Exploring bio-inspired systems:
a synergy between multiscale experimental and
Computational approaches
Wien, September 4, 2024 - September 7, 2024

Guiding colloidal SAT-assembly

The power of patchyness

!

Francesco Sciortino, Sapienza Universita' di Roma

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Educated Guess Design

Triblock Janus Particles

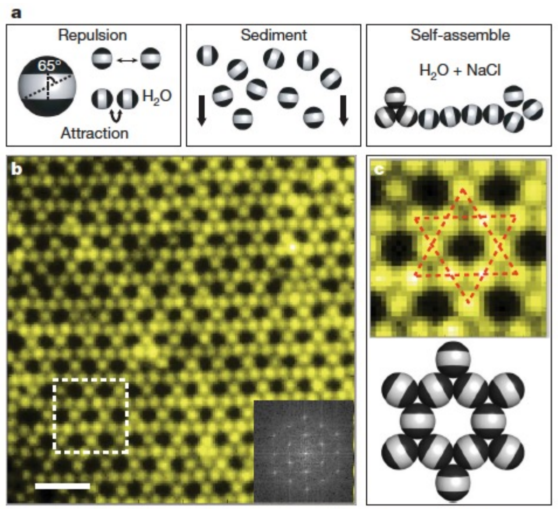


Figure 1 | Colloidal kagome lattice after equilibration. **a**, Triblock Janus spheres hydrophobic on the poles (black, with an opening angle of 65°) and charged in the equator section (white), are allowed to sediment in deionized water. Then NaCl is added to screen electrostatic repulsion, allowing self-assembly by short-range hydrophobic attraction. **b**, Fluorescence image of a colloidal kagome lattice (main image) and its fast Fourier transform image (bottom right). Scale bar is 4 μm. The top panel in **c** shows an enlarged view of the dashed white rectangle in **b**. Dotted red lines in **c** highlight two staggered triangles. The bottom panel in **c** shows a schematic illustration of particle orientations.

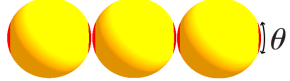
LETTER

doi:10.1038/nature09713

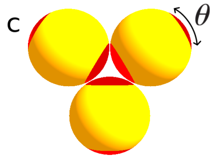
Directed self-assembly of a colloidal kagome lattice

Qian Chen[1], Sung Chul Bae[1] & Steve Granick[1,2,3]

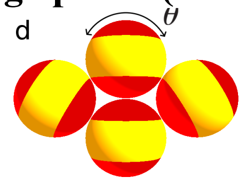
Small patch (chains)



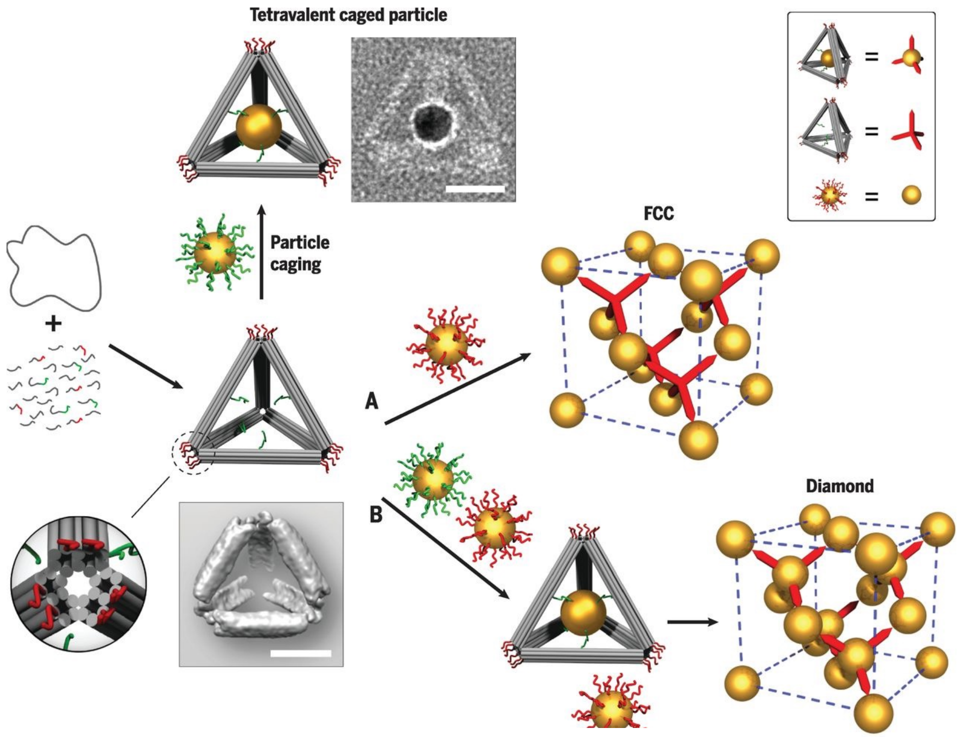
Medium patch (Kagome)



Large patch (dense crystal)



DNA wireframe origami

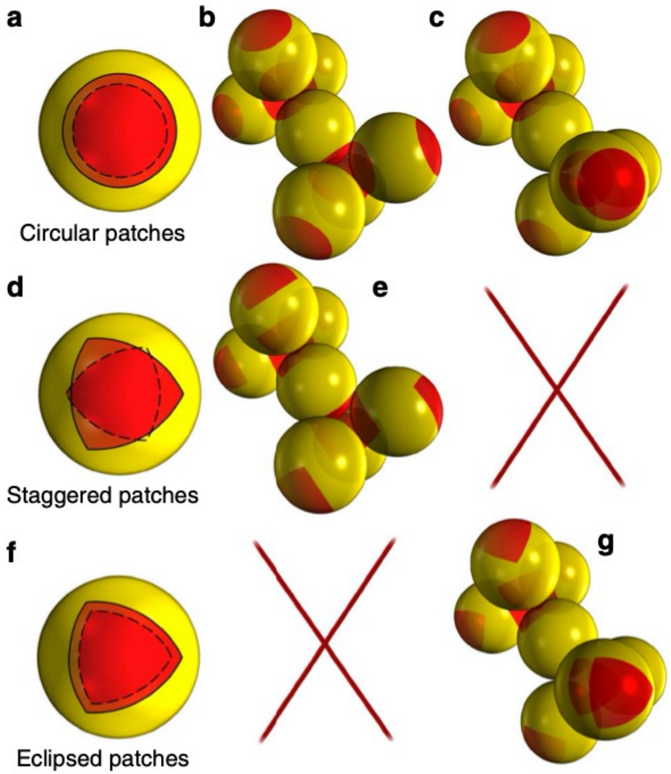


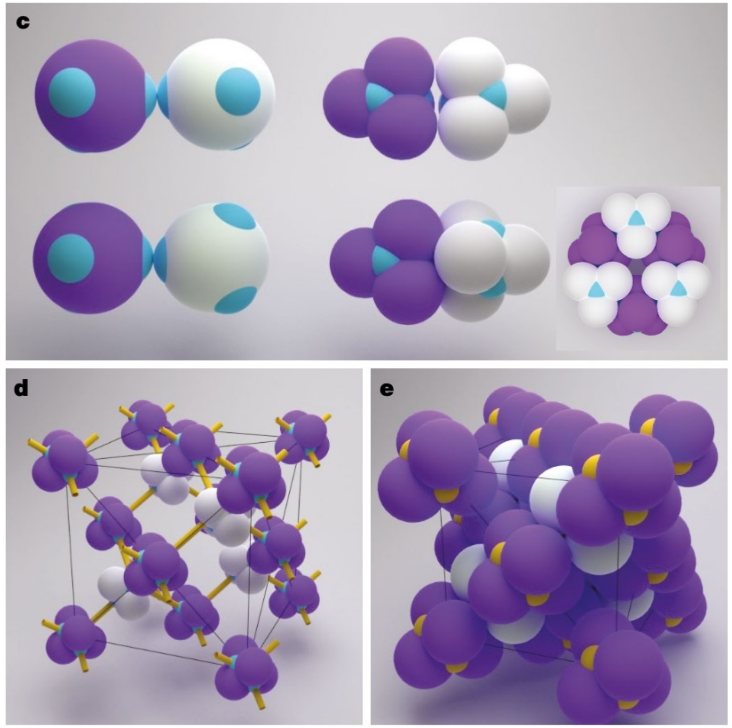
MATERIALS SCIENCE

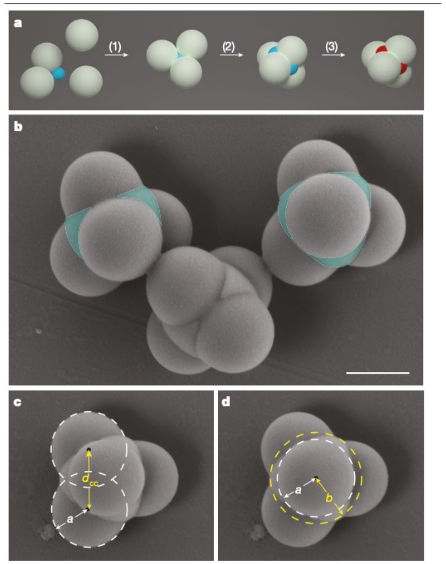
Diamond family of nanoparticle superlattices

Wenyan Liu,[1] Miho Tagawa,[2] Huolin L. Xin,[1] Tong Wang,[3] Hamed Emamy,[4] Huilin Li,[3,5] Kevin G. Yager,[1] Francis W. Starr,[4] Alexei V. Tkachenko,[1] Oleg Gang[1*]

Educated Guess Design







ARTICLE

Received 12 Mar 2012 | Accepted 21 Jun 2012 | Published 24 Jul 2012 DOI: 10.1038/ncomms1968

Patterning symmetry in the rational design of colloidal crystals

Flavio Romano[1,*] & Francesco Sciortino[2,*]

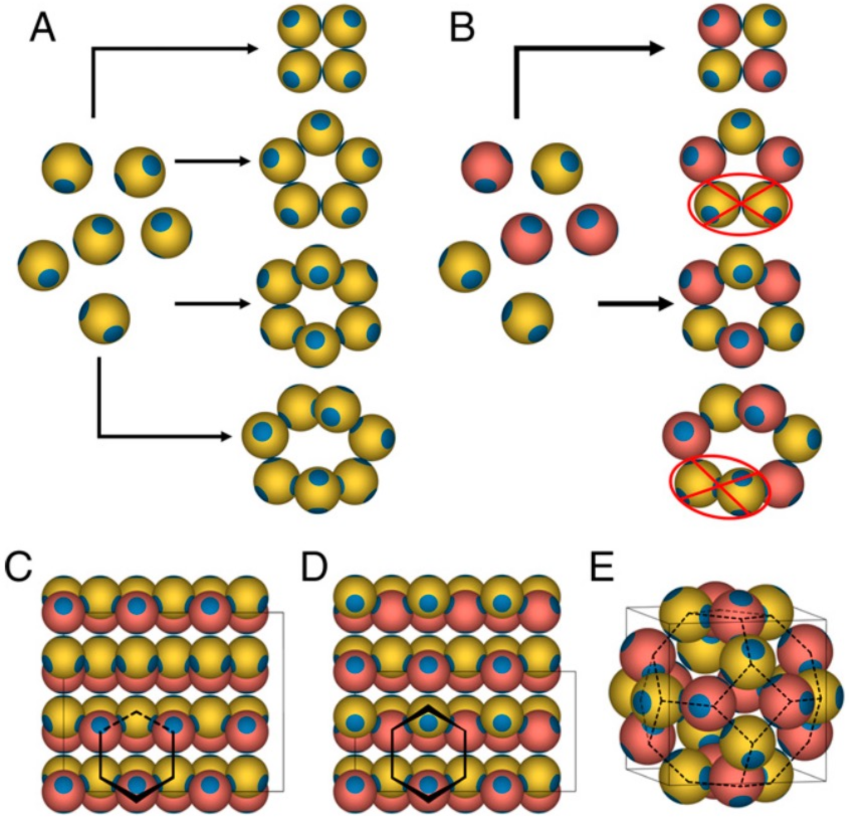
Colloidal diamond

https://doi.org/10.1038/s41586-020-2718-6

Received: 9 February 2020

Mingxin He[1,2], Johnathon P. Gales[2], Étienne Ducrot[2,3], Zhe Gong[4], Gi-Ra Yi[5], Stefano Sacanna[4✉] & David J. Pine[1,2✉]

Educated Guess Design



Facile self-assembly of colloidal diamond from tetrahedral patchy particles via ring selection

Andreas Neophytou[a], Dwaipayan Chakrabarti[a,1], and Francesco Sciortino[b,1]

[a]School of Chemistry, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom; and [b]Dipartimento di Fisica, Sapienza Università di Roma, 00185 Roma, Italy

Potential pitfalls typically encountered in self- assembly

* metastable states that can compete with the final product;

(cubic diamond, hexagonal diamond, clathrates)

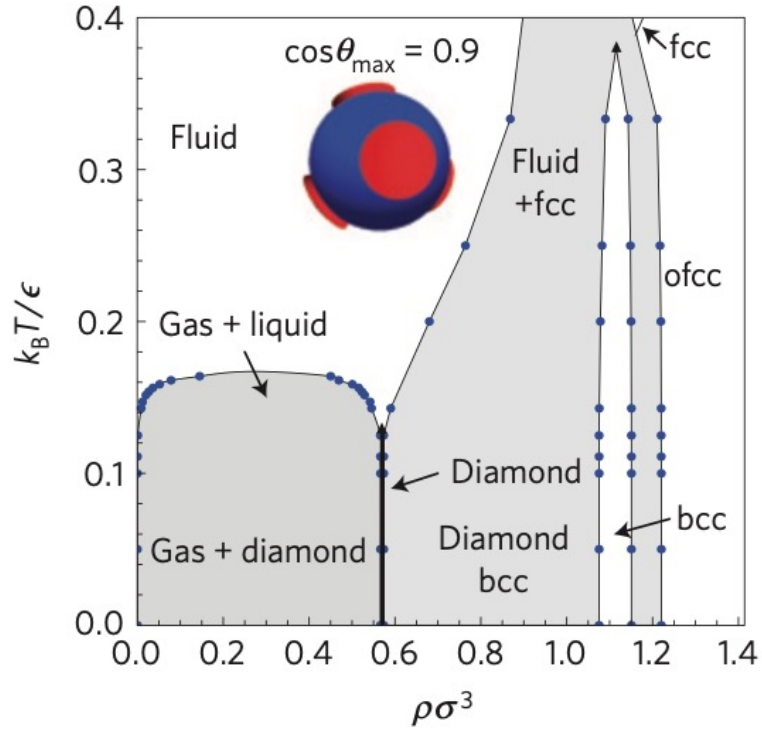
Potential pitfalls typically encountered in self- assembly

* metastable states that can compete with the final product;

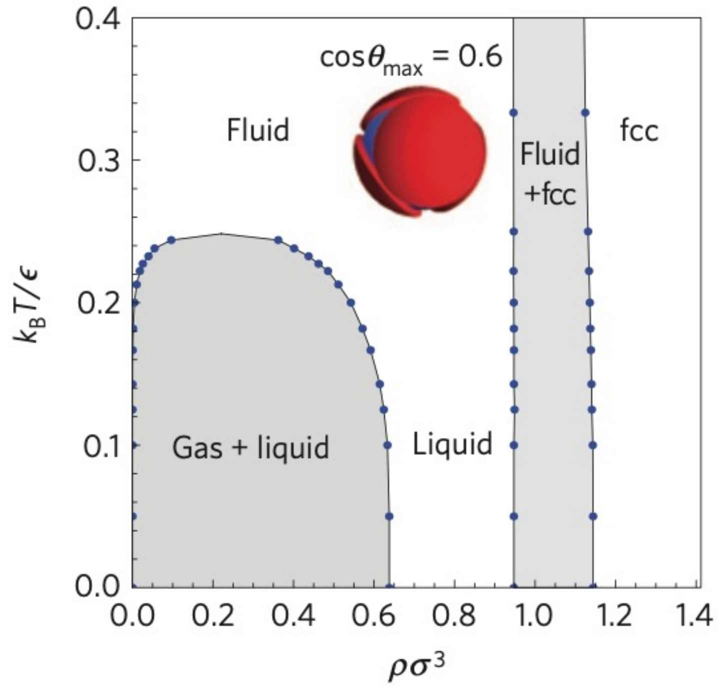
* dynamically arrested states (kinetic traps);

(glasses, gels)

Tetrahedral patchy particles phase diagram – role of the potential parameters



Directional bonds



Flexible bonds

LETTERS
nature physics
PUBLISHED ONLINE: 4 AUGUST 2013 | DOI: 10.1038/NPHYS2693

Liquids more stable than crystals in particles with limited valence and flexible bonds

Frank Smallenburg* and Francesco Sciortino

The *SAT assembly* way

From patchy particles to finite size, periodic and a-periodic selected structures

The *SAT assembly* way

Use patchy colloidal particles as building blocks

Define as logical variable the lattice and the particle properties

Write the solution in term of CLAUSES which need to be satisfied

Find with a SAT-solver the possible solutions

Designing Patchy Interactions to Self-Assemble Arbitrary Structures

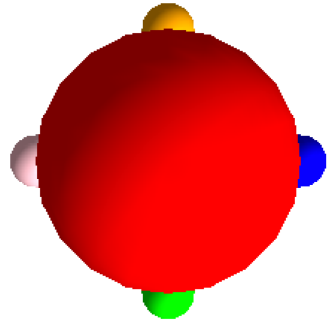
Flavio Romano, John Russo, Lukáš Kroc, and Petr Šulc
Phys. Rev. Lett. **125**, 118003 – Published 10 September 2020

J. Phys.: Condens. Matter **34** (2022) 354002 (11pp)

SAT-assembly: a new approach for designing self-assembling systems

John Russo[1,*], Flavio Romano[2,3], Lukáš Kroc[4], Francesco Sciortino[1], Lorenzo Rovigatti[1,5] and Petr Šulc[4,*]

Why Patchy particles as building blocks ?



