Experimental statistics and stochastic modeling of stick-slip dynamics in a sheared granular fault

Alberto Petri

Istituto dei Sistemi Complessi - CNR
Dipartimento di Fisica, Università La Sapienza Roma, Italy

Summary. — These lectures illustrate laboratory experiments investigating a spring-slider system shearing a granular bed. When the shear rate is low enough, the dynamics is intermittent, displaying a chaotic stick-slip motion which can only be described statistically. This is an instance, among many others, of systems exhibiting intermittent and erratic activity, in the form of *avalanches* characterized by self–similar fluctuations of physical quantities in a wide range of values. In analogy with equilibrium systems, it is thought that such properties originate from the vicinity of some critical transition, and therefore that systems microscopically very different could display similar and universal statistical properties. Investigation of such systems is thence not only interesting in itself, but can be of help in understanding the dynamics of a wider class of phenomena and in devising effective models for their description.

PACS `45.70.-n` – Granular systems.
PACS `91.55.Jk` – Fractures and faults.
PACS `05.40.-a` – Fluctuation phenomena, random processes, noise, and Brownian motion.
PACS `05.65.+b` – Self-organized systems.

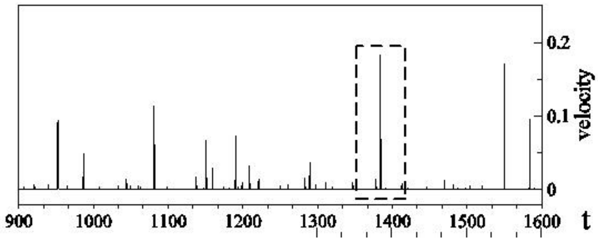

Fig. 1. – An example of intermittent signal supplied by the velocity time series of a spring-slider shearing a granular bed.

1. – Motivations

Laboratory experiments on the response of a granular bed subjected to a shear stress can be performed in different ways and with different aims. One possibility is to aim at replicating what happens within real seismic faults, trying to reproduce analogous conditions, with inherent difficulties especially for what concerns fault pressure and extension. Another possibility, which is the one we explore, aims at finding whether the relevant mechanisms of the dynamics are general enough to be apply also at different systems whose phenomenology displays very similar features. This would allow to understand several situations of interest, including those not easily accessible. These lectures illustrate and discuss the results of laboratory experiments performed within this latter point of view, which is essentially the point of view of statistical physics. For those who are not familiar with this area of physics, and perhaps with its language and basic concepts, we shall recall in the first part some notions and some methods that will also serve to introduce concepts useful to the second part, where we shall describe the experimental results. In the third part we shall try to better understand what is seen in experiments with the aid of some simple stochastic models.

1˙1. *Crackling noise*. – Many systems in the natural world react to solicitations in a discontinuous and impulsive manner, displaying random intermittence of the activity and strong fluctuations of physical quantities. An instances of this phenomenology are earthquakes, which share this feature with many natural phenomena such as fractures [1], crack propagation [2], plastic deformation [3], structural phase transitionss [4], rain precipitation [5], and others.

Besides intermittency, these diverse phenomena share several other tatistical properties. In particular, physical quantities display often long range correlations and self-similar distributions, i.e. power laws, in a wide range of values. Such properties raise the question whether some common or similar mechanism could trigger the occurrence of events. This is one main reason for investigating such systems, which have also been associated for their "crackling noise" [6]. Figure **1**˙1 shows the velocity of a spring-slider shearing a granular medium as function of time. This is an example of crackling noise and will be object of these lectures.

Another hint that similar dynamics could be at the base of the observed phenomenology, common to different systems, comes from the study of the critical phase transition. It is in this context that the concept of universality has been developed and observed: systems that are microscopically very different, can display similar and universal statistical features in their critical properties. In this chapter the main properties characterizing critical phenomena will be shortly recalled, with in mind their possible connection with the "crackling noise" systems.

1˙2. *The point of view of the statistical physics*. – The study of critical phase transitions makes use of the tools of the statistical mechanics. This branch of physics is usually concerned with systems made by many components (e.g. molecules) whose trajectories are practically impossible to follow singularly, and that therefore are treated statistically.

To make these ideas clearer let us consider a concrete example. Many phenomena present Gaussian statistics. An instance is the diffusion, like that of a colloidal particle in a liquid, known as Brownian motion. For each spatial component the dynamics of the particle can in principle be described by a Newton equation like

$$m\frac{d^2x}{dt^2} = -\mu\frac{dx}{dt} + f(\vec{x}) + F(\mathbf{x},\mathbf{v}), \tag{1}$$

where m and μ are the mass and the viscous damping of the particle. $f(\vec{x})$ are the external forces and $F(\mathbf{x},\mathbf{v})$ describes the interaction of the particle with the liquid molecules, depending on their coordinates and velocities $\mathbf{x}$ and $\mathbf{v}$. Being impossible to determine the motion of all the molecules, one can treat F like a random force with suitable properties [7]. One expects that such interaction force, describing the collision of the particle with the liquid molecules, has no preferred direction

$$\overline{F(t)} = 0 \tag{2}$$

and very short memory:

$$\overline{Fx} = 0, \tag{3}$$

where $\overline{...}$ indicates time, or ensemble, averages. Using these assumptions, and the equipartition principle

$$\overline{v}^2 = \frac{RT}{N},$$

where T is the absolute temperature, R the gas constant and N the Avogadro number, Langevin determined the strength of the random force:

$$\overline{F^2} = 2kT\mu, \tag{4}$$

(where $k = R/N$ is the Boltzmann constant).

From Eq. (1) also follows the Fick's law for diffusion. An easy way to see this is to consider Eq. (1) in the simpler case when external forces and inertia can be neglected:

$$\mu v = F(t). \tag{5}$$

In modern language the Langevin assumptions amount to take a force that only depend on time, with zero average

$$\overline{F(t)} = 0 \tag{6}$$

and no correlations

$$\overline{F_i(t')F_i(t)} = \Gamma\delta(t' - t), \tag{7}$$

i.e a white spectrum, with $\Gamma = 2kT\mu$. By integrating Eq. (5) it s straightforward to obtain (assuming the intial position in the origin)

$$\overline{x^2} = \frac{\Gamma}{\mu^2}t = Dt \tag{8}$$

with

$$D = \frac{\Gamma}{\mu^2} = \frac{2kT}{\mu} \tag{9}$$

In fact, the most important result is this formula relating the diffusion coefficient to the absolute temperature and the viscous damping of the particle. From Eq. (1) (it must be reminded that this relation was first derived by Einstein in a less straightforward way [8]). It can also be shown that the coordinate of the particle obeys a Gaussian probability distribution whose variance is indeed Dt [9]. This example shows how a general results can be derived by exploiting the statistical features of a system.

