Exercise:

Determine the tracking efficiency for charged pions as a function of momentum in the KLOE detector exploiting the decay:

$\phi \rightarrow \pi^+\pi^-\pi^0$

Proposed exercises

In DAFNE operations for KLOE-2 experiment:

Top-up injection
20 mA injections at a rate of 2 Hz with 60% duty cycle
Veto of KLOE-2 DAQ for 50ms at each single injection
Dead time DAQ 4 μs
Trigger rate ~ 8 kHz

Determine DAQ inefficiency

Goodness-of-fit test : P-value

Test of hypothesis H_0 (**null hypothesis**)

Fit done (best estimate of θ_i) => t* obtained for the test statistics

Suppose pdf of test statistics t known => $f(t|H_0)$

P-value $$p_0 = \int_{t^*}^{\infty} f\left(t/H_0\right) dt$$

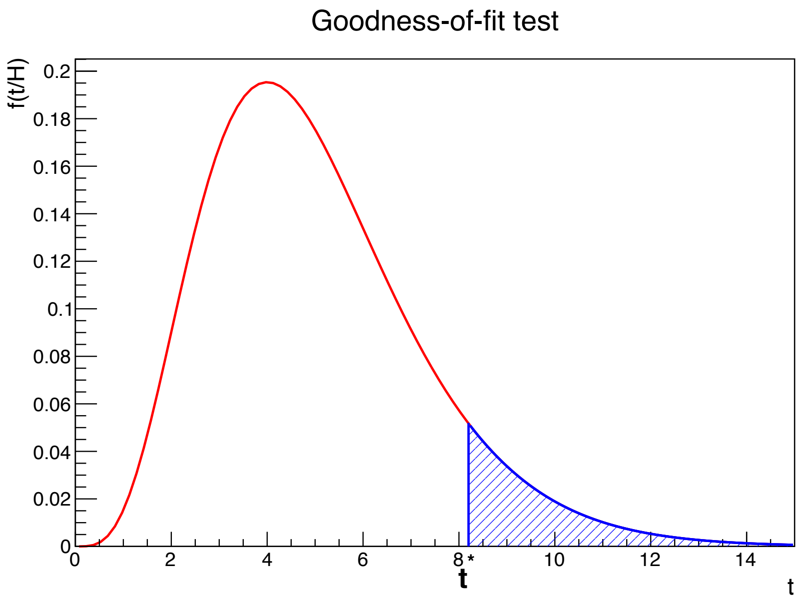

FIGURE 9. χ^2 distribution for 5 degrees of freedom. The case of $t^* = 8.2$ is illustrated. The blue hatched area correspond to the p_0 value.

Goodness-of-fit test : P-value

Meaning of P-value

$$p_0 = \int_{t^*}^{\infty} f(t/H_0)\, dt$$

Probability that - if H_0 is true - the result t of the experiment will fluctuate more than t*.

Repeating the experiment N times, p_0 is the fraction in which we get t>t*

$t > t^*$. If this number is low, either the hypothesis is wrong or there was an anomalous large fluctuation. In other words we are on the right tail of the distribution. So we can put a limit on the acceptable values of p_0: if p_0 is less than, say 5% or 1% we will reject the null hypothesis, if it is larger than the same limit we will say on the contrary that the null hypothesis is corroborated. The choice of the limit (5, 1 or 0.1%) depends on the nature of the problem, and on the degree we decide to be severe with the results we are considering.

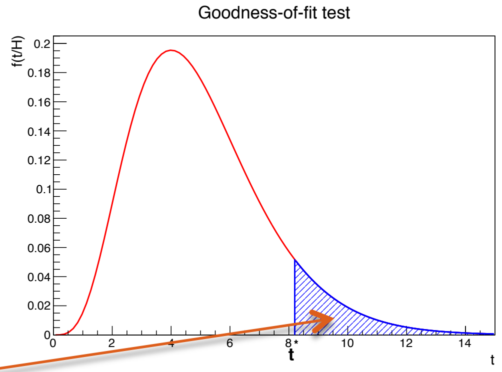

FIGURE 9. χ^2 distribution for 5 degrees of freedom. The case of $t^* = 8.2$ is illustrated. The blue hatched area correspond to the p_0 value.

$p_0 \approx 0$ => rejection of null H_0 hypothesis,
i.e. scarce agreement data-theory

Goodness-of-fit test : P-value

Meaning of P-value

$$p_0 = \int_{t^*}^{\infty} f\,(t/H_0)\,dt$$

f(t) pdf of t

g(F) pdf of F primitive of f

The P-value is a random variable itself uniformly distributed between 0 and 1:

$$g(F)dF = f(t)dt$$

by definition $dF/dt = f(t)$ $\quad g(F) = \frac{f(t)}{dF/dt} = \frac{f(t)}{f(t)} = 1$

All p-values are equally probable! e.g. $p_0 \approx 0$ or $p_0 \approx 1$

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If H_0 is true, if H_0 is false usually $p_0 \approx 0$.

