Measurements, uncertainties and probabilistic inference/forecasting

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"The only relevant thing is uncertainty – the extent of our knowledge and ignorance. The actual fact of whether or not the events considered are in some sense determined, or known by other people, and so on, is of no consequence"

(Bruno de Finetti)

What is measurement?

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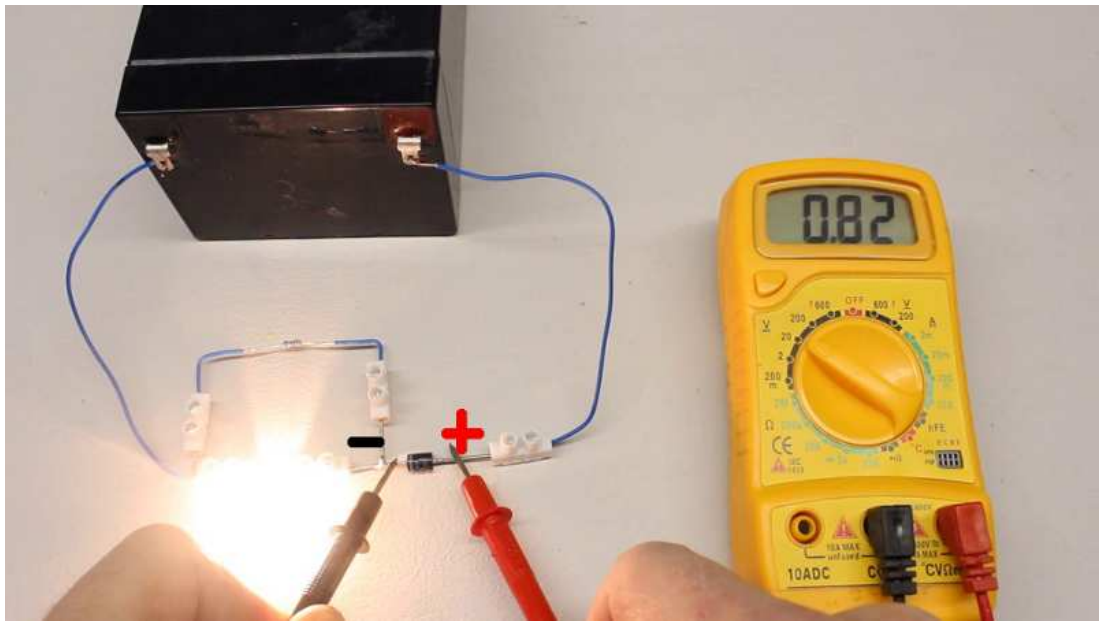

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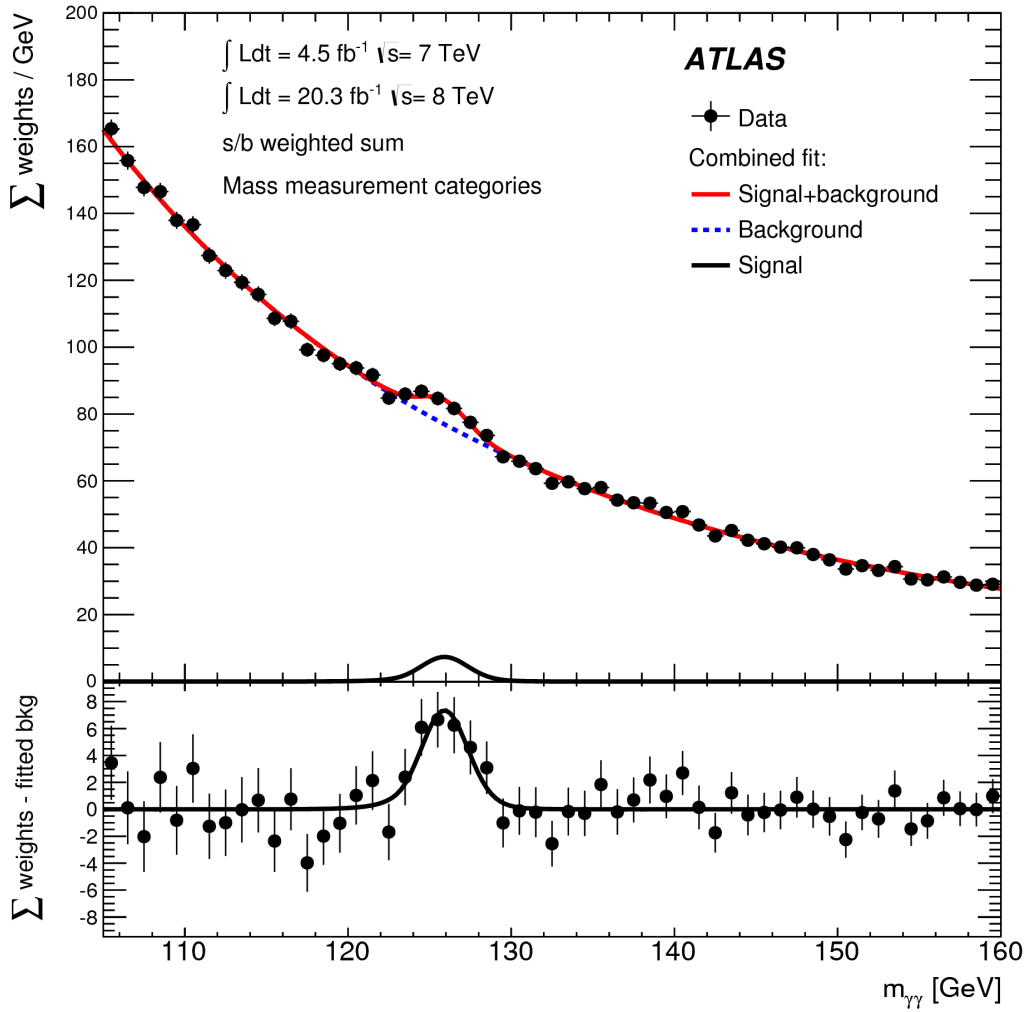
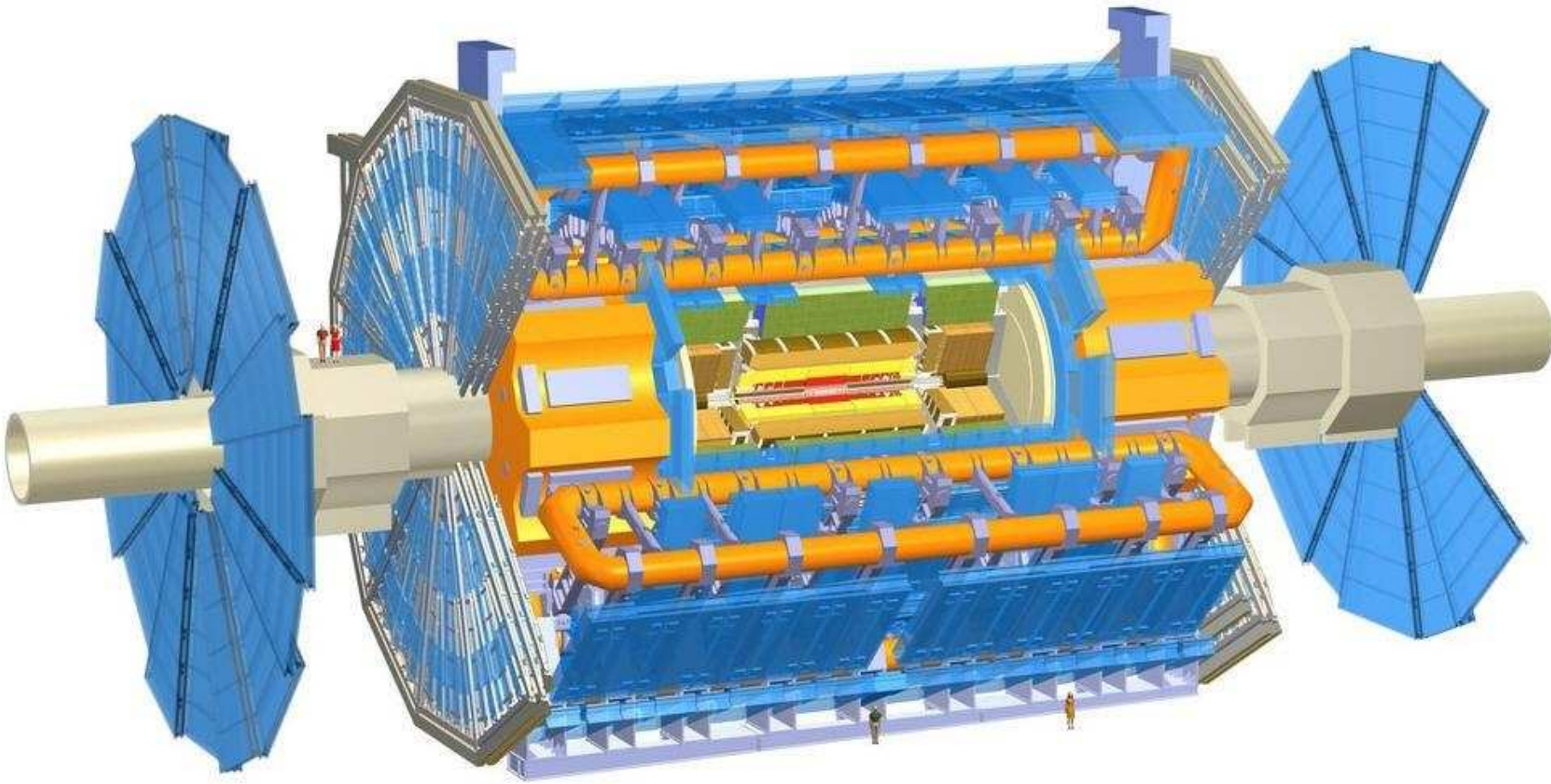
Higgs $\to \gamma\gamma$ (2012)



