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In particular, if $\sigma_p = \sigma_f = \sigma$, then

$$f(x_f\,|\,x_p, \sigma_p = \sigma_f = \sigma) = \frac{1}{\sqrt{2\,\pi}\,\sqrt{2}\,\sigma}\,\exp\left[-\frac{(x_f-x_p)^2}{2\,(\sqrt{2}\,\sigma)^2}\right]$$