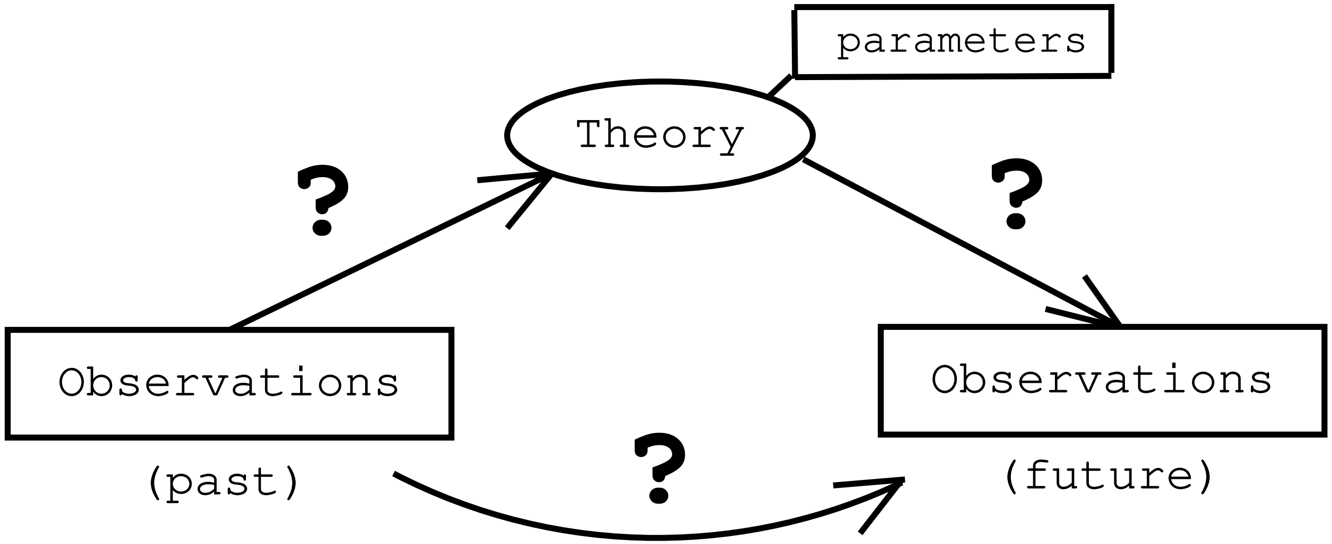
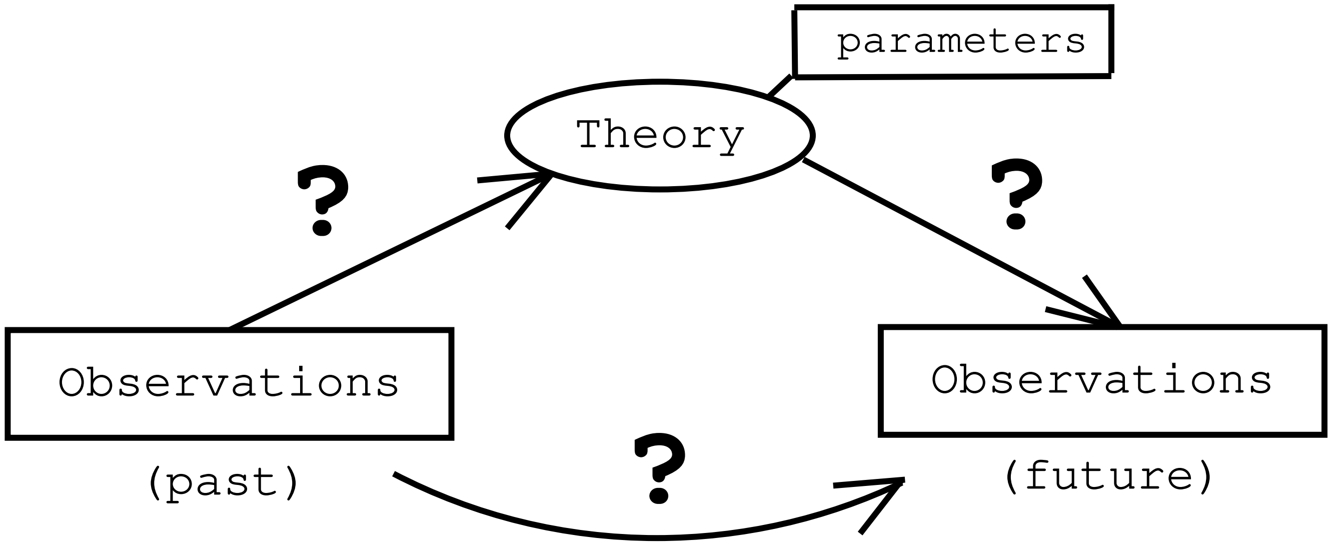
Uncertainty



⇒ Uncertainty:

1. Given the past observations, in general we are not sure about the parameters of the model (and/or the model itself)
2. Even if we were sure about theory and parameters, there could be internal ("noise", variables out of our control) or external effects (initial/boundary conditions, 'errors', etc) that make the forecasting uncertain.