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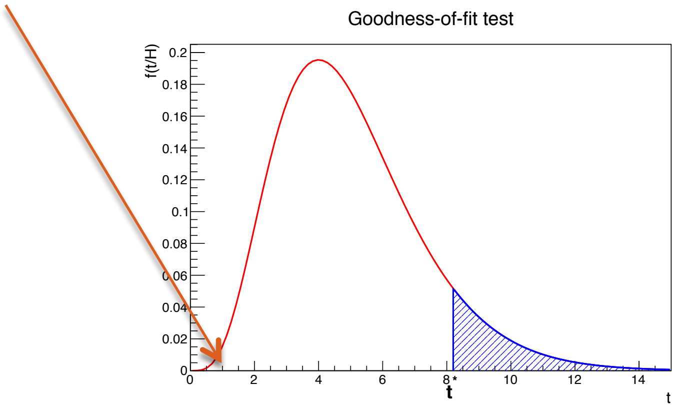
What if $p_0 \approx 1$?

$p_0 \approx 1$ => underfluctuations of experimental points or overestimate of the uncertainties , i.e. scarce self-consistency of data

2-tails test vs 1-tail test

e.g. Accept H_0 if $5\% < p_0 < 95\ \%$



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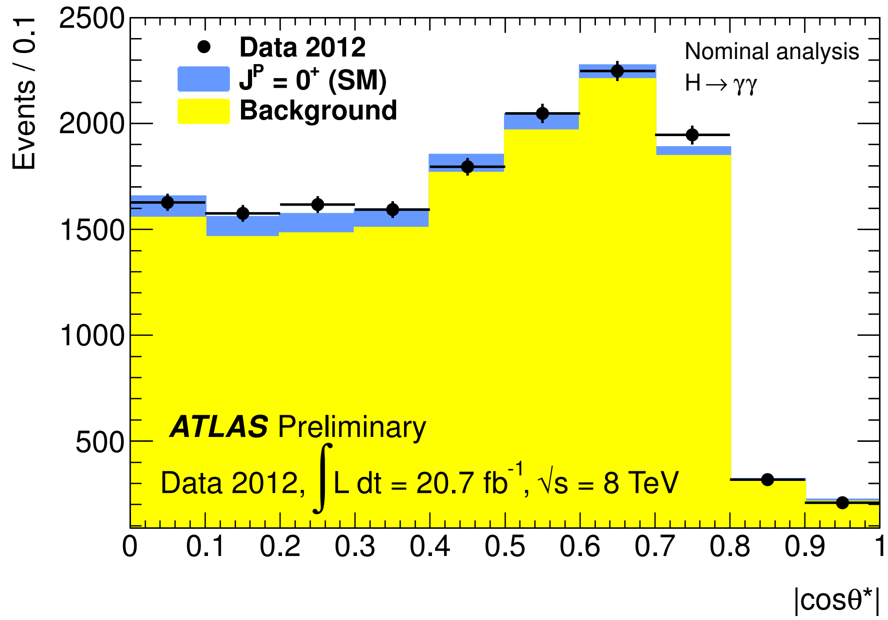
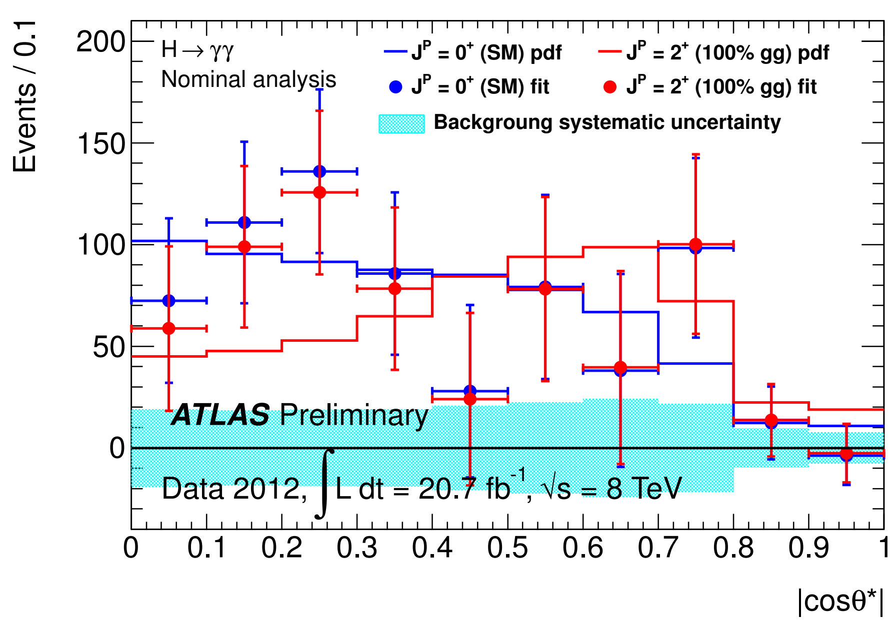
Example of two alternate hypotheses H_0 and H_1

