CPT- AND LORENTZ-SYMMETRY BREAKING: A REVIEW

Ralf Lehnert
Center for Theoretical Physics
Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A

Abstract

The breakdown of spacetime symmetries has recently been identified as a promising candidate signal for underlying physics, possibly arising through quantum-gravitational effects. This paper gives an overview over various aspects of CPT- and Lorentz-violation research. Particular emphasis is given to the interplay between CPT, Lorentz, and translation symmetry, mechanisms for CPT and Lorentz breaking, and the construction of a low-energy quantum-field description of such effect. This quantum field framework, called the SME, is employed to determine possible phenomenological consequences of CPT and Lorentz violation for neutral-meson interferometry.

1 Introduction

Although phenomenologically successful, the Standard Model of particle physics leaves unanswered a variety of theoretical questions. At present, a significant amount of theoretical work is therefore directed toward the search for an underlying theory that includes a quantum description of gravity. However, observational tests of such ideas face a major obstacle of practical nature: most quantum-gravity effects in virtually all leading candidate models are expected to be extremely small due to Planck-scale suppression. For example, low-energy measurements are likely to require sensitivities of at least one part in 10^{17}. This paper gives an overview of a recent approach to this issue that involves spacetime symmetries.

The presumed minute size of candidate quantum-gravity effects requires a careful choice of experiments. A promising idea that one may pursue is testing physical laws that satisfy three primary criteria. First, one should consider fundamental laws that are believed to hold *exactly* in established physics. Any measured deviations would then definitely indicate qualitatively new physics. Second, the likelihood of observing such effects is increased by testing laws that may be *violated* in credible candidate fundamental theories. Third, from a practical point of view, these laws must be amenable to *ultrahigh-precision* tests.

One example of a physics law that satisfies all of these criteria is CPT invariance. [1] As a brief reminder, this law requires that the physics remains unchanged under the combined operations of charge conjugation (C), parity inversion (P), and time reversal (T). Here, the C transformation connects particles and antiparticles, P corresponds to a spatial reflection of physics quantities through the coordinate origin, and T reverses a given physical process in time. The Standard Model of particle physics is CPT-invariant by construction, so that the first criterion is satisfied. With regards to criterion two, we mention that a variety of approaches to fundamental physics can lead to CPT violation. Such approaches include strings, [2] spacetime foam, [3] nontrivial spacetime topology, [4] and cosmologically varying scalars. [5] The third of the above criteria is met as well. Consider, for instance, the conventional figure of merit for CPT conservation in the kaon system: its value lies currently at 10^{-18}, as quoted by the Particle Data Group. [6]

Since the CPT transformation relates a particle to its antiparticle, one would expect that CPT invariance implies a symmetry between matter and antimatter. One can indeed prove that the magnitude of the mass, charge, decay rate, gyromagnetic ratio, and other intrinsic properties of a particle are exactly equal to those of its antiparticle. This proof can be extended to systems of particles and their dynamics. For instance, atoms and anti-atoms must exhibit identical spectra and a particle-reaction process and its CPT-conjugate process must possess the same reaction cross section. It follows that experimental matter–antimatter comparisons can serve as probes for the validity of CPT invariance. In particular, the extraordinary sensitivities offered by meson interferometry yield high-precision tools in this context.

This paper is organized as follows. Section 2 discusses the interplay of various spacetime symmetries. Two mechanisms for CPT and Lorentz breakdown in Lorentz symmetric underlying theories are reviewed in Sec. 3. The basic ideas behind the construction of the SME are contained in Sec. 4. Section 5 extracts CPT observables from the SME. In Sec. 6, we comment on CPT tests involving neutral-meson systems. A brief summary is presented in Sec. 7.

2 Spacetime symmetries and their interplay

Spacetime transformation fall into two classes: continuous and discrete. The continuous transformations include translations, rotations, and boosts. Examples of discrete transformations are C, P, and T discussed in the introduction. Suppose symmetry is lost under one or more of these transformations. It is then a natural question as to whether the remaining transformations can still remain symmetries, or whether the breaking of one set of spacetime symmetry is typically associated with the violation of other spacetime invariances. This sections contains a brief discussion of this issue.

Suppose translational symmetry is broken (one possible mechanism for this effect is discussed in the next section). Then, the generator of translations, which is the energy–momentum tensor $\theta^{\mu\nu}$, is typically no longer conserved. Would this also affect Lorentz symmetry? To answer this question, let us look at the generator for Lorentz transformations, which is given by the the angular-momentum tensor $J^{\mu\nu}$:

$$J^{\mu\nu} = \int d^3x \, \left(\theta^{0\mu}x^{\nu} - \theta^{0\nu}x^{\mu}\right). \tag{1}$$

Note that this definition contains the non-conserved energy–momentum tensor $\theta^{\mu\nu}$. It follows that in general $J^{\mu\nu}$ will exhibit a nontrivial dependence on time, so that the usual time-independent Lorentz-transformation generators do not exist. As a result, Lorentz symmetry is no longer assured. We see that (with the exception of special cases) translation-symmetry violation leads to Lorentz breakdown.

We next consider CPT invariance. The celebrated CPT theorem of Bell, Lüders, and Pauli states that CPT symmetry arises under a few mild assumptions through the combination of quantum theory and Lorentz invariance. If CPT symmetry is broken one or more of the assumptions necessary to prove the CPT theorem must be false. This leads to the obvious question which one of the fundamental assumptions in the CPT theorem should be dropped. Since both CPT and Lorentz invariance involve spacetime transformations, it is natural to suspect that CPT violation implies Lorentz-symmetry breakdown. This has recently been confirmed rigorously in Greenberg's "anti-CPT theorem," which roughly states that in any unitary, local, relativistic point-particle field theory CPT breaking implies Lorentz violation. [7, 8] Note, however, that the converse of this statement—namely that Lorentz breaking implies CPT violation—is not true in general. In any case, it follows that CPT tests also probe Lorentz invariance. *As a result, potential CPT violation in the kaon system would typically be direction and energy dependent.* We will confirm this result explicitly in Sec. 5. Other types of CPT violation would require further deviations from conventional physics.[1]

3 Sample mechanisms for spacetime-symmetry breaking

In the previous section, we have found that the violation of a particular spacetime symmetry can lead to the breaking of another spacetime invariance. However, the question of *how* exactly a translation-, Lorentz-, and CPT-invariant candidate theory can lead to the violation of a spacetime symmetry in the first place has thus far been left unaddressed. The purpose of this section is to provide some intuition about such mechanisms for spacetime-symmetry breaking in underlying physics. Of the various possible mechanisms mentioned in Sec. 1, we will focus on spontaneous CPT and Lorentz breakdown as well as CPT and Lorentz violation through varying scalars.