Uncertainty

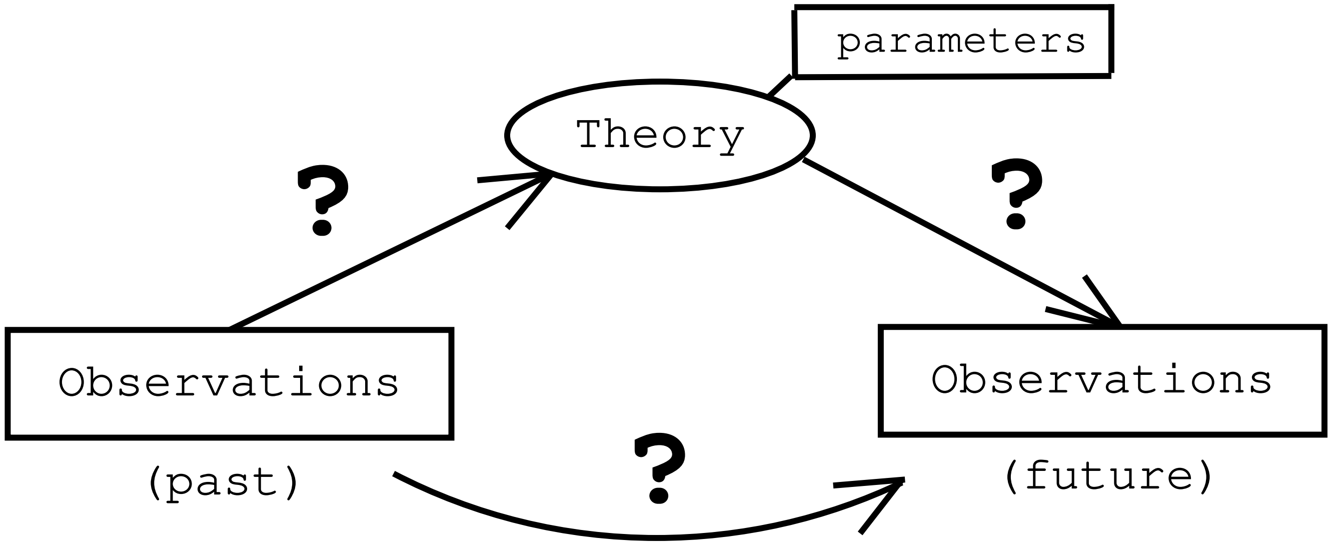


⇒ Uncertainty:

- No certainties, only probabilities

- $P(\Theta \,|\, X_{past})$
- $P(X_{future} \,|\, \Theta)$
- $P(X_{future} \,|\, X_{past})$

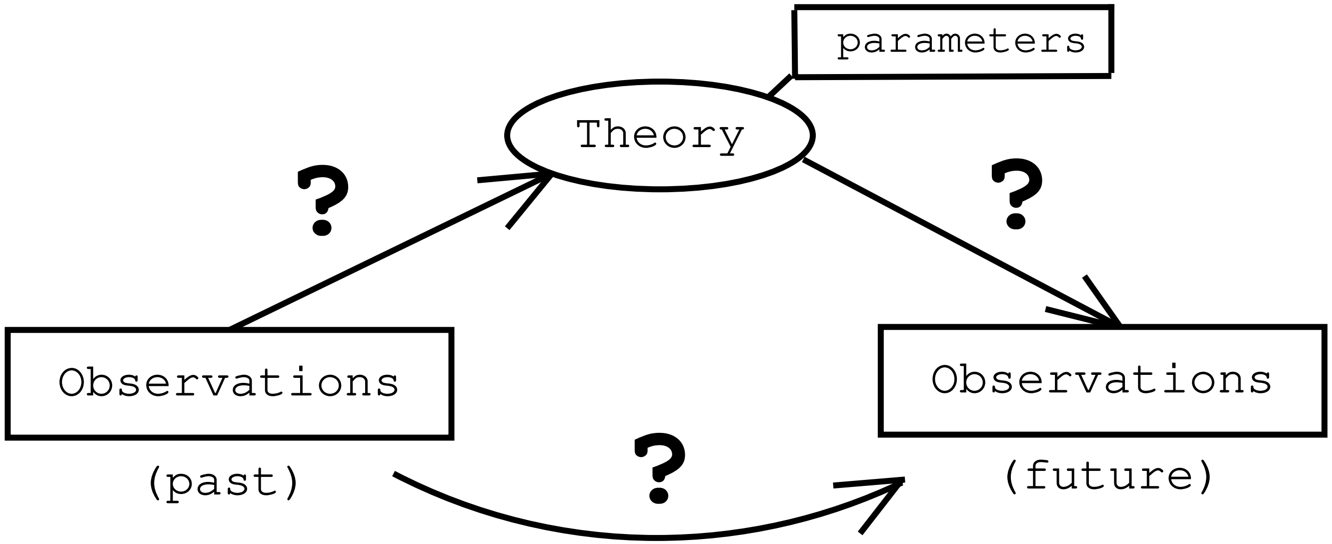
Deep source of uncertainty



Uncertainty:

Theory	— ?	$\longrightarrow$	Future observations
Past observations	— ?	$\longrightarrow$	Theory
Theory	— ?	$\longrightarrow$	Future observations

Deep source of uncertainty



Uncertainty:

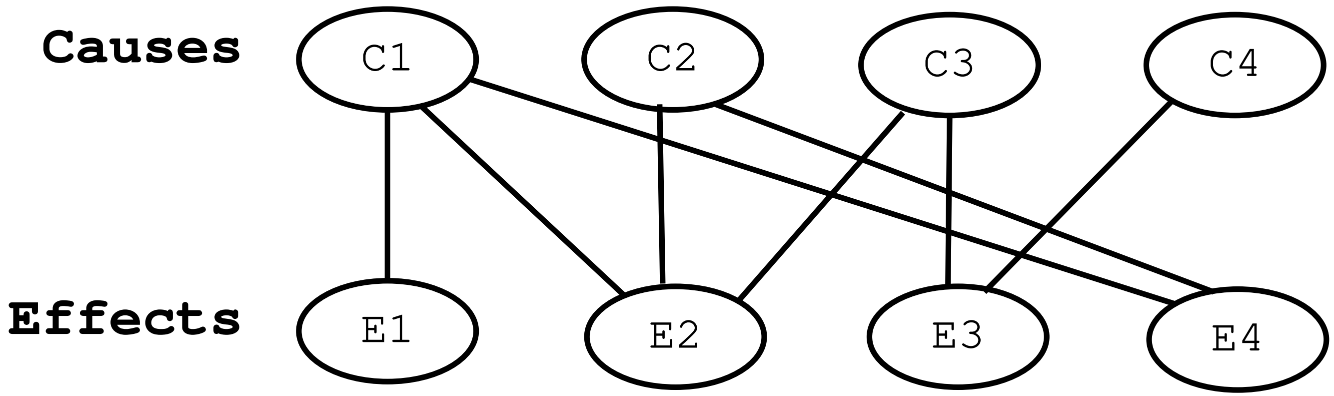
Theory	— ? ⟶	Future observations
Past observations	— ? ⟶	Theory
Theory	— ? ⟶	Future observations

