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CPT tests & the Bell-Steinberger relation

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- Introduction
- Time-evolution of the neutral kaon system
- Unitarity & the BS relation
- Present experimental status & future prospects
- Concluding remarks

▶ *Introduction*

CPT symmetry is linked to the basic mathematical tools that we use in particle physics:

QFT + Lorentz invariance + Locality $\Rightarrow$ CPT

These tools have intrinsic limitations [we are not able to include gravity in a consistent way] $\Rightarrow$ we should expect ~~CPT~~ at some level

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Phenomenologically driven search

[reference scale set by the most significant experimental bounds]

- Neutral kaon system ideal testing ground

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- With this system, the most powerful & simple phenomenological tool for CPT tests is provided by the Bell-Steinberger (BS) relation

Time-evolution of the neutral Kaon system:

$$i \frac{d}{dt} \begin{bmatrix} K^0 \\ \overline{K^0} \end{bmatrix} = [\, M - i\,\Gamma/2 \,] \begin{bmatrix} K^0 \\ \overline{K^0} \end{bmatrix}$$

Hermitian matrices

CPT invariance $\Rightarrow$ $M_{11} = M_{22}$ $\Gamma_{11} = \Gamma_{22}$

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$\cancel{CP}$ $\cancel{CPT}$

Diagonalization $\Rightarrow$

$$|K_S\rangle = N_S \left[\, |K_+\rangle + \varepsilon_S |K_-\rangle \,\right]$$
$$|K_L\rangle = N_L \left[\, |K_-\rangle + \varepsilon_L |K_+\rangle \,\right]$$

$$\varepsilon_{L,S} = \varepsilon_M \pm \Delta$$

$$\Delta = \frac{1}{2} \frac{(m_{K^0} - m_{\bar{K}^0}) - \frac{i}{2}(\Gamma_{K^0} - \Gamma_{\bar{K}^0})}{(m_L - m_S) + \frac{i}{2}(\Gamma_S - \Gamma_L)} [1 + \mathcal{O}(\epsilon)]$$

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phase-convention dependent quantities

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$\Delta\Gamma=0$

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$$\left| \frac{m_{K^0} - m_{\bar{K}^0}}{m_{K^0}} \right| \approx 3 \times 10^{-14} \; |\, \mathrm{Im}\, \Delta \,|$$

Unitarity and the BS relation:

Even if CPT is violated,
we can assume that unitarity
[=the conservation of prbability]
is conserved

$$\Gamma_{K^0} = \Gamma_{11} = \sum_f \mathcal{A}(K^0 \to f)\mathcal{A}(K^0 \to f)^*$$

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Expressing the decay amplitudes in the K_L-K_S basis and using the defintions of ε_M and Δ

$$\boxed{\left[\frac{\Gamma_L + \Gamma_S}{\Gamma_S - \Gamma_L} + i \tan\phi_{SW}\right] \frac{\mathrm{Re}(\epsilon_M) - i\mathrm{Im}(\Delta)}{1 + |\epsilon_M^2|} = \frac{1}{\Gamma_S - \Gamma_L} \sum_f \mathcal{A}_L(f)\mathcal{A}_S(f)^*}$$

Exact relation (phase convention independent, no approximations) in the CPT limt

[only Δ has been treated as a small parameter and expanded to 1st non trivial order]

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Exp. Inputs
[$m_{L,S}$ & $\Gamma_{L,S}$]

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2 physical outputs

Im(Δ) ≠ 0 could only be due to

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A marvelous deep tool to tests basic principles of particle physics

Present experimental status & future prospects:

$$\alpha_f = \frac{1}{\Gamma_S} A_L(f) A_S(f)^* = \eta_f \mathcal{B}(K_S \to f)$$

contributions expected within the SM [for $\mathrm{Re}(\varepsilon_M) = 1.60 \times 10^{-3}$]

