The role of $\sigma_{\mathrm{hadronic}}$ for future precision physics

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Outline of Talk:

① **Introduction**

② $\alpha(M_Z)$ **in precision physics**

③ **Evaluation of** $\alpha(M_Z)$

④ **A look at the** e^+e^-**–data**

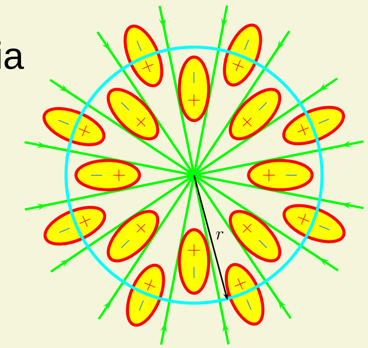
⑤ **Evaluation of** $a_\mu \equiv (g-2)_\mu/2$

⑥ **Remark on** τ**–decay spectral functions**

⑦ **Iso-spin breaking corrections in** τ **vs.** e^+e^-

⑧ **Status and Outlook**

① Introduction



Non-perturbative hadronic effects in electroweak precision observables, main effect via

effective finestructure "constant" $\alpha(E)$

(charge screening by vacuum polarization)

Of particular interest:

$$\alpha(M_Z) \textbf{ and } a_\mu \equiv (g-2)_\mu/2$$

- **electroweak effects (leptons etc.) calculable in perturbation theory**
- **strong interaction effects (hadrons/quarks etc.) perturbation theory fails**

$\Longrightarrow$ **Dispersion integrals over** e^+e^-**–data**

encoded in

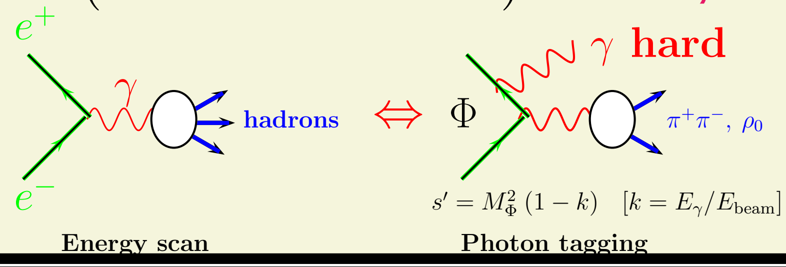
$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

Errors of data $\Longrightarrow$ **theoretical uncertainties !!!**

The art of getting precise results from non-precision measurements !

New challenge for precision experiments on $\sigma(e^+e^- \to hadrons)$ **KLOE, BABAR, ...**

$\sigma_{\mathrm{hadronic}}$ **via radiative return:**



② $\alpha(M_Z)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

α , G_μ, M_Z most precise input parameters

50% non-perturbative relationship

$\Updownarrow$

precision predictions $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ best effective input parameters for VB physics (Z,W) etc.

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$$

$$\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}$$

$$\frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}$$

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 1.6 \div 6.8 \times 10^{-4} \quad (\text{present : lost } 10^5 \text{ in precision!})$$

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5.3 \times 10^{-5} \quad (\texttt{TESLA requirement})$$

LEP/SLD: $\sin^2\Theta_{\rm eff} = (1 - g_{Vl}/g_{Al})/4 = 0.23148 \pm$ **0.00017**

$\delta\Delta\alpha(M_Z) = 0.00036 \quad \Rightarrow \quad \delta\sin^2\Theta_{\rm eff} =$ **0.00013**

affects Higgs mass bounds, precision tests and new physics searches!!!

For perturbative QCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ Lattice-QCD

Indirect

Higgs boson mass "measurement"

$m_H = 88^{+53}_{-35}$ **GeV**

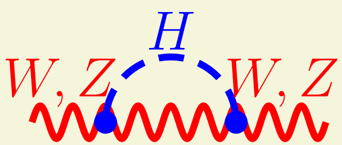
$e^+e^- \to \tau$: $\Rightarrow \delta m_H \sim -19$ **GeV**

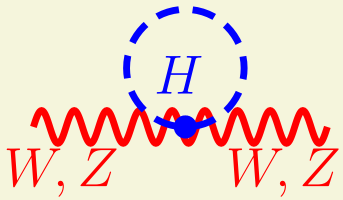
Direct lower bound:

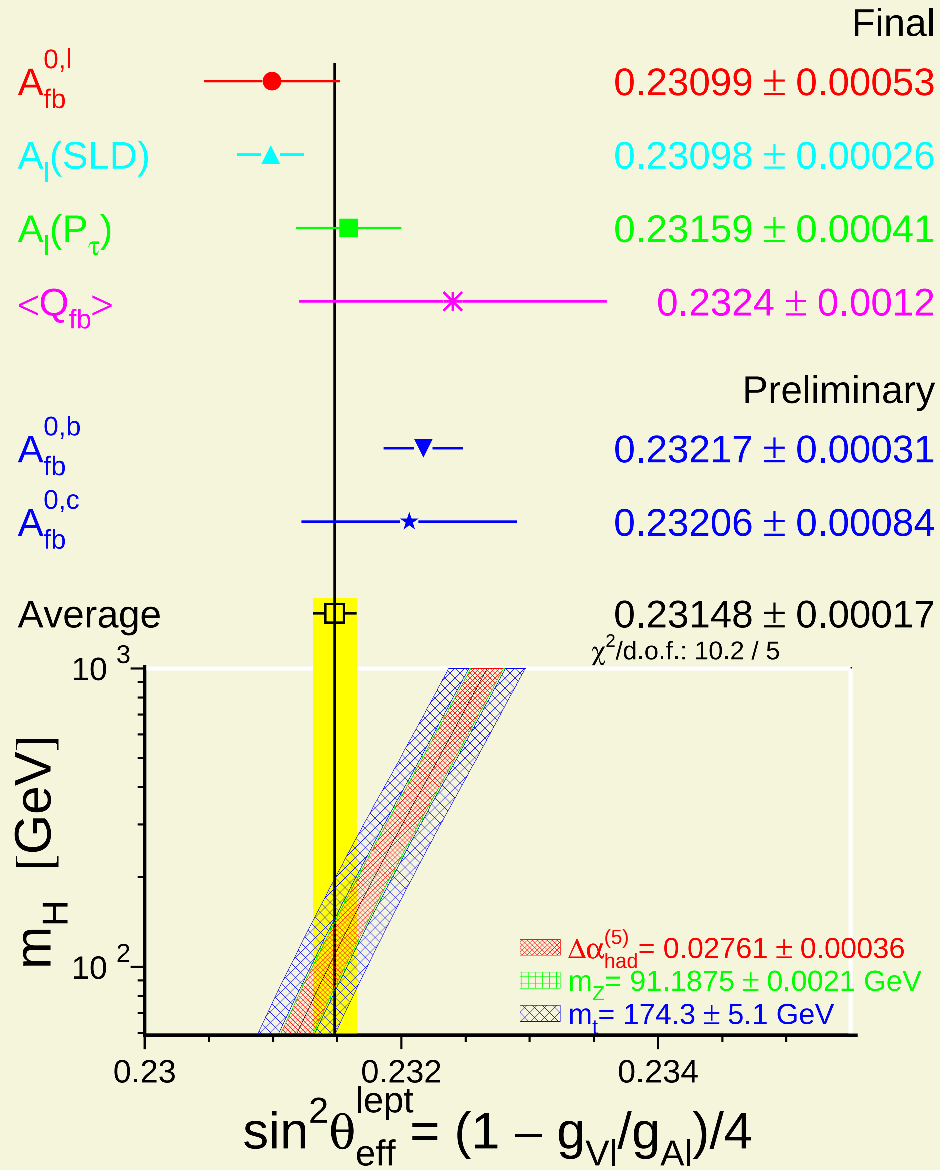
$m_H > 114$ **GeV at 95% CL**

Indirect upper bound:

$m_H < 193$ **GeV at 95% CL**



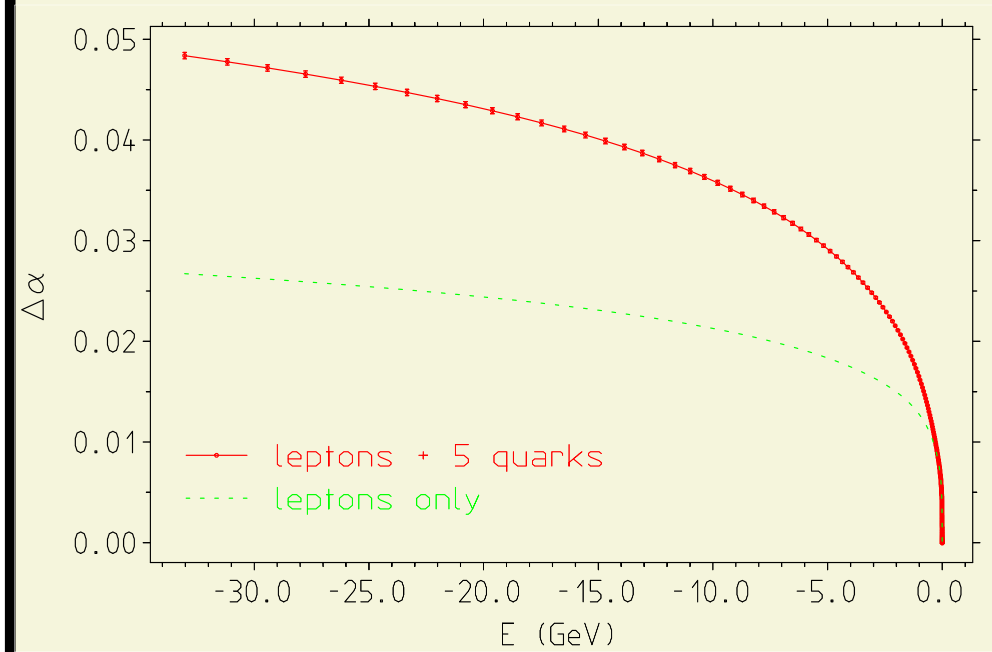


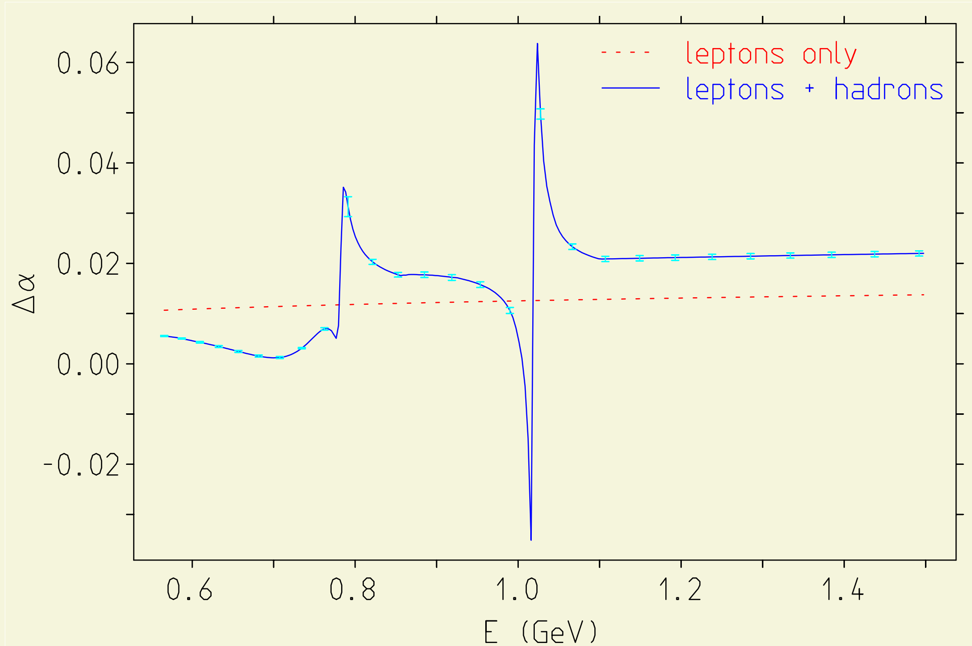


(LEP Electroweak Working Group: D. Abbaneo et al. 03)

- **Need to know running of $\alpha_{\rm QED}$ very precisely.**

  Large corrections, steeply increasing at low E



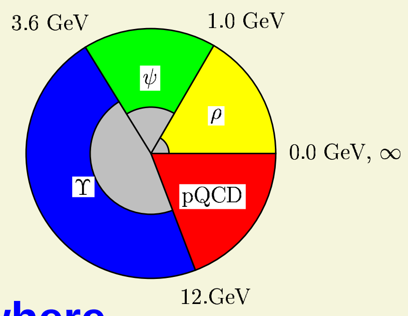


The running of α. The "negative" E axis is chosen to indicate space-like momentum transfer. The vertical bars at selected points indicate the uncertainty. In the time-like region the resonances lead to pronounced variations of the effective charge (shown in the $\rho-\omega$ and ϕ region).

