LIMITS ON ELECTRON COMPOSITENESS FROM BHABHA SCATTERING AT PEP AND PETRA

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A combined analysis of the published data on the Bhabha scattering differential cross section at $\sqrt{s} \sim 30\ GeV$ is performed in order to search for deviations from the Standard Model predictions, due to an internal structure of the electron. Particular care has been put to find a more appropriate way to quantify the result than the usual $\Lambda^{\pm}$ limits. We find that the combination of several experiments can be sensitive up to $\sim 5\ TeV$ (VV coupling) and that no deviation from the point-like structure of the electron shows up compatibly with this resolution power.

INTRODUCTION

The results of the electron positron storage rings have provided in the past years, with the increasing center of mass energy, fundamental tests of the Standard Model. In particular, the study of the differential cross section of Bhabha scattering, which involves only the electron both in the initial and in the final state, allows the most precise test of the point-like nature of the electron. In fact, any structure of the electron would give rise to a kind of form factor, which would affect the total and (in particular) the differential cross section. One of the possible sources of such a deviation would be a substructure inside the electron. In this case, to the usual Feynman diagrams describing the Bhabha scattering, one should add those originating from the exchange or the interaction of the hypothetical objects constituing the electron. At present energies, much lower than the typical scale of these phenomena, the new interaction would appear as an extra contact term in the EW lagrangian.

Eichten et al.[1] have provided a model independent parametrization of the compositeness lagrangian, motivated by the fact that "in any model, in which one or both chiral components of the fermion is composite, there must occur flavor-diagonal helicity-conserving contact interaction of the form $\mathcal{L}_{eff} = \frac{g^2}{2\Lambda^2}(\eta_{\mathrm{LL}} j_L j_L + \eta_{\mathrm{RR}} j_R j_R + \eta_{\mathrm{RL}} j_L j_R)$". j_L and j_R denote the left-handed and right-handed fermion currents. The parameter Λ characterizes the mass scale of compositeness, the interaction is assumed to be strong $(g^2/4\pi = 1)$ and the overall sign of the lagrangian is left free. Combinations of $\eta_i = 0, \pm 1$ can be made in order to describe interactions involving the products of left-left (LL), right-right (RR), axial-axial (AA) or vector-vector (VV) currents. In the literature these interactions are designated with an index indicating the overall sign of the lagrangian. So we have: $LL^{\pm} : \eta_{\mathrm{LL}} = \pm 1, \eta_{\mathrm{RR}} = \eta_{\mathrm{RL}} = 0$; $RR^{\pm} : \eta_{\mathrm{RR}} = \pm 1, \eta_{\mathrm{LL}} = \eta_{\mathrm{RL}} = 0$; $VV^{\pm} : \eta_{\mathrm{LL}} = \eta_{\mathrm{RR}} = \eta_{\mathrm{RL}} = \pm 1$; $AA^{\pm} : \eta_{\mathrm{LL}} = \eta_{\mathrm{RR}} = -\eta_{\mathrm{RL}} = \pm 1$.

At present energies no evidence for a structure of the electron has been found and several experiments[2,5,6,3,4,7] have presented results on the lower limits of the scale parameter Λ for each coupling. Conventionally this is given with indexes showing the coupling (with sign) to which it refers (e.g. Λ_{+}^{VV}). The limits from present day e^+e^- experiments are in the TeV region, with values up 7 TeV[7]. However, we do not agree on how the analysis is sometimes performed and, more in general, on the standard way to present the experimental result. In fact, often the operative procedure used to calculate the limits is not described, and sometimes even the numerical values of the published limits seem to disagree with the figures to which they refer. Moreover it is not possible to combine statistically the published limits of the different experiments to get a stronger constrain on the point-like nature of the electron. This turns out in a bad general tendency to quote only the highest limits[8], that often are nothing but the largest statistical fluctuaction, as it will be discussed later.

The purpose of this analysis[9] is twofold:

- find the proper way to present the experimental results that will allow in the future an easy comparison with other data;

- make a combined homogeneous analysis of the published data in order to increase the experimental sensitivity to the EW parameters of the electron and to any deviation from its point-like nature.

