EUROPHYSICS LETTERS 10 May 1993

Europhys. Lett., 22 (5), pp. 333-339 (1993)

Quantum Limit in Resonant Vacuum Tunnelling Transducers.

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(received 5 November 1992; accepted in final form 12 March 1993)

PACS. 03.65 – Quantum theory; quantum mechanics.
PACS. 06.70M – Transducers.
PACS. 73.40G – Tunnelling: general.

Abstract. – We propose an electromechanical transducer based on a resonant-tunnelling configuration that, with respect to the standard tunnelling transducers, allows larger tunnelling currents while using the same bias voltage. The increased current leads to a decrease of the shot noise and an increase of the momentum noise which determine the quantum limit in the system under monitoring. Experiments with micromachined test masses at 4.2 K could show dominance of the momentum noise over the Brownian noise, allowing observation of quantum-mechanical noise at the mesoscopic scale.

Recently a novel electromechanical transducer based upon vacuum tunnelling of electrons has been proposed to detect displacements of a macroscopic mass [1]. A variation of the distance between the test mass and a tip changes the tunnelling current and whenever small fractions of the current are appreciable, corresponding displacements of the test mass, which are small fractions of the De Broglie wavelength of the tunnelling electrons, are also detectable. The relevance of this new class of transducers has been emphasized expecially concerning detection of gravitational waves using bar antennae [1,2], design of quantum standard of current in metrology [3] and study of quantum-mechanical noise at the mesoscopic scale [4].

Vacuum tunnelling transducers are intrinsically quantum limited [5]. The small output capacitance allows to neglect the back-action noise due to the amplifier following the transducer in the detection chain with respect to the quantum uncertainties coming from the tunnelling process in itself. In this last process two uncorrelated sources of noise have been identified. Firstly, the shot noise due to the discrete nature of the electric charge is responsible for a position uncertainty of the test mass. Secondly, the fluctuations in the momentum imparted by the electrons to the test mass give rise to a momentum uncertainty of the test mass. The product of these two quantities is of the order of $\hbar/2$ reaching exactly this value in the case of a transducer schematized by a square-well barrier [6].

Brownian noise arising from the coupling of the test mass to the environment usually dominates over the quantum noise and destroys the quantum properties of the test mass. Suppression of the Brownian noise contribution is crucial for improving the sensitivity of position transducers until the standard quantum limit is reached and eventually surpassed as required in high-precision experiments in general relativity [7]. Moreover, repeated monitoring at a quantum level of sensitivity of a single degree of freedom of a macroscopic mass is relevant to understand quantum measurement theory [8]. It is therefore important to study mechanisms for which the quantum noise can be made dominant with respect to the Brownian noise. In this letter we propose the use of resonant vacuum tunnelling transducers to achieve such a goal. We will apply the uncertainty principle to a double barrier in which resonant tunnelling occurs and we will compare the noise figures to the corresponding non-resonant case.

Let us consider a tunnelling transducer driven by an incident current I, *i.e.* I is the current which should flow in the device if the tip and the test mass were in contact. Due to this current, during a sampling time Δt the number of electrons which attempt to tunnel across the vacuum gap, the number of incident electrons hereafter, is given by

$$N = \frac{I}{e}\Delta t. \tag{1}$$

In a first stage we suppose that all the incident electrons have the same energy E, after we will discuss the case of a biased device with electrons having Fermi distribution. Let $T(E, l)$ be the transmission coefficient at energy E for a distance l between the tip and the test mass. A fraction T of the N incident electrons gives rise to a measured tunnelling current $I_\mathrm{T} = TI$. Due to the discrete nature of the charge carriers a shot noise in the measured tunnelling current I_T inversely proportional to $\sqrt{N}$ arises. The test mass position is inferred by means of the tunnelling current through the dependence of the transmission coefficient on the distance l and a shot noise position uncertainty $\Delta\lambda$ for the test mass also arises [6]:

$$\Delta\lambda^2 = \frac{\Delta l^2}{N} = \frac{1}{N}T(1-T)\left|\frac{\partial T}{\partial l}\right|^{-2}. \tag{2}$$