Channel	$B(K_S)$	$B(K_L)$	$10^5 \times \alpha_f^{SM}$
$\pi^+\pi^-(\gamma)$	0.69	2.1×10^{-3}	110.8 + 105.1 i
$\pi^0\pi^0$	0.31	9.3×10^{-3}	49.2 + 46.6 i
$\pi^\pm e^\mp \nu$	6.7×10^{-4}	0.39	0.22 + 0.00 i
$\pi^\pm \mu^\mp \nu$	4.7×10^{-4}	0.27	0.17 + 0.00 i
$\pi^0\pi^0\pi^0$	1.9×10^{-9}	0.21	0.06 + 0.06 i
$\pi^+\pi^-\pi^0$	2.7×10^{-7}	0.12	0.04 + 0.04 i
$\pi^+\pi^-\gamma_{DE}$	$\sim 10^{-5}$	$\sim10^{-5}$	< 0.01
others	$< 10^{-5}$	$< 10^{-4}$	< 0.01

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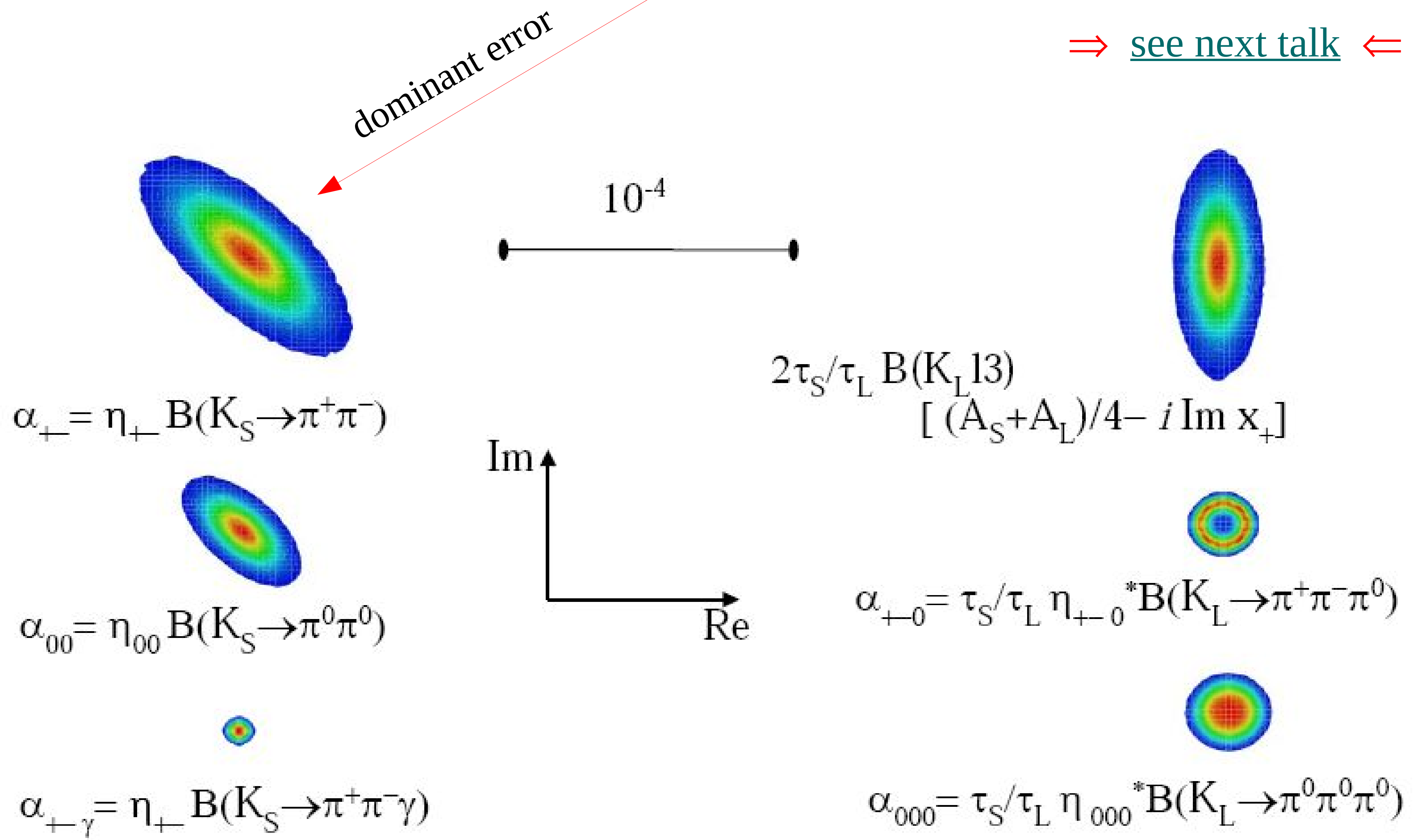
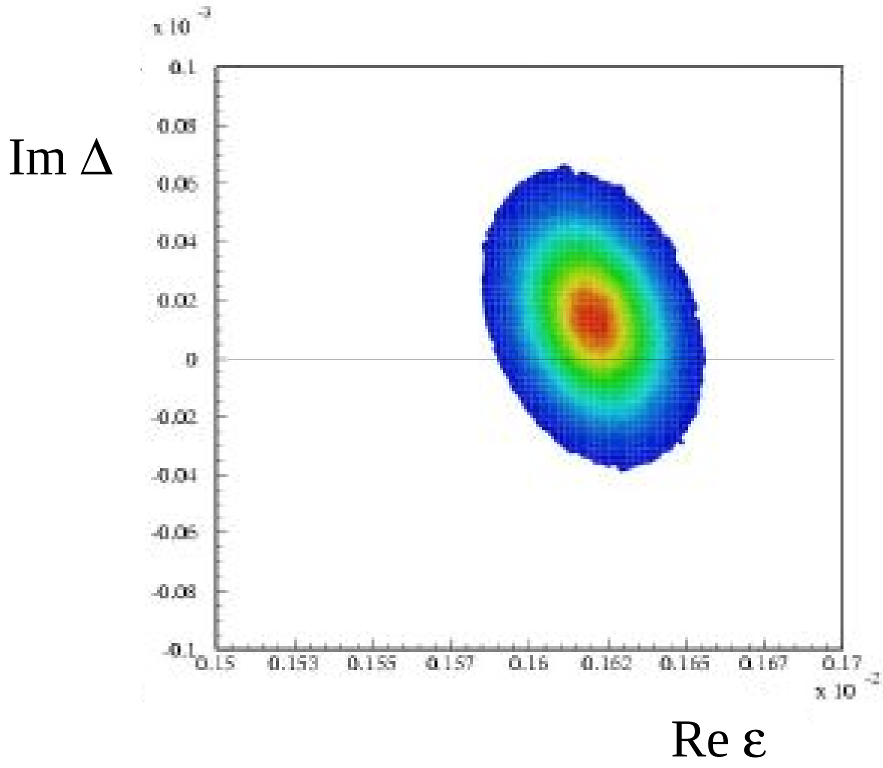
Recent KLOE re-analysis, combining new data on several BR's & τ_L with less-recent interference data by KTeV (ϕ_{+-} & ϕ_{00}) & CPLEAR (Kl3)