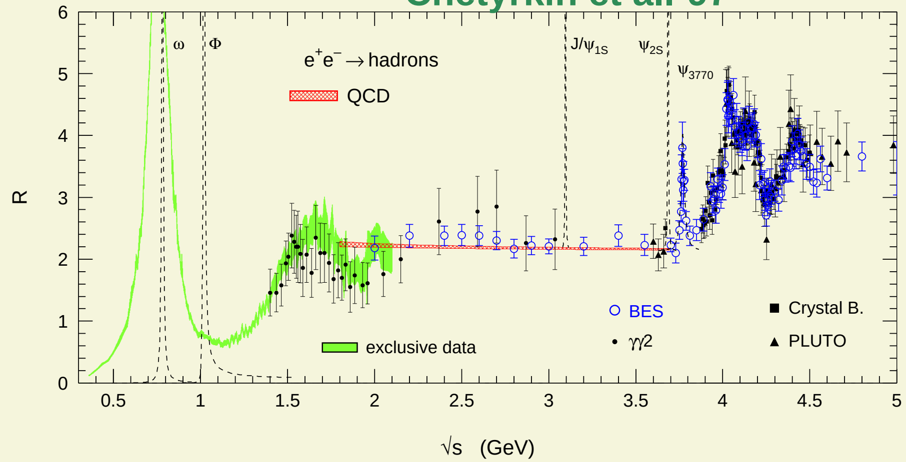
③ Evaluation of $\alpha(M_Z)$

Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s)$ can be evaluated in terms of $\sigma(e^+e^- \to {\rm hadrons})$ data via dispersion integral:

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\int_{4m_\pi^2}^{E^2_{\rm cut}} \!\!\!\!\!\! P \; ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \int_{E^2_{\rm cut}}^{\infty} \!\!\!\!\!\! P \; ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

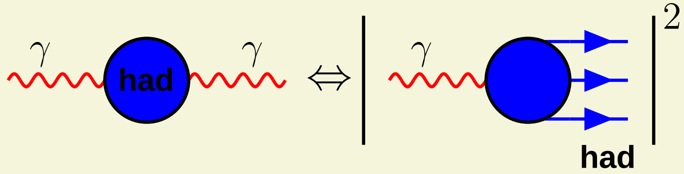


Compilation: Davier, Eidelman et al. 02
Theory = pQCD: Groshny et al. 91, Chetyrkin et al. 97

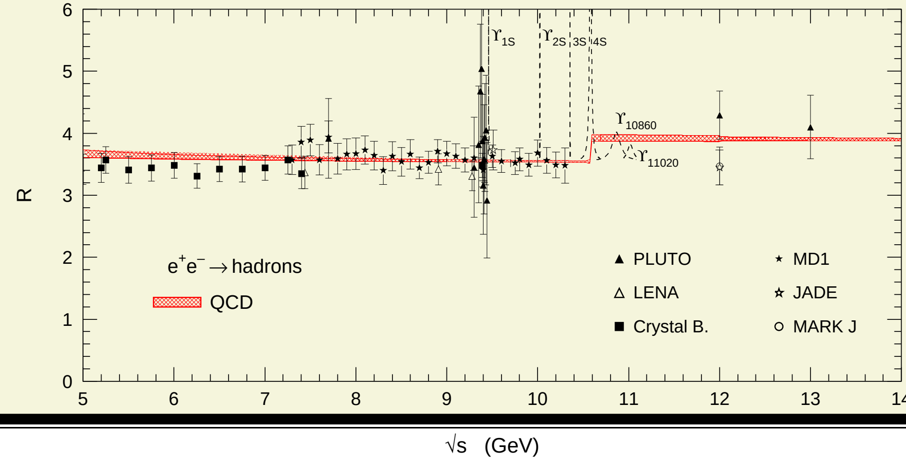


where

$$R_\gamma(s) \equiv \frac{\sigma^{(0)}(e^+e^- \to \gamma^* \to {\rm hadrons})}{\frac{4\pi\alpha^2}{3s}}$$



$\Pi'_\gamma(q^2)$ $\sim \sigma^{\rm had}_{\rm tot}(q^2)$

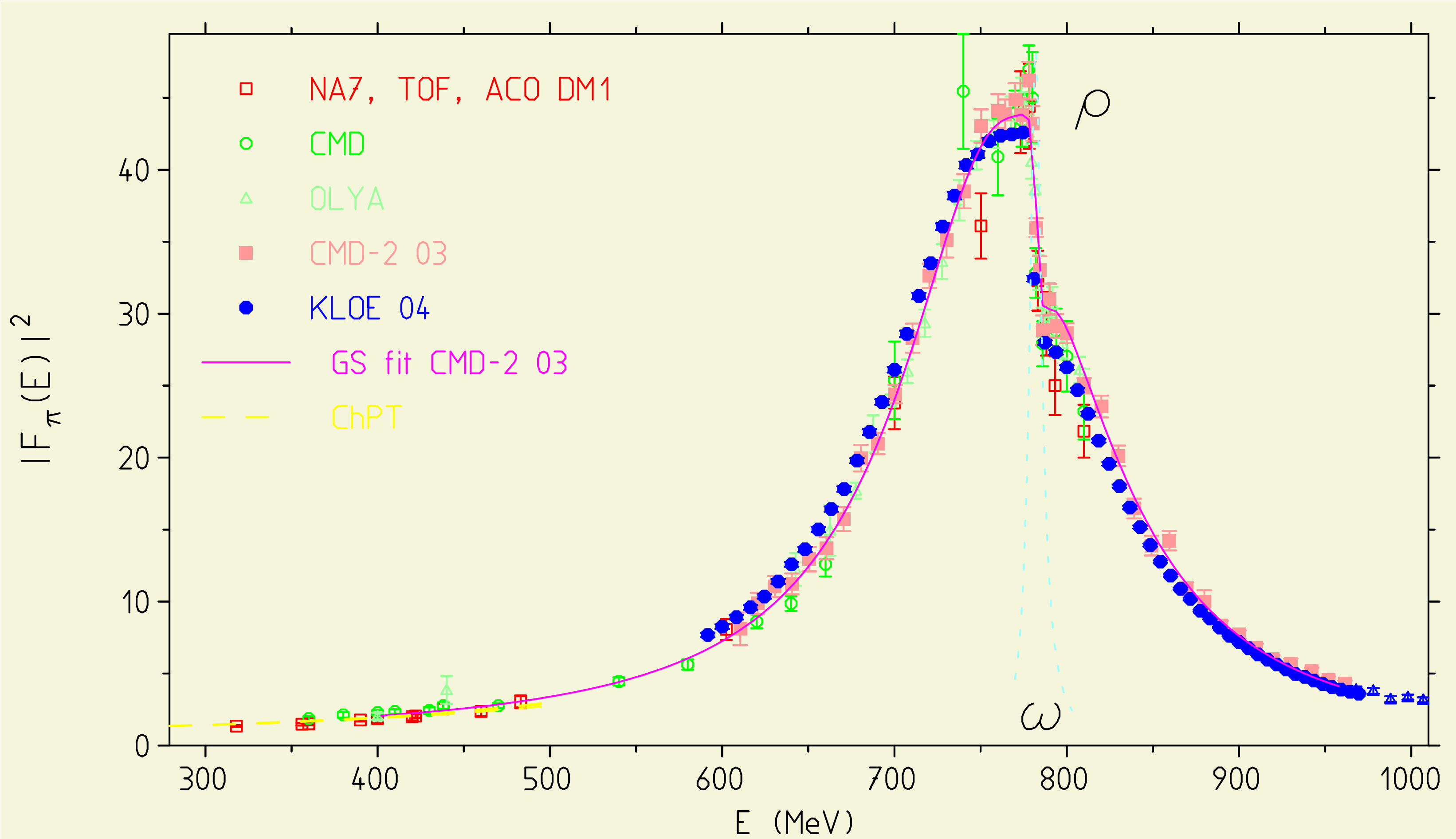


Evaluation FJ 2005 update: at $M_Z = 91.19$ GeV

- $R(s)$ data up to $\sqrt{s} = E_{cut} = 5$ GeV
  and for Υ resonances region between 9.6 and 13 GeV
- perturbative QCD from 5.0 to 9.6 GeV
  and for the high energy tail above 13 GeV