Two-photon *invariant mass*

What is measurement?

ATLAS Experiment at LHC (CERN, Geneva)



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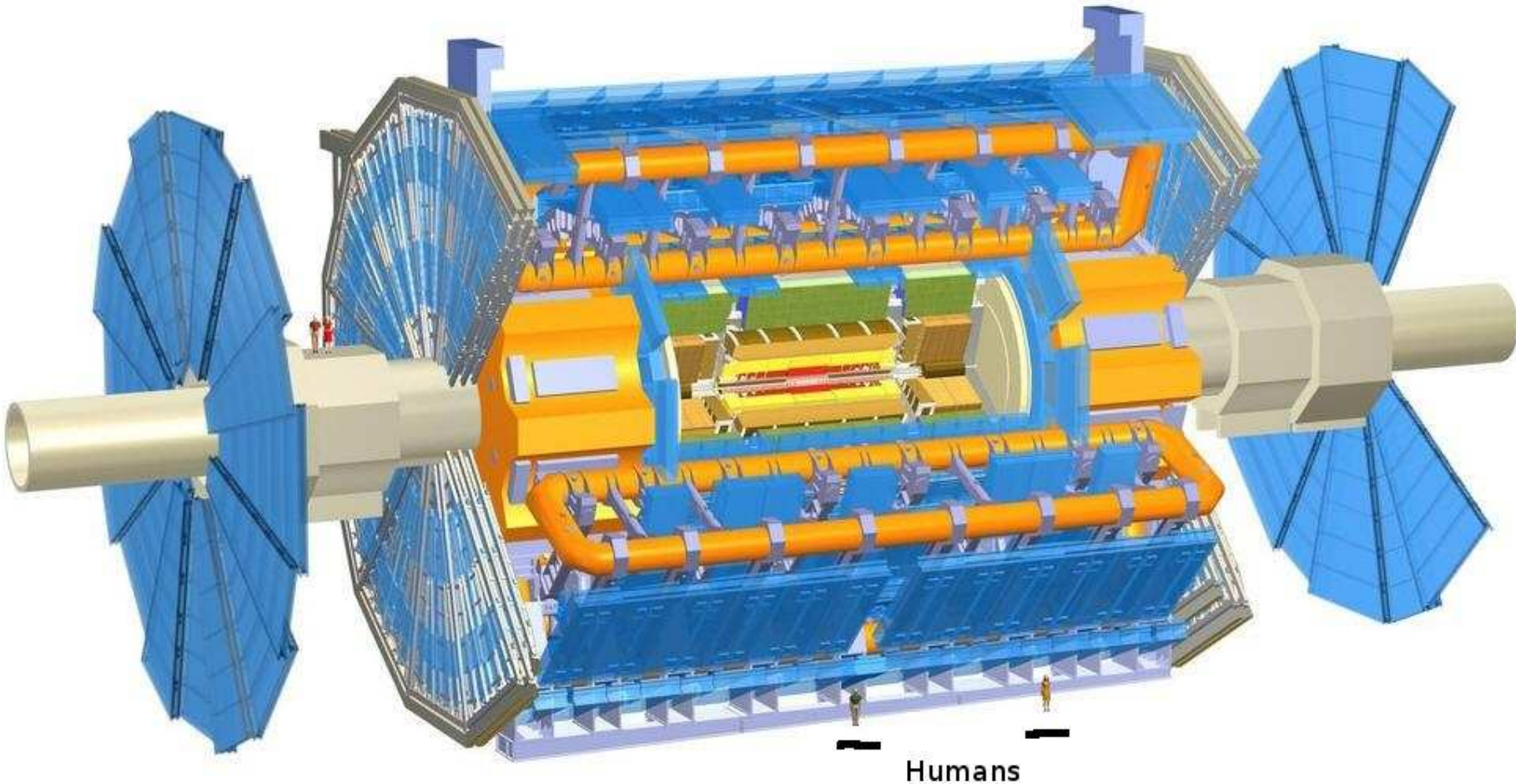
ATLAS Experiment at LHC [length: 46 m; ⌀ 25 m]



