# Constraining the Higgs boson mass through the combination of direct search and precision measurement results*

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## Abstract

We show that the likelihood ratio of Higgs search experiments is a form to report the experimental results suitable to be combined with the information from precision measurements to obtain a joint constraint on the Higgs mass. We update our previous combined analysis using the new results on direct searches and recent precision measurements, including also the Z° leptonic partial width result. The method is also improved to take into account small non linearity effects in the theoretical formulae. We find an expected value for the Higgs mass around 160-170 GeV with an expectation uncertainty, quantified by the standard deviation of the distribution, of about 50-60 GeV. The 95% probability upper limit comes out to be around 260-290 GeV.

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# 1 Introduction

The Higgs boson is still the missing particle of the Standard Model (SM) picture and a considerable effort has been devoted to search for evidence of it. Unfortunately, till now all direct search experiments have been unsuccessful. However, the impressive amount of data collected at LEP, SLC, and the Tevatron allows to probe the quantum structure of the SM, thereby providing indirect information about the Higgs mass. While the negative outcome of the Higgs searches at LEP is usually reported as a combined 95% Confidence Level (C.L.) lower bound, the virtual Higgs effects are analyzed through a $\chi^2$ fit to the various precision observables that allows a 95% C.L. upper bound to be derived. In Ref.[1] we proposed a method to combine the information on the Higgs boson coming from direct searches (that we indicate generically as *dir.*) with that obtained from precision measurements (*ind.*), in order to derive a probability density function (p.d.f.) for its mass

$$f(m_H \,|\, \text{“data”}, \text{“SM”}) \equiv f(m_H \,|\, dir.\ \&\ ind.)$$

conditioned by both kind of experimental results under the assumption of validity of the SM. The heart of our method is the use of the likelihood of the Higgs search experiments normalized to its value in the case of pure background, the so called likelihood ratio $\mathcal{R}$, to further constraint the p.d.f. for the Higgs mass obtained employing only the precision physics data, $f(m_H \,|\, ind.)$.

In this paper we would like to recall the main features of our method and present an updated analysis. With respect to Ref.[1] we improve our analysis in several aspects: i) we use the exact $\mathcal{R}$ for searches up to $\sqrt{s} = 196$ GeV as provided by the LEP Higgs Working Group [2]. ii) We include as observables most sensitive to the Higgs boson mass not only the effective mixing parameter, $\sin^2\theta_{eff}^{lept} \equiv s^2_{eff}$, and the $W$ boson mass, $M_W$, but also the $Z^\circ$ leptonic partial width, $\Gamma_\ell$. iii) We take into account small non linearity effects in the theoretical formulae. iv) We use the most recent results on the various precision observables.

# 2 Role of the likelihood ratio in reporting results of searches

We begin by discussing the role of the likelihood ratio in constraining the Higgs mass. Let us assume that through the information coming from precision measurements we have obtained $f(m_H \,|\, ind.)$. A natural question to ask is then how this p.d.f. should be modified in order to take into account the knowledge that the Higgs boson has not been observed at LEP for center of mass energies up to the highest available. To answer this question let us discuss first an ideal case. We consider a search for Higgs production in association with a particle of negligible width in an experimental situation of "infinite" luminosity, perfect efficiency and no background whose outcome was no candidate. In this situation we are sure that all mass values below a sharp kinematical limit $M_K$ are excluded. This implies that: a) the p.d.f. for $M_H$ must vanish below $M_K$; b) above $M_K$ the relative probabilities cannot change, because there is no sensitivity in this region, and then the experimental results cannot give information over there. For example, if $M_K$ is 110 GeV, then $f(200\,\mathrm{GeV})/f(120\,\mathrm{GeV})$ must remain constant before and after the new piece of information is included. In this ideal case we have then

$$f(m_H \,|\, dir.\,\&\, ind.) = \begin{cases} 0 & m_H < M_K \\ \frac{f(m_H \,|\, ind.)}{\int_{M_K}^{\infty} f(m_H \,|\, ind.)\,\mathrm{d}m_H} & m_H \geq M_K\,, \end{cases} \tag{1}$$

where the integral at denominator is just a normalization coefficient.

