Inferring vaccine efficacies and their uncertainties
A simple model implemented in JAGS/rjags

(Based on a work with Alfredo Esposito)

Giulio D'Agostini

Università di Roma La Sapienza e INFN
Roma, Italy

Inferring vaccine efficacies and their uncertainties
A simple model implemented in JAGS/rjags

(Based on a work with Alfredo Esposito)

Giulio D'Agostini

Università di Roma La Sapienza e INFN
Roma, Italy

"It is scientific only to say what is more likely
and what is less likely"

(R. Feynman)

Preamble

- The very reason of this ‘exercise’ has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.

Preamble

- The very reason of this ‘exercise’ has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)

Preamble

- The very reason of this ‘exercise’ has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty

Preamble

- The very reason of this 'exercise' has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty is not a scientific result.

Preamble

- The very reason of this 'exercise' has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty is not a scientific result.
- In absence of detailed information, we are used to get an idea of the uncertainty by the *rounding of the result*.

Preamble

- The very reason of this 'exercise' has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty is not a scientific result.
- In absence of detailed information, we are used to get an idea of the uncertainty by the *rounding of the result*.
  - providing the third decimal digit implies an uncertainty of that order of magnitude

Preamble

- The very reason of this 'exercise' has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty is not a scientific result.
- In absence of detailed information, we are used to get an idea of the uncertainty by the *rounding of the result*.
  - providing the third decimal digit implies an uncertainty of that order of magnitude ($\pm 0.1\%$, ..., $\pm 0.3\%$, ..., $\pm 0.5\%$, ...)

Preamble

- The very reason of this 'exercise' has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty is not a scientific result.
- In absence of detailed information, we are used to get an idea of the uncertainty by the *rounding of the result*.
  - providing the third decimal digit implies an uncertainty of that order of magnitude ($\pm 0.1\%$, ..., $\pm 0.3\%$, ..., $\pm 0.5\%$, ...)
  - a simple exercise showed that such a high accuracy would imply a number of infected in the vaccine group ranging from hundreds to thousands.

Preamble

- The very reason of this 'exercise' has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty is not a scientific result.
- In absence of detailed information, we are used to get an idea of the uncertainty by the *rounding of the result*.
  - providing the third decimal digit implies an uncertainty of that order of magnitude ($\pm 0.1\%, \ldots, \pm 0.3\%, \ldots, \pm 0.5\%, \ldots$)
  - a simple exercise showed that such a high accuracy would imply a number of infected in the vaccine group ranging from hundreds to thousands. ???

Preamble

- The very reason of this 'exercise' has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty is not a scientific result.
- In absence of detailed information, we are used to get an idea of the uncertainty by the *rounding of the result*.
  - providing the third decimal digit implies an uncertainty of that order of magnitude ($\pm 0.1\%$, ..., $\pm 0.3\%$, ..., $\pm 0.5\%$, ...)
  - a simple exercise showed that such a high accuracy would imply a number of infected in the vaccine group ranging from hundreds to thousands. ???
- Then, when we read that they were only 5, a rough calculation based on physicists $\sqrt{n}$ rule of thumb gave us a standard uncertainty of $\approx 2\%$.

Preamble

- The very reason of this 'exercise' has been listening/reading in the media the 94.5% vaccine efficacy by Moderna.
  (To make it clear, we had never done it if we had just heard of the 95% result of Pfizer.)
- Because of education and profession, for us a result not accompanied by an uncertainty is not a scientific result.
- In absence of detailed information, we are used to get an idea of the uncertainty by the *rounding of the result*.
  - providing the third decimal digit implies an uncertainty of that order of magnitude ($\pm 0.1\%, \ldots, \pm 0.3\%, \ldots, \pm 0.5\%, \ldots$)
  - a simple exercise showed that such a high accuracy would imply a number of infected in the vaccine group ranging from hundreds to thousands. ???
- Then, when we read that they were only 5, a rough calculation based on physicists $\sqrt{n}$ rule of thumb gave us a standard uncertainty of $\approx 2\%$.
- At the beginning we thought we could not do better, due to the limited data, but indeed *we succeded*.

Outline

- Measurements and related uncertainty.

Outline

- Measurements and related uncertainty.
- Importance of the models:

Outline

- Measurements and related uncertainty.
- Importance of the models:
  - measuring is nothing but inferring parameters of models;

Outline

- Measurements and related uncertainty.
- Importance of the models:
  - measuring is nothing but inferring parameters of models;
  - at the end of the game, there will be values of parameters we believe more and values we believe less

Outline

- Measurements and related uncertainty.
- Importance of the models:
  - measuring is nothing but inferring parameters of models;
  - at the end of the game, there will be values of parameters we believe more and values we believe less

    "It is scientific only to say what is more likely and what is less likely" *(R. Feynman)*

Outline

- Measurements and related uncertainty.
- Importance of the models:
  - measuring is nothing but inferring parameters of models;
  - at the end of the game, there will be values of parameters we believe more and values we believe less

    "It is scientific only to say what is more likely and what is less likely" (R. Feynman)

  - Bayesian networks and MCMC machinery to handle them (details beyond the purpose of our work).

Outline

- Measurements and related uncertainty.
- Importance of the models:
  - measuring is nothing but inferring parameters of models;
  - at the end of the game, there will be values of parameters we believe more and values we believe less

    "It is scientific only to say what is more likely and what is less likely" *(R. Feynman)*

  - Bayesian networks and MCMC machinery to handle them (details beyond the purpose of our work).
  - Simple examples.

Outline

- Measurements and related uncertainty.
- Importance of the models:
  - measuring is nothing but inferring parameters of models;
  - at the end of the game, there will be values of parameters we believe more and values we believe less

    "It is scientific only to say what is more likely and what is less likely" *(R. Feynman)*

  - Bayesian networks and MCMC machinery to handle them (details beyond the purpose of our work).
  - Simple examples.
- **Simplified model** to treat the limited information in our hand

Outline

- Measurements and related uncertainty.
- Importance of the models:
  - measuring is nothing but inferring parameters of models;
  - at the end of the game, there will be values of parameters we believe more and values we believe less

    "It is scientific only to say what is more likely and what is less likely" (R. Feynman)

  - Bayesian networks and MCMC machinery to handle them (details beyond the purpose of our work).
  - Simple examples.
- **Simplified model** to treat the limited information in our hand (but nevertheless we are confident that it is ok, at least for the main result of interest $\rightarrow$ vaccine efficacy).

Outline

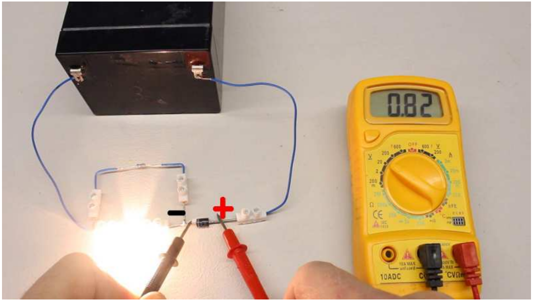
- Measurements and related uncertainty.
- Importance of the models:
  - measuring is nothing but inferring parameters of models;
  - at the end of the game, there will be values of parameters we believe more and values we believe less

    "It is scientific only to say what is more likely and what is less likely" (R. Feynman)

  - Bayesian networks and MCMC machinery to handle them (details beyond the purpose of our work).
  - Simple examples.
- **Simplified model** to treat the limited information in our hand (but nevertheless we are confident that it is ok, at least for the main result of interest $\rightarrow$ vaccine efficacy).
- Results and comparisons with Moderna and Pfizer claims.

What is measurement?

What is measurement?

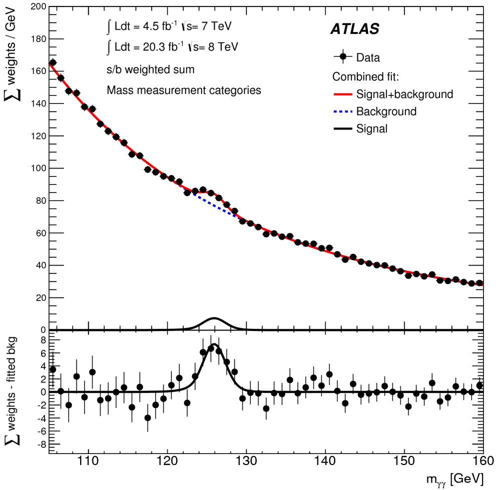


What is measurement?

What is measurement?

What is measurement?

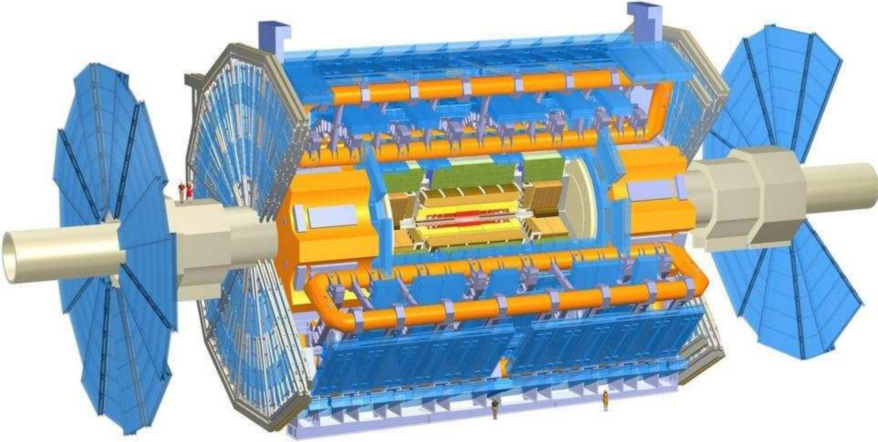
Higgs $\rightarrow \gamma\gamma$ (2012)



Two-photon *invariant mass*

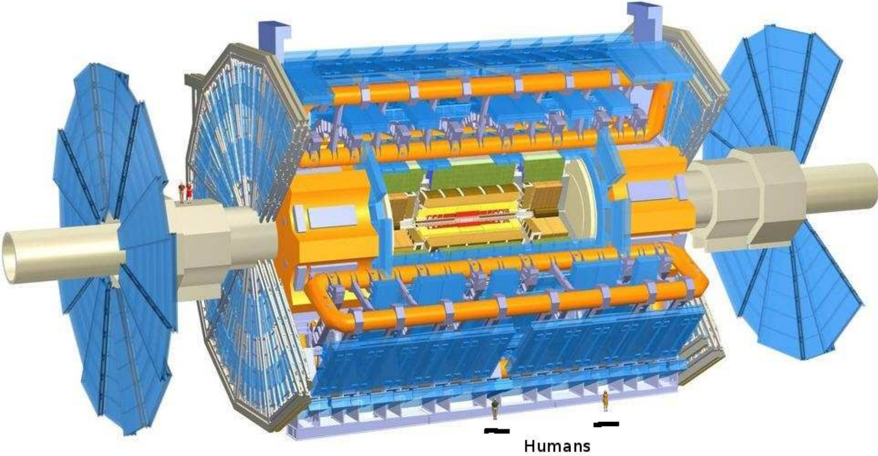
What is measurement?