$\Longrightarrow$ **Uncertainty about causal connections**

CAUSE $\Longleftrightarrow$ **EFFECT**

Causes $\rightarrow$ effects

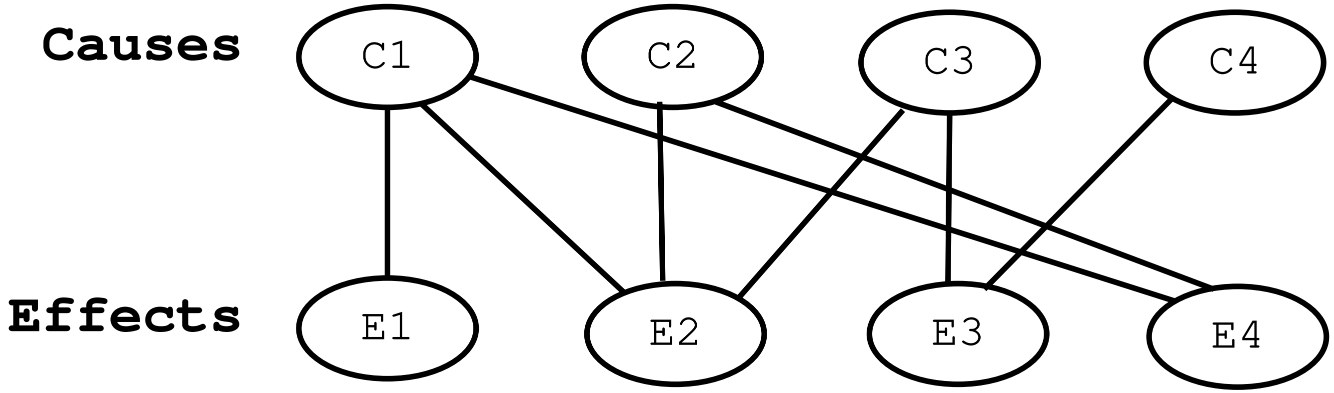
The same *apparent* cause might produce several,different effects



Given an observed effect, we are not sure about the exact cause that has produced it.

Causes → effects

The same *apparent* cause might produce several,different effects



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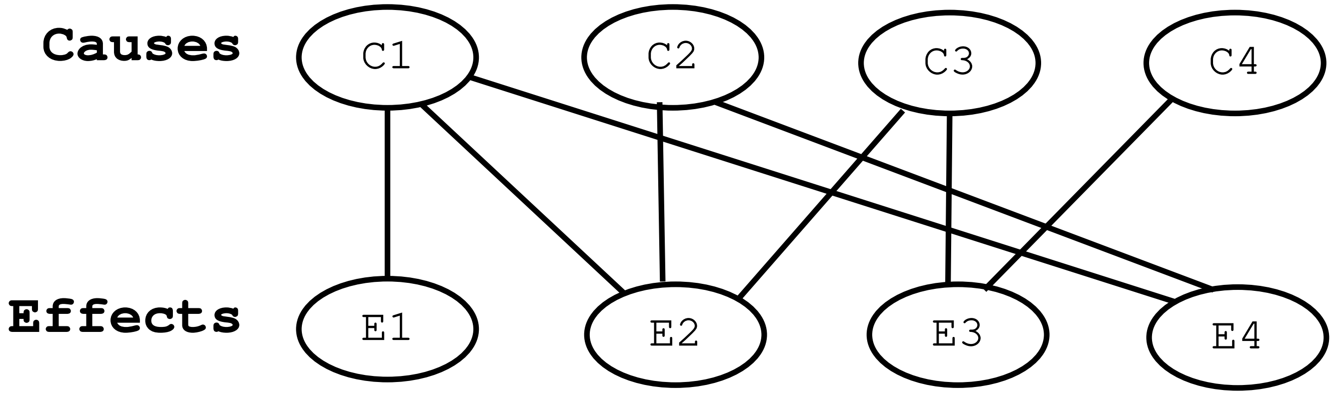


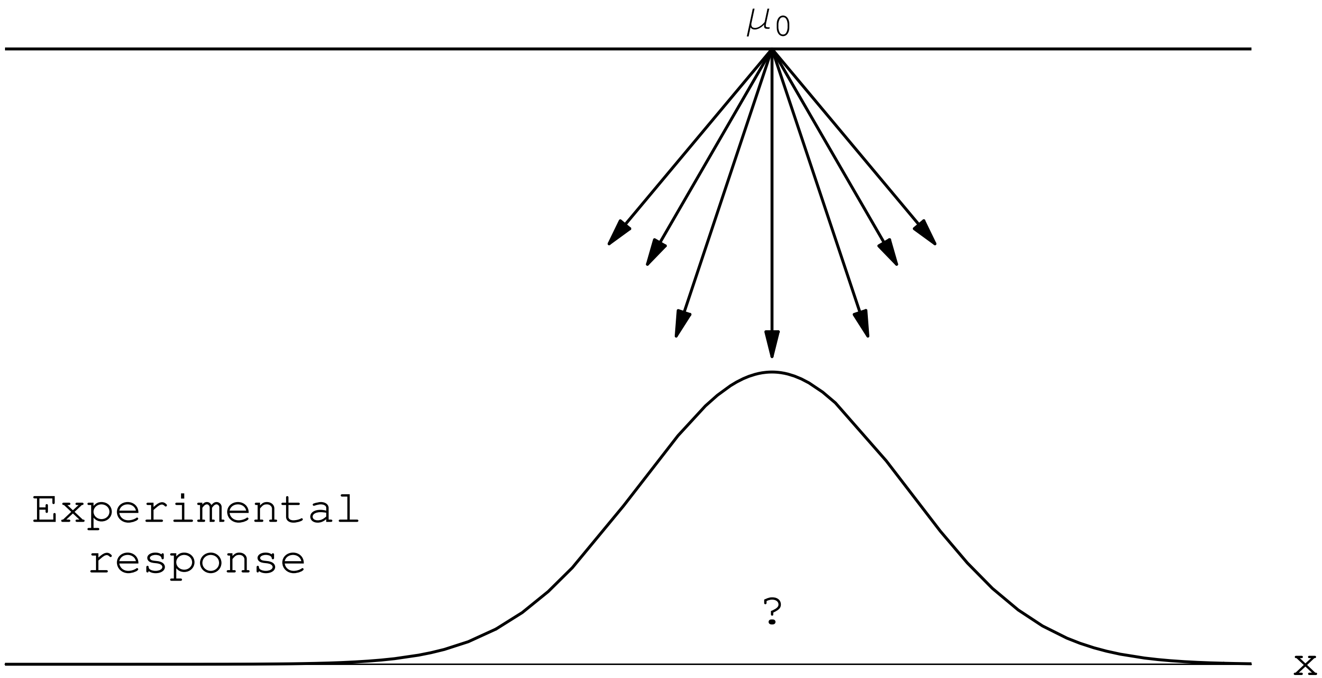
Given an observed effect, we are not sure about the exact cause that has produced it.

