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Quantum Zeno effect with the Feynman–Mensky path-integral approach

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A model for the quantum Zeno effect based upon an effective Schrödinger equation originated by the path-integral approach is developed and applied to a two-level system simultaneously stimulated by a resonant perturbation. It is shown that inhibition of stimulated transitions between the two levels appears as a consequence of the influence of the meter whenever measurements of energy, either continuous or pulsed, are performed at quantum level of sensitivity. The generality of this approach allows one to qualitatively understand the inhibition of spontaneous transitions as the decay of unstable particles, originally presented as a paradox of the quantum measurement theory.

The quantum measurement theory has been developed in the thirties to understand the problems which arise when quantum mechanical formalism is interpreted and confronted with its macroscopic limit. Some of the debates originated by the matching between quantum and classical worlds assumed the form of paradoxes and have been discussed in terms of ideal experiments. This is the case of the so-called quantum Zeno effect [1–5], i.e., the inhibition of the free evolution of a system subjected to continuous measurements. In the original example of Misra and Sudarshan an unstable particle whose trajectory is continuously monitored should never be observed to decay. Due to recent technological progress in the measurement of physical quantities, especially in quantum optics, in the physics of superconducting coherent devices such as SQUIDs, and in experimental gravitation, some of these ideal experiments can be actually performed [6]. Claims of the observation of the quantum Zeno effect have been reported by Itano et al. [7], which observed freezing of the stimulated transition probability in a two-level system subjected to frequent measurements of the population of a level.

The original interpretation of the authors in terms of the quantum Zeno effect has been debated both with philosophical considerations on the concept of measurement and with detailed calculations (among the exploding literature on the debate originated by ref. [7] we cite, without any pretence of completeness, ref. [8]). In this Letter we make use of a measurement model which allows one to discuss the general features of a measurement operation independent of the particular measuring apparatus used to perform it. As a result of this analysis the quantum Zeno effect turns out to be just an example of the influence of the meter when measurements are performed on a system in the quantum regime of sensitivity.

The model is based upon restriction of the Feynman paths to the measurement result by introducing a measure functional in the space of the paths [9,10]. In this method the effect of the meter on the measured system is taken into account giving the output of the measurement and the accuracy with which it has been performed. No explicit degrees of freedom of the meter are introduced, and this makes the considerations quite independent of the particular type of measuring apparatus. The model has been applied to understand the accuracy of measurements of the position in non-linear systems, monitored in a continuous [11] and impulsive way [12]. In the case of the quantum Zeno effect as investigated in ref. [7] we are interested to measure the energy of a system under the simultaneous effect of an external potential responsible for stimulated transitions. This makes the Feynman paths in the phase space a very adequate tool. Let us suppose that a continuous measurement of the energy with result E is performed for a time τ (for simplicity the result is considered constant) using an instrument with error ΔE. The restriction to the paths around the measurement result is obtained by introducing a functional weighting the paths, for instance with a Gaussian measure,

$$w_{[E]} = \exp[-\langle (H_0 - E)^2\rangle/\Delta E^2], \tag{1}$$

where $\langle\ \rangle$ indicates time-average of the argument between 0 and τ. The kernel for the propagation of the system under continuous monitoring of its energy is modified by the weight functional to

$$K_{[E]}(q',\tau;q,0) = \int d[q]\,d[p]\exp\left(\frac{i}{\hbar}\int_0^\tau [p\dot{q} - H_0(q,p)]\,dt\right) w_{[E]}, \tag{2}$$

which can be rewritten as the usual kernel of a new system in which the effect of the measurement has been taken into account through an effective Hamiltonian

$$H_{\mathrm{eff}} = H_0 - i\frac{\hbar}{\tau\Delta E^2}(H_0 - E)^2 . \tag{3}$$

The non-Hermitian nature of the effective Hamiltonian is due to the selective measurement that restricts the possible future results [13]. According to this selection the state of the measured system loses its normalization. During the measurement the evolution of the system, supposed to be in a pure state $|\psi(0)\rangle$ at the beginning of the measurement, is given by the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H_{\mathrm{eff}}|\psi(t)\rangle . \tag{4}$$