More formally, this result can be obtained making explicit use of the Bayes' theorem. Applied to our problem, the theorem can be expressed as follows (apart from a normalization constant):

$$f(m_H \,|\, dir.\,\&\, ind.) \propto f(dir. \,|\, m_H) \cdot f(m_H \,|\, ind.)\,, \tag{2}$$

where $f(dir \,|\, m_H)$ is the so called likelihood. In the idealized example we are considering now, $f(dir \,|\, m_H)$ can be expressed in terms of the probability of observing zero candidates in an experiment sensitive up to a $M_K$ mass for a given value $m_H$, or

$$f(dir. \,|\, m_H) = f(\text{"zero cand."} \,|\, m_H) = \begin{cases} 0 & m_H < M_K \\ 1 & m_H \geq M_K\,. \end{cases} \tag{3}$$

In fact, we would expect an "infinite" number of events if $M_H$ were below the kinematical limit. Therefore the probability of observing nothing should be zero. Instead, for $M_H$ above $M_K$, the condition of vanishing production cross section and no background can only yield no candidates.

Consider now a real life situation. In this case the transition between Higgs mass values which are impossible to those which are possible is not so sharp. In fact because of physical reasons (such as threshold effects and background) and experimental reasons (such as luminosity and efficiency) we cannot be really sure about excluding values close to the kinematical limit, nevertheless the ones very far from $M_K$ are ruled out. Furthermore, the kinematical limit is in general not sharp. In the case of Higgs production at LEP the dominant mode is the Bjorken process $e^+e^- \to H + Z^\circ$. Indeed, this reaction does not have a sharp kinematical limit at $\sqrt{s} - M_Z$ (minus a negligible kinetic energy), due to the large total width of the $Z^\circ$. The effective kinematical limit ($M_{K_{eff}}$) depends on the available integrated luminosity and could reach up to the order of $\approx \sqrt{s} - M_Z + \mathcal{O}(10\,\mathrm{GeV})$ for very high luminosity. Thus, in a real life situation we expect the ideal step function likelihood of Eq. (3) to be replaced by a smooth curve which goes to zero for low masses. Concerning, instead, the region of no experimental sensitivity, $M_H \gtrsim M_{K_{eff}}$, the likelihood is expected to go to a value independent on the Higgs mass that however is different from that of the ideal case, i.e. 1, because of the presence of the background.

In order to combine the various pieces of information easily it is convenient to replace the likelihood by a function that goes to 1 where the experimental sensitivity is lost [3]. Because constant factors do not play any role in the Bayes' theorem this can be achieved by dividing the likelihood by its value calculated for very large Higgs mass values where no signal is expected, i.e. the case of pure background. This likelihood ratio, $\mathcal{R}$, can be seen as the counterpart, in the case of a real experiment, of the step function of Eq. (3). Therefore, the Higgs mass p.d.f. that takes into account both direct search and precision measurement results can be written as

$$f(m_H \,|\, dir.\,\&\,ind.) = \frac{\mathcal{R}(m_H)\, f(m_H \,|\, ind.)}{\int_0^\infty \mathcal{R}(m_H)\, f(m_H \,|\, ind.)\, \mathrm{d}m_H}\ . \tag{4}$$

In Eq. (4) $\mathcal{R}$, namely the information from the direct searches, acts as a shape distortion function of $f(m_H \,|\, ind.)$. As long as $\mathcal{R}(m_H)$ is 1, the shape (and therefore the relative probabilities in that region) remains unchanged, while $\mathcal{R}(m_H) \to 0$ indicates regions where the p.d.f. should vanish. A *conventional* limit can be derived by the $\mathcal{R}$ function alone in the transition region between the region of firm exclusion ($\mathcal{R} \to 0$) and the region of insensitivity ($\mathcal{R} \to 1$). However this limit can only have the meaning of a 'sensitivity bound' [4], and cannot be a probabilistic limit which tells us how much we are confident that the Higgs mass is above a certain value. To express consistently our confidence we need to pass necessarely through (4).

One should notice that $\mathcal{R}(m_H)$ can also assume values larger than 1 for Higgs mass values below the kinematical limit. This situation corresponds to a number of observed candidate events larger than the expected background. In this case the role played by $\mathcal{R}(m_H)$ is to stretch $f(m_H \,|\, ind.)$ below the effective kinematical limit and this might even prompt a claim for a discovery if $\mathcal{R}$ becomes sufficiently large for the probability of $M_H$ in that region to get very close to 1.

