# ON THE QUANTUM-GRAVITY PHENOMENOLOGY OF MULTIPARTICLE STATES

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**Abstract**

We discuss some general expectations concerning the structure of multiparticle states in the quantum-gravity realm, and we introduce the first elements of a toy model which could be used as guidance in the estimate of some associated effects. We also provide a brief review of "quantum gravity phenomenology" and comment on how the study of multiparticle states could contribute to the overall development of this field.

## 1 Introduction

The "quantum-gravity problem" has been discussed for more than 70 years [1] assuming that no guidance could be obtained from experiments. Indeed, it is not unlikely that experiments might never give us any clear lead toward quantum gravity, especially if our intuition concerning the role of the tiny Planck length ($\sim 10^{-35} m$) in setting the magnitude of the characteristic effects of the new theory turns out to be correct. But over the past decade or so a growing number of research groups is working hard [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] at trying to find ways to uncover experimentally some manifestations of quantum gravity, even if the new effects were really so small.

Our estimate that the quantum-gravity corrections should be very small in low-energy experiments is based on our experience with other similar situations; in fact, we expect that the Planck scale, since it is the energy scale where the current theories appear to break down, should also govern the magnitude of quantum-gravity corrections to the analysis of processes involving particles with energies smaller than the Planck scale. For example in processes involving two particles both with energy $E$ the magnitude of the new effects should be set by some power of the ratio between $E$ and the Planck scale $E_p$ ($\sim 10^{28} eV$). Since in all cases accessible to us experimentally $E/E_p$ is extremely small, this is a key challenge for quantum-gravity phenomenology. A challenge which however can be dealt with also relying on experience with other analogous situations in physics: for example, as emphasized in Ref. [2], ongoing studies of proton stability from the grandunification perspective and early 1900s studies of Brownian motion could be described as facing a very similar challenge.

In the second part of these notes we shall review some key results obtained in quantum-gravity phenomenology. We intend to convey the point that this phenomenology has already established some (however humble but) valuable constraints for quantum-gravity model building, but these constraints are essentially confined to the behaviour of isolated particles, or systems of particles interacting for a very short time. In the next section we argue that some key hints for the search of quantum gravity might be uncovered in the study of the evolution over time of certain types of multiparticle states. And in Section 3 we introduce the first elements of a toy model which could be used as guidance in the estimate of some peculiar Planck-scale effects for multiparticle states.

Our model is at present too crude to make definite predictions, but our intuition is that, when fully developed, it could be sensitively probed through the study of certain multi-kaon systems, such as the states of two neutral kaons produced by decay of the $\phi$ resonance. Indeed, as we shall discuss, the primary source of inspiration for the toy model discussed in Section 3 is the framework based on the $\kappa$-Minkowski noncommutative spacetime, which already inspired a picture for CPT-violation mechanism [22] which could be tested in studies of neutral-kaon systems.

## 2 A perspective on multiparticle states in the quantum-gravity realm

It is probably fair to say that we are still rather far from a comprehensive solution of the quantum-gravity problem. We do have some proposals, such as String Theory and Loop Quantum Gravity, that provide tentative solutions for some (but not all) aspects of the problem, but these theories still have absolutely no support in experimental and from a robust conservative scientific perspective must therefore be viewed as mere theoretical speculations.

In more than 70 years of work on the quantum gravity problem the community has developed some intuition for features to expect in the quantum-gravity realm, such as the mentioned expected role of the Planck scale in setting the magnitude of effects, and, although of course this intuition must be treated cautiously (with no less caution than the one that should be adopted in relying on String Theory or Loop Quantum Gravity), it is natural to use this intuition as guidance for at least some of our efforts searching for experimentally-established facts about the quantum-gravity realm. In this section we intend to discuss briefly (our perspective on) the part of this intuition that concerns the relationship between the structure of one-particle states and the structure of multiparticle states.

In our current (pre-quantum-gravity) theories one obtains multiparticle states from single-particle states by a standard use of the trivial tensor product of Hilbert spaces, but there is (conceptual/theoretical) evidence that this recipe might not be applicable in the quantum-gravity realm. This expectation emerges not really from a single robust argument but rather from the fact that various lines of reasoning on multiparticle states all appear to suggest that novel features must be introduced.

A first observation which we should report here relies on our present understanding of gravity in 2+1 spacetime dimensions. 2+1D gravity is a topological field theory, rather similar to the Chern-Simons gauge theories that can be considered in a 2+1D spacetime. Especially for the case of a Chern-Simons theory with a $U(1)$ gauge field the literature is very large and it is well established that multiparticle states are not obtained by standard tensor product of single-particle states. The particle excitations of the Chern-Simons gauge field are the so-called "anyons", and it has emerged that for any given Hamiltonian governing the evolution of the anyon system it actually makes sense to treat as completely separate problems each of the n-anyon sectors: there is no simple recipe for obtaining two-particle states from single-particle states, or for obtaining three-particle states from the acquired knowledge of two-particle states. In this anyon example the complexity of multiparticle states is such that one cannot meaningfully introduce some creation-annihilation operators capable of producing from a vacuum state the different n-anyon sectors.

Some of our intuition for multiparticle states in the quantum-gravity originates from familiarity with this multianyon problem [33]. This intuition is directly applicable to 2+1D gravity, and might play an (however indirect) role also in 3+1D gravity, at least when viewed as a "broken topological field theory": guided by the known facts about 2+1D gravity one could set up 3+1D as a theory which is itself "topological up to correction terms".

As an example of argument suggesting complexity for the construction of multiparticle states without relying on the peculiarities of 2+1D spacetimes, we find useful here to mention one aspect of the quantum-gravity problem, which is often set aside but universally acknowledged. The differences between the gravity field and, say, the electromagnetic field are such that for quantum gravity it appears to be necessary to contemplate an in-principle obstruction [1, 34, 35] for a full decoupling of "apparatus" from "system". It is well-established that electromagnetism admits a limiting procedure such that (in the limit) the apparatus actually establishes facts about the system without interfering/affecting the evolution of the system, but the Equivalence Principle (by identifying the inertial mass and the gravitational charge) appears to provide an obstruction for this limiting procedure. And this opens at least an opportunity for complexity in the construction of multiparticle states: it appears to be rather plausible that the relationship between the way in which the apparatus "interferes" with a single-particle system and the way in which the apparatus "interferes" with a two-particle system might be more complex than what is codified in a standard tensor-product rule.

While not often discussed in papers and seminars, these issues for multiparticle states in quantum gravity are rather widely acknowledged. For example, some careful readers from the community of researchers involved in neutral-kaon studies (which is one of the communities toward which we are hoping to direct these notes, because of the possible use of neutral kaons in the investigation of the features discussed in the next section) might have noticed that the debate on the choice of parametrization for the phenomenology of Planck-scale-induced CPT violation [36, 37, 38, 39, 40] reflects in part some differences in the intuition for the structure of multiparticle states.