1˙3. *Critical phenomena.* – Actually the crackling noise statistics of intermittent systems is very far from that of a Gaussian process. It is generally characterized by strongly asymmetric distributions of many physical quantities, usually close to power laws. We will see below that also the Gutenberg-Richter statistics for earthquakes magnitude and the Omori's law for the aftershocks number can be considered examples of this phenomenology. So the questions are where do power laws come from, and why should they be universal.

In equilibrium critical phase transitions some power laws are observed which originate from long range correlations in the system. Power laws are self-similar, that is they have no characteristic scales , as opposite to, e.g., exponential or Gaussian distributions, whose scales are characterized by some parameter. They generally describe self-similar (fractal) objects, Such an absence of characteristic scales was first observed in critical phase transitions and revealed the existence of universal behaviors in nature, i.e. not

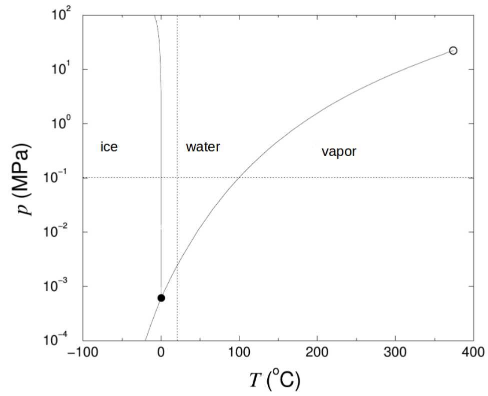

Fig. 2. – Phase diagram of water.

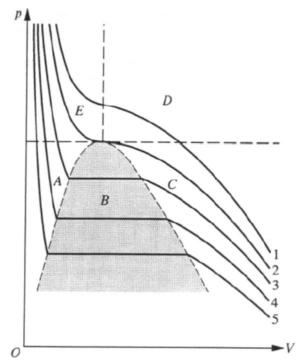

Fig. 3. – Phase diagram drawn from Van der Walls equation. Lines indicates isotherms. Within the shadowed region, B, the two pases A and C coexist. The region terminates above T_c, indicated by the line 2.

depending on many details of the system. This suggests the presence of some equivalent critical transition also in non equilibrium systems.

To see how a critical behavior emerges in equilibrium systems let us consider a very familiar substance like water. In certain conditions of pressure and temperature, liquid and gaseous (vapor) phases, so as liquid and solid (ice) phases, can coexist. This situation is resumed in the phase diagram in Fig. 2a), where the coexistence is marked by lines. There is also a point (black dot) where all the lines cross (the triple point) and another where the liquid–vapor line ends,. This is the *critical point*, where the two phases merge into an opalescent fog. This special fog is made of vapor bubbles of any size containing liquid drops of any size, which in turn contain vapor and so on [10]. The system is a fractal, proposing a same landscape at every length scale.

The existence of a critical point is predicted by the Van der Walls equation [11], relating temperature, pressure and volume at thermodynamic equilibrium. The coexistence curves derived by this equation (supplemented with the Maxwell construction) are shown in Fig 2b). Each line corresponds to a different temperature. The system is liquid in the region A, gas in the region C and the two phases coexist in region B. This region shrinks as temperature increases, until it reduces to a point at the critical temperature T_c.

1˙4. *Universality*. – The bell-shaped region of coexistence is common to many systems, which in fact display a critical point. Close to this point many thermodynamics quantities display power law dependences on the distance from the critical point. For instance the temperature distance ΔT from the critical temperature, T_c, that is $\Delta T = T - T_c$, or from the specific critical volume v_c: $\Delta v = v - v_c$. These dependencies of some standard thermodynamic quantities are reported in Table 1. The astonishing fact is that many different systems display very similar values of the exponents characterizing these

specific heat	specific volume	isothermal compressibility	pressure
$c_v \propto\mid \Delta T \mid^{-\alpha}$	$v_G - v_L \propto\mid \Delta T \mid^{\beta}$	$\kappa_T \propto\mid \Delta v \mid^{-\gamma}$	$\mid p - p_c \mid\propto\mid \Delta v \mid^{\delta}$

TABLE I. – *Critical dependence of some thermodynamic quantities*

dependencies, which therefore appear to be *universal*, as can be seen in Fig. 4 where the values for some substances are reported. Universality can also be seen in a famous plot by Guggenheim [12], Fig 5, where experimental points for different substances on the coexistence line are reported. All points fall on the same bell-shaped curve. It is also seen that in nature many different kind of transitions display a critical behavior [11]. After many contributes from several scholars, the concept of universality was definitely made clear by K. Wilson, who gained the Nobel prize in 1982 for identifying, by means of the renormalization group theory, which ingredients (like space dimensionality and symmetries in the interactions) make different systems fall into a same universality class.

The liaison between power laws and critical behavior strongly suggests that some similar mechanism can rule different out of equilibrium, e.g. dissipative, systems and that universality classes could exist as well in this context. Simplifying a bit it could be said that when algebraic relations are observed, they could correspond to some fractal features due to the criticality of the system. However the identification of universality classes in out of equilibrium systems is still an open problem [13, 14] which is far from being solved even for simple model systems.

	α	β	γ	δ
^3He	0,11	0,36	1,19	4,1
^4He	0,13	0,36	1,18	
Ne		0,33	1,25	
Ar	0,13	0,34	1,21	
Kr		0,36	1,18	
Xe	0,11	0,33	1,23	
H_2		0,33	1,19	
O_2	0,12	0,35	1,25	
N_2		0,33	1,23	
CO_2	0,11	0,32	1,24	
SF_6	0,11	0,32	1,28	
C_2H_4		0,33	1,18	4,4
C_2H_6	0,12	0,34		

Fig. 4. – Critical exponents of Tab.1 for different phase transitions.

