Order, Chaos, Irreversibility, Entropy: rethinking an old dualism.

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Is a theory of motion possible ?

Is it necessary ?

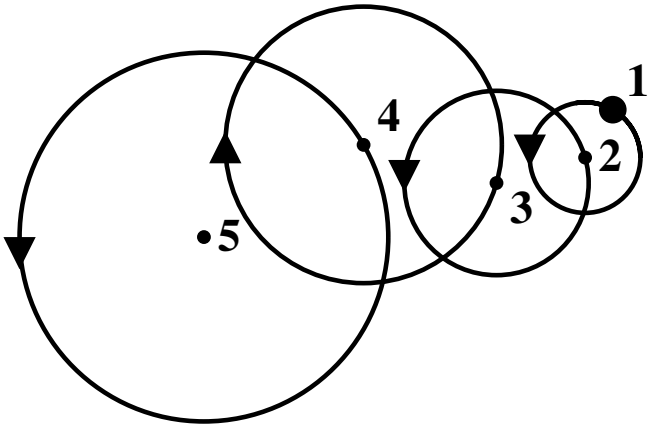
Two different conceptions of motion in Physics:
regular or *chaotic*.

Both *deterministic*: *i.e.* present determines future (and past)
Both *reversible*: if a *daemon* reversed all velocities ⇒ motion would exactly retrace itself.

In pre-Hellenistic Greece problem was posed under influence of Astronomy. Stars and planets seemed predictably moving.

Star motion seems at first sight *circular uniform motion*; at a closer analysis is *composition* of uniform rotatory motions and *such were all other motions* conceived. Meaning?

Imagine a wheel whose center is on the rim of a second wheel whose center is on a 3d wheel ... and each wheel rotatates at (its own) constant speed: point on first wheel rim



Awckward? was taken very seriously!!

ARISTOTELES and PLATO explicit request. HYPPARCOS astronomy entirely based on deferents and epicycles

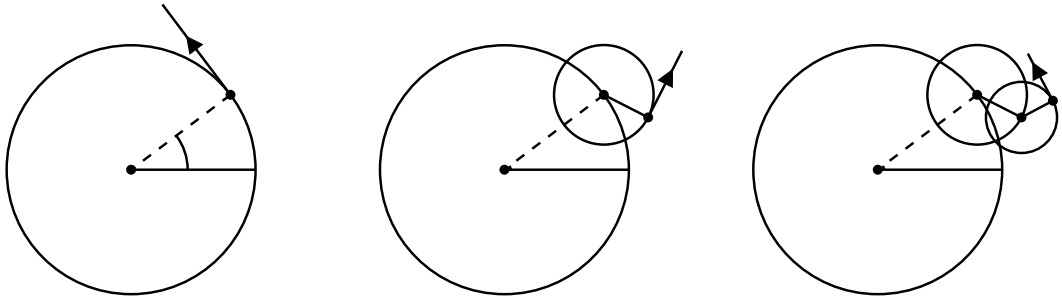


Fig.1 Motion on a deferent, or on an epicycle with center rotating on a deferent, or on an epicycle with center moving on a second epicycle whose center moves on a deferent.

Add epicycles → "*phenomena are saved*" ⇒ Sky ephemerides for future observations.

Remarkable simplicity: number of cycles and epicycles and their speeds needed *relatively small*

HYPPARCUS arranged the observation data known at his time in the above scheme.

Likely: clearly equivalent to assuming existence of a small number of *motors* rotating at constant speed put in motion an array of levers forming clockwork carrying the planet.

PTOLEMY made in fact wide use of the equivalence leading to believe that he had abandoned the Aristotelean principle that *any* motion is a composition of circular uniform motions

