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Entanglement and Bell's Inequality in the Neutral Kaon System

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for

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Motivation

Composite quantum system in pure or mixed state

nonlocal — contextual features J.S. Bell

entanglement

Basis for quantum communication and teleportation,
quantum information and computing ⟶ new area in physics

Aim: understand features of entanglement

phenomenological ⟶ conceptual ⟶ mathematical aspects

elementary particles — massive, internal symmetries, decays

$K^0\bar{K}^0$ – system strangeness interesting systems
$B^0\bar{B}^0$ – system beauty

Stability of quantum system

understand decoherence — entanglement loss

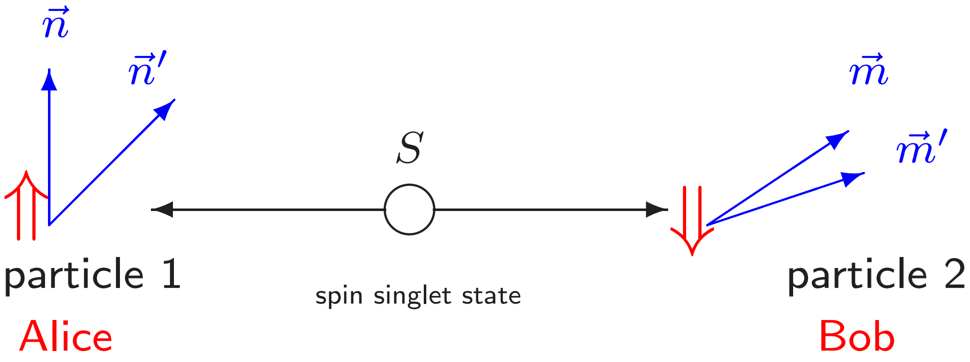
Part I

Bell inequality

Bell's Theorem

Bell's Theorem 1964

J.S. Bell: *"... in a certain experimental situation all LRT (local realistic theories) are incompatible with QM."*



EPR–type experiment

Expectation value

$A(n,\lambda)$ values of observable $A^{QM}(n) \longrightarrow \vec{\sigma}_{(1)} \cdot \vec{n}$

$B(m,\lambda)$ $B^{QM}(m) \longrightarrow \vec{\sigma}_{(2)} \cdot \vec{m}$

λ...hidden variable n, m...quantisation directions $|A|$, $|B| \leq 1$

$$A(n,\lambda) = \begin{cases} +1 & \Uparrow \\ 0 & \text{no detection} \\ -1 & \Downarrow \end{cases} \qquad B(m,\lambda) = \begin{cases} +1 & \Uparrow \\ 0 & \text{no detection} \\ -1 & \Downarrow \end{cases}$$

particle 1 at Alice particle 2 at Bob

Expectation value for combined spin measurement

$$E(n,m) = \int d\lambda\,\rho(\lambda)\underset{\substack{\uparrow \\ m}}{A(n,\lambda)}\underset{\substack{\uparrow \\ n}}{B(m,\lambda)} \qquad \text{with} \qquad \int d\lambda\,\rho(\lambda) = 1$$

independent of m n Bell's locality hypothesis

Bell inequality

Expectation value in terms of probabilities

$$\begin{aligned} E(n,m) &= P(n\Uparrow, m\Uparrow) + P(n\Downarrow, m\Downarrow) - P(n\Uparrow, m\Downarrow) - P(n\Downarrow, m\Uparrow) \\ &= -1 + 4\,P(n\Uparrow, m\Uparrow) \end{aligned}$$