This uncertainty has been expressed in terms of the uncertainty Δl due to a single electron incident at energy E. At the same time the N incident electrons impart a momentum uncertainty $\Delta\pi$ to the test mass [6]:

$$\Delta\pi^2 = N\Delta p^2, \tag{3}$$

where Δp is the test mass momentum uncertainty due to a single electron incident at energy E. On the basis of ref. [4] $\Delta p^2 = (J_p^\mathrm{t}/J_\mathrm{in})^2 - J_{p^2}^\mathrm{t}/J_\mathrm{in}$, where J_in is the incident electron flux and J_p^t and $J_{p^2}^\mathrm{t}$ are the momentum and momentum-squared fluxes transferred by the electron to the test mass, *i.e.* the momentum and the momentum-squared fluxes evaluated in the vacuum zone.

The two quantum noise sources increase the energy of the test mass. If we schematize the test mass by a harmonic oscillator at rest with mass M and angular frequency ω the energy increase in the sampling time Δt will be

$$\Delta\varepsilon = \frac{\Delta\pi^2}{2M} + \frac{1}{2}M\omega^2\Delta\lambda^2. \tag{4}$$

This can be considered as the exchange of energy between the test mass (measured object) and the electrons (meter) due to the quantum measurement process in the time Δt.

Superimposed to the two quantum noise sources there is the Brownian motion of the test mass coupled to the external environment. Taking into account also this contribution [9] the total variation of the test mass energy in the time Δt will be

$$\Delta\varepsilon = \frac{\Delta\pi^2}{2M} + \frac{1}{2}M\omega^2\Delta\lambda^2 + \frac{k_B\theta}{Q}\omega\Delta t, \tag{5}$$

where θ is the thermodynamical temperature of the reservoir schematizing the external environment and Q the mechanical quality factor expressing the relaxation of the harmonic oscillator in the thermal bath. The total energy introduced by the quantum measurement process and by the thermodynamical reservoir may be converted into a total effective displacement $\Delta\xi$, representing the sensitivity of the transducer for a measurement of duration Δt, through the relation $\Delta\varepsilon = M\omega^2\Delta\xi^2$

$$\Delta\xi^2 = \frac{\Delta l^2}{2N} + N\left(\frac{\Delta p^2}{2M^2\omega^2} + \frac{k_B\theta e}{M\omega QI}\right). \tag{6}$$

In this formula the dependence upon the number of incident electrons has been emphasized to show that both for small values of N, when the shot noise is dominant, and for large values of N, when the sum of the momentum uncertainty and of the thermal noise is dominant, large values of the effective displacement are obtained. A minimum value for the effective displacement will be achieved in an intermediate situation:

$$\Delta\xi^2_{\text{opt}} = \frac{\Delta l\,\Delta p}{M\omega}\sqrt{1 + \frac{2k_B\theta M\omega e}{Q\Delta p^2 I}}. \tag{7}$$

This optimal sensitivity corresponds to the quantum limit when the thermal contribution is negligible, *i.e.* when

$$\frac{2k_B\theta M\omega e}{Q\Delta p^2 I} \ll 1. \tag{8}$$

For instance in a square-well tunnelling transducer we have exactly $\Delta l\,\Delta p = \hbar/2$ [4, 6] and when the inequality (8) holds we get the standard quantum limit $\Delta\xi^2_{\text{opt}} = \hbar/2M\omega$, for an optimal number of incident electrons $N_{\text{opt}} = \hbar M\omega/2\Delta p^2$.

The requirement of thermal noise negligible with respect to quantum momentum noise is difficult to satisfy for a single-barrier tunnelling transducer [4]. Due to the small value of the single-electron momentum uncertainty Δp one has to use a large incident current I, a high mechanical quality factor Q, a small test mass M and a low temperature θ in order to satisfy the inequality (8). A different situation arises in a resonant tunnelling transducer. In fig. 1 we show a possible scheme for a transducer based upon a double-barrier potential. A double junction or an impurity zone is grown on the surface of the test mass producing an effective potential barrier against the current flow. When the tip is put near the test mass a resonant double barrier is achieved. At a resonance energy the transmission coefficient $T = 4T_1T_2/(T_1+T_2)^2$ of the double barrier may be expressed in terms of the transmission coefficients T_1 and T_2 of each single barrier [10]. A variation of the distance l between the tip and the test mass changes the transmission coefficient T_1. For $T_2 \gg T_1$, as in the situations we will analyse in the following, the simple relationship holds

$$\frac{\partial T}{\partial l} \simeq \frac{T}{T_1}\frac{\partial T_1}{\partial l}. \tag{9}$$

With respect to the single-barrier case we have at resonance a single-electron position uncertainty smaller by a factor T/T_1 and, because of $\Delta l\,\Delta p \simeq \hbar/2$, a larger single-electron momentum uncertainty.