# 3 Higgs mass inference from precision measurements

We are going to construct $f(m_H \,|\, ind.)$ employing the three observables, $s^2_{eff}$, $M_W$ and $\Gamma_\ell$. These quantities are the most sensitive to the Higgs mass and also very accurate measured. The most convenient way to approach the problem is to make use of the simple parameterization proposed in Ref.[5] and updated in Ref.[6] where $s^2_{eff}$, $M_W$ and $\Gamma_\ell$ are written as functions of $M_H$, $M_t$, $\alpha_s$ and the hadronic contribution to the running of the electromagnetic coupling:

$$s^2_{eff} = (s^2_{eff})_\circ + c_1 A_1 + c_2 A_2 - c_3 A_3 + c_4 A_4\,, \tag{5}$$

$$M_W = M_W^\circ - d_1 A_1 - d_5 A_1^2 - d_2 A_2 + d_3 A_3 - d_4 A_4\,, \tag{6}$$

$$\Gamma_\ell = \Gamma_\ell^\circ - g_1\, A_1 - g_5\, A_1^2 - g_2\, A_2 + g_3\, A_3 - g_4\, A_4\,. \tag{7}$$

In the above equations $A_1 \equiv \ln(M_H/100\,\mathrm{GeV})$, $A_2 \equiv [(\Delta\alpha)_h/0.0280 - 1]$, $A_3 \equiv [(M_t/175\,\mathrm{GeV})^2 - 1]$ and $A_4 \equiv [(\alpha_s(M_Z)/0.118 - 1]$, where $M_t$ is the top quark mass, $\alpha_s(M_Z)$ is the strong coupling constant and $(\Delta\alpha)_h$ is the five-flavor hadronic contribution to the QED vacuum polarization at $q^2 = M_Z^2$. $(s^2_{eff})_\circ$, $M_W^\circ$ and $\Gamma_\ell^\circ$ are (to excellent approximation) the theoretical results obtained at the reference point $(\Delta\alpha)_h = 0.0280$, $M_t = 175\,\mathrm{GeV}$, and $\alpha_s(M_Z) = 0.118$ while the values of the coefficients $c_i$, $d_i$ and $g_i$ are reported in Tables 3-5 of Ref.[6] for three different renormalization scheme. Formulae (5–7) are very accurate for $75 \lesssim M_H \lesssim 350$ GeV with the other parameters in the ranges $170 \lesssim M_t \lesssim 181$ GeV, $0.0273 \lesssim (\Delta\alpha)_h \lesssim 0.0287$, $0.113 \lesssim \alpha_s(M_Z) \lesssim 0.123$. In this case they reproduce the exact results of the calculations of Refs.([7],[8],[6]) with maximal errors of $\delta s^2_{eff} \sim 1 \times 10^{-5}$, $\delta M_W \lesssim 1$ MeV and $\delta\Gamma_\ell \lesssim 3$ KeV, which are all very much below the experimental accuracy. Outside the above range, the deviations increase but remain very small for larger Higgs mass, reaching about $3 \times 10^{-5}$, 3 MeV, and 4 KeV at $M_H = 600$ GeV for $s^2_{eff}$, $M_W$, $\Gamma_\ell$, respectively.

Formula (5) can be seen as providing an indirect measurement of $A_1 = [s^2_{eff} - (s^2_{eff})_\circ - c_2 A_2 + c_3 A_3 - c_4 A_4]/c_1$, while (6) and (7) of the quantities $Y = M_W^\circ - M_W - d_2 A_2 + d_3 A_3 - d_4 A_4$ and $Z = \Gamma_\ell^\circ - \Gamma_\ell - g_2 A_2 + g_3 A_3 - g_4 A_4$, respectively, all three variables being described by Gaussian p.d.f.'s.

In Ref.[1] we considered only the relations (5) and (6) and used the fact that the non linearity effect given by the $d_5$ coefficient is small (in the $\overline{MS}$ scheme $d_1 = 5.79 \cdot 10^{-2}$, $d_5 = 8.0 \cdot 10^{-3}$) to linearize (6) in $A_1$ in order to directly obtain a $A_1$ determination also from $M_W$. Here, instead, we take into account exactly this non linearity effect, in view also of the fact that the uncertainty on $M_W$ shrank. In Fig. 1 we plot the p.d.f. of $A_1$ using formulae (5–7) and the input quantities specified in the next section. The inference from $s^2_{eff}$, $M_W$, and $\Gamma_\ell$, separately, and from their combination is shown. The quadratic expression in (6) and (7) for $M_W$ and $\Gamma_\ell$, respectively, gives rise to an unphysical peak on the left side of the plot for values of the Higgs mass where formulae (5–7) are not valid. Neverthless, as shown in the bottom plot, when the total combination is taken these unphysical peaks disappear and the inference obtained is concentrated in the physical region.

