Bayesian Reasoning in Physics: Prinicples and Applications

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This nine-hour-long tutorial will be presented at LANL, March 1-3, 1999. D'Agostini gave this course at CERN in May, 1998. His extensive course notes may be found on the web Course Notes. Other papers on Bayesian analysis in physics may be found on his own web site: web site.

- The course will consist of three one and one half hour tutorials per day. Each tutorial will be followed by an hour and one half of exercises and discussion
- Addressed to physicists, experimental, as well theoretical.
- Background assumed: familiarity with "standard" probability and statistics (however, important concepts will be recalled)

INTRODUCTION

- Criticism of the ``usual'' methods for the evaluation of measurement uncertainty and for comparison of physical hypotheses.
- Comparison of the different approaches to probability: ``classical'', frequentist, logical and subjective.
- Probability as ``degree of belief'': odds in betting, penalization rule and coherence.
- Role and meaning of conditional probability.
- Bayes' theorem: standard and Bayesian use.

REVIEW OF CONCEPTS CONCERNING RANDOM VARIABLES

- Distributions of discrete and continuous variables.
- Mean and variance.
- Binomial, Poisson, uniform, normal and exponential distributions.
- Several variables: marginal and conditional distribution;
Bayes' theorem for continuous variables;
covariance and correlation coefficient;
bivariate normal distribution: terms, applications and cautions.
- Central limit theorem.

STATISTICAL INFERENCE

- Credibility intervals vs. confidence interval: example of normally distributed observable.
- Subjective interpretation of (frequentist) confidence intervals.
- BIPM/ISO recommendation on the measurement uncertainty.
- Meaning of the frequentist hypothesis tests and common misunderstanding.
- Bayesian inference and Bayesian theory of measurement uncertainty.
- Choice of priors and likelihoods.
- Bertrand paradox, Maximum Entropy and subjective priors.
- Comparison of Bayesian and Maximum Likelihood methods.

APPLICATIONS

1) Gaussian likelihood:
- inference of the mean;
- dependence of results from the choice of the prior;
- combination of results;
- combination of "incompatible" results;
- measurements at the edge of the physical region.
- combination of uncertainty due to statistical and systematic errors:
- types of uncertainty: scale offset; global uncertainty; correlations among several measurements; multiple calibrations; case of known systematic error;
- general case.

3) Binomial likelihood:
- general solution: f(p|x);
- physical and probabilistic interpretation of "p": concept of exchangeability;
- limit to normal;
- special cases: f(p|x=0) and f(p|x=n): upper and lower limits.

4) Poisson likelihood:
- general solution: f(lambda|x) and limit to normal;
- special case: f(lambda|x=0) and upper limits;
- dependence of the limits from the prior;
- combination of upper limits;
- counting experiments in presence of background.

5) Linearization:
- estimators of the true values and of their linear correlations;
- combination of uncertainty due to not exactly known systematic errors;
- correlations introduced by not exactly known systematic errors;
- Type A and Type B uncertainty (BIPM/ISO);
- evaluation of Type B uncertainties;

- propagation of measurement uncertainties;
- possible problems caused by improper use of linearizations and of other approximated methods;
- use and misuse of the covariance matrix to fit correlated data.

6) Unfolding experimental distributions:
- naive approach: bin to bin correction;
- mention some unfolding methods;
- Bayesian unfolding.

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