Spontaneous CPT and Lorentz breaking. The mechanism of spontaneous symmetry violation is well established in various subfields of physics, such as the physics of elastic media, condensed-matter physics, and elementary-particle theory. From a theoretical viewpoint, this mechanism is very attractive because the invariance is essentially violated through a non-trivial ground-state solution. The underlying dynamics of the system, which is governed by the hamiltonian, remains completely invariant under the symmetry. To gain intuition about spontaneous Lorentz and CPT violation, we will consider three sample systems, whose features will gradually lead us to a better understanding of the effect. Figure 1 contains an illustration supporting these three examples.

We first look at classical electrodynamics. Any electromagnetic-field configuration is associated with an energy density $V(\vec{E},\vec{B})$, which is given by

$$V(\vec{E},\vec{B}) = \tfrac{1}{2}\left(\vec{E}^2+\vec{B}^2\right) . \tag{2}$$

[1]One could consider violations of unitarity, so that the usual quantum-mechanical probability conservation no longer holds. See, for example, J. Bernabeu *et al.*contribution to this handbook.

Here, we have employed natural units, and $\vec{E}$ and $\vec{B}$ denote the electric and magnetic fields, respectively. Equation (2) determines the field energy of any given solution of the Maxwell equations. Note that if the electric field, or the magnetic field, or both are nonzero in some spacetime region, the energy stored in these fields will be strictly positive. The field energy can only vanish when both $\vec{E}$ and $\vec{B}$ are zero everywhere. The ground state (or vacuum) is usually identified with the lowest-energy configuration of a system. We see that in conventional electromagnetism the configuration with the lowest energy is the field-free one, so that the Maxwell vacuum is empty (disregarding Lorentz- and CPT-symmetric quantum fluctuations).

Second, let us consider the Higgs field, which is part of the phenomenologically very successful Standard Model of particle physics. As opposed to the electromagnetic field, the Higgs field is a scalar. In what follows, we may adopt some simplifications without distorting the features important in the present context. The expression for the energy density of our Higgs scalar ϕ in situations with spacetime independence is given by

$$V(\phi) = (\phi^2 - \lambda^2)^2 \,. \tag{3}$$

Here, λ is a constant. As in the electrodynamics case discussed above, the lowest possible field energy is zero. Note, however, that this configuration *requires* ϕ to be non-vanishing: $\phi = \pm\lambda$. It therefore follows that the vacuum for a system containing a Higgs-type field is not empty; it contains, in fact, a constant scalar field $\phi_{vac} \equiv \langle\phi\rangle = \pm\lambda$. In quantum physics, the quantity $\langle\phi\rangle$ is called the vacuum expectation value (VEV) of ϕ. One of the physical effects caused by the VEV of the Standard-Model Higgs is to give masses to many elementary particles. We remark that $\langle\phi\rangle$ is a scalar and does *not* select a preferred direction in spacetime.

We finally take a look at a vector field $\vec{C}$ (the relativistic generalization is straightforward) not contained in the Standard-Model. Clearly, there is no observational evidence for such a field at the present time, but fields like $\vec{C}$ frequently arise in approaches to more fundamental physics. In analogy to the Higgs case, we take its expression for energy density in cases with constant $\vec{C}$ to be