⇒ <u>see next talk</u> ⇐





KLOE preliminary:

$$\mathrm{Re}\,\varepsilon = (160.2 \pm 1.3) \times 10^{-5}$$
$$\mathrm{Im}\,\Delta = (1.2 \pm 1.9) \times 10^{-5}$$

CPLEAR:

$$\mathrm{Re}\,\varepsilon = (164.9 \pm 2.5)\; 10^{-5}$$
$$\mathrm{Im}\,\Delta = (2.4 \pm 5.0)\; 10^{-5}$$

$$|\,\mathrm{Im}\,\Delta\,| < 4.3 \times 10^{-5} \;\; (95\%\mathrm{CL})$$

$$\left| m_{K^0} - m_{\bar{K}^0} \right| < 6.5 \times 10^{-19}\,\mathrm{GeV}$$

[assuming CPT violation only in the mass matrix]

$$(m_{K^0} - m_{\bar{K}^0}) - \tfrac{1}{2}(\Gamma_{K^0} - \Gamma_{\bar{K}^0}) = (1.8 \pm 2.8) \times 10^{-19}\,\mathrm{GeV}$$

[or, more generally, without any assumption about the possible source of CPTV]

Concluding remarks:

- The Bell-Steinberger relation is a very powerful tool which allow us to probe some of the basic principles of fundamental interactions [CPT + Unitarity] $\Rightarrow$ very valuable to improve its experimental tests
- Significant step forward thanks to the recent data by KLOE (time-independent)
- Future improvements require dedicated interference measurements, especially in the 2π channels (ϕ_{+-})