ATLAS Experiment at LHC (CERN, Geneva)



What is measurement?

ATLAS Experiment at LHC [length: 46 m; ⌀ 25 m]

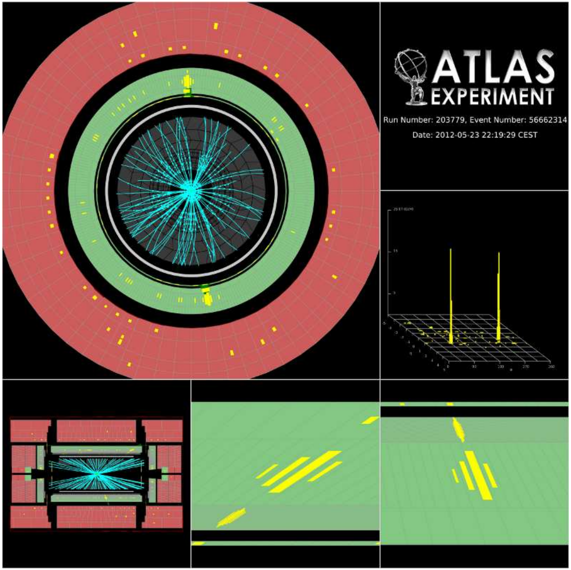


≈ 3000 km cables

≈ 7000 tonnes

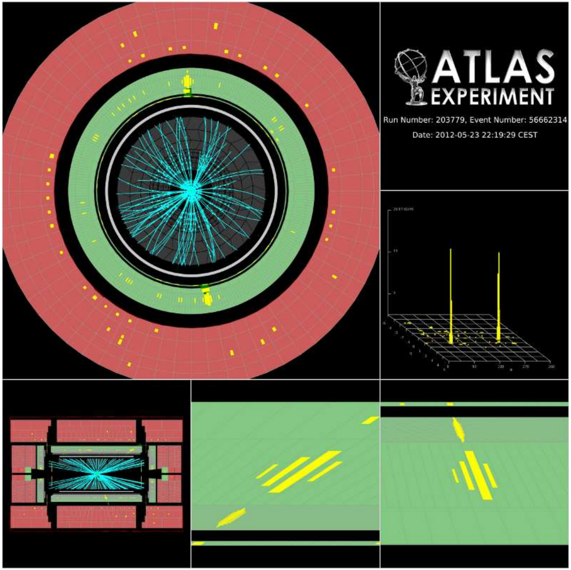
≈ 100 millions electronic channels

What is measurement?



Two flashes of ‘light’ (2 γ’s) in a ‘noisy’ environment.

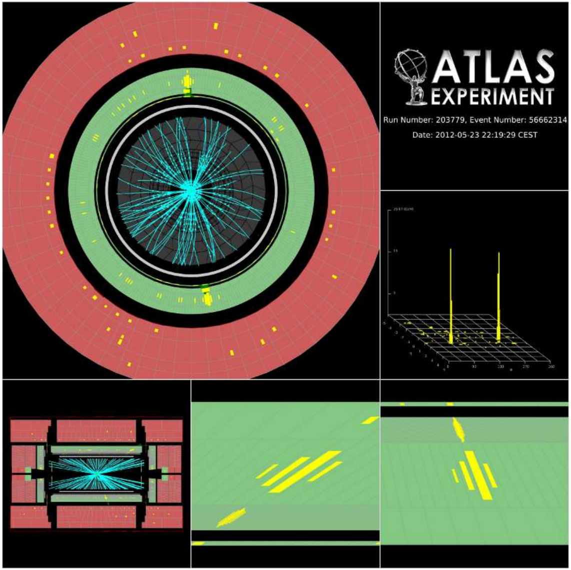
What is measurement?



Two flashes of 'light' (2 γ's) in a 'noisy' environment.

Higgs $\to \gamma\gamma$?

What is measurement?

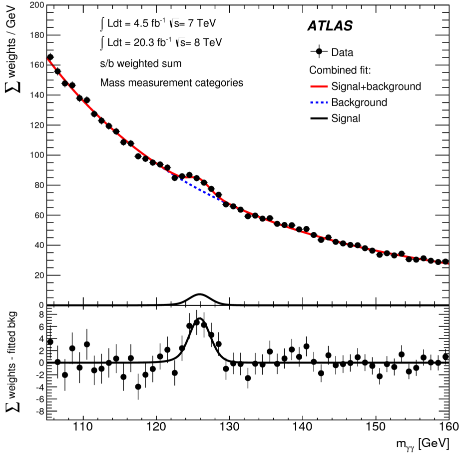


Two flashes of 'light' (2 γ's) in a 'noisy' environment.

Higgs $\rightarrow \gamma\gamma$? Probably not. . .

What is measurement?

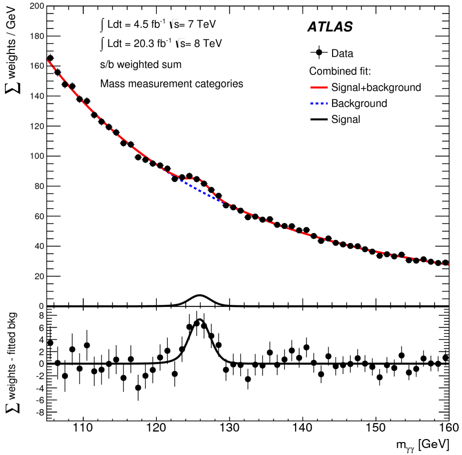
Higgs $\rightarrow \gamma\gamma$



$\Rightarrow \begin{cases} \text{Mass value} \\ \text{Production rate} \end{cases}$

What is measurement?

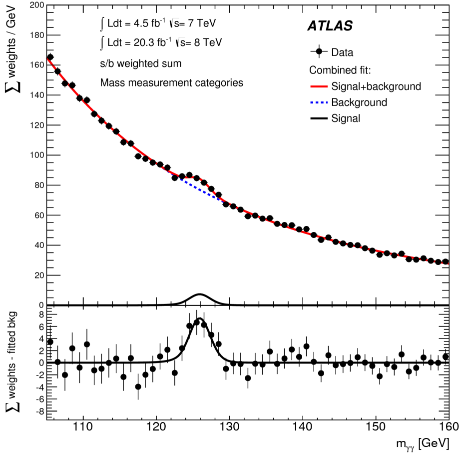
Higgs $\to \gamma\gamma$



$\Rightarrow$ Mass value
Production rate
(with uncertainties)

What is measurement?

Higgs $\rightarrow \gamma\gamma$



$\Rightarrow$ { Mass value
Production rate }
(with uncertainties)

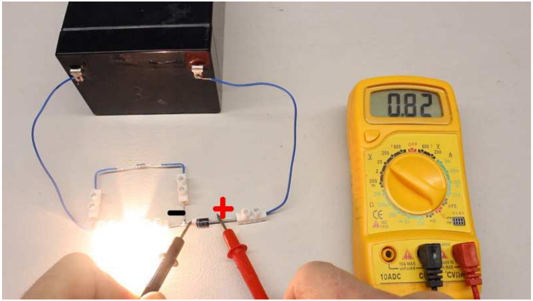
Quite indirect measurements of something we do not “see”!

Can we “see” physics quantities?

But, can we see our mass?

Can we "see" physics quantities?

. . . or a voltage?



Can we "see" physics quantities?

. . . or our blood pressure?

Can we “see” physics quantities?

Certainly not!

Can we “see” physics quantities?

Certainly not!

. . . although for some quantities we can have

a ‘vivid impression’ (in the David Hume’s sense)

Measuring a mass on a scale

Equilibrium:

$$\begin{aligned} mg - k\Delta x &= 0 \\ \Delta x &\rightarrow \theta \rightarrow \text{scale reading} \end{aligned}$$

(with 'g' *gravitational acceleration;* 'k' spring constant.)

Measuring a mass on a scale

Equilibrium:

$$mg - k\Delta x = 0$$
$$\Delta x \rightarrow \theta \rightarrow \text{scale reading}$$

(with 'g' *gravitational acceleration;* 'k' spring constant.)

From the reading to the value of the mass:

$$\text{scale reading} \xrightarrow[\textit{given } g,\ k,\ \text{"etc."}\ldots]{} m$$

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\text{given } g,\, k,\, \text{``etc.''}\ldots]{} m$$

Dependence on 'g': $$g \stackrel{?}{=} \frac{GM_{\oplus}}{R_{\oplus}^2}$$

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\text{given } g,\ k,\ \text{“etc.”}\ldots]{} m$$

Dependence on 'g': $$g \stackrel{?}{=} \frac{GM_{\oplus}}{R_{\oplus}^2}$$

- Position is usually not at "$R_{\oplus}$" from the Earth center;
- Earth not spherical...
- ...not even ellipsoidal...
- ...and not even homogeneous.
- Moreover we have to consider centrifugal effects
- ...and even the effect from the Moon

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\textit{given } g,\ k,\ \text{“etc.”}\ldots]{} m$$

Dependence on ‘g’: $$g \stackrel{?}{=} \frac{GM_{\oplus}}{R_{\oplus}^2}$$

- Position is usually <u>not</u> at “$R_{\oplus}$” from the Earth center;
- Earth not spherical. . .
- . . . not even ellipsoidal. . .
- . . . and not even homogeneous.
- Moreover we have to consider centrifugal effects
- . . . and even the effect from the Moon

Certainly not to watch our weight

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\text{given } g,\ k,\ \text{"etc."}\ldots]{} m$$

Dependence on 'g': $$g \stackrel{?}{=} \frac{GM_\oplus}{R_\oplus^2}$$

- Position is usually <u>not</u> at "$R_\oplus$" from the Earth center;
- Earth not spherical...
- ...not even ellipsoidal...
- ...and not even homogeneous.
- Moreover we have to consider centrifugal effects
- ...and even the effect from the Moon

Certainly not to watch our weight

But think about it!