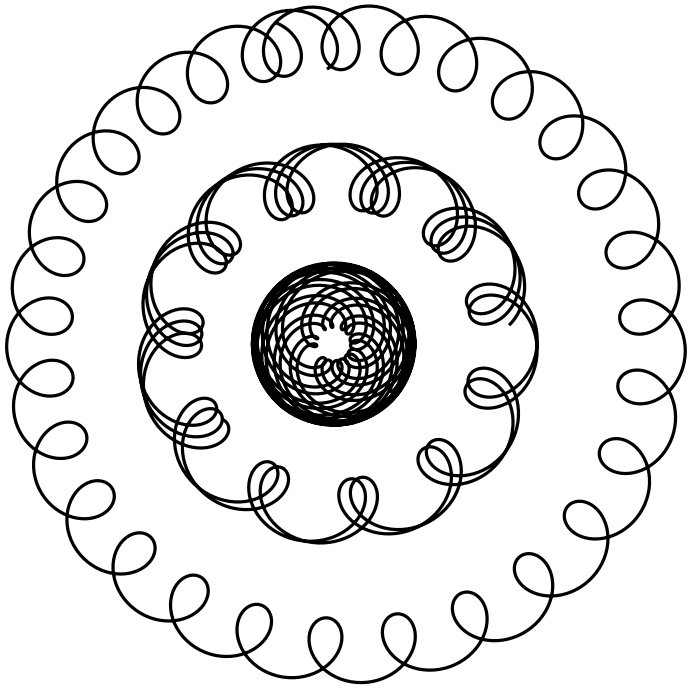


Fig.2: *Geocentric motion of the external planets (Mars, Jupiter and Saturn) seen from Earth with respect to the fixed stars: Ptolemaic system.*

Hellenism decadence → levers–epicycles equivalence "forgotten". Together with theoretical analysis behind Ptolemaic astronomy.

After ~ 200 AC no systematic method for ephemerides!

COPERNICUS considered that PTOLEMY had betrayed ARISTOTELES and PLATO principles. In his *Commentariolus*:

"Nevertheless, what PTOLEMY *and many others legated us here and there about such questions, although mathematically acceptable, did not seem for this reason not to give rise to doubts and difficulties ...". "So that such an explanation seemed to be neither sufficiently complete nor sufficiently conform to a rational criterion...". "Having therefore realized this I often meditated on whether, by chance, a more rational system of circles could be found with which it would be possible to explain every possible apparent inequality;* [with respect to composition of circular motions] *I mean circles all moving upon themselves with uniform motion* **as demanded** *by the law on absolute motion.*

Young COPERNICUS returns to circles with centers rotating upon circles: systems of levers, eccentrics and equants of Ptolemaic astronomy abandoned. For the first time since a millennium rebuilds *ab initio* a model for the solar system following a "rational" method. At the end of his life did not surpass in precision PTOLEMY. But showed how to build systematically a perfectible theory of planetary motions.

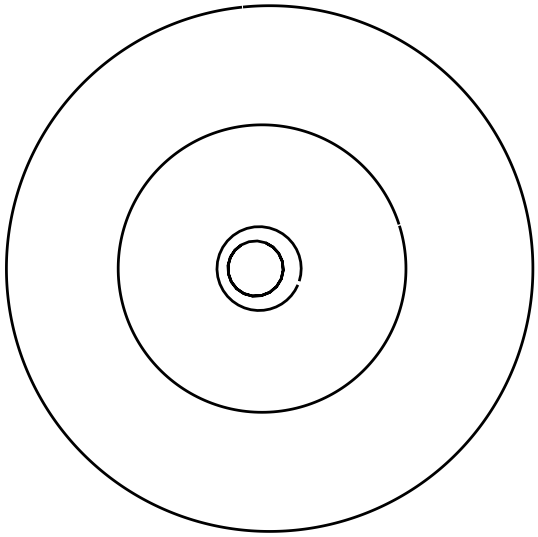


Fig.3. *The external planets in the Heliocentric theory. In spite of the appearances and of the evident greater simplicity* Copernicus*' theory was not more precise than* Ptolemy*'s.*

A major contribution opening the path to Tycho Brahe, Kepler.

Short parenthesis: in *Almagest* no trace of the methods but such methods had to exists. Difficult to think only empirical. Ptolemy book is "ephemerides catalog" (like the modern "American Astronomical Almanac"): the underlying theory is not explained, set of presciptions to "save the phenomena".

Scientific decadence following the departure from Alexandria of the Hellenistic scientists due to the tragic political events of the V century (Jews' expulsion, Hypathia's murder, religious riots ...) caused preservation only of what was considered "useful", according to criteria heard again as a sinister knell at the onset of the XXI century

Kepler and Newton, based on Copernican methodology and Galileo's work, did not diminish the epicyclical viewpoint. Quite the contrary, as shown by Laplace.