## 3 A simple toy model

To give some substance to the arguments presented in the previous section we now intend to introduce the first elements of a possible toy model for the description of a class of effects which could characterize multiparticle states at the Planck scale. This toy model is loosely inspired by the results of our investigations [41, 42, 43, 44, 45, 46] of field theories in $\kappa$-Minkowski spacetime, an example of "noncommutative spacetime" (spacetime with noncommuting coordinates) characterized by the following commutators of spacetime coordinates [47, 48, 49]

$$[x_j, x_0] = i\lambda x_j \, , \qquad [x_k, x_j] = 0 \, , \tag{1}$$

where $\lambda$ is an observer-independent length scale usually expected to be of the order of the minute Planck length ($\sim 10^{-35}m$). Preliminary evidence suggests (but are still inconclusive [50] on the fact) that the observer independence of the noncommutativity parameter may result in the necessity to describe symmetry transformations somehow in terms of the $\kappa$-Poincaré Hopf algebra [51, 47, 48] (rather than the Poincaré, or other, Lie algebra), but this will not be used explicitly in our reasoning. It is however important for us that the role played by the $\kappa$-Poincaré Hopf algebra in the structure of theories in $\kappa$-Minkowski spacetime, has led to a peculiar proposal for the law of composition of momenta, and it is this deformed law of composition of momenta that provides the key ingredient of our rudimentary toy model.

The phenomenological scheme for quantum fields that we intend to describe in this section is only loosely based on our work on $\kappa$-Minkowski [41, 42, 43, 44, 45, 46] partly because of the present limitations of our understanding of quantum field theories in $\kappa$-Minkowski and partly because of the hope that, by not borrowing too much from detailed aspects of the $\kappa$-Minkowski, we might have a chance to gain an intuition for the properties of multiparticle states which is of wider relevance for Planck-scale theories. It appears indeed plausible (but it is difficult to test this conjecture presently because of the huge mathematical complexity of some of these frameworks) that structures at least somewhat similar to the ones we contemplate here might arise not only in $\kappa$-Minkowski but also in other approaches to the Planck scale problem, perhaps most notably the Loop Quantum Gravity approach [52, 53, 54, 55].

Even for the understanding of classical field theories in $\kappa$-Minkowski, while some noteworthy results have been obtained [41, 43, 44, 45], several issues still remain to be clarified. And in the analysis of Ref. [46], which does provide a proposal for quantum fields in $\kappa$-Minkowski and is the main source of intuition for the scheme here considered, one encounters structures that are somewhat more complex that the simplified scheme we are here using as illustrative example. Readers who would consider contributing to further development of this scheme should therefore consider Ref. [46] as a natural entry point into the literature devoted to the issues that must deal with in attempting to discuss more rigorously the relevant framework.

The first ingredient of our construction is the assumption that the non-commutativity properties of single-particle states of given fourmomentum $k_\mu$ would be in agreement with the ones of the much studied [41, 43, 46, 48] time-ordered plane waves on $\kappa$-Minkowski space-time

$$|\Psi_{\vec{k}} > \;\leftrightarrow\; e^{i\vec{k}\cdot\vec{x}} e^{-i\omega^+(\vec{k})x_0} \tag{2}$$

in which $\omega^+(\vec{k})$ represents the (real) positive root of the equation

$$0 = -m^2 + (2/\lambda)^2 \sinh^2(\lambda\omega/2) - \vec{k}^2 \exp(\lambda\,\omega) \tag{3}$$

This "on-shell condition" (3) comes from the form of the deformed Klein-Gordon equation one adopts in $\kappa$-Minkowski, which in turn is dictated by the form of the mass-Casimir of the relevant Hopf algebra of symmetries [41, 43, 46, 47, 48]:

$$(2/\lambda)^2 \sinh^2(\lambda P_0/2) - \vec{P}^2 \exp(\lambda\, P_0) \tag{4}$$

with $P_\mu$ the energy-momentum[1] operator, i.e. $\{P_0,\, \vec{P}\}|\Psi_{\vec{k}} >= \{\omega^+(\vec{k}), \vec{k}\}|\Psi_{\vec{k}} >$.

The other structure for which we take inspiration from the $\kappa$-Minkowski literature is a candidate for the total momentum of a two-particle state. From the observation that the commutators (1) imply

$$e^{i\vec{k}\cdot\vec{x}}\, e^{-i\omega^+(\vec{k})x_0} \;\; e^{i\vec{q}\cdot\vec{x}}\, e^{-i\omega^+(\vec{q})x_0} = e^{i(\vec{k}+\vec{q}\, e^{-\lambda\omega^+(\vec{k})})\cdot\vec{x}}\, e^{-i(\omega^+(\vec{k})+\omega^+(\vec{q}))\, x_0} \tag{5}$$

[1]The identification of $P_\mu$ with the energy-momentum observable is a key point in which we are to be considered only loosely inspired by the $\kappa$-Minkowski literature. There is rather robust evidence that these operators $P_\mu$ should appear in the relevant formulas for energy-momentum, but they might be have to be combined with other structures (see, *e.g.*, Ref. [46]).

one is led to suggesting the possibility that

$$\{\vec{K}^{tot}, K_0^{tot}\} = \{\vec{k} + \vec{q}\, e^{-\lambda\omega^+(\vec{k})},\, \omega^+(\vec{k}) + \omega^+(\vec{q})\} \equiv \{\vec{k}\dot{+}\vec{q},\, \omega^+(\vec{k}) + \omega^+(\vec{q})\} \quad (6)$$

where $\dot{+}$, such that $\vec{k}\dot{+}\vec{q} \equiv \vec{k} + \vec{q}\, e^{-\lambda\omega^+(\vec{k})}$ is a nonabelian addition rule based on (5).

Within this setup it is obvious that the description of multiparticle states must require new structures with respect to the usual construction. Let us consider for example a state with two indistinguishable particles in a 1+1-dimensional $\kappa$-Minkowski spacetime. If we measure the energy-momentum of each of the two particles the indistinguishability would require a description of the state of the following form

$$|\Psi^{(2)}_{\{k,\, q\,\}} >= \frac{1}{\sqrt{2}}\,(\,|\psi_k > \otimes |\psi_q > \,+ |\psi_q > \otimes |\psi_k >) \quad (7)$$

However, this already exposes a peculiarity: according to (6) the state (7) obtained by "indistinguishability symmetrization" based on the information gained by measurement of the energy-momentum of the two particles is not an eigenstate of total energy-momentum. In fact the action of $P_0$ on both the states $|\psi_q > \otimes |\psi_k >$ and $|\psi_k > \otimes |\psi_q >$ gives the eigenvalue $\omega^+(\vec{q}) + \omega^+(\vec{k})$, while the action of $P$ on $|\psi_q > \otimes |\psi_k >$, which gives $P\, |\psi_q > \otimes |\psi_k > = (q + k\, e^{-\lambda\omega^+(q)})\, |\psi_q > \otimes |\psi_k >$, differs from the action of $P$ on $|\psi_k > \otimes |\psi_q >$, which gives $P\, |\psi_k > \otimes |\psi_q > = (k + q\, e^{-\lambda\omega^+(k)})\, |\psi_k > \otimes |\psi_q >$.