≈ 3000 km cables

≈ 7000 tonnes

≈ 100 millions electronic channels

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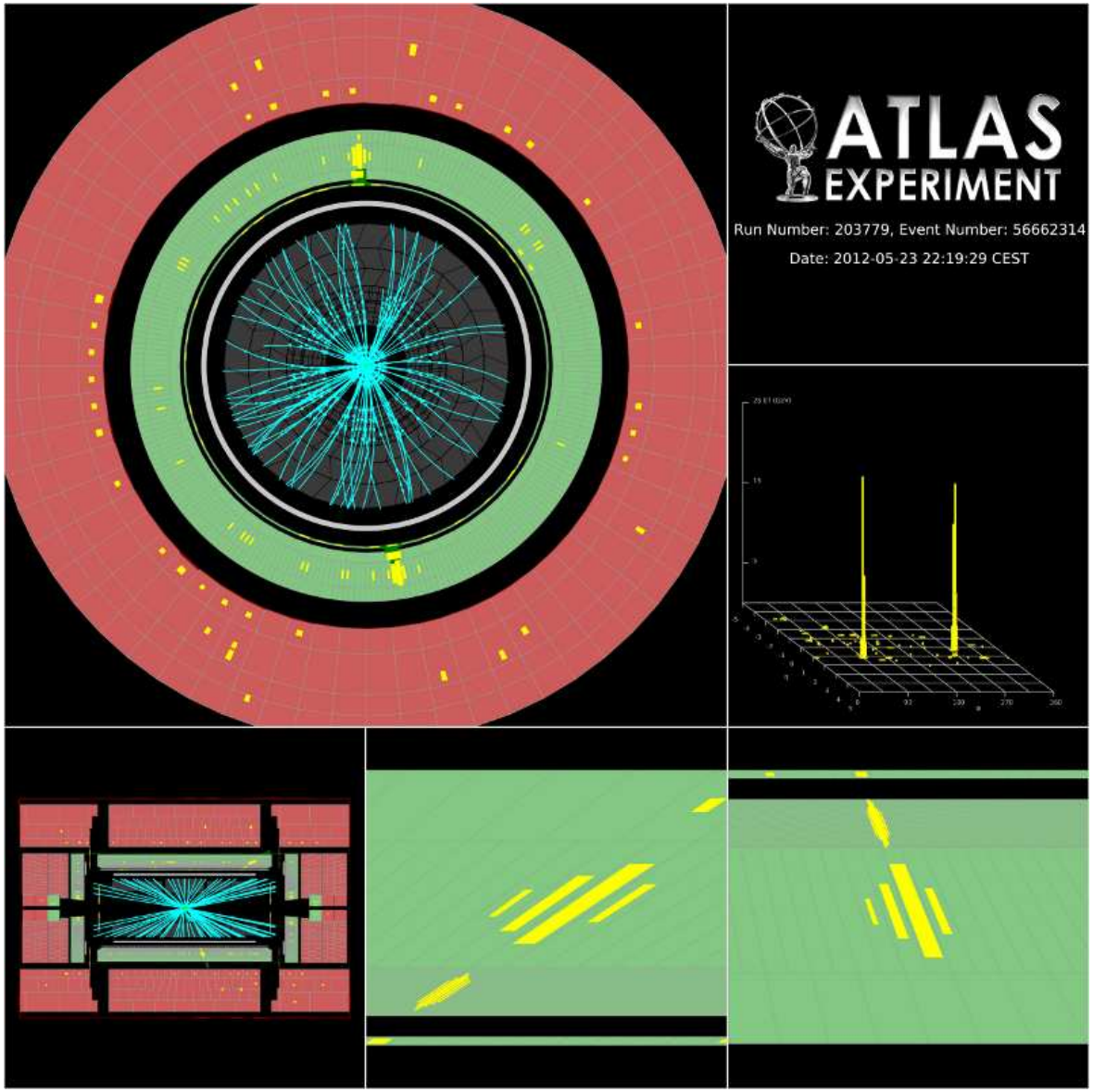

Two flashes of 'light' (2 γ's) in a 'noisy' environment.

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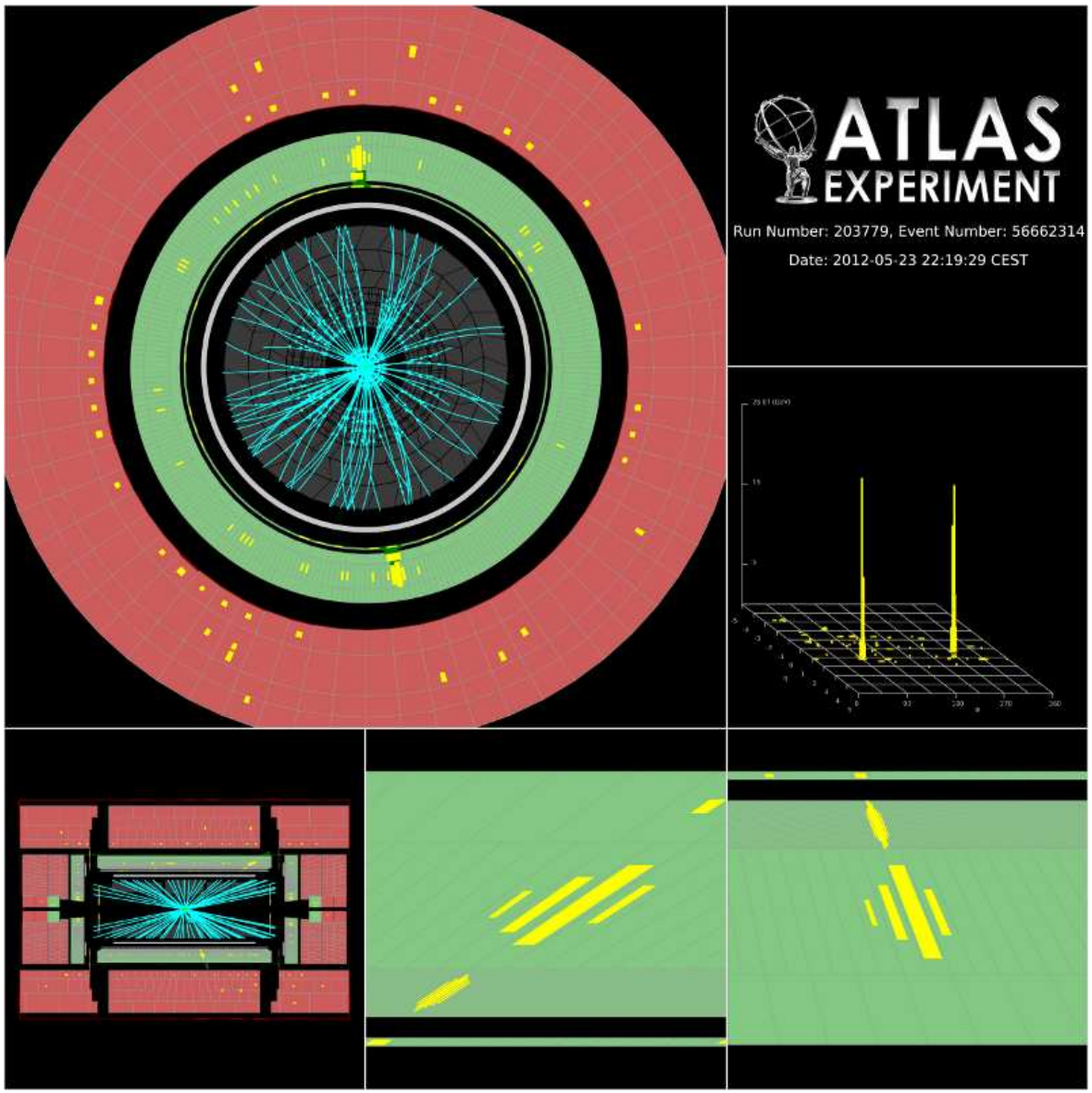