$$\mathbf{E_2} \Rightarrow \{C_1,\ C_2,\ C_3\}?$$

$\rightarrow$ Probability of causes

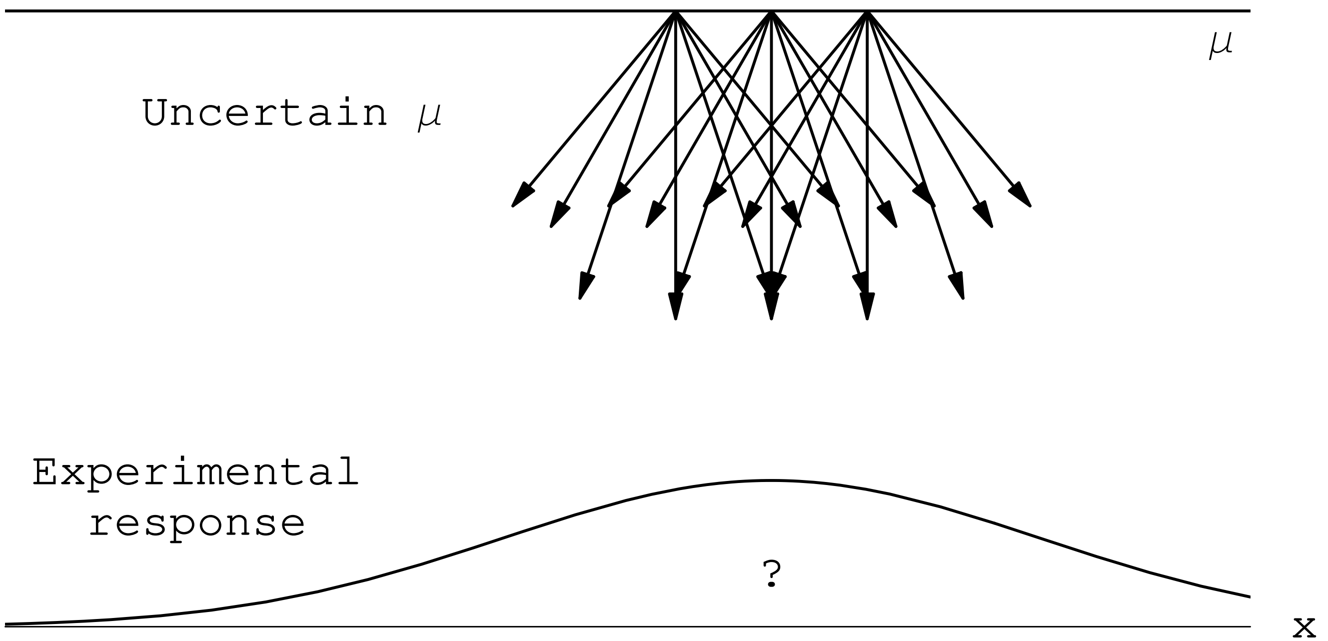
"the essential problem of the experimental method"

From ‘true value’ to observations



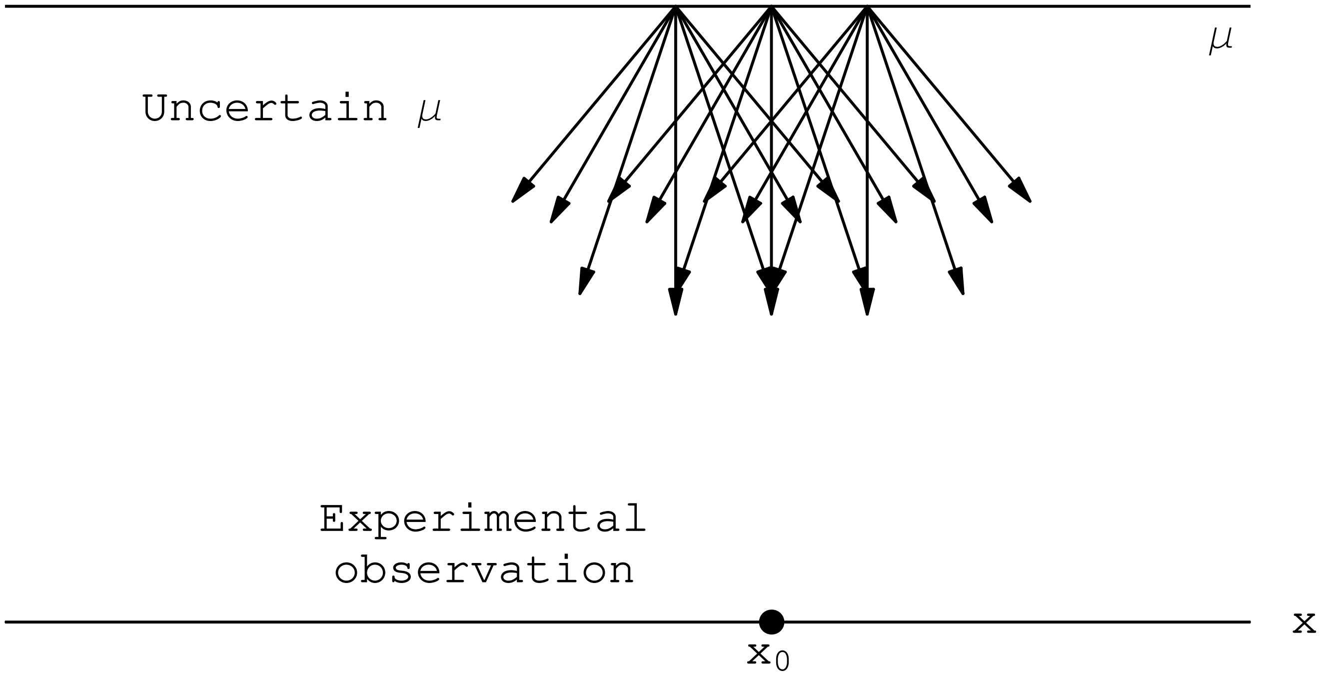
Given μ (exactly known) we are uncertain about x