This paper is organized as follows: we first summarize the used cross-section formulae, then we describe the analysis method and finally we give the preliminary results based on the PEP and PETRA data.

CROSS SECTION FORMULAE

The SM cross section to lowest order, as calculated by Budny [10], has been extended by Eichten et al. [1] to incorporate composite models, assuming a four-fermion contact interaction with a helicity-conserving (V, A) structure of the currents. For unpolarized beams the differential cross section for the reaction $e^+e^- \to e^+e^-$ can be written in the form

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8s} \cdot \left\{4B_1 + B_2(1-\cos\theta)^2 + B_3(1+\cos\theta)^2\right\}, \tag{1}$$

with

$$B_1 = \left(\frac{s}{t}\right)^2 \left|1 + (g_V^2 - g_A^2)\xi + \frac{\eta_{\mathrm{RL}} t}{\alpha\Lambda^2}\right|^2, \qquad B_2 = \left|1 + (g_V^2 - g_A^2)\chi + \frac{\eta_{\mathrm{RL}} s}{\alpha\Lambda^2}\right|^2, \tag{2}$$

$$B_3 = \frac{1}{2}\left|1 + \frac{s}{t} + (g_V + g_A)^2\left(\frac{s}{t}\xi + \chi\right) + \frac{2\eta_{\mathrm{LL}} s}{\alpha\Lambda^2}\right|^2 + \frac{1}{2}\left|1 + \frac{s}{t} + (g_V - g_A)^2\left(\frac{s}{t}\xi + \chi\right) + \frac{2\eta_{\mathrm{RR}} s}{\alpha\Lambda^2}\right|^2, \tag{3}$$

$$\chi = \frac{G_F\, M_Z^2}{2\sqrt{2}\,\pi\alpha} \cdot \frac{s}{s - M_Z^2 + iM_Z\Gamma}, \qquad \xi = \frac{G_F\, M_Z^2}{2\sqrt{2}\,\pi\alpha} \cdot \frac{t}{t - M_Z^2 + iM_Z\Gamma}. \tag{4}$$

Here α is the QED fine structure constant, θ is the polar scattering angle measured between the incoming and the outgoing electron, and s and t are the usual Mandelstam variables, related to each other through $t = -s/2(1-\cos\theta)$. In the standard $SU(2)_L \times U(1)$ model the weak contributions are described by the vector coupling $g_V = -1/2 + 2\sin^2\theta_W$, the axial vector coupling $g_A = -1/2$, the weak mixing angle $\sin^2\theta_W$ and propagator terms given by the Fermi constant G_F, the Z^0 mass M_Z, the Z^0 width Γ and α. For calculations within the SM we use $\sin^2\theta_W = 0.23$ and $M_Z = 91.2\ GeV$ and $\Gamma = 2.5\ GeV$[11]. The chosen parametrization is not sensitive to Γ and to the exact value of M_Z because, at our energies, M_Z^2 dominates in the denominator in Eqs. (5) and (6), thus cancelling largely the M_Z^2 in the numerator.

The pure QED case can be recovered by setting g_V, g_A and all η's to zero. Traditionally any departure from QED has been parametrized by inserting time-like and space-like form factors at the respective vertices with cut-off parameters $\Lambda_{\mathrm{QED}\pm}$[12] $F_t(q^2) = 1 \mp \frac{q^2}{q^2 - \Lambda^2_{\mathrm{QED}\pm}}$, where q^2 is s or t. The Λ_{QED} parameters are connected (see [13] for detailed formulae) to the compositeness parameter of vector-vector coupling through:

$$\Lambda_{\mathrm{QED}} = \sqrt{\alpha} \cdot \Lambda_{VV}. \tag{5}$$

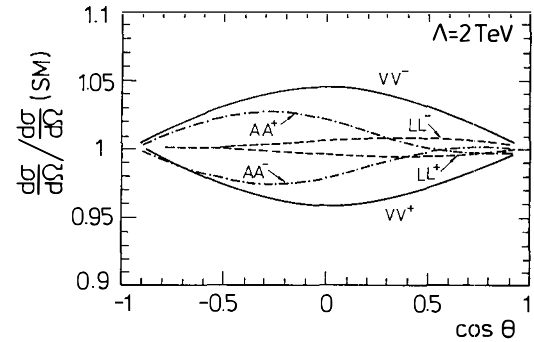

Figure 1: Expected deviations form SM of Bhabha differential cross section due to electron compositeness.