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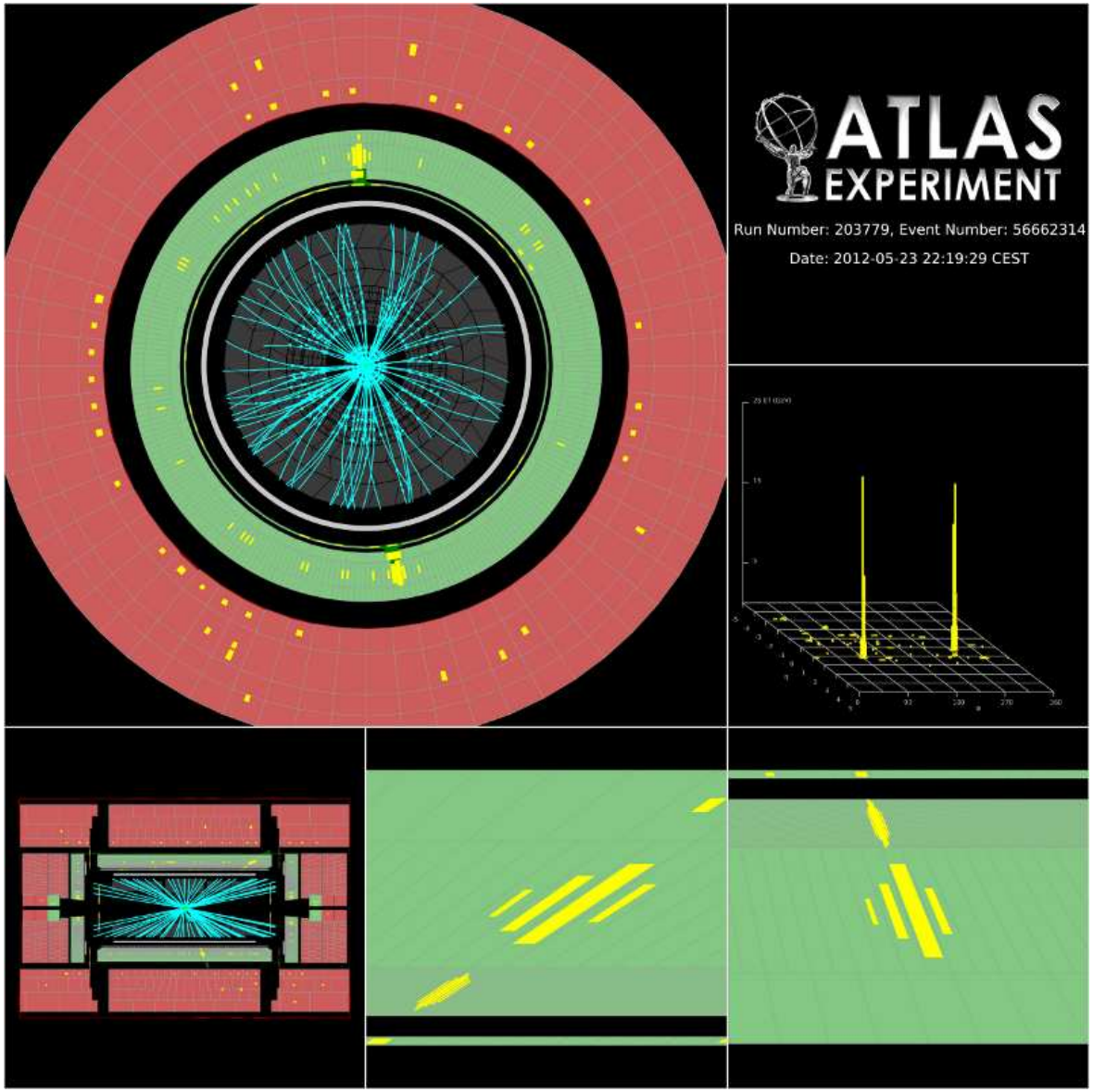

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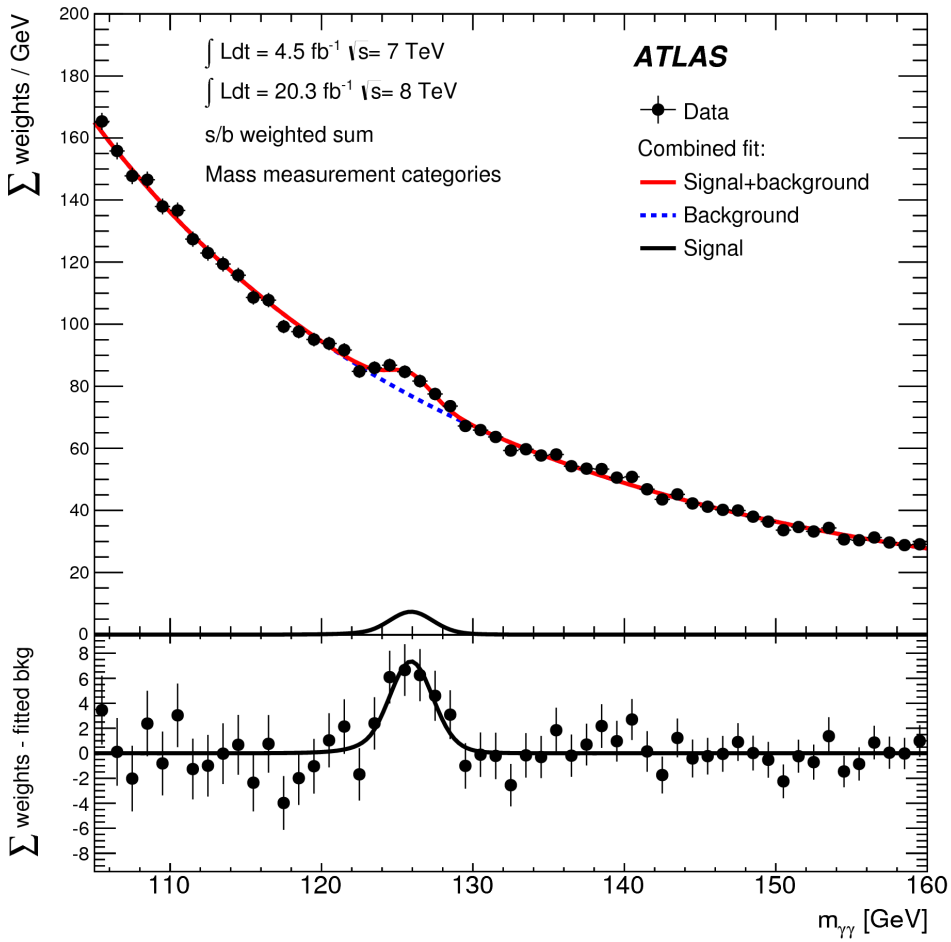

$\Rightarrow$ { Mass value
Production rate

What is measurement?

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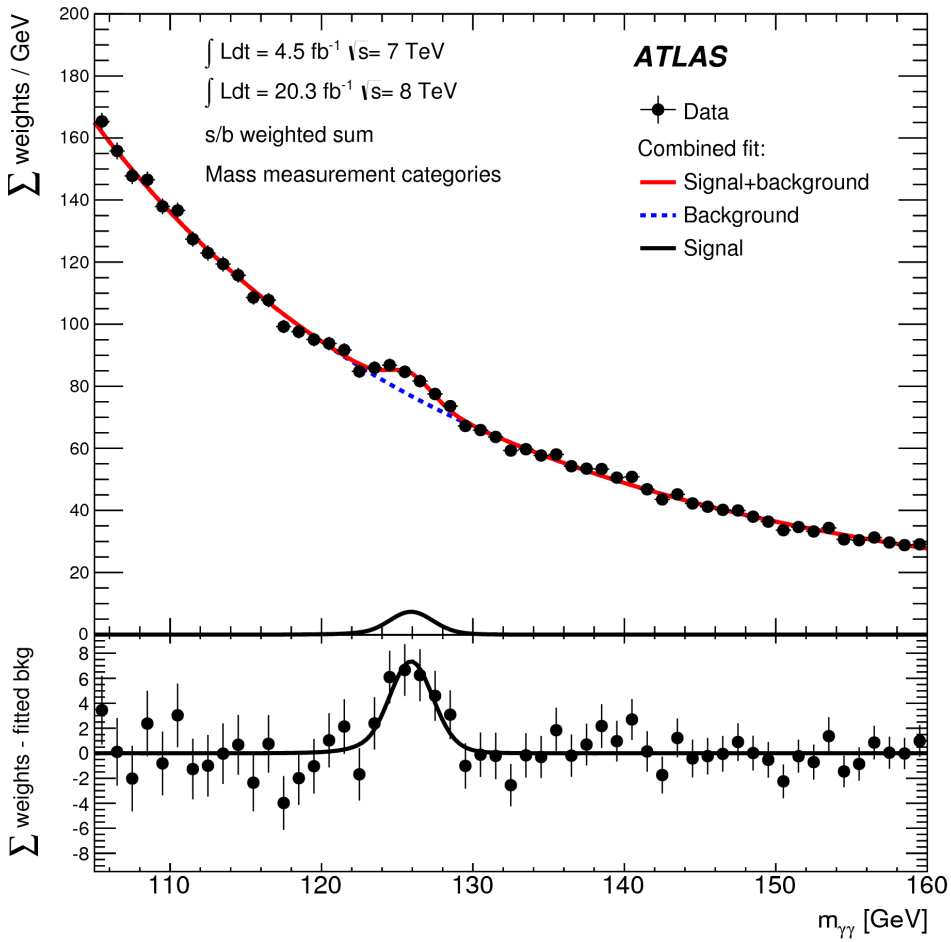