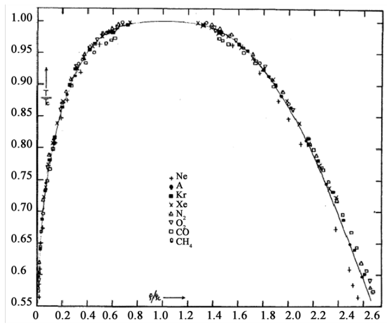

Fig. 5. – Region of coexistence (From Guggenheim (1945)[12]).

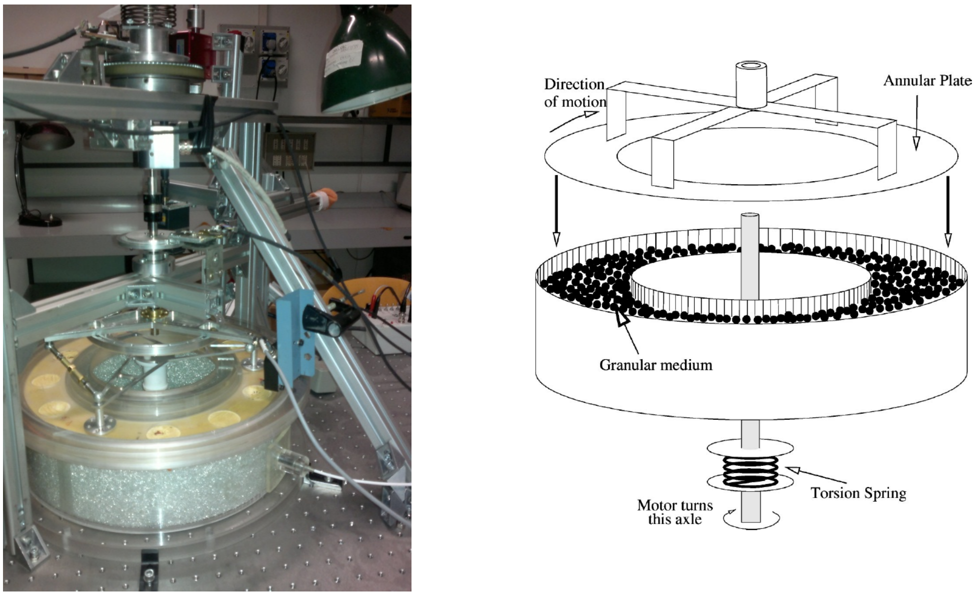

Fig. 6. – An image of the laboratory granular system and its schematic.

2. – Sheared granular matter in laboratory experiments

2·1. *The laboratory set up.* – We have realized a laboratory version of the spring-slider system [15]. A circular geometry has been chosen because it allows for long lasting experiments, and consequently large statistics. The system, in its present version, consists of a PPMI annular channel (see Fig. 6) of inner radius $r = 12.5$ cm and outer $R = 19.2$ cm. The channel is 12 cm height and filled almost completely with glass beads of diameter ranging between 1 and 2.5 mm, dependning on the experiment. The beads can be sheared by a horizontal plate laying on their top and inserted in the channel. The plate has a few layer of grains glued to its lower face, in order to better drag the underlying granular medium, and is free to move vertically. This implies that in our experiments the medium can change volume under a nominal pressure of $p = Mg/[\pi(R^2 - r^2)]$, if M is the mass of the plate. The plate is put into rotation by a motor to which it is connected via a torsion spring, and two optical encoders supply the angular positions of motor, $\theta_0(t)$, and plate, $\theta(t)$, with a spatial resolution of $3 \cdot 10^{-5}$ rad. From θ_0 and θ one derives the respective instantaneous velocities, $\omega_0(t)$ and $\omega(t)$. The instantaneous frictional torque $\tau(t)$ is obtained through the motion equation of the plate

$$\tau = -\kappa(\theta_0 - \theta) - I\dot{\omega}, \tag{10}$$

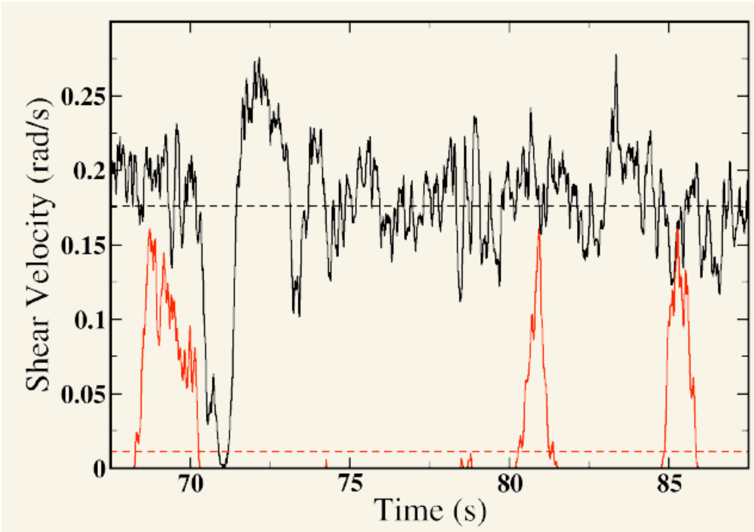

Fig. 7. – Velocity signal in stick slip (black-higher line) and continuous sliding (red-lower line).

where κ is the spring constant and I the plate inertia. All quantities are acquired at a sampling rate form 10 to 1000 Hz, depending on the experiment, recorded by a computer and then processed and analyzed.

2˙2. *Distribution of dynamical quantities*. – When the motor speed and the spring constant are small enough, one observes high intermittent motion (Fig. **1**˙1) where the plate alternates slipping, in which it explores a broad range of velocities, with sticking, in which it stays at rest for unpredictable times, as shown in Fig. (**1**˙1) More details of this *stick-slip* regime can be seen in the sample of plate velocity time series shown in Fig. 7. The lower curve (red online) shows the instantaneous plate velocity for some slips. In the same figure the higher curve (black online) displays the velocity during continuous slide, occurring for larger values of driving speed and/or spring constant.