In the two-body decay **H → γ γ**, the spin information is extracted from the distribution of the polar angle $\theta*$ of the photons with respect to the z-axis of the Collins-Soper frame.

$$\cos\theta^* = \frac{\sinh(\eta_{\gamma_1} - \eta_{\gamma_2})}{\sqrt{1 + \left(p_{\mathrm{T}}^{\gamma\gamma}/m_{\gamma\gamma}\right)^2}} \cdot \frac{2p_{\mathrm{T}}^{\gamma_1} p_{\mathrm{T}}^{\gamma_2}}{m_{\gamma\gamma}^2}$$

With this choice, the impact of initial state radiation is expected to be minimized and a better discrimination power compared to other choices of axis, such as the beam axis or the boost axis of the particle, is achieved. A spin-0 particle decays isotropically in its rest frame; before any acceptance cuts, the distribution dN/d cos $\theta*$ is thus uniform. The corresponding distribution for a spin-2 particle follows a combination of Wigner functions for the production and decay whose probabilities are specified in particular models.





Example of two alternate hypotheses H_0 and H_1

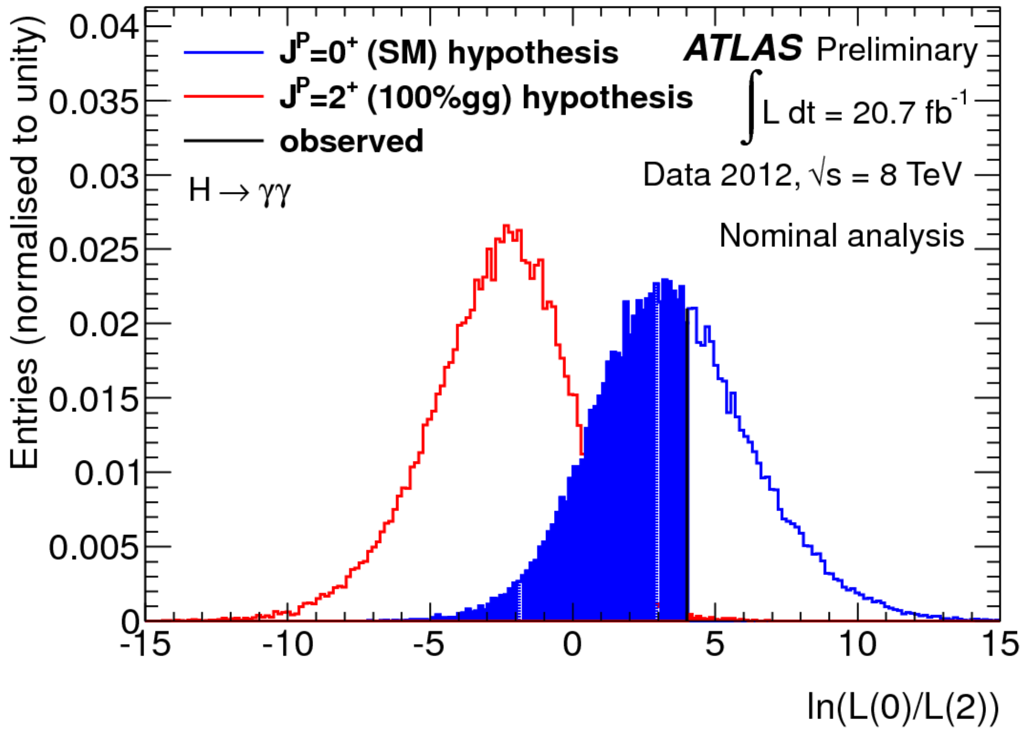


FIGURE 10. One of the results of the ATLAS experiment for the study of the spin of the Higgs boson. The pdf's of the test statistics q (defined as the logarithm of the likelihood ratio) are shown for two alternative hypotheses: spin 0 and spin 2. The black vertical line corresponds to the experimental value of the test statistics. The blue hatched area is the 1-p-value. (taken from ATLAS Collaboration, ATLAS-CONF-2013-029).

Two alternate hypotheses H_0 and H_1

Define t_{cut}

If $t^* < t_{cut}$ => accept the null hypothesis

If $t^* > t_{cut}$ => accept the alternate hypothesis

By applying a cut we accept type-I and type-II errors (similarly to single events…)

$$\alpha = \int_{t_{cut}}^{\infty} f(t/H_0)dt$$
$$\beta = \int_{-\infty}^{t_{cut}} f(t/H_1)dt$$

Apply Neyman-Pearson lemma, i.e. construct a Likelihood ratio variable as best test statistics