$$V(\vec{C}) = (\vec{C}^2 - \lambda^2)^2 \,. \tag{4}$$

Just as in the previous two examples, the lowest-possible energy is exactly

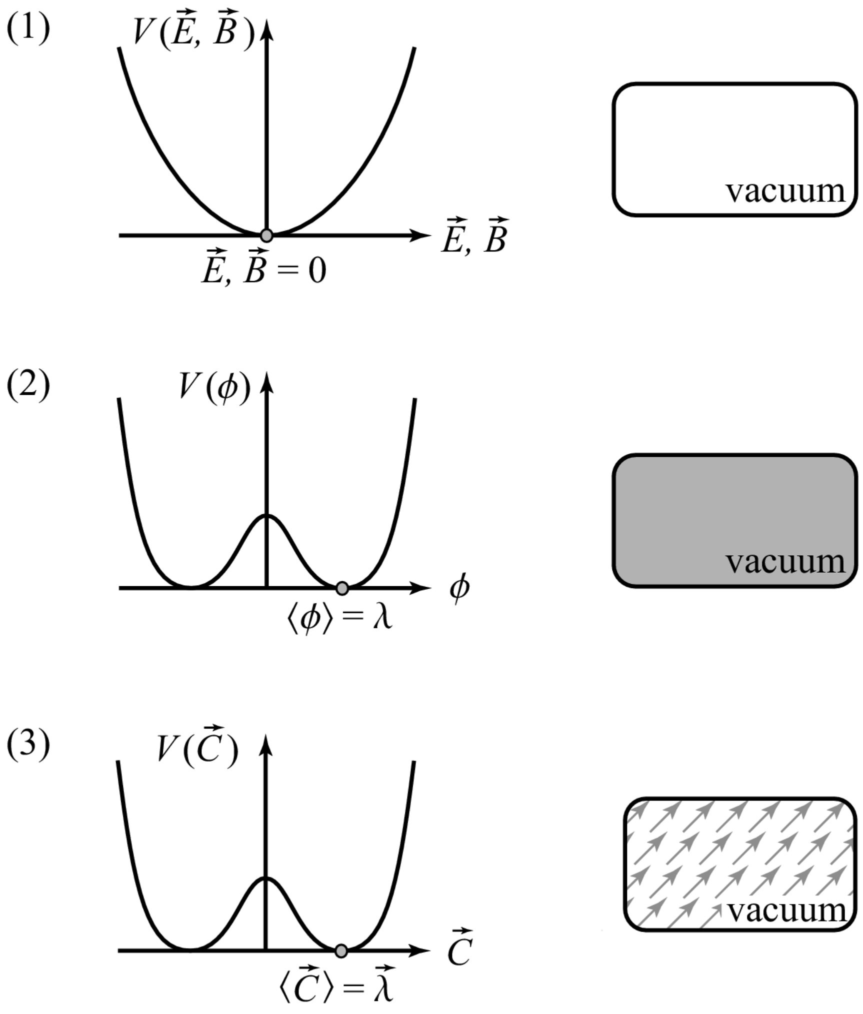

Figure 1: Spontaneous symmetry violation. In conventional electromagnetism (1), the lowest-energy state is attained for zero $\vec{E}$ and $\vec{B}$ fields. The vacuum remains essentially field free. For the Higgs-type field (2), interactions lead to an energy density $V(\phi)$ that forces a non-vanishing value of ϕ in the ground state. The vacuum is filled with a scalar condensate shown in gray. CPT and Lorentz invariance still hold (other, internal symmetries may be violated though). Vector fields occurring, for example, in string theory (3) can exhibit interactions similar to those of the Higgs requiring a nonzero field value in the lowest-energy state. The VEV of a vector field selects a preferred direction in the vacuum, which violates Lorentz and possibly CPT symmetry.

zero. As for the Higgs, this lowest energy configuration requires a nonzero $\vec{C}$. More specifically, we must demand $\vec{C}_{vac} \equiv \langle\vec{C}\rangle = \vec{\lambda}$, where $\vec{\lambda}$ is any constant vector satisfying $\vec{\lambda}^2 = \lambda^2$. Again, the vacuum does not remain empty, but it contains the VEV of our vector field. Because we have only considered constant solutions $\vec{C}$, $\langle\vec{C}\rangle$ is also spacetime independent (x dependence would lead to positive definite derivative terms in Eq. (4) raising the energy density). The true vacuum in the above model therefore contains an intrinsic direction determined by $\langle\vec{C}\rangle$ *violating rotation invariance and thus Lorentz symmetry.* We remark that interactions leading to energy densities like those in Eq. (4) are absent in conventional renormalizable gauge theories, but can be found in the context of strings, for example.

Spacetime-dependent scalars. A varying scalar, regardless of the mechanism driving the variation, typically implies the breaking of spacetime-translation invariance. [5] In Sec. 2 we have argued that translations and Lorentz transformations are closely linked in the Poincaré group, so that translation-symmetry violation typically leads to Lorentz breakdown. In the remainder of this section, we will focus on an explicit example for this effect.

Consider a system with a varying coupling $\xi(x)$ and scalar fields ϕ and Φ, such that the lagrangian $\mathcal{L}$ contains a term $\xi(x)\,\partial^\mu\phi\,\partial_\mu\Phi$. We may integrate the action for this system by parts (e.g., with respect to the first partial derivative in the above term) without affecting the equations of motion. An equivalent lagrangian $\mathcal{L}'$ would then be

$$\mathcal{L}' \supset -K^\mu\phi\,\partial_\mu\Phi\,. \tag{5}$$

Here, $K^\mu \equiv \partial^\mu\xi$ is an external nondynamical 4-vector, which selects a preferred direction in spacetime violating Lorentz symmetry. Note that for variations of ξ on cosmological scales, K^μ is constant locally to an excellent approximation—say on solar-system scales.

Intuitively, the violation of Lorentz symmetry in the presence of a varying scalar can be understood as follows. The 4-gradient of the scalar must be nonzero in some spacetime regions. This 4-gradient then selects a preferred direction in such regions (see Fig. 2). Consider, for instance, a particle that interacts with the scalar. Its propagation properties might be different in the directions parallel and perpendicular to the gradient. But physically inequivalent directions imply the violation of rotation invariance. Since rotations are contained in the Lorentz group, Lorentz symmetry must be broken.

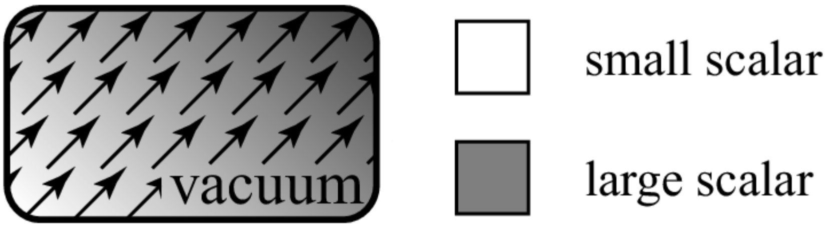

Figure 2: Lorentz violation through varying scalars. The background shade of gray corresponds to the value of the scalar: the lighter regions are associated with smaller values of the scalar. The gradient represented by the black arrows picks a preferred direction in the vacuum. It follows that Lorentz invariance is violated.

4 The Standard-Model Extension

To determine general low-energy manifestations of CPT and Lorentz violation and to identify specific experimental signatures for these effects, a suitable test model is needed. Many Lorentz tests are motivated and analyzed in purely kinematical frameworks allowing for small violations of Lorentz invariance. Examples are Robertson's framework and its Mansouri–Sexl extension, as well as the c^2 model and phenomenologically constructed modified dispersion relations. However, the CPT properties of these test models are unclear, and the lack of dynamical features severely restricts their scope. For this reason, the SME, already mentioned in Sec. 1, has been developed. The present section gives a brief review of the ideas behind the construction of the SME.

We begin by arguing in favor of dynamical rather than kinematical test models. The construction of a dynamical test framework is constrained by the demand that known physics must be recovered in certain limits, despite some residual freedom in introducing dynamical features compatible with a given set of kinematical rules. In addition, it appears difficult and may even be impossible to develop an effective theory containing the Standard Model with dynamics significantly different from that of the SME. We also mention that kinematical analyses are limited to only a subset of potential Lorentz-breakdown signatures from fundamental physics. From this perspective, it seems to be desirable to implement explicitly dynamical features of sufficient generality into test models for CPT and Lorentz symmetry.