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\textit{given } g,\, k,\, \text{“etc.”}\ldots]{} m$$

Dependence on 'k':

- temperature
- non linearity
- ...

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\textit{given } g,\ k,\ \text{“etc.”}\ldots]{} m$$

Dependence on 'k':

- temperature
- non linearity
- ...

Δx $\rightarrow \theta \rightarrow$ scale reading:

- left to your imagination...

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\text{given } g,\ k,\ \text{“etc.”}\ldots]{} m$$

Dependence on 'k':

- temperature
- non linearity
- ...

$\boldsymbol{\Delta}$x $\rightarrow \theta \rightarrow$ scale reading:

- left to your imagination...

+ randomic effects:

- stopping position of damped oscillation;
- variability of all quantities of influence (in the ISO-GUM sense);
- reading of analog scale.

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\text{given } g,\ k,\ \text{“etc.”}\ldots]{} m$$

Dependence on 'k':

- temperature
- non linearity
- ...

Δx $\rightarrow \theta \rightarrow$ scale reading:

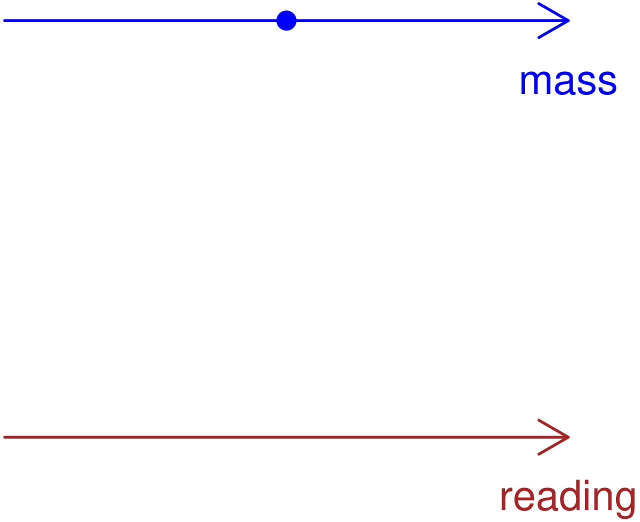
- left to your imagination...

+ randomic effects:

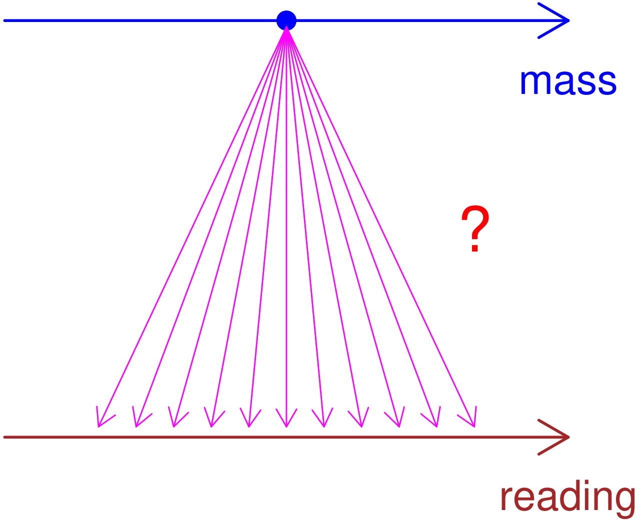
- stopping position of damped oscillation;
- variability of all quantities of influence (in the ISO-GUM sense);
- reading of analog scale.

$\Rightarrow$ m??

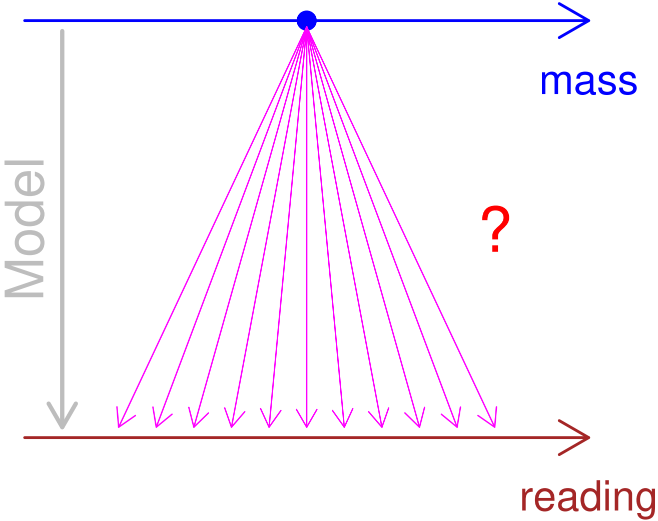
Mass ⟶ Reading



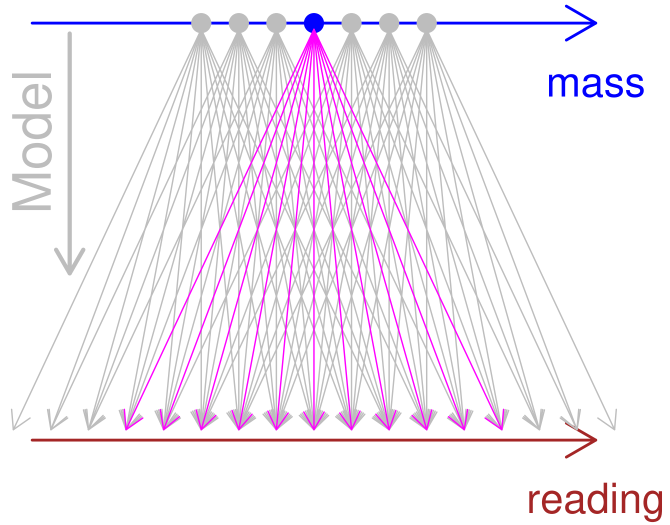
Mass $\longrightarrow$ Reading



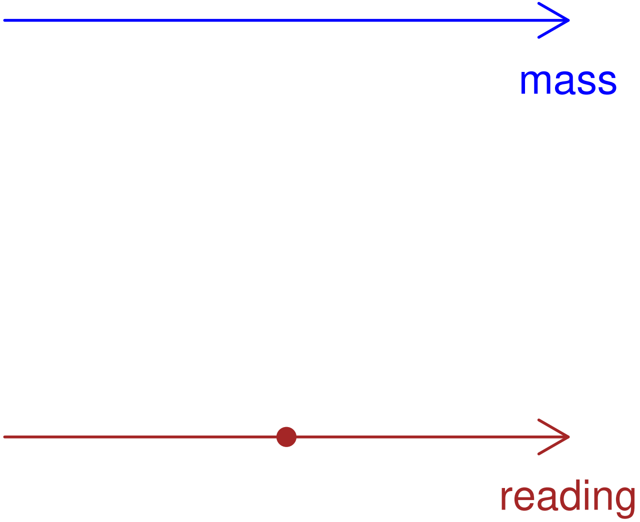
Mass $\longrightarrow$ Reading



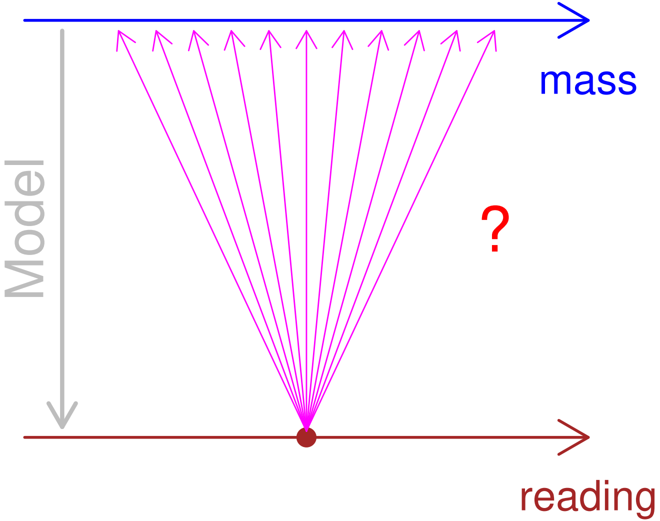
Mass ⟶ reading



Reading ⟶ 'true' mass

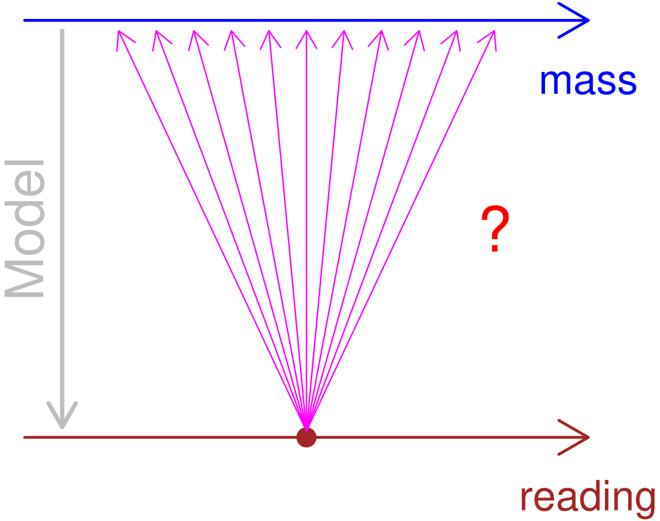


Reading ⟶ 'true' mass



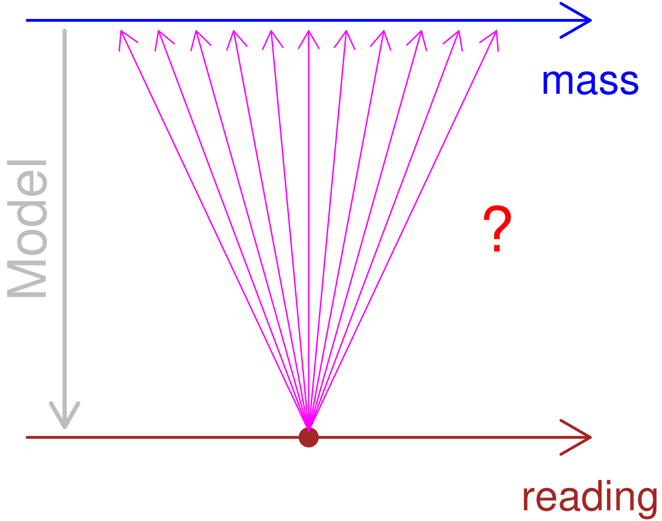
Data uncertainty?