(i) Can be experimentally realized (colloids, origami)

(ii) The thermodynamic behaviour of these models is very well understood (Wertheim theory)

(iii) Computationally efficient (this is required to observe self-assembly phenomena in silico).

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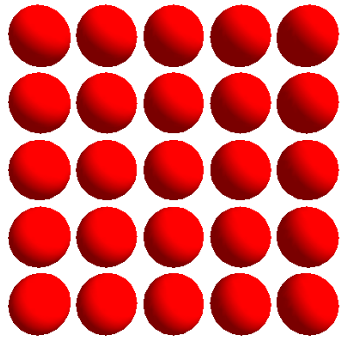
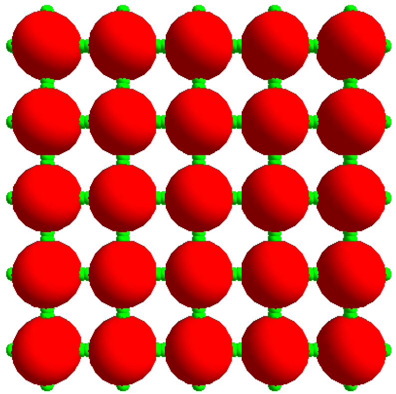
SAT Assembly Steps:



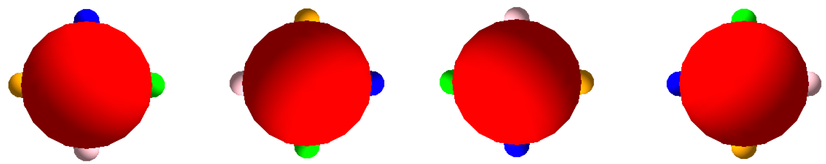
Pick the target (square) lattice ($0<i<L$ lattice sites)

Define the connections (slots) – 4 per particle --   (patchy particles)



Define the number of different patchy particles and the number of different colours (N_p, N_c)

Define orientations



(some of) The logical variables defined to solve the coloring process

$x^{Lattice}_{l,p,o}$ True if lattice site l is occupied by particle type p with orientation o

$x^{Patch\ Colour}_{p,s,c}$ True if patch number s of particle p has color c

$x^{ColourInteraction}_{c_i,c_j}$ True if color c_i interact with color c_j

$x^{DesignVariables}_{l,s,c}$ True if the slot s of lattice site l has colour c

10^5 variables

(some) Clauses to solve the coloring process (NOT $\bar{}$ and OR $\vee$)

Each site is occupied by only one particle type with specified orientation

$$C^{L}_{l,p_i,o_i,p_j,o_j} = \bar{x}^{Lattice}_{l,p_i,o_i} \vee \bar{x}^{Lattice}_{l,p_j,o_j}.$$

Each color can be complementary to only one other color

$$C^{\text{int}}_{c_i,c_j,c_k} = \bar{x}^{ColourInteraction}_{c_i,c_j} \vee \bar{x}^{ColourInteraction}_{c_i,c_k}.$$

Neighboring positions l_i and l_j connected by slots s_i and s_j must have color c_i and c_j that can bind to each other

J. Phys.: Condens. Matter **34** (2022) 354002 (11pp) https://dc

SAT-assembly: a new approach for designing self-assembling systems

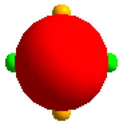
John Russo[1,*], Flavio Romano[2,3], Lukáš Kroc[4], Francesco Sciortino[1], Lorenzo Rovigatti[1,5] and Petr Šulc[4,*]

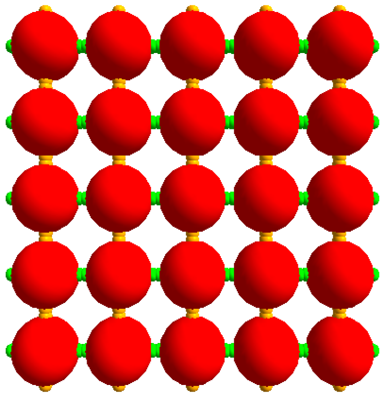
# of colors must be N_c and # of particles must be N_p

number of clauses 10^5

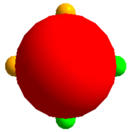
Some solutions:

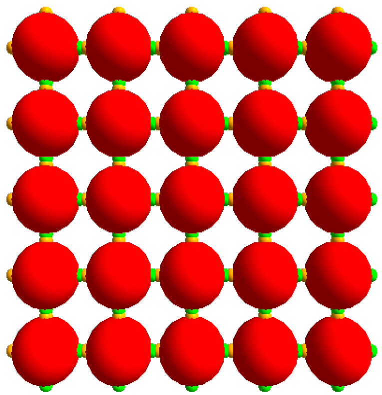
N_p=1, N_c=2 G-G, O-O



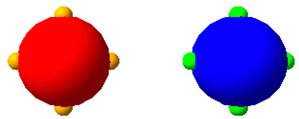


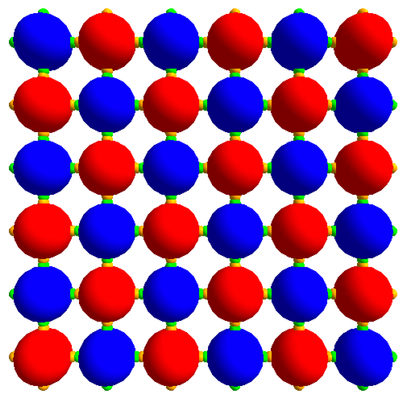
N_p=1, N_c=2 G-O



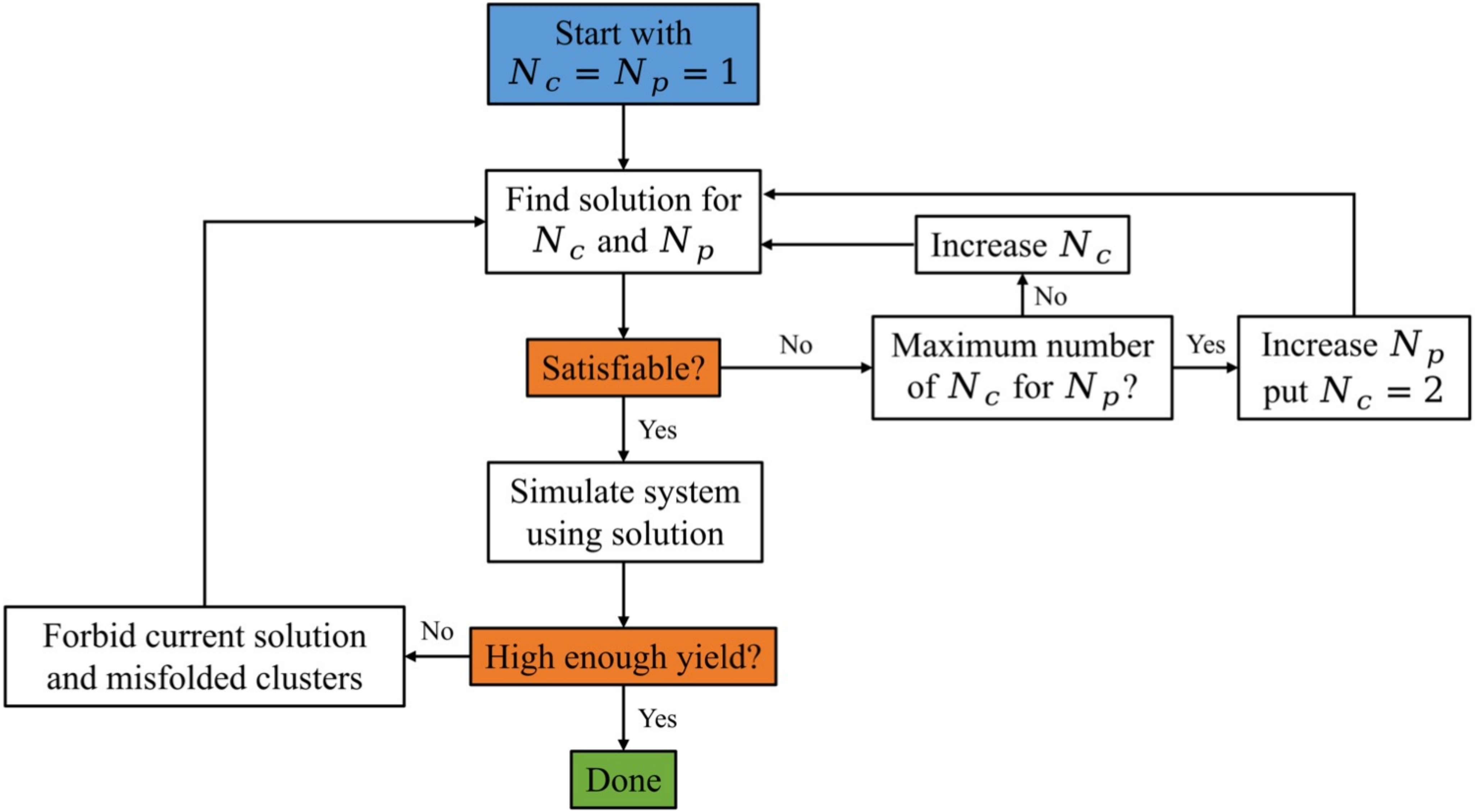


N_p=2, N_c=2 G-O



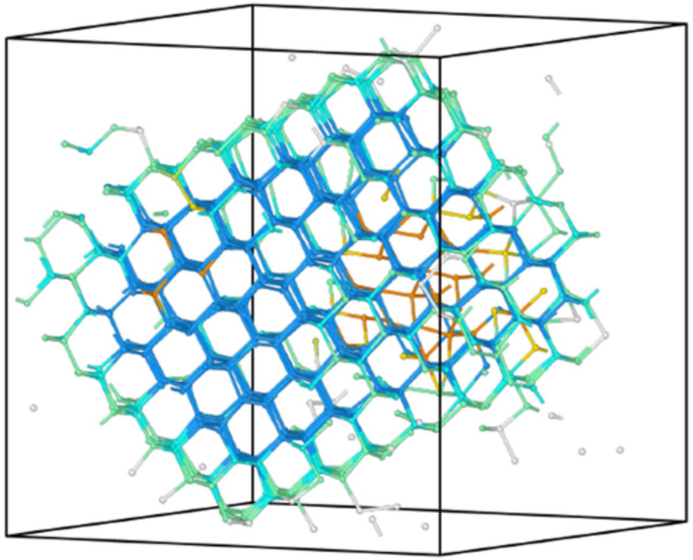


The SAT-ASSEMBLY pipeline



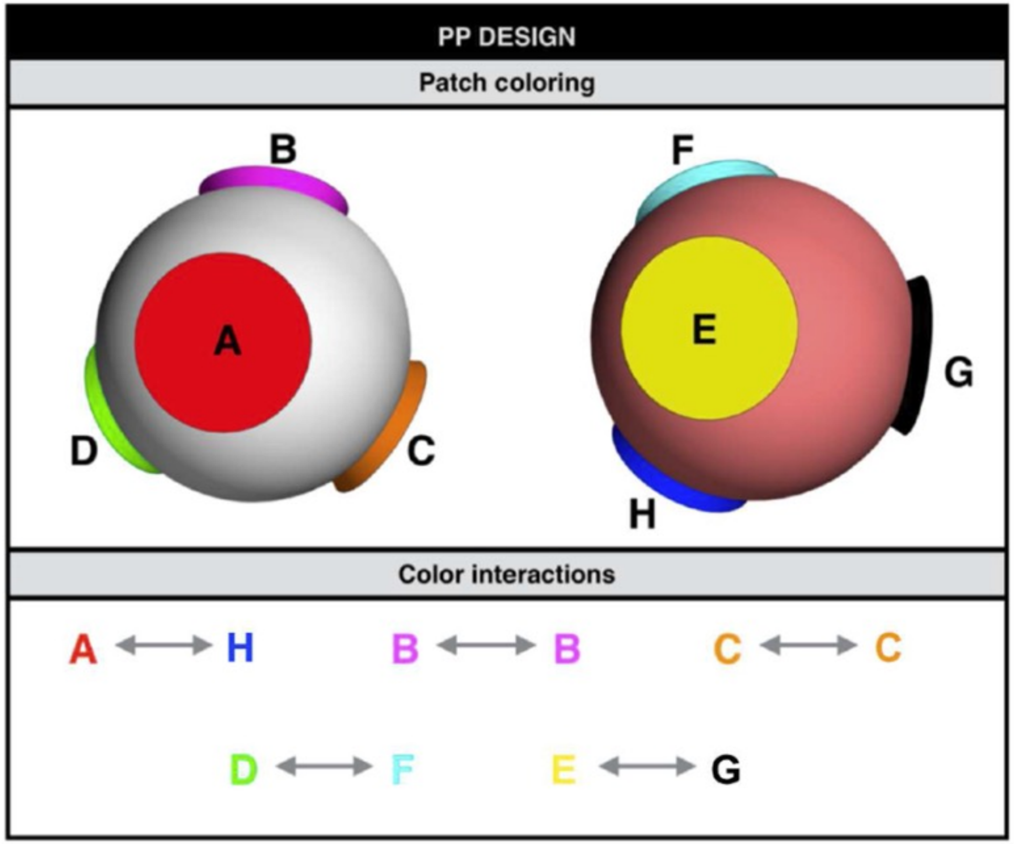
Cubic (and only cubic) diamond

$N_p=2, N_c=8$



A simple solution to the problem of self-assembling cubic diamond crystals†

Lorenzo Rovigatti, *[ab] John Russo, *[a] Flavio Romano, [cd] Michael Matthies, [e] Lukáš Kroc [e] and Petr Šulc *[e]



Two-step nucleation in a binary mixture of patchy particles

Cite as: J. Chem. Phys. 158, 154502 (2023); doi: 10.1063/5.0140847
Submitted: 31 December 2022 • Accepted: 24 March 2023 •
Published Online: 17 April 2023

Camilla Beneduce,[1] Diogo E. P. Pinto,[1] Petr Šulc,[2,3] Francesco Sciortino,[1] and John Russo[1,a)]

Self Assembly in Multi component systems….