ANALYSIS METHOD

In the formulae for the cross-section shown above one observes that the SM prediction are recovered if Λ goes to infinity. Instead a finite value would indicate the presence of electron compositeness. In Fig. 1 we give the deviations from the Standard Model for the different couplings (LL and RR are indistinguishable at the energies of interest) for a scale value of $2\ TeV$. One notices that the highest sensitivity is coming from the VV coupling and that the overall sign of the lagrangian does not always produce the same sign of the deviation of the differential cross section with respect to the SM.

The fact that one wants to check the compatibility with infinity of the Λ's leads usually to numerical problems in the fit. Our approach is based on two simple observations:

- One can see from Eqs. (2),(3) and (4) that the experimental data are sensitive to q^2/Λ^2 with q^2 equal to s or t and then it is natural to expand the cross-section in serie of $\epsilon = 1/\Lambda^2$:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}(SM) + \epsilon f_C(\theta, q^2, ...) \tag{6}$$

  where f_C ia a function which depends from the couplings.

- For any coupling, the cross section formulae have a mathematical continuation from Λ^+ to Λ^- through infinity, which can better treated as a continuation through zero from ϵ^+ to ϵ^-, i.e. for small ϵ :

$$\frac{d\sigma}{d\Omega}(\epsilon^\pm) - \frac{d\sigma}{d\Omega}(SM) = -(\frac{d\sigma}{d\Omega}(\epsilon^\mp) - (\frac{d\sigma}{d\Omega}(SM)) \tag{7}$$

  For example, if a fit to the data of the VV^+ coupling yields $\epsilon^+ = \epsilon_0 \pm \sigma_\epsilon$, the fit of VV^- will give $\epsilon^+ = -\epsilon_0 \pm \sigma_\epsilon$, with the same σ_ϵ.

The advantage of quoting as experimental result the fitted value of ϵ with its standard deviation, instead of just 95 % C.L. limits, is that its meaning is unambiguos and its distribution is gaussian, so that different experimental results can be easily averaged. There is also another problem related to the determination of the 95 % C.L. limits of $\Lambda^{\pm}$. They are obtained as $\Lambda^{\pm} = 1/\sqrt{1.64\sigma_\epsilon \pm \epsilon_0}$. If ϵ_0 deviates from zero by more than $1.64\sigma_\epsilon$, one of the two limits becomes unphysical, while the other one will support the compositeness. Statistically this should happen to about 10 % of the published Λ's. The fact that checking through all published and preliminary papers on the subject (included similar analysis on the compositeness of other fermions) this does not show up, means that some obscure treatement of the data has been performed. Moreover it is clear that, often, high limits do not mean a strong sensitivity of the data, but just an accidental value of $|\epsilon_0|$ close to $1.64\sigma_\epsilon$.

It is interesting to note what is the physical meaning of σ_ϵ. Its square root $r = \sqrt{\sigma_\epsilon}$ has the physical dimension of a distance, and can have the meaning of the spatial *resolution power* of the experiment. $\lambda = 1/\sqrt{\sigma_\epsilon}$ has a similar meaning as resolution power in the mass scale. So, it is clear that limits of $\epsilon < r$ (or equivalently $\Lambda > \lambda$), i.e. below the sensitivity of the measurement, don't make much sense. Notice also that r and λ, related to the second derivative of the χ^2 around its minimum, depend only on the combination of the statistical and normalization errors of the experiments and from the sensitivity of the measurable quantity on the physical parameter. As in any physical instrument, they don't depend on the numerical values of the actual data. Following this approach we will quote for the single experiment and for their combination the resolution power in the mass scale and the fitted value of ϵ.

