Details of the calculations

$$f(\mu \,|\, \underline{x}, \underline{\sigma}, f_0(\mu) = k) \quad \propto \quad \prod_i \exp\left[-\frac{(x_i-\mu)^2}{2\,\sigma_i^2}\right]$$

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$$\propto\; \exp\left[-\sum_i \frac{(x_i - \mu)^2}{2\,\sigma_i^2}\right]$$

Details of the calculations

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\begin{aligned}
f(\mu \,|\, \underline{x}, \underline{\sigma}, f_0(\mu) = k) &\propto \prod_i \exp\left[-\frac{(x_i-\mu)^2}{2\,\sigma_i^2}\right] \\
&\propto \exp\left[-\sum_i \frac{(x_i-\mu)^2}{2\,\sigma_i^2}\right] \\
&\propto \exp\left[-\frac{1}{2}\sum_i \frac{x_i^2 - 2x_i\mu + \mu^2}{\sigma_i^2}\right]
\end{aligned}
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Details of the calculations

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f(\mu \,|\, \underline{x}, \underline{\sigma}, f_0(\mu) = k) &\propto \prod_i \exp\left[-\frac{(x_i-\mu)^2}{2\,\sigma_i^2}\right] \\
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&\propto \exp\left[-\frac{1}{2}\sum_i \left(\frac{x_i^2}{\sigma_i^2} - 2\,\frac{x_i}{\sigma_i^2}\,\mu + \frac{\mu^2}{\sigma_i^2}\right)\right]
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$$

$$
\propto \;\; \exp\left[-\frac{1}{2}\cdot\frac{\sum_i 1/\sigma_i^2}{\sum_i 1/\sigma_i^2}\cdot\left(\sum_i \frac{x_i^2}{\sigma_i^2} - 2\left(\sum_i \frac{x_i}{\sigma_i^2}\right)\mu + \left(\sum_i \frac{1}{\sigma_i^2}\right)\mu^2\right)\right]
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Details of the calculations

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$$\propto\; \exp\left[-\frac{1}{2}\sum_i \left(\frac{x_i^2}{\sigma_i^2} - 2\,\frac{x_i}{\sigma_i^2}\,\mu + \frac{\mu^2}{\sigma_i^2}\right)\right]$$

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$$\propto\; \exp\left[-\frac{1}{2}\cdot\left(\sum_i 1/\sigma_i^2\right)\cdot\left(\overline{x^2} - 2\,\overline{x}\,\mu + \mu^2\right)\right]$$

Details of the calculations

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f(\mu \,|\, \underline{x}, \underline{\sigma}, f_0(\mu) = k) \;&\propto\; \prod_i \exp\left[-\frac{(x_i-\mu)^2}{2\,\sigma_i^2}\right] \\
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\propto\; \exp\left[-\frac{1}{2}\cdot\frac{\sum_i 1/\sigma_i^2}{\sum_i 1/\sigma_i^2}\cdot\left(\sum_i \frac{x_i^2}{\sigma_i^2} - 2\left(\sum_i \frac{x_i}{\sigma_i^2}\right)\mu + \left(\sum_i \frac{1}{\sigma_i^2}\right)\mu^2\right)\right]
$$

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$$

$$
\propto\; \exp\left[-\frac{\overline{x^2} - 2\,\overline{x}\,\mu + \mu^2}{2/(\sum_i 1/\sigma_i^2)}\right]
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Details of the calculations

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$$

$$
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$$

$$
\propto \exp\left[-\frac{\overline{x^2} - 2\,\overline{x}\,\mu + \mu^2}{2/(\sum_i 1/\sigma_i^2)}\right] \propto \exp\left[-\frac{-2\,\overline{x}\,\mu + \mu^2}{2\,\sigma_C^2}\right]
$$

Inferring μ from n 'independent' measurements

$$f(\mu \,|\, \underline{x}, \underline{\sigma}, f_0(\mu) = k) \quad \propto \quad \exp\left[-\frac{-2\,\overline{x}\,\mu + \mu^2}{2\,\sigma_C^2}\right]$$

Inferring μ from n 'independent' measurements

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\begin{aligned}
f(\mu \,|\, \underline{x}, \underline{\sigma}, f_0(\mu) = k) &\propto \exp\left[-\frac{-2\,\overline{x}\,\mu + \mu^2}{2\,\sigma_C^2}\right] \\
&\propto \exp\left[-\frac{\overline{x}^2 - 2\,\overline{x}\,\mu + \mu^2}{2\,\sigma_C^2}\right]
\end{aligned}
$$

Inferring μ from n 'independent' measurements

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\begin{aligned}
f(\mu \,|\, \underline{x}, \underline{\sigma}, f_0(\mu) = k) &\propto \exp\left[-\frac{-2\,\overline{x}\,\mu + \mu^2}{2\,\sigma_C^2}\right] \\
&\propto \exp\left[-\frac{\overline{x}^2 - 2\,\overline{x}\,\mu + \mu^2}{2\,\sigma_C^2}\right] \\
&\propto \exp\left[-\frac{(\mu - \overline{x})^2}{2\,\sigma_C^2}\right]
\end{aligned}
$$

(having used the technique of complementing the exponential) with

$$
\begin{aligned}
\overline{x} &= \frac{\sum_i x_i/\sigma_i^2}{\sum_i 1/\sigma_i^2} \\
\overline{x^2} &= \frac{\sum_i x_i^2/\sigma_i^2}{\sum_i 1/\sigma_i^2} \\
\sigma_C^2 &= \frac{1}{\sum_i 1/\sigma_i^2}
\end{aligned}
$$