Gas → Liquid → Crystal

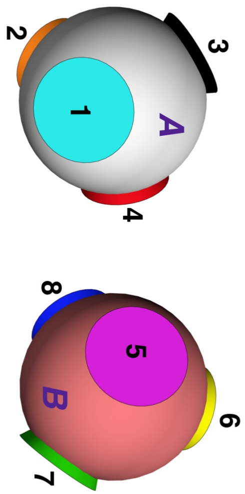
Consumption of the minority component

Azeotropy: when a multicomponent system behaves as a one component one

When coexisting phases have the same

N_p=2, N_c=8

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

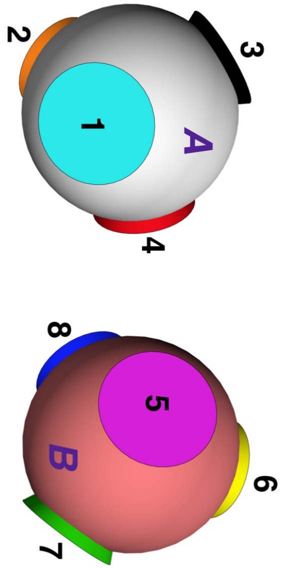
One 1 in each row (*azeotropic at equimolar*)

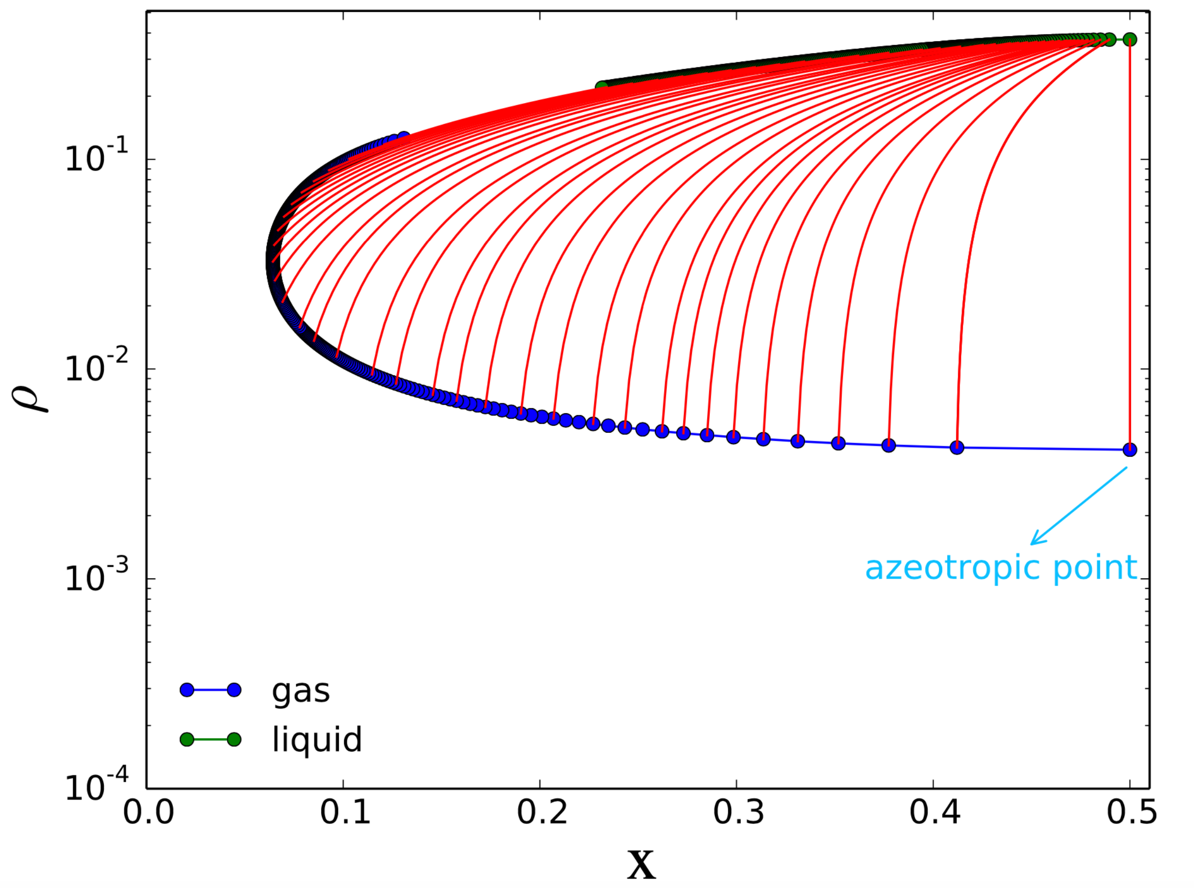
Two 1 in each row (but on different particles, *azeotropic at all compositions*)

Why and How to include azeotropy in the design of self-assembling systems

Camilla Beneduce,[1] Francesco Sciortino,[1] Petr Šulc,[2] and John Russo[1]

Theoretical predictions based on Wertheim theory (case: azeotropic at equimolar)

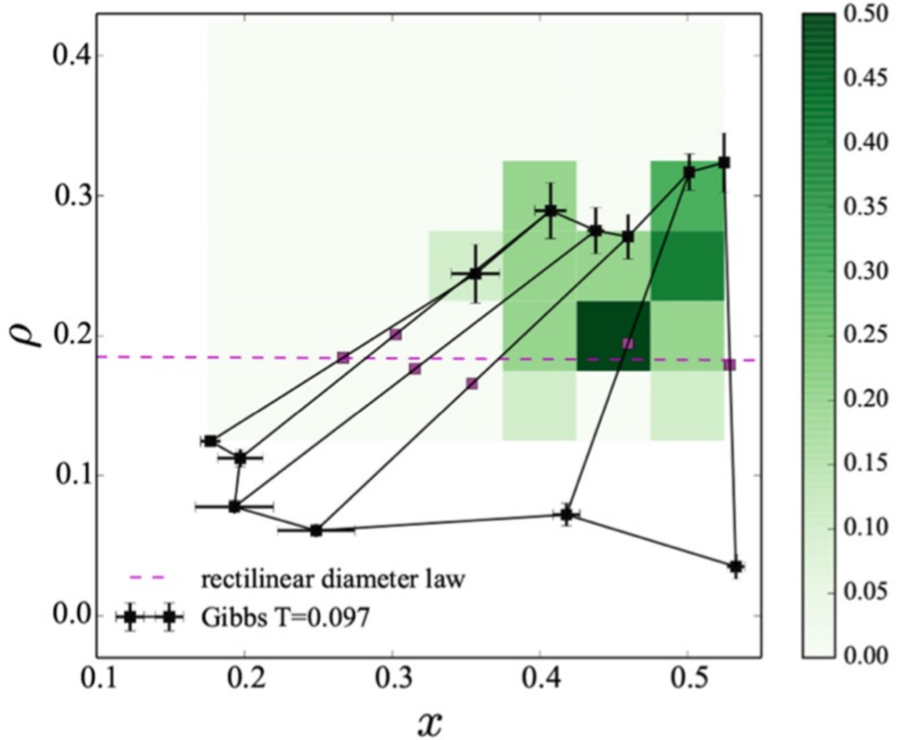


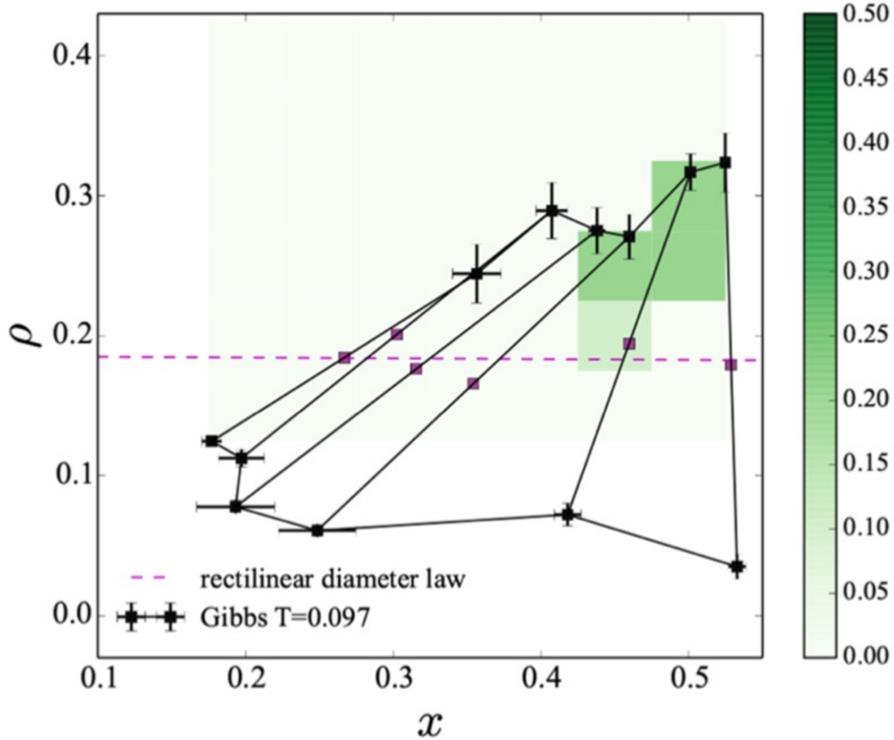


Why and How to include azeotropy in the design of self-assembling systems

Camilla Beneduce,[1] Francesco Sciortino,[1] Petr Šulc,[2] and John Russo[1]

Numerical results: Azeotropy favours nucleation !





Two-step nucleation in a binary mixture of patchy particles

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Camilla Beneduce,[1] Diogo E. P. Pinto,[1] Petr Šulc,[2,3] Francesco Sciortino,[1] and John Russo[1,a)]

Finite size clusters

Archimedean polyhedral shells that can be assemble from building blocks with a coordination number of five:

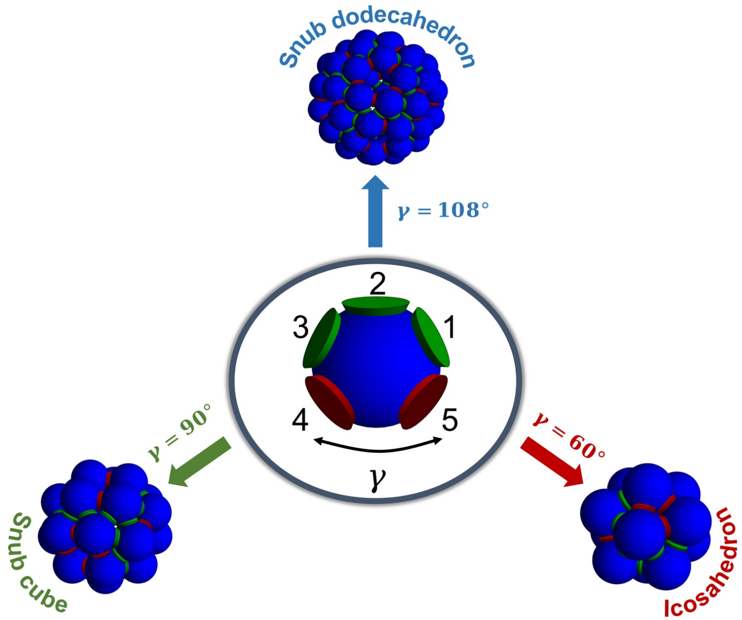
the 12-particle *icosahedron*,

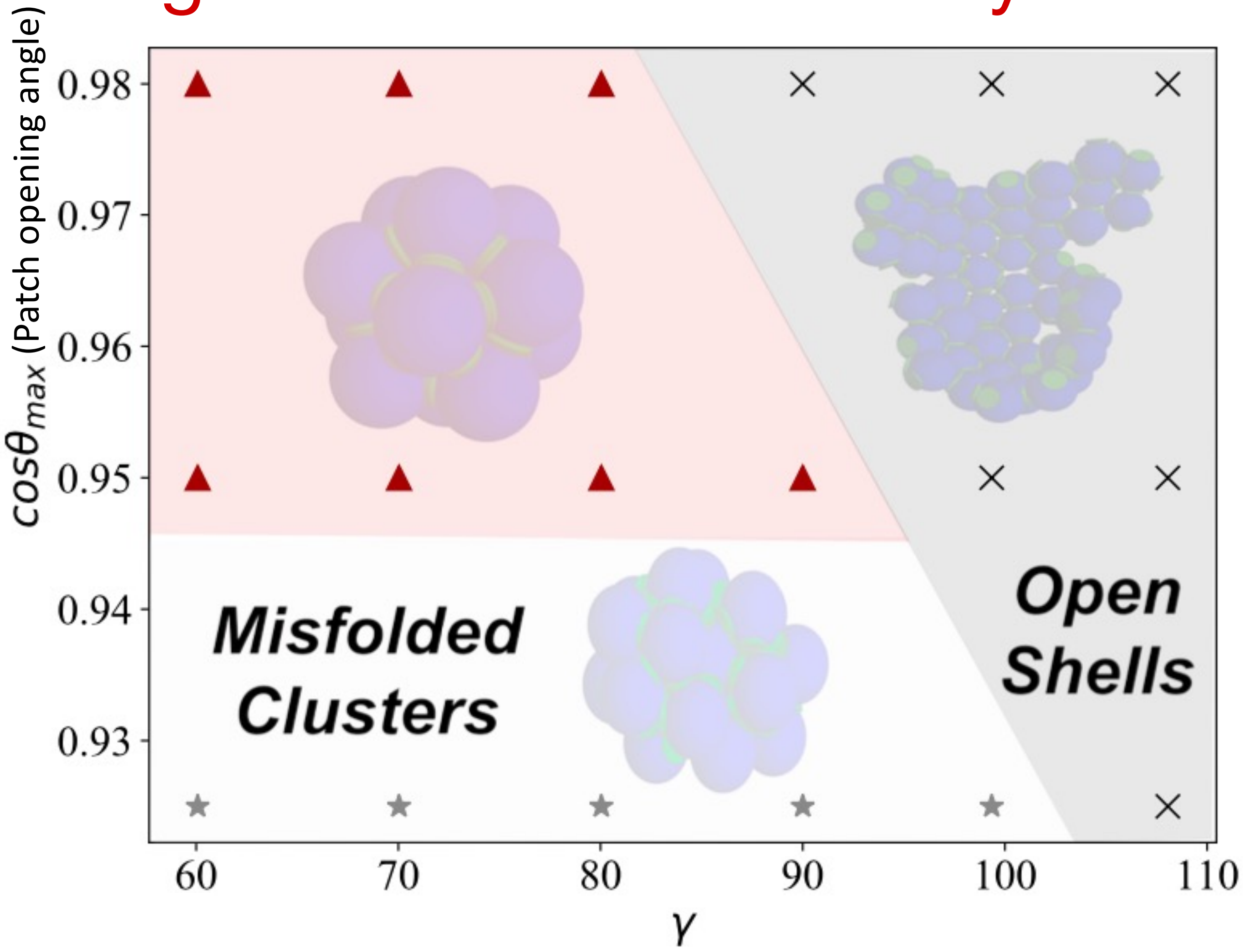
the 24-particle *snub cube*,

the 60- particle *snub dodecahedron*.