construct Bell inequality of Wigner–type for 3 different quantization directions

$$P(n,m) \leq P(n,n') + P(n',m) \qquad \text{BI}$$

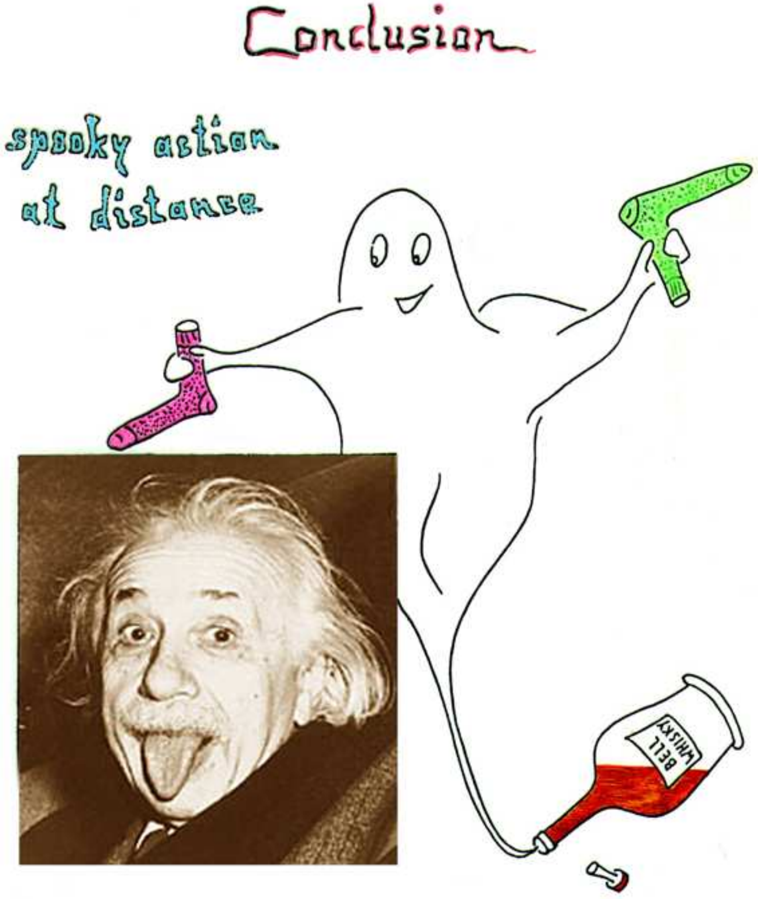
Quantum mechanics: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\Uparrow_n\rangle|\Downarrow_m\rangle - |\Downarrow_n\rangle|\Uparrow_m\rangle\right)$ spin entangled state

Result: QM violates BI **!** no separable state

Conclusion: QM nonlocal **!**

Experiments: Measurements of polarization of entangled photons
in accordance with predictions of QM **!**

Spooky



Literature – Introduction to BI's

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Part II

Bell inequality for strange mesons

QM of K-mesons

Strangeness eigenstates

$$S\,|K^0\rangle \;=\; +|K^0\rangle \qquad\qquad S\,|\bar{K}^0\rangle \;=\; -|\bar{K}^0\rangle$$

CP-transformation $\qquad$ P... parity $\quad$ C... charge conjugation

$$CP\,|K^0\rangle \;=\; -|\bar{K}^0\rangle \qquad\qquad CP\,|\bar{K}^0\rangle \;=\; -|K^0\rangle$$

CP eigenstates

$$\begin{aligned} CP\,|K_1^0\rangle &= +|K_1^0\rangle \qquad & |K_1^0\rangle &= \tfrac{1}{\sqrt{2}}\left(\,|K^0\rangle - |\bar{K}^0\rangle\,\right) \\ CP\,|K_2^0\rangle &= -|K_2^0\rangle \qquad & |K_2^0\rangle &= \tfrac{1}{\sqrt{2}}\left(\,|K^0\rangle + |\bar{K}^0\rangle\,\right) \end{aligned}$$

strong interactions: CP conservation

weak interactions: small CP violation $\qquad$ almost conserved

K-meson decays

K–decay
$$K_S \approx \; K_1^0 \longrightarrow 2\pi \qquad \tau_S \approx 10^{-10} \text{ sec} \qquad \text{short-lived}$$
$$K_L \approx \; K_2^0 \longrightarrow 3\pi \qquad \tau_L \approx 5 \cdot 10^{-8} \text{ sec} \qquad \text{long-lived}$$

physical masses m_S, m_L with $\Delta m = m_L - m_S = 3.5 \cdot 10^{-6}$ eV small