The $A_1$, $Y$ and $Z$ determination are clearly correlated, therefore one has to built a covariance matrix. This can be easily done because formulae (5–7) are linear in the common terms $\underline{X} \equiv \{M_t,\, \alpha_s(M_Z),\, (\Delta\alpha)_h\}$. The likelihood of our indirect measurements $\underline{\Theta} \equiv \{A_1,\, Y,\, Z\}$ is then a three dimensional correlated normal with covariance matrix

$$V_{ij} = \sum_l \frac{\partial \Theta_i}{\partial X_l} \cdot \frac{\partial \Theta_j}{\partial X_l} \cdot \sigma^2(X_l) \tag{8}$$

or

$$f(\underline{\theta} \,|\, \ln(m_H)) \propto e^{-\chi^2/2} \tag{9}$$

where $\chi^2 = \underline{\Delta}^T \mathbf{V}^{-1} \underline{\Delta}$ with $\underline{\Delta}^T = \{a_1 - \ln(m_H/100),\, y - d_1 \ln(m_H/100) - d_5 \ln^2(m_H/100),\, z - g_1 \ln(m_H/100) - g_5 \ln^2(m_H/100)\}$. Using Bayes' theorem the likelihood (9) can be turned into a p.d.f. through the choice of a prior. The natural choice is a uniform prior in $\ln(m_H)$, since it is well understood that radiative corrections measure this quantity (for this reason the likelihood (9) has been expressed in terms of $\ln(m_H)$). Moreover, this choice recovers the result of Ref. [1], which was obtained as uncertainty propagation without explicit use of a prior on the Higgs mass. The uniform prior in $\ln(m_H)$ implies that $f(\ln(m_H/100) \,|\, ind.)$ is just the normalized likelihood (9). Using standard probability calculus we can express our results as a p.d.f. of $M_H$:

$$f(m_H \,|\, ind.) = \frac{m_H^{-1}\, e^{-(\chi^2/2)}}{\int_0^\infty m_H^{-1}\, e^{-(\chi^2/2)}\, \mathrm{d}m_H}. \tag{10}$$

The theoretical coefficients, $c_i$, $d_i$, $g_i$, depends on the renormalization scheme in which the relevant calculations are done and their numerical spread is usually taken as an indication of the theory uncertainty of the