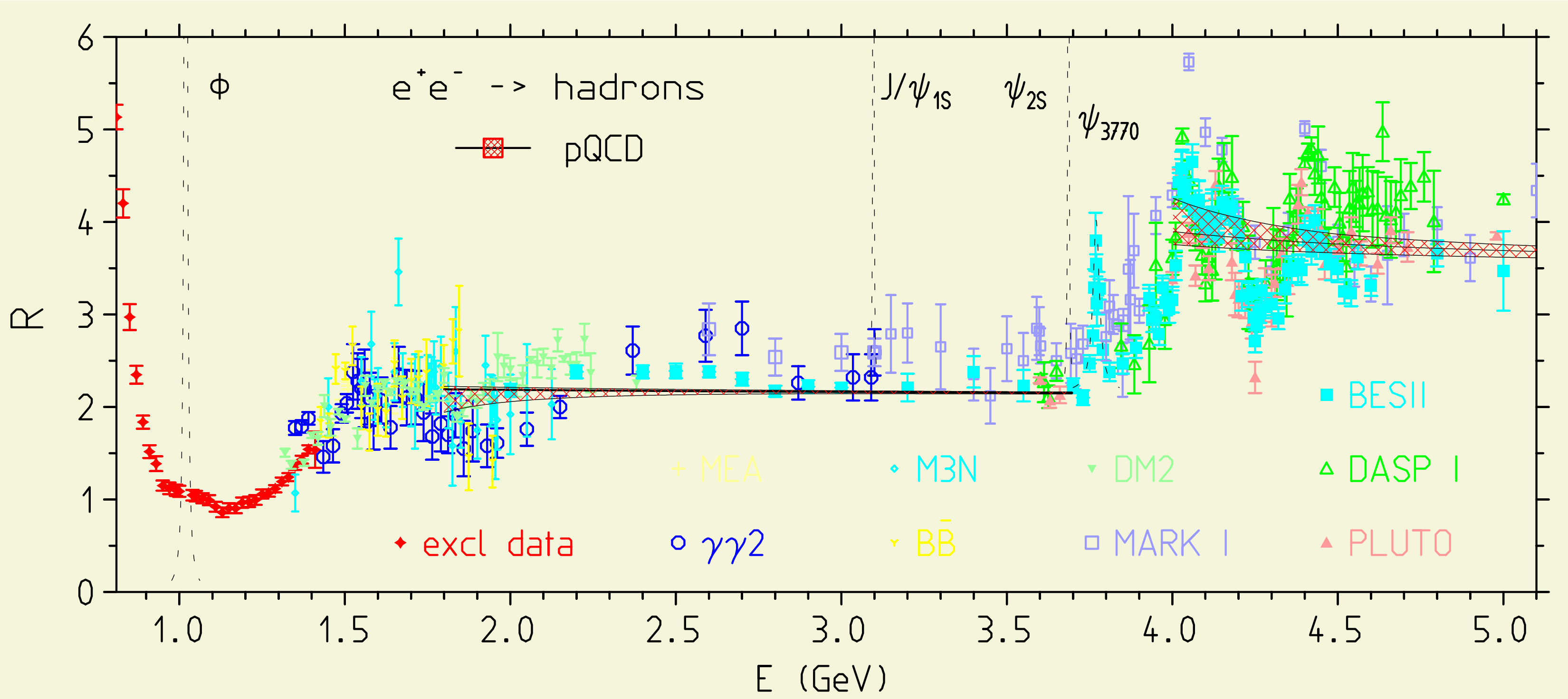
$$\Delta\alpha^{(5)}_{\mathrm{hadrons}}(M_Z^2) = 0.027773 \pm 0.000354$$
$$0.027664 \pm 0.000173 \quad \text{Adler}$$
$$\alpha^{-1}(M_Z^2) = 128.922 \pm 0.049$$
$$128.937 \pm 0.024 \quad \text{Adler}$$

④ A look at the e^+e^-–data

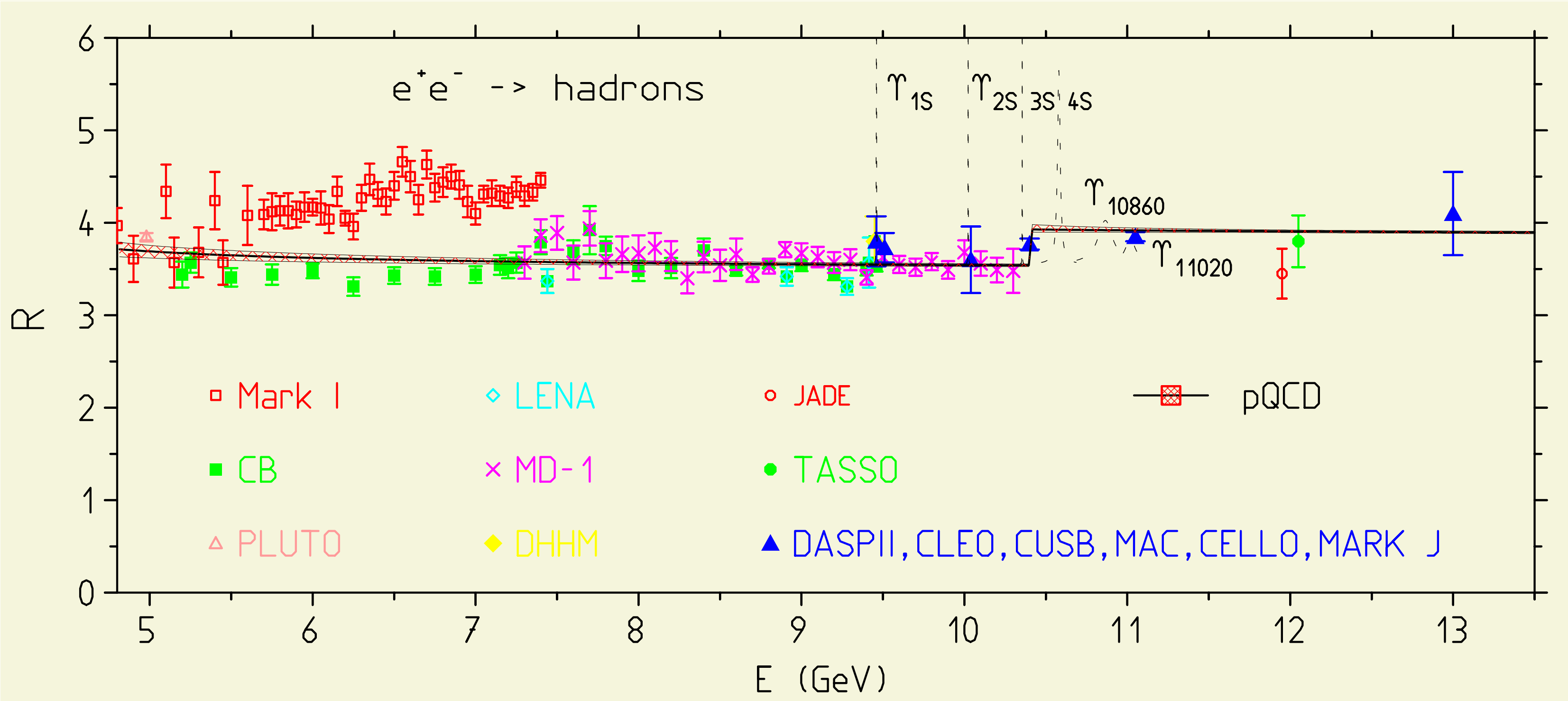


Below 1 GeV deviations between data sets much larger than errors claimed by experiments! CMD-2 vs. KLOE vs. SND somewhat confusing; will be settled by ongoing experiments

- e^+e^- **overall look 1 to 5 GeV**



- e^+e^- **overall look 5 to 13 GeV**



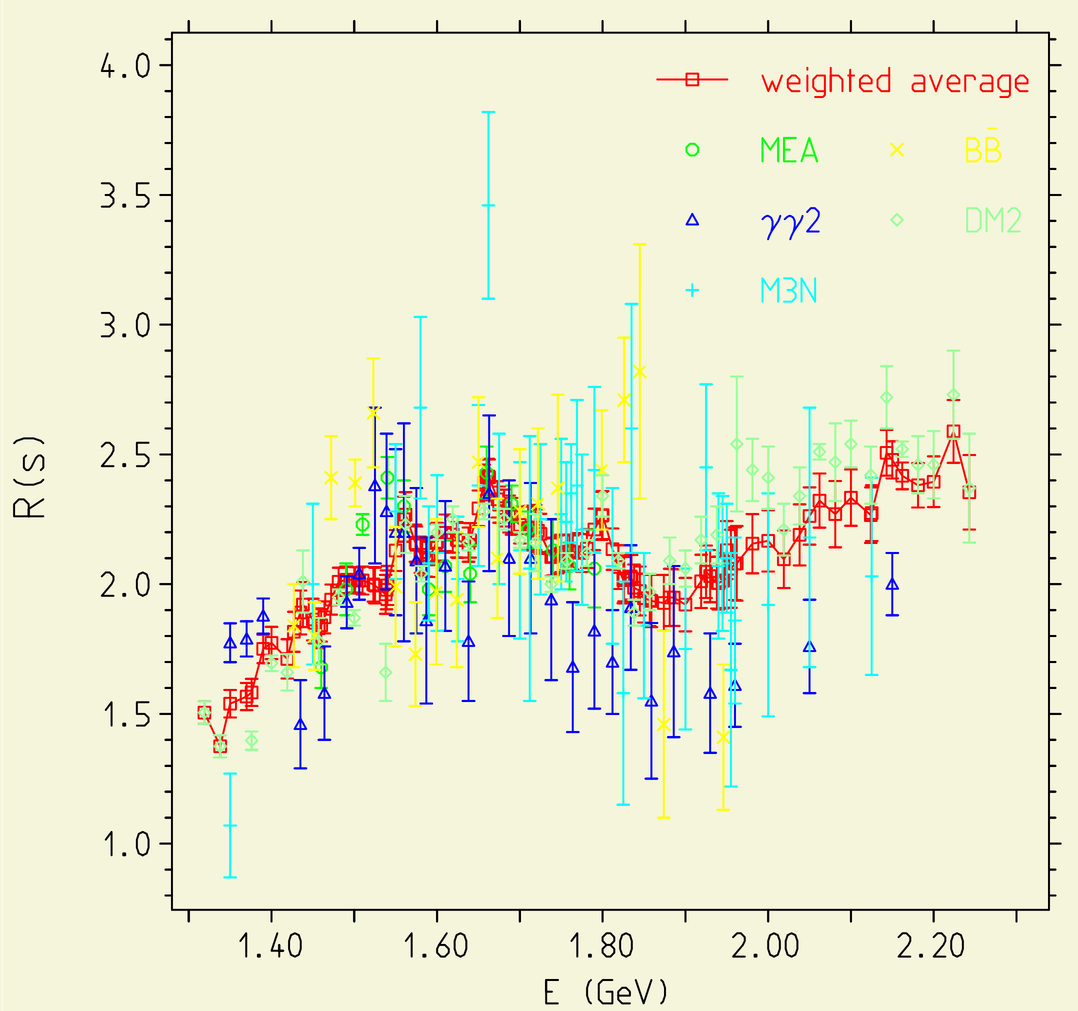
- **Most problematic e^+e^- region now 1.4-2.2 GeV. Quality of data poor typically 20% systematics**

Hagiwara et al.: take

inclusive $\gamma\gamma2$ data only

(say its more consistent with pQCD)

get error reduced



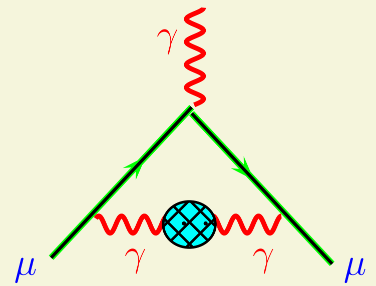
final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ (stat) (syst)	$\Delta\alpha^{(5)}_{\rm had}(-s_0)$	[tot]	rel	abs
ρ	(0.28, 0.99)	33.14 (0.26) (0.29)	30.48 (0.25) (0.27)	[0.36]	1.2%	19.3%
ω	(0.42, 0.81)	3.02 (0.04) (0.08)	2.75 (0.03) (0.07)	[0.08]	2.8%	1.0%
ϕ	(1.00, 1.04)	4.74 (0.07) (0.11)	4.07 (0.06) (0.09)	[0.11]	2.7%	1.8%
J/ψ		11.73 (0.56) (0.61)	4.16 (0.20) (0.19)	[0.28]	6.6%	11.7%
Υ		1.27 (0.05) (0.07)	0.07 (0.00) (0.00)			
had	(0.99, 2.00)	17.21 (0.09) (0.55)	12.56 (0.06) (0.42)	[0.42]	3.4%	26.2%
had	(2.00, 3.10)	15.69 (0.06) (0.46)	7.88 (0.04) (0.25)	[0.25]	3.2%	9.3%
had	(3.10, 3.60)	5.31 (0.11) (0.10)	1.90 (0.04) (0.04)	[0.06]	3.0%	0.5%
had	(3.60, 9.46)	51.48 (0.25) (3.00)	8.40 (0.04) (0.44)	[0.44]	5.3%	28.8%
had	(9.46,13.00)	18.59 (0.25) (1.36)	0.90 (0.01) (0.07)	[0.07]	7.8%	0.7%
pQCD	(13.0,∞)	115.57 (0.00) (0.12)	1.09 (0.00) (0.00)			
data	(0.28,13.00)	162.19 (0.74) (3.44)	73.18 (0.34) (0.75)			
total		277.76 (0.74) (3.44)	74.26 (0.34) (0.75)	[0.82]	1.1%	100%

Table 1: Results for $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)^{\rm data}\cdot 10^4$ and $\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}\cdot 10^4$ ($\sqrt{s_0}=2.5$ GeV).