CP-violation $K_L \longrightarrow 2\pi$ small

short-lived, long-lived states $|\varepsilon| \approx 10^{-3}$ CP-violating parameter

$$|K_S\rangle = \frac{1}{N}\left(p\,|K^0\rangle \;-\; q\,|\bar{K}^0\rangle \right) \qquad p = 1 + \varepsilon, \quad q = 1 - \varepsilon$$
$$|K_L\rangle = \frac{1}{N}\left(p\,|K^0\rangle \;+\; q\,|\bar{K}^0\rangle \right) \qquad N^2 = |p|^2 + |q|^2$$

decaying states evolve in time ⟶ Weisskopf-Wigner approximation

$$|K_{\substack{S\\L}}(t)\rangle \;=\; e^{-i\lambda_{\substack{S\\L}} t}|K_{\substack{S\\L}}\rangle \quad \text{with} \quad \lambda_{\substack{S\\L}} = m_{\substack{S\\L}} - \frac{i}{2}\Gamma_{\substack{S\\L}} \quad \text{and} \quad \Gamma_{\substack{S\\L}} \sim \tau_{\substack{S\\L}}^{-1}$$

$\Longrightarrow$ time evolution for K^0 and $\bar{K}^0$

“Quasi-spin” of K-mesons

	$K_0 \sim$	$\Uparrow$	$\bar{K}^0 \sim$	$\Downarrow$	2-dim Hilbertspace
strangeness	$+1$	up	-1	down	

operators in “quasi-spin” space: $S \sim \sigma_3$ $\quad CP \sim -\sigma_1$ $\quad \cancel{C}P \sim \sigma_2$

Hamiltonian $\quad H = M - \frac{i}{2}\Gamma = \frac{1}{2}\left(a \cdot \mathbb{1} + \vec{h} \cdot \vec{\sigma}\right)$ with $|K_{\substack{S\\L}}\rangle$ eigenstates

Analogy

K-meson	spin-$\frac{1}{2}$	photon
$\|K^0\rangle$	$\|\Uparrow\rangle_z$	$\|V\rangle$
$\|\bar{K}^0\rangle$	$\|\Downarrow\rangle_z$	$\|H\rangle$
$\|K_S\rangle$	$\|\Rightarrow\rangle_y$	$\|L\rangle = \frac{1}{\sqrt{2}}(\|V\rangle - i\|H\rangle)$
$\|K_L\rangle$	$\|\Leftarrow\rangle_y$	$\|R\rangle = \frac{1}{\sqrt{2}}(\|V\rangle + i\|H\rangle)$

Bell inequality for K-mesons

consider $K^0\bar{K}^0$ system in analogy to $\Uparrow\Downarrow$ system
construct Bell inequality á la Wigner

choose: fix time — vary quasi-spin of K-meson rotation in quasi-spin space

for BI we need 3 different "angles" – quasi-spins: $|K_S\rangle\,,\ |\bar{K}^0\rangle\,,\ |K_1^0\rangle$ choice

$\Longrightarrow$ Bell inequality of Wigner-type

$$P(K_S,\bar{K}^0)\ \leq\ P(K_S,K_1^0)\ +\ P(K_1^0,\bar{K}^0)$$

contains unphysical CP-even state $|K_1^0\rangle$ P... probability
But!
BI $\Longrightarrow$ inequality on physical CP-parameter — experimentally testable !
how does it come ?

