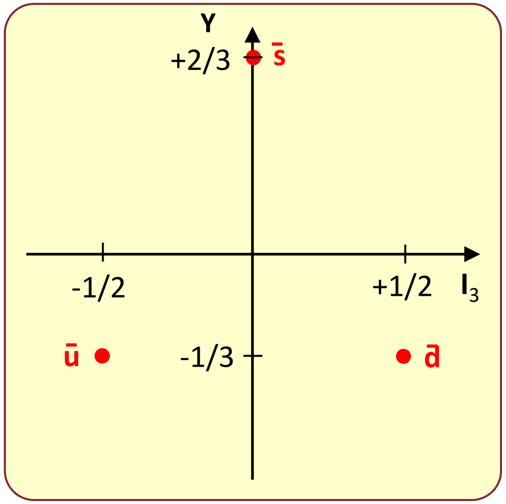
16. The fundamental representations are triplets (→ quarks), from which higher multiplets (→ hadrons) are derived :

    mesons: $3 \otimes \bar{3} = 1 \oplus 8$;

    baryons: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$.

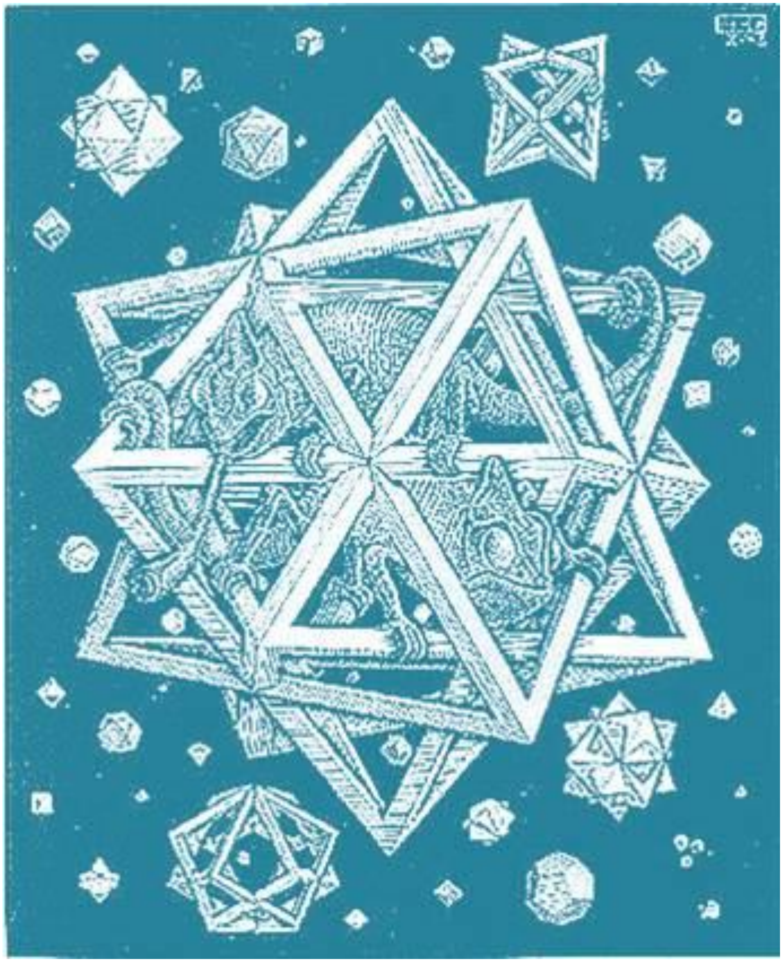
17. This purely mathematical scheme has two relevant applications:
    a. "flavour SU(3)", $SU(3)_F$ with Y_F and I_{3F} for the quarks uds – this symmetry is approximate (i.e. "broken");
    b. "color SU(3)", $SU(3)_C$ with Y_C and I_{3C} for the colors rgb; this symmetry is exact.





References

1. e.g. [BJ, 8];
2. large overlap with [FNSN, 7]
3. isospin and SU(3) : [IE, 2];
4. group theory : [IE, app C];
5. color + eightfold way : [IE, 7-8]
6. G.Salmè – appunti.



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End of chapter 1