② Evaluation of $a_\mu \equiv (g-2)_\mu/2$

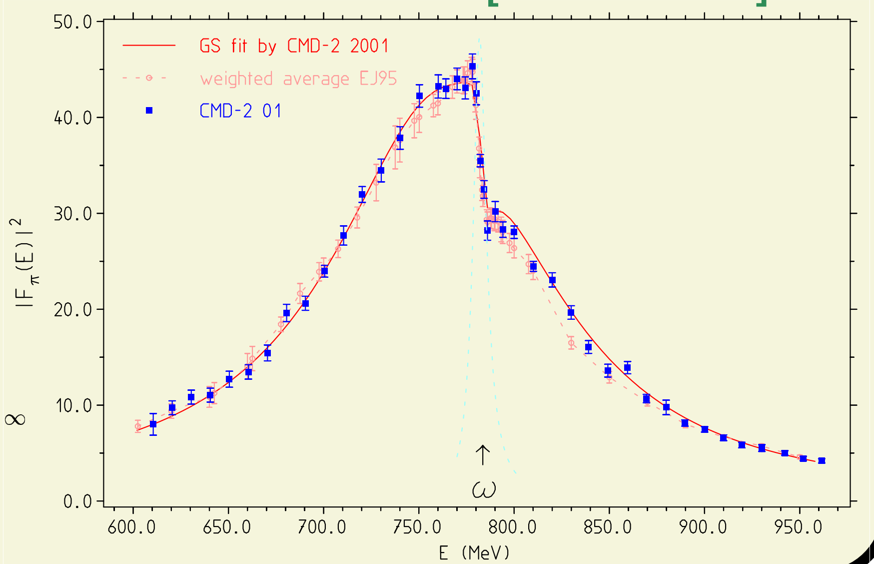
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to {\rm hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds \, \frac{R_\gamma^{\rm data}(s)\,\hat{K}(s)}{s^2} + \int\limits_{E^2_{\rm cut}}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\,\hat{K}(s)}{s^2} \right)$$



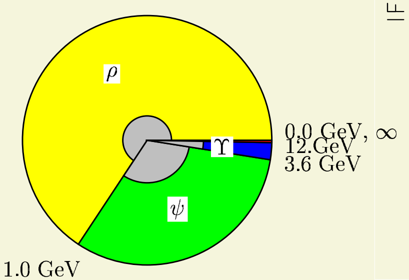
- **Experimental error implies theoretical uncertainty!**
- **Low energy contributions enhanced: $\sim 67\%$ of error on $a_\mu^{\rm had}$ comes from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$**

Data: Akhmetshin et al. 02 [Collab. CMD-2]



$$a_\mu^{\rm had(1)} = (695.5 \pm 8.6)\,10^{-10}$$

e^+e^-**–data based**

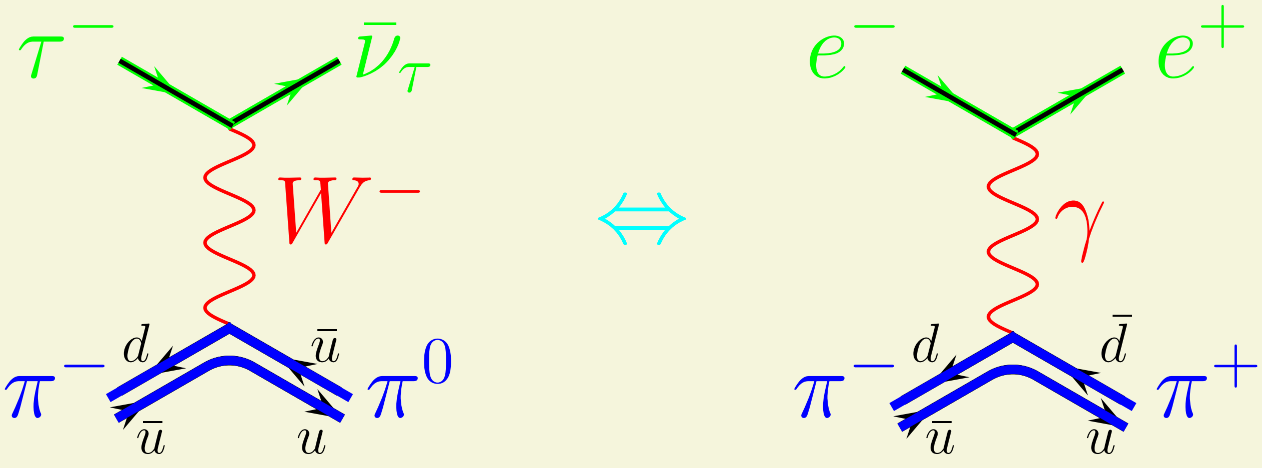


final state	range (GeV)	δa_μ (stat) (syst)	[tot]	rel	abs
ρ	(0.28, 0.99)	500.71 (5.07) (5.69)	[7.62]	1.5%	84.7%
ω	(0.42, 0.81)	37.99 (0.46) (1.03)	[1.13]	3.0%	1.9%
ϕ	(1.00, 1.04)	36.07 (0.50) (0.83)	[0.97]	2.7%	1.4%
J/ψ		8.97 (0.42) (0.40)	[0.58]	6.5%	0.5%
Υ		0.11 (0.00) (0.01)	[0.01]	9.1%	0.0%
had	(0.99, 2.00)	67.75 (0.45) (2.54)	[2.58]	3.8%	9.7%
had	(2.00, 3.10)	22.06 (0.12) (0.89)	[0.90]	4%	1.2%
had	(3.10, 3.60)	4.06 (0.08) (0.08)	[0.11]	2.8%	0.0%
had	(3.60, 9.46)	14.43 (0.07) (0.75)	[0.75]	5.2%	0.8%
had	(9.46,13.00)	1.30 (0.02) (0.10)	[0.10]	7.8%	0.0%
pQCD	(13.0,∞)	1.53 (0.00) (0.00)			
data	(0.28,13.00)	693.44 (5.15) (6.49)			
total		694.97 (5.15) (6.49)	[8.28]	1.19%	100%

Table 2: Results for $\delta a_\mu^{\rm data} \cdot 10^{10}$.

⑤ Remark on τ–decay spectral functions

The isovector part of $\sigma(e^+e^- \to \mathrm{hadrons})$ may be calculated by a isospin rotation from τ–decay spectra (to the extend that CVC is valid)



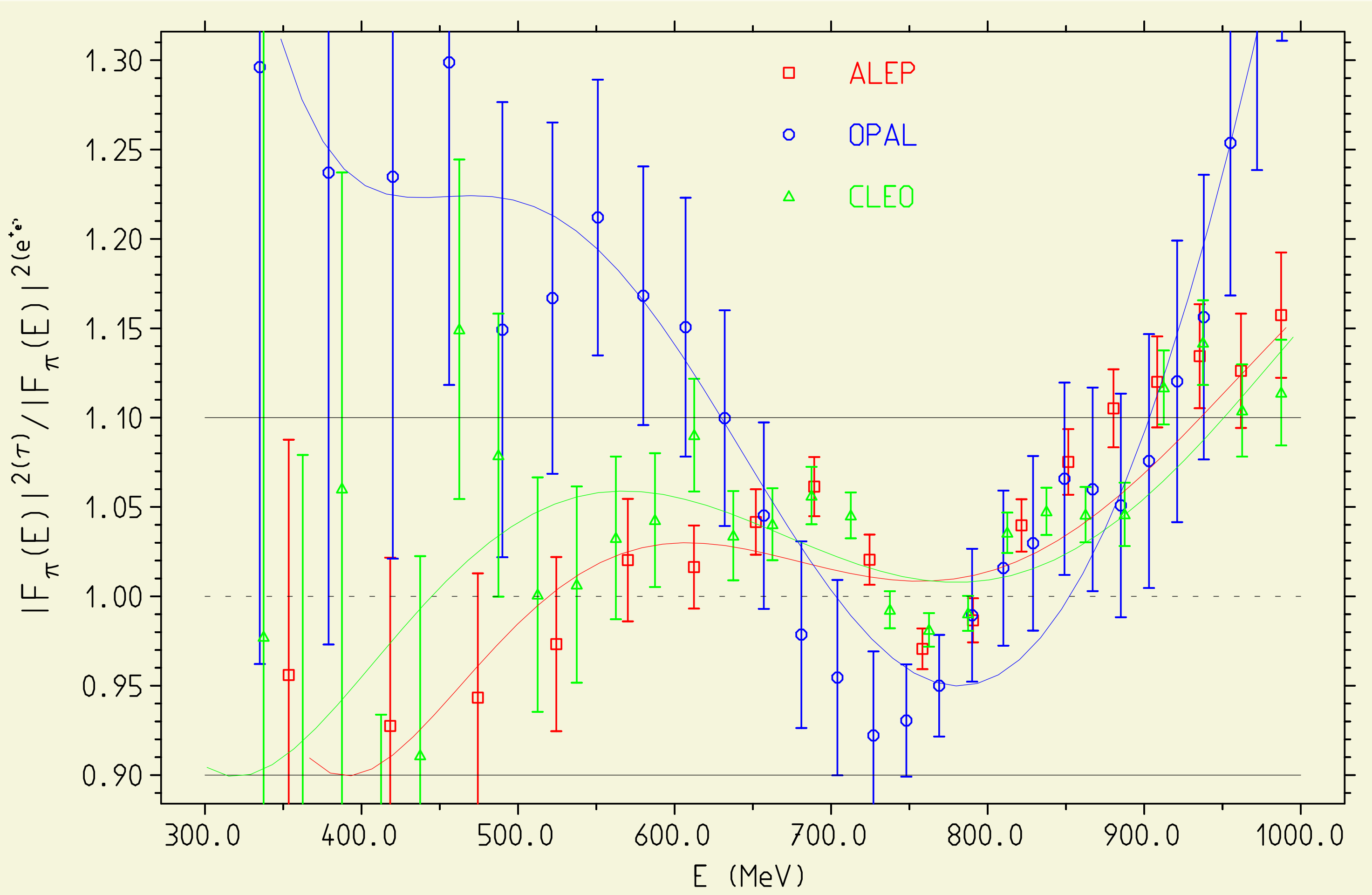
$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+e \to X^0$

X^- and X^0 are hadronic states related by iso-spin rotation.