From 'true value' to observations



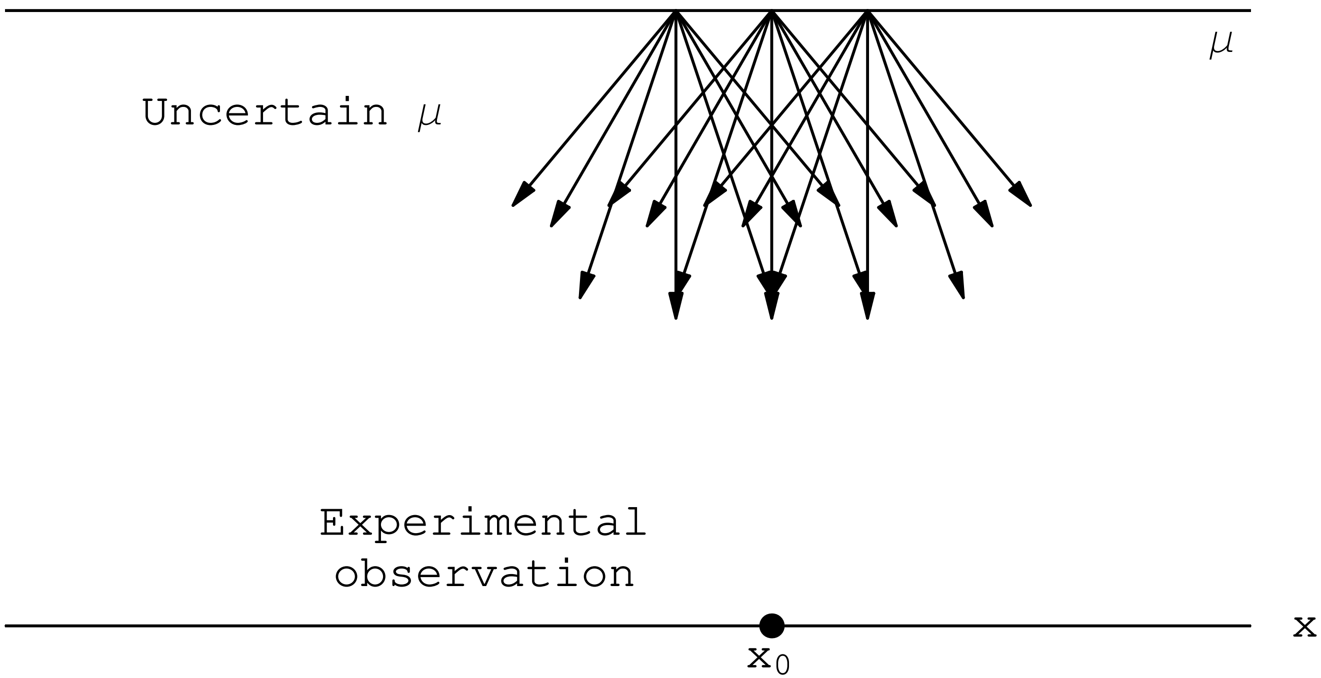
Uncertainty about μ makes us more uncertain about x

. . . and back: Inferring a true value



The observed data is <u>certain</u>: $\rightarrow$ 'true value' uncertain.

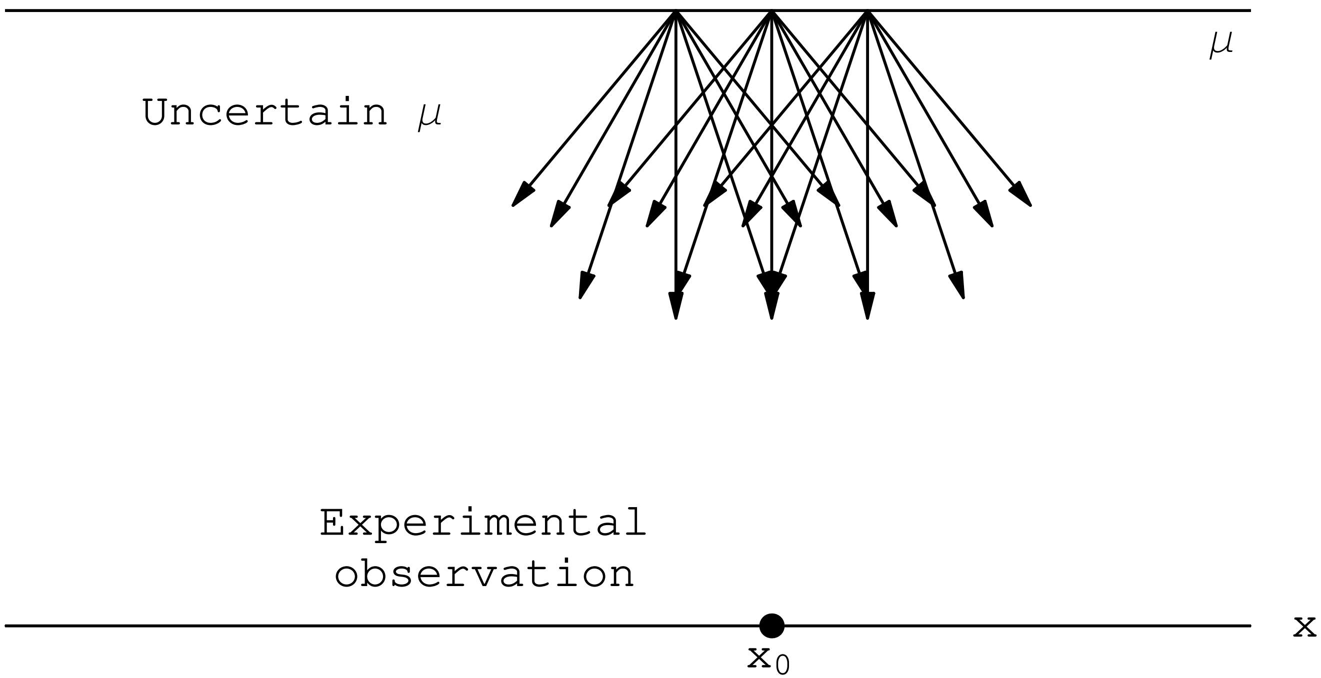
. . . and back: Inferring a true value



The observed data is certain: → 'true value' uncertain.

"data uncertainty" ?

. . . and back: Inferring a true value



The observed data is <u>certain</u>: → 'true value' uncertain.

"data uncertainty" ? Data corrupted?

