Inferring keys and balls

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Prevision and prevision uncertainty

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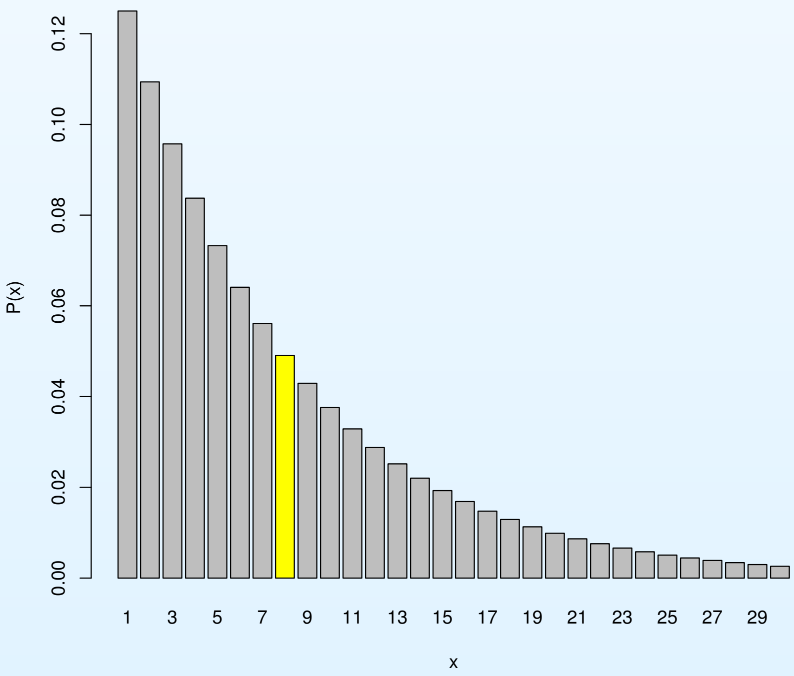
$\mathsf{E}[X] = 1/p$
$\sigma(X) = \sqrt{1-p}/p$

$p = 1/8$:

$$\mathsf{E}[X] = 8$$
$$\sigma(X) = 7.5$$



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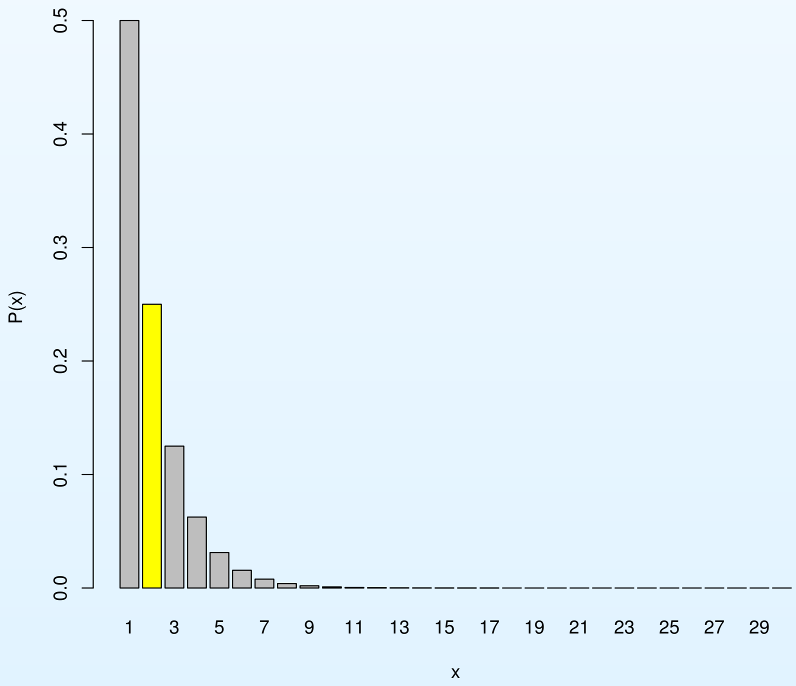
$\mathsf{E}[X] = 1/p$
$\sigma(X) = \sqrt{1-p}/p$

$p = 1/2$:

$$\mathsf{E}[X] = 2$$
$$\sigma(X) = 1.4$$



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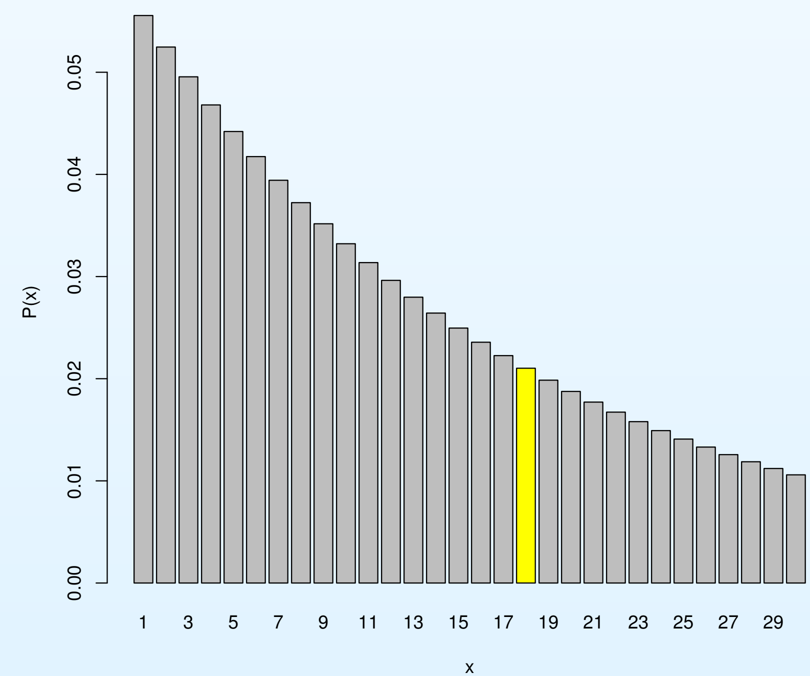
$$\text{Variance}(X) = \sum_x (x - \mathsf{E}[X])^2\, f(x) \longrightarrow \sigma^2(X) \to \sigma = \sqrt{\sigma}$$