The e^+e^- cross–section is then given by

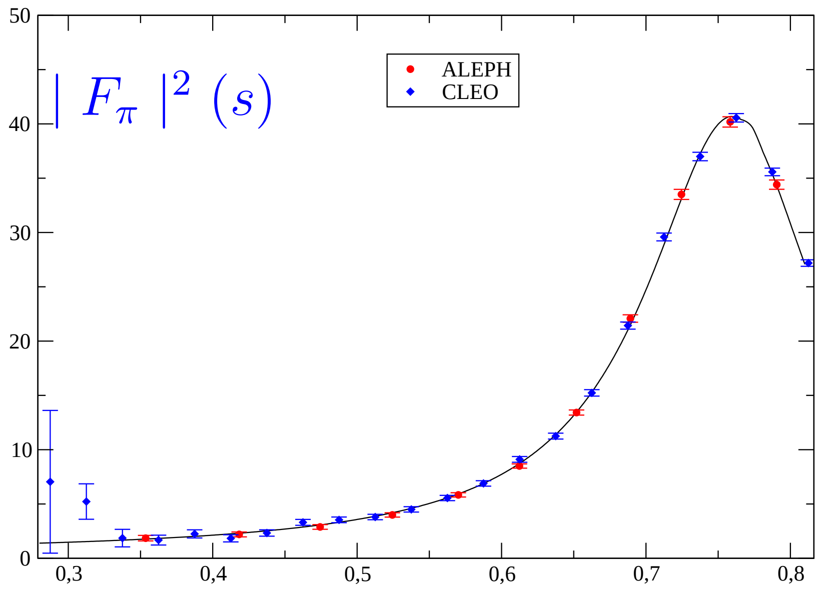
$$\sigma^{I=1}_{e^+e^-\to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \leq M_\tau$$

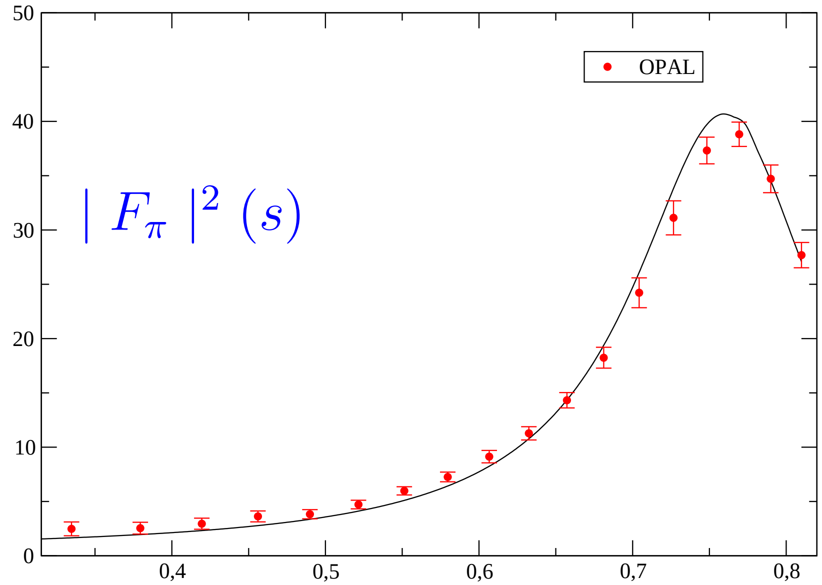
in terms of the τ spectral function v_1.



ALEP
OPAL
CLEO
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
300.0
400.0
500.0
600.0
700.0
800.0
900.0
1000.0
E (MeV)
$|F_\pi(E)|^{2(\tau)}/|F_\pi(E)|^{2(e^+e^-)}$

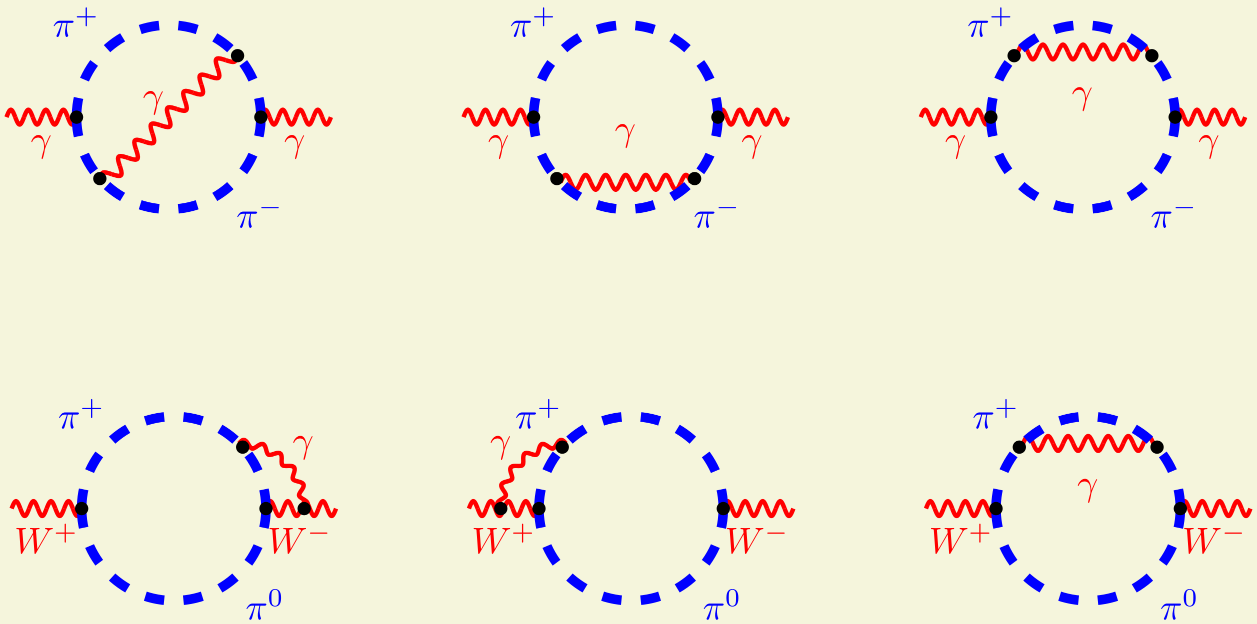
NA7+CMD-2 e^+e^-–data analytized+unitarized+chiral limit: solid line vs. τ –data





✣ Iso-spin breaking in τ vs. e^+e^-:

FSR correction in τ–decay: inclusive approach?



QED corrections are obviously not related by a iso-spin rotation and must be subtracted before CVC arguments can be applied

⑥ Iso-spin breaking corrections in τ vs. e^+e^-

Cirigliano ea al., hep-ph/0104267, hep-ph/0212386

$$a_\mu^{\rm vacpol} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{-infty} ds\, K(s)\, \sigma^{(0)}_{e^+e^-\to{\rm hadrons}}(s)$$

$$\sigma^{(0)}_{\pi\pi} = \left[\frac{K_\sigma(s)}{K_\Gamma(s)}\right] \frac{d\Gamma_{\pi\pi[\gamma]}}{ds} \times \frac{R_{\rm IB}(s)}{S_{\rm EW}}$$

$$K_\sigma(s) = \frac{G_F^2\, |V_{ud}|^2\, m_\tau^3}{384\pi^3} \left(1-\frac{s}{m_\tau^2}\right)^2 \left(1+2\,\frac{s}{m_\tau^2}\right)$$

$$K_\sigma(s) = \frac{\pi\alpha^2}{3s}$$

Iso-spin breaking correction in

$$R_{\rm IB}(s) = \frac{1}{G_{\rm EM}(s)} \frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^+\pi^0}} \left|\frac{F_V(s)}{f_+(s)}\right|^2$$

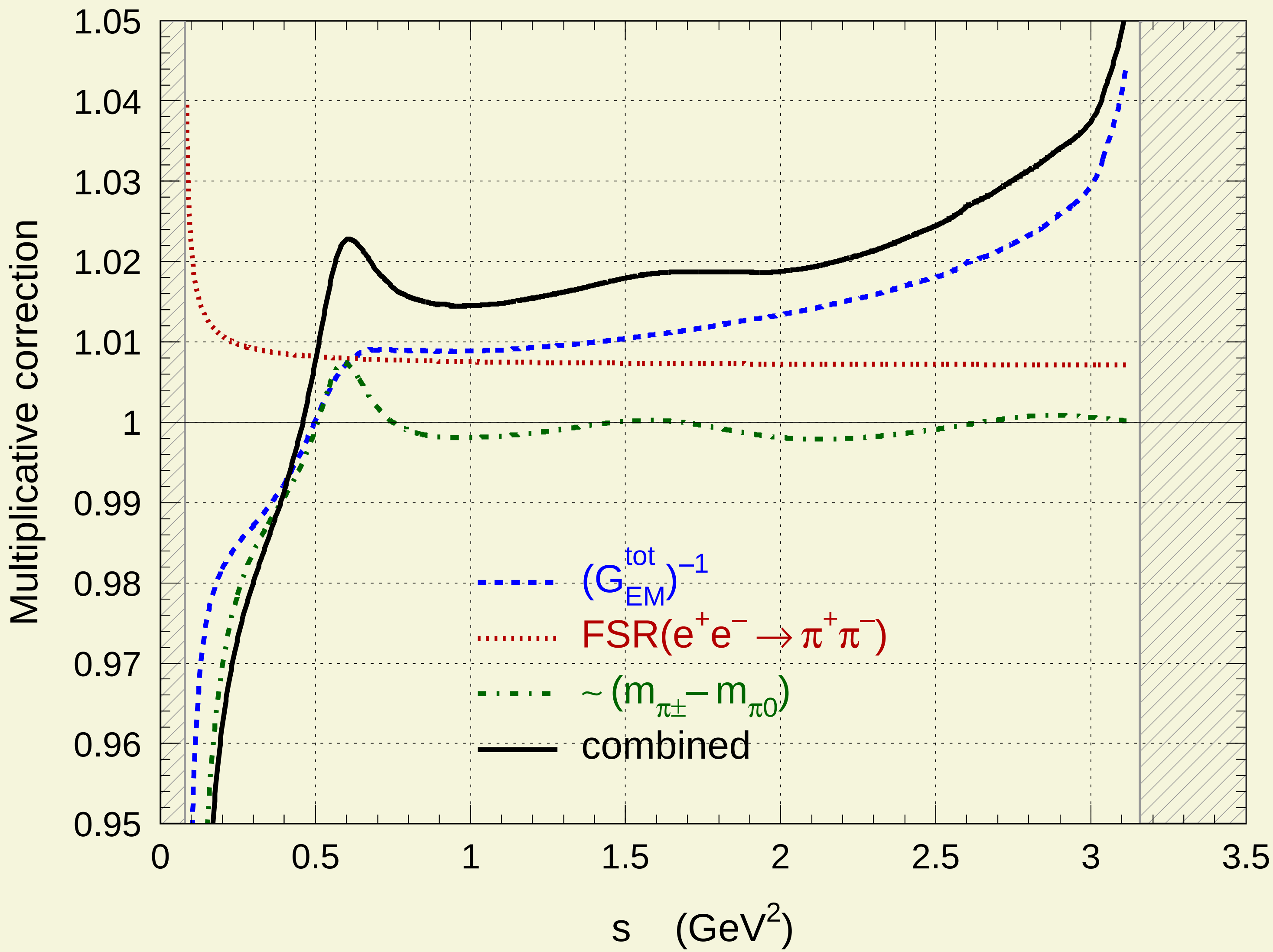
Summary of results:

Contributions to $\Delta a_\mu^{\rm vacpol}$ from various sources of iso-spin violation (in units of 10^{-11}) for different values of $t_{\rm max}$ (in units of GeV^2) .