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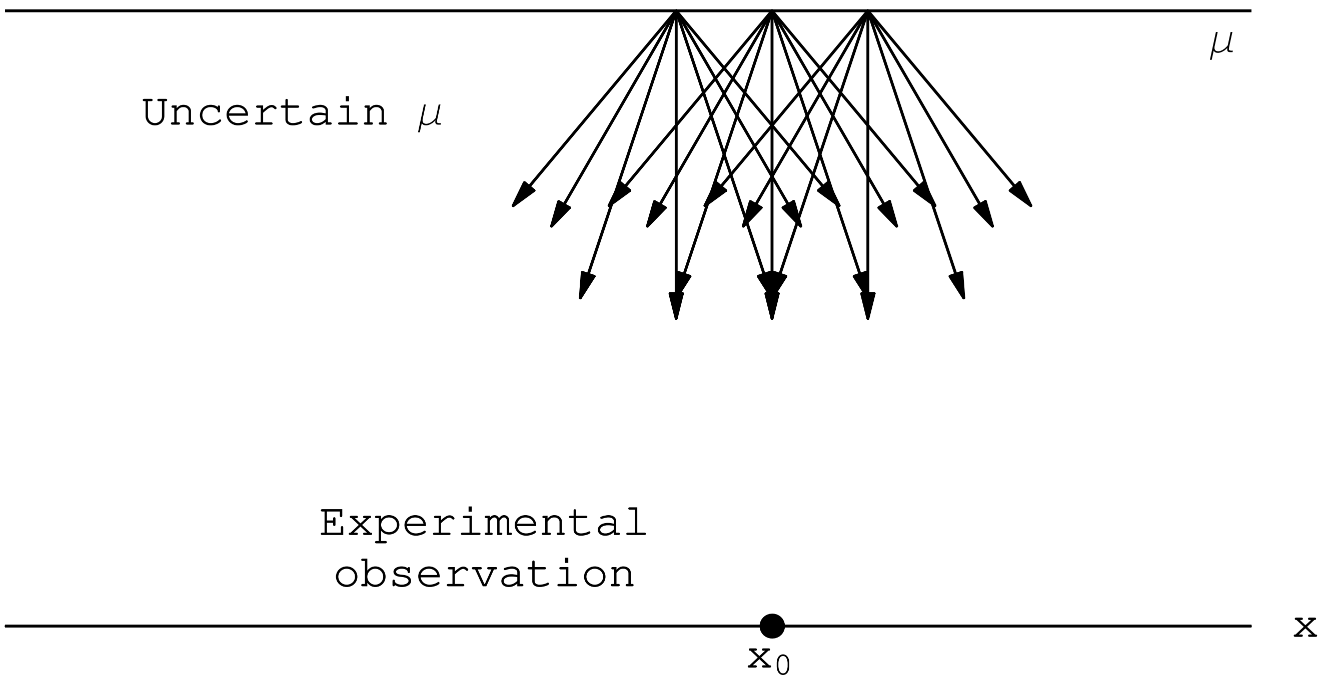
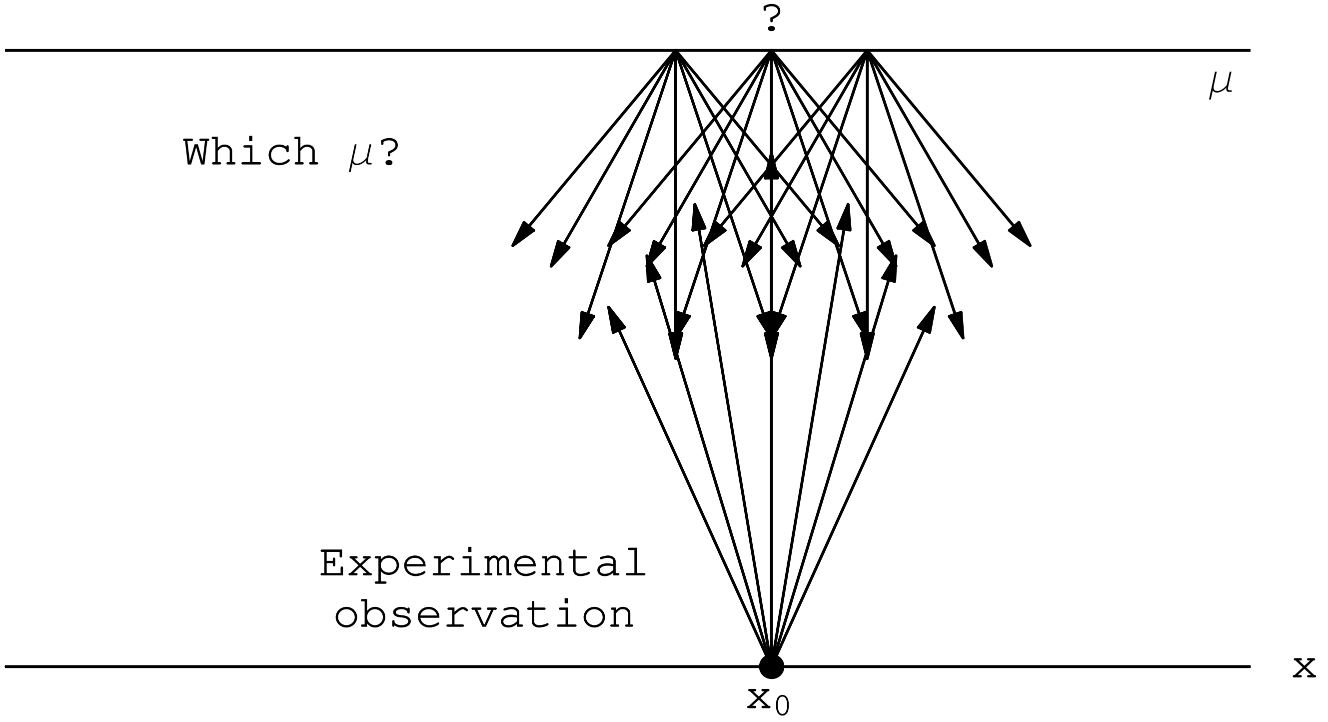


The observed data is <u>certain</u>: $\rightarrow$ 'true value' uncertain.