Reading ⟶ ‘true’ mass



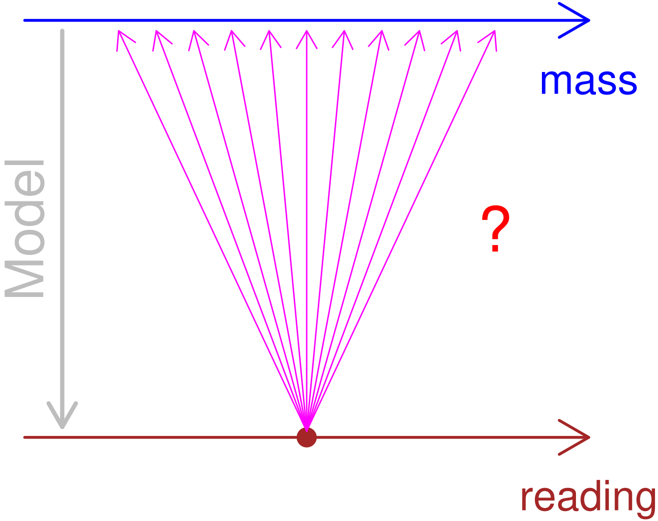
Data uncertainty? ???

Reading $\longrightarrow$ 'true' mass



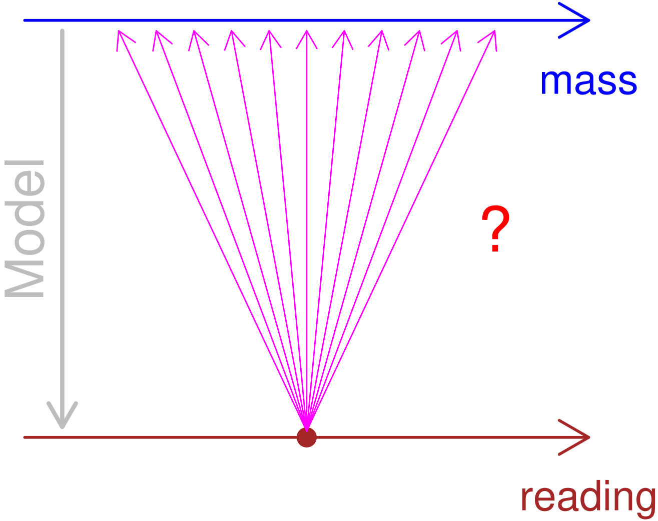
Data uncertainty? ??? Are the data corrupted?

Reading ⟶ ‘true’ mass



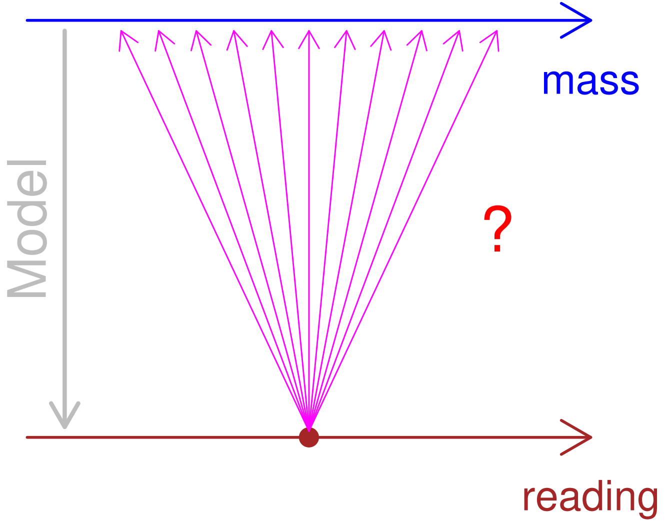
Data uncertainty? **???** Are the data corrupted?
In that case the data are . . . **the corrupted data**!

Reading ⟶ 'true' mass



Data uncertainty? **???** Are the data corrupted?
In that case the data are . . . **the corrupted data**!
What is uncertain is m, or whatever we are interested in.

Reading $\longrightarrow$ ‘true’ mass



Data uncertainty? **???** Are the data corrupted?
In that case the data are . . . **the corrupted data**!
What is uncertain is m, or whatever we are interested in.
$\rightarrow$ Model parameter(s)

What is measurement

- Measurement is not only related to the use of instruments.

What is measurement

- Measurement is not only related to the use of instruments.
- It can be also based in 'counting objects'

What is measurement

- Measurement is not only related to the use of instruments.
- It can be also based in 'counting objects'
- What is important is to build up a model
  that 'relates' (often in a complicate, probabilistic way)

model parameter(s) $\leftrightarrow$ empirical observations

What is measurement

- Measurement is not only related to the use of instruments.
- It can be also based in 'counting objects'
- What is important is to build up a model
  that 'relates' (often in a complicate, probabilistic way)

$$\text{model parameter(s)} \leftrightarrow \text{empirical observations}$$

(Reading a value on a device is the simplest direct measurement

What is measurement

- Measurement is not only related to the use of instruments.
- It can be also based in 'counting objects'
- What is important is to build up a model
  that 'relates' (often in a complicate, probabilistic way)

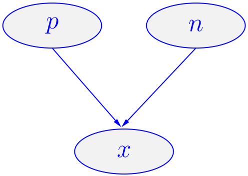
model parameter(s) $\leftrightarrow$ empirical observations

(Reading a value on a device is the simplest direct measurement, although 'getting the value' of the quantity of interest, including the uncertainty to associate to it, might be not that trivial.)

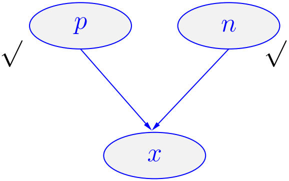
Simple cases based on binomial distribution

Simple cases based on binomial distribution

Model connecting the variables of interest:

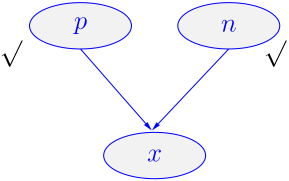


Graphical models of the typical problems

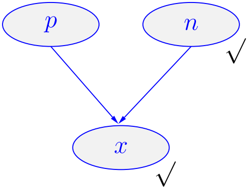


$\rightarrow \; f(x \,|\, n, p)$

Graphical models of the typical problems



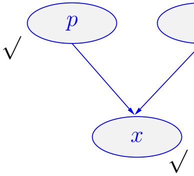
$\rightarrow \; f(x \mid n, p)$



$\rightarrow \; f(p \mid n, x)$

Extending the model

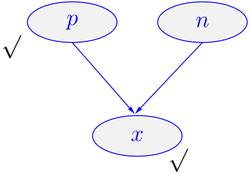
Uncertain n



$\rightarrow f(n \,|\, p, x)$

Extending the model

Uncertain n

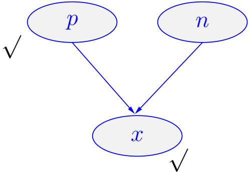


$\rightarrow \; f(n \,|\, p, x)$

But in this case we need some (usually indirect) knowledge about p

Extending the model

Uncertain n

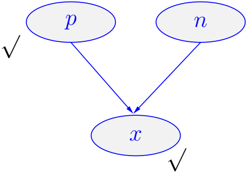


$\rightarrow\ f(n\,|\,p, x)$

But in this case we need some (usually indirect) knowledge about p
(Usually we do not calculate p from the proportion of white balls!)

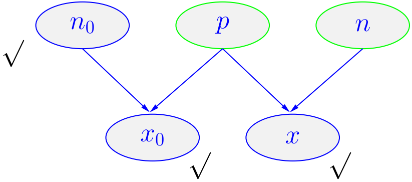
Extending the model

Uncertain n



$\rightarrow f(n \mid p, x)$

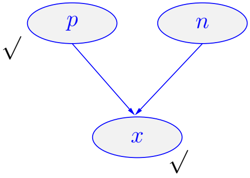
But in this case we need some (usually indirect) knowledge about p
(Usually we do not calculate p from the proportion of white balls!)



$\rightarrow f(p, n \mid n_0, x_0, x)$

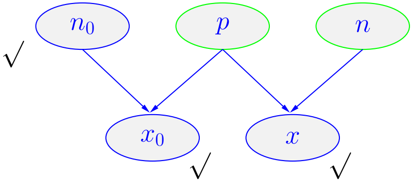
Extending the model

Uncertain n



$\rightarrow\ f(n\,|\,p, x)$

But in this case we need some (usually indirect) knowledge about p
(Usually we do not calculate p from the proportion of white balls!)

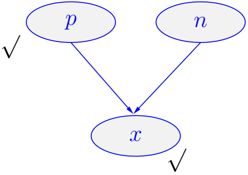


$\rightarrow\ f(p, n\,|\,n_0, x_0, x)$

$\rightarrow\ f(p\,|\,n_0, x_0, x)$

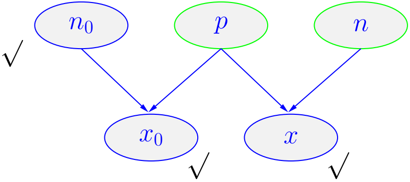
Extending the model

Uncertain n



$\rightarrow\ f(n \mid p, x)$

But in this case we need some (usually indirect) knowledge about p
(Usually we do not calculate p from the proportion of white balls!)



$\rightarrow\ f(p, n \mid n_0, x_0, x)$

$\rightarrow\ f(p \mid n_0, x_0, x)$

But what is *n*?

Extending the model

In Physics we are usually not interested in the numbers we do see, but in those to which we assign a 'physical meaning'.

Extending the model

In Physics we are usually not interested in the numbers we do see, but in those to which we assign a 'physical meaning'.

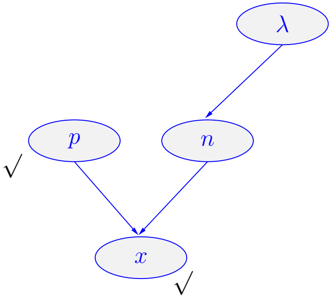
- When we say "we are uncertain on numbers", we do not mean that we are uncertain on the numbers we 'see' in our detector, but to 'other numbers'.
- Typically $n \longleftrightarrow \lambda$.

Extending the model

In Physics we are usually not interested in the numbers we do see, but in those to which we assign a ‘physical meaning’.