This invites us to ask how a framework with these peculiarities should describe the case in which for a system of two identical particles one measures the total energy-momentum of the system. For on-shell particles the measurement of the total momentum $\{K^{tot}, K_0^{tot}\}$ translates into constraints of the type

$$\begin{aligned} K^{tot} &= p'\dot{+}p'' \\ K_0^{tot} &= \omega^+(p') + \omega^+(p'') \end{aligned} \quad (8)$$

These admit as solutions two possible pairs of on-shell momenta, and it is easy to establish a relationship between these two solutions: denoting by $\{k, \omega^+(k)\}, \{q, \omega^+(q)\}$ a first solution the second solution is related to the first by

$$\begin{aligned} \{\tilde{q}_0;\, \tilde{q}\} &= \{\omega^+(q\, e^{-\lambda\omega^+(k)}),\, q\, e^{-\lambda\omega^+(k)}\} \\ \{\tilde{k}_0;\, \tilde{k}\} &= (\omega^+(k\, e^{\lambda\tilde{q}_0}),\, k\, e^{\lambda\tilde{q}_0}) \end{aligned} \quad (9)$$

From this we infer that this state selected by a total-energy-momentum measurement should have the form

$$|\Psi^{(2)}_{\{K^{tot}\}}>=\frac{1}{\sqrt{2}}\left(|\psi_k>\otimes|\psi_q>+|\psi_{\tilde{q}}>\otimes|\psi_{\tilde{k}}>\right) \tag{10}$$

Evidently just like the state of two particles with definite energy-momenta turned out not to be an eigenstate of total momentum, this state of two particles with definite total energy-momentum does not provide a sharp prediction for the energy-momentum of the individual particles.

We have therefore produced a scheme with very peculiar relationship between one-particle states and multiparticle states, which in particular introduces a sort of new uncertainty principle: there is an incompatibility between measurements of total energy-momentum and measurements of the individual energy-momenta of particles. Sharp measurements of total energy-momentum introduce an irreducible uncertainty in the individual energy-momenta, and sharp measurements of individual energy-momenta introduce an irreducible uncertainty in the total energy-momentum.

We exposed this peculiarities thinking of identical particles, but it seems clear that they are structural to the tensor product of Hilbert spaces, so related (though possibly different) peculiarities should be expected also for distinguishable particles.

While it might be difficult to test directly the new energy-momentum-measurement uncertainty principle, it might be possible to test, *e.g.*, Eq. (10) by looking in data analysis for a sort of contamination by an unexpected state. The analysis could be inspired by the following perspective on Eq. (10)

$$\begin{aligned}|\Psi^{(2)}_{\{K^{tot}\}}> &= \frac{1}{\sqrt{2}}\left(|\psi_k>\otimes\ |\psi_q>+|\psi_{\tilde{q}}>\otimes\ |\psi_{\tilde{k}}>\right) = \\ &= \frac{1}{\sqrt{2}}\left(|\psi_k>\otimes\ |\psi_q>+|\psi_q>\otimes\ |\psi_k>+|\Delta>\right)\end{aligned} \tag{11}$$

with $|\Delta>\equiv|\psi_{\tilde{q}}>\otimes|\psi_{\tilde{k}}>-|\psi_q>\otimes|\psi_k>$. Clearly the limitations of the theoretical basis of our toy model do not allow us at present to characterize $|\Delta>$ sharply enough to be of real help for phenomenology. But the situation should improve gradually as we develop a better understanding of the $\kappa$-Minkowski (and possibly other) framework. In particular, it will be interesting to establish if this theory framework ends up having at least a partial overlap with the phenomenological proposal for multi particle states put forward in Ref. [56].

## 4 On other areas of quantum-gravity phenomenology

The phenomenology for multiparticle states discussed in the previous sections would be in many ways complementary to the topics so far studied in an ongoing effort searching for some first experimental manifestation of effects with quantum-gravity origin. Indeed these previous "quantum-gravity phenomenology" [2] studies mainly focused on the behaviour of isolated particles, or systems of particles interacting for a very short time. In this section we intend to review briefly some of these topics previously considered from the quantum-gravity-phenomenology perspective, also hoping to provide some intuition for the potential relevance of the studies of multiparticle states discussed in the previous sections.

Of course, the first concern for quantum-gravity phenomenology was to show that it was really possible to test experimentally some effects introduced genuinely at the Planck scale. This is by now well established, and we discuss one explicit example in Subsection 4.1.

In Subsection 4.2 we comment on the possibility for quantum-gravity phenomenology to actually falsify theories (something worth our efforts even when the results of experiments are negative, rather than merely keep trying to catch lucky through a positive/discovery result).

Subsection 4.3 is devoted to a (incomplete but representative) list of effects that should be considered as candidate quantum-gravity effects, and a brief descriptions of the experiments and/or observations which are being analyzed as opportunities to provide related insight.

The rest of this section focuses on the most studied area of quantum-gravity phenomenology, the one that concerns the possibility of Planck-scale departures from Poincaré (Lorentz) symmetry.

### 4.1 Quantum-Gravity Phenomenology exists

Task number one for any phenomenology (usually an easy task but a challenging one here) is to show that effects of the type that could be expected from the relevant class of theories could be seen. The key source of pride for quantum-gravity phenomenologists comes from the fact that over these past few years, and over a time that indeed spanned over only a handful of years, we managed to change the perception of quantum-gravity research from the traditional "no help from experiments possible" to the present intuition, shared by most workers in the field, that these effects could be seen. We might need some luck to actually see them, but clearly it is not possible. There is therefore a legitimate phenomenology to be developed for quantum gravity.

Once task one is accomplished it is important to show that the type of observations that are doable not only provide opportunities to luckily stumble upon a manifestation of the new theory, but actually the data could be used to falsify candidate theories. This task two clearly requires much more of task one, both at the level of our understanding of the theories and for what concerns the quality of the data and their phenomenological analysis.

Concerning task one it is of course significant that over these past few years several authors have shown in different ways and for different candidate Planck-scale effects that, in spite of the horrifying smallness of these effects, some classes of doable experiments and observations could see the effects. Just to make absolutely clear the fact that effects genuinely introduced at the Planck scale could be seen, let us just exhibit here one very clear illustrative example.

The Planck-scale effect we consider is codified by the following energy-momentum (dispersion) relation

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 \left( \frac{E^2}{E_p^2} \right) \ , \tag{12}$$

where $E_p$ denotes again the Planck scale ($E_p = 1/L_p \sim 10^{28} eV$) and $\eta$ is a phenomenological parameter. This is a good choice because convincing the reader that we are dealing with an effect introduced genuinely at the Planck scale is in this case effortless. It is in fact well known (see, *e.g.*, Ref. [57]) that this type of $E_p^{-2}$ corrections to the dispersion relation can result from discretization of spacetime on a lattice with $E_p^{-1}$ lattice spacing[2].

