Linear combinations of uncertain numbers A first exercise

Imagine we have measured the two sides of an A4 paper, obtaining

$$a = 29.73 \pm 0.03 \,\mathrm{cm}$$

 $b = 21.45 \pm 0.04 \,\mathrm{cm}$.

1. Evaluate (expected values, standard uncertainty and correlation)

• their difference
$$(d = a - b)$$
;

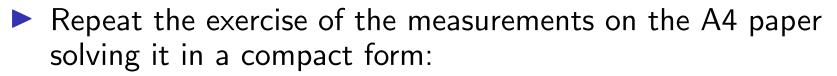
• their sum (s = a + b);

assuming $\rho(a, b) = 0$ or $\rho(a, b) = +0.8$.

- 2. Evaluate the same quantities by Monte Carlo simulation.
- 3. Repeat points 1. and 2. changing $\sigma(a)$: 0.03, 0.04, 0.05 cm.

Previous exercise reviewed

Exercise



• write the covariance matrix (with the two values of ρ);

write the transformation matrix C;

• apply the formula $V_Y = C V_X C^T$.

Linearization exercise: Extending the A4 paper example

Imagine we have measured the two sides of an A4 paper, obtaining

 $a = 29.73 \pm 0.03 \,\mathrm{cm}$ $b = 21.45 \pm 0.04 \,\mathrm{cm}$.

Evaluate (expected values, standard uncertainty and correlation)

- perimeter, p = 2a + 2b;
- Area, A = a b;
- diagonal, $d = \sqrt{a^2 + b^2}$

assuming both $\rho(a, b) = 0$ and $\rho(a, b) = +0.8$.

► Write directly *C* and then use the matrix formalism.

Linearization of monomial expressions of independent variables Exercise

Imagine we want to measure g with a pendulum:

$$T = 2\pi \sqrt{rac{I}{g}}$$

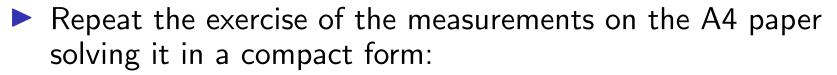
from which it follows

$$g = (2\pi)^2 I T^{-2}$$

Q.: How precisely we have to measure I and T if we require they contribute equally to r_g , that we want to keep $\leq 1\%$? Try...

Previous exercise reviewd

Exercise



• write the covariance matrix (with the two values of ρ);

write the transformation matrix C;

• apply the formula $V_Y = C V_X C^T$.

Problems on multidimensional conditioning

Imagine the angles of a triangle have been measured, resulting in

 $\begin{array}{rcl} \alpha & = & 24.2 \pm 0.5 \ \mathrm{degrees} \\ \beta & = & 65.3 \pm 0.6 \ \mathrm{degrees} \\ \gamma & = & 89.1 \pm 0.8 \ \mathrm{degrees} \,. \end{array}$

 \Rightarrow Condition on $\alpha+\beta+\gamma=$ 180 degrees.

Old problem:

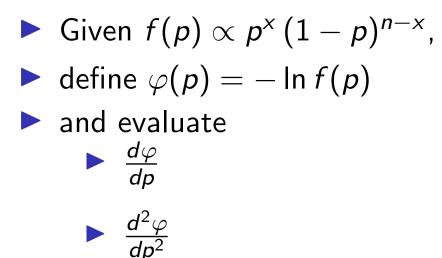
$$egin{array}{rcl} \mu_1&=&10.1\pm0.5\,{
m u}\ \mu_2&=&5.0\pm1.0\,{
m u}\
ho(\mu_1,\mu_2)&=&-0.80\,. \end{array}$$

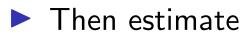
- 1. recondition on $\mu_1 + \mu_2 = 14.00 \, \text{u}$;
- 2. recondition on $\mu_1/\mu_2 = 2.20$.

Inferring the "Bernoulli's p"

Approximate solution using the 'Gaussian trick'

Exercise





• $E(p) \approx p_m$ from minimum;

• $\sigma^2(p)$ from second derivative at the minimum.