Experiment

consider transition amplitudes

$$\langle \bar{K}^0 | K_S \rangle = -\frac{q}{N} \qquad \langle \bar{K}^0 | K_1^0 \rangle = -\frac{1}{\sqrt{2}} \qquad \langle K_S | K_1^0 \rangle = \frac{1}{\sqrt{2}N}(p^* + q^*)$$

BI $\Longrightarrow$ optimal Inequality for complex weights p, q of $|K_S\rangle$, $|\bar{K}^0\rangle$, $|K_1^0\rangle$

$$|\,p\,| \;\leq\; |\,q\,|$$ experimentally testable **!**

Experiment:

semileptonic decay of strange mesons — quark level

$$K^0(d\bar{s}) \longrightarrow \pi^-(d\bar{u}) \quad l^+ \quad \nu_l \qquad\qquad \bar{s} \longrightarrow \bar{u} \quad l^+ \quad \nu_l$$

$$\bar{K}^0(\bar{d}s) \longrightarrow \pi^+(\bar{d}u) \quad l^- \quad \bar{\nu}_l \qquad\qquad s \longrightarrow u \quad l^- \quad \bar{\nu}_l$$

$\Longrightarrow$ l^+ tags K^0 in K_L state

l^- tags $\bar{K}^0$ $l = \mu, e$

$|p|^2 \ldots$ probability for K^0 in K_L

$|q|^2 \ldots$ probability for $\bar{K}^0$ in K_L

charge asymmetry

$$\delta = \frac{\Gamma(K_L \to \pi^- l^+ \nu_l) - \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)}{\Gamma(K_L \to \pi^- l^+ \nu_l) + \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)} \quad = \quad \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}$$

Conclusion

$\xrightarrow{|p|\leq|q|}$ Bell inequality for δ

$\delta \leq 0$ Experiment: $\delta_{exp} = (3.27 \pm 0.12) \cdot 10^{-3}$

BI violated **!**

- consider 2 BI's $\delta \leq 0$ and $\delta \geq 0$ $\bar{K}^0 \longrightarrow K^0$, $p \longleftrightarrow q$

$\Longrightarrow$ $\delta = 0$ CP conservation

in contradiction to experiment **!**

Conclusion

LRT are only compatible with strict CP conservation in $K^0\bar{K}^0$ mixing **!**

$\delta \neq 0$ $\Longleftrightarrow$ $K^0\bar{K}^0$ entanglement

CP violation nonlocal — contextual

Literature – BI's for K-mesons

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Part III

Decoherence of entangled strangeness

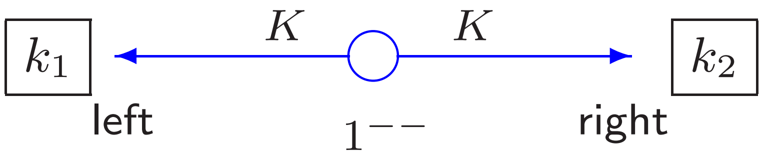
Entangled strangeness

how to measure possible decoherence in entangled state **?**

$\Longrightarrow$ information on quality of entangled state

Practical procedure:

assume creation of entangled kaon state – propagates in time



QM

$$|\psi(t_l, t_r)\rangle \;=\; \frac{N_{SL}}{\sqrt{2}}\left\{|K_S(t_l)\rangle_l \otimes |K_L(t_r)\rangle_r - |K_L(t_l)\rangle_l \otimes |K_S(t_r)\rangle_r\right\}$$

detect quasi-spin: $|k_1\rangle_l$ on left side $\longleftrightarrow$ $|k_2\rangle_r$ on right side

Decoherence parameter

Probability

$$\Big|\langle k_1|\langle k_2|\psi(t_l,t_r)\rangle\Big|^2 =$$

$$\frac{|N_{SL}|^2}{2}\Big\{\big|\langle k_1|K_S(t_l)\rangle_l\big|^2\big|\langle k_2|K_L(t_r)\rangle_r\big|^2 + \big|\langle k_1|K_L(t_l)\rangle_l\big|^2\big|\langle k_2|K_S(t_r)\rangle_r\big|^2$$

$$-\,2\ \underbrace{(1-\zeta)}_{\text{modification}}\ \Re e\Big[\langle k_1|K_S(t_l)\rangle_l^*\langle k_2|K_L(t_r)\rangle_r^*\langle k_1|K_L(t_l)\rangle_l\langle k_2|K_S(t_r)\rangle_r\Big]\Big\}$$

decoherence parameter as measure

$$\underset{\text{pure QM}}{\underset{\nearrow}{0}} \le \zeta \le \underset{\text{total decoherence}}{\underset{\nwarrow}{1}}$$