If such a modified dispersion relation is part of a framework where the laws

[2]The idea of a rigid lattice description of spacetime is not really one of the most advanced for quantum-gravity research, but this consideration is irrelevant for task one: in order to get this phenomenology started we first must establish that the sensitivities we have are sufficient for effects as small as typically obtained from introducing structure at the Planck scale. The smallness of the effect in (13) is clearly representative of the type of magnitude that quantum-gravity effects are expected to have, and the fact that it can also be obtained from a lattice with $E_p^{-1}$ spacing confirms this point.

of energy-momentum conservation are unchanged one easily finds [3, 4, 5, 6] significant implications for the cosmic-ray spectrum. In fact, the "GZK cutoff", a key expected feature of the cosmic-ray spectrum, is essentially given by the threshold energy for cosmic-ray protons to produce pions in collisions with CMBR photons. In the evaluation of the threshold energy for $p + \gamma_{CMBR} \rightarrow p + \pi$ the correction term $\eta \vec{p}^2 E^2/E_p^2$ of (13) can be very significant. Whereas the classical-spacetime prediction for the GZK cutoff is around $5 \cdot 10^{19} eV$, at those energies the Planck-scale correction to the threshold turns out [3, 4, 5, 6] to be of the order of $\eta E^4/(\epsilon E_p^2)$, where $\epsilon$ is the typical CMBR-photon energy. For positive values of $\eta$, even somewhat smaller[3] than 1, this amounts to an observably large shift of the threshold energy, which should easily be seen (or excluded) once the relevant portion of the cosmic-ray spectrum becomes better known, with observatories such as the Pierre Auger Observatory.

Of course, the same effect is present and is even more significant if instead of a $E_p^{-2}$ correction one introduces in the dispersion relation a correction of $E_p^{-1}$ type.

### 4.2 Falsifying theories

Arguments such as the one offered in the previous subsection clearly show that this phenomenology has a right to existence. Task one is settled. We do have at least a chance (perhaps slim, but this is not the point here) to see Planck-scale effects, and if we ever do see one such effect it will be wonderful. But a phenomenology should also be valuable when it does not find the effects it looks for, by setting limits on (and in some cases ruling out) corresponding theories. Have we proven that quantum-gravity phenomenology can rule out Planck-scale theories?

Of course the phenomenology will be based on some "test theories" and the parameters of the test theories will be increasingly constrained as data become available. But beyond the level of test theories there is the truly sought level of "theories", models which are not merely introduced (as is the case of test theories) as a language used in mapping the progress of experimental limits on some effects, but rather models which are originally motivated by some ideas for the solution of the quantum-gravity problem. And in order to falsify one such theory we need to prove experimentally the absence of an effect which has

[3] Of course the quantum-gravity intuition for $\eta$ is $\eta \sim 1$.

been rigorously established to be a necessary consequence of the theory. These are the ingredients of the task two described above. But the theories used in quantum-gravity research are so complex that we can rarely really establish that a given effect is necessarily present in the theory. What usually happens is that we find some "theoretical evidence" for the effect in a given quantum-gravity theory and then we do the phenomenology of that effect using some test theories. The link from theory to effect is too weak to be used in reverse: we are usually not able to say that the absence of the effect really amounts to ruling out the theory.

Think for example of Loop Quantum Gravity. Because of the so-called "classical-limit problem" at present one is never really able to use that theory to provide a definite prediction for an effect to be looked for by experimentalists. And for String Theory the situation might be worse, at least in the sense that one might not even be able to hope better things for the future: it is in fact at present not clear whether string theory is in principle able to make any definite predictions, since the formalism is so flexible, so capable to say anything, that it is feared to amount basically to saying nothing.

So concerning task two the situation does not look very healthy, but the problem resides on the theory side, not the phenomenology side.

If indeed, at least for now, we cannot falsify Loop Quantum Gravity and String Theory, can we at least falsify some other theory used in quantum-gravity research? It is of course extremely important for quantum-gravity phenomenology to find one such example. If we do find a first example then we can legitimately hope that the falsifiability of more and more theories will gradually be achieved. Theories in the $\kappa$-Minkowski noncommutative spacetime considered in Section 3 could turn out to be falsifiable, and the needed mathematics is probably within our reach.

### 4.3 Concerning quantum-gravity effects and the status of Quantum-Gravity theories

At the present stage of investigation of the quantum-gravity problem it is actually not so obvious how to identify candidate quantum-gravity effects. Analogous situations in other areas of physics are usually such that there are a few new theories which have started to earn our trust by successfully describing some otherwise unexplained data, and then often we let those theories guide us toward new effects that should be looked for. The theories we have for quantum gravity, in spite of all their truly remarkable mathematical beauty, and their extraordinary contribution to the investigation of the conceptual sides of the quantum-gravity problem, cannot (yet) claim any success in the experimental realm. Moreover, even if we wanted to used them as guidance for experiments the complexity of these theories proves to be a formidable obstruction. What we can do with these theories (and we must be content with it since we do not have many alternatives) is to look at their general structure and use this as a source of intuition for the proposal of a few candidate effects.

A similar type of path toward the identification of some candidate quantum-gravity effects is the one based on the analysis of the general structure of the quantum-gravity problem itself. It happens to be the case that by looking at the type of presently-unanswered questions for which quantum-gravity is being sought, one is automatically led to considering a few candidate effects.

Of course these ideas suggested from our perception of the structure of the quantum-gravity problem and from our analysis of the general structure of some proposed quantum-gravity theories could well turn out to be completely off the mark, but it still makes sense to investigate these ideas.

### *4.3.1 Planck-scale departures from classical spacetime symmetries*

From the general structure of the quantum-gravity problem, which clearly provides at least some encouragement to considering discretized (or otherwise "quantized") spacetimes, one finds encouragement to consider departures from classical spacetime symmetries. Consider for example the Minkowski limit, the one described by the classical Minkowski spacetime in current theories. There is a duality one-to-one relation between the classical Minkowski spacetime and the classical (Lie-) algebra of Poincaré symmetrie. Poincaré transformations are smooth arbitrary-magnitude classical transformations and it is rather obvious that they should be put under scrutiny [58] if the classical description of spacetime is replaced by a quantized/discretized one.

### *4.3.2 Planck-scale departures from CPT symmetry*

Perhaps the most intelligible evidence of a Planck-scale effect would be a violation of CPT symmetry. CPT symmetry is in fact protected by a theorem in our current (Minkowski-limit) theories, mainly as a result of locality and

Poincaré symmetry. The fact that the structure of the quantum-gravity problem invites us to consider spacetimes with some element of nonlocality and/or departures from Poincaré symmetry clearly opens a window of opportunity for Planck-scale violations of CPT symmetry.

### *4.3.3 Distance fuzziness and spacetime foam*

The fact that the structure of the quantum-gravity problem suggests that the classical description of spacetime should give way to a nonclassical one at scales of order the Planck scale has been used extensively as a source of inspiration concerning the proper choice of formalism for the solution of the quantum-gravity problem, but for a long time (decades) it had not inspired ideas relevant for phenomenology. The description that came closer to a physical intuition for the effects induced by spacetime nonclassicality is Wheeler's "spacetime foam", which however does not amount to a definition (at least not a scientific/operative definition). A few years ago one of us proposed [9] a physical/operative definition of (at least one aspect of) spacetime fuzziness/foam, which makes direct reference to interferometry. According to this definition the fuzziness/foaminess of a spacetime is established on the basis of an analysis of strain noise in interferometers set up in that spacetime. In achieving their remarkable accuracy modern interferometers must deal with several classical-physics strain noise sources (*e.g.*, thermal and seismic effects induce fluctuations in the relative positions of the test masses). And importantly strain noise sources associated with effects due to ordinary quantum mechanics are also significant for modern interferometers (the combined minimization of *photon shot noise* and *radiation pressure noise* leads to a noise source which originates from ordinary quantum mechanics). The operative definition of fuzzy/foamy spacetime advocated in Ref. [9] characterizes the corresponding quantum-gravity effects as an additional source of strain noise. A theory in which the concept of distance is fundamentally fuzzy in this operative sense would be such that the read-out of an interferometer would still be noisy (because of quantum-gravity effects) even in the idealized limit in which all classical-physics and ordinary-quantum-mechanics noise sources are completely eliminated.