$$\mathsf{E}[X] = 1/p$$
$$\sigma(X) = \sqrt{1-p}/p$$

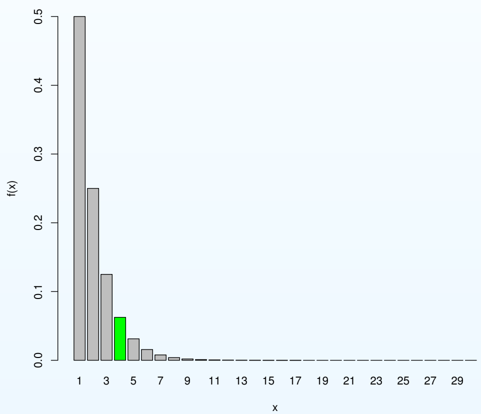
$p = 1/18$:

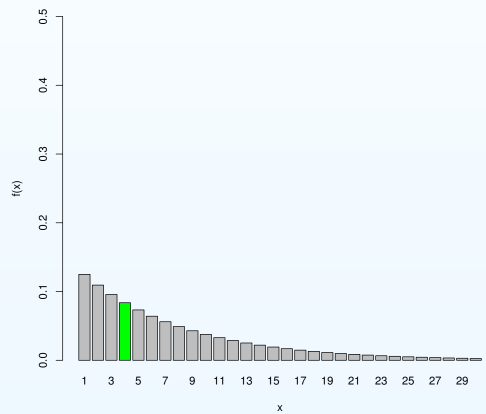
$$\mathsf{E}[X] = 18$$
$$\sigma(X) = 17.5$$

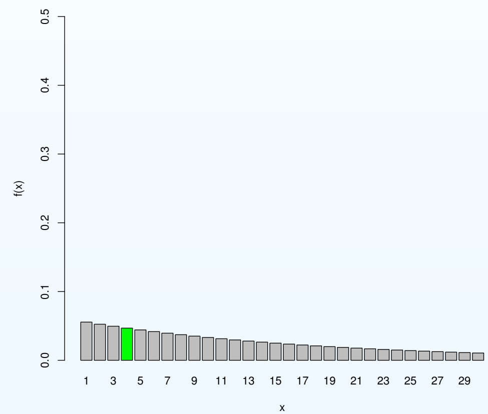


Inverse problem

The success happened in 4-th trial: how many keys?







$$P(H_i\,|\,x) \;\propto\; f(x\,|\,H_i)\,P_0(H_i)$$
$$P(p_i\,|\,x) \;\propto\; f(x\,|\,p_i)\,P_0(p_i)$$
$$\frac{P(p_i\,|\,x)}{P(p_i\,|\,x)} = \frac{f(x\,|\,H_i)}{f(x\,|\,p_i)}\,\frac{P_0(p_i)}{P_0(p_i)}$$

$f(x\,|\,p_i) = 0.0625;\ 0.0837;\ 0.0468 \rightarrow$ Data favor $p = 1/8$.

Probability of hypotheses Vs ad hoc arguments

In the case the three values of p where equally likely

$$\begin{aligned} f(p=1/2\,|\,x=4) &= 0.32 \\ f(p=1/8\,|\,x=4) &= 0.43 \\ f(p=1/18\,|\,x=4) &= 0.24\,. \end{aligned}$$

It does not matter that

- $x=4$ is 1.41σ, 0.53σ and 0.80σ from the expected values of the three p_i;
- the (one tail) p-values are 12.5%, 41% and 20%

Probability of hypotheses depends on the probability of what we have actually seen given each hypothesis (and on priors), but not on the probability to observe something that is rarer of what we have seen (very strange!).

Inferring numbers

Inference about H_i can be translated into inference about p_i, where p_i is the *parameter of the model*.

Parametric inference

Use probability function formalism

$$f(p\,|\,\text{data}) \propto f(\text{data}\,|\,p) \cdot f_0(p)$$

Extend p or our model to the continuum, imagine a box with a large number of balls, a *proportion* p of them is white.

$\longrightarrow$ infer p on the base of the observations (and of what we believed p might be).

Inferring a proportion

A) Think at a geometric distribution problem

$$
\begin{aligned}
f(x\,|\,p) &= (1-p)^{x-1}p \\
f(p\,|\,x=4) &\propto (1-p)^{3}p\cdot f_0(p) \\
f(p\,|\,x=4, f_0(p)=k) &= 20\,p\,(1-p)^3
\end{aligned}
$$

B) Or a binomial?

$f(x\,|\,n,p) \propto (1-p)^{n-x}p^x$
with $n=4$, $x=1$

$\Rightarrow$ Same solution!

($\Rightarrow$ Likelihood principle)