From these time series quantities such as slip extension, s, duration T and instantaneous velocity $\omega(t)$ can be extracted (see inset of Fig. 8), and their statistical distribution estimated. An example of the resulting distribution for the slip size is reported in the main panel of Fig. 8. It is seen that, as expected, it is not simply peaked around a typical value but, on the contrary, it is essentially monotonic. It is close to a power law and well described by

$$p(s) \simeq s^{\alpha} f(\frac{s}{s_0}).$$

Similar equations describe the distributions of T and v. The function f represent a physical cut-off and for a purely critical dynamics is an exponential function. The present system sometimes also presents a bump at high values of s and of the other quantities.

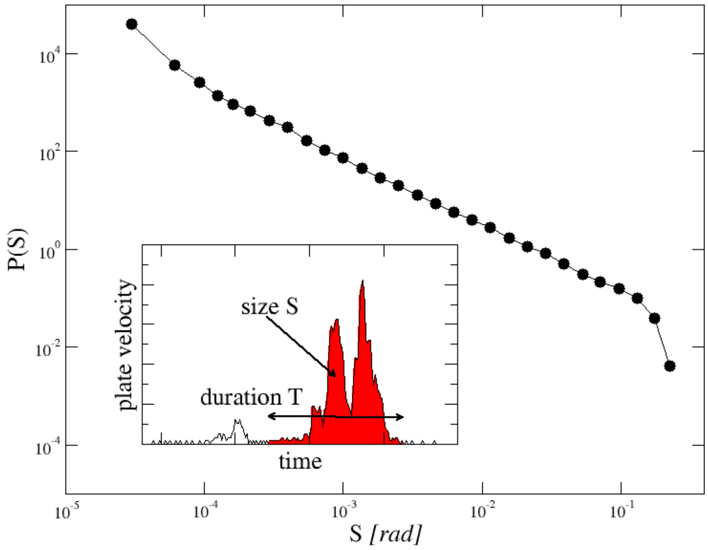

Fig. 8. – Experimental probability distribution of slip size s. In the inset an exemplification of the quantities s, T and v.

This is mainly related to the characteristic system time, proportional to $\sqrt{I/\kappa}$, but possibly also to other issues that can drive the system away from criticality. These possibility will be discussed below, talking about theoretical models.

It can be useful to remark that power laws underly also the celebrated Gutemberg-Richter law [16], relating the occurrence of seismic events to their magnitude:

$$P(m) \propto 10^{-bm},$$

where $P(m)$ is the probability of occurrence of an event with magnitude larger than m. In fact magnitude is related to the seismic moment M:

$$m = \frac{2}{3} \log M + c,$$

with $M \simeq \mu A s$, where A is the slipping area and μ the shear modulus. For given A and μ one gets from the above equations the cumulatative distribution of slips larger than s: $P(s) \propto s^{-\beta}$ with $\beta = \frac{2}{3}b$, and thus the density

$$p(s) = -\frac{dP(s)}{ds} \simeq s^{-(1+\beta)}.$$

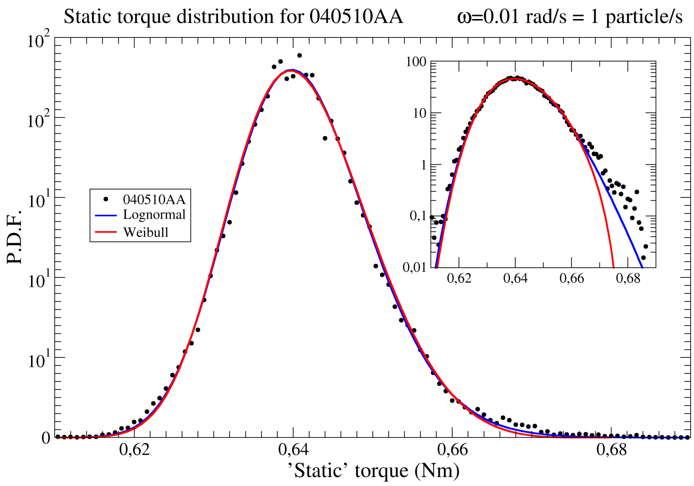

Fig. 9. – Static torque probability distribution in the stick regime.

For the typical value(1) $b \approx 1$ this implies $\beta \approx 1.7$, not far from what observed in laboratory (see e.g. Fig. 2 in [17]). This power law behavior is a manifestation of criticality in earthquakes, as it is the Omori's law, and several other regularities observed in seismic activity [16]. A recent review on this subject can be found in [18].

Another interesting quantity is the frictional torque. The first important thing to observe is that it is a fluctuating quantity. This aspect of friction is essential and is at the base of the observed critical behavior, as it will be discussed below. Friction exhibits two phases: The loading one, in which the plate stills and the spring is stretched by the motor causing a linear increase of the torque, and the slipping one, where friction strongly fluctuates (examples of these fluctuations are shown in [19]). As a stochastic quantity the friction torque can only be characterized statistically. Figure 9 shows the experimental probability distribution for the static torque in the stick slip regime. It is seen that, at variance with the previous quantities, it is not right winged. Nevertheless it is far from Gaussian and still asymmetric, with an exponential tail at large values. This signals the presence of strong correlations, as discussed in [20], which are not present in the continuous sliding phase, where the torque distribution shrinks and attains a Gaussian profile [15]. A similar behavior is observed for the dynamic friction [15].

(1) Measured values have been criticized, e.g. Tinti and Mulargia Bull. Seismol. Soc. Amer., 75, 1681 (1985) and revised, .e.g Y.Y. Kagan, Tectonophysics 490, 103 (2010))

3. – A stochastic model for the slider motion

Different models have been devised and investigated in the effort of understanding and reproducing the erratic and intermittent activity displayed by earthquakes an many other phenomena. The first of such models to our knowledge is that of Burridge and Knopoff [21]. The model is an idealization of a frictional elastic interface. It reproduces several features of earthquakes and has been extensively investigated [22]. A different approach to this kind of phenomena was proposed at the end of the eighties by Bak, Tang and Wiesenfeld [23] who, using a sand-pile as metaphor, proposed the concept of Self Organized Criticality (SOC). The concept gained a wide popularity and has been followed by a huge quantity of work [24]. Nevertheless the concept of critical self-organization itself has been criticized [25] and still presents many open problems [13, 14], like the existence of well defined universality classes and the way it can be connected with real systems in terms of specific dynamics, parameter values, etc.. The sand–pile, and the model inspired to it, belong to the category of stochastic cellular automata, to which belong even other kinds of models more directly related to real systems. For instance the Fiber Bundle Model and its derivations [26] which together with other statistical models [27] aims at the description of fractures or cracks [28]. There is no room here for reviewing these models in the due manner and in the following we shall focus on a different approach [29], which appears more suitable to describe the dynamics observed in the experiments..