### 4.3.4 Decoherence

For approaches to the quantum-gravity problem which assume that, in merging with General Relativity, Quantum Mechanics should be revised one of the most popular effects is decoherence. This may be also motivated using heuristic arguments, based mainly on quantum field theory in curved spacetimes, which suggest that black holes radiate thermally, with an associated "information-loss problem".

### 4.3.5 Planck-scale departures from the Equivalence Principle

Various perspectives on the quantum-gravity problem appear to suggest departures from one or another (stronger or weaker) form of the Equivalence Principle. For brevity let me just summarize here my preferred argument, which is based on the observation that locality is a key ingredient of the present formulation of the Equivalence Principle. In fact, the Equivalence Principle ensures that (for same initial conditions) two point particles would go on the same geodesic independently of their mass. But it is well established that this is not applicable to extended bodies, and presumably also not applicable to "delocalized point particles" (point particles whose position is affected by uncontrolled uncertainties). If spacetime structure is such to induce an irreducible limit on the localization of particles it would seem then natural to expect some departures from the Equivalence Principle.

### 4.3.6 Critical-dimension SuperString Theory

The most popular realization of String Theory, with the adoption of supersymmetry and the choice of working in a "critical" number of spacetime dimensions, has given a very significant contribution to the conceptual aspects of the debate on quantum gravity, perhaps most notably the fact that, indeed thanks to research on string theory, we now know that quantum gravity might well be a perturbatively renormalizable theory (whereas this was once thought to be impossible). But for what concerns the prediction of physical effects string theory has not proven (yet?) to be rich. In spite of all the noteworthy mathematical structure that are needed for the analysis of string theory, from a wider perspective this is the approach that by construction assumes that the solution to the quantum-gravity problem should bring about a rather limited amount of novelty. In particular, string theory is still introduced in a classical Minkowski spacetime and it is still a genuinely quantum-mechanical theory. None of the effects possibly due to spacetime quantization are therefore necessarily expected and all the departures-from-quantum-mechanics effects, like decoherence effects, are also not expected.

But on the other hand, as mentioned, string theory is turning out to be a remarkably flexible formalism, and therefore, while one can structure things in such a way that nothing interestingly new happens, one can also mould the formalism in such a way to have some striking new effects[4], and effects that fit within some intuitions concerning the quantum-gravity problem. In particular there is a known scheme for having violations of the equivalence principle [18], and by providing a vacuum expectation value for a relevant antisymmetric tensor one can give rise [59] to departures from Poincaré symmetry (together with the emergence of an effective spacetime noncommutativity).

#### *4.3.7 Loop Quantum Gravity*

The only other approach with contributions to the conceptual debate on the quantum-gravity problem of significance comparable to the ones of the string-theory approach is Loop Quantum Gravity. In particular, it is thanks to work on Loop Quantum Gravity that we now know that quantum gravity might fully preserve the diffeomorphism invariance of General Relativity (whereas this was once thought to be impossible). But also Loop Quantum Gravity, while excelling in the conceptual arena, has its difficulties providing predictions to phenomenologists. While String Theory may be perceived as frustratingly flexible, one might perhaps say that at the present stage of development Loop Quantum Gravity appears not to have even the needed room to maneuver it down to the mundane arena of corrections to General Relativity and corrections to the Standard Model of particle physics. As a result of the much debated "classical-limit problem", in a certain sense Loop Quantum Gravity provides a candidate description of everything but does not provide an explicit description of anything. One may attempt however (and several groups have

[4]One of the most noteworthy possibilities is the one of "large extra dimensions". This gives rise to a peculiar brand of quantum-gravity phenomenology, which is not governed by the Planck scale. In these notes we intend to focus on Planck-scale effects.

indeed attempted to do this) to infer from the general structure of the theory some ideas for candidate Loop-Quantum-Gravity effects. In particular, several studies [8, 19] have argued that the type of discretization of spacetime observables usually attributed to Loop Quantum Gravity could be responsible for Planck-scale departures from Lorentz symmetry. This hypothesis also finds encouragement [20] in light of the role apparently played by noncommutative geometry in the description of certain aspects of the theory.

Of course, as long as the "classical-limit problem" is not solved, the evidence of departures from Lorentz symmetry in (the Minkowski limit [21] of) Loop Quantum Gravity must be considered weak, and any attempt to give a concrete formulation of these effects will have to rely at one point or another on heuristics. This remains a very valuable exercise for quantum-gravity phenomenology, since it gives us ideas on effects that are worth looking for, but clearly at present phenomenologists are not given any chance of falsifying Loop Quantum Gravity.

From the phenomenology perspective there is more than the Lorentz-symmetry issue at stake in the "classical-limit problem": it is not unlikely that structures relevant for CPT symmetry and the Equivalence Principle are also present, and Loop Quantum Gravity could be a natural context where to develop a physical intuition for spacetime foam.

#### *4.3.8 Approaches based on noncommutative geometry*

Noncommutative spacetimes so far have been considered has opportunities to look at specific aspects of the quantum-gravity problem (whereas string theory and loop quantum gravity attempt to provide a full solution). It is perhaps fair to say that the most significant findings emerged in attempts to describe the Minkowski limit [21] of quantum-gravity. One might say that these studies look at one half of the quantum-gravity problem, the quantum-spacetime aspects. Because of the double role of the gravitational field, which in some ways is just like another field given in spacetime but it is also governs the structure of spacetime, in quantum-gravity research one ends up considering two types of quantization: some sort of quantization of gravitational interactions and some sort of quantization of spacetime structure. At present one might say that only within the Loop Quantum Gravity approach we are truly exploring both aspects of the problem. String Theory, as long as it is formulated in a classical (background) spacetime, focuses in a sense on the quantization of the gravitational interaction, and sets aside the possible "quantization" of spacetime[5]. And the reverse is true of mainstream research on spacetime noncommutativity, which provides a way to quantize spacetime, but, at least for this early stages of development, does not provide a description of gravitational interactions.

The analysis of noncommutative deformations of Minkowski spacetime has provided some intuition for what could be the fate of (Minkowski-limit/Poincaré) symmetries at the Planck scale. And also valuable for the development of quantum-gravity phenomenology is the fact that in some cases, such as the $\kappa$-Minkowski noncommutative spacetime, it is reasonable to hope that these studies will soon provide truly falsifiable predictions.