Let $|n\rangle$ and E_n be the eigenvectors and the eigenvalues of the Hamiltonian H_0 of the unmeasured system, the state $|\psi(t)\rangle$ of the measured system can be expanded in the base $\{|n\rangle\}$

$$|\psi(t)\rangle = \sum_n c_n(t)|n\rangle . \tag{5}$$

By substituting (5) in (4) an evolution equation for the coefficients c_n is obtained,

$$\frac{dc_n}{dt} = \left(-i\frac{E_n}{\hbar} - \frac{(E_n - E)^2}{\tau\Delta E^2}\right)c_n, \tag{6}$$

which is solved to get

$$c_n(t) = \exp\left(-i\frac{E_n}{\hbar}t - \frac{(E_n - E)^2}{\tau\Delta E^2}t\right)c_n(0) . \tag{7}$$

We note that the coefficients c_n are suppressed by the real exponential factor with a time constant $\tau\Delta E^2/(E_n - E)^2$, i.e., the state $|\psi(t)\rangle$ collapses around the measurement result. This last is not necessarily an eigenvalue E_n of the unmeasured system due to the classical uncertainty of the meter (for a detailed discussion of the classical properties of the meter see ref. [14]). If the measurement result is some definite eigenvalue E_m and $\Delta E \ll E_n - E_m$, the state of the measured system collapses to $|\psi(\tau)\rangle = c_m(\tau)|m\rangle$ at the end of the measurement and its squared norm, namely $|c_m(0)|^2$, is the probability associated to the initial state to get the measured result $E = E_m$.

The transition between different levels is obtained under the action of an appropriate external perturbation $V(t)$ which is added to the effective Hamiltonian of eq. (3). The decomposition of the state $|\psi(t)\rangle$ in terms of the same eigenstates $|n\rangle$ of the unmeasured and unperturbed system can be used again. The evolution equation for the coefficient c_n contains also a term proportional to the perturbation strength and nondiagonal in the index n,

$$\frac{\mathrm{d}c_n}{\mathrm{d}t} = \left(-\mathrm{i}\frac{E_n}{\hbar} - \frac{(E_n - E)^2}{\tau \Delta E^2}\right) c_n - \mathrm{i}\sum_k \frac{V_{nk}}{\hbar} c_k, \tag{8}$$

where $V_{nk} = \langle n|V(t)|k\rangle$.

A particularly simple picture is obtained for a two-level system with energies E_1 and E_2. Assuming a perturbation potential $V_{11} = V_{22} = 0$ and $V_{12} = V_{21}^* = V_0 \exp[\mathrm{i}\omega(t - t_0)]$ with V_0 real, the solution of the system (8) is

$$c_1(t) = \exp\left(-\mathrm{i}\frac{E_1}{\hbar}t - \frac{(E_1 - E)^2}{\tau\Delta E^2}t + \mathrm{i}qt\right)\left(c_1(0)\cos(wt) + \frac{qc_1(0) + \exp(\mathrm{i}\omega t_0)pc_2(0)}{\mathrm{i}w}\sin(wt)\right), \tag{9}$$

$$c_2(t) = \exp\left(-\mathrm{i}\frac{E_2}{\hbar}t - \frac{(E_2 - E)^2}{\tau\Delta E^2}t - \mathrm{i}qt\right)\left(c_2(0)\cos(wt) - \frac{qc_2(0) - \exp(-\mathrm{i}\omega t_0)pc_1(0)}{\mathrm{i}w}\sin(wt)\right), \tag{10}$$

where $p = V_0/\hbar$, $2q = \omega - (E_2 - E_1)/\hbar + 2\mathrm{i}\Omega$ with $\Omega = [(E_2 - E)^2 - (E_1 - E)^2]/2\tau\Delta E^2$ and $w = \sqrt{q^2 + p^2}$. In order to evidence the Zeno effect in a specific example let us suppose initially the system to be in the state $|1\rangle$, the perturbation to be resonant, i.e., $\hbar\omega = E_2 - E_1$, and the result of the continuous measurement to be $E = E_1$. The probability $P_1(t)$ to find the system at time $t \leqslant \tau$ in the state $|1\rangle$ is

$$P_1(t) = \frac{|c_1(t)|^2}{|c_1(t)|^2 + |c_2(t)|^2} = \left(1 + \left|\frac{V_0/\hbar}{\Omega + w\cot(wt)}\right|^2\right)^{-1}, \tag{11}$$