The educated guess design : misfolded and low yield

$N_c=1 \quad N_p=1$





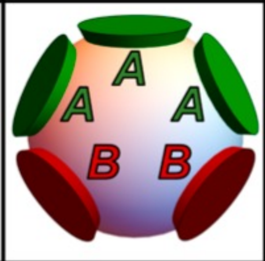
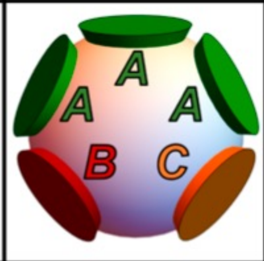
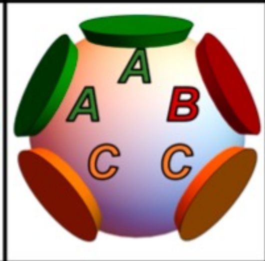
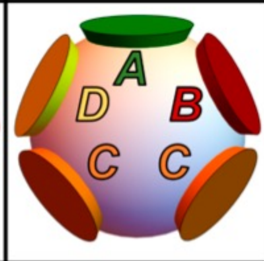
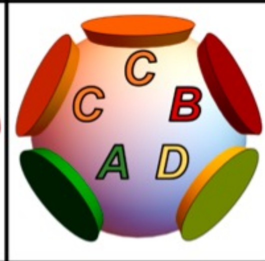
(angle between patch 4 and 5 – see figure)

PNAS RESEARCH ARTICLE | PHYSICS

Design strategies for the self-assembly of polyhedral shells

Diogo E. P. Pinto[a], Petr Šulc[b,c], Francesco Sciortino[a], and John Russo[a,1]

N_p=1,N_c=2,3,4,5 :   Colouring improves:

Solution	***C2(1)***	***C2(2)***	***C3(1)***	***C3(2)***	***C4(1)***	***C4(2)***	***C5***
Design							
Interaction	A↔A B↔B	A↔A B↔B	A↔A B↔C	A↔A B↔B C↔C	A↔D B↔B C↔C	A↔D B↔B C↔C	A↔A B↔C D↔E

Increasing the Number of Colours Increases the Yield.

Decreasing the Symmetry of the Building Block Increases the Yield.

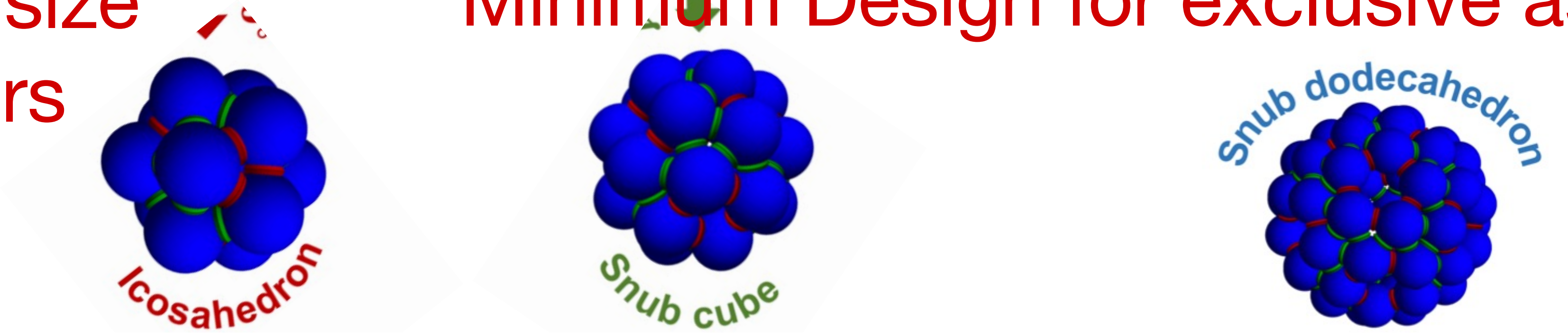
PNAS RESEARCH ARTICLE | PHYSICS

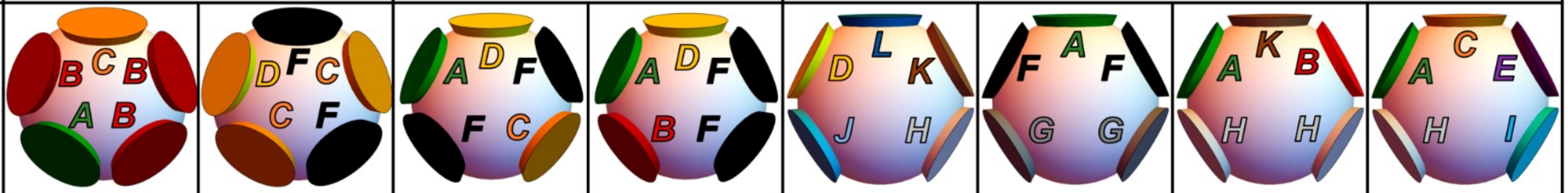
Design strategies for the self-assembly of polyhedral shells

Diogo E. P. Pinto[a], Petr Šulc[b,c], Francesco Sciortino[a], and John Russo[a,1]

Finite size clusters

Minimum Design for exclusive assembly



Solution	Icosahedron	Snub cube	Snub dodecahedron
Design			
Interaction	$N_c=5$ F↔F B↔C A↔D	$N_c=5$ F↔F B↔C A↔D	$N_c=11$ A↔F G↔H I↔J K↔K D↔D B↔C E↔L

PNAS RESEARCH ARTICLE | PHYSICS

Design strategies for the self-assembly of polyhedral shells

Diogo E. P. Pinto[a], Petr Šulc[b,c], Francesco Sciortino[a], and John Russo[a,1]

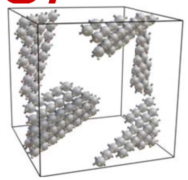
SAT-assembly and the experiments….

A complete pipeline

Tetrastack:

SAT-assembly and the experiments….

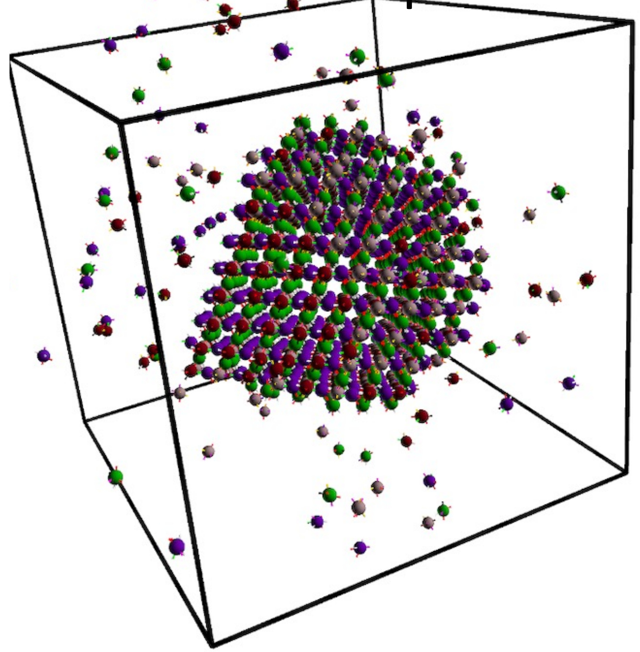
Tetrastack: N_p=4, N_c=24 (Maximum number of colors)



N_p=1 (kinetic traps, alternative structure)

N_p=2 OK but with same-particle bonds

N_p=4 OK with no same-particle bonds



PHOTONIC BANDGAPS

Inverse design of a pyrochlore lattice of DNA origami through model-driven experiments

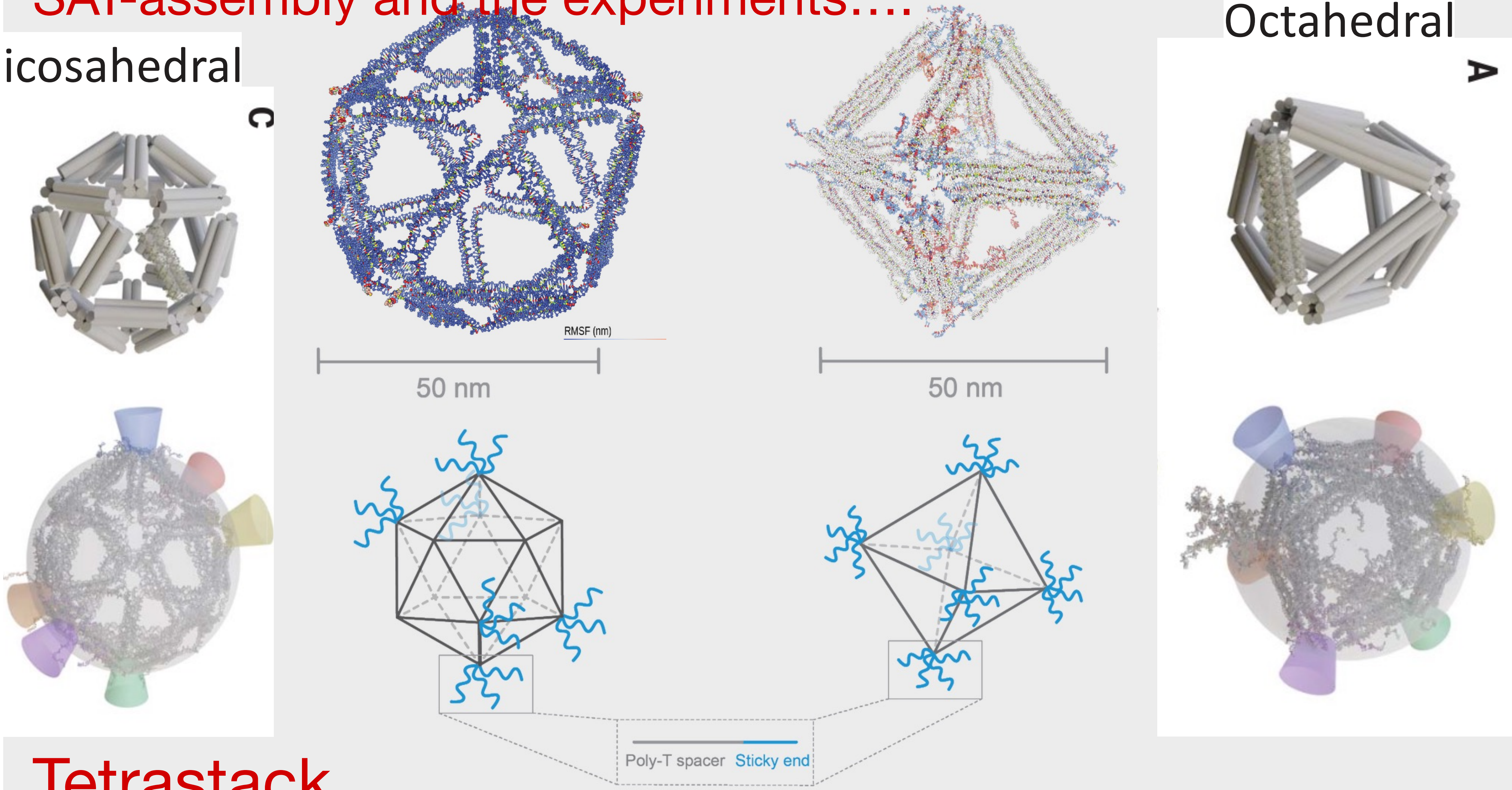
Hao Liu[1], Michael Matthies[1], John Russo[2], Lorenzo Rovigatti[2], Raghu Pradeep Narayanan[1,3], Thong Diep[1], Daniel McKeen[4], Oleg Gang[4,5,6], Nicholas Stephanopoulos[1], Francesco Sciortino[2], Hao Yan[1], Flavio Romano[7,8], Petr Šulc[1,9]*

Liu *et al.*, *Science* **384**, 776–781 (2024) 17 May 2024

SAT-assembly and the experiments….