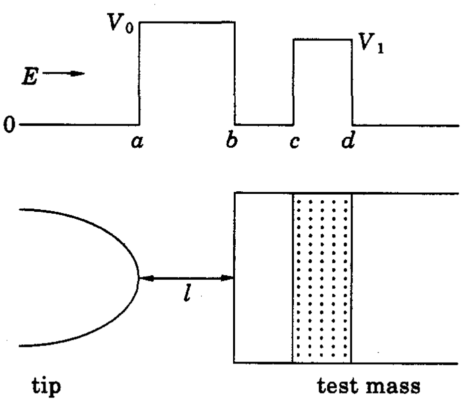

Fig. 1

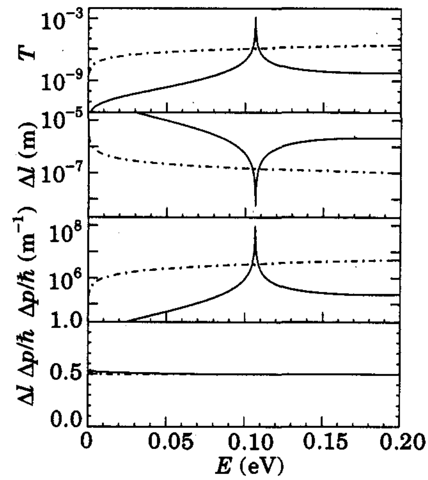

Fig. 2

Fig. 1. – Schematic view of a resonant tunnelling transducer and corresponding effective one-dimensional double-barrier potential.

Fig. 2. – Comparison of various quantities *vs.* the energy E of the incident electrons for a resonant double-barrier (solid) and a single-barrier (dot-dashed) transducer. From the top to the bottom, respectively: transmission coefficient T, position uncertainty Δl, momentum uncertainty Δp, both due to tunnelling of a single electron, and position-momentum uncertainty product in units of $\hbar$. The example corresponds to the choice of GaAs tip and test mass having an AlAs barrier and parameters (see fig. 1) $b-a=20$ Å, $c-b=50$ Å, $d-c=20$ Å, $V_0=4$ eV, $V_1=1$ eV and effective electron mass $m=0.1\,m_e$. The single-barrier case is obtained with the same parameters except $d-c=0$.

A quantitative comparison of a resonant tunnelling transducer to a non-resonant one is shown in fig. 2. In the former case around the resonance energy the position and momentum uncertainties transferred by a single incident electron to the test mass have a value respectively smaller and larger than in the corresponding non-resonant case. At the same time their product remains close to $\hbar/2$. It should be observed that for both resonant and non-resonant cases the shape of the momentum uncertainty Δp closely follows the shape of the transmission coefficient T. Indeed we can define a momentum uncertainty per single tunnelling electron as $\Delta p_T^2 = \Delta p^2/T \simeq 2mV_0$ which only depends upon the vacuum barrier V_0 and the effective electron mass m. This consideration allows to rewrite the inequality (8) in terms of the tunnelling current I_T and of Δp_T^2 instead of the corresponding quantities for the incident electron

$$\frac{2k_B\,\theta M\omega e}{Q\Delta p_T^2 I_T} \ll 1\,. \tag{10}$$

It is evident that this inequality is better satisfied for a resonant configuration because in this case a larger tunnelling current I_T for a given incident current I may be obtained.

The influence of a Brownian noise source is shown in fig. 3. A non-negligible Brownian noise with a ratio $\theta/Q = 10^{-5}$ K causes a worsening of the optimal sensitivity $\Delta\xi^2_{opt}$ by a factor 10^2 with respect to the case $\theta/Q=0$ in a non-resonant transducer. On the other hand, in the same conditions a resonant transducer working at the resonance energy retains an optimal sensitivity close to the quantum limit.