$\Rightarrow$ { Mass value, Production rate (with uncertainties)

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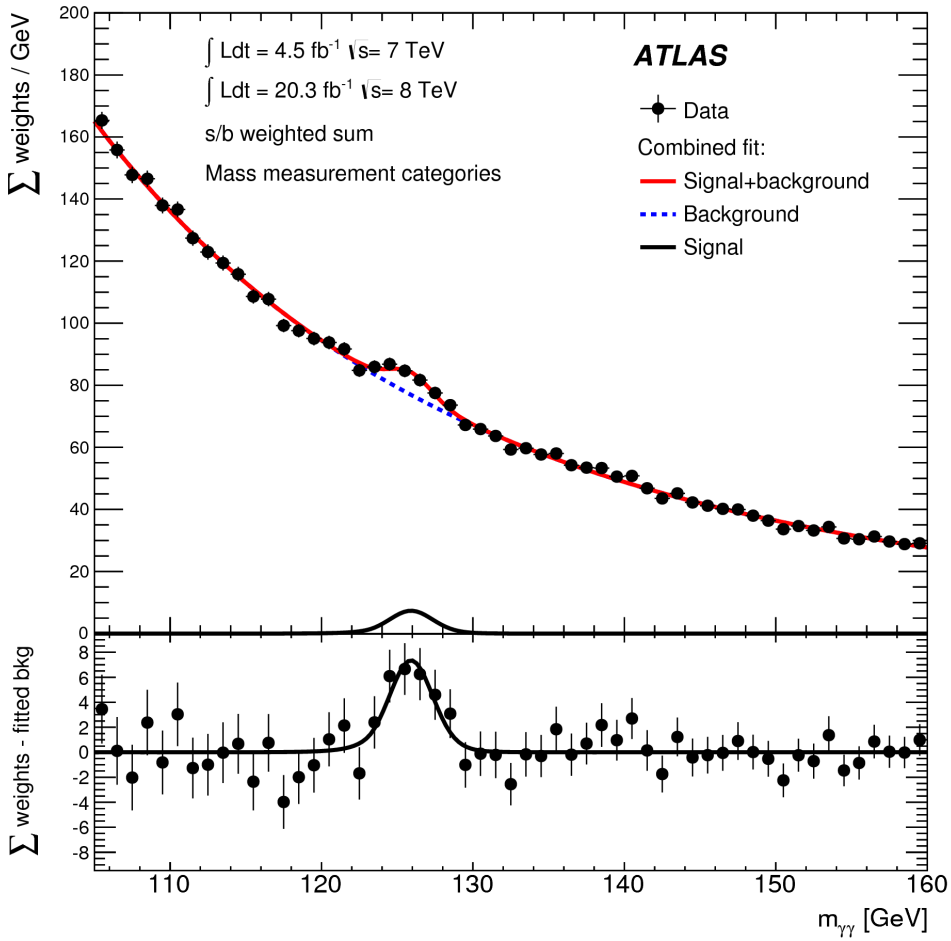

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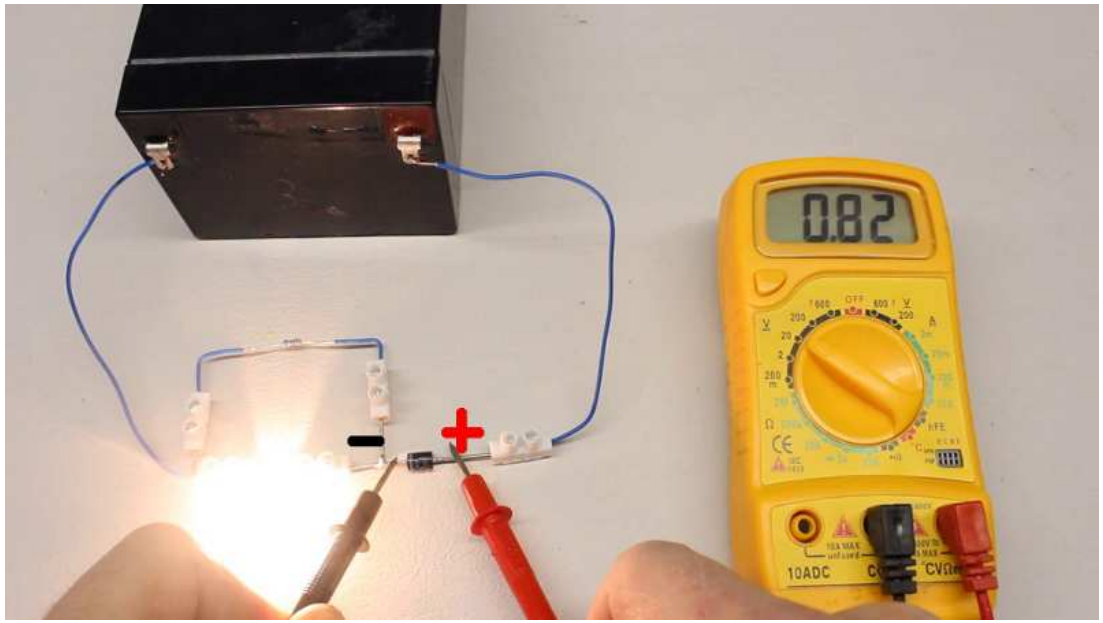
Quite indirect measurements of something we do not "see"!

Can we “see” physics quantities?

But, can we see our mass?

Can we “see” physics quantities?

. . . or a voltage?



Can we "see" physics quantities?

. . . or our blood pressure?

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Certainly not!

Can we "see" physics quantities?

Certainly not!

. . . although for some quantities we can have

a 'vivid impression' (in the David Hume's sense)

Measuring a mass on a scale

Equilibrium:

$$\begin{aligned} mg - k\Delta x &= 0 \\ \Delta x &\rightarrow \theta \rightarrow \text{scale reading} \end{aligned}$$

(with 'g' *gravitational acceleration;* 'k' spring constant.)

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From the reading to the value of the mass:

$$\text{scale reading} \xrightarrow[\text{given } g,\ k,\ \text{“etc.”}\ldots]{} m$$

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- Position is usually not at “$R_{\oplus}$” from the Earth center;
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- ...not even ellipsoidal...
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- Moreover we have to consider centrifugal effects
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But think about it!

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$\Rightarrow$ m??