The generality of the SME. To appreciate the generality of the SME, we review the main cornerstones of its construction. [9] Starting from the conventional Standard-Model lagrangian $\mathcal{L}_{\mathrm{SM}}$, Lorentz-breaking modifications $\delta\mathcal{L}$ are added:

$$\mathcal{L}_{\mathrm{SME}} = \mathcal{L}_{\mathrm{SM}} + \delta\mathcal{L} \; . \tag{6}$$

Here, the SME lagrangian is denoted by $\mathcal{L}_{\mathrm{SME}}$. The correction term $\delta\mathcal{L}$ is constructed by contracting Standard-Model field operators of any dimensionality with Lorentz-violating tensorial coefficients that describe a nontrivial vacuum with background vectors or tensors originating from the presumed effects in the underlying theory. Examples of such effects were discussed in the previous section. To guarantee coordinate independence, these contractions must give coordinate Lorentz scalars. It becomes thus apparent that all possible contributions to $\delta\mathcal{L}$ give the most general effective dynamical description of Lorentz breakdown at the level of observer Lorentz-invariant unitary quantum field theory. For simplicity, we have focused on nongravitational physics in the above line of reasoning. We remark, however, that the complete SME also contains an extended gravity sector.

Potential Planck-scale features, such as non-pointlike elementary particles or a discrete spacetime, are unlikely to invalidate the above effective-field-theory approach at currently attainable energies. On the contrary, the phenomenologically successful Standard Model is widely believed to be an effective-field-theory limit of more fundamental physics. If underlying physics indeed leads to minute Lorentz-violating effects, it would seem contrived to consider low-energy effective models outside the framework of effective quantum field theory. We finally remark that the necessity for a low-energy description beyond effective field theory is also unlikely to arise in the context of candidate fundamental models with novel Lorentz-*symmetric* aspects, such as additional particles, new symmetries, or large extra dimensions. Lorentz-invariant modifications can therefore be implemented into the SME, if needed. [10]

Advantages of the SME. The SME permits the identification and direct comparison of virtually all currently feasible experiments searching for Lorentz and CPT violation. Furthermore, certain limits of the SME correspond to classical kinematics test models of relativity (such as the previously mentioned Robertson's framework, its Mansouri-Sexl extension, or the

c^2 model). [11] Another advantage of the SME is the possibility of implementing further desirable features besides coordinate independence. For instance, one can choose to impose spacetime-translation invariance, SU(3)×SU(2)×U(1) gauge symmetry, power-counting renormalizability, hermiticity, and pointlike interactions. These demands further restrict the parameter space for Lorentz violation. One could also adopt simplifying choices, such as a residual rotational invariance in certain coordinate systems. This latter assumption together with additional simplifications of the SME has been considered in the literature. [12]

Analyses performed within the SME. At present, the flat-spacetime limit of the minimal SME has provided the basis for numerous investigations of CPT and Lorentz violation involving mesons, [13, 14, 15, 16, 17, 18, 19] baryons, [20, 21, 22] electrons, [23, 24, 25] photons, [26, 11] muons, [27] and the Higgs sector. [28] Studies involving the gravity sector have recently also been performed. [29] We remark that neutrino-oscillation experiments offer the potential for discovery. [9, 30, 31] CPT and Lorentz tests with mesons will be discussed further in the next section.

5 CPT and Lorentz tests with mesons

Some of the CPT and Lorentz tests listed in the previous section involve some form of antimatter. As pointed out earlier, certain matter–antimatter comparisons are extremely sensitive to CPT violations because CPT symmetry connects particles and antiparticles. This idea can be adopted for studies with mesons. Neutral-meson oscillations are essentially controlled by the energy difference between the meson and its antimeson. Although the SME contains the same mass parameter for quarks and antiquarks, these particles are affected differently by the CPT- and Lorentz-violating background. This allows the dispersion relations for mesons and antimesons to differ, so that mesons and antimesons can have distinct energies. This effect is potentially observable with interferometric methods. The present section contains a more detailed discussion of this idea.

We begin by recalling that any neutral-meson state is a linear combination of the Schrödinger wave functions for the meson P^0 and its antimeson $\overline{P^0}$. If this state is viewed as a two-component object $\Psi(t)$, its time evolution is controlled by a 2×2 effective hamiltonian Λ according to the Schrödinger-type equation [32] $i\partial_t\Psi = \Lambda\Psi$. Although the effective hamiltonian Λ is different for each neutral-meson system, we use a single symbol here for simplicity. The eigenstates $|P_a\rangle$ and $|P_b\rangle$ of Λ are the physical propagating states of the neutral-meson system. They exhibit the usual time evolution

$$|P_a(t)\rangle = \exp(-i\lambda_a t)|P_a\rangle \ , \quad |P_b(t)\rangle = \exp(-i\lambda_b t)|P_b\rangle \ , \tag{7}$$

where the complex parameters λ_a and λ_b are the eigenvalues of Λ. They can be written in terms of the physical masses m_a, m_b and decay rates γ_a, γ_b of the propagating particles:

$$\lambda_a \equiv m_a - \tfrac{1}{2}i\gamma_a \ , \quad \lambda_b \equiv m_b - \tfrac{1}{2}i\gamma_b \ . \tag{8}$$

For convenience, one usually works with the sum and difference of the eigenvalues instead:

$$\begin{aligned} \lambda &\equiv \lambda_a + \lambda_b = m - \tfrac{1}{2}i\gamma \ , \\ \Delta\lambda &\equiv \lambda_a - \lambda_b = -\Delta m - \tfrac{1}{2}i\Delta\gamma \ . \end{aligned} \tag{9}$$

Here, we have defined $m = m_a + m_b$, $\Delta m = m_b - m_a$, $\gamma = \gamma_a + \gamma_b$, and $\Delta\gamma = \gamma_a - \gamma_b$.

The effective hamiltonian Λ is a 2×2 complex matrix, and as such it contains eight real parameters for the neutral-meson system under consideration. Four of these correspond to the two masses and decay rates. Among the remaining four parameters are three that determine the extent of indirect CP violation in the neutral-meson system and one that is an unobservable phase. Indirect CPT violation in this system occurs if and only if the difference $\Delta\Lambda \equiv \Lambda_{11} - \Lambda_{22}$ of the diagonal elements of Λ is nonzero. It follows that Λ contains two real parameters for CPT breakdown. On the other hand, indirect T violation occurs if and only if the magnitude of the ratio $|\Lambda_{21}/\Lambda_{12}|$ of the off-diagonal components of Λ differs from 1. The effective hamiltonian therefore contains also one real parameter for T violation.

