Euclidean φ^4 fiekd in $2D$

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Physics: provide example of relativistic quantum theory

Mathematics: show exist of (non trivial) Markov fields on R^2

d=2: many examples ("$\lambda\varphi_x^4, \lambda P(\varphi_x), \lambda e^{\varepsilon\varphi_x}, \lambda\cos\varphi_x, \ldots$")

d=3: only $\lambda\varphi_x^4$

Formal

$$P(d\varphi) = \frac{\exp(-\frac{1}{2}\int_{R^2}((\partial\varphi_x)^2+\varphi_x^2)d^2x)\,d\varphi}{Z}$$

$\equiv$ the Gaussian process $P_0(d\varphi)$ with covariance C_{xy}

$$\langle\,\varphi_x\varphi_y\,\rangle = (\frac{1}{-\Delta+1})_{x,y} = \begin{cases} \sim e^{-|x-y|} & if\ |x-y|>1 \\ \sim \log|x-y|^{-1} & if\ |x-y|<1 \end{cases}$$

$\varphi_x \in H_{-\frac{1}{2}-\varepsilon}(R^2)_{loc}$ contains length scales: 0 and 1:

hence ∞-many: $1, \frac{1}{2}, \frac{1}{4}, \ldots, 2^{-n}, \ldots$

$$P_\lambda(d\varphi) \stackrel{def}{=} P_0(d\varphi) \frac{\exp -\lambda \int_{R^2} \varphi_x^4 d^2x}{Z_\lambda}$$

Regularization: infrared $\longleftrightarrow R^2 \to \Lambda$: e.g. $\Lambda = T^2$

ultraviolet a scale $2^{-N} \longleftrightarrow$ replace $\frac{1}{-\Delta+1}$ by

$$\frac{1}{-\Delta+1} - \frac{1}{-\Delta+2^{2N}} \equiv \left(\frac{1}{-\Delta+1} - \frac{1}{\Delta+2^2}\right) +$$

$$+\left(\frac{1}{-\Delta+2^2} - \frac{1}{-D+2^3}\right) + \ldots + \left(\frac{1}{-\Delta+2^{2(N-1)}} - \frac{1}{-\Delta+2^{2N}}\right)$$

$$= \sum_{k=0}^{N-1} \left(\frac{1}{-\Delta+2^{2k}} - \frac{1}{-\Delta+2^{2(k+1)}}\right) \equiv \sum_{k=0}^{N-1} \overline{C}_{2^k x, 2^k y}$$

$$\overline{C}_{\xi,\eta} = \frac{1}{(2\pi)^2} \int \frac{e^{ik(\xi-\eta)} d^2k}{(1+k^2)(1+4k^2)}$$