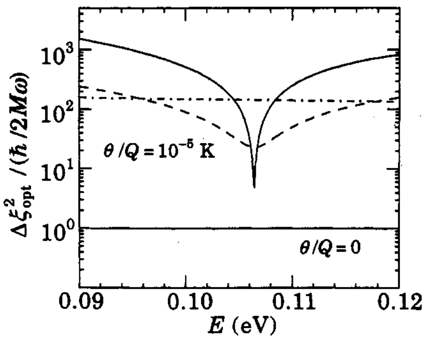

Fig. 3

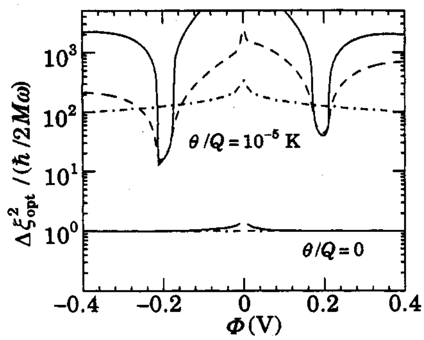

Fig. 4

Fig. 3. – Optimal sensitivity $\Delta\xi^2_{\rm opt}$ *vs.* the energy E of the incident electrons for a resonant double barrier with coherent tunnelling (solid) and in the presence of sequential tunnelling (dashed, $\gamma = 0.95$) and single-barrier (dot-dashed) transducer in the case of two different values of the ratio θ/Q (the two curves are undistinguishable for $\theta/Q = 0$). We have chosen $M = 10^{-10}$ kg, $\omega = 2\pi 10^5\ {\rm s}^{-1}$, $I = 1$ A ($I_{\rm T} \simeq 10^{-6}$ A and $I_{\rm T} \simeq 10^{-8}$ A for the single and double barriers, respectively) and the parameters of the barriers as in fig. 2.

Fig. 4. – Optimal sensitivity $\Delta\xi^2_{\rm opt}$ *vs.* the bias voltage Φ between the tip and the test mass for a resonant double barrier with coherent tunnelling (solid) and in the presence of sequential tunnelling (dashed, $\gamma = 0.95$) and a single-barrier (dot-dashed) transducer in the case of two different values of the ratio θ/Q. We have chosen a Fermi energy $E_{\rm F} = 0.02$ eV, a transverse surface $S = 10^{-9}\,{\rm m}^2$ ($I_{\rm T} \simeq 10^{-6}$ A and $I_{\rm T} \simeq 10^{-4}$ A for the single and double barriers, respectively) and all the other parameters as in fig. 3.

In a more realistic approach one has to consider not only the effect of the Brownian noise but also the partial loss of quantum-mechanical coherence due to inelastic scattering. If the escape time of the electrons from the well $\tau_{\rm esc}$ is longer than the inelastic-scattering time $\tau_{\rm i}$ the coherent tunnelling current decreases and it is only partially compensated by a sequential tunnelling current [11]. In the situation described in fig. 3 we have $\tau_{\rm esc} \approx [2(c-b)/v]\,2/(T_1 + T_2) = 10^{-11}$ s, where v is the velocity of the electrons at the resonance. If inelastic phonon scattering is assumed to dominate, we estimate $\tau_{\rm i} \approx 10^{-13}$ s. The loss of coherence can be included by introducing a phenomenological factor γ which is related to the damping of the wave function due to inelastic scattering and expressed in terms of $\tau_{\rm i}$ through the relationship $2\tau_{\rm i} = [2(c-b)/v]/(1-\gamma)$ [12]. By using the estimated value of $\tau_{\rm i}$ in our configuration, we get $\gamma = 0.95$ and the effect of the decoherence is shown in fig. 3 as a less pronounced peak which still allows for an order of magnitude improvement when compared to the single-barrier configuration. Such improvements, both in the fully coherent configuration (*i.e.* $\gamma = 1$) and in the presence of inelastic scattering, shown in fig. 3 are limited to an energy range of the order of the resonance energy width.

