FT and Irreversibility time scale

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How much irreversible a process from μ_0 to μ_∞ ?

Need "Entropy creation": via **FT**

Chaotic Hypothesis: *Motion on the attractor can be regarded as "hyperbolic" (Anosov)*
(Ruelle 73, Cohen-G 95)

($Ambitious \rightarrow$ identification btwnn Equilibrium and Nonequilibrium $\Rightarrow$ ergodic)

+ *reversibility*, *i.e.* $I^2 = 1$, $IS_t = S_{-t}I \rightarrow$ **FT**

Be $F_1(x), \ldots, F_n(x)$ even/odd: $F_i(Ix) = \pm F_i(x)$
Let $\sigma(x) = -divergence(x)$ *and* $\sigma_+ \stackrel{def}{=} \langle \sigma \rangle_{SRB}$

Given τ *and "patterns"* $t \to \varphi_i(t)$, $t \in (-\frac{\tau}{2}, \frac{\tau}{2})$

$$p = \frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \frac{\sigma(S_t x)}{\sigma_+} dt$$

"dimensionless phase space contraction"

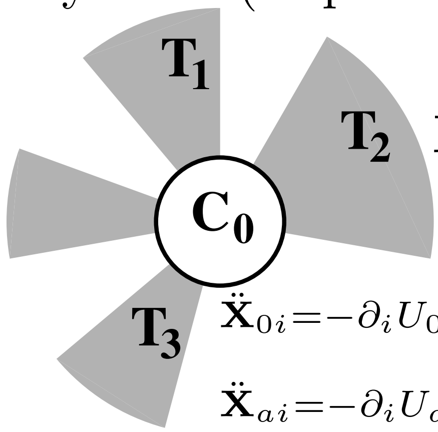
$$\frac{P_\tau(\forall j\ F_j(S_t x) \sim \varphi_j(t), p)}{P_\tau(\forall j\ F_j(S_t x) \sim \pm\varphi_j(-t), -p)} = e^{p\sigma_+\tau + O(1)},\ \ |p| < p^*$$

P_τ *= SRB probability; or (*$n = 0$*)*

$$\frac{P_\tau(p)}{P_\tau(-p)} = e^{\zeta(p)\tau} \ \textit{and}\ \zeta(-p) = \zeta(p) - p\sigma_+$$

FT ↑↑↑↑↑ (Cohen,Evans,Morriss (exp) Cohen,G (theory)

Physics ? (Experiment or Simulation).



$\mathbf{X}_0, \mathbf{X}_1, \ldots, \mathbf{X}_n$

$$\ddot{\mathbf{X}}_{0i} = -\partial_i U_0(\mathbf{X}_0) - \sum_a \partial_i U_a(\mathbf{X}_0, \mathbf{X}_i) + \mathbf{F}_i$$

$$\ddot{\mathbf{X}}_{ai} = -\partial_i U_a(\mathbf{X}_a) - \partial_i U_a(\mathbf{X}_0, \mathbf{X}_i) - \alpha_a \dot{\mathbf{X}}_a$$

$$\alpha_a = Gauss\ for\ K_a = \frac{1}{2}\sum_i \dot{\mathbf{X}}_{ia}^2 = \frac{3}{2} k_B T_a\, N_a$$

$$\alpha_a = (exercise) = \frac{W_a - \dot{U}_a}{3N_a k_B T_a}$$

$$W_a = -\sum \dot{\mathbf{X}}_a \cdot \boldsymbol{\partial}_{\mathbf{X}_a} U_a(\mathbf{X}_0, \mathbf{X}_a),\ W_a = \dot{Q}_a = \text{heat}$$

Divergence

$\sigma(\mathbf{X}) = \scriptstyle{(exercize)}$ $= \sigma_0(\mathbf{X}) + \dot{U}(\mathbf{X})$

$\sigma_0(\mathbf{X}) = \sum_{a=1}^{n} \frac{\dot{Q}_a}{k_B T_a} \frac{3N_a - 1}{3N_a} = \sum_{a=1}^{n} \frac{\dot{Q}_a}{k_B T_a}$

$U(\mathbf{X}) = \sum_{a=1}^{n} \frac{\dot{U}_a}{k_B T_a} \frac{3N_a - 1}{3N_a}$

If volume $V^N \times R^{3N}$ changes

$\sigma^\Gamma(\mathbf{X}) = \sigma_0(\mathbf{X}) + \dot{U} - N \frac{\dot{V}_t}{V_t}$

EPR,Ja 99: ∞-thermostats, no K_a-constraint

Assume U_a bounded and thermostats *efficient*

$\lim_{t\to\infty} \frac{1}{t} \int_0^t F(S_t\mathbf{X}) = \int_{\mathcal{F}} F(\mathbf{Y})\, \mu(d\mathbf{Y})$, then

$\sigma_+ = \langle \sigma \rangle_\mu \equiv \langle \sigma_0 \rangle + \lim_{t\to\infty} \frac{U(t)-U(0)}{t} \equiv \langle \sigma_0 \rangle$

CH $\to$ FT : $\zeta(-p) = \zeta(p) - p\sigma_+$

$p = p_0 + \frac{U(t)-U(0)}{t\sigma_+} \Rightarrow FT\ for\ p_0!$

$\frac{U(t)-U(0)}{t} \to 0$ on huge time scale ! (CvZ,BGGZ) but σ_+ and p_0 bear no more reference to thermostats and have time scale of $O(surface)$

Def Irreversibility time scale for process Π

$\varepsilon(X) \stackrel{def}{=} \sigma_0(\mathbf{X}) - N\frac{\dot{V}_t}{V_t}$

Quasi static $\Pi:\ \mathbf{F}(t) = \mathbf{F}_0 + (1 - e^{-\gamma t})(\mathbf{F}_\infty - \mathbf{F}_0)$

Stationary μ_0 evolves to μ_t and to μ_∞.

$\mu_{srb,t}$ = SRB parameters "frozen" at value taken at time t. Let *irreversibility time scale* for Π

$\frac{1}{\tau(\Pi)} \stackrel{def}{=} \frac{1}{N^2} \int_0^\infty \left(\langle\varepsilon_t\rangle_{\mu_t} - \langle\varepsilon_t^0\rangle_{\mu_{srb,t}}\right)^2 dt$

If quasi static $\tau(\Pi) = O(\gamma^{-1}\log\gamma^{-1}) \xrightarrow[\gamma\to 0]{} \infty$ (*i.e.* $\to$ reversible)

If "Joule-Thomson" with vol. doubling at speed w it is $\tau(\Pi) = O(\frac{L}{w})$

If "Joule-Thomson" with volume *suddenly* larger $\tau(\Pi) = 0$

If "Joule-Thomson" with volume side from L to $L(1+\delta)$ at speed w: $\tau(\Pi) = \frac{L}{w\delta}$