3˙1. *The friction force*. – To model the experimental spring-slider system one can start from Eq. (10). The friction force however is known only through its experimental realization. It looks random and in principle can be a function of many variables such as position. velocity, acceleration, $F(\theta, \omega, \dot{\omega}, ...)$, and others like state or memory variables. In analogy with other friction laws one can first attempt to put into evidence a definite dependence on the instantaneous velocity. As seen above, friction is a fluctuating quantity and presents complicated trajectories even as function of just the instantaneous plate speed, (as shown for instance in Fig. 2 of [17]). One possible way out is to average over many slips. That is to consider the mean value of friction for a given fixed instantaneous velocity. Despite the apparent chaoticity of the friction trajectories a regular pattern emerges, showing a well defined dependence, also characterized by a velocity weakening, visible in Fig.10. In order to account for the stochastic component one can be inspired by the Langevin dynamics discussed in Sec. 1, and assume that the friction is the sum of a deterministic velocity dependent term, $\tau_d(\omega)$ and a random term, τ_r:

$$\tau = +\tau_d(\omega) + \tau_r. \tag{11}$$

As for the Brownian motion the aim of the random term is to describe in an effective way the interaction of the probe, in this case the plate, with the medium, i.e. the granular bed, without resort to the motion of each individual particle. The friction exerted by the granular medium on the plate is due to the action in the medium of force chains which

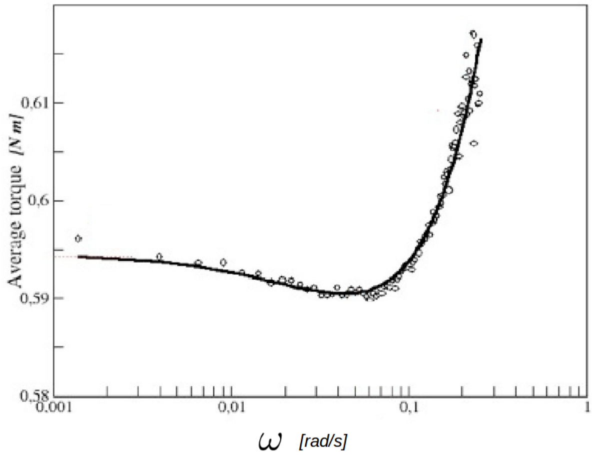

Fig. 10. – Average friction as function of the instantaneous velocity.

transmit the stress. The strength of these chains evolves in a continuous manner along with the configuration during the motion, while in the stick phases it does not change sensibly. Thus in this case the time dependence is replaced by an angular dependence. The simplest assumption that can be made is that, along the plate trajectory, the strength variations result from the sum of independent increments. This can be formalized as

$$\frac{d\tau_r}{d\theta} = \eta(\theta), \tag{12}$$

where η is a white noise satisfying the usual properties (6) of zero average

$$\overline{\eta(\theta)} = 0 \tag{13}$$

and δ correlation

$$\overline{\eta(\theta')\eta(\theta)} = r\delta(\theta' - \theta), \tag{14}$$

while and r is the strength of the fluctuations. This amounts to say that the friction force performs a Brownian motion [29], like the one described in Sec. 1.

The above form is coherent with the fact that friction cannot be a white noise itself, since this would imply that its value at $\theta + d\theta$ would be totally independent from its value at θ, which looks highly unphysical. On the other hand a simple random walk can also lead to unphysical consequences since, as we also know from Sec. 1, the average

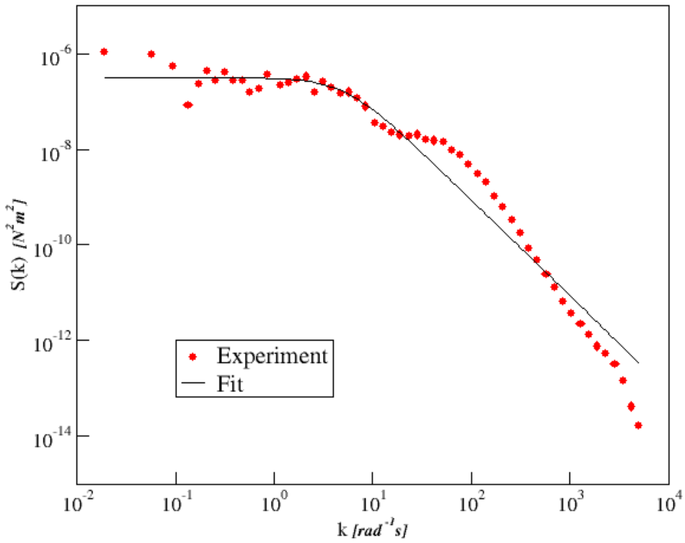

Fig. 11. – Spectrum of the dynamic torque with a Lorentzian fit.

fluctuations for this process diverge linearly with the parameter (which in this case is the angular coordinate θ). Forces must be limited, and this can be obtained by subjecting them to some bounding potential. At the lowest order it can be assumed to be harmonic so that the friction torque obeys the equation

$$\frac{d\tau_r}{d\theta} = r\eta(\theta) - a\tau_r. \tag{15}$$

This process is named after Uhlembeck and Ornstein [9] and the parameter a represents the inverse of its correlation length (a correlation angle in the present case).

In order to test this hypothesis and determine the relevant parameters r and a one resorts to the experimental data. By subtracting $\tau_d(\omega)$ from the experimental time series of $\tau(\omega)$, one obtains the series for $\tau_r(\theta)$. Performing a Fourier transform (in angle) one gets the power spectrum of the signal, as function of the wavenumber, an instance of which is shown in Fig 11. It is seen that its shape is well described by a Lorentian, and the values of r and a can be determined by a best fit.