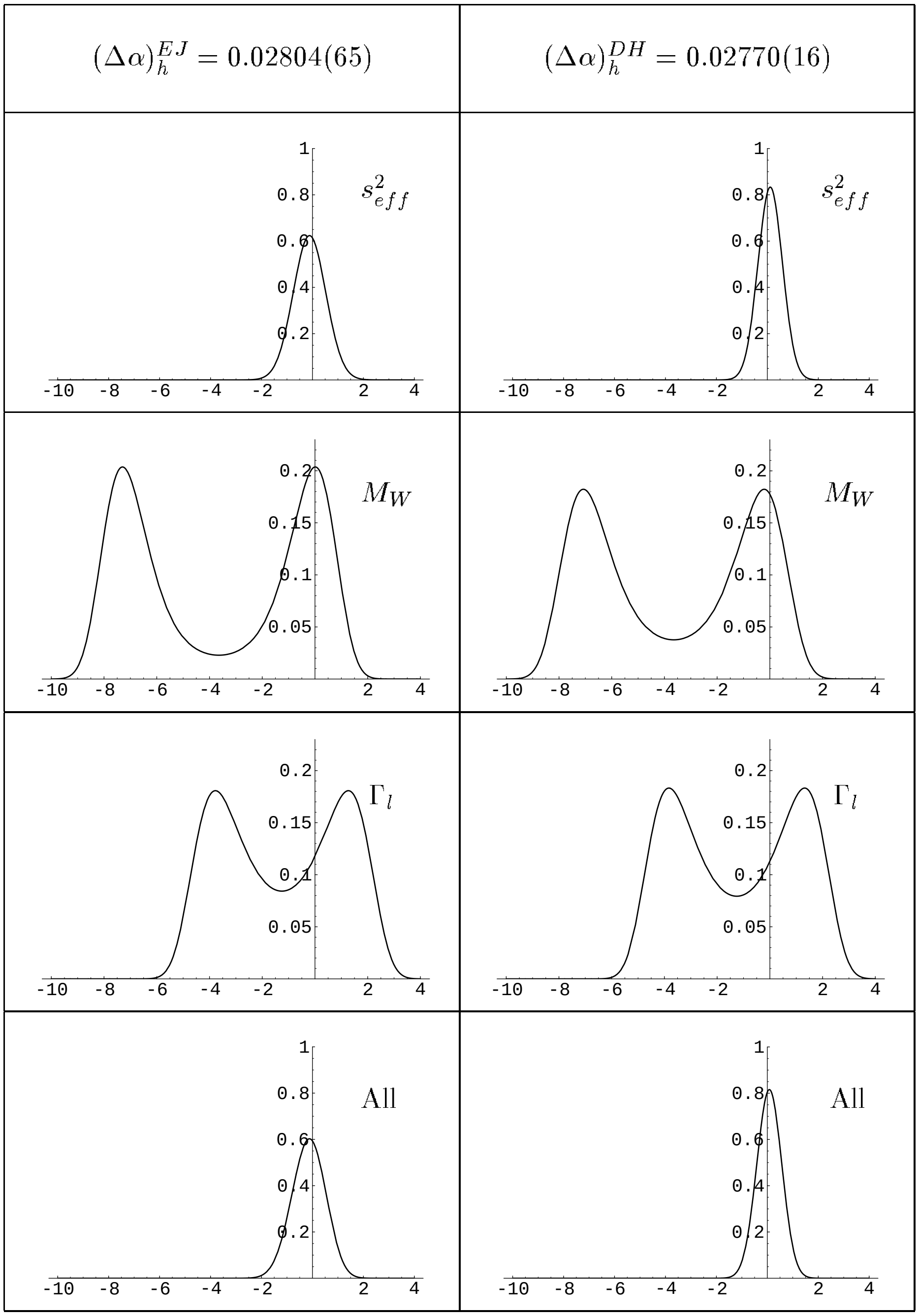


Figure 1: Probability density function of $A1 \equiv \ln(M_H/100\,\mathrm{GeV})$ obtained from $s^2_{eff}$, $M_W$, $\Gamma_l$ and their combination for different values of $(\Delta\alpha)_h$ (see comment on the text about low mass peaks).

calculations. A way to take into account this uncertainty is to consider different inferences, each conditioned by a given set of parameters, labelled by $R_i$. For each renormalization scheme $R_i$ we can construct a $f(m_H \,|\, ind., R_i)$ and obtain a p.d.f. "integrated" over the possible schemes, through

$$f(m_H \,|\, ind.) = \sum_i f(m_H \,|\, ind., R_i) \cdot f(R_i)\,, \tag{11}$$

where $f(R_i)$ is the probability assigned to each scheme ($f(R_i) = 1/3 \;\forall\, i$). The calculation of expectation value and variance is then straightforward or

$$\begin{aligned} \mathrm{E}[M_H] &= \frac{1}{3}\sum_i \mathrm{E}[M_H \,|\, R_i] & (12)\\ \sigma^2(M_H) &= \frac{1}{3}\sum_i \sigma^2(M_H \,|\, R_i) + \frac{1}{3}\sum_i \mathrm{E}^2[M_H \,|\, R_i] - \mathrm{E}^2[M_H] & (13)\\ &= \frac{1}{3}\sum_i \sigma^2(M_H \,|\, R_i) + \sigma_E^2\,, & (14) \end{aligned}$$

where $\sigma_E$ indicates the standard deviation calculated from the dispersion of the expected values. In Ref. [1] we have shown that results almost identical are obtained if one employs for the coefficients $c_i$, $d_i$ an average value and a standard deviation evaluated from the dispersion of the values obtained from the various renormalization schemes.