- When we say “we are uncertain on numbers”, we do not mean that we are uncertain on the numbers we ‘see’ in our detector, but to ‘other numbers’.
- Typically $n \longleftrightarrow \lambda$.

Assuming for a while p well known and focusing on ‘n’:

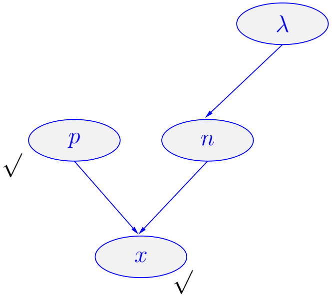


Extending the model

In Physics we are usually not interested in the numbers we do see, but in those to which we assign a 'physical meaning'.

- When we say "we are uncertain on numbers", we do not mean that we are uncertain on the numbers we 'see' in our detector, but to 'other numbers'.
- Typically $n \longleftrightarrow \lambda$.

Assuming for a while p well known and focusing on 'n':



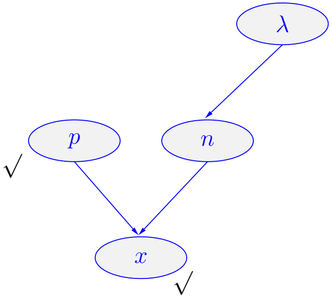
But λ is not really physical.

Extending the model

In Physics we are usually not interested in the numbers we do see, but in those to which we assign a 'physical meaning'.

- When we say "we are uncertain on numbers", we do not mean that we are uncertain on the numbers we 'see' in our detector, but to 'other numbers'.
- Typically $n \longleftrightarrow \lambda$.

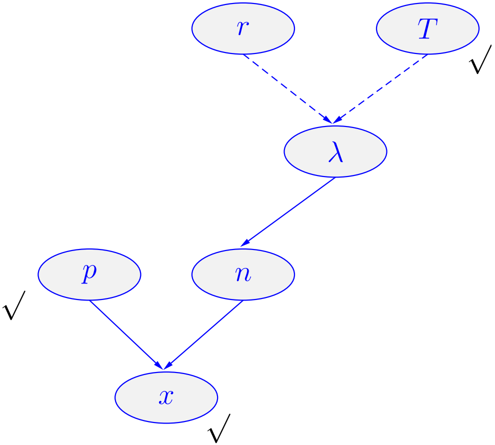
Assuming for a while p well known and focusing on 'n':



But λ is not really physical. What is physical is the intensity of the Poisson process (r) $\longrightarrow$ $\lambda = r \cdot T$

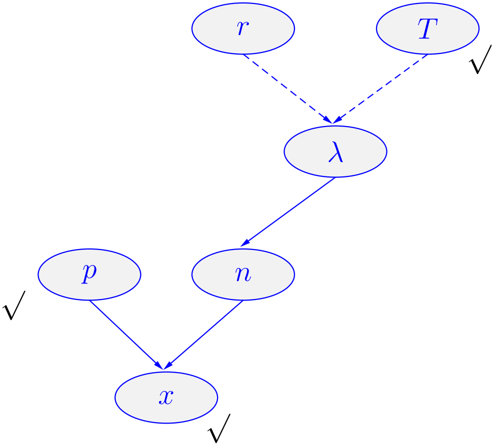
Extending the model

$\boldsymbol{\lambda} = r \cdot T$:



Extending the model

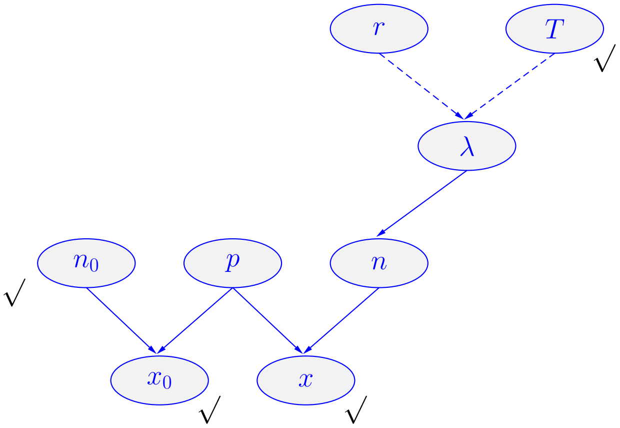
$\boldsymbol{\lambda} = r \cdot T$:



(Dashed arrows used in literature for deterministic links)

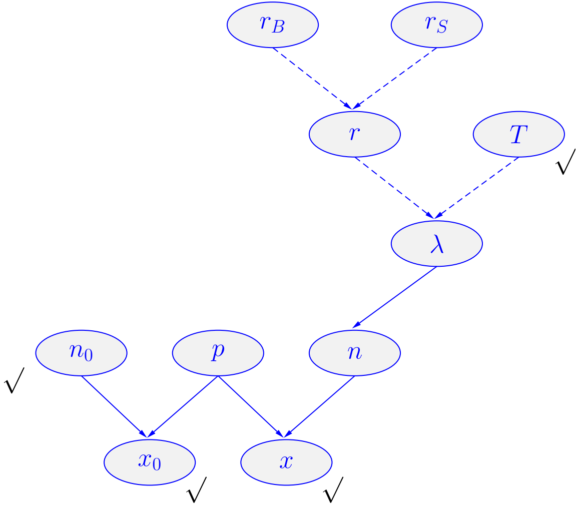
Extending the model

Remembering that *p* was got from a measurement:



Extending the model

The rate r gets contributions from signal and background

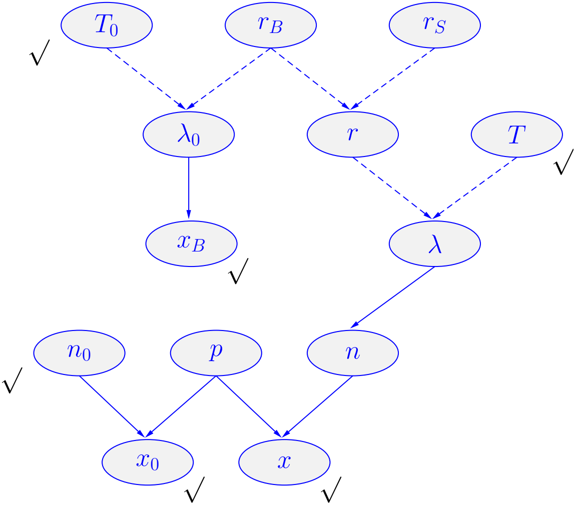


Extending the model

But we need some independent knowledge of the background

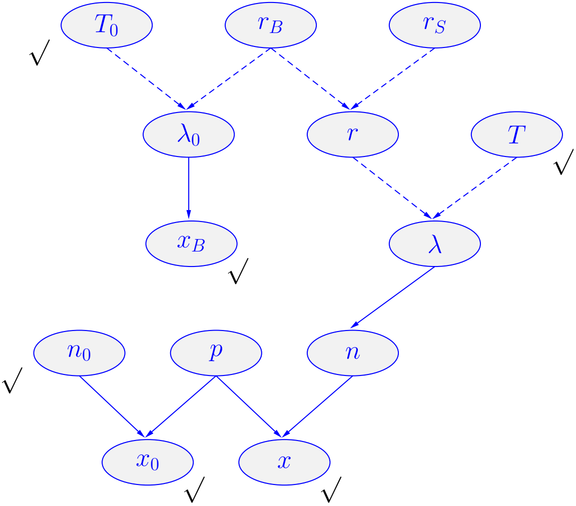
Extending the model

But we need some independent knowledge of the background



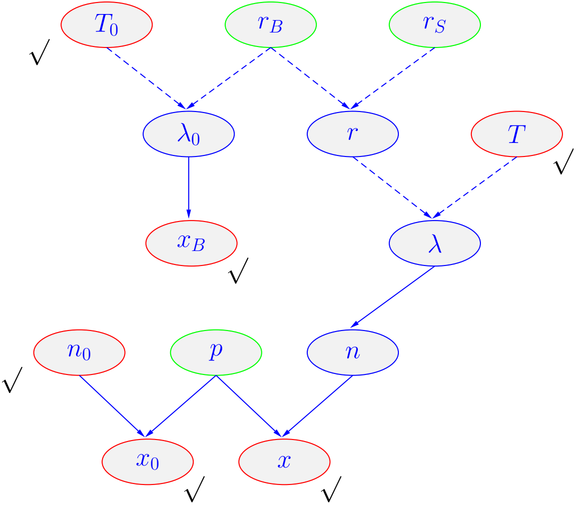
Extending the model

But we need some independent knowledge of the background

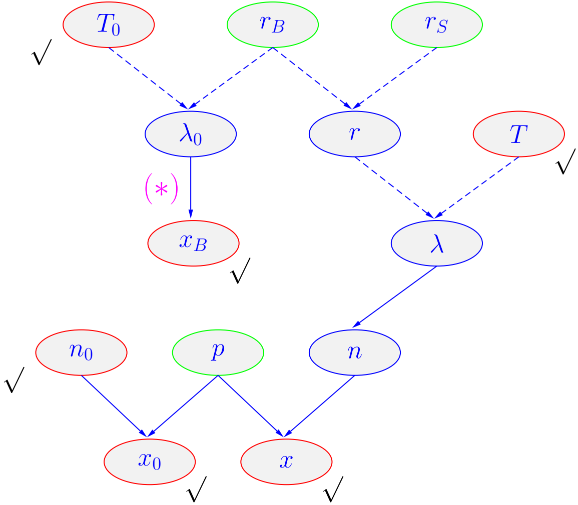


(T_0 and T assumed to be measured with sufficient accuracy)