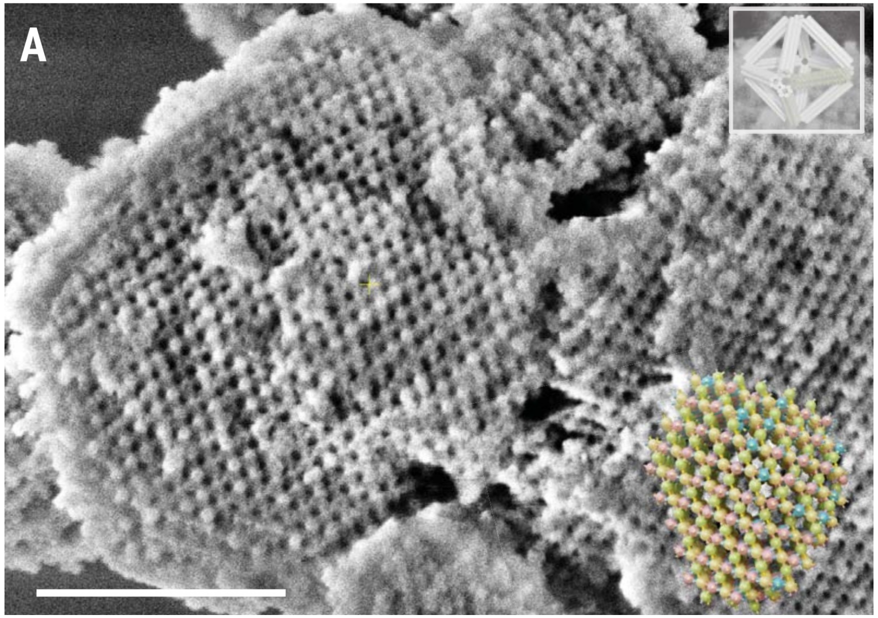
icosahedral

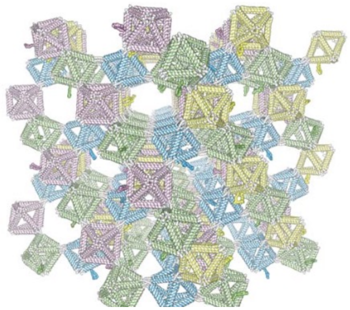
Octahedral



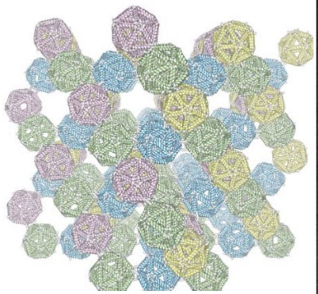
Tetrastack

Silica-coated
SEM images

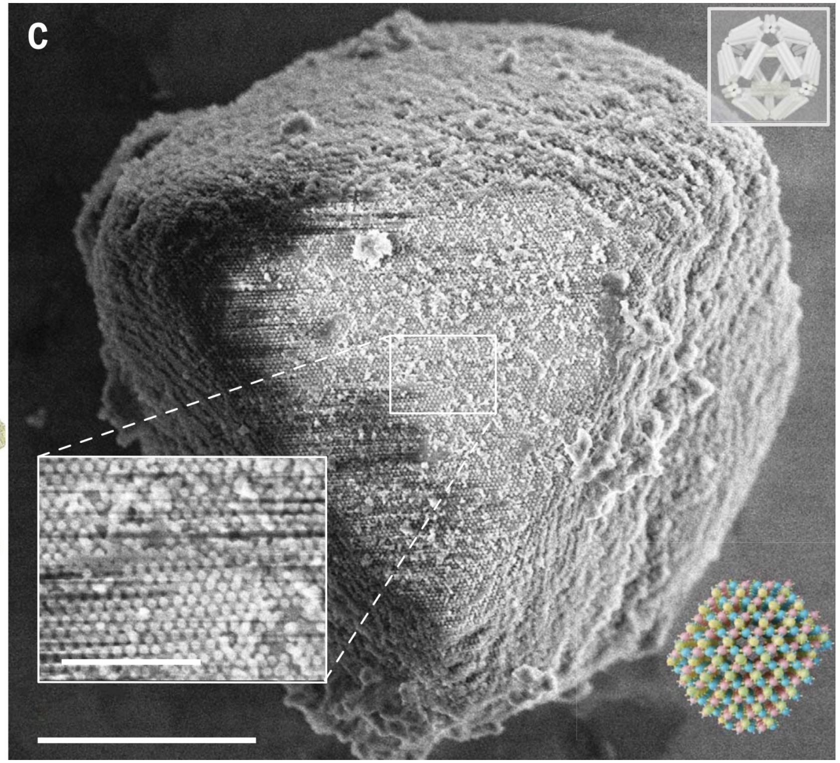




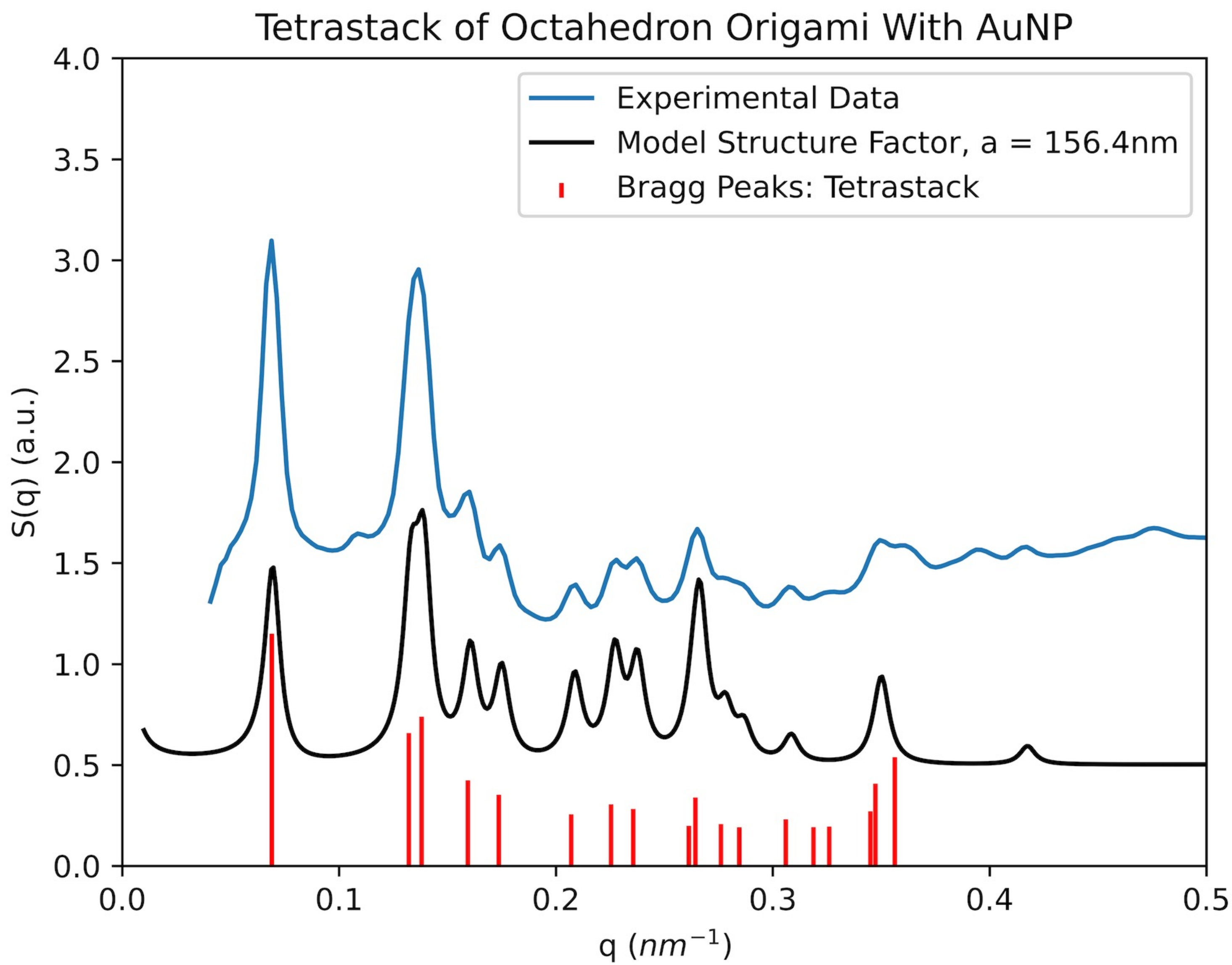
Octahedral



icosahedral



Micron-scale



Tetrastack of Octahedron Origami With AuNP
Experimental Data
Model Structure Factor, a = 156.4nm
Bragg Peaks: Tetrastack
S(q) (a.u.)
q (nm^{-1})
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.0
0.1
0.2
0.3
0.4
0.5

Article

How to design an icosahedral quasicrystal through directional bonding

https://doi.org/10.1038/s41586-021-03700-2

Eva G. Noya[1]✉, Chak Kui Wong[2], Pablo Llombart[1] & Jonathan P. K. Doye[2]✉

Received: 13 August 2020

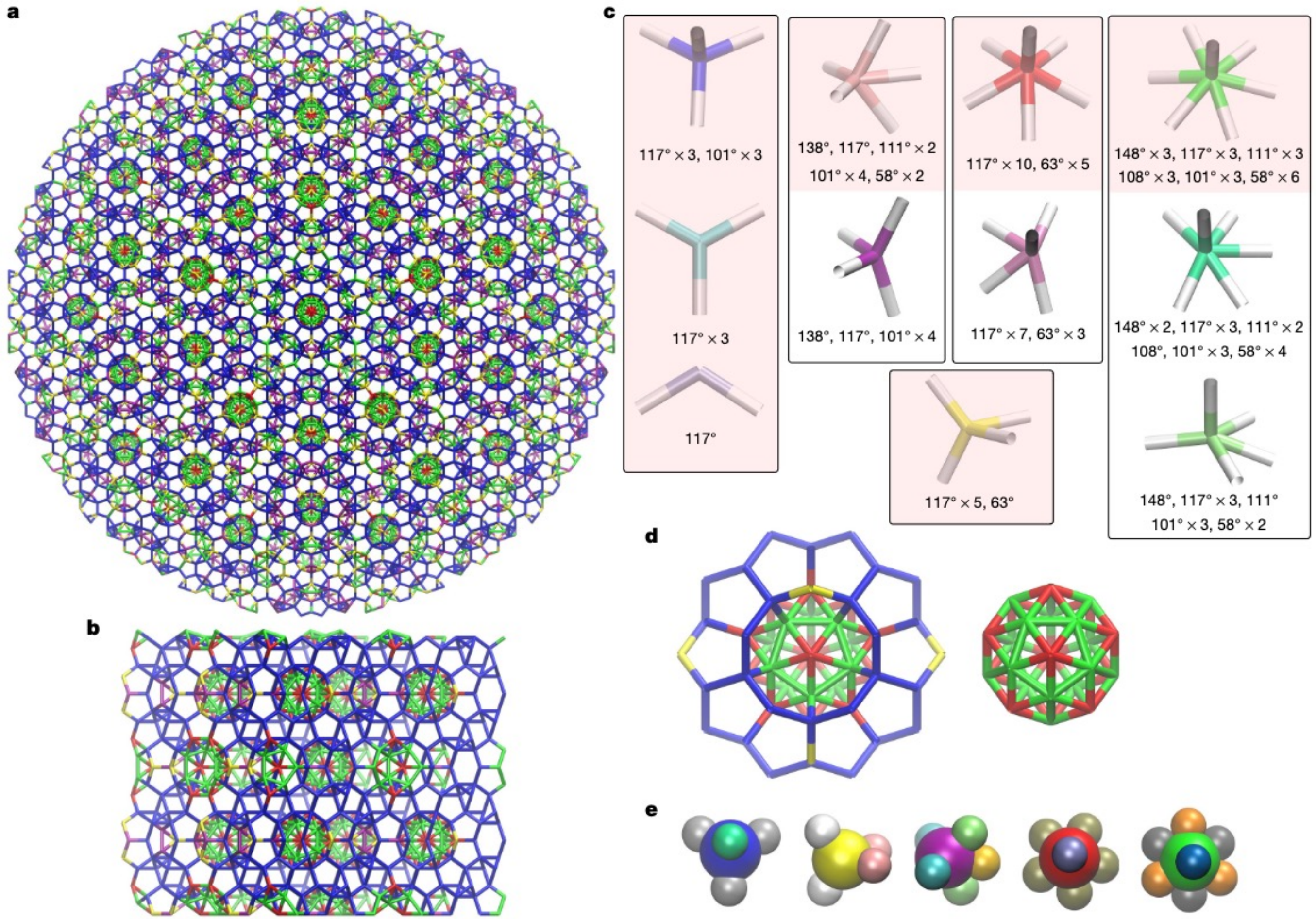


Fig. 1 | From ideal quasicrystal to patchy-particle design. a, Target ideal body-centred IQC structure. **b**, The corresponding 3/2 rational approximant (obtained by the cut-and-project method). **c**, Geometry of the local environments in the ideal quasicrystal organized in classes that are characterized by the same local geometry but with some dangling bonds. Those local environments also present in the 3/2 rational approximant are shaded in red. **d**, Structure of the icosahedral clusters forming the IQC. The inner shell is a 32-particle triacontahedra, which is surrounded by a shell of 50 four-patch particles. **e**, Model patchy particles used for the assembly of the body-centred 5*P* IQC.

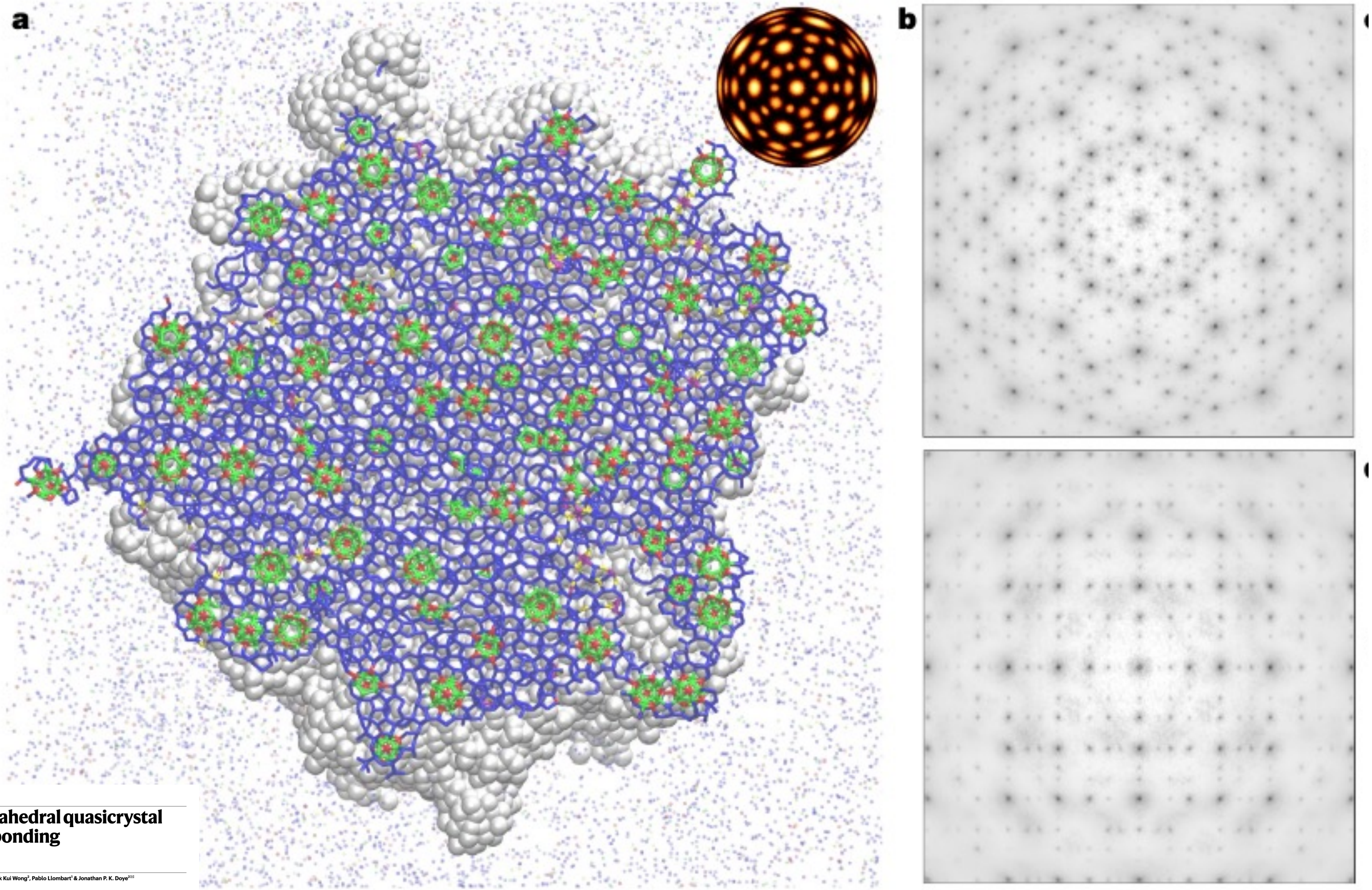
arXiv > cond-mat > arXiv:2407.17212

Condensed Matter > Soft Condensed Matter

[Submitted on 24 Jul 2024]

A one-component patchy-particle icosahedral quasicrystal

Eva G. Noya, Jonathan P.K. Doye



Article

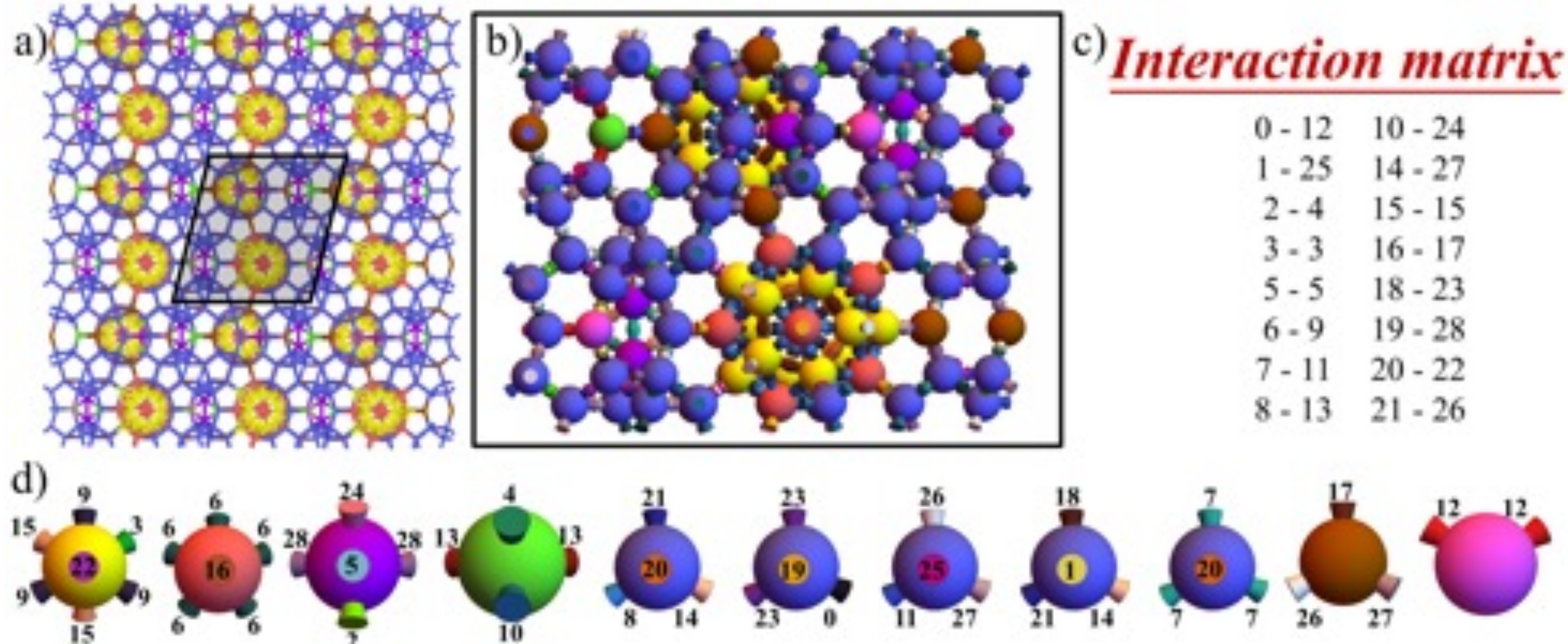
How to design an icosahedral quasicrystal through directional bonding

https://doi.org/10.1038/s41586-021-03700-2

Eva G. Noya[1✉], Chak Kui Wong[2], Pablo Llombart[1] & Jonathan P. K. Doye[2✉]

Received: 13 August 2020

Proof of concept: Can SAT-assembly solve the problems with colours instead of torsion ?