## 4 Results

The experimental inputs we use to construct $f(m_H \,|\, ind.)$ are [9]: $s^2_{eff} = 0.23151 \pm 0.00017$, $M_W = 80.394 \pm 0.042$ GeV, $\Gamma_\ell = 83.96 \pm 0.09$ MeV, $M_t = 174.3 \pm 5.1$GeV, $\alpha_s(M_Z) = 0.119 \pm 0.003$. Concerning $(\Delta\alpha)_h$ in the recent years there has been a lot of activity on this subject with several evaluations. They can be classified as of two types: i) the most phenomenological analyses, that rely on the use of all the available experimental data on the hadron production in $e^+\,e^-$ annihilation and on perturbative QCD (pQCD) for the high energy tail ($E \geq 40$ GeV) of the dispersion integral. The reference value in this approach is $(\Delta\alpha)_h^{EJ} = 0.02804 \pm 0.00065$ [10]. ii) The so called "theory driven" analyses [11, 12], that differ from the previous most phenomenological ones mainly by the use of pQCD down to energies of the order of a few GeV and by the treatment of old experimental data in regions where pQCD is not applicable. The combination of these two factors gives a result for $(\Delta\alpha)_h$ that differ from type i) one by a drastically reduced uncertainty but, at the same time, a lower central value. The most stringent evaluation of these theory oriented analyses is $(\Delta\alpha)_h^{DH} = 0.02770 \pm 0.00016$ [12], that we use as reference value for this kind of approach. At the moment there is no definite argument for choosing one or other of the two approaches. The results are absolutely compatible to each other. However, the numerical difference between central values and uncertainties is such that it prevents an easy estimation of the effect of choosing one value instead of the other. For these reasons we decided to present our results for the values of $(\Delta\alpha)_h$ given by $(\Delta\alpha)_h^{EJ}$ and $(\Delta\alpha)_h^{DH}$ separately.

The values of the $\mathcal{R}$ function that enters in Eq. (4) has been provided by the LEP Higgs Working Group [2]: they take into account the Higgs searches by all four LEP collaborations for center of mass energy up to $\sqrt{s} = 196$ GeV.

Table 1 summarizes the result of our analysis in terms of various convenient parameters of the distribution. We present the two cases $(\Delta\alpha)_h = (\Delta\alpha)_h^{EJ}$ and $(\Delta\alpha)_h = (\Delta\alpha)_h^{DH}$ and report all values in TeV to reduce the number of digits to the significant ones. The shape of the p.d.f. with and without the inclusion of the direct search information is presented in Fig. 2. From this figure one can notice that, in the case of $f(ind. \,|\, m_H)$, the use of a higher central value for $(\Delta\alpha)_h$ (i.e. $(\Delta\alpha)_h^{EJ}$) tends to concentrate more the probability towards smaller values of $M_H$. As a consequence the analysis based on $(\Delta\alpha)_h^{EJ}$ gives results for the standard deviation of the $M_H$ p.d.f. and 95 % probability upper limit, $M_H^{95}$, very close to those obtained using $(\Delta\alpha)_h^{DH}$ regardless the fact that the uncertainty on $(\Delta\alpha)_h^{EJ}$ is approximately 4 times larger than that of $(\Delta\alpha)_h^{DH}$. As expected, the inclusion of the direct search information in the Higgs mass probability analysis drifts the p.d.f. towards higher values of $M_H$, changing its shape such that the probability of $M_H$ values below 110 GeV drops to $\approx 9\,\%$. The various parameters of the distribution (expected value, standard deviation, mode $(\hat{M}_H)$ and median $(M_H^{50})$) are not very sensitive to the values of the hadronic contribution to the vacuum polarization. Also in both cases, $\approx 80\,\%$ of the probability is concentrated in the region $M_H < 0.20$ TeV. Instead the choice of $(\Delta\alpha)_h$ affects the tail of the distribution with $(\Delta\alpha)_h^{EJ}$ producing a much longer one.