A more realistic model

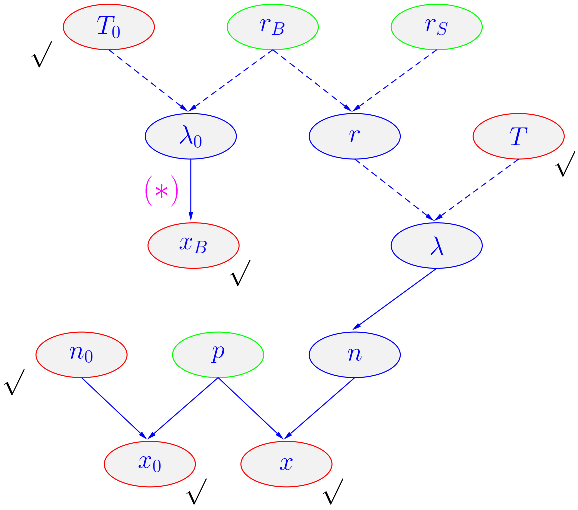


A more realistic model



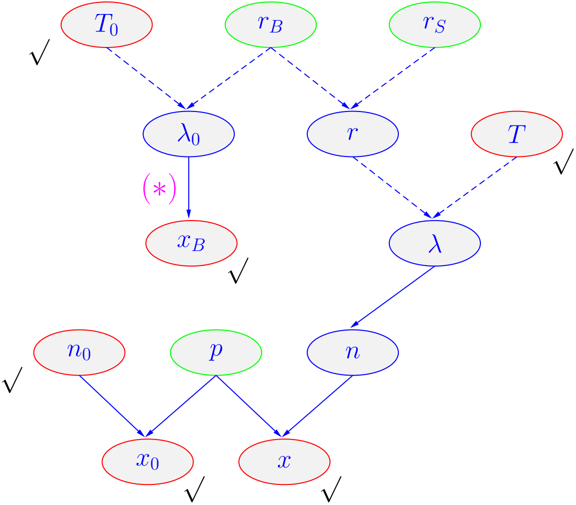
(*) Assuming unity efficiency

A more realistic model



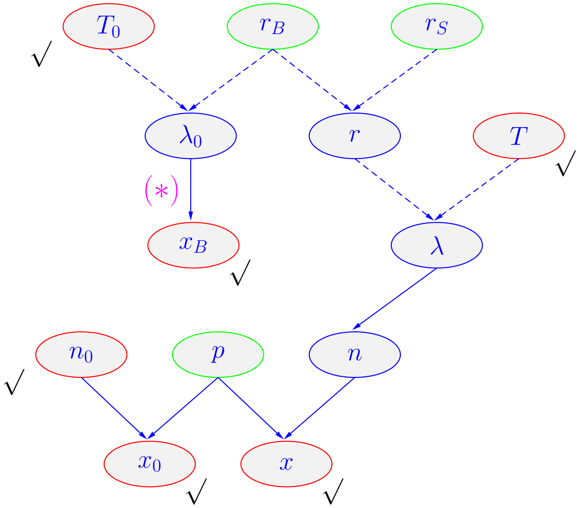
Nowadays, once you are able to write down the graphical model you have done more than 50% route towards the solution!

A more realistic model



Nowadays, once you are able to write down the graphical model you have done more than 50% route towards the solution! **How?**

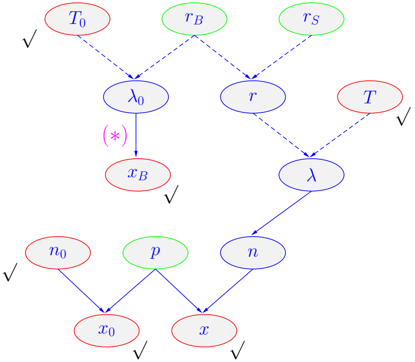
A more realistic model



$\Rightarrow$ probability distribution of uncertain variables

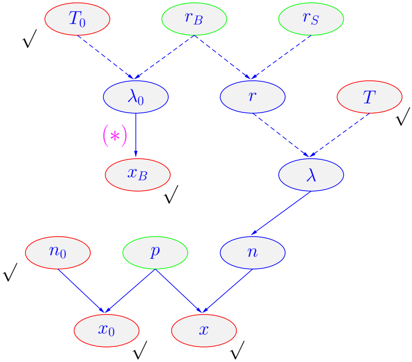
$\rightarrow f(p, r_s, r_B \mid n_0, x_0, x, x_B, T, T_0)$

Probabilistic approach



Steps needed

Probabilistic approach

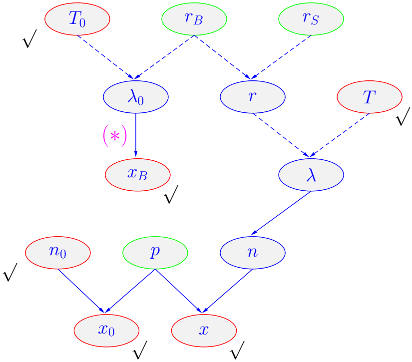


Steps needed (conceptually easy):

1. write down the joint probability function of all variables:

$$f(r_B, r_S, \lambda_0, r, \ldots, x_0, x)$$

Probabilistic approach

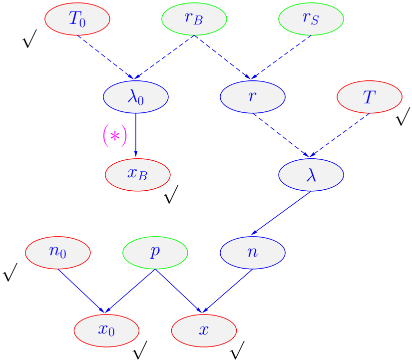


Steps needed (conceptually easy):

1. write down the joint probability function of all variables:

$$f(r_B, r_S, \lambda_0, r, \ldots, x_0, x) \equiv f(\cdots)$$

Probabilistic approach



Steps needed (conceptually easy):

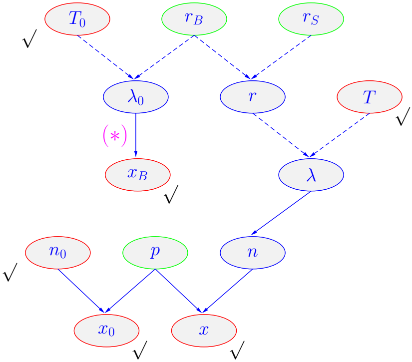
1. write down the joint probability function of all variables:

$$f(r_B, r_S, \lambda_0, r, \ldots, x_0, x) \equiv f(\cdots)$$

2. condition on what is known/assumed:

$$f(r_B, r_S, \lambda_0, r, \ldots \mid T_0, T, \ldots, x_0, x) = \frac{f(\cdots)}{f(n_0, x_0, x, \ldots)}$$

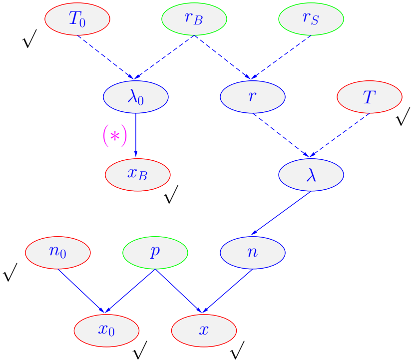
Probabilistic approach



Steps needed (conceptually easy):

1. write down the joint probability function of all variables:
$$f(r_B, r_S, \lambda_0, r, \ldots, x_0, x) \equiv f(\cdots)$$
2. condition on what is known/assumed:
$$f(r_B, r_S, \lambda_0, r, \ldots \mid T_0, T, \ldots, x_0, x) = \frac{f(\cdots)}{f(n_0, x_0, x, \ldots)}$$
3. marginalize:

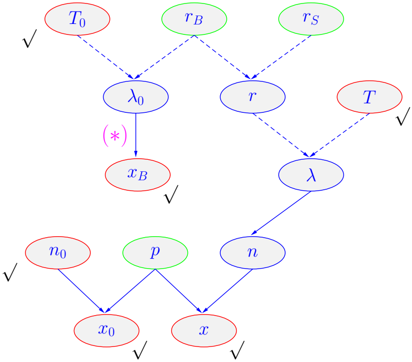
Probabilistic approach



Steps needed (conceptually easy):

1. write down the joint probability function of all variables:
$$f(r_B, r_S, \lambda_0, r, \ldots, x_0, x) \equiv f(\cdots)$$
2. condition on what is known/assumed:
$$f(r_B, r_S, \lambda_0, r, \ldots \mid T_0, T, \ldots, x_0, x) = \frac{f(\cdots)}{f(n_0, x_0, x, \ldots)}$$
3. marginalize: $\rightarrow\ f(r_B, r_S, p \mid T_0, T, \ldots, x_0, x)$

Probabilistic approach

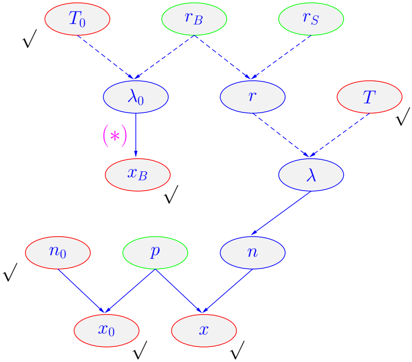


Steps needed (conceptually easy):

1. write down the joint probability function of all variables:
$$f(r_B, r_S, \lambda_0, r, \ldots, x_0, x) \equiv f(\cdots)$$
2. condition on what is known/assumed:
$$f(r_B, r_S, \lambda_0, r, \ldots \mid T_0, T, \ldots, x_0, x) = \frac{f(\cdots)}{f(n_0, x_0, x, \ldots)}$$
3. marginalize: $\rightarrow \; f(r_B, r_S, p \mid T_0, T, \ldots, x_0, x)$

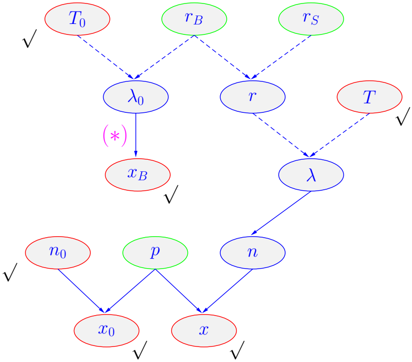
But in practice?

Probabilistic approach



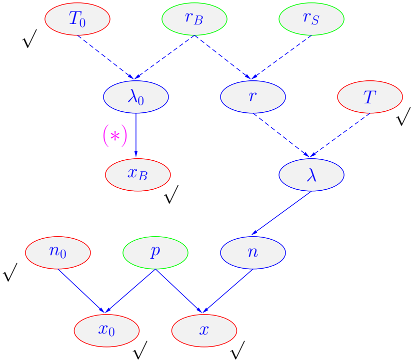
Mission impossible?

Probabilistic approach



Mission impossible? Non quite. . .

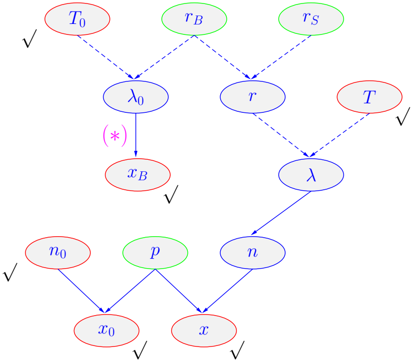
Probabilistic approach



Mission impossible? Non quite...