where

$$w = \sqrt{(V_0/\hbar)^2 - \Omega^2} \tag{12}$$

and

$$\Omega = \frac{(E_2 - E_1)^2}{2\tau\Delta E^2}. \tag{13}$$

A three-dimensional plot of P_1 versus time and measurement error ΔE is shown in fig. 1. When the measurement error is large we have $V_0/\hbar \gg \Omega$ and the system oscillates between levels 1 and 2 with Rabi frequency $2V_0/\hbar$. In the opposite limit of accurate measurements, when w is imaginary, an overdamped regime is achieved in which transitions are inhibited. A critical damping is observed when $w = 0$. In this case the probability P_1 approaches asymptotically to the value $\frac{1}{2}$ when $\tau \gg \hbar/V_0$. This critical damping is obtained at a measurement error

$$\Delta E_{\mathrm{crit}} = (E_2 - E_1)\sqrt{\hbar/2V_0\tau}, \tag{14}$$

which defines the borderline between the Rabi-like behaviour ($\Delta E > \Delta E_{\mathrm{crit}}$) and the Zeno-like inhibition ($\Delta E < \Delta E_{\mathrm{crit}}$) in terms of the instrumental accuracy of the meter. It should be observed that also in the Rabi regime the effect of the meter already appears as a deformation of the simple harmonic law for the probability P_1 with an evident change in its Fourier spectrum.

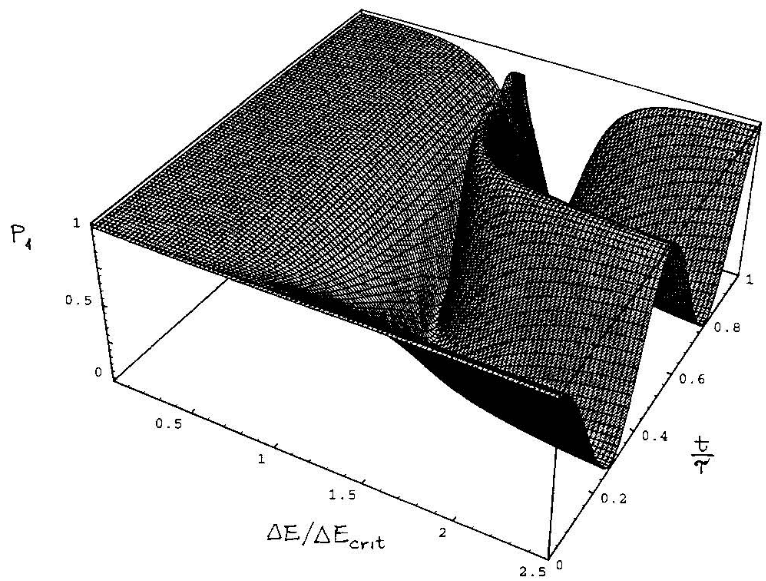

Fig. 1. Probability P_1 to observe a two level system in the state $|1\rangle$ during the measurement interval $0 < t < \tau$ and versus the normalized measurement error $\Delta E/\Delta E_{crit}$. The system is in the state $|1\rangle$ at time $t = 0$ and the result of the measurement between 0 and τ is $E = E_1$ constant. The transition from the underdamped regime with Rabi-like oscillations to the overdamped regime with Zeno inhibition is evident when decreasing ΔE below ΔE_{crit}. We put $E_2 - E_1 = V_0 = \hbar = 1$ and $\tau = 2\pi\hbar/V_0$.

In ref. [15] plots similar to fig. 1 were obtained integrating the optical Bloch equations for a three-level system and extracting the evolution of a two-level subsystem. In the same paper the effect of the measurement was expressed in terms of the coupling to a perturbation term. Here we have a more general picture of the meter which is taken into account by simply giving the measurement result and the accuracy with which it has been determined.

We want also to point out that, although for simplicity the case of a constant string of results equal to E_1 has been considered in our specific example, the formalism allows one to deal with any possible result $E(t)$ realizing a possible history of the energy measurement associated to a single quantum trajectory of the system [15,16].