arXiv > cond-mat > arXiv:2407.19968

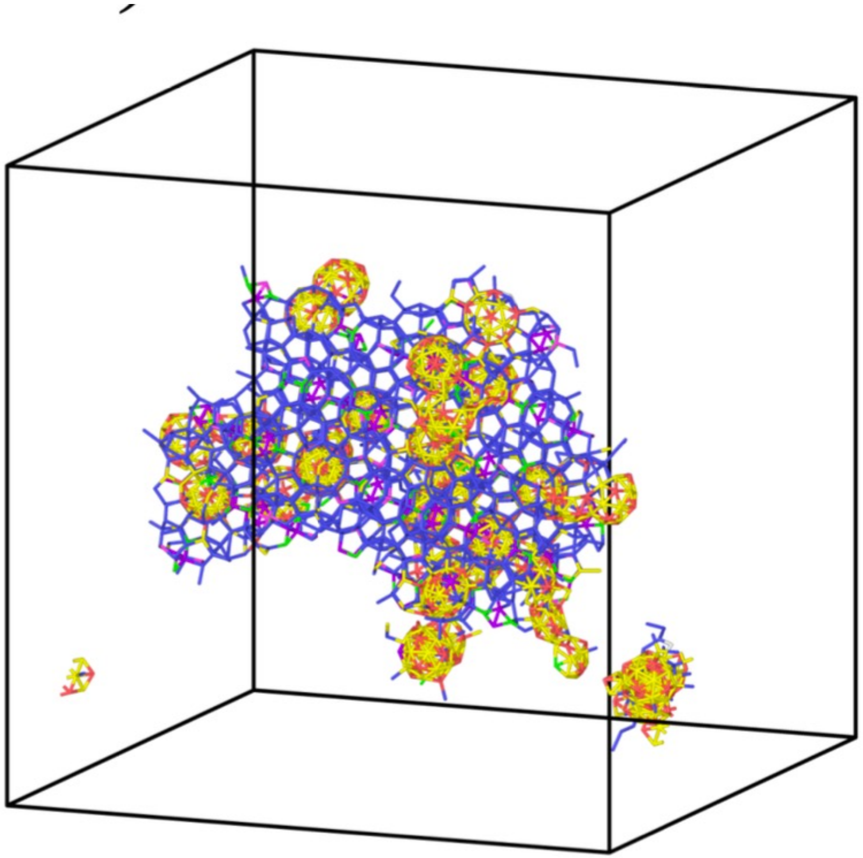
Condensed Matter > Soft Condensed Matter

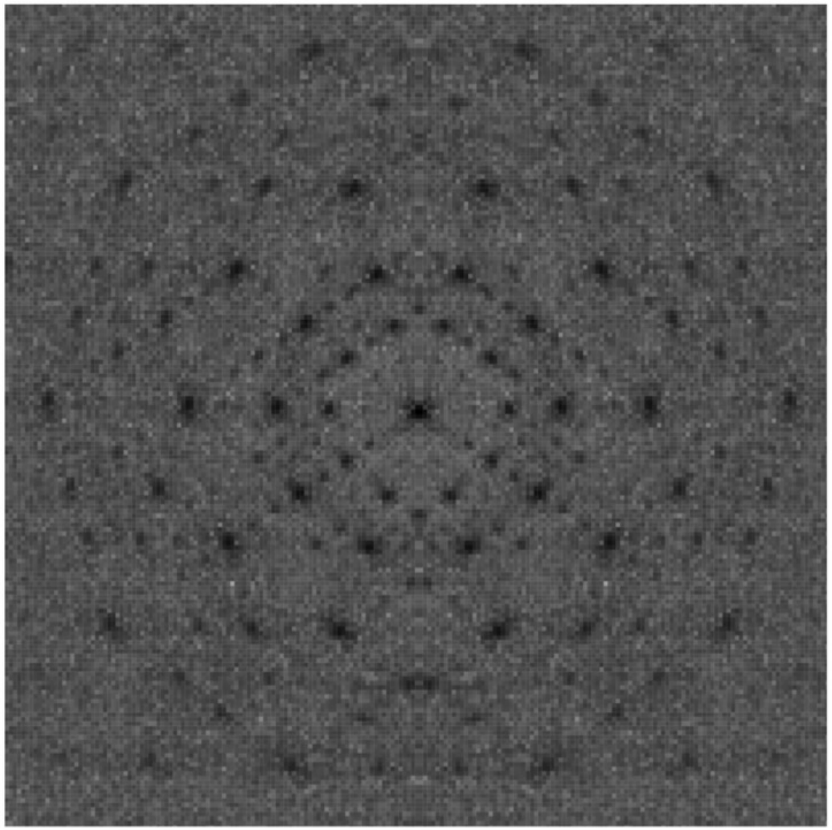
[Submitted on 29 Jul 2024]

Automating Blueprints for Colloidal Quasicrystal Assembly

Diogo E. P. Pinto, Petr Šulc, Francesco Sciortino, John Russo

Spontaneous formation of the approximant





N_p=11 N_c=29

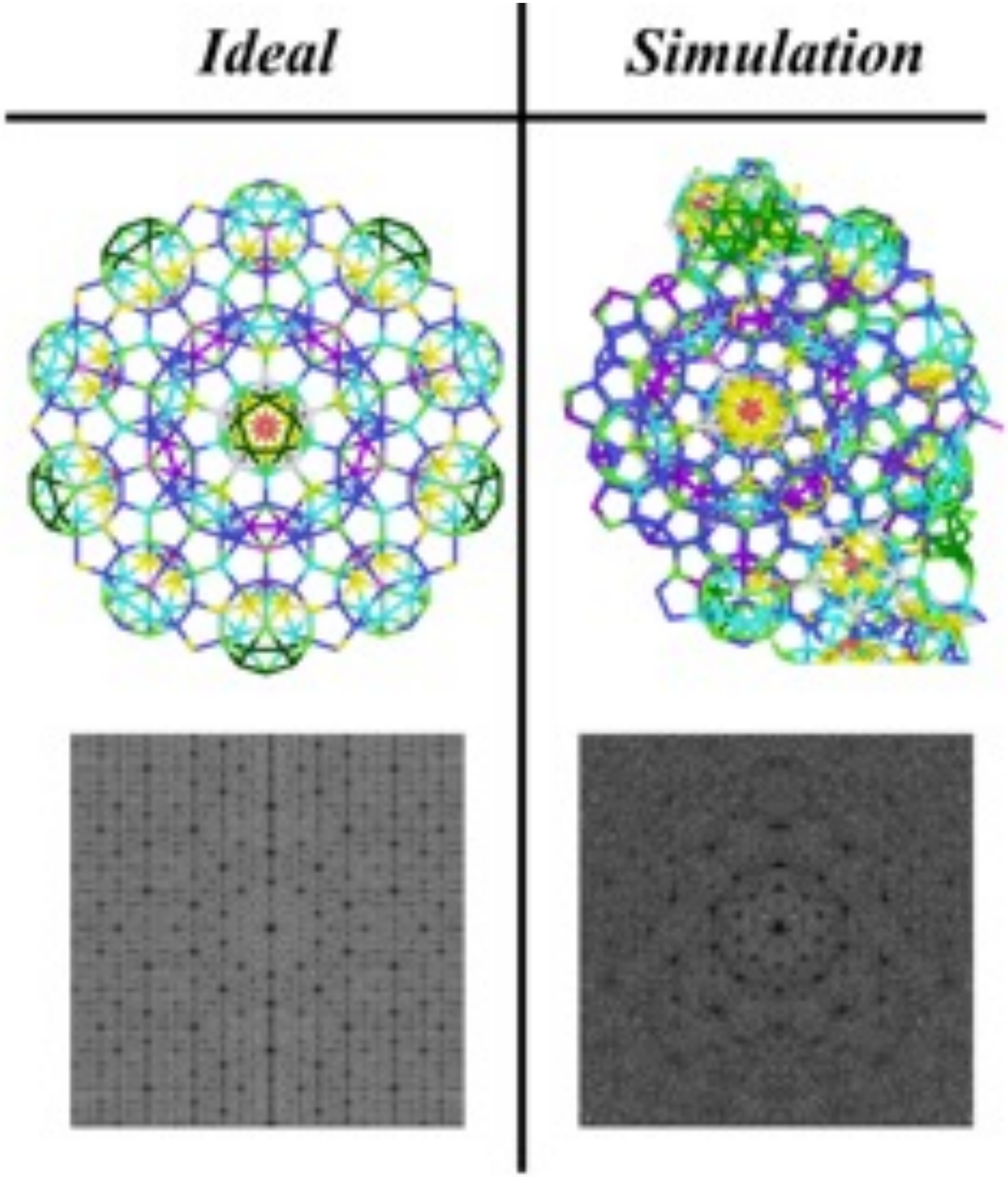
arXiv > cond-mat > arXiv:2407.19968

Condensed Matter > Soft Condensed Matter

[Submitted on 29 Jul 2024]

Automating Blueprints for Colloidal Quasicrystal Assembly

Diogo E. P. Pinto, Petr Šulc, Francesco Sciortino, John Russo



portion of an ideal quasicrystal with $N = 1538$

$N_p=28$ $N_c=27$

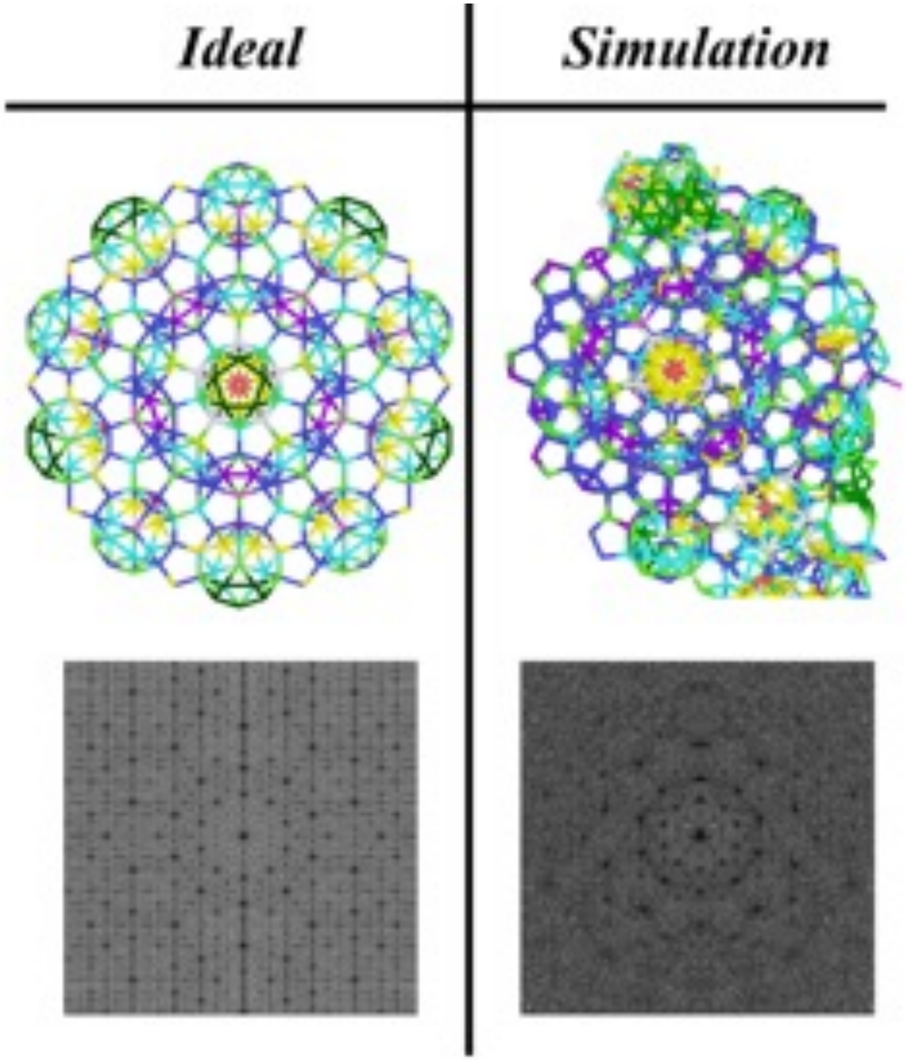
arXiv > cond-mat > arXiv:2407.19968

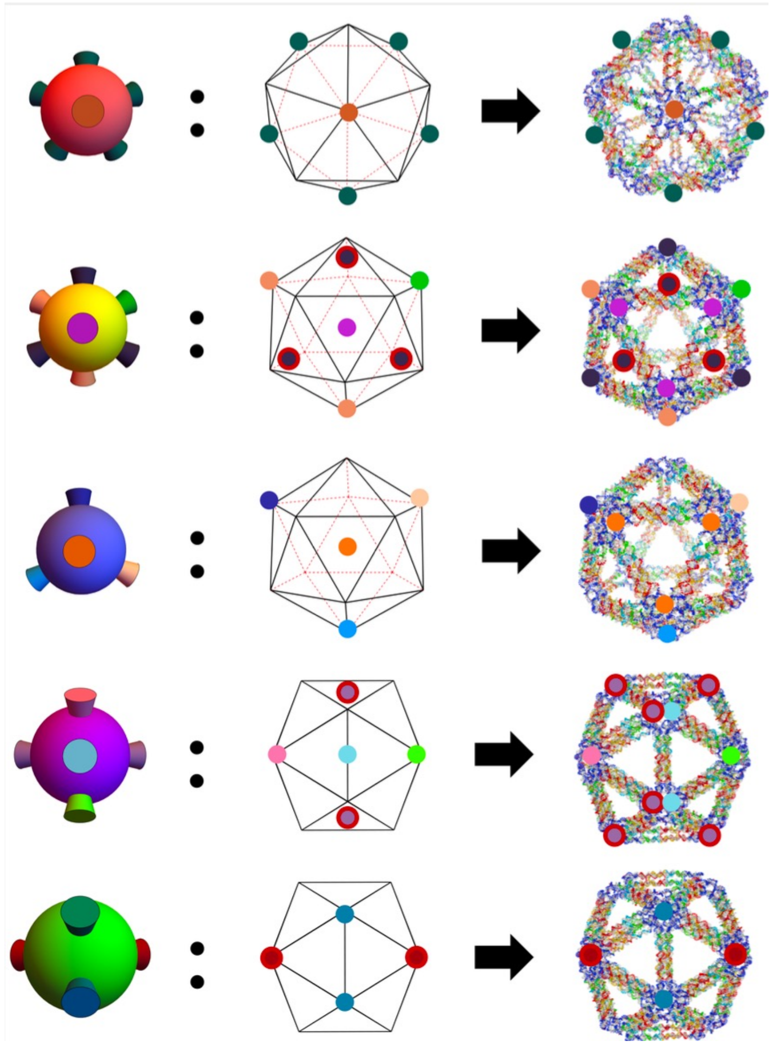
Condensed Matter > Soft Condensed Matter

[Submitted on 29 Jul 2024]

Automating Blueprints for Colloidal Quasicrystal Assembly

Diogo E. P. Pinto, Petr Šulc, Francesco Sciortino, John Russo


Ideal
Simulation



Take home messages

SAT-Assembly appears to be a valuable method to solve the self-assembly process based on patchy particle's building blocks

Ability to eliminate solutions presenting kinetic traps
Or ending in competing structures
Or encoding experimental requests

Possibility to encode the results into DNA-origami wireframe particles

Take home messages

More colors, easier self-assembly

Less symmetry, easier self-assembly

Azeotropic composition, easier self-assembly

Take home messages

More colors, easier self-assembly

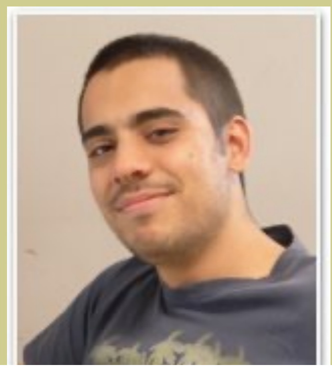
Less symmetry, easier self-assembly

Azeotropic composition, easier self-assembly

THANKS FOR LISTENING !