Various explicit parametrizations of Λ are possible. However, for the heavy meson systems D, B_d, B_s, less is known about CPT and T violation than for the K system. It is therefore desirable to employ a general parametrization of the effective hamiltonian Λ that is independent of phase conventions, [33] valid for arbitrary-size CPT and T breaking, model independent, and expressed in terms of mass and decay rates insofar as possible. Such a convenient parametrization can be achieved by writing two diagonal elements of

Λ as the sum and difference of two complex numbers, and the two off-diagonal elements as the product and ratio of two other complex numbers: [19]

$$\Lambda = \tfrac{1}{2}\Delta\lambda \begin{pmatrix} U+\xi & VW^{-1} \\ VW & U-\xi \end{pmatrix} . \qquad (10)$$

In this definition, $UVW\xi$ are dimensionless complex numbers. The requirement that the trace of Λ is tr $\Lambda = \lambda$ and that its determinant is $\det\Lambda = \lambda_a\lambda_b$ fixes the complex parameters U and V:

$$U \equiv \lambda/\Delta\lambda\ , \quad V \equiv \sqrt{1-\xi^2}\ . \qquad (11)$$

The CPT and T properties of the effective hamiltonian (10) are now determined in the complex numbers $W = w\exp(i\omega)$ and $\xi = \mathrm{Re}\,\xi + i\mathrm{Im}\,\xi$. Of the four real components, the phase angle ω of W is physically irrelevant. The remaining three components are physical, with $\mathrm{Re}\,\xi$ and $\mathrm{Im}\,\xi$ describing CPT violation and the modulus $w \equiv |W|$ of W governing T breaking. Their relation to the components of Λ are

$$\xi = \Delta\Lambda/\Delta\lambda\ , \quad w = \sqrt{|\Lambda_{21}/\Lambda_{12}|}\ . \qquad (12)$$

CPT conservation requires $\mathrm{Re}\,\xi = \mathrm{Im}\,\xi = 0$, while T conservation requires $w = 1$. The eigenstates of Λ, which are the physical states of definite masses and decay rates, can also be obtained in a straightforward way. [19]

We remark in passing that the $w\xi$ formalism above can be related to other formalisms used in the literature provided appropriate assumptions about the phase conventions and the smallness of CP violation are made. [19] For example, in the K system the widely adopted [32] formalism involving ϵ_K and δ_K depends on the phase convention, and it can be applied only if CPT and T violation are small. Under this assumption and in a special phase convention, δ_K is related to ξ_K by $\xi_K \approx 2\delta_K$.

Thus far, we have discussed the phenomenological description of neutral-meson oscillations with particular emphasis on CPT violation. We next review how the phenomenological CPT-breaking parameters above are connected to coefficients in the SME. Since the minimal SME is a relativistic unitary quantum field theory, it satisfies the conditions for Greenberg's "anti-CPT theorem," which states that CPT breaking must come with Lorentz violation. Without any calculations we can therefore conclude already at this point that δ_K, for example, cannot be constant. In particular, it will typically be direction dependent. This fact is further illustrated in Fig. 3.

The leading CPT-breaking contributions to Λ can be calculated perturbatively in the coefficients for CPT and Lorentz violation that appear in the SME. These corrections are expectation values of CPT- and Lorentz-violating interactions in the hamiltonian for the theory, [17] evaluated with the unperturbed wave functions $|P^0\rangle$, $|\overline{P^0}\rangle$ as usual. Note that the hermiticity of the perturbation hamiltonian ensures real contributions.

To determine an expression for the parameter $\xi_K \approx 2\delta_K$, one needs to find the difference $\Delta\Lambda = \Lambda_{11} - \Lambda_{22}$ of the diagonal terms of Λ. A calculation within the SME gives [19]

$$\Delta\Lambda \approx \beta^\mu \Delta a_\mu \ , \tag{13}$$

where $\beta^\mu = \gamma(1, \vec{\beta})$ is the four-velocity of the meson state in the observer frame. In this equation, we have defined $\Delta a_\mu = r_{q_1} a_\mu^{q_1} - r_{q_2} a_\mu^{q_2}$, where $a_\mu^{q_1}$, $a_\mu^{q_2}$ are coefficients for CPT and Lorentz breaking for the two valence quarks in the P^0 meson. These coefficients have mass dimension one, and they arise from lagrangian terms of the form $-a_\mu^q \overline{q} \gamma^\mu q$, where q specifies the quark flavor. The quantities r_{q_1}, r_{q_2} characterize normalization and quark-binding effects. [17]

We see that among the consequences of CPT and Lorentz breakdown are the 4-velocity and hence 4-momentum dependence of observables, as expected from our above considerations involving the "anti-CPT theorem." It follows that the standard assumption of a constant parameter ξ for CPT violation fails under the very general condition of unitary quantum field theory. In particular, the presence of the 4-velocity in Eq. (13) implies that CPT observables will typically vary with the magnitude and orientation of the meson momentum. This can have major consequences for experimental investigations, since the meson momentum spectrum and angular distribution now contribute directly to the determination of the experimental CPT reach.

An important effect of the 4-momentum dependence is the appearance of sidereal variations in some CPT observables: the vector $\Delta\vec{a}$ is constant, while the Earth rotates in a celestial equatorial frame. Because a laboratory frame is employed for the derivation of Eq. (13), and since this frame is rotating, observables can exhibit sidereal variations. This is schematically depicted in Fig. 3. To display explicitly this sidereal-time dependence, one can transform