1. $f(\cdots)$ can be 'usually' be written using the 'chain rule'

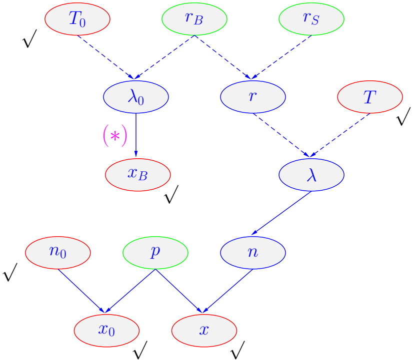
Probabilistic approach



Mission impossible? Non quite...

1. $f(\cdots)$ can be 'usually' be written using the 'chain rule',
   e.g. $f(x, y, z) = f(x \mid y, z) \cdot f(y \mid z) \cdot f(z)$

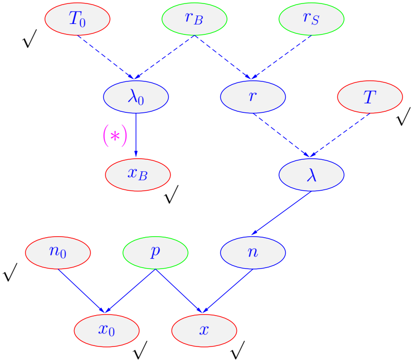
Probabilistic approach



Mission impossible? Non quite...

1. $f(\cdots)$ can be 'usually' be written using the 'chain rule',
   e.g. $f(x, y, z) = f(x \,|\, y, z) \cdot f(y \,|\, z) \cdot f(z)$
2. $f(n_0, x_0, x, \ldots)$ is simply a constant

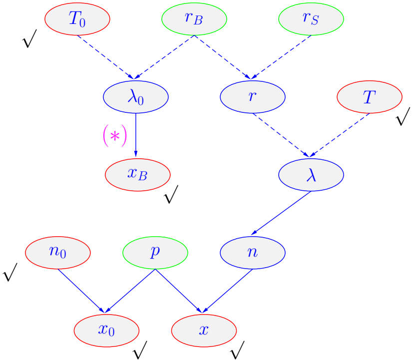
Probabilistic approach



Mission impossible? Non quite...

1. $f(\cdots)$ can be 'usually' be written using the 'chain rule', e.g. $f(x, y, z) = f(x \,|\, y, z) \cdot f(y \,|\, z) \cdot f(z)$
2. $f(n_0, x_0, x, \ldots)$ is simply a constant (indeed usually very difficult to evaluate', but just a numeric constant)

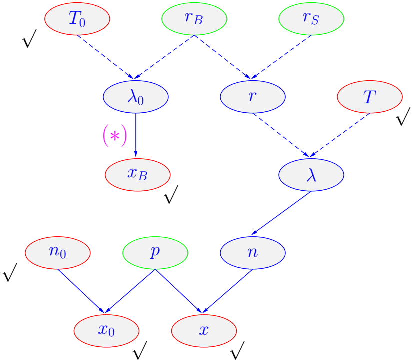
Probabilistic approach



Mission impossible? Non quite...

1. $f(\cdots)$ can be 'usually' be written using the 'chain rule',
   e.g. $f(x,y,z) = f(x\,|\,y,z)\cdot f(y\,|\,z)\cdot f(z)$
2. $f(n_0, x_0, x, \ldots)$ is simply a constant (indeed usually very difficult to evaluate', but just a numeric constant):
   $\tilde{f}(r_B, r_S, \lambda_0, r, \ldots \mid T_0, T, \ldots, x_0, x) \propto f(\cdots\ \cdots)$

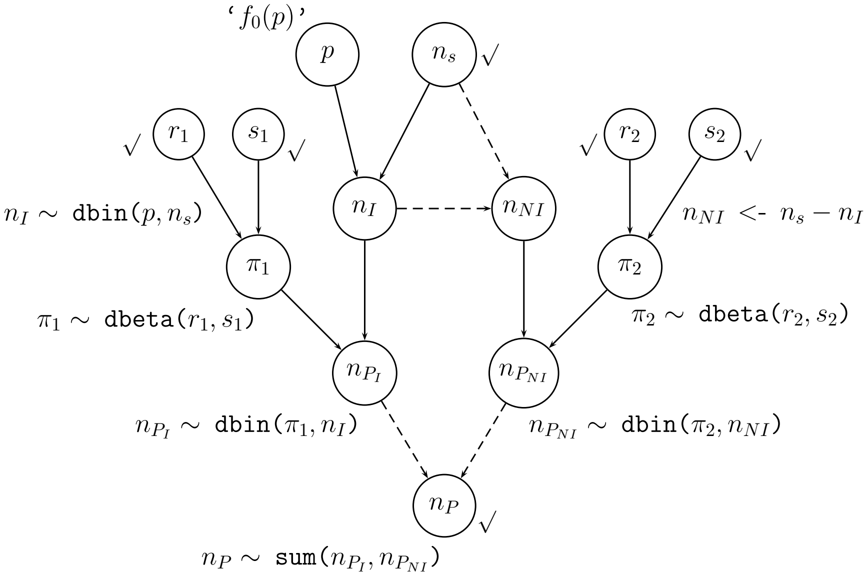
Probabilistic approach



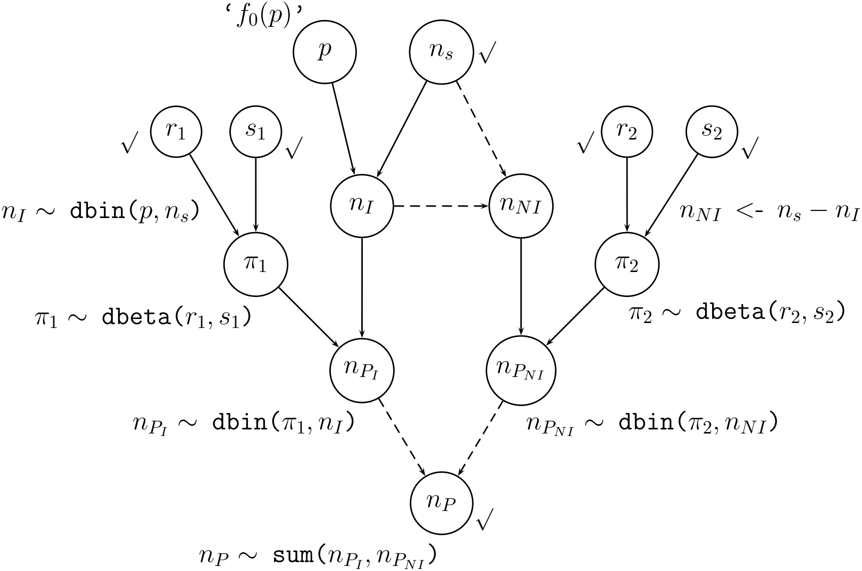
Mission impossible? Non quite...

1. $f(\cdots)$ can be 'usually' be written using the 'chain rule',
   e.g. $f(x,y,z) = f(x\,|\,y,z)\cdot f(y\,|\,z)\cdot f(z)$
2. $f(n_0, x_0, x, \ldots)$ is simply a constant (indeed usually very difficult to evaluate', but just a numeric constant):
   $\tilde{f}(r_B, r_S, \lambda_0, r, \ldots\,|\,T_0, T, \ldots, x_0, x) \propto f(\cdots\ \cdots)$
3. let the 'dirty work' be done by MCMC *tools*.

Model for random sampling (arXiv:2009.04843 [q-bio.PE])

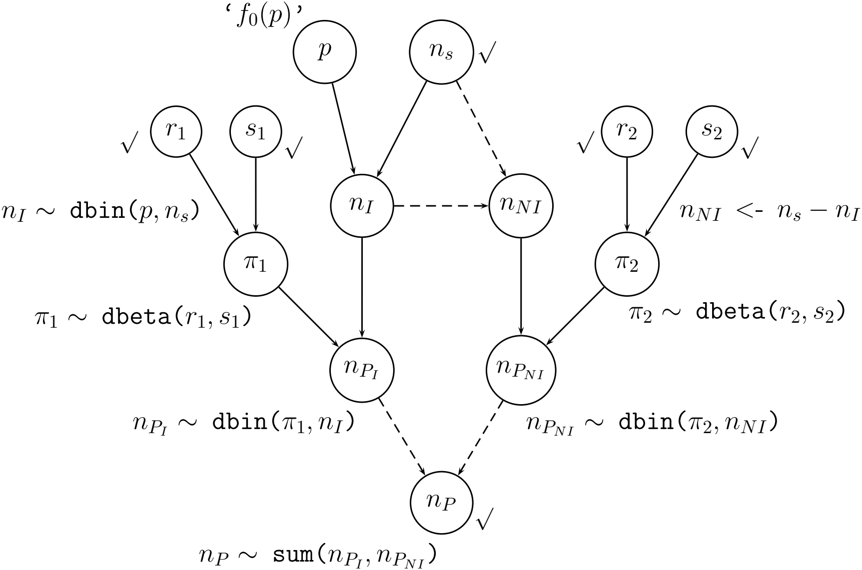


Model for random sampling (arXiv:2009.04843 [q-bio.PE])



$\Rightarrow \; f(p \,|\, n_s, n_P, \ldots)$

Model for random sampling (arXiv:2009.04843 [q-bio.PE])



$\Rightarrow$ $f(p \,|\, n_s, n_P, \ldots)$: How?

$\Rightarrow f(p \,|\, n_s, n_P, \ldots)$: How?

- In principle 'easy':

$\Rightarrow f(p \,|\, n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule

$\Rightarrow f(p \,|\, n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);

$\Rightarrow f(p \mid n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);
  2. evaluate the pdf of 'unobserved' nodes conditioned on the observed nodes (those with '$\surd$') in the above Bayesian Network;

$\Rightarrow f(p \,|\, n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);
  2. evaluate the pdf of 'unobserved' nodes conditioned on the observed nodes (those with '$\surd$') in the above Bayesian Network;
  3. marginalize.