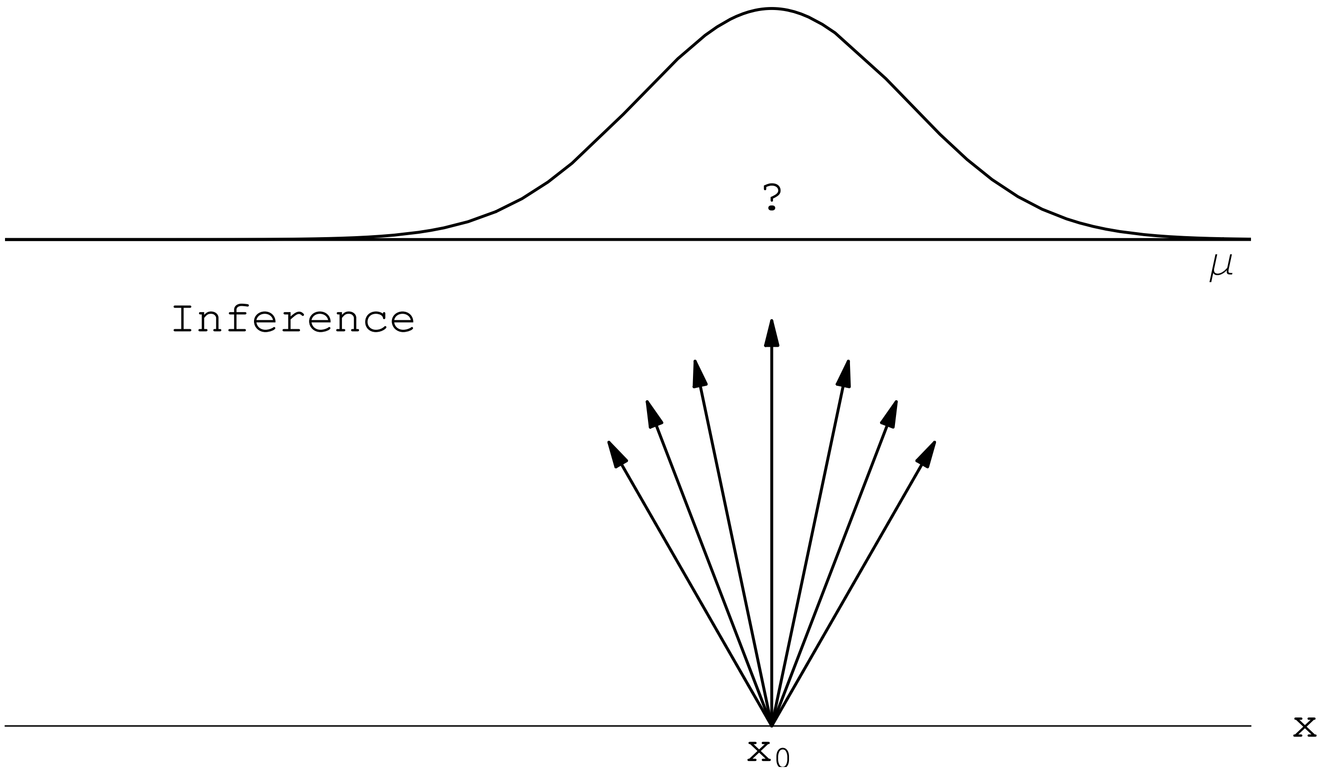
"data uncertainty" ? Data corrupted?
Even if the data were corrupted, the <u> data </u> were the corrupted data**!!** . . .

. . . and back: Inferring a true value



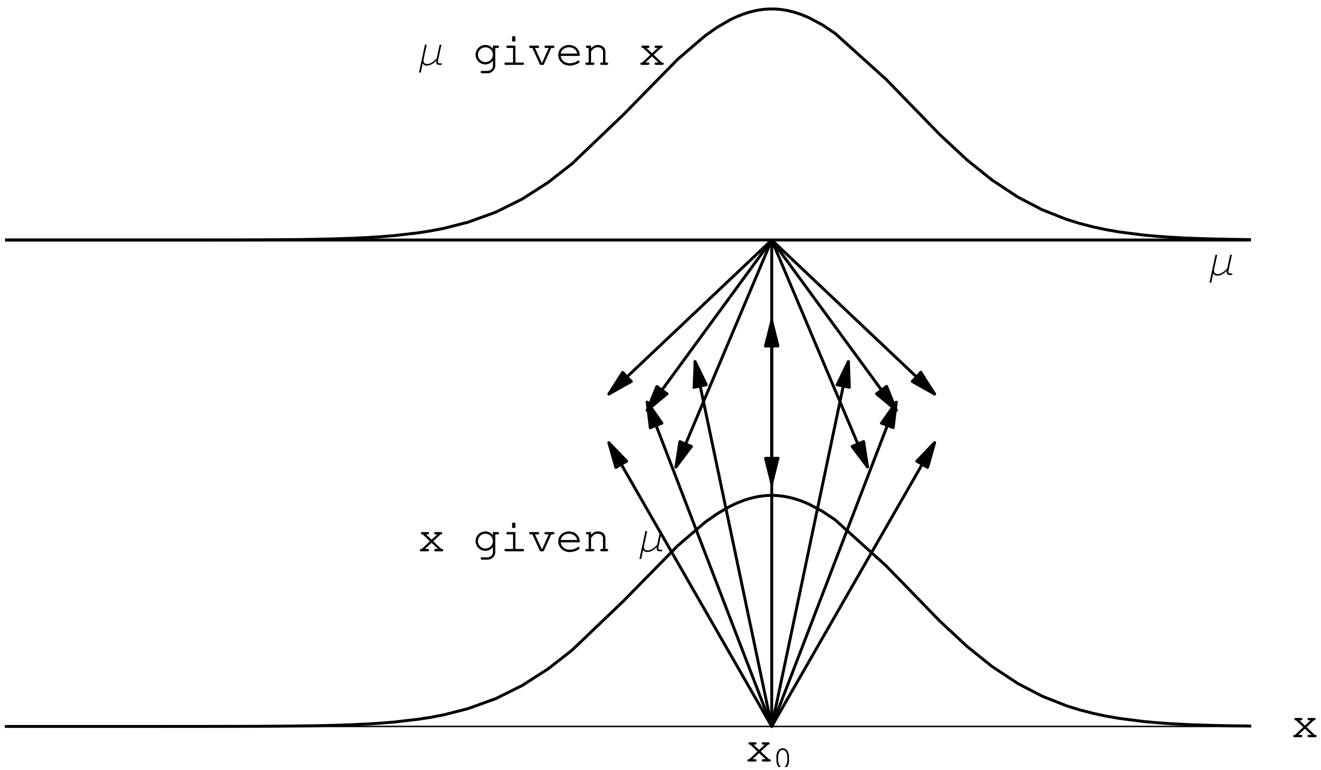
Where does the observed value of x comes from?

. . . and back: Inferring a true value



We are now uncertain about μ, given x.

. . . and back: Inferring a true value



Note the symmetry in reasoning.