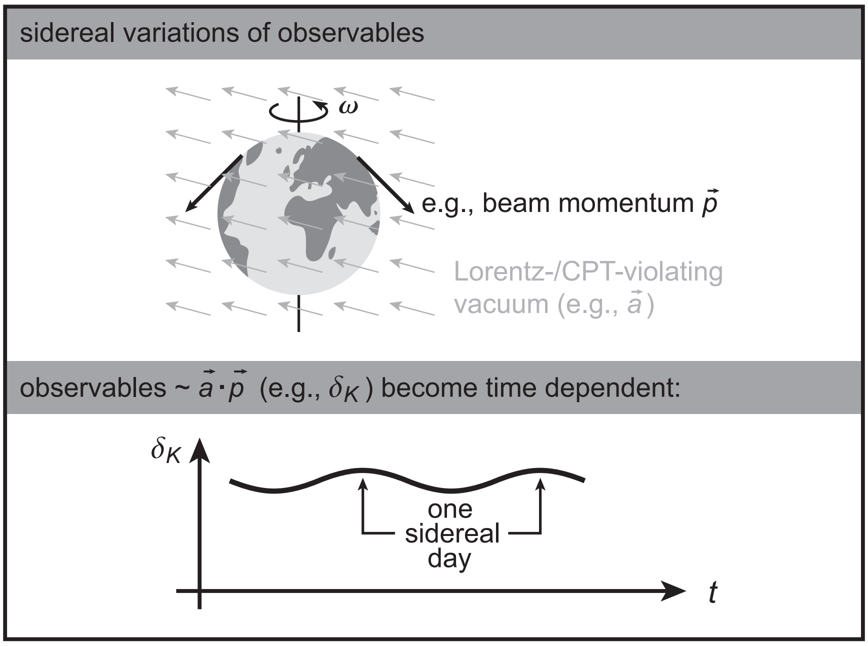

Figure 3: Sidereal variations. Experiments are typically associated with an intrinsic direction. For instance, particle-accelerator experiments have a characteristic beam direction determined by the set-up of the accelerator. As the Earth rotates, this direction will change because the accelerator is attached to the Earth. In the above figure, a beam direction $\vec{p}$ pointing south is shown at two times separated by approximately 12 hours (black arrows). The angle between the Lorentz-violating background (gray $\vec{a}$ arrows) and the orientation of the beam direction is clearly different at these two times. An observable, such as the phase δ_K, may for example acquire a correction $\sim \vec{p}\cdot\vec{a}$ that leads to the shown sidereal modulation.

the expression (13) for $\Delta\Lambda$ from the laboratory frame to a nonrotating frme. To this end, let us denote the spatial basis in the laboratory frame by $(\hat{x}, \hat{y}, \hat{z})$ and that in the nonrotating frame by $(\hat{X}, \hat{Y}, \hat{Z})$. We next choose the $\hat{z}$ axis in the laboratory frame for maximal convenience. For instance, the beam direction is a natural choice for the case of collimated mesons, while the collision axis could be adopted in a collider. We further define the nonrotating-frame basis $(\hat{X}, \hat{Y}, \hat{Z})$ to be consistent with celestial equatorial coordinates, with $\hat{Z}$ aligned along the Earth's rotation axis. For the observation of sidereal variations we must have $\cos\chi = \hat{z} \cdot \hat{Z} \neq 0$. It then follows that $\hat{z}$ precesses about $\hat{Z}$ with the Earth's sidereal frequency Ω. The complete transformation between the two bases can be found in the literature. [20] In particular, any coefficient $\vec{a}$ for Lorentz breakdown with laboratory-frame components (a^1, a^2, a^3) possesses nonrotating-frame components (a^X, a^Y, a^Z). This transformation determines the time dependence of $\Delta\vec{a}$ and hence the sidereal variation of $\Delta\Lambda$. The entire momentum and sidereal-time dependence of the CPT-breaking parameter ξ in any P system can then be extracted.

To give an explicit expression for the final answer for ξ, define θ and ϕ to be standard polar coordinates about the $\hat{z}$ axis in the laboratory frame. In general, the laboratory-frame 3-velocity of a P meson can then be written as $\vec{\beta} = \beta(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. It follows that the magnitude of the momentum obeys $p \equiv |\vec{p}| = \beta m_P \gamma(p)$, where $\gamma(p) = \sqrt{1 + p^2/m_P^2}$ as usual. In terms of these quantities and the sidereal time $\hat{t}$, the result for ξ takes the form [19]

$$\begin{aligned}
\xi &\equiv \xi(\hat{t}, \vec{p}) \equiv \xi(\hat{t}, p, \theta, \phi) \\
&= \frac{\gamma(p)}{\Delta\lambda} \big\{ \Delta a_0 + \beta \Delta a_Z (\cos\theta\cos\chi - \sin\theta\cos\phi\sin\chi) \\
&\qquad + \beta \big[\Delta a_Y (\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi) \\
&\qquad\qquad - \Delta a_X \sin\theta\sin\phi \big] \sin\Omega\hat{t} \\
&\qquad + \beta \big[\Delta a_X (\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi) \\
&\qquad\qquad + \Delta a_Y \sin\theta\sin\phi \big] \cos\Omega\hat{t} \big\} . \qquad (14)
\end{aligned}$$

The experimental challenge is the measurement the four independent coefficients Δa_μ for CPT breakdown allowed by quantum field theory. The result (14) shows that suitable binning of data in sidereal time, momentum magnitude, and orientation has the potential to extract four independent constraints from any observable with a nontrivial ξ dependence. Note that each one of the neutral-meson systems may have different values of these coefficients. As a result of the distinct masses and decay rates, the physics of each system is distinct. A complete experimental study of CPT breaking requires four independent measurements in each system.

6 Experiments

To date, various CPT tests with neutral mesons have been analyzed within the SME. Other current and future experiments offer the possibility to tighten these existing constraints or extract bounds on other CPT-violation coefficients in the SME. This section contains a brief account of this topic with focus on the KLOE or KLOE-II detectors.

As argued in the previous section, a key issue in the analysis of experimental data is magnitude of the meson momentum and its orientation relative to the CPT- and Lorentz-violating coefficient Δa^μ. The orientation depends on the experimental set-up, so that different experiments are sensitive to different combinations of Δa^μ components. One important parameter is the beam direction, which is usually fixed with respect to the laboratory. Since the Earth, and thus the laboratory, rotates with respect to Δa^μ, the beam direction relative to Δa^μ is determined by the date and the time of the day. This requires time binning for any neutral-meson experiment with sensitivity to $\Delta\vec{a}$.