Furry–Schrödinger

spontaneous factorization of wavefunction

Aim: determine range of ζ by experimental data

Experiment

consider as detected particles

like-strangeness (K^0, K^0) , $(\bar{K}^0, \bar{K}^0)$ and unlike-strangeness $(K^0, \bar{K}^0)$, $(\bar{K}^0, K^0)$

Asymmetry of Probabilities directly sensitive to interference term

$$A(t_l, t_r) \;=\; \frac{P_{\text{unlike}}(t_l, t_r) - P_{\text{like}}(t_l, t_r)}{P_{\text{unlike}}(t_l, t_r) + P_{\text{like}}(t_l, t_r)}$$

$$A_\zeta(t_l, t_r) \;=\; (1-\zeta)\, A^{\text{QM}}(t_l, t_r) \qquad \text{with} \quad A^{\text{QM}}(t_l, t_r) \;=\; \frac{\cos \Delta m\, \Delta t}{\cosh(\frac{1}{2} \Delta\Gamma \Delta t)}$$

$A_{exper} \Longrightarrow \zeta_{exper}$ CPLEAR experiment $p\bar{p} \longrightarrow K^0 \bar{K}^0$

$\Longrightarrow$ result for decoherence parameter range

$$\zeta = 0.13^{+0.16}_{-0.15}$$

$\swarrow$ $\searrow$

close to 0 1 far away

QM total decoherence, LRT

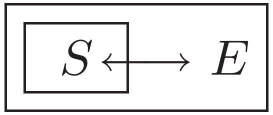
Message: Interference effect of massive system over macro distances $\approx$ 7 cm **!**

Theory of decoherence

Open quantum system — system S interacts with environment E

dissipation: energy flow

decoherence: mixing of states



$S \longleftrightarrow E$

Quantum master equation for density matrix $\rho = |\psi\rangle\langle\psi|$

$$\frac{d\rho}{dt} = -iH\rho + i\rho H^\dagger - D[\rho]$$

Model for decoherence dissipator — projectors to eigenstates of H

$$D[\rho] = \lambda \left(P_i \rho P_j + P_j \rho P_i \right) \quad \text{with} \quad P_j = |e_j\rangle\langle e_j| \quad (j = 1, 2)$$

calculate probabilities: $P_\lambda(K^0 t_l, K^0 t_r)$, $P_\lambda(K^0 t_l, \bar{K}^0 t_r)$, . . .

comparison $\zeta \longleftrightarrow \lambda$ decoherence parameters

$\Longrightarrow$ Relation as test of the model, is very characteristic **!**

$$\zeta(t_l, t_r) = 1 - e^{-\lambda \min(t_l, t_r)}$$

Entanglement measure

Measure for entanglement via entropy of system

von Neumann entropy for pure states $S\left(\rho(t)\right) = -\operatorname{Tr}\{\,\rho(t)\,\log_2 \rho(t)\,\}$

Entanglement of formation E $\longleftrightarrow$ concurrence C for mixed states Bennett

$$E(\rho) = \min \sum_i p_i\, S(\rho_i^l) \equiv E(C) \qquad \text{with} \qquad 0 \leq E, C \leq 1$$

average entanglement of pure states, least expected entanglement of ensemble

Loss of entanglement Bertlmann-Durstberger-Hiesmayr

$$1 - C\left(\rho(t)\right) = \zeta(t)$$

$$1 - E\left(\rho(t)\right) \doteq \frac{1}{\ln 2}\,\zeta(t) \doteq \frac{\lambda}{\ln 2}\, t$$

Proposition ↑ ↑

- loss of entanglement $=$ decoherence parameter

measuring ζ or λ $\Longrightarrow$ $1 - E$ entanglement loss quantitative **!**

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