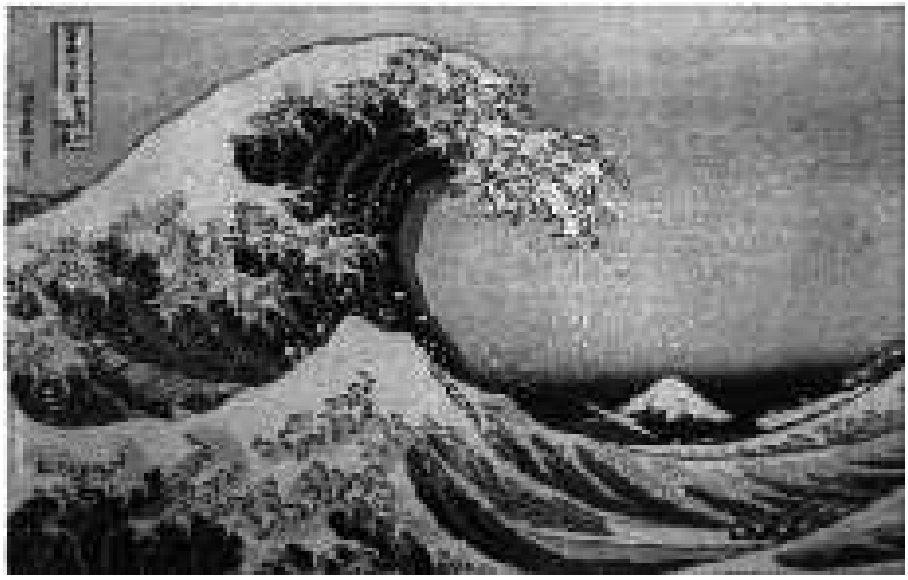


Fig.4: *Even though it might seem difficult to maintain until recent times it has been proposed that this type of motion could be seen as composed by uniform circular motions.*

But this can be assumed on a scientific basis as witnessed by a "recent" book of theoretical Physics by Landau and Lifshitz printed in the 1950's.

Turbulence theory is due to the increasing number of "motors" (or "epicycles") which, as the forcing increases, start rotating generating a motion which is *apparently* disordered

At the triumph of regular motions BOLTZMANN and POINCARÈ, on the eve of the XX Century started bringing evidence that not all motions could be represented so.

A characteristics of regular motions, technically called *quasi periodic*, is their *predictability*: table, 1-th 2 columns.

quasi periodic motion		chaotic motion"	
Difference	time	Difference	time
$2D$	1	$2D$	1
$4D$	3	$4D$	2
$8D$	7	$8D$	3
$16D$	15	16D	4
...	...	...	...
$1024D$	1023	$1024D$	10
...	...	...	...
$\sim 1.000.000D$	$\sim 1.000.000$	$\sim 1.000.000D$	~ 20

With Poincarè: clear existence of motions for which the table looks strikingly different, 2-d 2 columns.

Motions of the second kind, *chaotic*, are unpredictable on the same time scale their observation. A small variation in the initial data is geometrically amplified to too large for a forecast.

Chaotic motion dominates the phenomena involving mechanical systems, even the simple ones. Even in planetary motions which, on astronomical time sales (*i.e.* of millions of years), are subject to inequalities completely unexpected on the basis of the Ptolemaic-Laplacean theories.

Newtonian physics, which predicts both chaotic and regular motions on a case by case basis, implies that also chaotic motions are deterministic, although it is difficult to predict the future from the present. Furthermore both kind of motions share the property that the reversed motions, *i.e.* motions in which velocities are systematically opposite to the ones observed in a given motion are also perfectly possible.

Therefore both conceptions are in sharp contrast with certain very familiar aspects of reality. We know well that certain phenomena develop only in one direction.

If a daemon changed the velocity of the eight skies (the seven planets and the fixed stars) by inverting it, it is possible that no one would be surprised seeing the motion of the skies proceeding backwards: no problem for our understanding of reality would arise. However seeing a bucket of water become warmer in half of it and cooler in the other half would seem strange, to say the least: *but it would not contradict the laws of Newtonian mechanics.*

In conclusion *ancient and modern conceptions of motion* appear to be incompatible with easy empirical observations.

Contradiction of central interest for BOLTZMANN. It is possible (*in principle*) to adjust initial data and produce surprising results, like a spontaneous creation of a temperature difference between two halves of a water container.

However such states have an extremely short life and very soon are transformed into microscopic states with the usual equilibrium properties (constant temperature in the last example).

Subsequently the system continues its microscopic evolution lasting for a duration really unimaginable (measured in a simple example, by THOMSON and later by BOLTZMANN, by a multiple of *the age of the Universe* represented by 1 *followed by more than a billion of billions of* 0*'s*).

Therefore irreversibility of the approach to equilibrium is possible because reversibility is a phenomenon which can only be observed on time scales which are not observable because of their length: reversible dynamics can generate *apparently* irreversible dynamics,