A very simple experiment

Let's make an experiment

A very simple experiment

Let's make an experiment

- Here
- Now

A very simple experiment

Let's make an experiment

- Here
- Now

For simplicity

- μ can assume only six possibilities:

$$\mathbf{0}, \mathbf{1}, \ldots, \mathbf{5}$$

- x is binary:

$$\mathbf{0}, \mathbf{1}$$

$$[\,(1,2);\ \text{Black/White};\ \text{Yes/Not};\ \ldots]$$

A very simple experiment

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For simplicity

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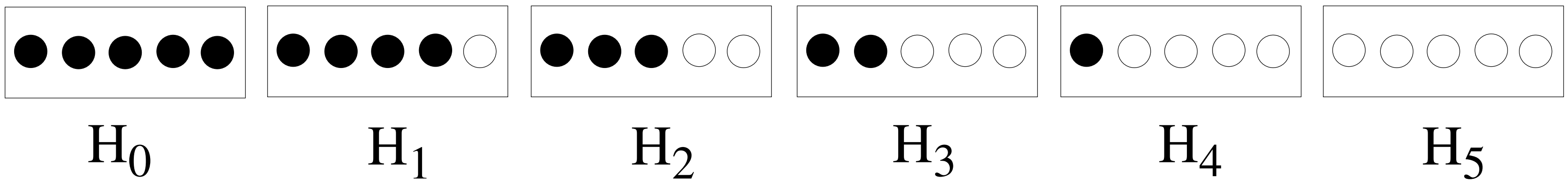
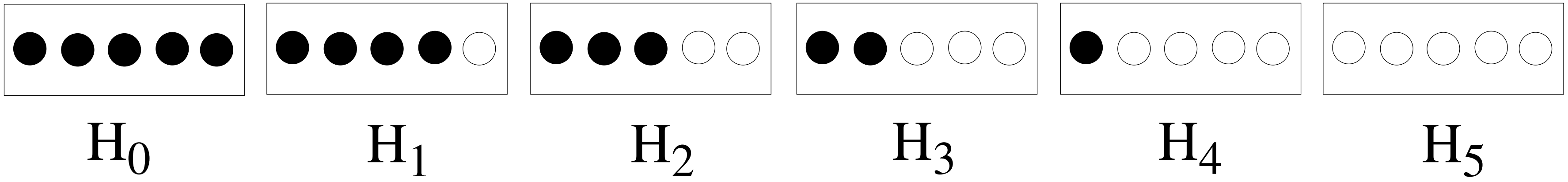
⇒ Later we shall make μ continuous.

Which box? Which ball?



Let us take randomly one of the boxes.

Which box? Which ball?



Let us take randomly one of the boxes.

We are in a state of uncertainty concerning several *events*, the most important of which correspond to the following questions:

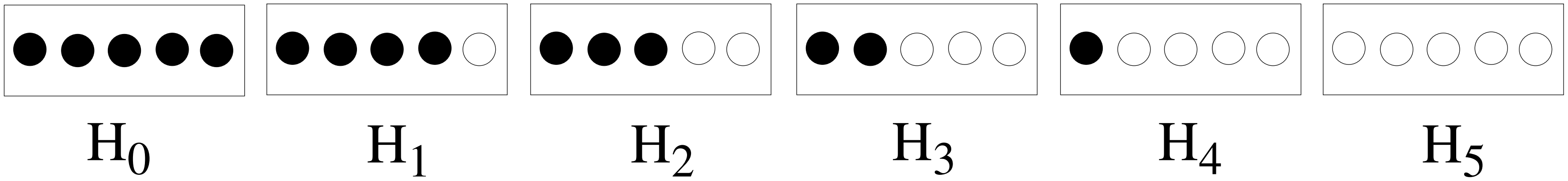
(a) Which box have we chosen, $H_0, H_1, \ldots, H_5$?

(b) If we extract randomly a ball from the chosen box, will we observe a white ($E_W \equiv E_1$) or black ($E_B \equiv E_2$) ball?

Our certainties:

$$\begin{aligned} \cup_{j=0}^{5} H_j &= \Omega \\ \cup_{i=1}^{2} E_i &= \Omega\,. \end{aligned}$$

Which box? Which ball?



Let us take randomly one of the boxes.

- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation

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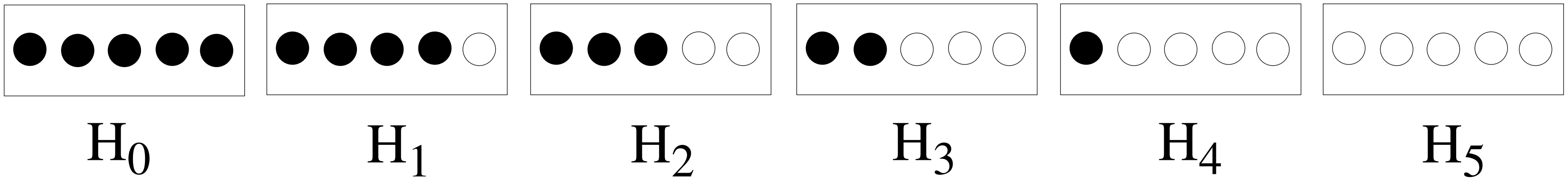

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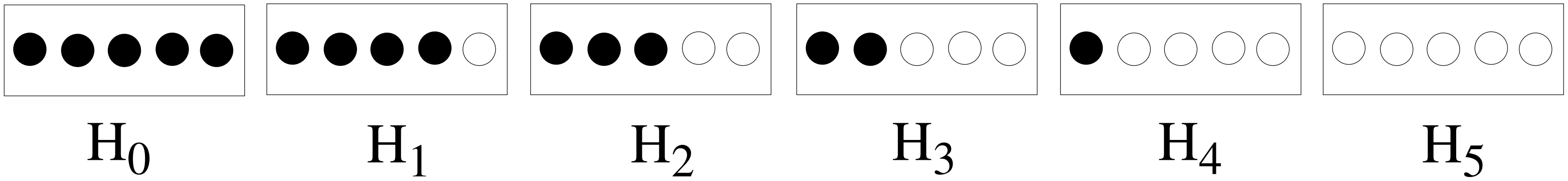

Let us take randomly one of the boxes.

- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation
  - Can we do it *quantitatively*, in an 'objective way'?
- And after a sequence of extractions?

The toy inferential experiment

The aim of the experiment will be to guess the content of the box without looking inside it, only extracting a ball, record its color and reintroducing in the box

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This toy experiment is conceptually very close to what we do in the pure and applied sciences

⇒ try to guess what we cannot see (the electron mass, a magnetic field, etc)

...from what we can see (somehow) with our senses.

The rule of the game is that we are not allowed to watch inside the box! (As we cannot open and electron and read its properties, unlike we read the MAC address of a PC interface.)