$\Rightarrow f(p \,|\, n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);
  2. evaluate the pdf of 'unobserved' nodes conditioned on the observed nodes (those with '$\surd$') in the above Bayesian Network;
  3. marginalize.
- In practice we do it by Markov Chain Monte Carlo sampling:

$\Rightarrow f(p \mid n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);
  2. evaluate the pdf of 'unobserved' nodes conditioned on the observed nodes (those with '$\surd$') in the above Bayesian Network;
  3. marginalize.
- In practice we do it by Markov Chain Monte Carlo sampling:
  - we use the JAGS program interfaced to R via rjags.
    (Details, including scripts, in the paper)

$\Rightarrow f(p \,|\, n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);
  2. evaluate the pdf of 'unobserved' nodes conditioned on the observed nodes (those with '$\surd$') in the above Bayesian Network;
  3. marginalize.
- In practice we do it by Markov Chain Monte Carlo sampling:
  - we use the JAGS program interfaced to R via rjags. (Details, including scripts, in the paper)

(But in the case of the random sampling showed above we did the effort of getting an exact result).

$\Rightarrow f(p \,|\, n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);
  2. evaluate the pdf of 'unobserved' nodes conditioned on the observed nodes (those with '$\surd$') in the above Bayesian Network;
  3. marginalize.
- In practice we do it by Markov Chain Monte Carlo sampling:
  - we use the JAGS program interfaced to R via rjags.
    (Details, including scripts, in the paper)

(But in the case of the random sampling showed above we did the effort of getting an exact result).

Note: from the probabilistic point of view no real distinction between inference and predictions:

$\Rightarrow f(p \,|\, n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);
  2. evaluate the pdf of 'unobserved' nodes conditioned on the observed nodes (those with '$\surd$') in the above Bayesian Network;
  3. marginalize.
- In practice we do it by Markov Chain Monte Carlo sampling:
  - we use the JAGS program interfaced to R via rjags. (Details, including scripts, in the paper)

(But in the case of the random sampling showed above we did the effort of getting an exact result).

Note: from the probabilistic point of view no real distinction between inference and predictions:

observed nodes $\rightarrow$ unobserved nodes

$\Rightarrow f(p \mid n_s, n_P, \ldots)$: How?

- In principle 'easy':
  1. write down the joint pdf (prob. density function) using a convenient chain rule (bottom-up);
  2. evaluate the pdf of 'unobserved' nodes conditioned on the observed nodes (those with '$\surd$') in the above Bayesian Network;
  3. marginalize.
- In practice we do it by Markov Chain Monte Carlo sampling:
  - we use the JAGS program interfaced to R via rjags.
    (Details, including scripts, in the paper)

(But in the case of the random sampling showed above we did the effort of getting an exact result).

Note: from the probabilistic point of view no real distinction between inference and predictions:

observed nodes $\rightarrow$ unobserved nodes

$\rightarrow$ Probability Theory not sensitive to their meaning!

Vaccine efficacy: a simplified model

Some initial difficulties, due to uncertainy in the models, having acccess only to a couple of numbers.

Vaccine efficacy: a simplified model

Some initial difficulties, due to uncertainy in the models,
having acccess only to a couple of numbers.
Solved introducing a catch-all term embedding
the many real life variables, apart being vaccinated or not.

Vaccine efficacy: a simplified model

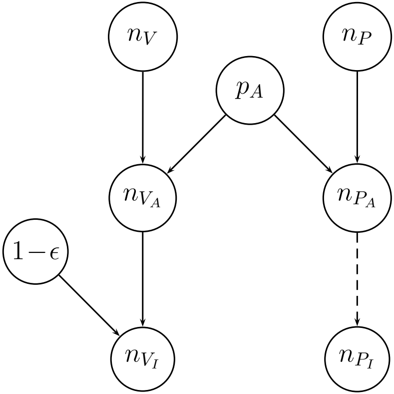
Some initial difficulties, due to uncertainy in the models,
having acccess only to a couple of numbers.
Solved introducing a catch-all term embedding
the many real life variables, apart being vaccinated or not.
⇒ '*assault probability*' p_A.

Vaccine efficacy: a simplified model

Some initial difficulties, due to uncertainy in the models, having acccess only to a couple of numbers.

Solved introducing a catch-all term embedding the many real life variables, apart being vaccinated or not.

⇒ '*assault probability*' p_A.



ϵ: efficacy

Vaccine efficacy

Jags model

Vaccine efficacy

Jags model (like describing the terms of the chain rule!)

```
model {
  nP.I   ~ dbin(pA, nP)              # 1.
  nV.A   ~ dbin(pA, nV)              # 2.
  pA     ~ dbeta(1,1)                # 3.
  nV.I   ~ dbin(ffe, nV.A)           # 4.  [ ffe = 1 - eff ]
  ffe    ~ dbeta(1,1)                # 5.
  eff    <- 1 - ffe                  # 6.
  }
```

Vaccine efficacy

Jags model (like describing the terms of the chain rule!)

```
model {
  nP.I   ~ dbin(pA, nP)            # 1.
  nV.A   ~ dbin(pA, nV)            # 2.
  pA     ~ dbeta(1,1)              # 3.
  nV.I   ~ dbin(ffe, nV.A)         # 4.  [ ffe = 1 - eff ]
  ffe    ~ dbeta(1,1)              # 5.
  eff    <- 1 - ffe                # 6.
  }
```

Sensitive data:

Moderna: `nV.I = 5`, `nP.I = 90`;

Pfizer: `nV.I = 8`, `nP.I = 162`.

Vaccine efficacy

Jags model (like describing the terms of the chain rule!)

```
model {
  nP.I   ~ dbin(pA, nP)             # 1.
  nV.A   ~ dbin(pA, nV)             # 2.
  pA     ~ dbeta(1,1)               # 3.
  nV.I   ~ dbin(ffe, nV.A)          # 4.   [ ffe = 1 - eff ]
  ffe    ~ dbeta(1,1)               # 5.
  eff    <- 1 - ffe                 # 6.
  }
```

Sensitive data:

Moderna: `nV.I = 5`, `nP.I = 90`;

Pfizer: `nV.I = 8`, `nP.I = 162`.

Less sensitive data (even factors 1/10 or 1/100 are irrelevant!):

Moderna: `nV = nP = 15000`;

Pfizer: `nV = nP = 20000`.

Results

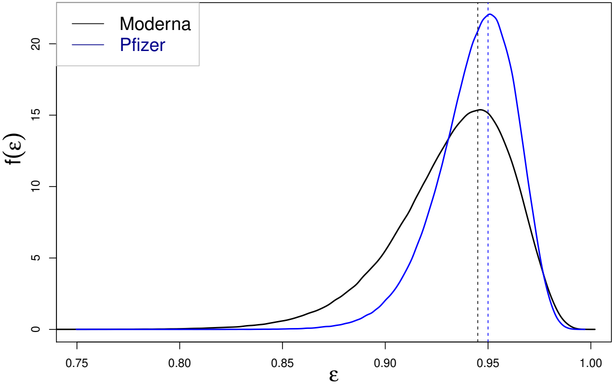
1. Real time run of JAGS

Results

1. Real time run of JAGS $\rightarrow$ watch

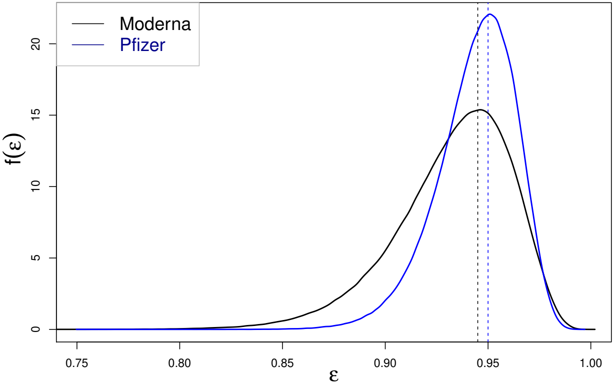
Results

1. Real time run of JAGS → watch
2. Comparison of $f(\epsilon \,|\, \text{Moderna})$ vs $f(\epsilon \,|\, \text{Pfizer})$



Results

1. Real time run of JAGS → watch
2. Comparison of $f(\epsilon\,|\,\text{Moderna})$ vs $f(\epsilon\,|\,\text{Pfizer})$



3. Summaries:

	mean $\pm$ stand. unc.	centr. 95% cred. int.	$P(\epsilon \geq 0.9)$
Moderna	0.933 ± 0.029	$[0.866, 0.976]$	0.872
Pfizer	0.944 ± 0.019	$[0.900, 0.975]$	0.976

Conclusions

- The recent announcements by Moderna and Pfizer give some hope of coping effectively with the pandemic.

Conclusions

- The recent announcements by Moderna and Pfizer give some hope of coping effectively with the pandemic.
- The efficacy they quote is in perfect agreement with a full probabilistic analysis providing the probability distribution of the quantity of interest.

Conclusions

- The recent announcements by Moderna and Pfizer give some hope of coping effectively with the pandemic.
- The efficacy they quote is in perfect agreement with a full probabilistic analysis providing the probability distribution of the quantity of interest.
- However, we insist on the point that providing results without uncertainty is definetely not scientific

Conclusions

- The recent announcements by Moderna and Pfizer give some hope of coping effectively with the pandemic.
- The efficacy they quote is in perfect agreement with a full probabilistic analysis providing the probability distribution of the quantity of interest.
- However, we insist on the point that providing results without uncertainty is definetely not scientific, because

  "It is scientific only to say what is more likely and what is less likely" (R. Feynman)

Conclusions

- The recent announcements by Moderna and Pfizer give some hope of coping effectively with the pandemic.
- The efficacy they quote is in perfect agreement with a full probabilistic analysis providing the probability distribution of the quantity of interest.
- However, we insist on the point that providing results without uncertainty is definetely not scientific, because

  "It is scientific only to say what is more likely and what is less likely" *(R. Feynman)*

- Our contribution is essential on methodological matter, and we astain from any comment on the several related issues.

Paper available on
`https://www.roma1.infn.it/~dagos/prob+stat.html`

Conclusions

- The recent announcements by Moderna and Pfizer give some hope of coping effectively with the pandemic.
- The efficacy they quote is in perfect agreement with a full probabilistic analysis providing the probability distribution of the quantity of interest.
- However, we insist on the point that providing results without uncertainty is definetely not scientific, because

  "It is scientific only to say what is more likely and what is less likely" *(R. Feynman)*

- Our contribution is essential on methodological matter, and we astain from any comment on the several related issues.

Paper available on
`https://www.roma1.infn.it/~dagos/prob+stat.html`

Bottom line: learn *model thinking* and MCMC (based tools) and you will have an extra gear!