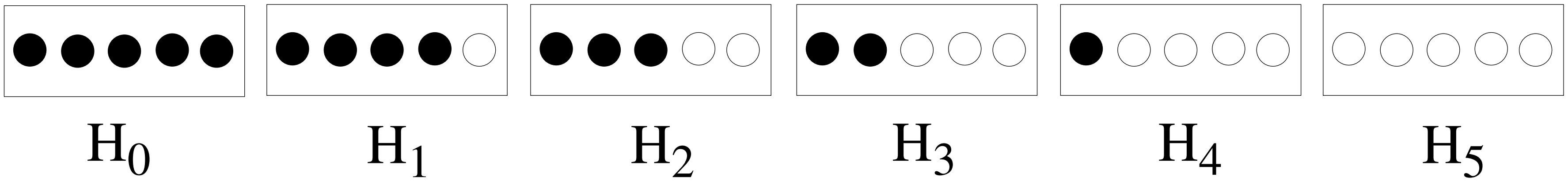
Application to the six box problem



Remind:

- $E_1 = \text{White}$
- $E_2 = \text{Black}$

Collecting the pieces of information we need

Our tool:

$$\boxed{P(H_j \mid E_i, I) = \frac{P(E_i \mid H_j, I)}{P(E_i \mid I)} P(H_j \mid I)}$$

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- $P(E_i \mid H_j, I)$:

$$\begin{aligned} P(E_1 \mid H_j, I) &= j/5 \\ P(E_2 \mid H_j, I) &= (5-j)/5 \end{aligned}$$

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Our prior belief about H_j

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Probability of E_i under a well defined hypothesis H_j
It corresponds to the 'response of the apparatus' in measurements.
$\rightarrow$ likelihood (traditional, rather confusing name!)

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Probability of E_i taking account all possible H_j

$\rightarrow$ How much we are confident that E_i will occur.

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Probability of E_i taking account all possible H_j
$\rightarrow$ How much we are confident that E_i will occur.
(taking into account all possible hypotheses H_j)

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But it easy to prove that $P(E_i \mid I)$ is related to the other ingredients, usually easier to ‘measure’ or to assess somehow, though vaguely

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'decomposition law': $P(E_i \mid I) = \sum_j P(E_i \mid H_j, I) \cdot P(H_j \mid I)$
($\rightarrow$ Easy to check that it gives $P(E_i \mid I) = 1/2$ in our case).