In a fixed-target measurement at high enough energies, the momenta of the produced mesons are aligned with the beam direction to a good approximation, and no further directional information in addition to the time stamp of the event needs to be recorded. These experiments typically involve uncorrelated mesons, which further simplifies their conceptual analysis. We have $\beta_\mu \Delta a^\mu = (\beta^0 \Delta a^0 - \Delta\vec{a}_\parallel \cdot \vec{\beta}_\parallel) - (\Delta\vec{a}_\perp \cdot \vec{\beta}_\perp)$, where $\parallel$ and $\perp$ are taken with respect to the Earth's rotation axis. We see that in principle all four components of Δa^μ can be determined: the $\perp$ components via their sidereal variations and the sidereally constant components in the first parentheses via their dependence on the momentum magnitude. However, under our initial assumption of high energies the variation of $|\vec{\beta}|$ with the energy is tiny, which makes it difficult to disentangle the individual components Δa^0 and $\Delta\vec{a}_\parallel$. On the other hand, high energies are associated with large boost factors, which increase the overall CPT reach for the other combinations of Δa^μ components.

These ideas have been applied in experiments with the K and D systems. For the K system, two independent CPT measurements of different combinations of the coefficients Δa_μ have been performed. [13, 19] One measurement constrains a linear combination of Δa_0 and Δa_Z to about 10^{-20} GeV, and the other bounds a combination of Δa_X and Δa_Y to 10^{-21} GeV. These experiments were performed with mesons highly collimated in the laboratory frame. In this case, ξ simplifies because the 3-velocity takes the form $\vec{\beta} = (0, 0, \beta)$. Binning in $\hat{t}$ yields sensitivity to the equatorial components Δa_X, Δa_Y. On the other hand, averaging over $\hat{t}$ eliminates these components altogether.

For the D-meson system, two independent bounds have been obtained by the FOCUS experiment. [14] They constrain a linear combination of Δa_0 and Δa_Z to about 10^{-16} GeV, and they bound Δa_Y also to roughly 10^{-16} GeV. Notice that CPT constraints in the D system are unique in that the valence quarks involved are the u and the c, whereas the other neutral mesons involve the d, s, and b.

CPT measurements are also possible for correlated meson pairs in a symmetric collider. This experimental set-up is relevant for the KLOE and KLOE-II experiments at the Frascati laboratory, and it differs significantly from that in the previous paragraph. In particular, the energy dependence is essentially irrelevant: the kaon pairs are produced in the decay of ϕ quarkonium just above threshold leading to approximately monoenergetic kaons. Moreover, the boost factor does not substantially improve the CPT reach. On the other hand, the wide angular distribution of the kaons in the laboratory frame requires angular binning in addition to date/time binning to reconstruct the direction of β^μ with respect to Δa^μ. Moreover, the correlation of the meson pairs can give additional observational information. We will see that these two features would allow the extraction of independent constraints on four components of Δa^μ.

Consider a ϕ quarkonium state with $J^{PC} = 1^{--}$ decaying at time t in its rest frame into a correlated K-$\overline{K}$ pair.[2] Since the laboratory frame is unboosted relative to the quarkonium rest frame, the time t may be taken as the sidereal time. Subsequently, one of the kaons decays into f_1 at time $t + t_1$, while the other decays into f_2 at time $t + t_2$. Then, standard arguments yield

$R_{12}(\vec{p}, t, \overline{t}, \Delta t) =$

[2] The line of reasoning for B_d, B_s, and D mesons would be similar.

$$|\hat{N}|^2 e^{-\overline{\gamma}\overline{t}/2} \left[|\eta_1|^2 e^{-\Delta\gamma\Delta t/2} + |\eta_2|^2 e^{\Delta\gamma\Delta t/2} - 2|\eta_1\eta_2| \cos(\Delta m \Delta t + \Delta\phi) \right] \quad (15)$$

for the double-decay rate. In this equation, η_α denotes the following ratio of amplitudes $A(K_L \to f_\alpha)/A(K_S \to f_\alpha)$, and $\hat{N}$ is a normalization containing the factor $A(K_L \to f_1)A(K_S \to f_2)$. We have further defined $\overline{t} = t_1 + t_2$, $\Delta t = t_2 - t_1$, $\overline{\gamma} = \gamma_S + \gamma_L$, $\Delta\gamma = \gamma_L - \gamma_S$, and $\Delta\phi = \phi_1 - \phi_2$. The amplitudes $A(K_{L/S} \to f_\alpha)$ may be functions of the momentum $\vec{p}_1 = -\vec{p}_2 \equiv \vec{p}$ and the sidereal time t via a possible dependence on $\Delta\Lambda$. It follows that the effects of potential CPT violations in $R_{12}(\vec{p}, t, \overline{t}, \Delta t)$ are contained in η_α and $\hat{N}$.

A detailed study of the CPT signals from symmetric-collider experiments with correlated kaons requires analyses with expressions of the type (15) for various final states f_1, f_2. With sufficient experimental resolution, the dependence of certain decays on the two meson momenta $\vec{p}_1$, $\vec{p}_2$ and on the sidereal time t could be measured by appropriate data binning and analysis. We note that different asymmetries can be sensitive to distinct components of $\Delta\Lambda$, so that some care is required in such investigations.

Let us consider the sample case of double-semileptonic decays of correlated kaon pairs in a symmetric collider. Assuming the $\Delta S = \Delta Q$ rule, one can show that the double-decay rate $R_{l^+l^-}$ can be regarded as proportional to an expression depending on the ratio [19]

$$\left| \frac{\eta_{l^+}}{\eta_{l^-}} \right| \approx 1 - \frac{4\mathrm{Re}\,(i \sin\hat{\phi}\; e^{i\hat{\phi}})}{\Delta m} \gamma(\vec{p}) \Delta a_0 \quad . \qquad (16)$$

In this expression, $\hat{\phi} \equiv \tan^{-1}(2\Delta m/\Delta\gamma)$ is sometimes called the superweak angle. Note the absence of all angular and time dependence in Eq. (16). This fact arises because for a symmetric collider we have $\vec{\beta}_1 \cdot \Delta\vec{a} = -\vec{\beta}_2 \cdot \Delta\vec{a}$, which leads to a cancellation between the contributions from each kaon.