Unfortunately spacetime fuzziness, which is the primary motivation for most researchers to consider noncommutativity, frustratingly remains only vaguely characterized in current research on noncommutative spacetimes.

## 4.4 On the status of different areas of this phenomenology

### *4.4.1 Planck-scale modifications of Poincaré symmetries*

The most developed quantum-gravity-phenomenology research area is the one that considers the possibility of Planck-scale departures from Poincaré symmetry. We shall discuss in some detail these studies later in this section.

### *4.4.2 Planck-scale modifications of CPT symmetry and Decoherence*

The most studied opportunity to test CPT symmetry is provided by the neutral-kaon and the neutral-B systems [36, 37]. One finds that in these neutral-meson systems a Planck-scale departure from CPT symmetry could in principle be amplified. In particular, the neutral-kaon system hosts the peculiarly small mass difference between long-lived and the short-lived kaons $|M_L - M_S|/M_{L,S} \sim 7{\cdot}10^{-15}$, and there are scenarios of Planck-scale CPT violation in the literature [36] in which the inverse of this small number amplifies a small (Planck-

[5] Just like in noncommutative geometry one hopes one day to obtain also the quantization of the interaction, by introducing a suitable noncommutative-geometrodynamics, in approaches like string theory one may hope that the quantization of the interaction field may at advanced levels of analysis amount to spacetime quantization. Some string-theory results do encourage this hope [60] but the situation remains puzzling [61]

scale induced) CPT-violation effect. This in particular occurs in the most studied scenario for Planck-scale violations of CPT symmetry in the neutral-kaon system, in which the Planck-scale effects induce a difference between the terms on the diagonal of the $K^0$,$\bar{K^0}$ mass matrix. An analogous effect would be present in the neutral-B system but if the Planck-scale effect for the terms on the diagonal is momentum independent the best sensitivity is expected from studies of the neutral-kaon system. It is however not implausible [22] that the Planck-scale effects would introduce a correction to the diagonal terms of the neutral-meson mass matrix that depends on the momentum of the particle, and in this case, among the experiments currently done or planned, the best sensitivity would be obtained with the neutral-B system.

### *4.4.3 Distance fuzziness and spacetime foam*

The phenomenology of distance fussiness is being developed mainly in two directions: interferometry and observations of extragalactic sources.

In interferometry the debate [9, 10] involves a variety of phenomenological models and different perspectives on what is the correct intuition that one should implement. It is perhaps best here to just give the simplest observation that can provide encouragement for these studies. As we stressed above in interferometry it is natural to look for Planck-scale contributions to the strain noise. And it is noteworthy that strain noise is naturally described in terms [9] of a function of frequency $\rho(\nu)$ (a tool for spectral analysis) that carries dimensions of $Hz^{-1}$. If one was to make a naive dimensional estimate of Planck scale effects one could simply pose $\rho \sim L_p/c$, which at first might seem not too encouraging since it leads to a very small estimate of $\rho$: $\rho \sim 10^{-44} Hz^{-1}$. However, modern interferometers are achieving truly remarkable sensitivities, driven by their main objective of seeing classical gravity waves, and levels of $\rho$ as small as $10^{-44} Hz^{-1}$ are within their reach.

Another much discussed opportunity for constraining models of spacetime fuzziness is provided by the observation of extragalactic sources, such as distant quasars. Essentially it is argued [23, 24] that, given a wave description of the light observed from the source, spacetime fuzziness should introduce an uncertainty in the waves phase that cumulates as the wave travels, and for sufficiently long propagation times this effect should scramble the wave front enough to prevent the observation of interferometric fringes. Also in this case plausible estimates suggest that, in spite of the smallness of the Planck-scale effects, thanks to the amplification provided by the long propagation times the needed sensitivity might soon be within our reach.

#### *4.4.4 Decoherence*

The development of test theories for decoherence is of course a challenging area of quantum-gravity phenomenology, since the test theories must go beyond quantum mechanics. Let us just here mention Refs. [25] as a good entry point in the relevant literature, and stress that the neutral-kaon system, with its delicate balance of scales, is also considered [36, 25] to be our best opportunity for laboratory studies of Planck-scale-induced decoherence.

#### *4.4.5 Planck-scale departures from the Equivalence Principle*

As mentioned the quantum-gravity problem also provides motivation to contemplate departures from the Equivalence Principle, and in some approaches (in particular in String Theory) some structures suitable for describing departures from the Equivalence Principle are found. The phenomenology is very rich and in many ways goes well beyond the specific interests of quantum-gravity research: the Equivalence Principle continues to be placed under careful scrutiny especially because of its central role in General Relativity. Interested readers could consider as points of entrance in the relevant literature the overall review in Ref. [26] and, more specifically for departures from the Equivalence Principle within the string-theory approach, Ref. [18].

### 4.5 Aside on Doubly-Special Relativity

In preparation for the next subsection, which focuses on the phenomenology of Planck-scale departures from Poincaré symmetry, it might useful to provide here a short introduction to "doubly-special relativity" (DSR) [17], which recently has been often analyzed as an alternative to the standard scenario of Planck-scale effects that break Lorentz(/Poincaré) symmetry. The doubly-special-relativity scenario was introduced [17] as a sort of alternative perspective on the results on Planck-scale departures from Lorentz symmetry which had been reported in numerous articles [3, 4, 5, 6, 7, 8, 19] between 1997 and 2000. These studies were advocating a Planck-scale modification of the energy-momentum dispersion relation, usually of the form $E^2 =$

$p^2 + m^2 + \eta L_p^n p^2 E^n + O(L_p^{n+1} E^{n+3})$, on the basis of preliminary findings in the analysis of several formalisms in use for Planck-scale physics. The complexity of the formalisms is such that very little else was known about their physical consequences, but the evidence of a modification of the dispersion relation was becoming robust. In all of the relevant papers it was assumed that such modifications of the dispersion relation would amount to a breakup of Lorentz symmetry, with associated emergence of a preferred class of inertial observers (usually identified with the natural observer of the cosmic microwave background radiation).

The DSR idea was proposed [17] on the basis of a striking analogy between these developments and the developments which led to the emergence of Special Relativity, now more than a century ago. In Galilei Relativity there is no observer-independent scale, and in fact the energy-momentum relation is written as $E = p^2/(2m)$. As experimental evidence in favour of Maxwell equations started to grow, the fact that those equations involve a fundamental velocity scale appeared to require the introduction of a preferred class of inertial observers. But in the end we figured out that the situation was not demanding the introduction of a preferred frame, but rather a modification of the laws of transormation between inertial observers. Einstein's Special Relativity introduced the first observer-independent relativistic scale (the velocity scale $c$), its dispersion relation takes the form $E^2 = c^2p^2 + c^4m^2$ (in which $c$ plays a crucial role for what concerns dimensional analysis), and the presence of $c$ in Maxwell's equations is now understood as a manifestation of the necessity to deform the Galilei transformations.