The problem of finding the most convenient way to present the experimental result and that of giving limits are somehow different. In the first case the data are analysed under the hypothesis that the deviation from the SM can be described as a continuous function of ϵ. When, instead, we want to find the limit on a certain coupling we have to consider unphysical the continuation in the negative region. The second case is similar to that which arises when one has to give an upper mass limit to a particle, and a negative value occurs by statistical fluctuation. Unfortunately, no satisfactory approach exists to handle this problem and two techniques are suggested in the literature. One is based in the renormalization of the gaussian probability function inside the physical region[8,14], the other on the introduction of an artificial cut-off to the physical bound, eventually increased by the experimental resolution, when the limits calculated in the usual way fall below it[8].

COMBINED ANALYSIS OF PEP AND PETRA DATA

The data entering this work come from the following experiments: HRS[3] (29 GeV), MAC[4] (29 GeV), CELLO[2] (35 GeV), JADE[5] (35 and 44 GeV), PLUTO[6] (35 GeV) and TASSO[7] (35,38 and 44 GeV). All the data have been treated omogeneously. The normalization error of the individual experiments ranges between 1 and 3 %, as shown in Tab. 1. In this analysis they have been considered to be all independent. The influence of these errors on the results will be

Experiment	$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb^{-1})	σ_{norm} (%)	λ^{VV} (TeV)	λ^{AA} (TeV)	$\lambda^{LL/RR}$ (TeV)
HRS	29	165	0.9	3.3	2.7	1.5
MAC	29	128	1.5	3.3	2.7	1.1
CELLO	35	86	2.5	3.1	2.7	1.3
JADE 1	35	75	3.2	2.9	2.9	1.2
JADE 2	44	27	3.2	2.7	2.1	1.3
JADE 1+2				3.4	3.1	1.4
PLUTO	35	42	2.6	2.7	2.4	1.2
TASSO 1	35	175	3.0	3.2	3.2	1.4
TASSO 2	38	9	3.0	2.2	1.5	1.0
TASSO 3	44	37	3.0	3.0	2.5	1.3
TASSO 1+2+3				3.8	3.5	1.6
All Expt.s				5.2	4.5	2.2

Table 1: Resolution power for electron compositeness from Bhabha experiments.

discussed later.

We have taken into account for the global normalization error for each data set in two different ways. In one case all data sets have been fitted independently and the normalization error has been taken into account adding the term $(1-f)^2/\sigma_n^2$ to the χ^2, where f is the unkown normalization factor, treated as an extra free parameter in the fit. The global result is then obtained averaging the partial ones. We have also used the covariance matrix method[15] and , after having corrected by Monte Carlo for a small bias due to the fact that the elements of the covariance matrix do not correspond exactly to the definition of the normalization errors, the result comes to be identical.

Tab. 1 shows the resolution power in the scale mass of the individual experiments and of their combination. One can see that they range from 2.2 to 5.2 TeV depending on the coupling. The resolution power for the QCD cut-off parameter, abtainable from λ^{VV} through Eq. (5) is 440 GeV, equivalent to probe the SM down to 0.5 $10^{-18}m$. The results on $\epsilon = 1/\Lambda^2$, fitted with the positive sign of the coupling, are shown in Fig. 2. The combined analysis yields ϵ equal to $(0.060 \pm 0.037)TeV^{-2}$ for VV^+, $(-0.018 \pm 0.049)TeV^{-2}$ for AA^+ and $(-0.36 \pm 0.21)TeV^{-2}$ for LL^+/RR^+. The $\chi^2/d.o.f.$ is 165/158, 165/158 and 164/158 respectively, to be compared with 170/159 when calculated with respect to the SM.

We have checked the influence of the normalization errors on the result, varying all by -50% or $+100\%$ and studying the variation of the σ_ϵ. For the AA coupling there is no change at all, for VV it changes by -22% and $+13\%$ respectively and for LL by -28% and $+14\%$. This