The model described here allows one also to recover the case of measurement pulses which is the subject of the experiment described in ref. [7]. Let us consider the transition probability $P_{1\to2} = 1 - P_1(T)$ at the end of an on-resonance π pulse of duration $T = \pi V_0/2\hbar$ such that $P_{1\to2} = 1$ without measurements. During this time interval n pulsed measurements, each lasting $\Delta\tau$, can be performed on the system for a total measurement time $\tau = n\Delta\tau$. For example we choose a measurement strategy such that the kth measurement pulse is on from kT/n to $kT/n + \Delta\tau$ and $k = 0, 1, ..., n-1$. Moreover we choose $\Delta\tau/T = 10^{-2}$ so that the number of measurement pulses which can be performed in the range $1 \leqslant n \leqslant 100$, where for $n = 100$ the case of a continuous measurement is recovered. By iterating $2n$ times eqs. (9) and (10) alternatively with finite ΔE (during the measurement time interval) and infinite ΔE (during the following non-measurement time interval) we obtain the results shown in fig. 2. Independently upon the number of measurement pulses the Rabi-like behaviour is observed in the classical regime when $\Delta E > \Delta E_{crit} \equiv (E_2 - E_1)\sqrt{\hbar/2V_0T}$. In the quantum regime $\Delta E < \Delta E_{crit}$ the Zeno-inhibition increases with the number of measurement pulses until the ultimate limit given by the continuous measurement is achieved. This also shows that there is no contradiction between continuous and discrete measurements because in both the cases inhibition of the evolution is achieved provided the measurement is sufficiently accurate to perturb the system.

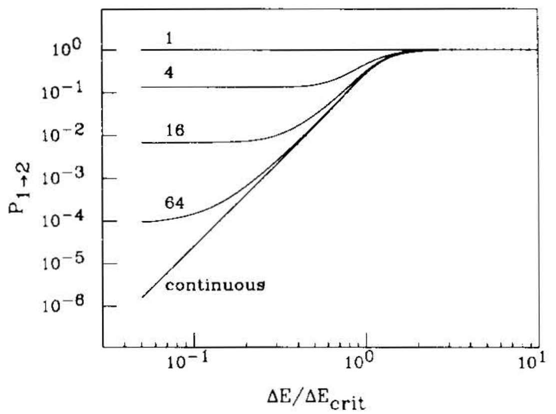

Fig. 2. Transition probability $P_{1\to2}$ at the end of an on-resonance π pulse versus the normalized measurement error $\Delta E/\Delta E_{\rm crit}$ for pulsed measurements with 1, 4, 16, 64 pulses and a continuous measurement.

In all these considerations there is no paradox: the observed system is coupled both to the external perturbation, characterized by the Rabi frequency $2V_0/\hbar$, and to the measurement system, characterized by the frequency Ω. Both the perturbation and the meter compete to influence the evolution of the observed system. Indeed one can choose also quantum measurement strategies which make the effect of the meter negligible. For instance an impulsive measurement of the energy each period of the Rabi oscillations should result in measurements of the occupation probability at the same instants of time unaffected by the meter itself, therefore realizing a quantum non-demolition monitoring of the population in level 1 [6,12,17].

The crucial role played by the meter accuracy in a measurement process also applies to the case of transitions due to spontaneous emission. In the original example of ref. [2] the track of an unstable particle is observed and the paradoxical inhibition of its decay is raised. Due to the poor spatial resolution in the knowledge of the particle track it is not surprising that quantum effects of the measurement are negligible, i.e., the particle is observed to decay, despite the microscopic nature of the measured system as already pointed out by Landau and Lifshitz [18]. The continuous monitoring of the position of a particle schematized as a harmonic or an anharmonic oscillator has been discussed [11]. It turns out that quantum effects of the measurement are important only below a critical accuracy of the order of the de Broglie wavelength of the oscillator $\Delta x_{\rm crit} \sim \sqrt{\hbar/2m\omega}$. In the case of the decay of elementary particles the critical accuracy discriminating the free decay from the Zeno-suppressed decay will be of the order of the Compton wavelength of the virtual intermediate vector bosons responsible for the decay. In the specific example given in ref. [3] a charged pion decays due to the coupling with the electroweak vector bosons $W^{\pm}$ which have Compton wavelength $\hbar/M_{\rm W}c \simeq 10^{-18}$ m: only having a particle detector with spatial resolution of such an order of magnitude the quantum Zeno effect could be observed, appearing as a sort of electroweak microcavity effect.

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