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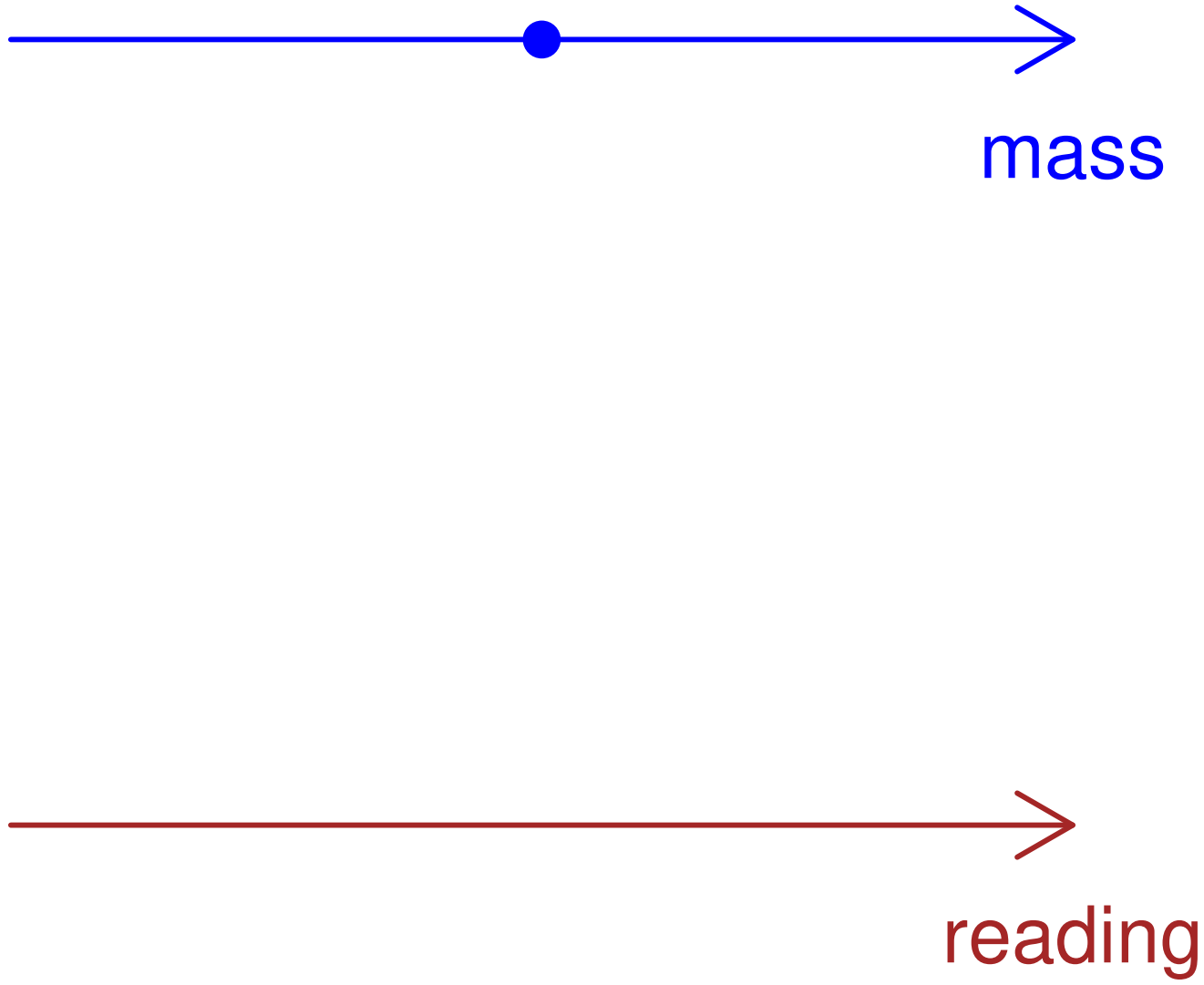

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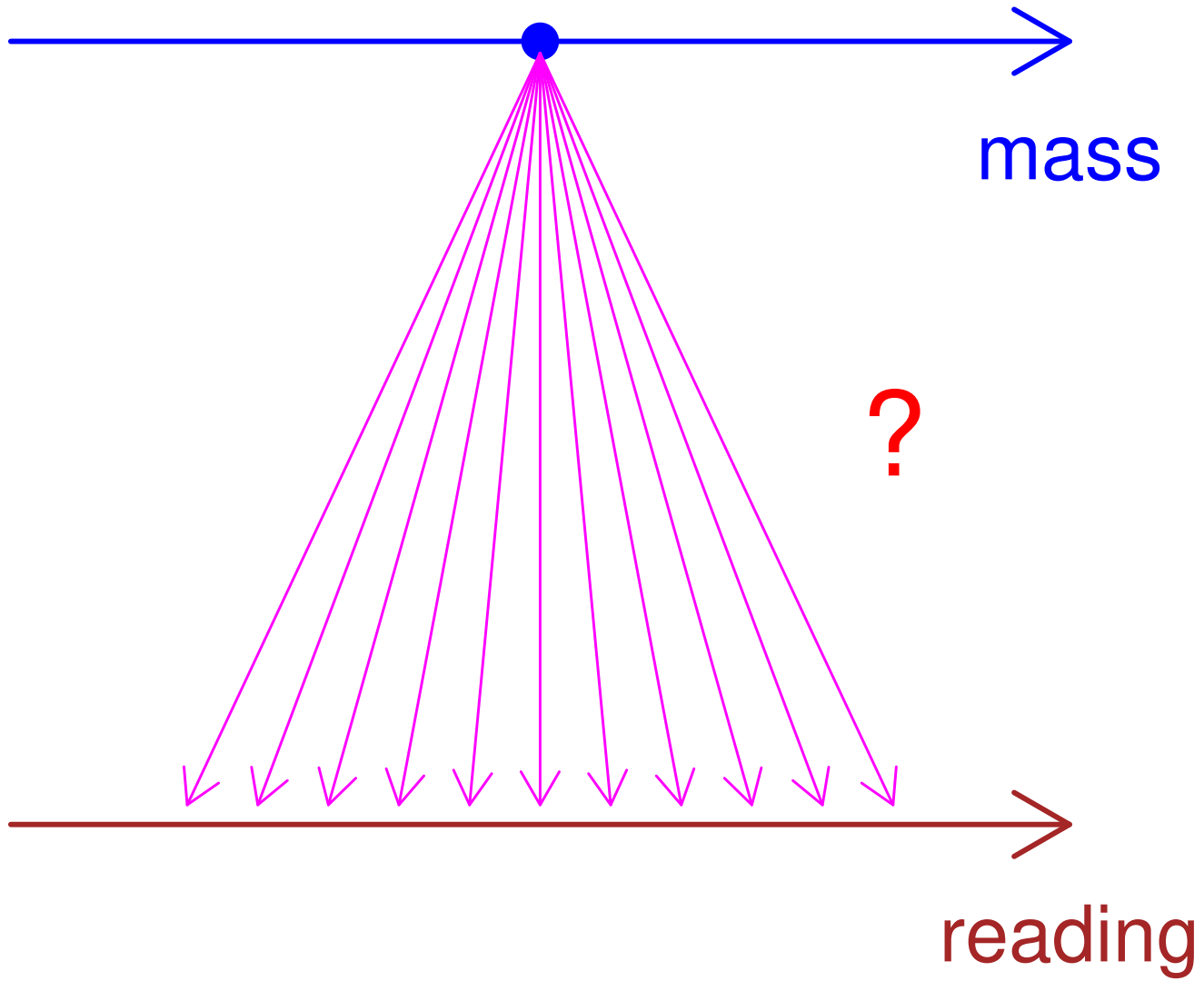