Refs. [17] argued that it is plausible that we might be presently confronted with an analogous scenario. Research in quantum gravity is increasingly providing reasons of interest in Planck-scale modifications of the dispersion relation, of the type mentioned above, and, while it was customary to assume that this would amount to the introduction of a preferred class of inertial frames (a "quantum-gravity ether"), the proper description of these new structures might require yet again a modification of the laws of transformation between inertial observers. The new transformation laws would have to be characterized by two scales ($c$ and $L_p$) rather than the single one ($c$) of ordinary Special Relativity.

The "historical motivation" described above leads to a scenario for Planck-scale physics which is not intrinsically equipped with a mathematical formalism for its implementation, but still is rather well defined. With Doubly-Special Relativity one looks for a transition in the Relativity postulates, which should be largely analogous to the Galilei → Einstein transition. Just like it turned out to be necessary, in order to describe high-velocity particles, to set aside Galilei Relativity (with its lack of any characteristic invariant scale) and replace it with Special Relativity (characterized by the invariant velocity scale $c$), it is at least plausible that, in order to describe ultra-high-energy particles, we might have to set aside Special Relativity and replace it with a new relativity theory, a DSR, with two characteristic invariant scales, a new small-length/large-momentum scale in addition to the familiar velocity scale.

A theory will be compatible with the DSR principles if there is complete equivalence of inertial observers (Relativity Principle) and the laws of transformation between inertial observers are characterized by two scales, a high-velocity scale and a high-energy/short-length scale. Since in DSR one is proposing to modify the high-energy sector, it is safe to assume that the present operative characterization of the velocity scale $c$ would be preserved: $c$ is and should remain the speed of massless low-energy particles[6]. Only experimental data could guide us toward the operative description of the second invariant scale $\lambda$, which may or may not be based on a deformed dispersion relation, but $\lambda$ is naturally guessed to be somewhere in the neighborhood of the Planck length $L_p$.

As a result of the "historical context" that led to the DSR idea most authors have explored the possibility that the second relativistic invariant be introduced through a modifications of the dispersion relation. This is a reasonable choice but it would be incorrect at present to identify (as often done in the literature) the DSR proposal with the proposal of observer-independent modifications of the dispersion relation. For example the dispersion relation might not be modified but there might instead be an observer-independent bound on the accuracy achievable in the measurement of distances.

In the search of a first example of formalism compatible with the DSR

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[6]Note however the change of perspective imposed by the DSR idea: within Special Relativity $c$ is the speed of all massless particles, but Special Relativity must be perceived as a low-energy theory (as viewed from the DSR perspective) and in taking Special Relativity as starting point for a high-energy deformation one is only bound to preserving $c$ as the speed of massless low-energy particles.

principles much work has been devoted to the study of the $\kappa$-Minkowski spacetime, which inspired our toy model (Section 3) for multiparticle-state phenomenology.

### 4.6 More on the phenomenology of departures from Poincaré symmetry

In this subsection we comment on some aspects of recent phenomenology work on departures from Poincaré symmetry, mostly as codified in modifications of the energy-momentum dispersion relation. We start by stressing that the same modified dispersion relation can be introduced in very different test theories, leading to completely different physical predictions.

#### *4.6.1 On the test theories with modified dispersion relation*

The majority (see, *e.g.*, Refs. [3, 4, 5, 6, 7, 8, 19]) of studies concerning Planck-scale modifications of the dispersion relation adopt the phenomenological formula

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 \left(\frac{E^n}{E_p^n}\right) + O(\frac{E^{n+3}}{E_{QG}^{n+1}}) \ , \tag{13}$$

with real $\eta$ (assumed to be of order $|\eta| \sim 1$) and integer $n$.

There is at this point a very large literature on the associated phenomenology, but actually some of the different phenomenological studies that compose this literature introduce this type of dispersion relation within different test theories. The limits obtained within different test theories are of course not to be compared. The same parametrization of the dispersion relation, if introduced within different test theories, actually gives rise to independent sets of parameters.

The potential richness of this phenomenology, for what concerns the development of test theories, mainly originates from the need to specify, in addition to the form of the dispersion relation, several other structural properties of the test theory.

It is necessary to state whether the theory is still “Hamiltonian”, at least in the sense that the velocity is obtained from the commutator with an Hamiltonian (for example, along the $x$ axis, $v \sim [x, H]$) and whether the Heisenberg commutator preserves its standard form ($[x, p] \sim \hbar$). Especially this second concern is rather significant since some of the heuristic arguments which are used to motivate the presence of modified dispersion relations at the Planck scale also suggest that the Heisenberg commutator should be correspondingly modified.

Then the test theory should formulate a law of energy-momentum conservation. We have discussed the example of The *kappa*-Minkowski which we considered is an example of spacetime that contributed to interest in modified dispersion relations and appears to be such to require also an accompanying modification of the law of energy-momentum conservation. And in particular a link between modification of the dispersion relation and associated modification of the law of energy-momentum conservation is required by the DSR principles (see below).

And one should keep clearly separate the test theories that intend to describe only kynematics and the ones that also adopt a scheme for Planck-scale dynamics. For example, in Loop Quantum Gravity and some noncommutative spacetimes which provided motivation for considering modifications of the dispersion relation, while we might be close to have a correct picture of kinematics it appears that we are still far from understanding Planck-scale corrections to dynamics[7]

An attempt to introduce a few examples of meaningful test theories has been reported in Ref. [27]. Here we shall be content with showing how in different phenomenological studies based on modified dispersion relations one ends up making assumptions about the points listed above.

#### *4.6.2 Photon stability*

It has been recently realized (see, *e.g.*, Refs. [28, 29, 30]) that when Lorentz symmetry is broken at the Planck scale there can be significant implications for certain decay processes. At the qualitative level the most significant novelty would be the possibility for massless particles to decay. Let us consider for example a photon decay into an electron-positron pair: $\gamma \to e^+e^-$. And let us analyze this process using the dispersion relation (13), for $n = 1$, with unmodified law of energy-momentum conservation. One easily finds a relation between the energy $E_\gamma$ of the incoming photon, the opening angle $\theta$ between the

[7]On the Loop Quantum Gravity side this is linked once again with the "classical limit problem", while for the relevant noncommutative spacetime the concern originates from the difficulties encountered in producing consistent theories of quantum matter fields in those spacetimes.

outgoing electron-positron pair, and the energy $E_+$ of the outgoing positron, which, for the region of phase space with $m_e \ll E_\gamma \ll E_p$, takes the form $\cos(\theta) = (A+B)/A$, with $A = E_+(E_\gamma - E_+)$ and $B = m_e^2 - \eta E_\gamma E_+(E_\gamma - E_+)/E_p$ ($m_e$ denotes of course the electron mass). The fact that for $\eta = 0$ this would require $cos(\theta) > 1$ reflects the fact that if Lorentz symmetry is preserved the process $\gamma \to e^+e^-$ is kinematically forbidden. For $\eta < 0$ the process is still always forbidden, but for positive $\eta$ and $E_\gamma \gg (m_e^2 E_p/|\eta|)^{1/3}$ one finds that $\cos(\theta) < 1$ in certain corresponding region of phase space.

The energy scale $(m_e^2 E_p)^{1/3} \sim 10^{13} eV$ is not too high for astrophysics. The fact that certain observations in astrophysics allow us to establish that photons of energies up to $\sim 10^{14} eV$ are not unstable (at least not noticeably unstable) could be used [28, 30] to set valuable limits on $\eta$.