3˙2. *Results from the model*. – Different forms can be equivalently assumed for describing the behavior of τ_d reproduced in Fig 16. We have resorted to one already employed in the literature for the solid-on-solid friction [30]:

$$\tau_d(\omega) = \tau_0 + \gamma \left\{ \omega - 2\omega_0 \ln(1 + \frac{\omega}{\omega_0}) \right\} \tag{16}$$

where τ_0 is the constant dynamical friction and ω_0 a characteristic speed. With this expression and (15) in Eq. (10) one can simulate numerically the stick slip motion of the plate employing the values of the parameters used in the experiment (plate inertia I, spring constant κ, motor driving angular speed ω_d) and those characterizing friction in Eqs. (15) and (16).

By repeating on the resulting time series the same analysis performed on the experimental series one can compute distributions for slip extension, duration, etc. Some results are shown in Fig 12. The different curves correspond to different driving speed of the plate and a good agreement is obtained in all cases.

4. – Criticality and its possible breakdown

4·1. *Where does criticality come from?*. – This question has not yet received definitive answers. It involves areas of non-equilibrium statistical physics which, at variance with thermodynamics, deals with systems where detailed balance is not observed and that usually include dissipative forces. As already recalled, a common feature of these systems is the existence of a non-equilibrium critical transition underlying the observed dynamics [6], that for granular matter could be the jamming transition [31, 32].

The model introduced above does not consider the motion of individual grains but only their global influence on the dynamic and is closely related to models for propagating fracture in disordered media [33, 27]. In this frame a large class of models presenting common, and in some cases universal, critical features are the depinning models. These models describe the motion of an elastic line or surface, representing the crack, driven by a uniform force: The motion takes place in the presence of a viscous force and of a disordered potential, which account for inhomogeneities and can exert a pinning force. When this force is larger than the driving force, the system stills, until the drive is made large enough. The depinnig transition is critical and presents all the features of criticality described above (power law distributions, correlations, etc.). These features are shared by other models describing different systems, and in certain cases give raise to identical critical exponents, distributions etc. (or at least compatible, when models be studied only numerically). One instance of these cases is the so called ABBM model.

4·2. *The ABBM model.* – A ferromagnet in an external magnetic field H is characterized by a certain amount of magnetization M. If the field changes, magnetization follows. However, on closer observation it can be seen that M as function of H is not smooth but rather irregular, displaying jumps, *avalanches*, of magnetization. These jumps were at first be revealed by their induced electromagnetic flux and are known under the name of Barkhausen effect, or noise, see e.g. [34]). In order to describe the Barkhausen noise, in [35] a model was derived from microscopic considerations, known as $ABBM$ model. Besides the application to material science, a very interesting point of the model is that it has been shown to be directly related with the depinning models mentioned above [34] and, in certain cases, it is described by the same equation [36]. One of this case is the *mean field* case, in which the acverage position of the interface is considered.

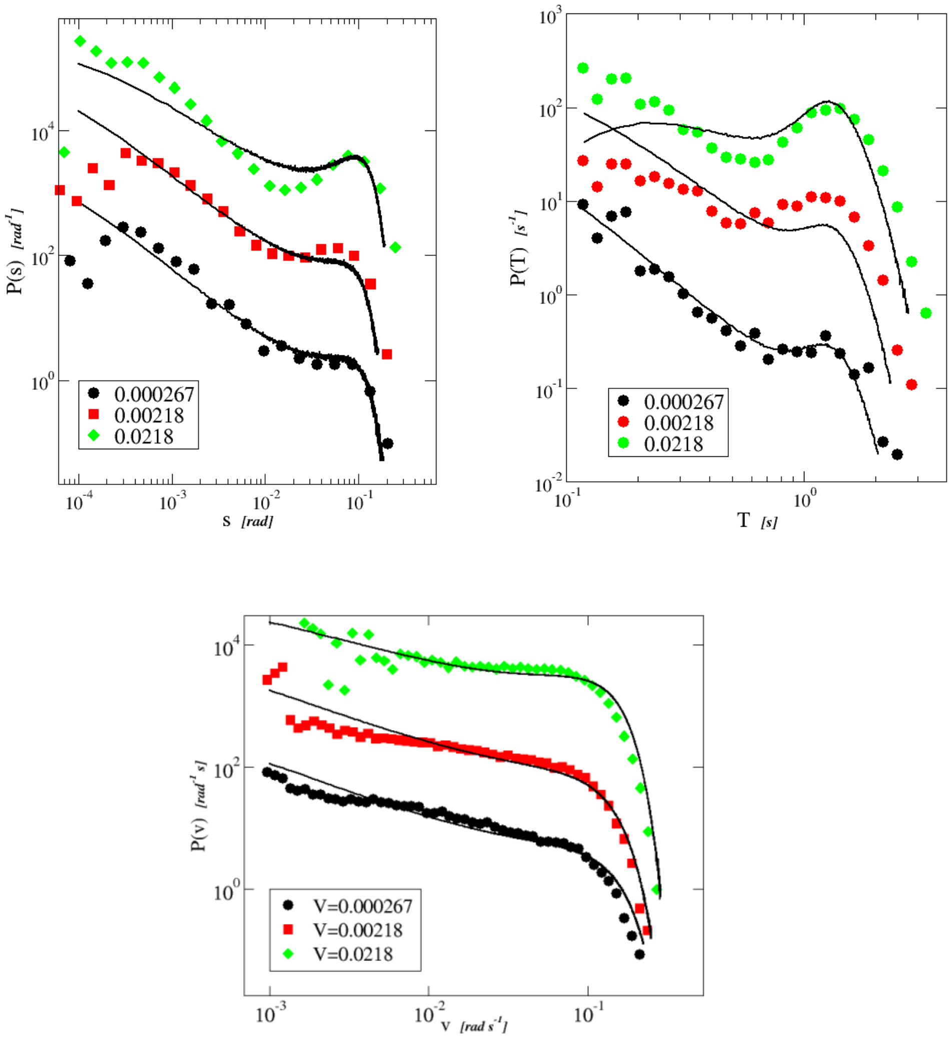

Fig. 12. – Distributions for slip size, velocity and duration from experiments (symbols) and simulation of the model (lines) from Eqs. (10), (11), (15) and (16). Three different driving speeds are considered.