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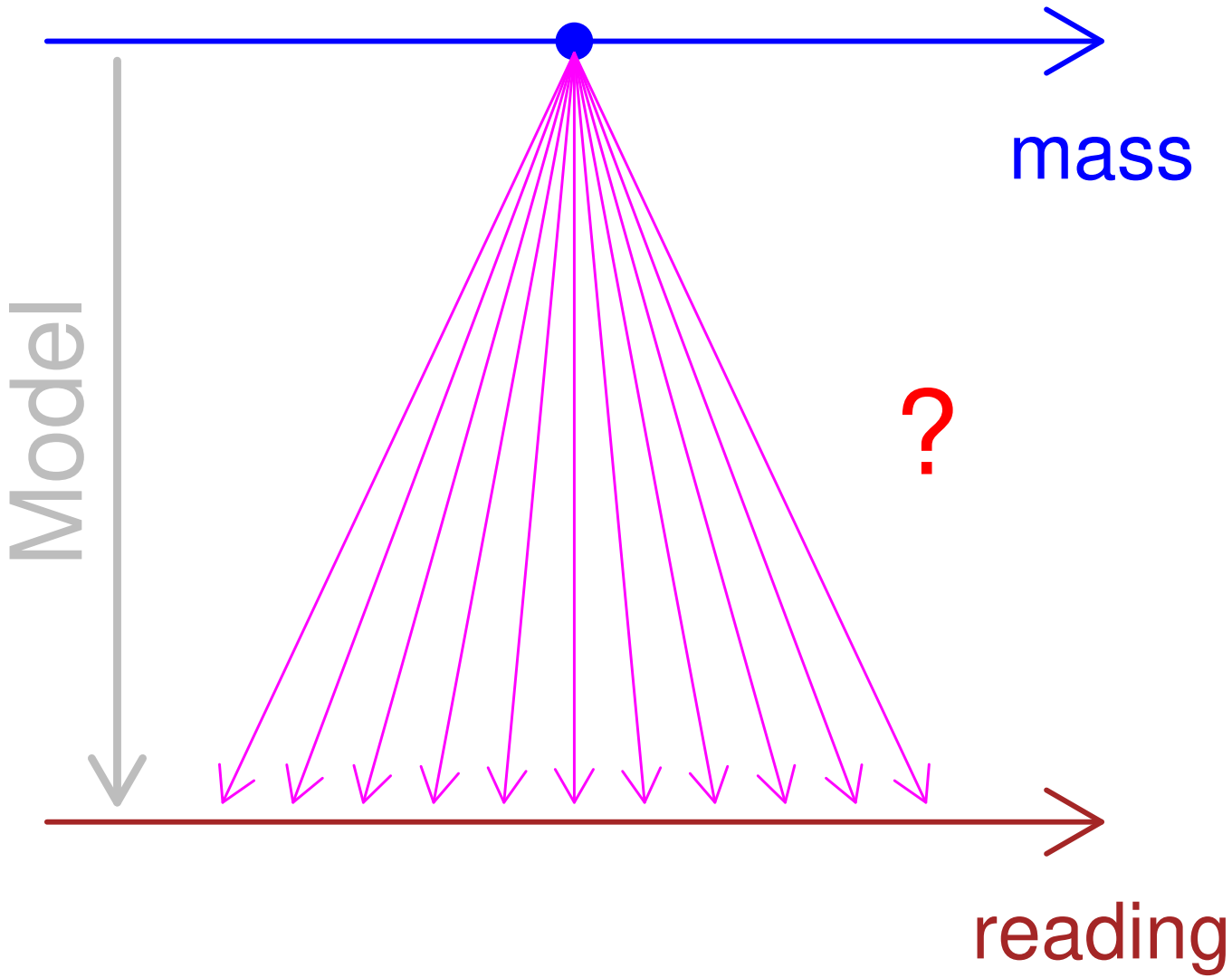

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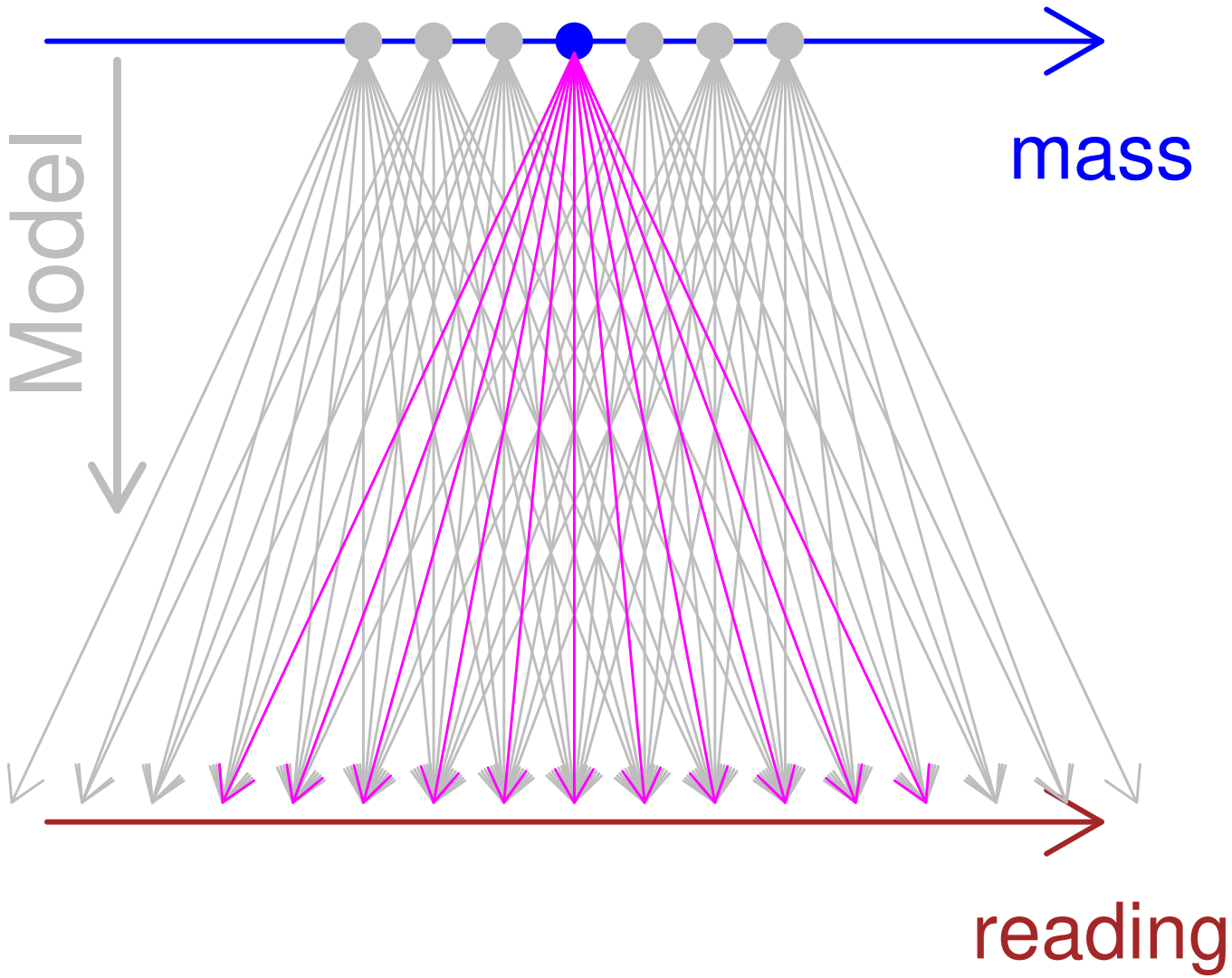