In this form for the double-decay rate $R_{l^+l^-}$, any angular and momentum dependence can therefore only enter through the overall factor of $|\hat{N}\eta_{l^-}|^2$. The measurement of such a normalizing factor is experimentally challenging. For example, the normalization factor would cancel in a conventional analysis to extract the physics using the usual asymmetry. Another obstacle is the line spectrum mentioned above, so that the dependence on $|\vec{p}|$ is unobservable. We conclude that the double-semileptonic decay channel is well suited to place a clean bound on the timelike parameter Δa_0 for CPT breakdown, and the experimental data may be collected for analysis without regard to their angular locations in the detector or their sidereal time stamps.

Apart from the double-semileptonic channel, there are also other decay possibilities for the two kaons. Among these are mixed double decays, in which only one of the two kaons has a ξ_K-sensitive mode. For such asymmetric decay products, there is no longer a cancellation of the spatial contributions of Δa^μ, and independent bounds on three of its components may become possible. One example for such a double-decay mode is a channel with one semileptonic prong and one double-pion prong. Note that in a conventional CPT analysis, a given double-decay mode of this type is inextricably connected with other parameters for CP violation. [34, 35, 36] However, in the present context the possibility of angular and time binning implies that clean tests of CPT breaking are feasible even for these mixed modes.

As a sample set-up, consider a detector with acceptance independent of the azimuthal angle ϕ. The distribution of mesons from the quarkonium decay is symmetric in ϕ, so the ξ_K dependence of a ϕ-averaged dataset is determined by

$$\begin{aligned} \delta_K^{\rm av}(|\vec{p}|,\theta,t) &\equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi\; \xi_K(\vec{p},t)/2 \\ &= \frac{i\sin\hat{\phi}\; e^{i\hat{\phi}}}{\Delta m}\gamma\,[\Delta a_0 + \beta\Delta a_Z\cos\chi\cos\theta \\ &\qquad +\beta\Delta a_Y\sin\chi\cos\theta\sin\Omega t \\ &\qquad +\beta\Delta a_X\sin\chi\cos\theta\cos\Omega t]\ . \end{aligned} \tag{17}$$

Inspection of this equation establishes that by measuring the θ and t dependences an experiment with asymmetric double-decay modes can in principle extract separate constraints on each of the three components of the parameter $\Delta\vec{a}$ for CPT breakdown. We remark that this result holds independent of other CP parameters that may appear because the latter neither possess angular nor time dependence. It follows that a combination of data from asymmetric double-decay modes and from double-semileptonic modes permits in principle the extraction of independent constraints on each of the four components of Δa_μ.

Similar arguments can be made for other experimental observables. Consider, for instance, the standard rate asymmetry for K_L semileptonic decays [6)]

$$\delta_l \equiv \frac{\Gamma(K_L \to l^+\pi^-\nu) - \Gamma(K_L \to l^-\pi^+\overline{\nu})}{\Gamma(K_L \to l^+\pi^-\nu) + \Gamma(K_L \to l^-\pi^+\overline{\nu})} \approx 2\mathrm{Re}\,\epsilon_K - \mathrm{Re}\,\xi_K(\vec{p},t) \; . \qquad (18)$$

Here, the symbol Γ denotes a partial decay rate, and violations of the $\Delta S = \Delta Q$ rule have been neglected. In principle, this asymmetry could also be investigated for angular and time dependencies, which would lead to bounds on Δa_μ. From the forward–backward asymmetry of this expression, a preliminary bound at the level of 10^{-17} GeV on the Δa_Z coefficient for the kaon can been obtained by KLOE. [37)] If confirmed, this would be the first clean constraint on this coefficient.

We finally mention another experimental set-up. Suppose the quarkonium is not produced at rest, but with a sufficient net momentum, such as in an asymmetric collider. Then, $\xi_1 + \xi_2$ does not cancel and could be sensitive to all four coefficients Δa_μ for the neutral-meson system under investigation. It follows that appropriate data binning would also allow up to four independent CPT measurements. The existing asymmetric B_d factories BaBar and BELLE would be able to undertake measurements of these types. [15)] Preliminary results from the BaBar experiment constrain various component combinations of Δa^μ for the B_d meson to about 10^{-13} GeV. [16)] We also mention that the same study does find a 2.2σ signal for sidereal variations. [16)] While this level of significance is still consistent with no effect, it clearly motivates further experimental CPT- and Lorentz-violation searches in neutral-meson systems.

7 Summary

Although both CPT and Lorentz invariance are deeply ingrained in the currently accepted laws of physics, there are a variety of candidate underlying theories that could generate the breakdown of these symmetries. The sensitivity attainable in matter–antimatter comparisons offers the possibility for CPT-breakdown searches with Planck precision. Lorentz-symmetry tests open an additional avenue for CPT measurements because CPT violation implies Lorentz violation.

A potential source of CPT and Lorentz breaking is spontaneous symmetry violation in string field theory. Because this mechanism is theoretically very attractive, and because strings show great potential as a candidate fundamental theory, this Lorentz-violation origin is particularly promising. CPT and Lorentz breaking can also originate from spacetime-dependent scalars: the gradient of such scalars selects a preferred direction in the effective vacuum. This mechanism for Lorentz violation might be of interest in light of recent claims of a time-dependent fine-structure parameter and the presence of time-dependent scalar fields in various cosmological models.

The leading-order CPT- and Lorentz-violating effects that would emerge from Lorentz-symmetry breaking in approaches to fundamental physics are described by the SME. At the level of effective quantum field theory, the SME is the most general dynamical framework for Lorentz and CPT violation that is compatible with the fundamental principle of unitarity. Experimental studies are therefore best performed within the SME.

Neutral-meson interferometry is an excellent high-sensitivity tool in experimental searches for Planck-scale physics. In the context of unitary quantum field theory, potential CPT violations come with Lorentz breaking, which then typically leads to direction- and energy-dependent CPT-violation observables. For Earth-based tests, this effect leads to sidereal variations, which typically requires momentum and time binning in experiments. Within the minimal SME, there are four independent coefficients for CPT breaking in each meson system. Observational constraints in the order of 10^{-13} down to 10^{-21} GeV have been obtained for a subset of these coefficients. In general, tests with neutral mesons bound parameter combinations of the SME inaccessible by other experiments. The KLOE and the planned KLOE-II experiments with their symmetric set-up offer unique opportunities for CPT tests along these lines. Such measurements would give further insight into the enigmatic kaon system, and they have the potential to probe Planck-scale physics.

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