The ABBM model in the mean field formulation can be described by the equation

$$\frac{dx}{dt} = -(x - ct) + F \tag{17}$$

with x a quantity directly proportional to the magnetization and c the rate of variation of the external field H. F is a random perturbation given by

$$\frac{dF}{dx} = \eta(x),$$

where η has the usual properties of the white noise, Eqs. (13) and (14). One can recognize the same evolution described by (12).

As anticipated, although not evident at first sight, the dynamics described by Eq. (17) is critical. Motion occurs intermittently and many physical quantities characterizing the avalanches (duration, size, velocity) are distributed as power laws. The importance of the model comes also from the fact that it is analytically soluble and all quantities are known exactly. Moreover it is possible to identify the origin of criticality in the self similar symmetry owned by the equation [34]. For all these reasons the model is considered paradigmatic and investigated in different contexts [37]. Moreover it also describes a kind of population dynamics (previously introduced by Feller [38] and the price evolution for several financial products [39]. More recently, the same equations have been adopted for describing other items, such as concentration of pollutants [**?**].

Concerning the stochastic dynamics of the slider described by Eq. (10), the is also a simplified model of this dynamics. In fact, by neglecting inertia in Eq. (11), in analogy with Eq. (5), and by moving the first term of the deterministic friction τ_d to the left-hand side, one is left with

$$\omega = -k(\theta - \omega_d t) + F_r,$$

where $k = \kappa/\gamma$, $v = \omega/\gamma$ and $F_r = \tau_r + \tau_d - \gamma\omega$. This is formally identical to Eq. (17), and it is easy to recognize that, by neglecting the logarithmic term in τ_d and by setting $a = 0$ in Eq. (15), a complete identity is attained.

4·3. *Breakdown of criticality.* – Models for granular and earthquake dynamics directly mappable into the original ABBM have been proposed and investigated [40]. On the other hand, how and to which extent the presence of different friction laws and inertia can affect the critical behavior and the universality properties of dynamics is still subject of investigation. The statistical distributions derived from our experiments, so as those reproduced by the related model, are not so clean power laws as those from the ABBM model [34]. Other deviations from criticality have been observed in [41] and [42].

In [37], some aspects of the effect of inertia on the original ABBM have been studied, while other aspects are under investigation [43]. In addition there are many features of the granular-slider dynamics that should be considered and that also the models discussed here does not take into consideration. One of these features is friction hysteresis. It is well known that in many instances dynamical friction is not a single valued function of velocity. Different friction laws addressing these problem have been proposed, among the most widely adopted being those by Dieterich [44] and Ruina [45], also known as *rate–and–state* laws. These laws were at first deduced and validated for solid–on–solid friction, and then adopted also for describing the friction of faults and granular gauges [46, 47, 48],

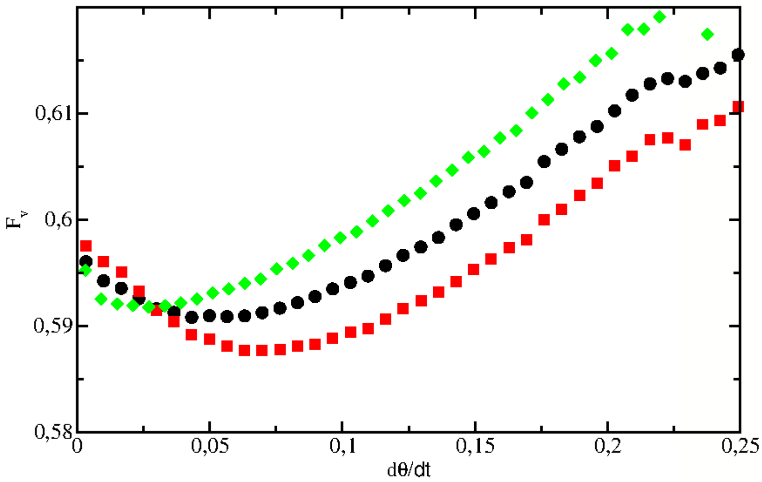

Fig. 13. – Friction torque measured in an experiment with imposed plate speed. Diamond (green on line) correspond to increasing speed and square (red on line) to decreasing speed. Bullets (black) are the average friction.

together with some simplified versions. It is thus clear that a better description of the systems has to take into account suitable friction laws. Attempts to supply Eq. (11) with a rate–and–state law were done in [19], where the behavior of friction as function of controlled plate speed has been measured while imposing a velocity ramp. An example of the resulting dynamical friction si shown in Fig. (13) It can be seen that, although maintaining weakening, friction behaves very differently in accelerating and decelerating phases, with a peculiar crossing value. Another important point concerns the correlation length a in Eq. (15). Since the granular medium is quasi–solid near the jamming, and quasi–liquid at high shear, it is clear that a is also a dynamical quantity that should be described by a suitable, shear rate dependent, constitutive equation. All these features can cause a breakdown of the critical behavior, at least for large avalanches, as emerging in recent experiments [41, 42].

5. – Summary and perspectives

These lectures have resumed some of the results produced in experiments on the stick-slip dynamics of a sheared laboratory fault, and some models aimed at describing and understanding the observed phenomenology. This has been done within a statistical physics approach and adopting simple stochastic models., because the very irregular and chaotic dynamics prevents detailed descriptions. At the same time, such an approach is also an effort to understand whether alike elementary mechanisms might underly several different natural phenomena that display apparently similar phenomenologies.

It is clear that all the features listed in the last section, like friction hysteresis, variable correlation length in the random friction, scaling breakdown, etc., are important in determining the slider motion even in a simplified approach as the one reported here. Their influence deserves to be more systematically investigated in both experiments and theoretical models. Especially these latter seem to be still rather scarce and very little evolved. At the same time the results discussed here show that also approaches based on general assumption can be quite effective in the description of systems with irregular dynamics.

* * *

The author acknowledges A. Baldassarri, F. Dalton and F. Leoni for some pictures.