John Russo

Diogo Pinto

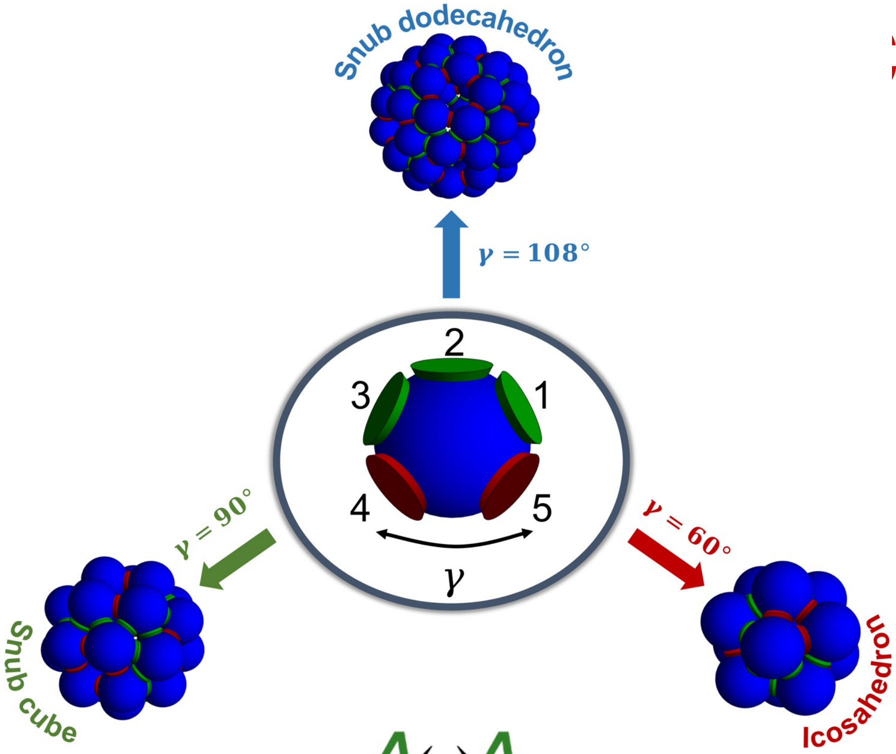


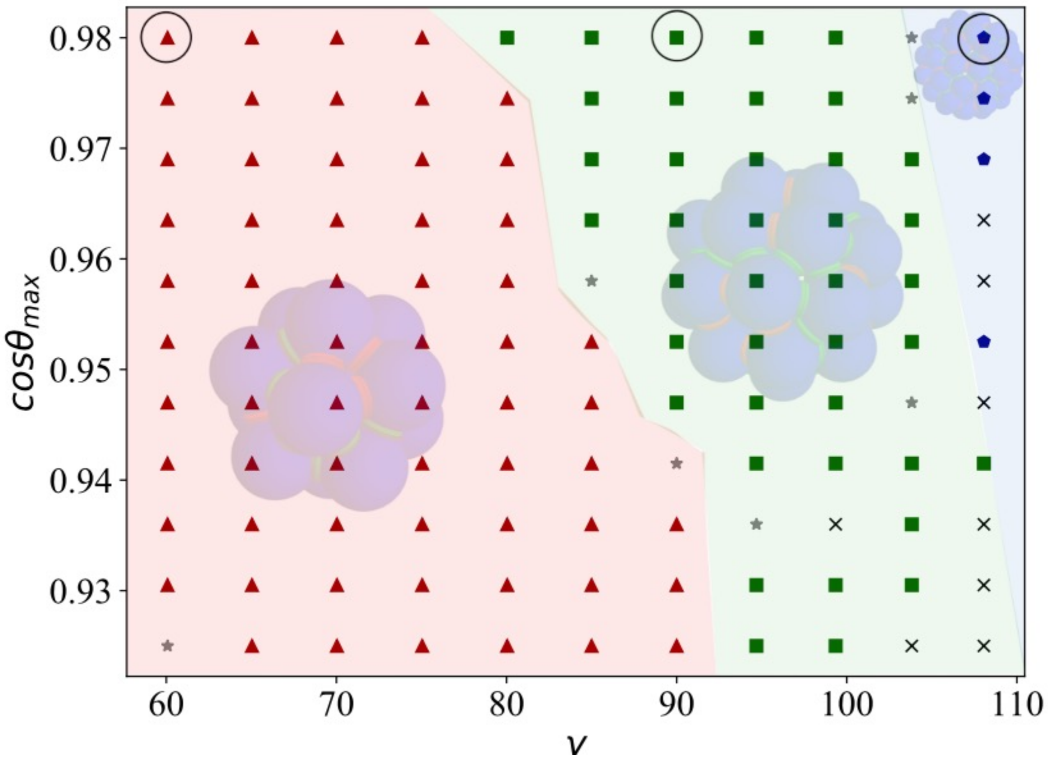
Lorenzo Rovigatti

Camilla Beneduce

Pets Sulc

olouring improves: $N_p=1, N_c=2$





Higher yield, no misfolded, competing structures.

PNAS RESEARCH ARTICLE | PHYSICS

Design strategies for the self-assembly of polyhedral shells

Diogo E. P. Pinto[a], Petr Šulc[b,c], Francesco Sciortino[a], and John Russo[a,1]

Sufficient condition for azeotropy $X_{\alpha}^{(i)} = X$

- the *bond exclusivity* condition. This rule generates azeotropic points at equimolar conditions;

- the *bond multiplicity* condition. This rule allows for azeotropic points at non-equimolar conditions;

- the *fully-connected bond* condition. This rule generates always-azeotropic mixtures, e.g. where the concentration remains the same during demixing for every point in the coexistence region.

Why and How to include azeotropy in the design of self-assembling systems

Camilla Beneduce,[1] Francesco Sciortino,[1] Petr Šulc,[2] and John Russo[1]

bond exclusivity

each patch has only one bonding partner (that can be itself in case of self-complementarity) among all patches of all species in the mixture.

Azeotropic at equal molar conditions

fully-connected bond

each patch can bind with N_s patches, each located on a different species.

Azeotropic at all molar conditions

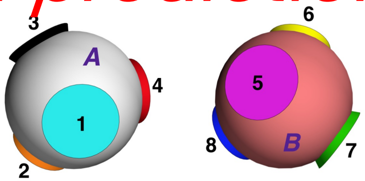
SAT-assembly and the experiments….

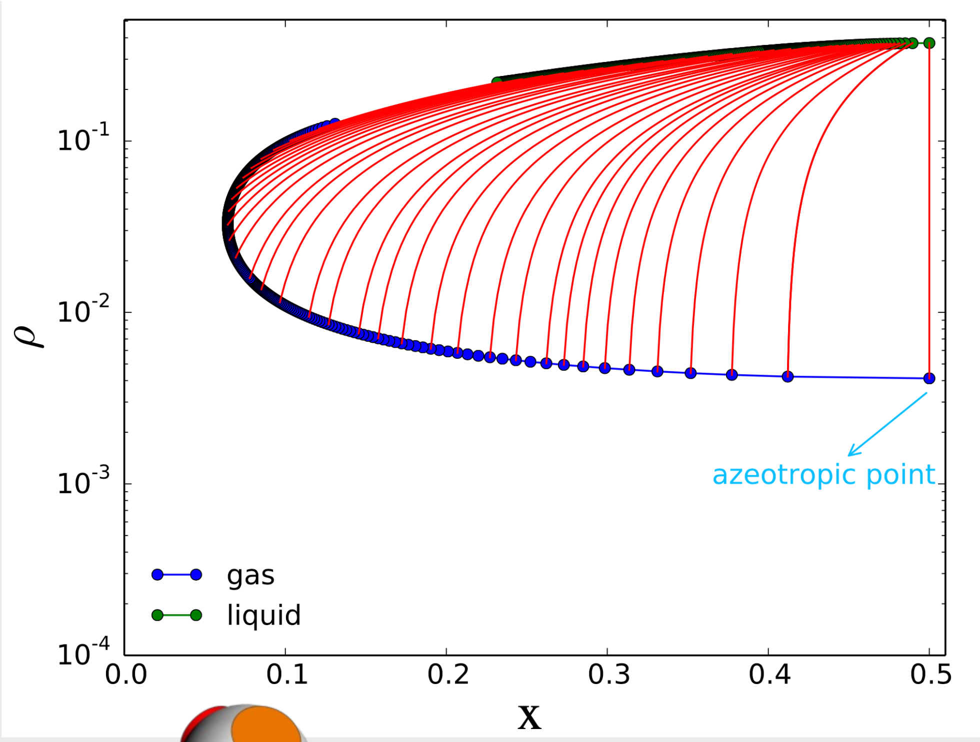
Tetrastack: N_p=4, N_c=24 (Maximum number)

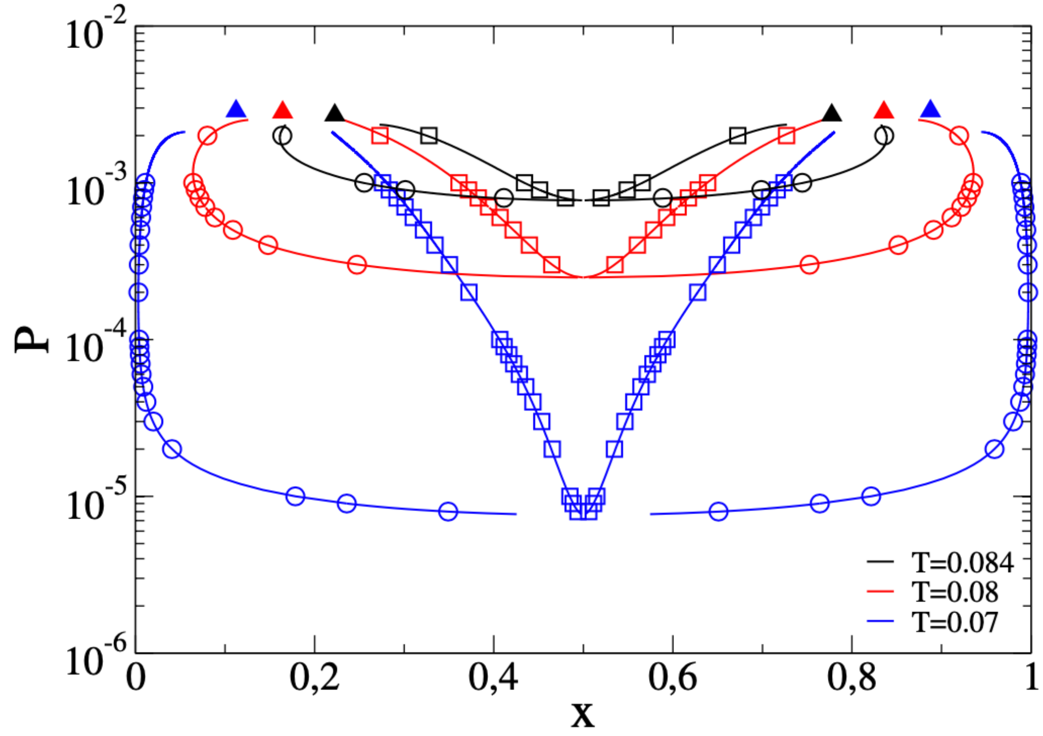
From model-driven design to experimental realization of tetrastack lattice with DNA nanotechnology

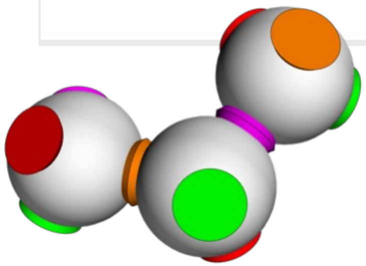
Hao Liu,[1] Michael Matthies,[1] John Russo,[2] Lorenzo Rovigatti,[2] Raghu Pradeep Narayanan,[1] Daniel McKeen,[3] Oleg Gang,[3] Nicholas Stephanopoulos,[1] Francesco Sciortino,[2] Hao Yan,[1] Flavio Romano,[4] and Petr Šulc[1]

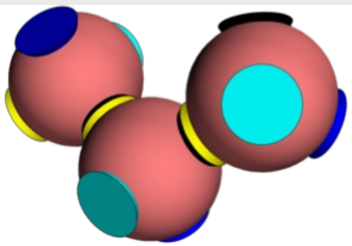
Theoretical predictions based on Wertheim theory
(case: azeotropic at equimolar)











Why and How to include azeotropy in the design of self-assembling systems

Camilla Beneduce,[1] Francesco Sciortino,[1] Petr Šulc,[2] and John Russo[1]

Potential pitfalls typically encountered in self- assembly

* metastable states that can compete with the final product;

* dynamically arrested states (kinetic traps);

* low aggregation rates;

* lack of knowledge of the underlying phase behaviour of the building-blocks, especially for mixtures with many components.

From an experimental point-of-view one need to consider:

* the difficulty in realizing bulk quantities of building blocks, their size (gravitational effect)

* the interaction polydispersity (promoting disordered arrest)

* The difficulty in controlling mechanical and molecular properties, such as softness and flexibility.

SAPIENZA
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Theoretical Approaches

Turning statistical physics models into materials design engines

Marc Z. Miskin[a,1], Gurdaman Khaira[b], Juan J. de Pablo[b,c], and Heinrich M. Jaeger[a]

[a]James Franck Institute and Department of Physics, The University of Chicago, Chicago, IL 60637; [b]Institute for Molecular Engineering, The University of Chicago, Chicago, IL 60637; and [c]Institute for Molecular Engineering, Argonne National Laboratory, Lemont, IL 60439

PHYSICAL REVIEW E **98**, 032611 (2018)

Programmable self-assembly of diamond polymorphs from chromatic patchy particles

Niladri Patra[1] and Alexei V. Tkachenko[2,*]

[1]*Department of Applied Chemistry, Indian Institute of Technology (Indian School of Mines), Dhanbad, Dhanbad 826004, India*
[2]*Center for Functional Nanomaterials, Brookhaven National Laboratory, Upton, New York 11973, USA*

THE JOURNAL OF PHYSICAL CHEMISTRY B

Cite This: *J. Phys. Chem. B* 2018, 122, 8462−8468

Inverse Design of Colloidal Crystals via Optimized Patchy Interactions

D. Chen,[†] G. Zhang,[†] and S. Torquato[*,†,‡,§,∥]

Annual Review of Chemical and Biomolecular Engineering

Machine Learning–Assisted Design of Material Properties

Sanket Kadulkar,[1] Zachary M. Sherman,[1]
Venkat Ganesan,[1] and Thomas M. Truskett[1,2]