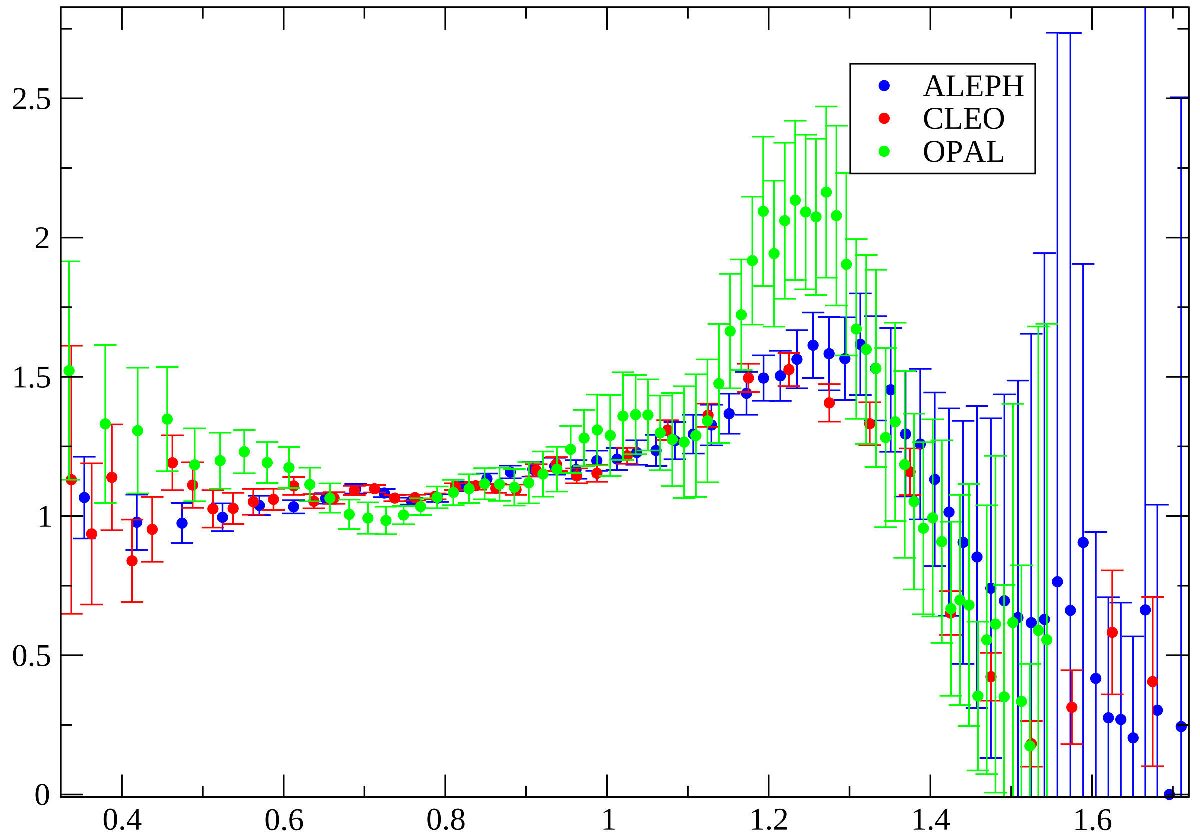
$t_{\rm max}$	$S_{\rm EW}$	KIN	EM	FF	$\delta a_\mu^{\rm IB}$
1	- 95	- 75	- 11	61 $\pm$ 26 $\pm$ 3	- 119
2	- 97	- 75	- 10	61 $\pm$ 26 $\pm$ 3	- 120
3	- 97	- 75	- 10	61 $\pm$ 26 $\pm$ 3	- 120

Isospin breaking effects: (Cirigliano et al.)

Surprisingly, after corrections at larger energies 10% deviations !



- **Comparison of τ–data:**

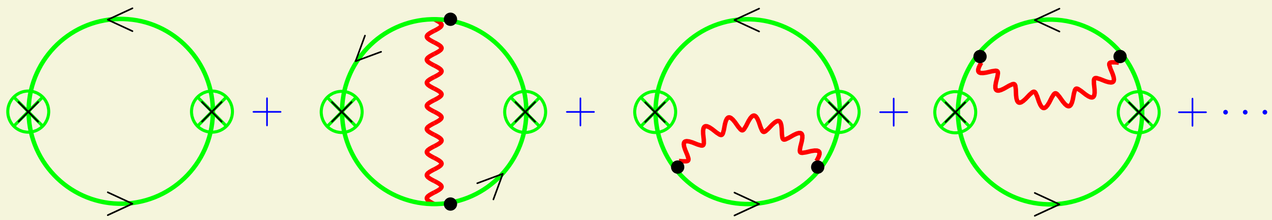


τ–data may be not so easy; DELPHI, L3 could not measure τ spectral–functions; ALEPH vs. OPAL no good agreement.

⑤ $\Delta\alpha^{\rm had}$ via the Adler function

Controlling pQCD via the Adler function

❶ **pQCD calculations of vacuum polarization amplitudes**



up to 4–loops massless — Groshny, Kataev, Larin 91

up to 3–loops massive — Chetyrkin, Kühn et al. 97

up to 2–loops massive BF–MOM RG — F. J., Tarasov 98

❷ **use old idea: testing non–perturbative effects with help of the Adler function**

Eidelman, F. J., Kataev, Veretin 98

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

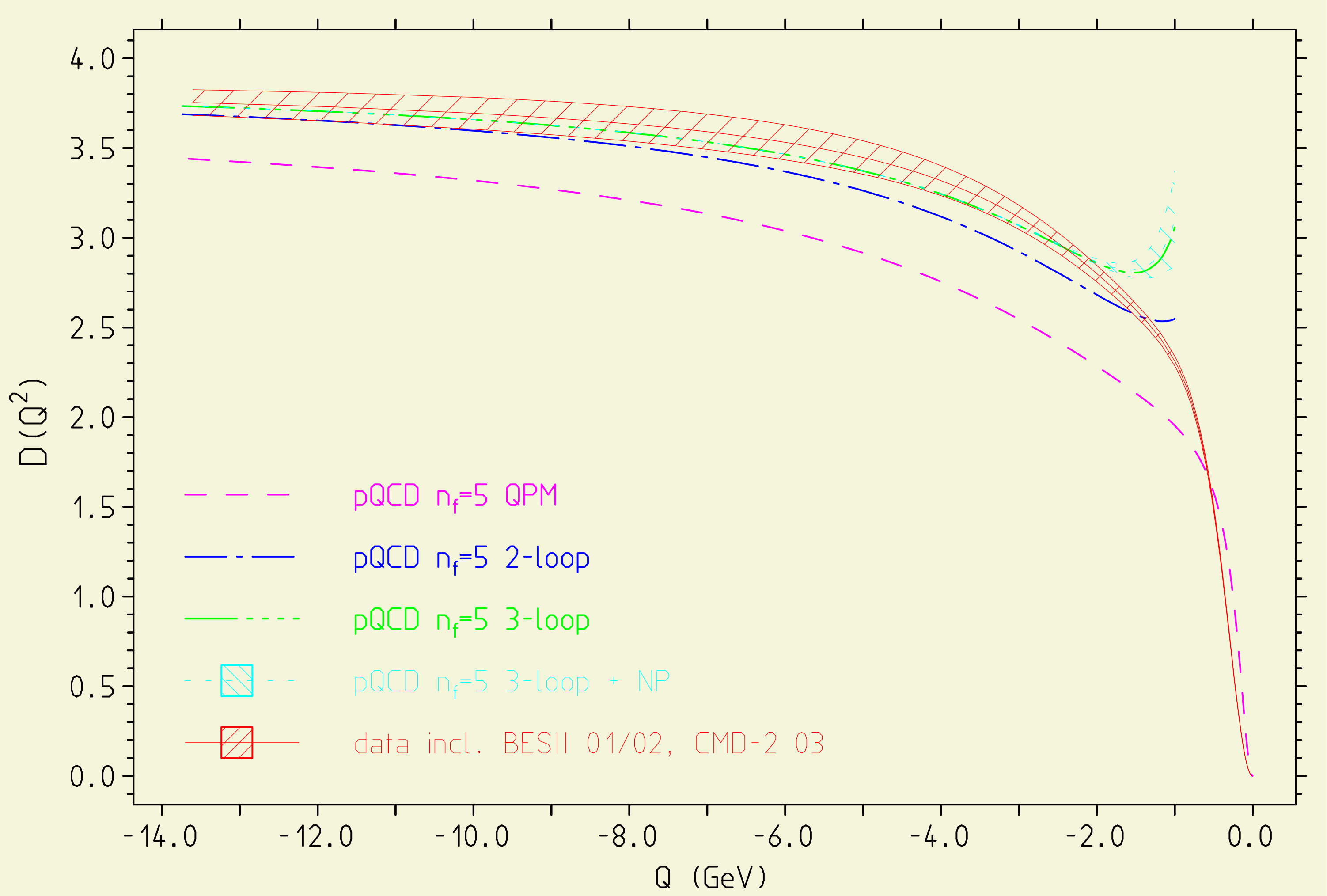
$$\Rightarrow \quad D(Q^2) \;=\; Q^2 \int\limits_{4m_\pi^2}^{\infty} ds \, \frac{R(s)}{(s+Q^2)^2}$$

pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

Conservative conclusion:

- **time-like approach: pQCD works well in "perturbative windows" 3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ (Kühn,Steinhauser)**
- **space-like approach: pQCD works well for $Q^2 = -q^2 > 2.5$ GeV (see plot) (EJKV 98/04)**

“Experimental” Adler–function versus theory (pQCD + NP)



(Eidelman, F.J., Kataev, Veretin 98, FJ 03 update (BES, CMD-2))

$\Rightarrow$ **pQCD works well to predict $D(Q^2)$ down to $s_0 = (2.5\,\textbf{GeV})^2$**

(not down to m_τ ! however);

use this to calculate

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int dQ'^2\,\frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.5\,\textbf{GeV})^2$: **(FJ 98/03)**

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007417 \pm 0.000086$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027613 \pm 0.000086 \pm 0.000149[0.000149]$$

The second error comes from the variation of the pQCD parameters. In square brackets the error if we assume the uncertainties from different parameters to be uncorrelated. The uncertainties coming from individual parameters are listed in the following table (masses are the pole masses):

parameter	range	pQCD uncertainty	total error
α_s	0.117 ... 0.123	0.000051	0.000155
m_c	1.550 ... 1.750	0.000087	0.000170
m_b	4.600 ... 4.800	0.000011	0.000146
m_t	170.0 ... 180.0	0.000000	0.000146
all correlated		0.000149	0.000209
all uncorrelated		0.000101	0.000178