REFERENCES

[1] PETRI A., PAPARO G., VESPIGNANI A., ALIPPI A. and COSTANTINI M., *Phys. Rev. Lett.*, **73** (1994) 3423.
[2] CALDARELLI G., TOLLA F. D. and PETRI A., *Phys. Rev. Lett.*, **77** (1996) 2503.
[3] DIMIDUK D. M., WOODWARD C., LESAR R. and UCHIC M. D., *Science*, **312** (2006) 1188.
[4] CANNELLI G., CANTELLI R. and CORDERO F., *Phys. Rev. Lett.*, **70** (1993) 3923.
[5] NEELIN J. D. and PETERS O., *Nature Physics*, **2** (2006) 393.
[6] SETHNA J. P., DAHMEN K. A. and MYERS C. R., *Nature*, **410** (2001) 242.
[7] LANGEVIN P., *C. R. Acad. Sci. (Paris)*, **146** (1908) .
[8] EINSTEIN A., *Ann. der Physik*, (1905) .
[9] GARDINER C., *Handbook of stochastic methods* (Springer-Verlag (Berlin) 1997.
[10] HUANG K., *Statistical mechanics* (John Wiley & Sons) 1987.
[11] OLIVEIRA M. J., *Equilibrium Thermodynamics* (Springer) 2017.
[12] GUGGENHEIM J., *Chem. Phys.*, **13** (1945) .
[13] PRUESSNER G., *Self-Organised Criticality: Theory, Models and Characterisation* (Cambridge University Press) 2012.
URL `http://books.google.it/books?id=TXKcpGqMSDYC`
[14] PETRI A. and PONTUALE G., *J. Stat. Mech.*, (2018) 063201.
[15] DALTON F., FARRELLY F., PETRI A., PIETRONERO L., PITOLLI L. and PONTUALE G., *Phys. Rev. Lett.*, **95** (2005) 138001.
[16] TURCOTTE D. R., *Fractals and Chaos in Geology and Geophysics* (Cambridge) 1997.
[17] CIAMARRA M. P., DALTON F., DE ARCANGELIS L., GODANO C., LIPPIELLO E. and PETRI A., *Tribology Letters*, **48** (2012) 89.
[18] DE ARCANGELIS L., GODANO C., GRASSO J. R. and LIPPIELLO E., *Physics Reports*, **628** (2016) 1.
[19] LEONI F., BALDASSARRI A., DALTON F., PETRI A., PONTUALE G. and ZAPPERI S., *Journal of Non-Crystalline Solids*, **357** (2010) 749.
[20] PETRI A., BALDASSARRI A., DALTON F., PONTUALE G., PIETRONERO L. and ZAPPERI S., *The European Physical Journal B - Condensed Matter and Complex Systems*, **64** (2008) 531.
[21] BURRIDGE R. and KNOPOFF L., *Bull. Seis. Soc. Amer.*, **341** (1967) 57.
[22] CARLSON J. M., LANGER J. S. and SHAW B. E., *Rev. Mod. Phys.*, **66** (1994) 657.
[23] BAK P., TANG C. and WIESENFELD K., *Phys. Rev. Lett.*, **59** (1987) 381.
URL `http://link.aps.org/doi/10.1103/PhysRevLett.59.381`

[24] Watkins N. W., Pruessner G., Chapman S. C., Crosby N. B. and Jensen H. J., *Space Science Reviews*, **198** (2016) 3.
[25] Dickman R., Muñoz M. A., Vespignani A. and Zapperi S., *Brazilian Journal of Physics*, **30** (2000) 27.
[26] Pradhan S., Hansen A. and Chakrabarti B. K., *Rev. Mod. Phys.*, **82** (2010) 499.
[27] Alava M. J., Nukala P. K. and Zapperi S., *Advances in Physics*, **55** (2006) 349.
[28] Pontuale, G., Colaiori, F. and Petri, A., *EPL*, **101** (2013) 16005.
URL http://dx.doi.org/10.1209/0295-5075/101/16005
[29] Baldassarri A., Dalton F., Petri A., Zapperi S., Pontuale G. and Pietronero L., *Physical Review Letters*, **96** (2006) 118002.
[30] Baumberger T. and Caroli C., *Advances in Physics*, **55** (2006) 279.
[31] Dauchot O., Marty G. and Biroli G., *Phys. Rev. Lett.*, **95** (2005) 265701.
[32] Ciamarra M. P., Lippiello E., Godano C. and de Arcangelis L., *Physical review letters*, **104** (2010) 238001.
[33] Dalton F., Petri A. and Pontuale G., *Journal of Statistical Mechanics: Theory and Experiment*, (2010) P03011.
[34] Colaiori F., *Advances in Physics*, **57** (2008) 287.
[35] Alessandro B., Beatrice C., Bertotti G. and Montorsi A., *J. Appl. Phys.*, **68** (1990) 2901.
[36] Bonamy D., Santucci S. and Ponson L., *Phys. Rev. Lett.*, **101** (2008) 045501.
[37] Le Doussal P., Petković A. and Wiese K. J., *Physical Review E*, **85** (2012) 061116.
URL http://link.aps.org/doi/10.1103/PhysRevE.85.061116
[38] Feller W., *Annals of mathematics*, **54** (1951) 173.
[39] Cox J., J.E. I. and Ross S., *Econometrica*, **53** (1985) 385.
[40] Dahmen K. A., Ben-Zion Y. and Uhl J., *Nature Phyisics*, (2011) 1957.
[41] Barés J., Wang D., Wang D., Bertrand T., O'Hern C. S. and Behringer R. P., *Phys. Rev. E*, **96** (2017) 052902.
URL http://arxiv.org/abs/1709.01012
[42] Baldassarri A., Annunziata M., Gnoli A., Pontuale G. and Petri A., *Breakdown of scaling and friction memory in the critical granular flow*, in preparation.
[43] Baldassarri A. and Petri A., in preparation.
[44] Dieterich J., *Journal of Geophysical Research: Solid Earth*, **99** (1994) 2601.
[45] Ruina A., *Journal of Geophysical Research: Solid Earth*, **88** (1983) 10359.
[46] Scholz C. H., *Nature*, **391** (1998) 37.
[47] Marone C., *Ann. Revs. Earth & Plan. Sci.*, **26** (1998) 643.
[48] Bizzarri A. and Cocco M., *Journal of Geophysical Research: Solid Earth*, **108** (2003) 2373.