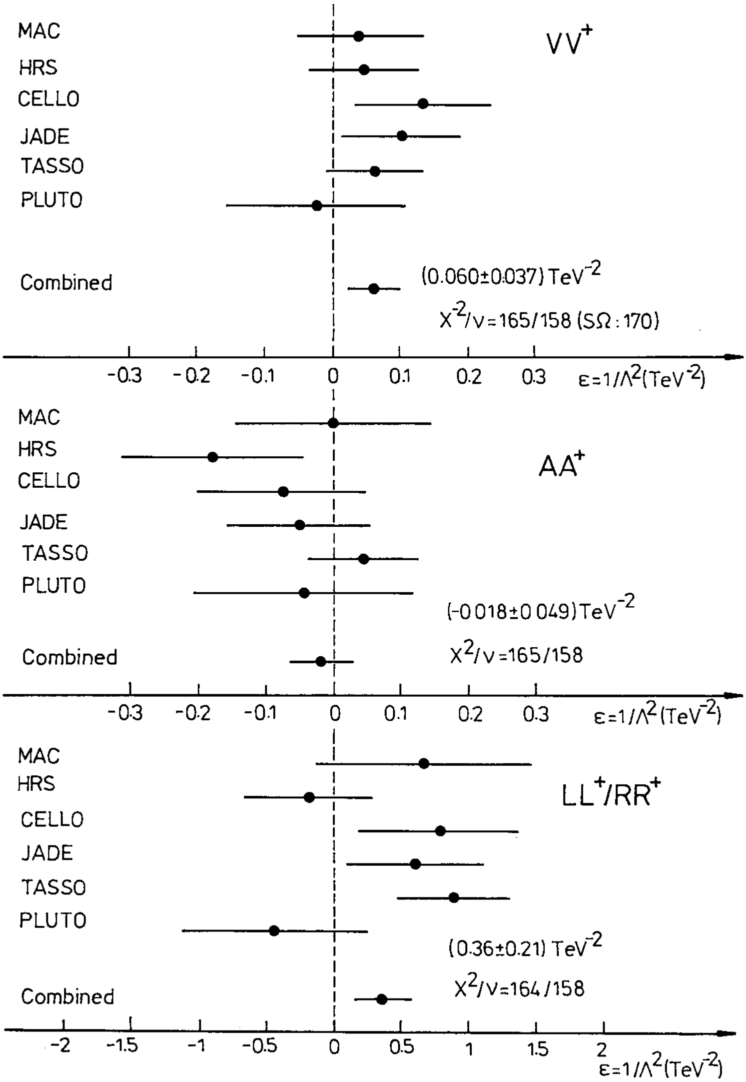

Figure 2: Fitted values of $\epsilon = 1/\Lambda^2$.

means that the results are determined essentially by the shape of the differential cross section. Moreover, the couplings which cause deviations from SM in the forward direction are those which depend more on the normalization, as can be easily understood.

Fig 3 shows the combined data (corrected with normalization factors obtained from a fit to the SM) compared with the expectated deviations from the compositeness. For comparison the scales Λ have been chosen equal to the typical resolution power of a single experiment and that of their combination (see. Tab. 1). The increased of resolution power obtained by the combination of several experiments is evident. The figure gives also a clear picture of the meaning of the resolution power, which puts a limit to the sensitivity of the experiment.

Even if somehow against the spirit of the work, we would like to give limits on $\Lambda^{\pm}$. We do it in three different ways, summarized in Tab. 2:

- Naïve 95 % C.L. limit: obtained from $\Lambda^{\pm} = 1/\sqrt{1.64\sigma_\epsilon \pm \epsilon_\bullet}$; this is reported only for the curiosity of the reader. One can can see that for LL^- the limit is unphysical and for VV^- a ridiculous limit is obtained. In these cases also the 99 % C.L. limit is given. Notice that it is not particulary significant that two out of the three couplings produce "strange" results, since they probe similar kinematical regions and are hence correlated.

- "95% C.L." limit with probability function renormalized on the physical region: we put the quote marks because the value of the confidence level obtained with this tecnique is not rigorously defined.

- Naïve 95 % C.L. limit with cut-off at the resolution power: this is a safe and pragmatic procedure, based on the idea that it is not possible to make measurements below the instrumental sensitivity.

One can see that the last two methods give similar results which can be summarized by stating that there is no evidence of electron compositeness up to mass scales camparible with the resolution power of the combination of the present data.