The largest uncertainty is due to the poor knowledge of the charm mass. I have taken errors to be 100% correlated. The uncorrelated error is also given in the table.

$$\Rightarrow \quad \delta\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.00015$$

New values: pQCD/SR (moment method) and Lattice QCD

Ref	$\alpha_s(M_Z)$	$\Lambda_{\overline{\rm MS}}^{N_f=0}$ [MeV]	$m_s(m_s)$ [MeV]	$m_c(m_c)$ [GeV]	$m_b(m_b)$ [GeV]
PDG	0.118(3)	-	-	-	-
Steinhauser	$0.124^{+0.011}_{-0.014}$	-	-	1.304(27)	4.191(51)
Rolf	-	238(19)[Q]	97(4)[Q]	1.301(34)[Q]	4.12(7)(4)[Q]

Lattice: waiting for full QCD results [unquenched], continuum limit performed

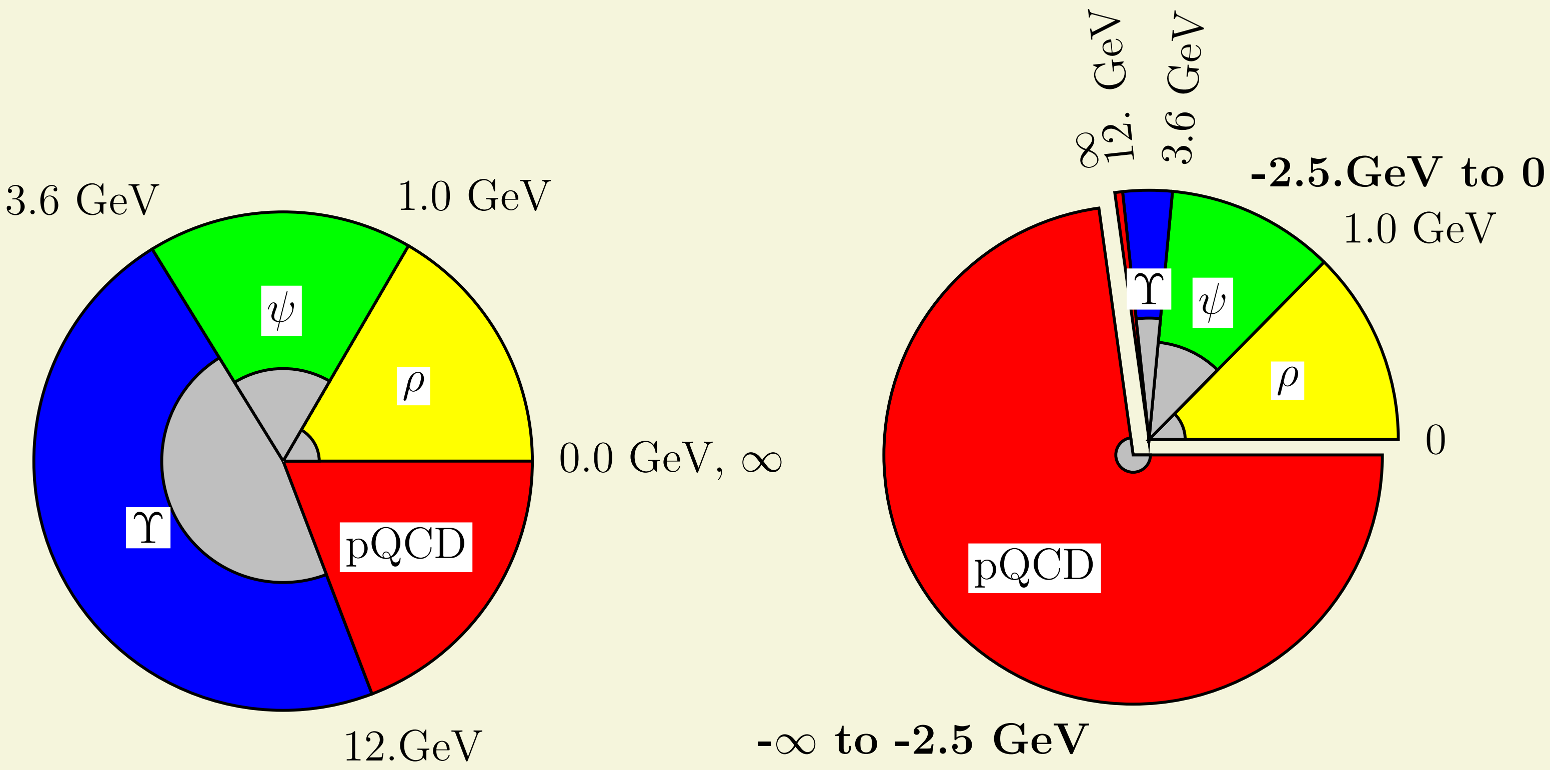
The virtues of this analysis are obvious:

- no problems with physical threshold and resonances
- pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.5 GeV).
- no manipulation of data, no assumptions about global or local duality.
- non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!

Distribution of hadronic contributions to $\Delta\alpha^{\mathrm{had}}$

e^+e^- data based approach:

Based on available data up to 12 GeV (FJ 03)



Comparison of the distribution of contributions and errors (shaded areas scaled up by 10) in the standard (left) and the Adler function based approach (right), respectively.

Contributions to

$$\tilde{a}_\mu^{\rm had} = a_\mu^{\rm had} \times 10^{10}$$

from exclusive channels

channel	$\tilde{a}_\mu^{\rm had}$	acc.
$\rho,\omega \to \pi^+\pi^-$	506	0.3%
$\omega \to 3\pi$	47	$\sim$ 1%
ϕ	40	$\downarrow$
$\pi^+\pi^-\pi^0\pi^0$	24	·
$\pi^+\pi^-\pi^+\pi^-$	14	·
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	5	10%
3π	4	$\downarrow$
K^+K^-	4	·
K_SK_L	1	·
$\pi^+\pi^-\pi^+\pi^-\pi^0$	1.8	·
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	0.5	·
$p\bar{p}$	0.2	·
$2\,{\rm GeV} \leq E \leq M_{J/\psi}$	22	
$M_{J/\psi} \leq E \leq M_\Upsilon$	20	
$M_\Upsilon < E$	$\lesssim 5$	

Daphne

Theoretical work with the aim

to calculate radiative corrections

at the level of precision as indicated

in the Table is in progress.

$\Downarrow$

$$\delta a_\mu^{\rm had} \lesssim 26 \times 10^{-11}$$

from

$\sqrt{s} \lesssim 2$ **GeV.**

⑦ Status and Outlook

- Future high precision experiments on $a_\mu = (g-2)/2$ (BNL/KEK project may gain factor 10?) and $\sin^2\Theta_{\rm eff}$, etc. (LEP/SLD-¿TSLA/ILC) imposed a lot of pressure to theory to improve (or find errors in) their calculations and, in particular, to reduce hadronic uncertainties which mainly reflect the experimental errors of $R(s)^{\rm exp}_{\rm had}$
- Experimental groups have reconsider older data to reduce errors (CMD-2); new data from BES (20% $\longrightarrow$ 7%) (2 GeV to 5 GeV), and τ data from ALEPH, OPAL, CLEO. The latter disagree with e^+e^- data in some regions at the 10% level and essentially lead to two "incompatible" prediction for $a_\mu^{\rm had}$. Also KLOE, CMD-2 and SND definitely not in satisfacory agreement.
- All kind of attempts to squeeze out of the old data more precise results; theory only partially can help. What is the appropriate "pseudo observable"?, What is missing (e.g., hard photon effects)?, What is double counted? Etc.
- Key role now for radiative return experiments on low energy hadronic cross sections: KLOE, BABAR,...; radiative corrections very crucial to get a precise answer. Theory: special effort by Karlsruhe group (Kühn et al.) to advance calculations.

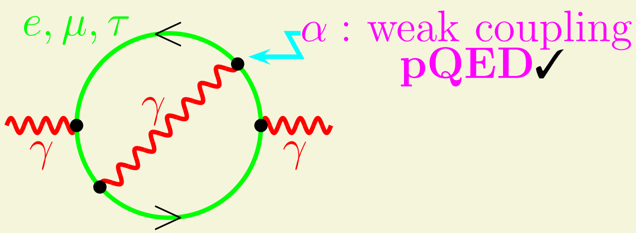
- $(g-2)_\mu$: need settle ρ region and in addition range 1.4 GeV to 2 GeV.
- Needs for linear collider (like TESLA/ILC): requires $\sigma_{\rm had}$ at 1% level up to the $\Upsilon \Rightarrow \delta\alpha(M_Z)/\alpha(M_Z) \sim 5 \times 10^{-5}$. At present would allow to get better Higgs boson mass limits.
- Future precision physics requires dedicated effort on $\sigma_{\rm had}$ experimentally as well as theoretically (radiative corrections, final state radiation from hadrons etc.)

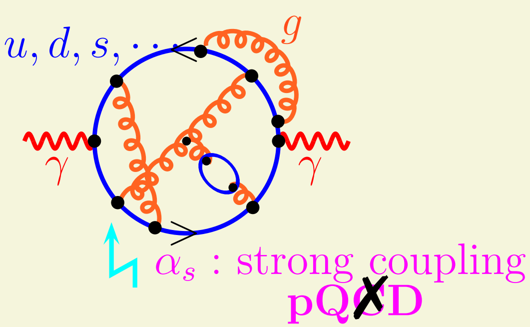
† Hadronic Contributions

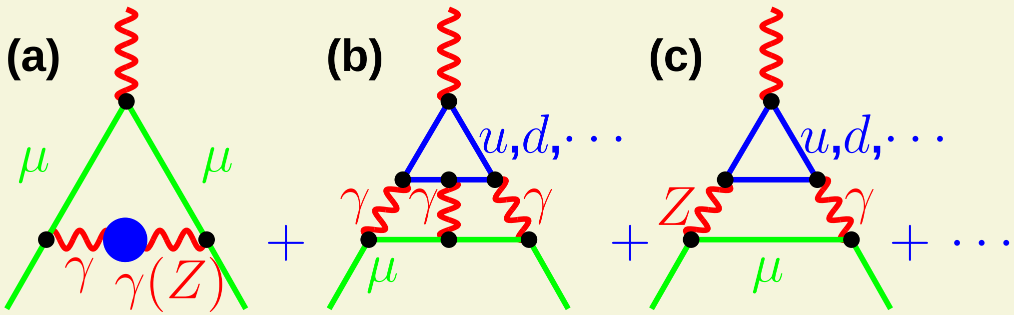
General problem in electroweak precision physics:

contributions from hadrons (quark loops) at low energy scales

Leptons **Quarks**







(a) **Hadronic vacuum polarization** $O(\alpha^2), O(\alpha^3)$

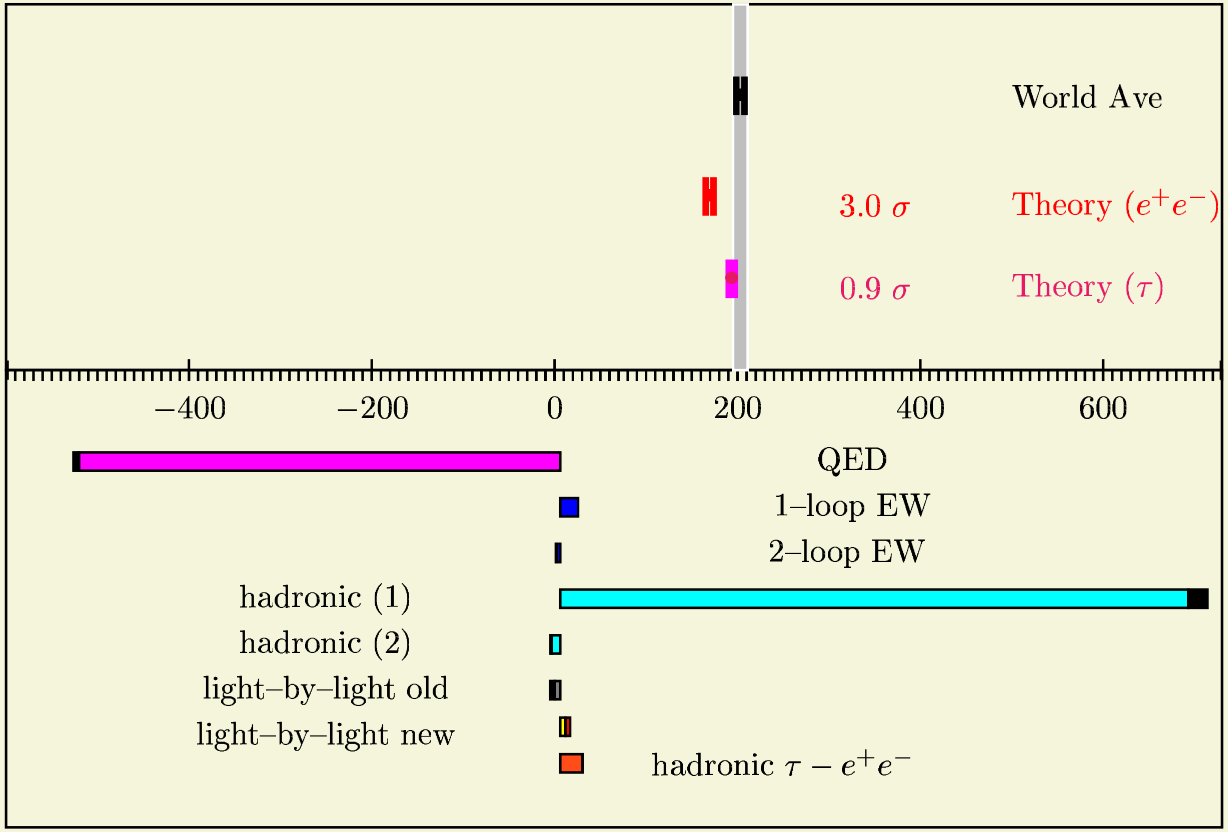
(b) **Hadronic light-by-light scattering** $O(\alpha^3)$

(c) **Hadronic effects in 2-loop EWRC** $O(\alpha G_F m_\mu^2)$

Light quark loops $\longrightarrow$ **Hadronic "blob"**

a_μ: type and size of contributions

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm had(1)} + a_\mu^{\rm had(2)} + a_\mu^{\rm weak(1)} + a_\mu^{\rm weak(2)} + a_\mu^{\rm lbl} \; (+a_\mu^{\rm new\ physics})$$



All kind of physics meets !

❑ $a_\mu^{\rm had}$ based on theory of the Pion form factor

Electromagnetic Form Factor constraint by analyticity, unitarity and chiral limit: [see also (Trocóniz and Yndurain 01 and others)]

$$F(s) = \exp \Delta(s) \times G_\omega(s) \times G(s)$$

- Omnès factor (cut due to 2π intermediate states)

  In elastic region curvature in $F(s)$ generated by these states is determined by P-wave phase shift $\delta(s)$ of $\pi\pi$ scattering

$$\Delta(s) = \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dx \delta(x)}{x(x-s)}$$

  using more accurate phase shifts relying on the Roy equation analysis Colangelo, Gasser, Leutwyler 01

  The $\pi\pi$ scattering phase shift is due to elastic rescattering of the pions in the fi nal state as illustrated by

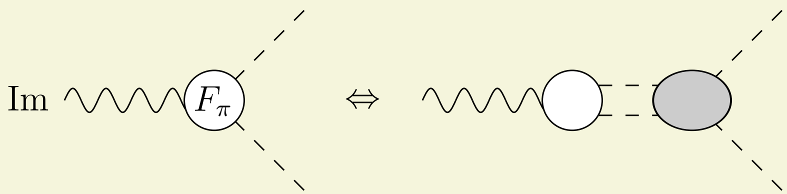


Figure 1: Final state interaction due to $\pi\pi \longrightarrow \pi\pi$ scatteing

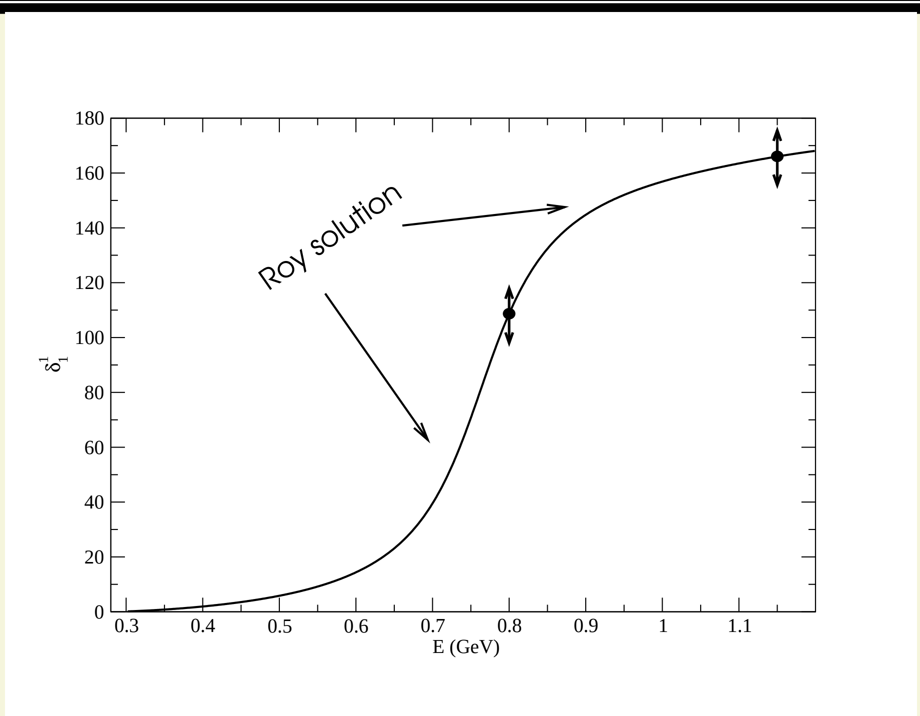


Figure 2: The $\pi\pi$ phase shift δ_1^1: the values of the phase at the two points are free; the Roy equation and chiral symmetry completely fi x the solution [from Colangelo et al]

Behavior of $\delta(s)$ in region below matching point $E_0 = 0.8$ GeV controlled by 3 parameters: 2 S-wave scattering length $a_0^0,\ a_0^2$ and $\phi \equiv \delta(E_0)$ phase at matching point. We treat ϕ as a free parameter and rely on very accurate predictions for $a_0^0,\ a_0^2$ from chiral perturbation theory.

- $\rho - \omega$–mixing contribution

$$G_\omega(s) = 1 + \varepsilon \frac{s}{s_\omega - s} + \ldots \qquad s_\omega = (M_\omega - \frac{1}{2} i \Gamma_\omega)^2$$

In fact: in order to get it real in space-like region, we replace it by dispersion integral with proper behavior at threshold (is inessential numerically for our purpose). $G_\omega(s)$ is (a) fully determined by $\varepsilon,\ M_\omega$ and Γ_ω and (b) in the experimental range $|G_\omega(s)|$ is very close to magnitude of the pole approximation

- Low energy singularities generated by states with 2 or 3 pions are accounted for by the first two factors of the "master equation" above. the function $G(s)$ represents the smooth background that contains the curvature generated by the remaining singularities. The 4 π channel opens at $s = 16\ M_\pi^2$ but phase space strongly suppresses the strength of the corresponding branch point singularity - a significant inelasticity only manifests itself for $s > s_{\rm in} = (M_\omega + M_\pi)^2$. Conformal mapping:

$$z = \frac{\sqrt{s_{\rm in} - s_1} - \sqrt{s_{\rm in} - s}}{\sqrt{s_{\rm in} - s_1} + \sqrt{s_{\rm in} - s}}$$

maps the plane cut along $s > s_{\rm in}$ onto the unit disk in the z–plane. It contains a free parameter s_1 - the value of s which gets maps into the origin. We find that if s_1 is taken in the vicinity of M_ρ^2, then the fit becomes rather insensitive to the details of the parametrization. In the following we set $s_1 = -1.0$ GeV^2. We approximate $G(s)$ by a n_P degree polynomial in z:

$$G_2(s) = 1 + \sum_{i=1}^{n_P} c_i\ (z^i - z_0^i)$$

where z_0 is the image of $s = 0$. The shift of z by $z \to z - z_0$ is required to preserves the charge normalization condition $G_2(0) = 1$. The form of the branch point singularity $(1 - s_{\rm in}/s)^{9/2}$ imposes four constraints on the polynomial; a non-trivial contribution from $G_2(s)$ thus requires a polynomial of fifth order at least. (work in progress with Caprini, Colangelo, Leutwyler)

P	χ^2/d.o.f.	$\chi^2_{\mathrm{CMD2/NA7}}$	$10^{10} a_\rho$	$10^{10} a_{2M_K}$	$\langle r^2 \rangle$(fm^2)
0	84.9/83	43.6 / 43.7	420.1 ± 2.1	489.5 ± 2.2	0.4254 ± 0.0020
5	78.4/82	35.9 / 42.6	423.8 ± 2.6	494.1 ± 2.7	0.4300 ± 0.0024
6	78.1/81	36.0 / 42.2	424.4 ± 2.8	494.7 ± 2.9	0.4339 ± 0.0051
7	73.5/80	31.7 / 42.2	423.4 ± 2.9	493.2 ± 3.0	0.4350 ± 0.0051
8	73.5/79	31.6 / 42.2	423.5 ± 5.7	493.4 ± 7.4	0.4347 ± 0.0052

Numerical results for fi ts to CMD-2 and (spacelike) NA7 data. The errors given are purely statistical.

To be compared with: 429.02 ± 4.95 (stat) from trapezoidal rule. Gain factor of 2 in precision in stat error!

Note on new KLOE result:

my old value:	694.75 (5.15) (6.83) [8.56]	
subtract cmd2:	389.36 (2.75) (2.59) [3.78]	extended to KLOE range
	305.39 (4.35) (6.32) [7.67]	
KLOE:	388.75 (0.52) (5.05) [5.08]	KLOE range: 591.6-969.5 MeV
add weighted	389.24 (1.14) (2.39) [2.65]	
my new value	694.63 (4.50) (6.76) [8.12]	