

UNITEXT

La Matematica per il 3+2

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Carlo Presilla

Complex Functions of a Variable

An Introduction with Solved Exercises

 Springer

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In memory of my parents

Preface

This book is intended as a first course in the theory of functions of a single complex variable for students who have already acquired the fundamental notions of real mathematical analysis. It is the expanded English version of an older text, published by Springer in Italian, based on the lectures I gave for several years at the University of Rome “La Sapienza” and at the Scuola Superiore di Catania.

The claim of mathematical rigor is combined with the idea that students should develop, at the same time, know-how skills. In fact, half of the book is devoted to the detailed solution of about 300 exercises illustrating, also with figures, the theory.

It’s a pleasure to thank Prof. Filippo Cesi for countless discussions and suggestions. I also thank Prof. Alberto Perelli for his critical observations on the second edition of the Italian book, especially regarding the proof of the Cauchy-Goursat Theorem. Finally, I acknowledge the invaluable help of various students in highlighting misprints and errors.

Rome, Italy
September 2025

Carlo Presilla

Declarations

Competing Interests The author has no competing interests to declare that are relevant to the content of this manuscript.

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