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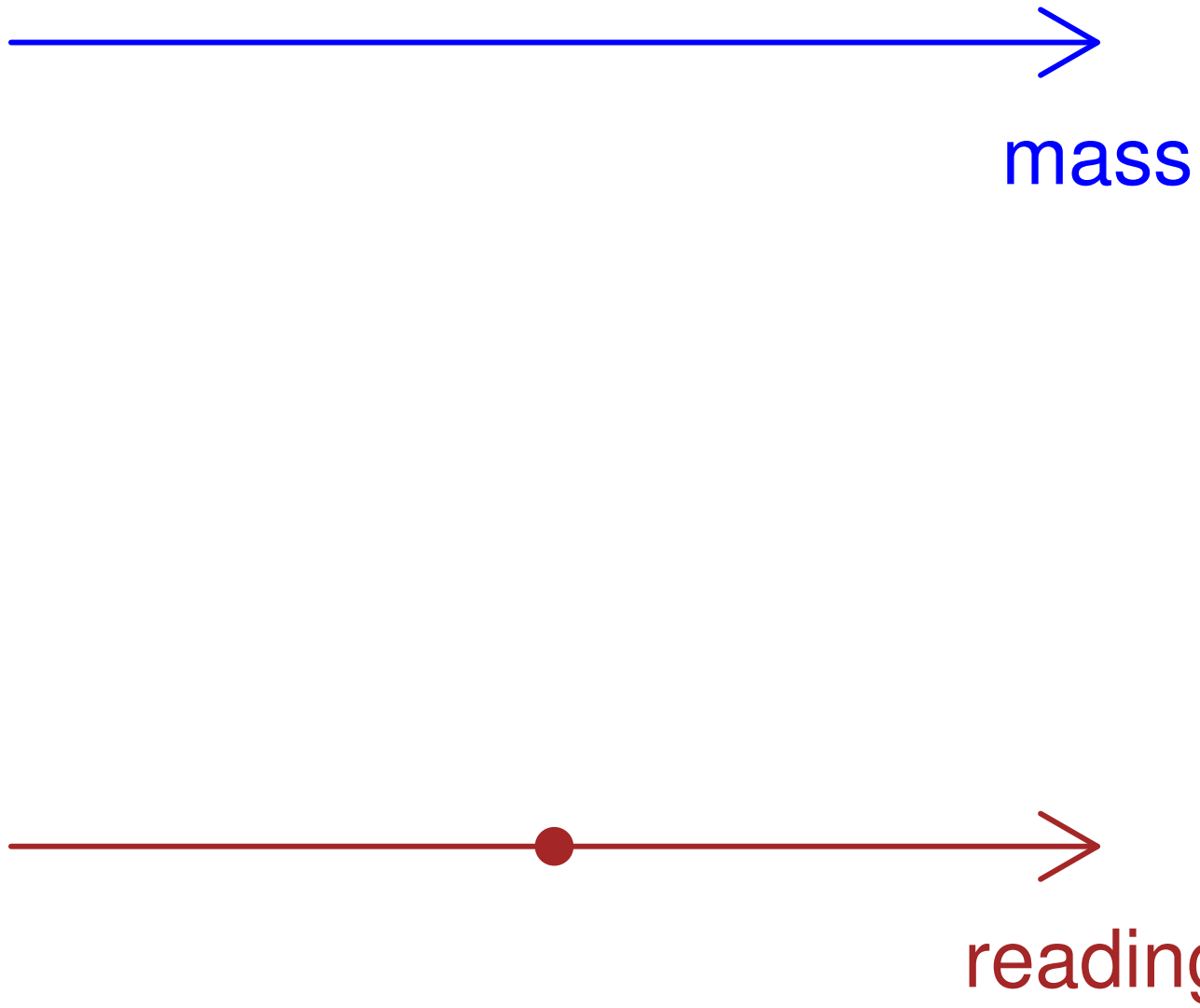

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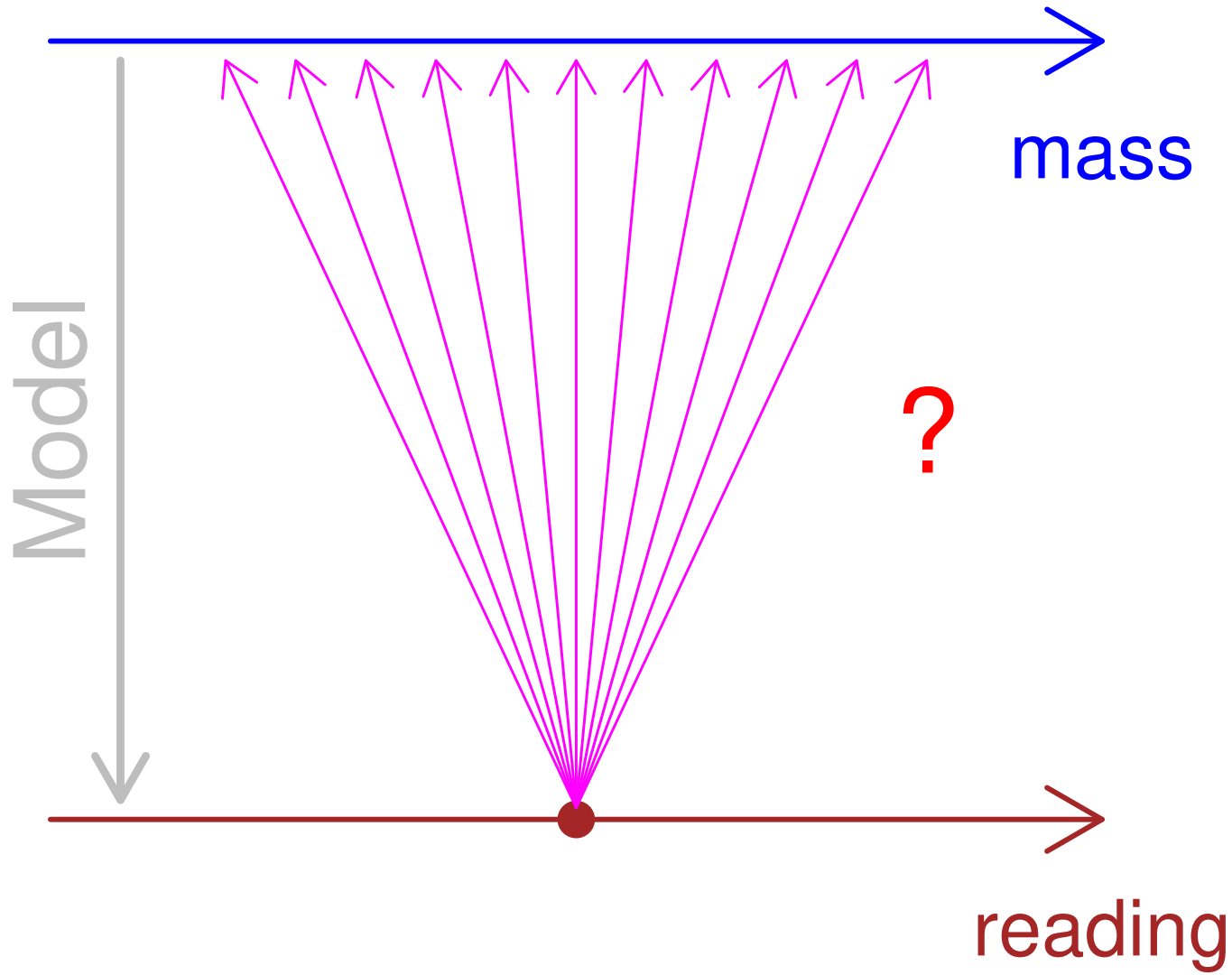

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5. *personal bias in reading analogue instruments;*

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Note

- Sources not necessarily independent
- In particular, sources 1-9 may contribute to 10 (e.g. not-monitored electric fluctuations)

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Error and *uncertainty* are not synonyms!

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