Production of ballistic electrons with energy spread smaller than the resonance energy width is within the current semiconductor technology capabilities [13]. A similar gain may also be obtained using a biased device in which a Fermi distribution of electrons gives rise to the tunnelling current. By integrating over the transverse states the energy distribution of the incident electrons, representing the differential form of eq. (1), is expressed by [14]

$$\frac{{\rm d}N}{{\rm d}E} = \frac{mSk_{\rm B}\theta}{2\pi^2\hbar^3}\ \ln\left[\frac{1+\exp[(E_{\rm F}-E)/k_{\rm B}\theta]}{1+\exp[(E_{\rm F}-E-e\Phi)/k_{\rm B}\theta]}\right]\Delta t, \tag{11}$$

where S is the transverse surface, Φ is the bias voltage and E_F the Fermi energy. This allows to evaluate the integrated shot noise position uncertainty

$$\Delta\lambda^2 = \int_0^\infty \frac{\mathrm{d}N}{\mathrm{d}E} T(1-T)\,\mathrm{d}E \left[\int_0^\infty \frac{\mathrm{d}N}{\mathrm{d}E} \left| \frac{\partial T}{\partial l} \right| \mathrm{d}E \right]^{-2}, \tag{12}$$

and the integrated momentum uncertainty

$$\Delta\pi^2 = \int_0^\infty \frac{\mathrm{d}N}{\mathrm{d}E} \Delta p^2\,\mathrm{d}E. \tag{13}$$

By repeating the same arguments of the monoenergetic case an optimal sensitivity may be obtained at a proper sampling time:

$$\Delta\xi_{\mathrm{opt}}^2 = \frac{\Delta\lambda\,\Delta\pi}{M\omega} \sqrt{1 + \frac{2k_B\,\theta M\omega}{Q\Delta\pi^2/\Delta t}}. \tag{14}$$

The product $\Delta\lambda\,\Delta\pi$ remains close to $\hbar/2$ and the quantum limit is reached when the second term inside the square root is negligible. Due to the constancy of the quantity $\Delta p_T^2 = \Delta p^2/T \simeq 2mV_0$ this condition is expressed again by the inequality (10) in terms of the tunnelling current I_T,

$$I_T = \frac{emSk_B\,\theta}{2\pi^2\hbar^3} \int_0^\infty \ln\left[\frac{1+\exp[(E_F - E)/k_B\,\theta]}{1+\exp[(E_F - E - e\Phi)/k_B\,\theta]}\right] T\,\mathrm{d}E. \tag{15}$$

In fig. 4 we show the dependence of the optimal sensitivity as a function of the bias voltage for the same thermal contributions of fig. 3 with and without the effect of inelastic scattering. Despite the integration over all the available electrons the improvement in the use of the resonant configuration at the proper bias voltage remains one order of magnitude with respect to the single-barrier situation. Moreover, the optimal sensitivity has a slight dependence upon the inelastic-scattering processes.

Resonant tunnelling may be relevant for improving the sensitivity of the tunnelling transducers proposed to detect gravitational waves. In this class of transducers the sensitivity is limited, apart from $1/f$ noise which strongly depends upon the materials used for the test mass and the tip, by the shot noise. Due to the increased tunnelling current in a resonant configuration the shot noise is decreased (1). For the same reason the momentum noise contribution is enhanced and studies of macroscopic quantum noise due to the interaction between the electrons and the test mass are more easily performed. From fig. 3 and 4 it turns out that experiments for detecting quantum noise at $\theta = 4.2$ K seem also feasible provided that a quality factor $Q \geqslant 10^6$ at that temperature can be achieved. Thus experimental studies of the quantization of a macroscopic degree of freedom of a micromachined test mass may open interesting prospects in mesoscopic mechanics. In particular, coherence properties of such single macroscopic oscillators, *e.g.* the creation of distinguishable states of a harmonic oscillator already proposed in a quantum optics framework [16], and its destruction through the influence of the reservoir can be investigated.

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(1) Recent measurements indicate that the shot noise in a double barrier is further reduced below the theoretically expected value, see [15].

Additional Remark.

After the submission we have been aware of an experiment performed with an optical resonator based on tunnelling of frustrated light which is the optical counterpart of the device we proposed here [17].

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We are grateful to M. F. BOCKO for stimulating comments and to M. BÜTTIKER for pointing us out ref. [15]. This work has been supported by INFN, Italy.

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