This is quite a striking result, which however should be reported with caution: this is not a strategy to set direct limits on the parameters of the dispersion relation, since the analysis very explicitly requires us to specify also the form of the energy-momentum conservation law. By changing the form of the law of energy-momentum conservation, for fixed form of the dispersion relation, one can indeed obtain very different results. This is best illustrated contemplating the possibility that such a dispersion relation be introduced within a DSR framework. First of all let us notice that any theory compatible with the DSR principle will have stable massless particles, so that by looking for massless-particle decay one could falsify the DSR idea. A threshold-energy requirement for massless-particle decay (such as the $E_\gamma \gg (m_e^2 E_p/|\eta|)^{1/3}$ mentioned above) cannot of course be introduced as an observer-independent law, and is therefore incompatible with the DSR principles.

An analysis of the stability of massless particles that is compatible with the DSR principles can be obtained by combining the modification of the dispersion relation with an associated modification of the laws of energy-momentum conservation. The form of the new law of energy-momentum conservation can be derived from the requirement of being compatible both with the DSR principles and with the modification of the dispersion relation [17], and in particular in the case of $a \to b + c$ decays one arrives at $E_\gamma \simeq E_+ + E_- - \lambda \vec{p}_+ \cdot \vec{p}_-$, $\vec{p}_\gamma \simeq \vec{p}_+ + \vec{p}_- - \lambda E_+ \vec{p}_- - \lambda E_- \vec{p}_+$. Using these in place of ordinary conservation of energy-momentum one ends up with a result for $\cos(\theta)$ which is still of the form $(A + B)/A$ but now with $A = 2E_+(E_\gamma - E_+) + \lambda E_\gamma E_+(E_\gamma - E_+)$ and

$B = 2m_e^2$. Evidently this formula always gives $\cos(\theta) > 1$, consistently with the fact that $\gamma \to e^+e^-$ is forbidden in DSR.

### *4.6.3 Threshold anomalies*

Another opportunity to investigate Planck-scale departures from Lorentz symmetry is provided by certain types of energy thresholds for particle-production processes that are relevant in astrophysics. This is a very powerful tool for quantum-gravity phenomenology, and in fact we already discussed the evaluation of the threshold energy for $p + \gamma_{CMBR} \to p + \pi$ as a key example in support of the fact that quantum-gravity phenomenology is worth doing.

Numerous quantum-gravity-phenomenology papers (see, *e.g.*, Refs.[3, 4, 5, 6]) have been devoted to the study of Planck-scale-modified thresholds, so the interested readers can find an abundance of related materials.

### *4.6.4 Time-of-travel analyses*

A wavelength dependence of the speed of photons is obtained from a modified dispersion relation, if one assumes the velocity to be still described by $v = dE/dp$. For the dispersion relation here considered one finds that at "intermediate energies" ($m < E \ll E_p$) the velocity law will take the form

$$v \simeq 1 - \frac{m^2}{2E^2} + \eta \frac{n+1}{2} \frac{E^n}{E_p^n} \ . \tag{14}$$

On the basis of this formula one would find that two simultaneously-emitted photons should reach the detector at different times if they carry different energy. And this time-of-arrival-difference effect can be significant[7] in the analysis of short-duration gamma-ray bursts that reach us from cosmological distances. For a gamma-ray burst it is not uncommon that the time travelled before reaching our Earth detectors be of order $T \sim 10^{17}s$. Microbursts within a burst can have very short duration, as short as $10^{-3}s$, and this means that the photons that compose such a microburst are all emitted at the same time, up to an uncertainty of $10^{-3}s$. Some of the photons in these bursts have energies that extend at least up to the $GeV$ range, and for two photons with energy difference of order $\Delta E \sim 1GeV$ a $\Delta E/E_p$ speed difference over a time of travel of $10^{17}s$ would lead to a difference in times of arrival of order $\Delta t \sim T\Delta\frac{E}{E_p} \sim 10^{-2}s$

which is significant (the time-of-arrival differences would be larger than the time-of-emission differences within a microburst).

It is well established that the sensitivities achievable [31] with the next generation of gamma-ray telescopes, such as GLAST [31], could allow to test very significantly (14) in the case $n = 1$, by possibly pushing the limit on $|\eta|$ far below 1. And, as we shall stress later, for the case $n = 2$ neutrino astronomy may lead to valuable insight [14, 15].

### *4.6.5 Synchrotron radiation*

As observed recently in Ref. [32], in the mechanism that leads to the production of synchrotron radiation a key role is played by the special-relativistic velocity law $v = dE/dp \simeq 1 - m^2/(2E^2)$. And an interesting observation is obtained by considering the velocity law (14) for the case $n = 1$. Assuming that all other aspects of the analysis of synchrotron radiation remain unmodified at the Planck scale, one is led [32] to the conclusion that, if $\eta < 0$, the energy/wavelength dependence of the Planck-scale term in (14) can affect the value of the cutoff energy for synchrotron radiation. This originates from the fact that according to (14), for $n = 1$ and $\eta < 0$, an electron cannot have a speed that exceeds the value $v_e^{max} \simeq 1 - (3/2)(|\eta| m_e/E_p)^{2/3}$, whereas in special relativity $v_e$ can take values arbitrarily close to 1. This may be used to argue that for negative $\eta$ the cutoff energy for synchrotron radiation should be lower than it appears to be suggested by certain observations of the Crab nebula [32].

In making use of this striking observation it is however important to notice that synchrotron radiation is due to the acceleration of the relevant electrons and therefore dynamics plays an implicit role in the derivation of the result [27]. From a field-theory perspective the process of synchrotron-radiation emission is described in terms of Compton scattering of the electrons with the virtual photons of the magnetic field, confirming the need to include a description of some aspects of dynamics and of energy-momentum conservation (for the vertices in the Compton-scattering analysis).

### *4.6.6 Neutrino observations*

In closing we find appropriate to spend a few words on a novel opportunity for quantum-gravity phenomenology: planned neutrino observatories, such as ICE-CUBE, are likely to be very valuable. This had already been timidly suggested in a few earlier papers [11, 12, 13] and should now gain some momentum in light of the analysis reported in Ref. [14] (also see Refs. [15, 16]), which proposes a definite and apparently doable programme of studies.

A key reason of interest in these neutrino studies is the possibility to use them in combination with gamma-ray studies to seek evidence of a spin dependence of the way in which conjectured quantum properties of spacetime affect particle propagation. And, even assuming that there is no such spin dependence (so that gamma rays and neutrinos could serve exactly the same purposes), neutrinos might well be then our best weapon for the study of certain candidate effects. This is due to the fact that it is actually easier to detect high-energy neutrinos (at or above $10^{14}eV$), rather than low energy ones, whereas it is expected that high-energy gamma rays (starting at energies of a few $TeV$) be absorbed by soft photons in the cosmic background. So neutrinos will effectively extend the energy range accessible to certain classes of studies, and energy is obviously a key factor for the sensitivity of quantum-gravity-phenomenology analyses.

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