CONCLUSIONS

The combination of several PEP and PETRA experiments reaches resolution power on the mass scales of the electron compositness of 2.2 , 4.5 and 5.2 TeV for hypothetical left-left, axial-axial and vector-vector coupling respectively. Within this resolutions no significant deviation from the Standard Model has been observed. A quantitative result on the agreement of the experimental data with the SM has been parametrized with $\epsilon = 1/\Lambda^2$, fitted to the positive overall sign of the compositness lagrangian. We obtain ϵ equal to (0.060 ± 0.037) TeV^{-2}, (-0.018 ± 0.049) TeV^{-2}, and (-0.36 ± 0.21) TeV^{-2} for VV, AA and LL/RR respectively.

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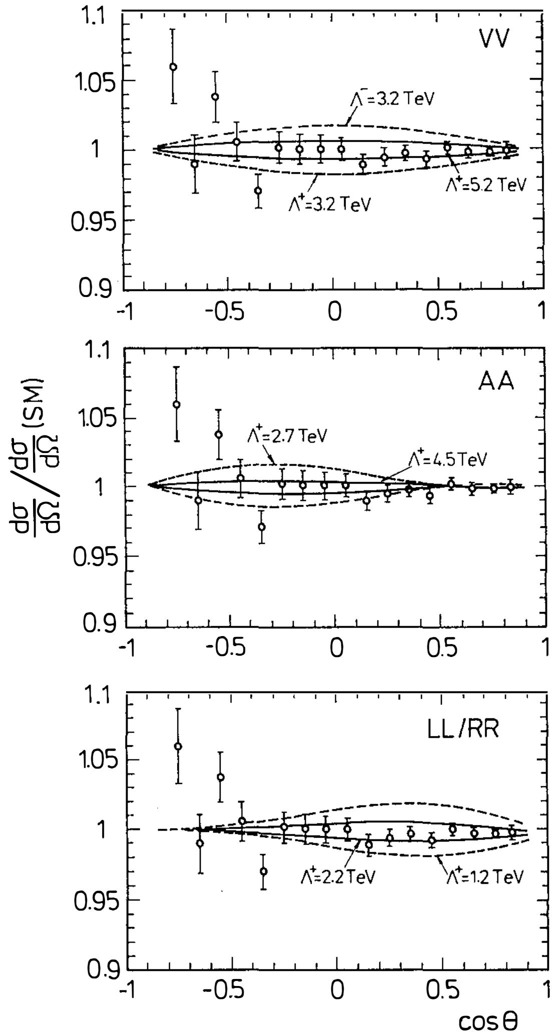

Figure 3: Averaged corrected data compared with the resolution power of the typical individual experiment and of the combined data.

Coupling	$1/\Lambda^2(TeV^{-2})$ $\lambda(TeV)$	naive 95% C.L.	"95% C.L." renorm. prob. on phys. reg.	95% C.L. + cut-off on sensitivity
VV	0.060 ± 0.037 5.2 $(\lambda_{QED} = 0.44\ TeV)$	$\Lambda^- > 2.9\ TeV$ $\Lambda^- > 38\ TeV(?!)$ $(6\ TeV, 99\%)$	$\Lambda^+ > 2.6\ TeV$ $\Lambda^- > 6.6\ TeV$	$\Lambda^+ > 2.9\ TeV$ $\Lambda^- > 5.2\ TeV(*)$
AA	-0.018 ± 0.049 4.5	$\Lambda^+ > 4.0\ TeV$ $\Lambda^- > 3.2\ TeV$	$\Lambda^+ > 3.4\ TeV$ $\Lambda^- > 3.0\ TeV$	$\Lambda^+ > 4.0\ TeV$ $\Lambda^- > 3.2\ TeV$
LL/RR	0.36 ± 0.21 2.2	$\Lambda^+ > 1.2\ TeV$ $\Lambda^- >??$ (2.8 TeV, 99%)	$\Lambda^+ > 1.2\ TeV$ $\Lambda^- > 2.0\ TeV$	$\Lambda^+ > 1.2\ TeV$ $\Lambda^- > 2.2\ TeV(*)$

Table 2: Results of the combined analysis. See the text for the meaning of the different limits. (*) Bound by the resolution power.

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