| | $(\Delta\alpha)_h = 0.02804(65)$ | | $(\Delta\alpha)_h = 0.02770(16)$ | |
|---|---|---|---|---|
| | $(ind.)$ | $\begin{pmatrix} ind. \\ + \\ dir. \end{pmatrix}$ | $(ind.)$ | $\begin{pmatrix} ind. \\ + \\ dir. \end{pmatrix}$ |
| $\mathbf{E[M_H]}$/TeV | 0.10 | **0.17** | 0.12 | **0.16** |
| $\sigma(\mathbf{M_H})$/TeV | 0.07 | **0.06** | 0.06 | **0.05** |
| $\hat{M}_H$/TeV | 0.06 | 0.11 | 0.09 | 0.11 |
| $\mathbf{M_H^{50}}$/TeV | 0.09 | **0.15** | 0.11 | **0.14** |
| $P(M_H \leq 0.11\,\mathrm{TeV})$ | 67 % | 8.9 % | 51 % | 8.5 % |
| $P(M_H \leq 0.13\,\mathrm{TeV})$ | 76 % | 34 % | 65 % | 35 % |
| $P(M_H \leq 0.20\,\mathrm{TeV})$ | 92 % | 79 % | 91 % | 83 % |
| $\mathbf{M_H^{95}}$/TeV; $P(M_H \leq M_H^{95}) \approx 0.95$ | 0.23 | **0.29** | 0.23 | **0.26** |
| $\mathbf{M_H^{99}}$/TeV; $P(M_H \leq M_H^{99}) \approx 0.99$ | 0.33 | **0.41** | 0.31 | **0.34** |
| $\begin{cases} M_1/\mathrm{TeV};\ P(M_H < M_1) \approx 0.16 \\ M_2/\mathrm{TeV};\ P(M_H > M_2) \approx 0.16 \end{cases}$ | $\begin{cases} 0.05 \\ 0.15 \end{cases}$ | $\begin{cases} 0.12 \\ 0.22 \end{cases}$ | $\begin{cases} 0.07 \\ 0.17 \end{cases}$ | $\begin{cases} 0.12 \\ 0.20 \end{cases}$ |

Table 1: Summary of the direct plus indirect information.

## 5 Conclusions

The likelihood ratio $\mathcal{R}$ is a form to report the experimental results that allows an easy combination of the information from Higgs search experiments with that coming from precision measurements and accurate calculations in order to constraint jointly the Higgs mass. The $\mathcal{R}$ function is very convenient for comparing and combining the various informations and it has also an intuitive interpretation because of its limit to the step function of the ideal case. It can be also seen as the p.d.f. for the Higgs mass in the case of complete lack of other information, i.e. when one assumes $f(m_H \,|\, ind.) = 1$. Although in this case the normalization integral in Eq. (4) is mathematically "infinite", still the relative probabilities of different intervals of mass regions

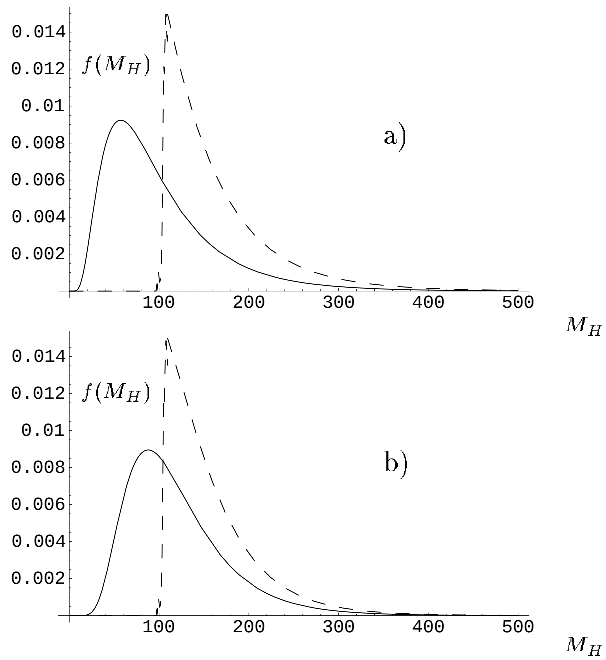


Figure 2: Probability distribution functions using only indirect information (solid line) and employing also the experimental results from direct searches (dashed one): a) $(\Delta\alpha)_h = 0.02804(65)$; b) $(\Delta\alpha)_h = 0.02770(16)$.

are perfectly well defined. However, it is obvious that it is not possible to evaluate from the $\mathcal{R}$ function alone a probabilistic lower limit. This can only be done when $\mathcal{R}$ is combined with $f(m_H \,|\, ind.)$ obtained from precision measurements, which has the important role of making large values of $M_H$ impossible, thus making the final p.d.f. normalizable.

The analysis we have performed clearly shows that a heavy Higgs scenario is highly disfavored, the data preferring a $\log_{10}(M_H/\mathrm{GeV}) \approx \mathcal{O}(2)$ . Note that our results are derived under the assumption of the validity of the SM and rely on the input experimental and theoretical quantities stated in the text. All these assumptions seem to us very reasonable. In particular, we don't consider strong evidence against the SM the fact that about one half of $f(m_H \,|\, ind.)$ is eaten up by the LEP direct search.

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