SCIENCE ADVANCES | RESEARCH ARTICLE

MATERIALS SCIENCE

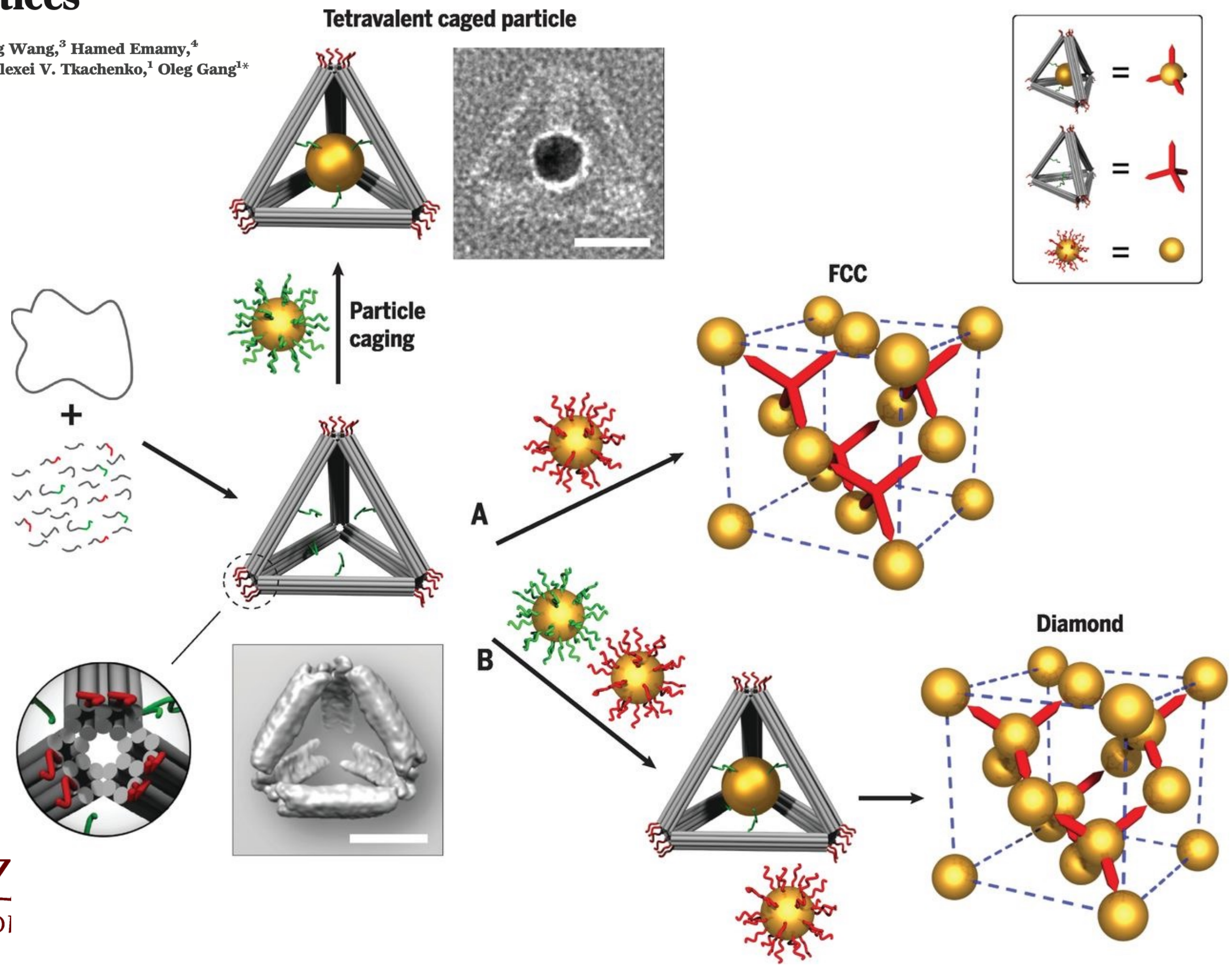
Inverse design of soft materials via a deep learning–based evolutionary strategy

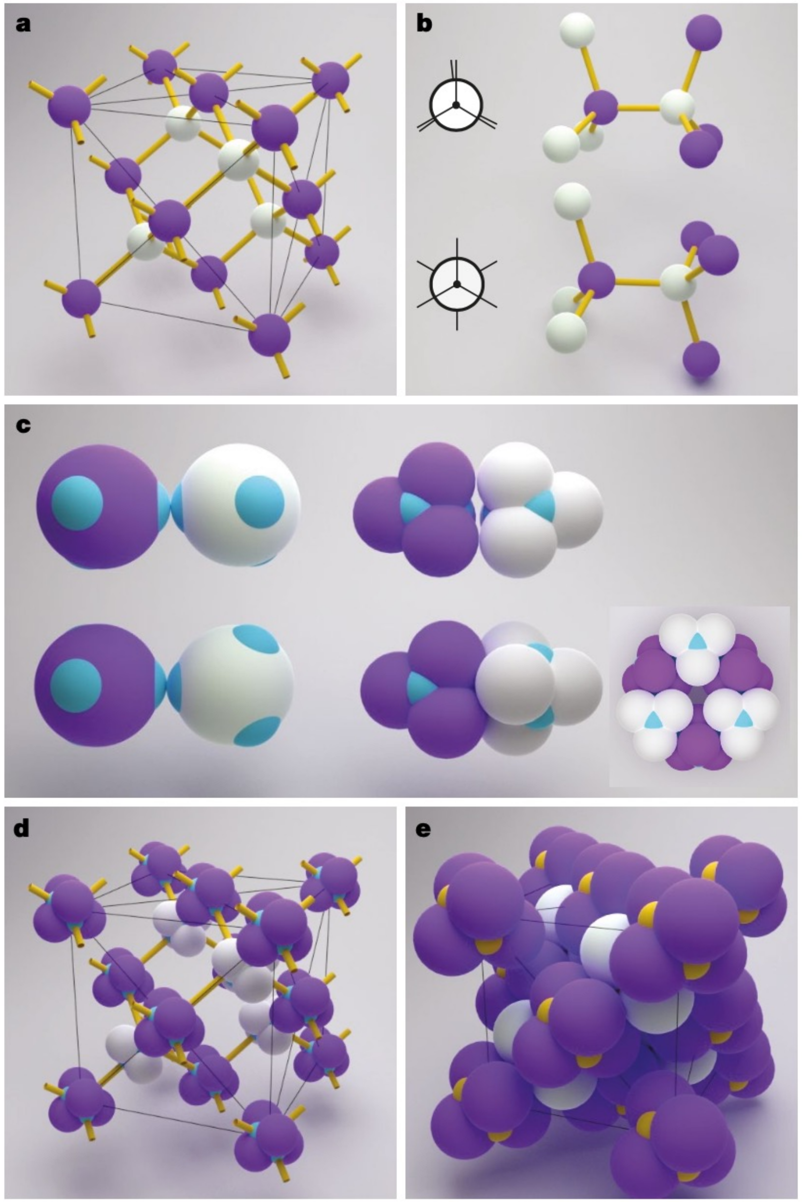
Gabriele M. Coli*†, Emanuele Boattini*†, Laura Filion, Marjolein Dijkstra

MATERIALS SCIENCE

Diamond family of nanoparticle superlattices

Wenyan Liu,[1] Miho Tagawa,[2] Huolin L. Xin,[1] Tong Wang,[3] Hamed Emamy,[4] Huilin Li,[3,5] Kevin G. Yager,[1] Francis W. Starr,[4] Alexei V. Tkachenko,[1] Oleg Gang[1]*





Colloidal diamond

https://doi.org/10.1038/s41586-020-2718-6

Received: 9 February 2020

Mingxin He[1,2], Johnathon P. Gales[2], Étienne Ducrot[2,3], Zhe Gong[4], Gi-Ra Yi[5], Stefano Sacanna[4]✉ & David J. Pine[1,2]✉

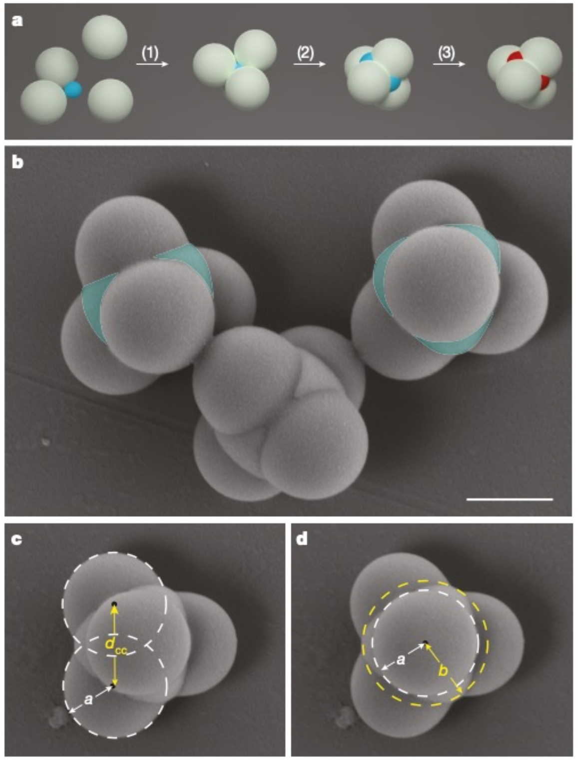


Fig. 2 | Synthesis of compressed tetrahedral patchy clusters. a, (1) Aggregation of four polystyrene particles (white) around a smaller oil droplet (blue), followed by (2) controlled deformation of the polystyrene particles with THF, which extrudes the central oil droplet. (3) The THF is then removed, the oil polymerized and coated with DNA to produce solid compressed tetrahedral clusters with DNA-coated patches (red). **b**, Scanning electron microscopy (SEM) image of compressed tetrahedral clusters. Some TPM patches are highlighted in light blue. Scale bar, 1 μm. **c**, The compression ratio $d_{cc}/(2a)$ is defined as the distance between the centres of the spherical lobes divided by

ARTICLE

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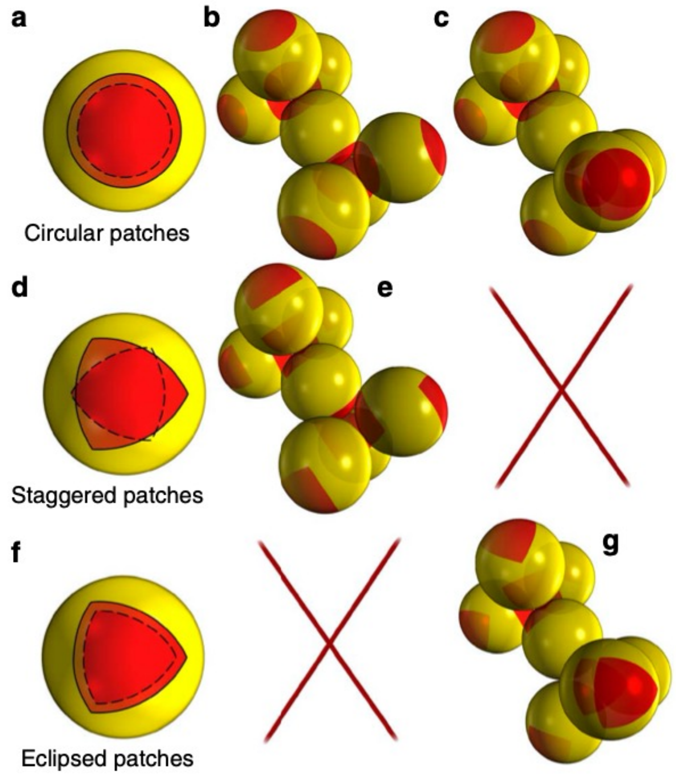
DOI: 10.1038/ncomms1968

Patterning symmetry in the rational design of colloidal crystals

Flavio Romano[1,*] & Francesco Sciortino[2,*]



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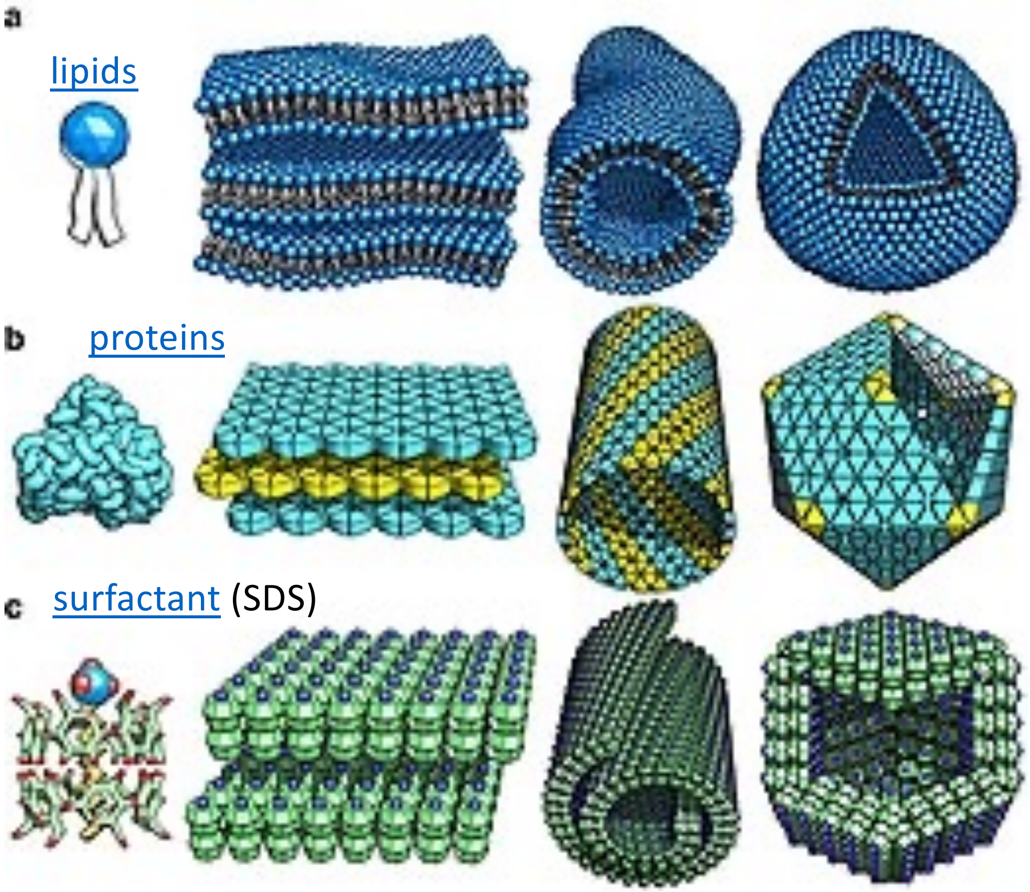
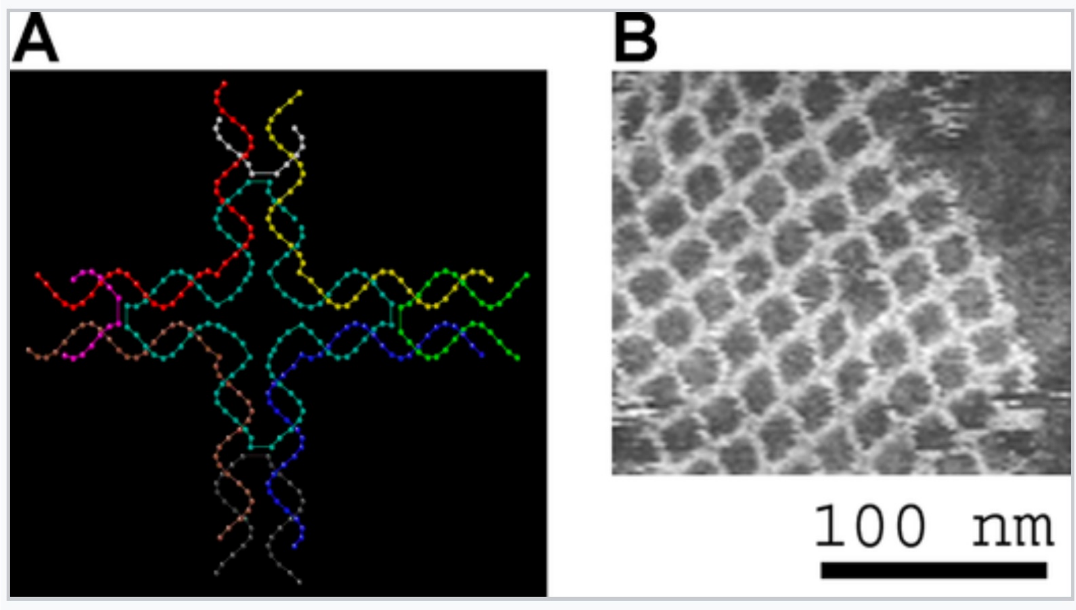
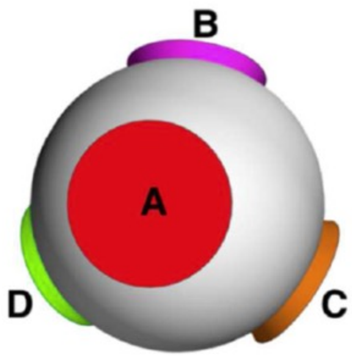


Wikipedia
The Free Encyclopedia

Self-assembly





$$\Delta_{\alpha\gamma} = \frac{V_b}{\pi\sigma^3/6}\left(e^{\beta\epsilon_{\alpha\gamma}} - 1\right)$$

$X_\alpha^{(i)} =$ probability that patch α of particle i is not bonded

$$X_\alpha^{(i)} = \left[1 + \phi \sum_{j=1,N_s} x^{(j)} \sum_{\gamma\in\Gamma(j)} X_\gamma^{(j)} \Delta_{\alpha\gamma}\right]^{-1}$$

(mass action law)

Why and How to include azeotropy in the design of self-assembling systems

Camilla Beneduce,[1] Francesco Sciortino,[1] Petr Šulc,[2] and John Russo[1]